

Floating Point Routines for the 6502

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Editor's Note: Although these routines are for the 6502, it would appear that one could generate equivalent routines for most of the "traditional" microprocessors, relatively easily, by following the flow of the algorithms given in the excellent comments included in the program listing. This is particularly true of the transcendental functions which were directly modeled after well-known and proven algorithms, and for which, the comments are relatively machine-independent.

These floating point routines allow 6502 users to perform most of the more popular and desired floating point and transcendental functions, namely:

- Natural Log - LOG
- Common Log - LOG10
- Exponential - EXP
- Floating Add - FADD
- Floating Subtract - FSUB
- Floating Multiply - FMUL
- Floating Divide - FDIV
- Convert Floating to Fixed - FIX
- Convert Fixed to Floating - FLOAT

They presume a four-byte floating point operand consisting of a one-byte exponent ranging from -218 through +127, and a 24-bit two's complement mantissa between 1.0 and 2.0.

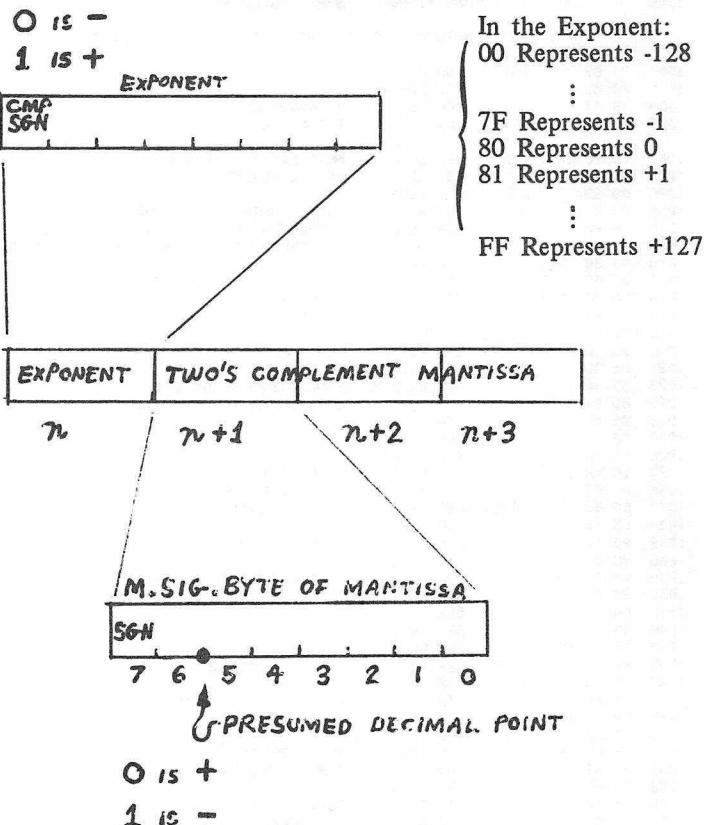
The floating point routines were done by Steve Wozniak, one of the principals in Apple Computer Company. The transcendental functions were patterned after those offered by Hewlett-Packard for their HP2100 minicomputer (with some modifications), and were done by Roy Rankin, a Ph.D. student at Stanford University.

There are three error traps; two for overflow, and one for prohibited logarithm argument. ERROR (1D06) is the error exit used in event of a non-positive log argument. OVFLW (1E3B) is the error exit for overflow occurring during calculation of e to some power. OVFL (1FE4) is the error exit for overflow in all of the floating point routines. There is no trap for underflow; in such cases, the result is set to 0.0.

All routines are called and exited in a uniform manner: The argument(s) are placed in the specified floating point storage locations (for specifics, see documentation preceeding each routine in the listing), then a JSR is used to enter the desired routine. Upon normal completion, the called routine is exited via a subroutine return instruction (RTS).

Note: The preceeding documentation was written by the Editor, based on phone conversations with Roy and studying the listing. There is a high probability that it is correct. However, since it was not written nor reviewed by the authors of these routines, the preceeding documentation may contain errors in concept or in detail.

- JCW, Jr.



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BASIC FLOATING POINT ROUTINES
FOR 6502 MICROPROCESSOR
BY R. RANKIN AND S. WOZNIAK

CONSISTING OF:
NATURAL LOG
COMMON LOG
EXPONENTIAL (E**X)
FLOAT FIX
FADD FSUB
FMUL FDIV

FLOATING POINT REPRESENTATION (4-BYTES)
EXPONENT BYTE 1
MANTISSA BYTES 2-4

MANTISSA: TWO'S COMPLIMENT REPRESENTATION WITH SIGN IN MSB OF HIGH-ORDER BYTE. MANTISSA IS NORMALIZED WITH AN ASSUMED DECIMAL POINT BETWEEN BITS 5 AND 6 OF THE HIGH-ORDER BYTE. THUS THE MANTISSA IS IN THE RANGE 1. TO 2. EXCEPT WHEN THE NUMBER IS LESS THAN 2**(-128).

EXPONENT: THE EXPONENT REPRESENTS POWERS OF TWO. THE REPRESENTATION IS 2'S COMPLIMENT EXCEPT THAT THE SIGN BIT (BIT 7) IS COMPLIMENTED. THIS ALLOWS DIRECT COMPARISON OF EXPONENTS FOR SIZE SINCE THEY ARE STORED IN INCREASING NUMERICAL SEQUENCE RANGING FROM \$00 (-128) TO \$FF (+127) (\$ MEANS NUMBER IS HEXADECIMAL).

REPRESENTATION OF DECIMAL NUMBERS: THE PRESENT FLOATING POINT REPRESENTATION ALLOWS DECIMAL NUMBERS IN THE APPROXIMATE RANGE OF 10**(-38) THROUGH 10**(38) WITH 6 TO 7 SIGNIFICANT DIGITS.

```
0003      ORG 3          SET BASE PAGE ADDRESSES
0003  EA      SIGN      NOP
0004  EA      X2        NOP
0005  00 00 00  M2      BSS 3      EXPONENT 2
0006  EA      X1        NOP
0007  00 00 00  M1      BSS 3      MANTISSA 2
0008  EA      M1        BSS 4      EXPONENT 1
0009  00 00 00  E      BSS 4      MANTISSA 1
000C      E          BSS 4      SCRATCH
0010      Z          BSS 4
0014      T          BSS 4
```

0018		SEXP	BSS 4		1DC1	BD CD 1D	L10	LDA LN10,X	
001C	00	INT	BSS 1		1DC4	95 04		STA X2,X	LOAD EXP/MANT2 WITH 1/LN(10)
		*			1DC6	CA		DEX	
1D00		*	ORG \$1D00	STARTING LOCATION FOR LOG	1DC7	10 F8		BPL L10	
		*			1DC9	20 77 1F		JSR FMUL	LOG10(X)=LN(X)/LN(10)
		*	NATURAL LOG OF MANT/EXP1 WITH RESULT IN MANT/EXP1		1DCC	60		RTS	
1D00	A5 09	LOG	LDA M1		1DCD	7E 6F	LN10	DCM 0.4342945	
1D02	F0 02		BEO ERROR			2D ED			
1D04	10 01		BPL CONT	IF ARG>0 OK	1DD1	00 5A	R22	DCM 1.4142136	SORT(2)
1D06	00	ERROR	BRK	ERROR ARG<=0		02 7A			
		*			1DD5	7F 58	LE2	DCM 0.69314718	LOG BASE E OF 2
1D07	20 1C 1F	CONT	JSR SWAP	MOVE ARG TO EXP/MANT2		89 0C			
1D0A	A5 04		LDA X2	HOLD EXPONENT	1DD9	00 52	A1	DCM 1.2920074	
1D0C	A0 00		LDY =#00			00 40			
1D0E	04 04		STY X2	SET EXPONENT 2 TO 0 (#00)	1DDD	01 AB	MB	DCM -2.6398577	
1D10	49 00		EOR =#00	COMPLIMENT SIGN BIT OF ORIGINAL EXPONENT		06 49			
1D12	05 0A		STA M1+1	SET EXPONENT INTO MANTISSA 1 FOR FLOAT	1DE1	00 6A	C	DCM 1.6567626	
1D14	A9 00		LDA =0			00 66			
1D16	05 09		STA M1	CLEAR MSB OF MANTISSA 1	1DE5	7F 40	MHLF	DCM 0.5	
1D18	20 2C 1F		JSR FLOAT	CONVERT TO FLOATING POINT		00 00			
1D1B	A2 03		LDX =3	4 BYTE TRANSFERS			*		
1D1D	05 04	SEXP1	LDA X2,X		1E00			ORG \$1E00	STARTING LOCATION FOR EXP
1D1F	95 10		STA Z,X	COPY MANTISSA TO Z			*		
1D21	05 00		LDA X1,X				*		EXP OF MANT/EXP1 RESULT IN MANT/EXP1
1D23	95 10		STA SEXP,X	SAVE EXPONENT IN SEXP			*		
1D25	0D 11 1D		LDA R22,X	LOAD EXP/MANT1 WITH SORT(2)	1E00	A2 03	EXP	LDX =3	4 BYTE TRANSFER
1D28	95 00		STA X1,X		1E02	0D 08 1E		LDA L2E,X	
1D2A	CA		DEX		1E05	95 04		STA X2,X	LOAD EXP/MANT2 WITH LOG BASE 2 OF E
1D2B	10 F0		BPL SEXP1		1E07	CA		DEX	
1D2D	20 4A 1F		JSR FSUB	Z-SORT(2)	1E08	10 F8		BPL EXP+2	
1D30	A2 03		LDX =3	4 BYTE TRANSFER	1E0A	20 77 1F		JSR FMUL	LOG2(E)*X
1D32	05 00	SAVET	LDA X1,X	SAVE EXP/MANT1 AS T	1E0D	A2 03		LDX =3	4 BYTE TRANSFER
1D34	95 14		STA T,X		1E0F	05 00	FSA	LDA X1,X	
1D36	05 10		LDA Z,X	LOAD EXP/MANT1 WITH Z	1E11	95 10		STA Z,X	STORE EXP/MANT1 IN Z
1D38	95 00		STA X1,X		1E13	CA		DEX	
1D3A	0D 11 1D		LDA R22,X	LOAD EXP/MANT2 WITH SORT(2)	1E14	10 F9		BPL FSA	SAVE Z=LN(2)*X
1D3D	95 04		STA X2,X		1E16	20 E8 1F		JSR FIX	CONVERT CONTENTS OF EXP/MANT1 TO AN INTEGER
1D3F	CA		DEX		1E19	A5 0A		LDA M1+1	
1D40	10 F0		BPL SAVET		1E1B	05 1C		STA INT	SAVE RESULT AS INT
1D42	20 50 1F		JSR FADD	Z+SORT(2)	1E1D	30		SEC	SET CARRY FOR SUBTRACTION
1D45	A2 03		LDX =3	4 BYTE TRANSFER	1E1E	E9 7C		SBC =124	INT-124
1D47	05 14	TM2	LDA T,X		1E20	A5 09		LDA M1	
1D49	95 04		STA X2,X	LOAD T INTO EXP/MANT2	1E22	E9 00		SBC =0	
1D4B	CA		DEX		1E24	10 15		BPL OVFLW	OVERFLOW INT>=124
1D4C	10 F9		BPL TM2		1E26	10		CLC	CLEAR CARRY FOR ADD
1D4E	20 9D 1F		JSR FDIIV	T=(Z-SORT(2))/(Z+SORT(2))	1E27	A5 0A		LDA M1+1	
1D51	A2 03		LDX =3	4 BYTE TRANSFER	1E29	69 70		ADC =120	ADD 120 TO INT
1D53	05 00	M1T	LDA X1,X		1E2B	A5 09		LDA M1	
1D55	95 14		STA T,X	COPY EXP/MANT1 TO T AND	1E2D	69 00		ADC =0	
1D57	95 04		STA X2,X	LOAD EXP/MANT2 WITH T	1E2F	10 00		BPL CONTIN	IF RESULT POSITIVE CONTINUE
1D59	CA		DEX		1E31	A9 00		LDA =0	INT<-120 SET RESULT TO ZERO AND RETURN
1D5A	10 F7		BPL M1T		1E33	A2 03		LDX =3	4 BYTE MOVE
1D5C	20 77 1F		JSR FMUL	T*T	1E35	95 00	ZERO	STA X1,X	SET EXP/MANT1 TO ZERO
1D5F	20 1C 1F		JSR SWAP	MOVE T*T TO EXP/MANT2	1E37	CA		DEX	
1D62	A2 03		LDX =3	4 BYTE TRANSFER	1E38	10 F8		BPL ZERO	
1D64	0D E1 1D	M1C	LDA C,X		1E3A	60		RTS	RETURN
1D67	95 00		STA X1,X	LOAD EXP/MANT1 WITH C			*		
1D69	CA		DEX		1E3B	00		OVFLW	BRK OVERFLOW
1D6A	10 F8		BPL M1C				*		
1D6C	20 4A 1F		JSR FSUB	T*T-C	1E3C	20 2C 1F	CONTIN	JSR FLOAT	FLOAT INT
1D6F	A2 03		LDX =3	4 BYTE TRANSFER	1E3F	A2 03		LDX =3	
1D71	0D DD 1D	M2MB	LDA MB,X		1E41	05 10	ENTD	LDA Z,X	
1D74	95 04		STA X2,X	LOAD EXP/MANT2 WITH MB	1E43	95 04		STA X2,X	LOAD EXP/MANT2 WITH Z
1D76	CA		DEX		1E45	CA		DEX	
1D77	10 F8		BPL M2MB		1E46	10 F9		BPL ENTD	
1D79	20 9D 1F		JSR FDIIV	MB/(T*T-C)	1E48	20 4A 1F		JSR FSUB	Z=Z-FLOAT(INT)
1D7C	A2 03		LDX =3	4 BYTE TRANSFER	1E48	A2 03		LDX =3	4 BYTE MOVE
1D7E	0D D9 1D	M2A1	LDA A1,X		1E4D	05 00	ZSAV	LDA X1,X	
1D81	95 04		STA X2,X	LOAD EXP/MANT2 WITH A1	1E4F	95 10		STA Z,X	SAVE EXP/MANT1 IN Z
1D83	CA		DEX		1E51	95 04		STA X2,X	COPY EXP/MANT1 TO EXP/MANT2
1D84	10 F8		BPL M2A1		1E53	CA		DEX	
1D86	20 50 1F		JSR FADD	MB/(T*T-C)+A1	1E54	10 F7		BPL ZSAV	
1D89	A2 03		LDX =3	4 BYTE TRANSFER	1E56	20 77 1F		JSR FMUL	Z*Z
1D8B	05 14	M2T	LDA T,X		1E59	A2 03		LDX =3	4 BYTE MOVE
1D8D	95 04		STA X2,X	LOAD EXP/MANT2 WITH T	1E5B	0D DC 1E	LA2	LDA A2,X	
1D8F	CA		DEX		1E5E	95 04		STA X2,X	LOAD EXP/MANT2 WITH A2
1D90	10 F9		BPL M2T		1E60	05 00		LDA X1,X	
1D92	20 77 1F		JSR FMUL	(MB/(T*T-C)+A1)*T	1E62	95 10		STA SEXP,X	SAVE EXP/MANT1 AS SEXP
1D95	A2 03		LDX =3	4 BYTE TRANSFER	1E64	CA		DEX	
1D97	0D E5 1D	M2MHL	LDA MHLF,X		1E65	10 F4		BPL LA2	
1D9A	95 04		STA X2,X	LOAD EXP/MANT2 WITH MHLF (.5)	1E67	20 50 1F		JSR FADD	Z*Z+A2
1D9C	CA		DEX		1E6A	A2 03		LDX =3	4 BYTE MOVE
1D9D	10 F8		BPL M2MHL		1E6C	0D E0 1E	LB2	LDA B2,X	
1D9F	20 50 1F		JSR FADD	+.5	1E6F	95 04		STA X2,X	LOAD EXP/MANT2 WITH B2
1DA2	A2 03		LDX =3	4 BYTE TRANSFER	1E71	CA		DEX	
1DA4	05 18	LDEXP	LDA SEXP,X		1E72	10 F8		BPL LB2	
1DA6	95 04		STA X2,X	LOAD EXP/MANT2 WITH ORIGINAL EXPONENT	1E74	20 9D 1F		JSR FDIIV	T=B2/(Z*Z+A2)
1DA8	CA		DEX		1E77	A2 03		LDX =3	4 BYTE MOVE
1DA9	10 F9		BPL LDEXP		1E79	05 00	DLOAD	LDA X1,X	
1DAB	20 50 1F		JSR FADD	+EXPX	1E7B	95 14		STA T,X	SAVE EXP/MANT1 AS T
1DAE	A2 03		LDX =3	4 BYTE TRANSFER	1E7D	0D E4 1E		LDA C2,X	
1DB0	0D D5 1D	MLE2	LDA LE2,X		1E80	95 00		STA X1,X	LOAD EXP/MANT1 WITH C2
1DB3	95 04		STA X2,X	LOAD EXP/MANT2 WITH LN(2)	1E82	05 10		LDA SEXP,X	
1DB5	CA		DEX		1E84	95 04		STA X2,X	LOAD EXP/MANT2 WITH SEXP
1DB6	10 F8		BPL MLE2		1E86	CA		DEX	
1DB8	20 77 1F		JSR FMUL	*LN(2)	1E87	10 F8		BPL DLOAD	
1DBB	60		RTS	RETURN RESULT IN MANT/EXP1	1E89	20 77 1F		JSR FMUL	Z*Z*C2
		*			1E8C	20 1C 1F		JSR SWAP	MOVE EXP/MANT1 TO EXP/MANT2
		*	COMMON LOG OF MANT/EXP1 RESULT IN MANT/EXP1		1E8F	A2 03		LDX =3	4 BYTE TRANSFER
1DBC	20 00 1D	LOG10	JSR LOG	COMPUTE NATURAL LOG	1E91	05 14	LTMP	LDA T,X	
1DBF	A2 03		LDX =3		1E93	95 00		STA X1,X	LOAD EXP/MANT1 WITH T
					1E95	CA		DEX	

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