# Homework 1: Applied Machine Learning - Linear | Logistic | SVM

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```
In [1]:
         import numpy as np
         import pandas as pd
         import seaborn as sns
         import matplotlib.pyplot as plt
         from numpy.linalg import inv
         %matplotlib inline
         from sklearn.model selection import train test split
         from sklearn.preprocessing import StandardScaler, MinMaxScaler
         from sklearn.preprocessing import OrdinalEncoder
         from sklearn.svm import LinearSVC, SVC
         from sklearn.metrics import accuracy score
In [2]:
         import warnings
         def fxn():
             warnings.warn("deprecated", DeprecationWarning)
         with warnings.catch warnings():
             warnings.simplefilter("ignore")
             fxn()
In [3]:
         pd.options.mode.chained_assignment = None
```

### **Part 1: Linear Regression**

In part 1, we will use **two datasets** to train and evaluate our linear regression model.

The first dataset will be a synthetic dataset sampled from the following equations:

```
\epsilon \sim \text{Normal}(0,3)
z = 3x + 10y + 10 + \epsilon
```

```
In [4]:
    np.random.seed(0)
    epsilon = np.random.normal(0, 3, 100)
    x = np.linspace(0, 10, 100)
    y = np.linspace(0, 5, 100)
    z = 3 * x + 10 * y + 10 + epsilon
```

To apply linear regression, we need to first check if the assumptions of linear regression are not violated.

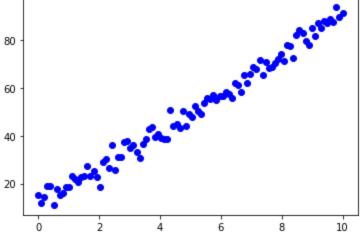
Assumptions of Linear Regression:

- Linearity: y is a linear (technically affine) function of x.
- Independence: the x's are independently drawn, and not dependent on each other.
- Homoscedasticity: the  $\epsilon$ 's, and thus the y's, have constant variance.
- Normality: the  $\epsilon$ 's are drawn from a Normal distribution (i.e. Normally-distributed errors)

These properties, as well as the simplicity of this dataset, will make it a good test case to check if our linear regression model is working properly.

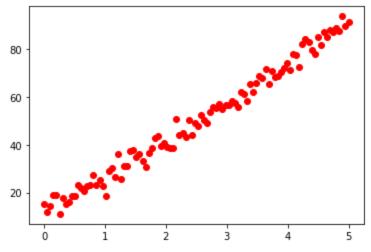
1.1. Plot z vs x and z vs y in the synthetic dataset as scatter plots. Label your axes and make sure your y-axis starts from 0. Do the independent and dependent features have linear relationship?

```
In [5]: ### Your code here
plt.plot(x,z, 'bo')
Out[5]: [<matplotlib.lines.Line2D at 0x7f88a00ac730>]
```



```
In [6]:
    ### Your code here
    plt.plot(y, z, 'ro')
    print("Yes, the independent and dependent features appear to have a linear relationship")
```

Yes, the independent and dependent features appear to have a linear relationship



1.2. Are the independent variables correlated? Use pearson correlation to verify? What would be the problem if linear regression is applied to correlated features?

```
In [7]:
### Your code here
print("Yes, the independent variables, x and y, are highly correlated. They have a Pearson
```

Yes, the independent variables, x and y, are highly correlated. They have a Pearson correlation coefficient of 1.0

The second dataset we will be using is an auto MPG dataset. This dataset contains various characteristics for around 8128 cars. We will use linear regression to predict the selling\_price label

```
In [8]: auto_mpg_df = pd.read_csv('Car details v3.csv')
# Dropping Torque column, there is information in this column but it will take some prepro
# The idea of the exercise is to familarize yourself with the basics of Linear regression.
auto_mpg_df = auto_mpg_df.drop(['torque'], axis = 1)
```

In [9]:

auto\_mpg\_df

Out[9]:		name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mileage	engine	1
	0	Maruti Swift Dzire VDI	2014	450000	145500	Diesel	Individual	Manual	First Owner	23.4 kmpl	1248 CC	
	1	Skoda Rapid 1.5 TDI Ambition	2014	370000	120000	Diesel	Individual	Manual	Second Owner	21.14 kmpl	1498 CC	
	2	Honda City 2017- 2020 EXi	2006	158000	140000	Petrol	Individual	Manual	Third Owner	17.7 kmpl	1497 CC	
	3	Hyundai i20 Sportz Diesel	2010	225000	127000	Diesel	Individual	Manual	First Owner	23.0 kmpl	1396 CC	
	4	Maruti Swift VXI BSIII	2007	130000	120000	Petrol	Individual	Manual	First Owner	16.1 kmpl	1298 CC	
	•••											
	8123	Hyundai i20 Magna	2013	320000	110000	Petrol	Individual	Manual	First Owner	18.5 kmpl	1197 CC	
	8124	Hyundai Verna CRDi SX	2007	135000	119000	Diesel	Individual	Manual	Fourth & Above Owner	16.8 kmpl	1493 CC	
	8125	Maruti Swift Dzire ZDi	2009	382000	120000	Diesel	Individual	Manual	First Owner	19.3 kmpl	1248 CC	
	8126	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	23.57 kmpl	1396 CC	
	8127	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	23.57 kmpl	1396 CC	

#### 8128 rows × 12 columns

```
In [10]:
```

```
dataTypeSeries = auto_mpg_df.dtypes
print('Data type of each column of Dataframe :')
print(dataTypeSeries)
```

```
Data type of each column of Dataframe : name % \left( \frac{1}{2}\right) =\frac{1}{2}\left( \frac{1}{2}\right) +\frac{1}{2}\left( \frac{1}{2}\right) +\frac{1}{2}\left(
```

```
year
                   int64
selling price
                   int64
km driven
                   int64
fuel
                  object
                  object
seller type
transmission
                  object
owner
                  object
mileage
                  object
                  object
engine
max power
                  object
                 float64
seats
dtype: object
```

#### 1.3. Missing Value analysis - Auto mpg dataset.

Are there any missing values in the dataset? If so, what can be done about it? Jusify your approach.

```
In [11]:
```

```
### Your code here

missing_df = auto_mpg_df.loc[auto_mpg_df.isnull().any(axis=1)]

print("Yes, there are missing values in the dataset, as shown below. There are several way print("I will choose to drop the rows that are missing data, because there are only a small display(missing_df)
fauto_mpg_df = auto_mpg_df.loc[auto_mpg_df.notnull().all(axis=1)]
#Removing an annoying value in "max_power" with no value, but not NULL (was incorrectly in fauto_mpg_df.drop(fauto_mpg_df[fauto_mpg_df['max_power'].str.len() < 5].index, inplace = ?
display(fauto_mpg_df)</pre>
```

Yes, there are missing values in the dataset, as shown below. There are several ways to handle missing values:

- Drop the columns/features (worst case)
- Drop the rows with missing values (roughly 2.5% of dataset)
- Replace the missing values with the mean/median

I will choose to drop the rows that are missing data, because there are only a small number of them

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mileage	engine	r
13	Maruti Swift 1.3 VXi	2007	200000	80000	Petrol	Individual	Manual	Second Owner	NaN	NaN	
31	Fiat Palio 1.2 ELX	2003	70000	50000	Petrol	Individual	Manual	Second Owner	NaN	NaN	
78	Tata Indica DLS	2003	50000	70000	Diesel	Individual	Manual	First Owner	NaN	NaN	
87	Maruti Swift VDI BSIV W ABS	2015	475000	78000	Diesel	Dealer	Manual	First Owner	NaN	NaN	
119	Maruti Swift VDI BSIV	2010	300000	120000	Diesel	Individual	Manual	Second Owner	NaN	NaN	
•••											
7846	Toyota Qualis Fleet A3	2000	200000	100000	Diesel	Individual	Manual	First Owner	NaN	NaN	

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mileage	engine	r
7996	Hyundai Santro LS zipPlus	2000	140000	50000	Petrol	Individual	Manual	Second Owner	NaN	NaN	
8009	Hyundai Santro Xing XS eRLX Euro III	2006	145000	80000	Petrol	Individual	Manual	Second Owner	NaN	NaN	
8068	Ford Figo Aspire Facelift	2017	580000	165000	Diesel	Individual	Manual	First Owner	NaN	NaN	
8103	Maruti Swift 1.3 VXi	2006	130000	100000	Petrol	Individual	Manual	Second Owner	NaN	NaN	
221 rov	vs × 12 cc	olumns									
			11	1 11					***	•	
	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mileage	engine	- 1 
0	Maruti Swift Dzire VDI	2014	450000	145500	Diesel	Individual	Manual	First Owner	23.4 kmpl	1248 CC	
	Skoda							Second	21 1/	1/102	

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mileage	engine	1
0	Maruti Swift Dzire VDI	2014	450000	145500	Diesel	Individual	Manual	First Owner	23.4 kmpl	1248 CC	
1	Skoda Rapid 1.5 TDI Ambition	2014	370000	120000	Diesel	Individual	Manual	Second Owner	21.14 kmpl	1498 CC	
2	Honda City 2017- 2020 EXi	2006	158000	140000	Petrol	Individual	Manual	Third Owner	17.7 kmpl	1497 CC	
3	Hyundai i20 Sportz Diesel	2010	225000	127000	Diesel	Individual	Manual	First Owner	23.0 kmpl	1396 CC	
4	Maruti Swift VXI BSIII	2007	130000	120000	Petrol	Individual	Manual	First Owner	16.1 kmpl	1298 CC	
•••	•••	•••			•••						
8123	Hyundai i20 Magna	2013	320000	110000	Petrol	Individual	Manual	First Owner	18.5 kmpl	1197 CC	
8124	Hyundai Verna CRDi SX	2007	135000	119000	Diesel	Individual	Manual	Fourth & Above Owner	16.8 kmpl	1493 CC	
8125	Maruti Swift Dzire ZDi	2009	382000	120000	Diesel	Individual	Manual	First Owner	19.3 kmpl	1248 CC	
8126	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	23.57 kmpl	1396 CC	

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mileage	engine	1
8127	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	23.57 kmpl	1396 CC	

#### 7906 rows × 12 columns

1.4. The features engine, max\_power and mileage have units in the dataset. In the real world if we have such datasets, we generally remove the units from each feature. After doing so, convert the datatype of these columns to float. For example: 1248 CC engine is 1248, 23.4 kmpl is 23.4 and so on.

Hint: Check for distinct units in each of these features. A feature might have multiple units as well. Also, a feature could have no value but have unit. For example 'CC' without any value. Remove such rows.

```
In [12]:
          ### Your code here
          m unit set = set()
          for mileage in fauto mpg df["mileage"]:
              m unit set.add(mileage[-5:])
          print("Distinct units in mileage:\t" + str(m_unit_set))
          p unit set = set()
          for power in fauto mpg df["max power"]:
              p unit set.add(power[-4:])
          print("Distinct units in max power:\t" + str(p unit set))
          e unit set = set()
          for power in fauto_mpg_df["engine"]:
              e unit set.add(power[-3:])
          print("Distinct units in engine:\t" + str(e_unit_set))
         Distinct units in mileage:
                                          { ' kmpl', 'km/kg'}
                                          { ' bhp ' }
         Distinct units in max power:
                                          {' CC'}
         Distinct units in engine:
In [13]:
          def change_mileage(mileage):
              if "km/kg" in mileage:
                  return float(mileage[:-6])*1.28
              elif "kmpl" in mileage:
                  return float(mileage[:-5])
              return mileage
          def change power(power):
              return float(power[:-4])
          def change engine(engine):
              return float(engine[:-3])
          def change km(km):
              return float(km)
          fauto mpg df["mileage"] = fauto mpg df["mileage"].apply(change mileage)
          fauto_mpg_df["max_power"] = fauto_mpg_df["max_power"].apply(change_power)
          fauto mpg df["engine"] = fauto mpg df["engine"].apply(change engine)
```

```
fauto_mpg_df["km_driven"] = fauto_mpg_df["km_driven"].apply(change_km)
display(fauto_mpg_df)
```

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mileage	engine
0	Maruti Swift Dzire VDI	2014	450000	145500.0	Diesel	Individual	Manual	First Owner	23.40	1248.0
1	Skoda Rapid 1.5 TDI Ambition	2014	370000	120000.0	Diesel	Individual	Manual	Second Owner	21.14	1498.0
2	Honda City 2017- 2020 EXi	2006	158000	140000.0	Petrol	Individual	Manual	Third Owner	17.70	1497.0
3	Hyundai i20 Sportz Diesel	2010	225000	127000.0	Diesel	Individual	Manual	First Owner	23.00	1396.0
4	Maruti Swift VXI BSIII	2007	130000	120000.0	Petrol	Individual	Manual	First Owner	16.10	1298.0
•••		•••						•••		
8123	Hyundai i20 Magna	2013	320000	110000.0	Petrol	Individual	Manual	First Owner	18.50	1197.0
8124	Hyundai Verna CRDi SX	2007	135000	119000.0	Diesel	Individual	Manual	Fourth & Above Owner	16.80	1493.0
8125	Maruti Swift Dzire ZDi	2009	382000	120000.0	Diesel	Individual	Manual	First Owner	19.30	1248.0
8126	Tata Indigo CR4	2013	290000	25000.0	Diesel	Individual	Manual	First Owner	23.57	1396.0
8127	Tata Indigo CR4	2013	290000	25000.0	Diesel	Individual	Manual	First Owner	23.57	1396.0

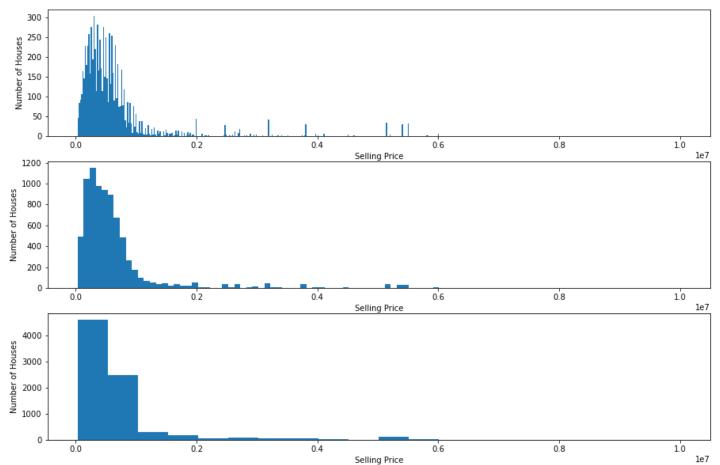
7906 rows × 12 columns

```
In [14]:
    auto_mpg_X = fauto_mpg_df.drop(columns=['selling_price'])
    auto_mpg_y = fauto_mpg_df['selling_price']
```

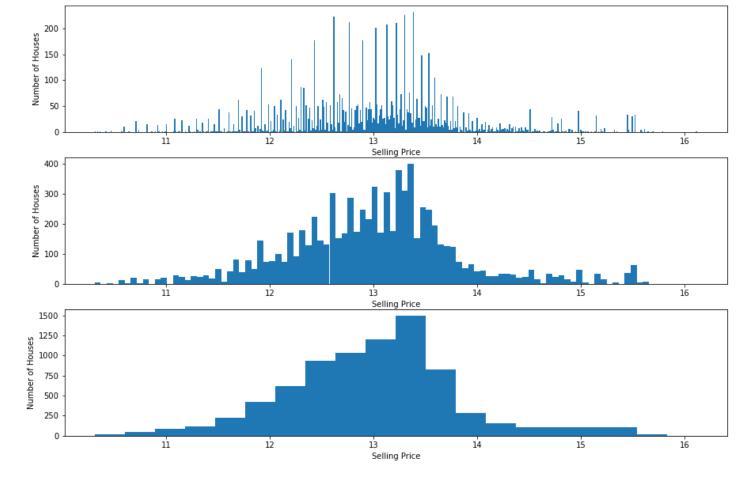
1.5. Plot the distribution of the label (selling\_price) using a histogram. Make multiple plots with different binwidths. Make sure to label your axes while plotting.

```
In [15]:
### Your code here
binSizes = [500,100,20]
figure, axs = plt.subplots(3, 1, figsize = (15, 10))
for index, binSize in enumerate(binSizes):
```

```
axs[index].hist(auto_mpg_y, bins=binSize)
axs[index].set(xlabel="Selling Price", ylabel="Number of Houses")
```

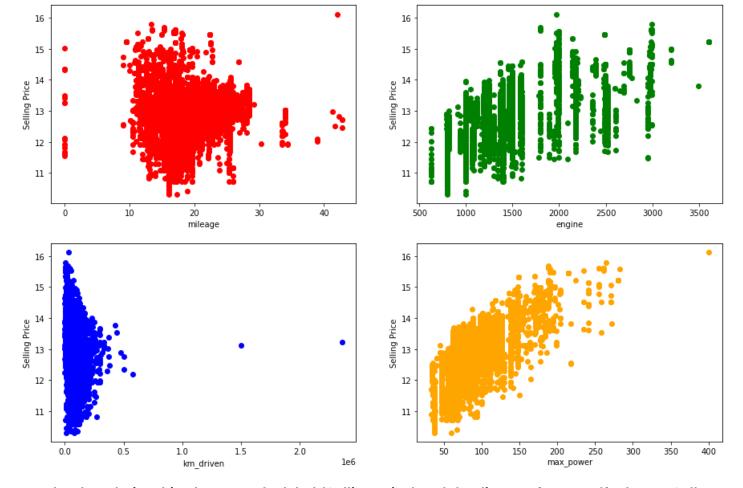


```
In [16]:
    binSizes = [500,100,20]
    figure, axs = plt.subplots(3, 1, figsize = (15, 10))
    for index, binSize in enumerate(binSizes):
        axs[index].hist(np.log(auto_mpg_y), bins=binSize)
        axs[index].set(xlabel="Selling Price", ylabel="Number of Houses")
    auto_mpg_y = np.log(auto_mpg_y)
```



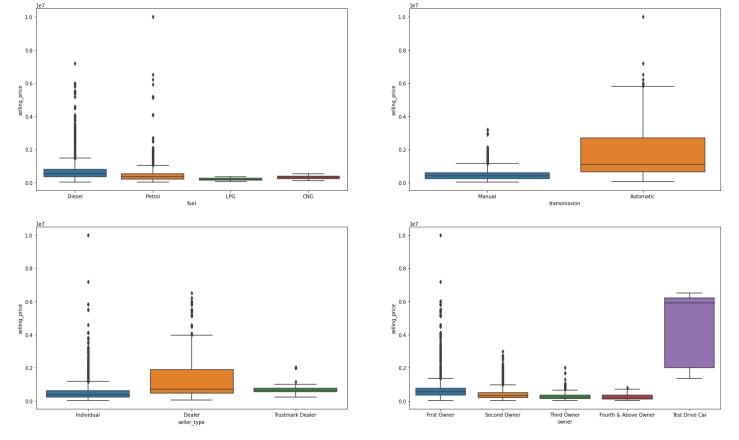
1.6. Plot the relationships between the label (Selling Price) and the continuous features (Mileage, km driven, engine, max power) using a small multiple of scatter plots. Make sure to label the axes. Do you see something interesting about the distributions of these features.

I can see that Engine and Max power seem to have a positive correlation with price



1.7. Plot the relationships between the label (Selling Price) and the discrete features (fuel type, Seller type, transmission) using a small multiple of box plots. Make sure to label the axes.

```
In [18]:
### Your code here
cat_ft = ['fuel', 'seller_type', 'transmission','owner']
figure, axes = plt.subplots(2, 2, figsize = (25, 15))
for index, feature in enumerate(cat_ft):
    xval = index%2
    yval = int(index/2)
    sns.boxplot(x=feature,y='selling_price', data=fauto_mpg_df, ax=axes[xval,yval])
#sns.boxplot(x='fuel',y='selling_price',data=fauto_mpg_df)
```



## 1.8. From the visualizations above, do you think linear regression is a good model for this problem? Why and/or why not?

```
In [19]:
    ### Your answer here
    print("Yes, I think linear regression is a good model for this problem because there seems
```

Yes, I think linear regression is a good model for this problem because there seems to be a strong linear correlation with some of the continuous features.

```
In [20]: auto_mpg_X['year'] = 2020 - auto_mpg_X['year']
In [21]: #dropping the car name as it is irrelevant.
    auto_mpg_X.drop(['name'],axis = 1,inplace=True)
    #check out the dataset with new changes
    auto_mpg_X.head()
```

Out[21]:		year	km_driven	fuel	seller_type	transmission	owner	mileage	engine	max_power	seats
	0	6	145500.0	Diesel	Individual	Manual	First Owner	23.40	1248.0	74.00	5.0
	1	6	120000.0	Diesel	Individual	Manual	Second Owner	21.14	1498.0	103.52	5.0
	2	14	140000.0	Petrol	Individual	Manual	Third Owner	17.70	1497.0	78.00	5.0
	3	10	127000.0	Diesel	Individual	Manual	First Owner	23.00	1396.0	90.00	5.0
	4	13	120000.0	Petrol	Individual	Manual	First Owner	16.10	1298.0	88.20	5.0

#### **Data Pre-processing**

1.9. Before we can fit a linear regression model, there are several pre-processing steps we should apply to the datasets:

1. Encode categorial features appropriately.

>>>Part 3/4: Scaling data and adding 1s

- 2. Split the dataset into training (60%), validation (20%), and test (20%) sets.
- 3. Standardize the columns in the feature matrices X\_train, X\_val, and X\_test to have zero mean and unit variance. To avoid information leakage, learn the standardization parameters (mean, variance) from X\_train, and apply it to X\_train, X\_val, and X\_test.
- 4. Add a column of ones to the feature matrices X\_train, X\_val, and X\_test. This is a common trick so that we can learn a coefficient for the bias term of a linear model.

```
In [22]:
          # 1. If no categorical features in the synthetic dataset, skip this step
          cat df = auto_mpg_X[cat_ft]
          cat df one hot = pd.get dummies(cat df, dummy na = False) #since we already removed N/A ve
          transformed_auto_df = auto_mpg_X.drop(columns=cat_ft)
          transformed auto df = transformed auto df.join(cat df one hot)
          print(">>>Part 1: One hot encoding")
          print("Shape of data before one-hot encoding:\t" + str(auto mpg X.shape))
          print("Shape of data after one-hot encoding:\t" + str(transformed auto df.shape))
          # 2. Split the dataset into training (60%), validation (20%), and test (20%) sets
          X dev, X test, y dev, y test = train test split(transformed auto df, auto mpg y, test size
          X train, X val, y train, y val = train test split(X dev, y dev, test size = 0.25, random s
          print("\n>>>Part 2: Train-test split")
          print("Shape of training data\nX train:\t" + str(X train.shape) + "\ny train:\t" + str(y t
          print("\nShape of validation data\nX val:\t" + str(X val.shape) + "\ny val:\t" + str(y val
          print("\nShape of test data\nX_test:\t" + str(X_test.shape) + "\ny_test:\t" + str(Y_test.shape)
          # 3. Standardize the columns in the feature matrices
          print("\n>>>Part 3/4: Scaling data and adding 1s")
          print("First row of X train before:")
          print(np.asarray([round(num, 8) for num in X_train.head(1).values.tolist()[0]]))
          scaler = StandardScaler()
          X_train_scaled = scaler.fit_transform(X_train) #fit and transform based on x_train, but or
          X val scaled = scaler.transform(X val)
          X test scaled = scaler.transform(X test)
          # 4. Add a column of ones to the feature matrices
          X train = np.hstack((X train scaled, np.ones((len(X train scaled),1))))
          X val = np.hstack((X val scaled, np.ones((len(X val scaled),1))))
          X_test = np.hstack((X_test_scaled, np.ones((len(X_test_scaled),1))))
          print("\nFirst row of X train after:")
          print(X train[0])
         >>>Part 1: One hot encoding
         Shape of data before one-hot encoding: (7906, 10)
         Shape of data after one-hot encoding: (7906, 20)
         >>>Part 2: Train-test split
         Shape of training data
         X train: (4743, 20)
                        (4743,)
         y train:
         Shape of validation data
         X val: (1581, 20)
         y val: (1581,)
         Shape of test data
         X_test: (1582, 20)
         y test: (1582,)
```

```
First row of X_train before:
[5.000e+00 1.700e+05 1.299e+01 2.494e+03 1.006e+02 7.000e+00 0.000e+00
1.000e+00 0.000e+00 0.000e+00 0.000e+00 1.000e+00 0.000e+00 0.000e+00
1.000e+00 1.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00]

First row of X_train after:
[-0.26360703 2.02884763 -1.5375452 2.04149403 0.24837876 1.63029724
-0.07566499 0.89825227 -0.06341936 -0.88076071 -0.4024058 0.45145
-0.17567827 -0.38526115 0.38526115 0.71753152 -0.14064385 -0.58587161
-0.02053903 -0.26261287 1. ]
```

At the end of this pre-processing, you should have the following vectors and matrices:

- Auto MPG dataset: auto\_mpg\_X\_train, auto\_mpg\_X\_val, auto\_mpg\_X\_test, auto\_mpg\_y\_train, auto\_mpg\_y\_val, auto\_mpg\_y\_test

#### **Implement Linear Regression**

Now, we can implement our linear regression model! Specifically, we will be implementing ridge regression, which is linear regression with L2 regularization. Given an  $(m \times n)$  feature matrix X, an  $(m \times 1)$  label vector y, and an  $(n \times 1)$  weight vector w, the hypothesis function for linear regression is:

$$y = Xw$$

Note that we can omit the bias term here because we have included a column of ones in our X matrix, so the bias term is learned implicitly as a part of w. This will make our implementation easier.

Our objective in linear regression is to learn the weights w which best fit the data. This notion can be formalized as finding the optimal w which minimizes the following loss function:

$$\min_{w} \|Xw - y\|_2^2 + \alpha \|w\|_2^2$$

This is the ridge regression loss function. The  $\|Xw-y\|_2^2$  term penalizes predictions Xw which are not close to the label y. And the  $\alpha\|w\|_2^2$  penalizes large weight values, to favor a simpler, more generalizable model. The  $\alpha$  hyperparameter, known as the regularization parameter, is used to tune the complexity of the model – a higher  $\alpha$  results in smaller weights and lower complexity, and vice versa. Setting  $\alpha=0$  gives us vanilla linear regression.

Conveniently, ridge regression has a closed-form solution which gives us the optimal w without having to do iterative methods such as gradient descent. The closed-form solution, known as the Normal Equations, is given by:

$$w = (X^T X + \alpha I)^{-1} X^T y$$

1.10. Implement a LinearRegression class with two methods: train and predict. You may NOT use sklearn for this implementation. You may, however, use np.linalg.solve to find the closed-form solution. It is highly recommended that you vectorize your code.

```
self.alpha = alpha
         self.w = None
    def train(self, X, y):
         '''Trains model using ridge regression closed-form solution
         (sets w to its optimal value).
        Parameters
         _____
        X : (m x n) feature matrix
         y: (m x 1) label vector
        Returns
         _____
        None
        Xt = X.T
        n = X.shape[1]
        aI = self.alpha*np.identity(n)
        XtX aI = Xt.dot(X)+aI
         # Instead of using this, which requires us to calculate the inverse (expensive),
         # we can use np.linalq.solve
         #inv = np.linalg.inv(XtX aI)
         \#self.w = (inv.dot(Xt)).dot(y)
         Xty = (X.T).dot(y)
         self.w = np.linalg.solve(XtX aI, Xty)
     def predict(self, X):
         '''Predicts on X using trained model.
        Parameters
         _____
         X: (m x n) feature matrix
        Returns
         _____
        y_pred: (m x 1) prediction vector
        y_pred = X.dot(self.w)
        return y pred
# Testing out functions
test = LinearRegression(100)
test.train(X train, y train)
print("Learned weights " + str(test.w.shape))
print(test.w)
print("Prediction on X val")
print(test.predict(X_val))
Learned weights (21,)
[-4.09105514e-01 -3.59799055e-02 4.17051067e-02 1.33640439e-01
  3.32619846e-01 2.53754213e-02 -1.40840276e-02 4.46308422e-02
  9.58560215e-04 -4.27190607e-02 1.96959163e-02 -2.37711312e-02
  1.22840006e-02 3.64759705e-02 -3.64759705e-02 2.82430390e-02
 -1.24441708 \\ e-02 \\ -1.14987723 \\ e-02 \\ 1.27283893 \\ e-02 \\ -2.81168947 \\ e-02
 1.27327414e+01]
Prediction on X_val
[13.36029746 12.20477388 12.89443746 ... 13.42375829 10.85155419
 14.24912115]
```

def init (self, alpha=0):

In [24]:

Train, Evaluate, and Interpret Linear Regression Model

Predictions:

Test Set Baseline MSE:

Actual:

1.11. A) Train a linear regression model ( $\alpha=0$ ) on the auto MPG training data. Make predictions and report the mean-squared error (MSE) on the training, validation, and test sets. Report the first 5 predictions on the test set, along with the actual labels.

```
In [25]:
          ### Your code here
          linearRegModel = LinearRegression(0)
          linearRegModel.train(X train, y train)
          def mse(y observed, y predicted):
              resid = np.square(y_observed-y_predicted)
              return np.mean(resid)
          print("Training Set MSE:\t" + str(mse(y train, linearRegModel.predict(X train))))
          print("Validation Set MSE:\t" + str(mse(y_val, linearRegModel.predict(X_val))))
          print("Test Set MSE:\t\t" + str(mse(y_test, linearRegModel.predict(X_test))))
          print("First 5 test set:")
          predictions = np.exp(linearRegModel.predict(X test))
          print("Predictions:\t" + str([round(x, 2) for x in predictions[:5]]))
          print("Actual:\t\t" + str([round(x, 2) for x in np.exp(y_test)[:5].tolist()]))
                               0.08697327056651748
         Training Set MSE:
         Validation Set MSE:
                               0.08385567301023064
         Test Set MSE:
                                 0.09755964235966472
         First 5 test set:
```

B) As a baseline model, use the mean of the training labels (auto\_mpg\_y\_train) as the prediction for all instances. Report the mean-squared error (MSE) on the training, validation, and test sets using this baseline. This is a common baseline used in regression problems and tells you if your model is any good. Your linear regression MSEs should be much lower than these baseline MSEs.

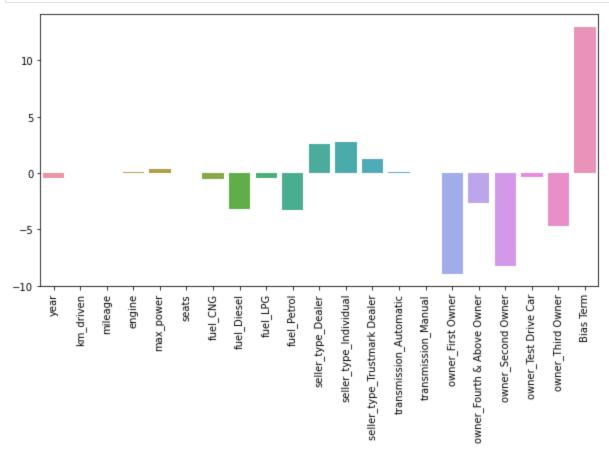
[434088.7, 434539.43, 259129.1, 310224.12, 661437.96] [501000.0, 440000.0, 140000.0, 476999.0, 620000.0]

1.12. Interpret your model trained on the auto MPG dataset using a bar chart of the model weights. Make sure to label the bars (x-axis) and don't forget the bias term! Use lecture 3, slide 15 as a reference. According to your model, which features are the greatest contributors to the selling price

0.6824405635131328

```
In [27]:
    ### Your code here
    features = list(transformed_auto_df.columns)
    features.append("Bias Term")
    fig = plt.figure(figsize = (10,5))
```

```
ax = sns.barplot(x=features, y=linearRegModel.w)
ax.tick_params(axis='x', rotation=90)
```

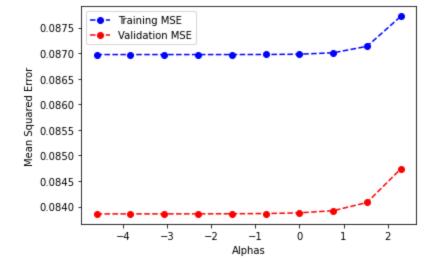


Tune Regularization Parameter  $\alpha$ 

Now, let's do ridge regression and tune the lpha regularization parameter on the auto MPG dataset.

1.13. Sweep out values for  $\alpha$  using alphas = np.logspace(-2, 1, 10). Perform a grid search over these  $\alpha$  values, recording the training and validation MSEs for each  $\alpha$ . A simple grid search is fine, no need for k-fold cross validation. Plot the training and validation MSEs as a function of  $\alpha$  on a single figure. Make sure to label the axes and the training and validation MSE curves. Use a log scale for the x-axis.

```
In [28]:
          ### Your code here
          alphas = np.logspace(-2, 1, 10)
          training MSEs = []
          validation MSEs = []
          for a in alphas:
              linearRegModel = LinearRegression(a)
              linearRegModel.train(X train, y train)
              training MSEs.append(mse(y train, linearRegModel.predict(X train)))
              validation_MSEs.append(mse(y_val, linearRegModel.predict(X_val)))
          plt.plot(np.log(alphas), training MSEs, '--bo', label = "Training MSE")
          plt.plot(np.log(alphas), validation MSEs, '--ro', label = "Validation MSE")
          plt.xlabel("Alphas")
          plt.ylabel("Mean Squared Error")
          plt.legend()
          plt.show()
```



Explain your plot above. How do training and validation MSE behave with decreasing model complexity (increasing  $\alpha$ )?

```
In [29]:
### Your answer here
print("Both seem to improve with decreasing model complexity, or by increasing alpha. This
```

Both seem to improve with decreasing model complexity, or by increasing alpha. This makes sense because simpler models usually perform better

1.14. Using the  $\alpha$  which gave the best validation MSE above, train a model on the training set. Report the value of  $\alpha$  and its training, validation, and test MSE. This is the final tuned model which you would deploy in production.

```
In [30]:
          ### Your code here
          bestAlpha = alphas[validation MSEs.index(max(validation MSEs))]
          linearRegModel = LinearRegression(bestAlpha)
          linearRegModel.train(X train, y train)
          print("Tuned, best alpha value:\t\t"+str(bestAlpha))
          print("Final Tuned Model Training Set MSE:\t" + str(mse(y train, linearRegModel.predict(X
          print("Final Tuned Model Validation Set MSE:\t" + str(mse(y val, linearRegModel.predict(X
          print("Final Tuned Model Test Set MSE:\t\t" + str(mse(y test, linearRegModel.predict(X test))
         Tuned, best alpha value:
         Final Tuned Model Training Set MSE:
                                                  0.0877232664184994
         Final Tuned Model Validation Set MSE:
                                                  0.08473980683625679
         Final Tuned Model Test Set MSE:
                                                  0.09836878876430162
```

### **Part 2: Logistic Regression**

**Gender Recognition by Voice and Speech Analysis** 

This dataset is used to identify a voice as male or female, based upon acoustic properties of the voice and speech.

```
In [31]:
           voice_df = pd.read_csv("voice-classification.csv")
           voice df.head()
Out[31]:
             meanfreq
                             sd
                                  median
                                               Q25
                                                        Q75
                                                                  IQR
                                                                           skew
                                                                                        kurt
                                                                                                sp.ent
              0.059781 0.064241 0.032027
                                           0.015071 0.090193 0.075122 12.863462
                                                                                             0.893369
                                                                                  274.402906
                                                                                                       0.4919
```

	meanfreq	sd	median	Q25	Q75	IQR	skew	kurt	sp.ent	sf
1	0.066009	0.067310	0.040229	0.019414	0.092666	0.073252	22.423285	634.613855	0.892193	0.51372
2	0.077316	0.083829	0.036718	0.008701	0.131908	0.123207	30.757155	1024.927705	0.846389	0.47890
3	0.151228	0.072111	0.158011	0.096582	0.207955	0.111374	1.232831	4.177296	0.963322	0.72723
4	0.135120	0.079146	0.124656	0.078720	0.206045	0.127325	1.101174	4.333713	0.971955	0.7835€

5 rows × 21 columns

#### **Data - Checking Rows & Columns**

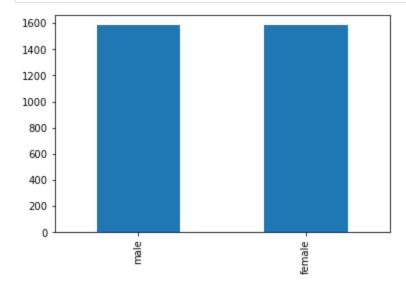
```
In [32]: #Number of Rows & Columns
print(voice_df.shape)

(3168, 21)
```

#### 2.1 What is the probability of observing different categories in the Label feature of the dataset?

This is mainly to check class imbalance in the dataset, and to apply different techniques to balance the dataset, which we will learn later.

```
In [33]: #code here
    #voice_df["label"]
    fig, ax = plt.subplots()
    voice_df['label'].value_counts().plot(ax=ax, kind='bar')
    plt.show()
    print("The categories in the Label feature seem balanced")
    print(voice_df['label'].value_counts())
```



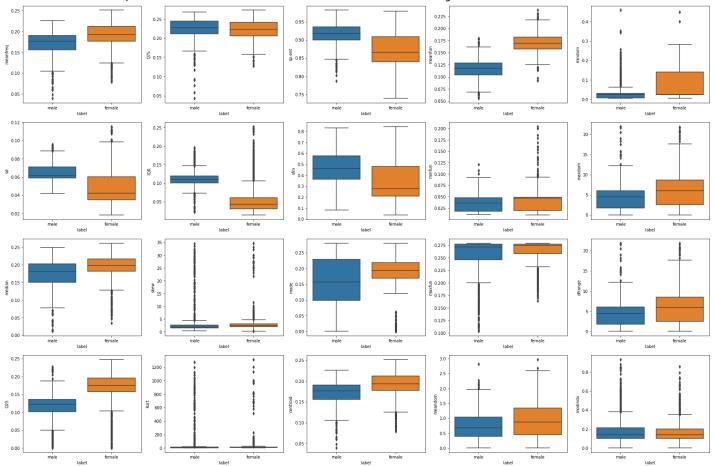
```
The categories in the Label feature seem balanced male 1584 female 1584 Name: label, dtype: int64
```

2.2 Plot the relationships between the label and the 20 numerical features using a small multiple of box plots. Make sure to label the axes. What useful information do this plot provide?

```
figure, axes = plt.subplots(4, 5, figsize = (30, 20))
features = list(voice_df.columns)[:20]
for index, feature in enumerate(features):
    xval = index%4
    yval = int(index/4)
```

```
sns.boxplot(x='label',y=feature, data=voice_df, ax=axes[xval,yval])
print("These plots provide a great deal of useful information. Because these boxplots all
```

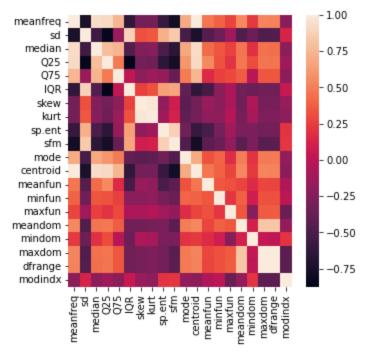
These plots provide a great deal of useful information. Because these boxplots all seem to have significant overlap, there is not a single feature that will make this categorization easier. However, some features do seem to show a more significant difference than others.



2.3 Plot the correlation matrix, and check if there is high correlation between the given numerical features (Threshold >=0.9). If yes, drop those highly correlated features from the dataframe. Why is necessary to drop those columns before proceeding further?

```
In [35]:
          corr = voice df.corr()
          f, ax = plt.subplots(figsize=(5, 5))
          sns.heatmap(corr)
          threshold = 0.9
          plt.show()
          def get redundant pairs(df):
              pairs_to_drop = set()
              cols = df.columns
              for i in range(0, df.shape[1]):
                  for j in range(0, i+1):
                      pairs_to_drop.add((cols[i], cols[j]))
              return pairs_to_drop
          def get_top_abs_correlations(df, threshold = 0.9):
              au corr = df.abs().unstack()
              print(type(au corr))
              labels_to_drop = get_redundant_pairs(df)
              au corr = au corr.drop(labels=labels to drop).sort values(ascending=False).to frame()
              au_corr = au_corr[au_corr[0] >= threshold]
              return au corr
          print("Top Absolute Correlations")
```

```
topCorr = get_top_abs_correlations(corr, 0.9)
display(topCorr)
toDrop = list(set([x[1] for x in topCorr.index.values]))
print(toDrop)
filtered_voice_df = voice_df.drop(labels=toDrop, axis=1)
print("It is important to remove these highly correlated features because multicolinearity
```



Top Absolute Correlations
<class 'pandas.core.series.Series'>

		0
meanfreq	centroid	1.000000
maxdom	dfrange	0.999838
skew	kurt	0.977020
median	centroid	0.925445
meanfreq	median	0.925445
	Q25	0.911416
Q25	centroid	0.911416

```
['dfrange', 'median', 'centroid', 'Q25', 'kurt']
```

It is important to remove these highly correlated features because multicolinearity may le ad to solutions that are varying and numerically unstable. It becomes difficult for the mo del to evaluate the relationship between an independent and dependent variable when independent variables change in unison.

#### Separating Features & Y variable from the processed dataset

Please note to replace the dataframe below with the new dataframe created after removing highly correlated features

```
'modindx'],
dtype='object')
```

#### 2.4 Apply the following pre-processing steps:

- 1) Use OrdinalEncoding to encode the label in the dataset (male & female)
- 2) Convert the label from a Pandas series to a Numpy (m x 1) vector. If you don't do this, it may cause problems when implementing the logistic regression model.
- 3) Split the dataset into training (60%), validation (20%), and test (20%) sets.
- 4) Standardize the columns in the feature matrices. To avoid information leakage, learn the standardization parameters from training, and then apply training, validation and test dataset.
- 5) Add a column of ones to the feature matrices of train, validation and test dataset. This is a common trick so that we can learn a coefficient for the bias term of a linear model.

```
In [37]:
          # 1/2. Encode the label in the dataset, and convert to numpy vector
          print(">>>Part 1/2: Ordinal encoding/Conversion to numpy")
          print("Data before ordinal encoding:\t")
          display(voice y.head())
          print("Data after ordinal encoding/conversion to numpy:")
          gender mapper = {"male":0, "female":1}
          voice y = voice y.replace(gender mapper)
          voice y = voice y.to numpy()
          print(voice y)
          # 3. Split the dataset into training (60%), validation (20%), and test (20%) sets
          X dev, X test, y dev, y test = train test split(voice X, voice y, test size = 0.2, random
          X_train, X_val, y_train, y_val = train_test_split(X_dev, y_dev, test_size = 0.25, random_s
          print("\n>>>Part 3: Train-test split")
          print("Shape of training data\nX train:\t" + str(X train.shape) + "\ny train:\t" + str(y t
          print("\nShape of validation data\nX val:\t\t" + str(X val.shape) + "\ny val:\t\t" + str(\sqrt{\sqrt{\text{}}}
          print("\nShape of test data\nX_test:\t\t" + str(X_test.shape) + "\ny_test:\t\t" + str(y_test.shape)
          # 4. Standardize the columns in the feature matrices
          print("\n>>>Part 4/5: Scaling data and adding 1s")
          print("First row of X train before:")
          print(np.asarray([round(num, 8) for num in X train.head(1).values.tolist()[0]]))
          scaler = StandardScaler()
          X train scaled = scaler.fit transform(X train) #fit and transform based on x train, but of
          X val scaled = scaler.transform(X val)
          X_test_scaled = scaler.transform(X_test)
          # 5. Add a column of ones to the feature matrices
          X train = np.hstack((X train scaled, np.ones((len(X train scaled),1))))
          X val = np.hstack((X val scaled, np.ones((len(X val scaled),1))))
          X test = np.hstack((X test scaled, np.ones((len(X test scaled),1))))
          print("\nFirst row of X train after:")
          print(X train[0])
```

```
>>>Part 1/2: Ordinal encoding/Conversion to numpy
Data before ordinal encoding:
0    male
1    male
2    male
3    male
```

```
4
    male
Name: label, dtype: object
Data after ordinal encoding/conversion to numpy:
[0 0 0 ... 1 1 1]
>>>Part 3: Train-test split
Shape of training data
X train:
         (1900, 15)
               (1900,)
y_train:
Shape of validation data
X val:
         (634, 15)
y val:
               (634,)
Shape of test data
               (634, 15)
X test:
y_test:
               (634,)
>>>Part 4/5: Scaling data and adding 1s
First row of X train before:
[0.20855907 0.03942026 0.22710645 0.03022489 3.1048892 0.84013854
 0.27603304 \ 0.21241379 \ 0.17152469 \ 0.04853387 \ 0.27745665 \ 1.44497283
 0.0234375 7.5703125 0.07572862]
First row of X train after:
[0.92810852 -1.04929751 \ 0.10142764 -1.25571826 -0.01439802 -1.1898484]
-0.73465153 \quad 0.59936436 \quad 0.8979924 \quad 0.60604727 \quad 0.6172792 \quad 1.14312763
-0.47088706 0.67858718 -0.8089197 1.
                                               ]
```

#### 2.5 Implement Logistic Regression

We will now implement logistic regression with L2 regularization. Given an  $(m \times n)$  feature matrix X, an  $(m \times 1)$  label vector y, and an  $(n \times 1)$  weight vector w, the hypothesis function for logistic regression is:

$$y = \sigma(Xw)$$

where  $\sigma(x)=\frac{1}{1+e^{-x}}$ , i.e. the sigmoid function. This function scales the prediction to be a probability between 0 and 1, and can then be thresholded to get a discrete class prediction.

Just as with linear regression, our objective in logistic regression is to learn the weights w which best fit the data. For L2-regularized logistic regression, we find an optimal w to minimize the following loss function:

$$\min_{w} \ -y^{T} \log(\sigma(Xw)) \ - \ (\mathbf{1} - y)^{T} \log(\mathbf{1} - \sigma(Xw)) \ + \ lpha \|w\|_{2}^{2}$$

Unlike linear regression, however, logistic regression has no closed-form solution for the optimal w. So, we will use gradient descent to find the optimal w. The (n x 1) gradient vector g for the loss function above is:

$$g = X^T \Big( \sigma(Xw) - y \Big) + 2 lpha w$$

Below is pseudocode for gradient descent to find the optimal w. You should first initialize w (e.g. to a (n x 1) zero vector). Then, for some number of epochs t, you should update w with  $w-\eta g$ , where  $\eta$  is the learning rate and g is the gradient. You can learn more about gradient descent here.

$$w=\mathbf{0}$$
 for  $i=1,2,\ldots,t$   $w=w-\eta g$ 

Implement a LogisticRegression class with five methods: train, predict, calculate\_loss, calculate\_gradient, and calculate\_sigmoid. You may NOT use sklearn for this implementation. It is highly recommended that you vectorize your code.

```
In [38]:
          class LogisticRegression():
              Logistic regression model with L2 regularization.
              Attributes
              _____
              alpha: regularization parameter
              t: number of epochs to run gradient descent
              eta: learning rate for gradient descent
              w: (n x 1) weight vector
              1.1.1
              def __init__(self, alpha, t, eta):
                  self.alpha = alpha
                  self.t = t
                  self.eta = eta
                  self.w = None
              def train(self, X, y):
                  '''Trains logistic regression model using gradient descent
                  (sets w to its optimal value).
                  Parameters
                  _____
                  X : (m \times n) feature matrix
                  y: (m x 1) label vector
                  Returns
                  losses: (t x 1) vector of losses at each epoch of gradient descent
                  self.w = np.zeros(X.shape[1])
                  losses = []
                  for i in range(self.t):
                      self.w = self.w - self.eta*self.calculate_gradient(X, y)
                      losses.append(self.calculate loss(X, y))
                  return losses
              def predict(self, X):
                  '''Predicts on X using trained model. Make sure to threshold
                  the predicted probability to return a 0 or 1 prediction.
                  Parameters
                  X : (m x n) feature matrix
                  Returns
                  _____
                  y_pred: (m x 1) 0/1 prediction vector
                  y_pred = self.calculate_sigmoid((X.dot(self.w)))
                  y_pred = np.rint(y_pred)
                  return y pred
                  ### Your code here
```

def calculate\_loss(self, X, y):

```
\label{eq:calculates} the logistic regression loss using X, y, w,
    and alpha. Useful as a helper function for train().
   Parameters
    -----
   X : (m x n) feature matrix
   y: (m x 1) label vector
   Returns
    loss: (scalar) logistic regression loss
    sigmoid Xw = self.calculate sigmoid(X.dot(self.w))
    firstTerm = -1*(y.T).dot(np.log(sigmoid_Xw))
    secondTerm = -1*((1-y).T).dot(np.log(1-sigmoid Xw))
   norm = 0
    for w i in self.w:
       norm += np.power(w_i, 2)
   norm = np.sqrt(norm)
    thirdTerm = self.alpha*(norm)
    loss = firstTerm + secondTerm + thirdTerm
    return loss
def calculate gradient(self, X, y):
    '''Calculates the gradient of the logistic regression loss
    using X, y, w, and alpha. Useful as a helper function
    for train().
   Parameters
    _____
   X : (m x n) feature matrix
   y: (m x 1) label vector
   Returns
    gradient: (n x 1) gradient vector for logistic regression loss
    inner = self.calculate sigmoid(X.dot(self.w))-y
    gradient = (X.T).dot(inner) + 2*self.alpha*self.w
    return gradient
def calculate sigmoid(self, x):
    '''Calculates the sigmoid function on each element in vector x.
    Useful as a helper function for predict(), calculate_loss(),
    and calculate gradient().
   Parameters
    _____
   x: (m x 1) vector
   Returns
    sigmoid x: (m x 1) vector of sigmoid on each element in x
    sigmoid x = 1 / (1 + np.exp(-x))
    return sigmoid x
    ### Your code here
```

A: Using your implementation above, train a logistic regression model (alpha=0, t=100, eta=1e-3) on the voice recognition training data. Plot the training loss over epochs. Make sure to label your axes. You should see the loss decreasing and start to converge.

B: Using alpha between (0,1), eta between (0, 0.001) and t between (0, 100), find the best hyperparameters for LogisticRegression. You can randomly search the space 20 times to find the best hyperparameters.

C. Compare accuracy on the test dataset for both the scenarios.

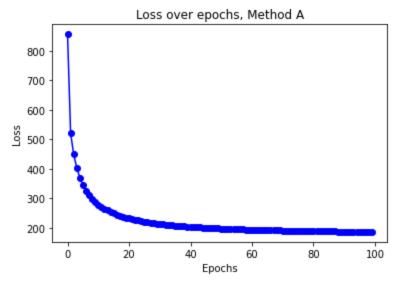
print("Best alpha value:\t\t"+str(bestAlpha))

```
In [39]:
          from sklearn.metrics import matthews corrcoef
          #Part A: Training with given hyperparameters
          presetAlpha, preset t, presetEta = 0, 100, 1e-3
          logRegA = LogisticRegression(presetAlpha, preset t, presetEta)
          lossesA = logRegA.train(X_train, y_train)
          plt.plot(lossesA, '-bo')
          plt.title("Loss over epochs, Method A")
          plt.xlabel("Epochs")
          plt.ylabel("Loss")
          plt.show()
          print("Alpha value:\t\t"+str(presetAlpha))
          print("Epoch length:\t\t"+str(preset t))
          print("Learning rate:\t\t"+str(presetEta))
          print("Loss:\t\t\t"+str(lossesA[-1]))
          #Part B: Finding the best hyperparameters by randomly searching the space
          import random
          import itertools
          alphas = np.logspace(-2, 0, 5)
          t = pochs = np.linspace(20, 100, 5)
          etas = np.logspace(-5, -3, 5)
          hparamSearchSpace = list(set(itertools.product(alphas,t_epochs, etas)))
          rand hParams = random.sample(hparamSearchSpace, 20)
          plt.title("Loss over epochs, Method B")
          plt.xlabel("Epochs")
          plt.ylabel("Loss")
          bestMCC = -np.inf
          #for alpha, t, eta in hparamSearchSpace: #could do this to search whole space
          for alpha, t, eta in rand hParams:
              logRegSearch = LogisticRegression(alpha, int(t), eta)
              thisLoss = logRegSearch.train(X train, y train)
              # After training model, use VALIDATION set prediction to choose hyperparameters
              # Choose to use MCC score instead of accuracy (takes false positives and false negative
              y pred = logRegSearch.predict(X val)
              thisAcc = 1-np.mean(np.abs(y_pred-y_val))
              thisMCC = matthews_corrcoef(y_val, y_pred)
              if thisMCC > bestMCC:
                  bestMCC = thisMCC
                  bestAcc = thisAcc
                  bestAlpha, best_t, bestEta, bestLosses = alpha, t, eta, thisLoss
                  bestModel = logRegSearch
          plt.plot(bestLosses, '-ro')
          plt.title("Loss over epochs, Method B")
          plt.xlabel("Epochs")
          plt.ylabel("Loss")
          plt.show()
```

```
print("Best epoch length:\t\t"+str(best_t))
print("Best learning rate:\t\t"+str(bestEta))
print("Best Loss:\t\t\t"+str(bestLosses[-1]))
print("Best (Validation) Accuracy:\t"+str(bestAcc))
print("Best (Validation) MCC:\t\t"+str(bestMCC))

#Part C: Comparing accuracy for both scenarios
yA_pred = logRegA.predict(X_test)
yB_pred = bestModel.predict(X_test)

A_accuracy = 1-np.mean(np.abs(yA_pred-y_test))
B_accuracy = 1-np.mean(np.abs(yB_pred-y_test))
print("\n\nScenario A's Accuracy on Test Dataset:\t"+str(A_accuracy))
print("Scenario B's Accuracy on Test Dataset:\t"+str(B_accuracy))
```



Alpha value: 0
Epoch length: 100
Learning rate: 0.001

Loss: 185.96690533096108

#### Loss over epochs, Method B Loss Epochs

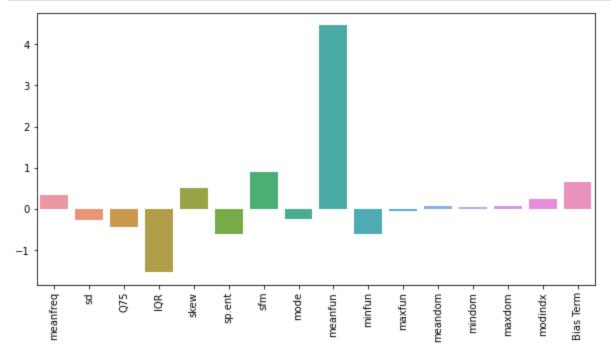
Best (Validation) Accuracy: 0.9668769716088328
Best (Validation) MCC: 0.9337496379837762

Scenario A's Accuracy on Test Dataset: 0.9794952681388013 Scenario B's Accuracy on Test Dataset: 0.9794952681388013

#### 2.7 Feature Importance

Interpret your trained model using a bar chart of the model weights. Make sure to label the bars (x-axis) and don't forget the bias term!

```
features = list(voice_X.columns)
features.append("Bias Term")
fig = plt.figure(figsize = (10,5))
ax = sns.barplot(x=features, y=logRegA.w)
ax.tick_params(axis='x', rotation=90)
```



# Part 3: Support Vector Machines - with the same Dataset

#### 3.1 Dual SVM

- A) Train a dual SVM (with default parameters) for both kernel="linear" and kernel="rbf") on the Voice Recognition training data.
- B) Make predictions and report the accuracy on the training, validation, and test sets. Which kernel gave better accuracy on test dataset and why do you think that was better?
- C) Please report the support vectors in both the cases and what do you observe? Explain

```
from sklearn.compose import make_column_transformer
from sklearn.pipeline import make_pipeline
print("Class Distribution:")
voice_y = pd.Series(voice_y)
print(voice_y.value_counts())
print("Dataset Size:")
print(voice_X.shape)
feature_names = list(voice_X.columns)
dev_X, test_X, dev_y, test_y = train_test_split(voice_X, voice_y, test_size=0.2, random_st
preprocess = make_column_transformer((StandardScaler(), feature_names))
pipe_linear = make_pipeline(preprocess, SVC(kernel='linear'))
pipe_rbf = make_pipeline(preprocess, SVC(kernel='rbf'))
```

```
pipe_linear.fit(dev_X, dev_y)
pipe_rbf.fit(dev_X, dev_y)

print(f"\nLinear Kernel Test Score:\t", pipe_linear.score(test_X, test_y))
print(f"RBF Kernel Test Score:\t", pipe_rbf.score(test_X, test_y))
print("\nThe RBF Kernel gave better accuracy on the test dataset. It is usually recommended.)
```

The RBF Kernel gave better accuracy on the test dataset. It is usually recommended to use a linear kernel when the number of features is large, and a non-linear RBF (Gaussian) kern el when the number of samples is large. This is because a linear kernel is never more accurate than a properly tuned Gaussian kernel, it is mainly used because it is faster on data sets with many features.

## 3.2 Using Kernel "rbf", tune the hyperparameter "C" using the Grid Search & k-fold cross validation. You may take k=5 and assume values in grid between 1 to 100 with interval range of your choice.

The C parameter tells the SVM optimization how much you want to avoid misclassifying each training example. For large values of C, the optimization will choose a smaller-margin hyperplane if that hyperplane does a better job of getting all the training points classified correctly. Conversely, a very small value of C will cause the optimizer to look for a larger-margin separating hyperplane, even if that hyperplane misclassifies more points.

Best Score: 0.9822446227128502 Best Params: {'C': 1.6237767391887217} Test Score: 0.9810725552050473