Vishal O. Sharma

Classic Endemic model[1]

$$ds/dt = -\beta is + \mu - \mu s, \qquad s(0) = s_o \geq 0,$$

$$di/dt = \beta is - (\gamma + \mu)\, i, \qquad i(0) = i_o \geq 0,$$

where, β = interaction rate

μ = death/born rate

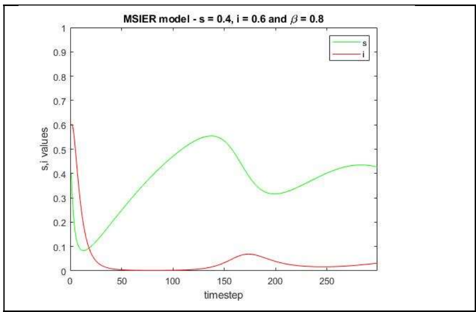
γ = cure rate

We assume, $\gamma = 0.3$, $\mu = 0.015$ and β changes from 0.8 to 1.6 by step 0.1 i.e. [0.8, 0.9, 1.0, … , 1.5, 1.6] and R= average number of people infected by an infectious individual.

Solving the differential equation of the Classic Endemic model given above and plotting for next 500 steps.

The differential equations can be solved in multiple ways. One of them is using Euler's equations.[2]

Thus, solving the equations through Euler's method in MATLAB, we get set of s and i points for different initial values of s and i. Plotting time dependencies graphs of s(t) and i(t):



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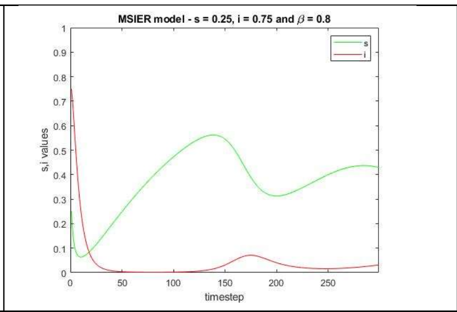

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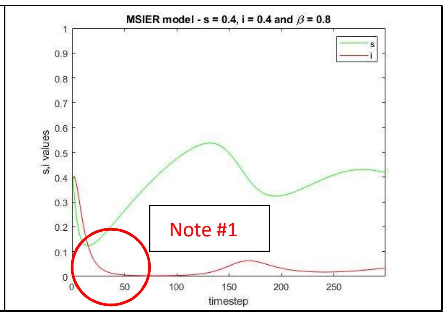

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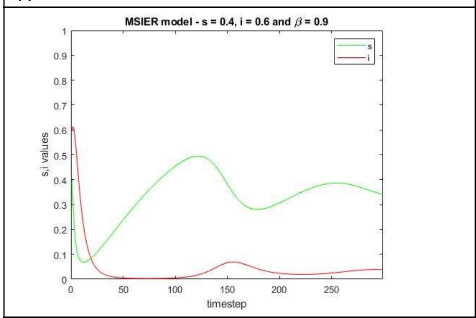

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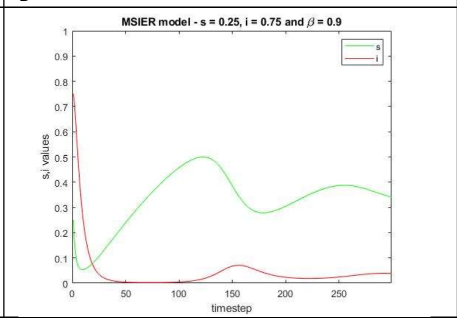

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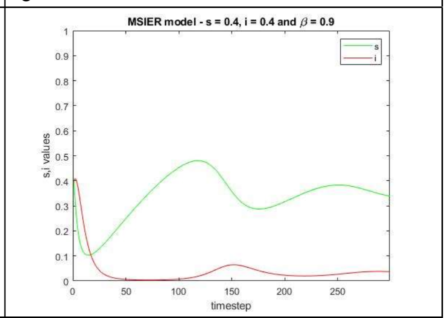

F

[1] The Mathematics of Infectious Diseases, Herbert W. Hethcote

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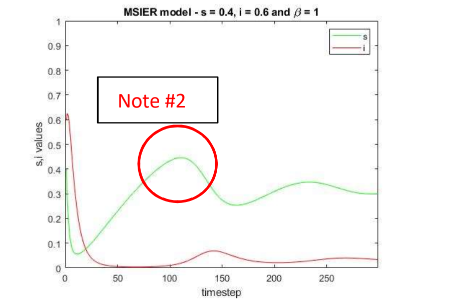

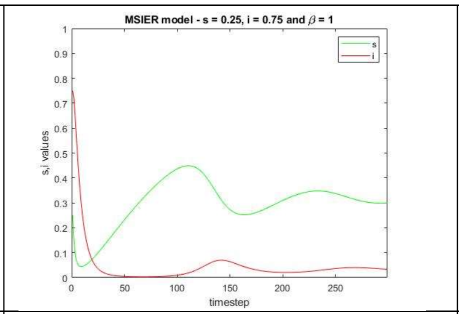

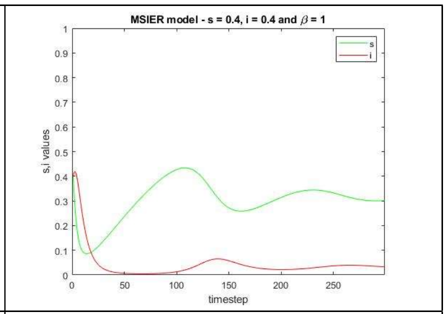

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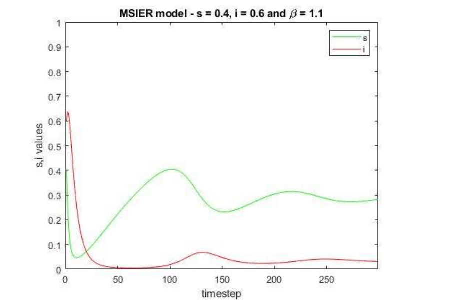

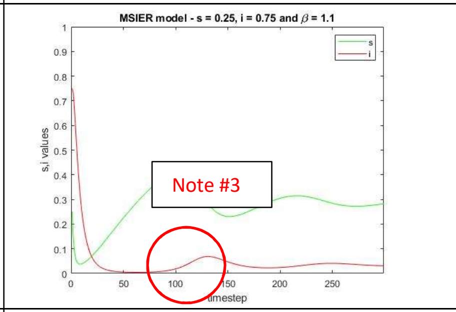

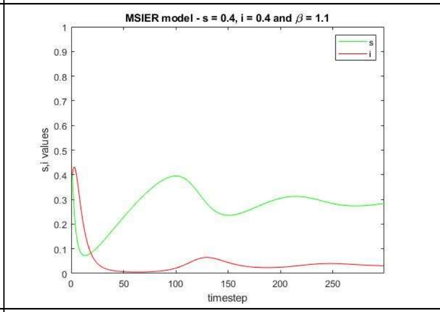

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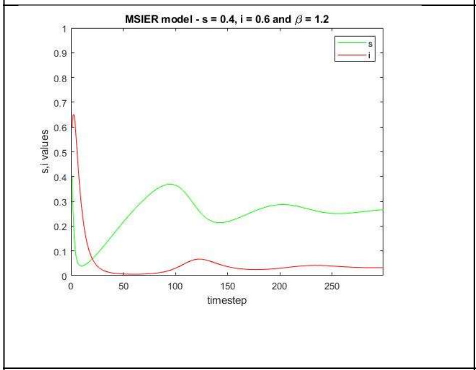

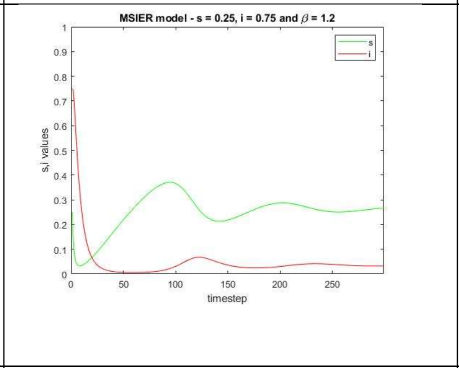

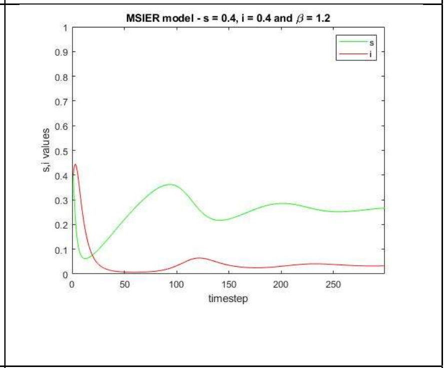

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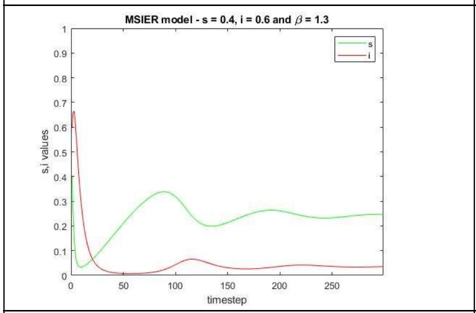

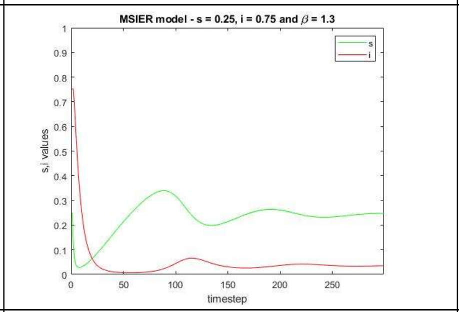

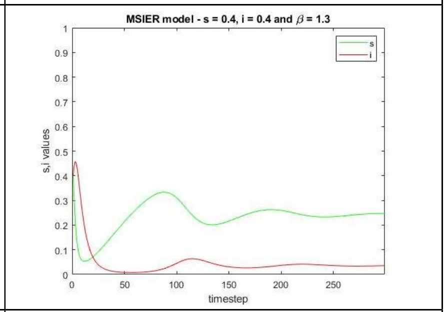

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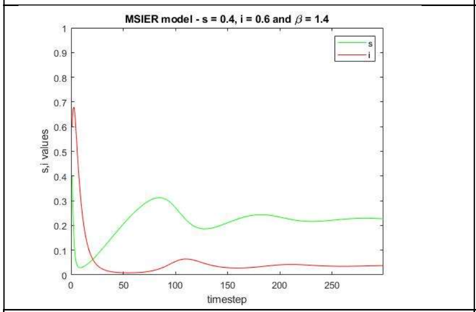

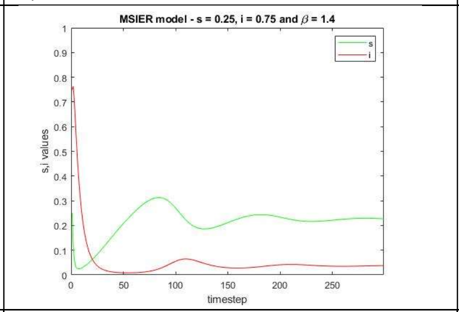

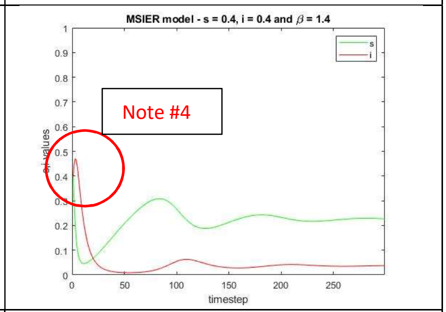

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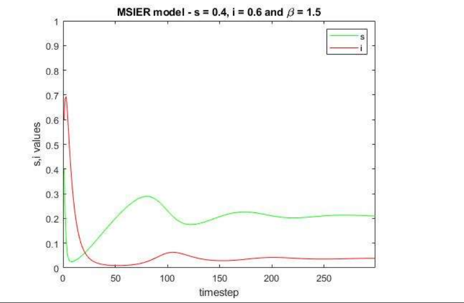

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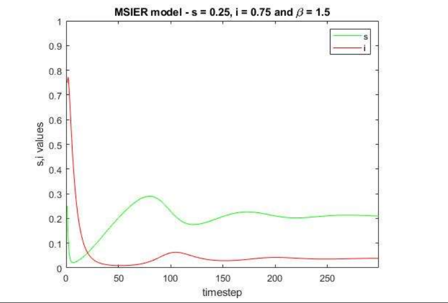

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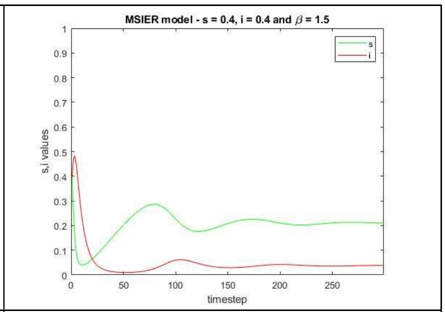

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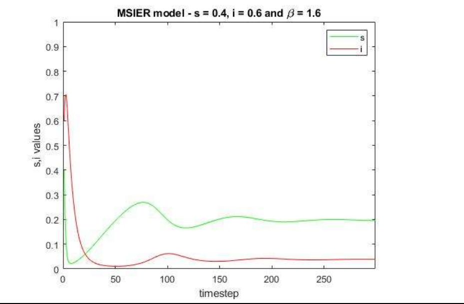

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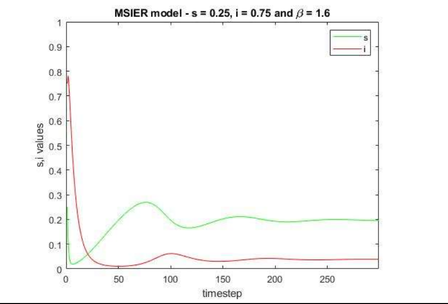

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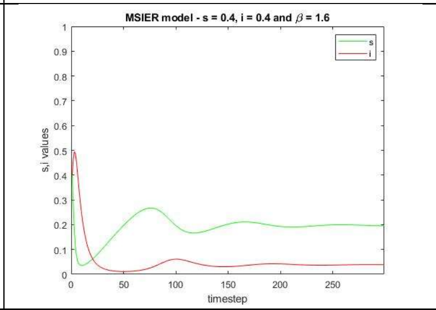


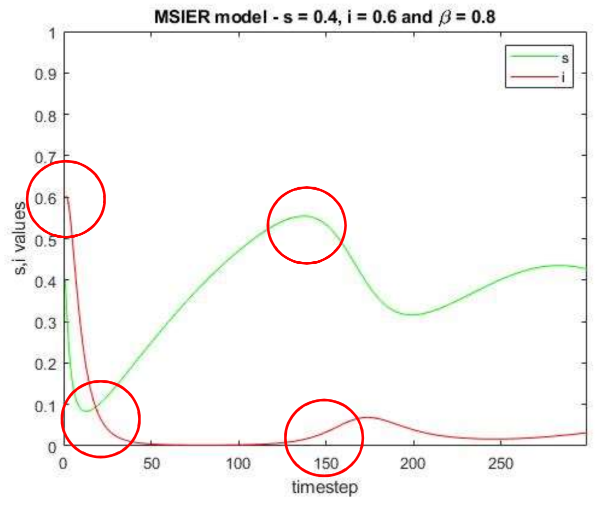
AA

Table 1a: Graphs for i(t) and s(t) at s = 0.25, 0.4, 0.4 and i = 0.75, 0.6, 0.4 respectively in each column. Each row has different value of β for three combinations of s and i which is increased from 0.8 till 1.6 with time step of 0.1.

Beginning with interaction rate (β) of 0.8 we can see from (A, B, C) the three different graphs follow a similar pattern of graph i.e. the infection (i) increases as the number of susceptible (s) increases. A slight shift towards the right with the infected is observed in the beginning which follows the similar pattern of susceptible. This shows that it takes few time steps for susceptible people to interact and fall victim.

A similar kind of graph can be seen after different iterations, i.e. just increasing the interaction rate. But the peak of the susceptible is now getting slightly shifted towards the left as well as number of infected people is also gradually increased. As we increase the interaction rate we can see the number of susceptible decreases while there is an increase in the number of infected people. Also, the potential peak amount of people to be not infected i.e. susceptible is degraded as they face more interaction with the infected people.

Four noticeable observations:



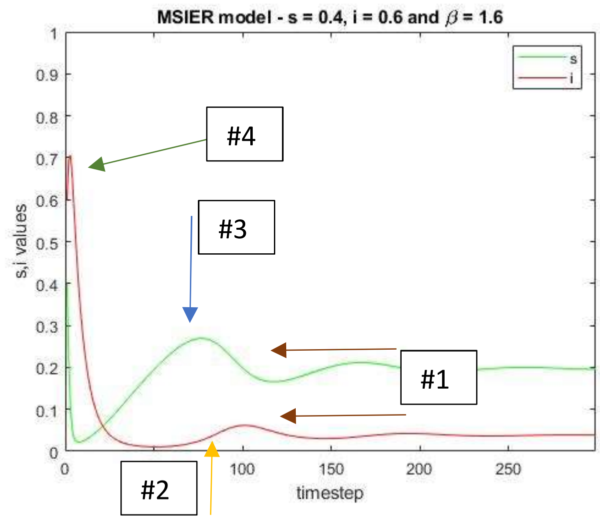


Table 1b: Noticeable important observations from graphs of s and i w.r.t. time t at beta = 0.8 and 1.6

Note #1: Change in susceptible

In the first graph we kept the interaction rate minimum and in the second graph we have kept the maximum interaction (as per the given situation). We can see that as the interaction has been increased the number of susceptible has drastically decreased. The peak number of susceptible is constrained compared to initial conditions.

Note #2: The increase in infected

As the interaction rate is increased it is expected that the amount of infected people will also increase. And interestingly the rate at which it is increased is quicker. More number of susceptible are infected in a small amount of time.

Note #3: The shift in susceptible

The susceptible graph shifts to left when interaction is increased, that means the potential peak number of susceptible in the initial low interaction case has been limited way early. It doesn't allow the susceptible number to rise.

Note #4: Sudden rise in the beginning

A very minute but an extremely important observation is the sudden rise of infected value in the initial time steps. This is especially observed from Table 1 when the rate of interaction is increased and higher number of susceptible are available in the environment.

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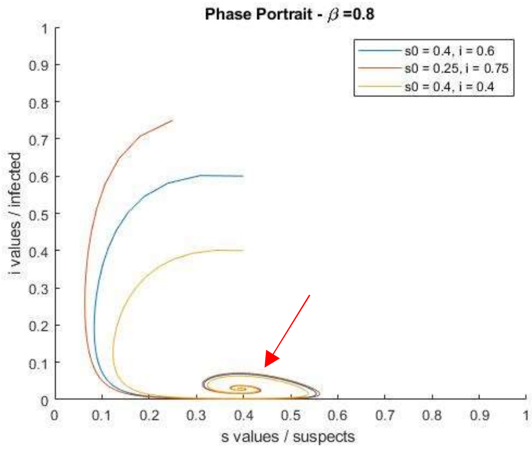
Phase Portrait:

Table 1c : Phase portrait for i vs s for beta value 0.8 to 1.6 with step of 0.1



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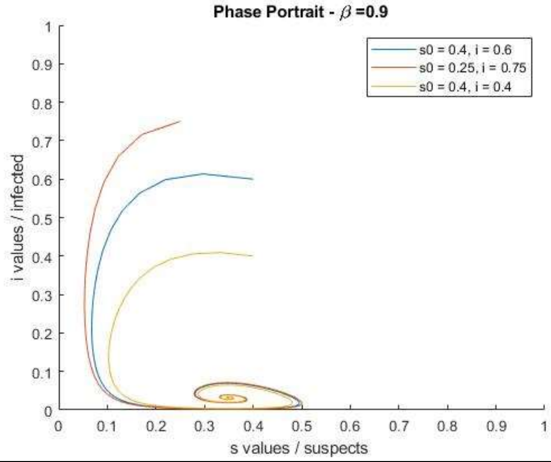

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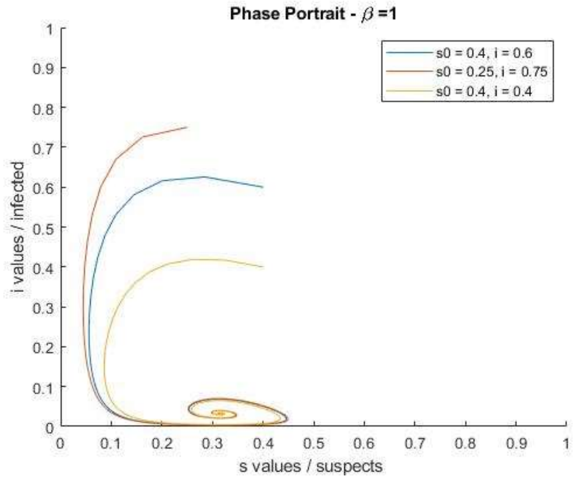
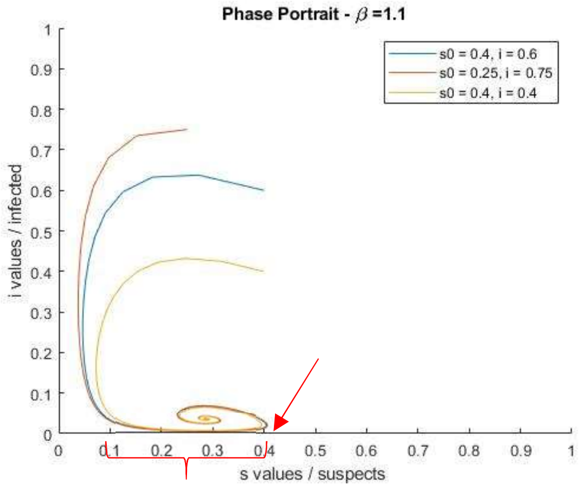


c



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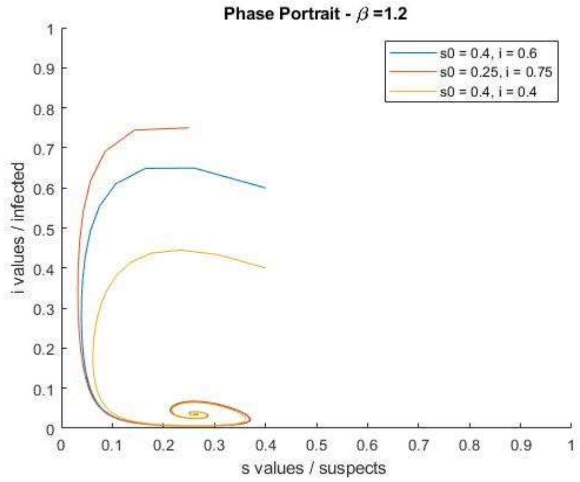

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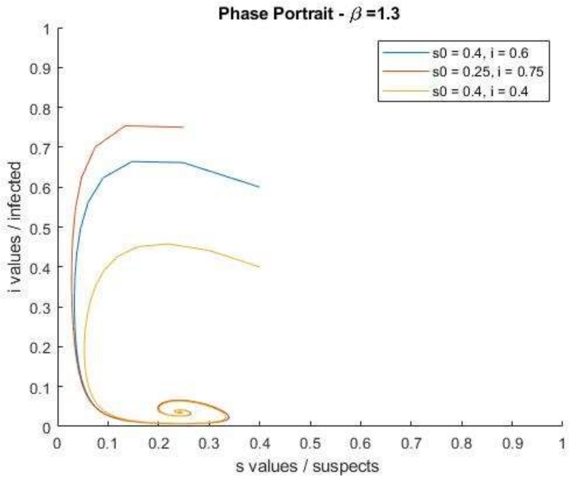

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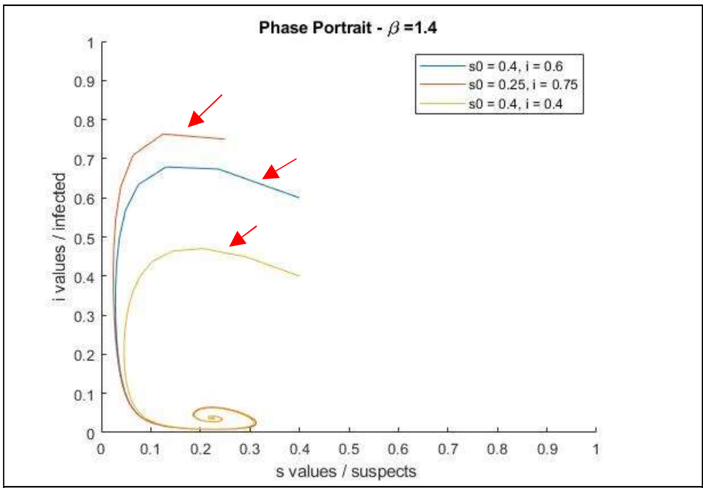

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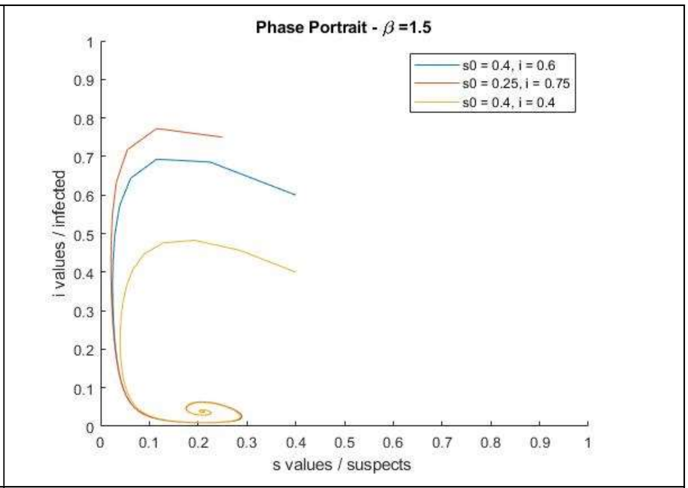

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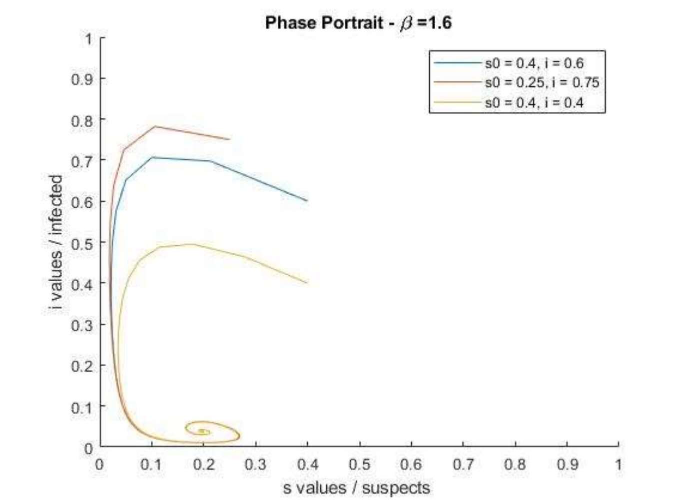

i

<u>Observations:</u>

1. The infection remains highest in the beginning having many suspects.

2. The rate of infection decreases with a decrease in rate of susceptible for all the three trajectories. This can be through cure from proper medication, early diagnosis or deaths.

3. As the number of susceptible increases later, the probability of having a contact with the infected individuals increases and which results into again rise of infection. Usually in cases when infection viruses go stronger and immune to current vaccination or less concern by few people for the viruses.

4. The observations discussed in points 1,2 and 3 is then repeated but is gradually cut-down in the amount of susceptible and infection. Since now, vaccination goes stronger and concern is made to have a check-up and diagnosis.

5. This is how a spiral shape is obtained which ends or goes in fraction at point R_0 i.e. a stable state where infection completely starts diminishing. This can be clearly seen from Figure 1c-a.

6. In figure 1c-d, as pointed a similar pattern can be recorded for all different values. The number of susceptible decreases as the interaction rate is being increased. We have discussed this phenomenon already above while discussing figure 1a. The susceptible decreases since the interaction is more.

7. The point 6 discussed above can be proved with the help of change observed in Figure 1c-g. As the interaction rate is gradually increased, gradual increase in people to get infected can be seen especially in the initial time.

Compared to Hethcote's paper

Table 1e: Comparison of output from Hethcote's paper and our calculations.

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<tr><th>Hethcote's result[1]</th><th>Our result</th></tr>
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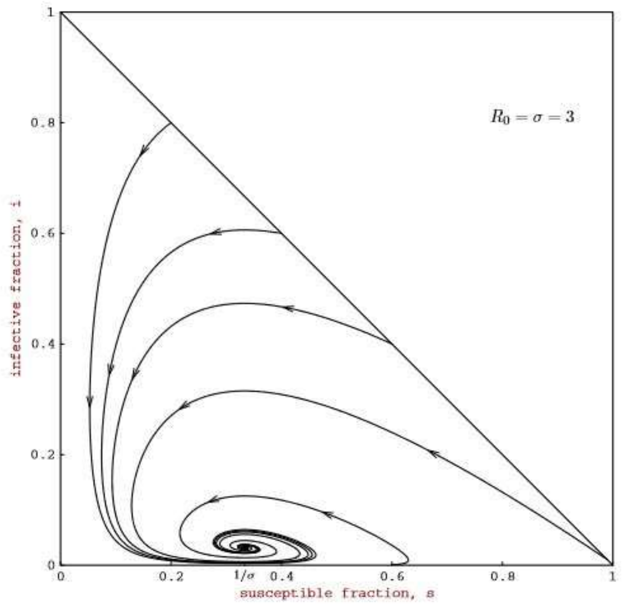
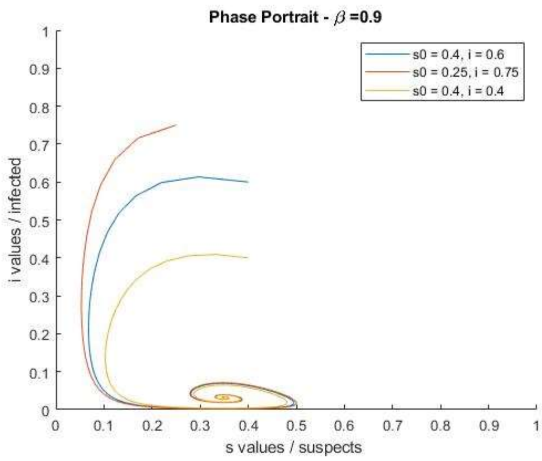


Fig. 6 *Phase plane portrait for the classic SIR endemic model with contact number $\sigma = 3$, average infectious period $1/\gamma = 3$ days, and average lifetime $1/\mu = 60$ days. This unrealistically short average lifetime has been chosen so that the endemic equilibrium is clearly above the horizontal axis and the spiraling into the endemic equilibrium can be seen.*
</td><td>


Phase portrait graph for susceptible and infected values, given interaction rate of 0.9
</td></tr>
<tr><td colspan="2">As can be clearly seen from output of the both the sides i.e. on left – Hethcote's output, and on right – our output, we get a similar output for the endemic model performed. It can be seen that irrespective of the values of different attributes, endemic tends to happen and follow a smiliar fashion of behaviour. It tends to appear spiral in shape and gradually tends to end at a point i.e. the stable state.</td></tr>
</table>