Homework Solutions

Problem 0. Find the value of the digit D if $47D4$ leaves a remainder of 2 when divided by 33. (Source: Intro the Number Theory)

Solution: $47D4$ leaving a remainder of 2 when divided by 33 is the same as saying that $47D4 - 2 = 47D2$ is divisible by 33. Since $33 = 3 \cdot 11$, an integer is divisible by 33 if and only if it is divisible by 3 and 11.
In order for $47D2$ to be divisible by 3, we must have

$$4 + 7 + D + 2 \equiv 0 \pmod{3}.$$

That is, $13 + D \equiv 0 \pmod 3$, or $1 + D \equiv 0 \pmod 3$, or $D \equiv 2 \pmod 3$.
In order for $47D2$ to be divisible by 11, we must have

$$2 - D + 7 - 4 \equiv 0 \pmod{11}.$$

That is, $5 - D \equiv 0 \pmod{11}$, or $D \equiv 5 \pmod{11}$.
The only digit satisfying $D \equiv 2 \pmod 3$ and $D \equiv 5 \pmod{11}$ is $\boxed{5}$. $\qquad\square$