2. GRID-BASED NUMERICAL METHODS FOR *PDE* SOLVING

- Start with something like: $\partial_t^2(u) = c^2\partial_x^2(u)$
  - Re-formulate to coupled, first-order in time system
    $\partial_t u = v$, $\partial_t v = c^2\partial_x^2 u$
- Use a uniform grid to define $u, v \to u_i \equiv u(x_i), v_i \equiv v(x_i)$
- Use finite difference (or finite volume) derivative formula for the spatial derivatives: $\partial_x^2 u \to \dfrac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2}$
- We now have $\partial_t u_i = v_i \equiv \mathrm{RHS}_u$
  $\partial_t v_i = c^2\dfrac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} \equiv \mathrm{RHS}_v$
  Discretize this in time & solve with ODE methods (e.g. RK4), as before (this is "Method of Lines")
- Apply the boundary condition to the boundary grid points (may need more than one)

3. GRID-BASED NUMERICAL METHODS FOR *COUPLED* PDE SOLVING