Lotka – Valterra equations – Predator vs Prey[1]

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$

Where x = number of prey,

y = number of predators,

α = exponential growth in population – used for preys,

δ = growth rate of predators,

β = death rate of preys,

γ = death rate of predators.

dx/dt = current growth in preys

dy/dt = current growth in predators

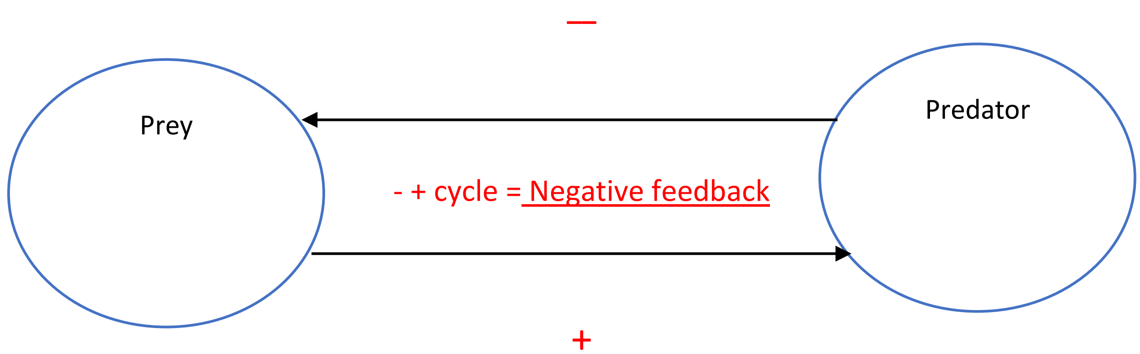


Figure 1a: Flow cycle of Predator-Prey competition.

Negative feedback:

As we can make out from the phenomenon of predator and prey, predator needs prey to survive. Therefore, more number of preys = more number of predators. But another analogy defines that more the predators = less number of preys available. Thus, this creates both positive and a negative cycle in a system. We can say that this system gives negative feedback.

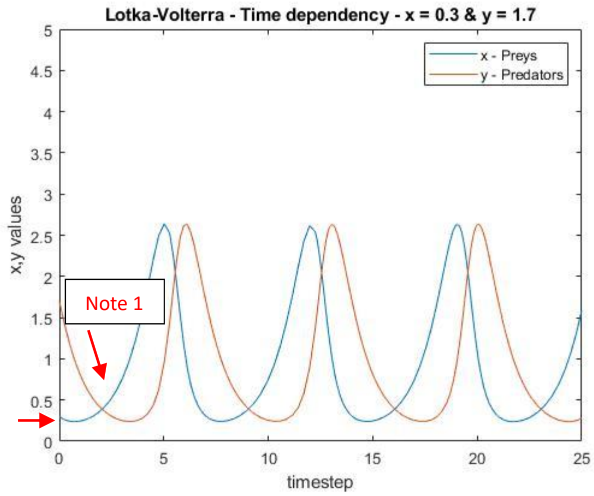
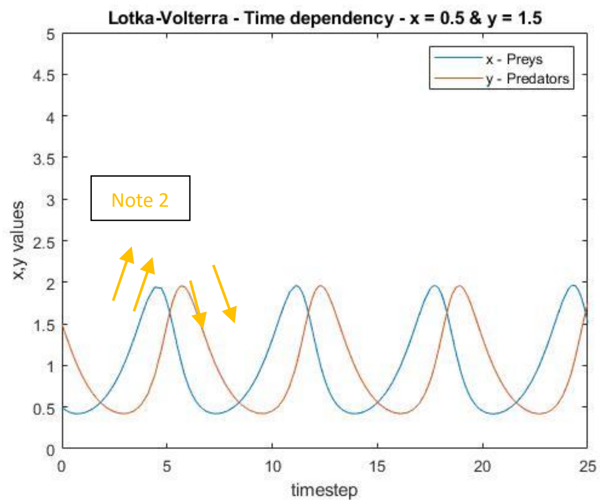
It means that the system just have one single steady stable state rather than multiple stable states. From this we can expect the system to work in *harmonic* fashion.

Differential equations can be easy solved using automatic integrating software/function. Solving the differential equations using ODE45 function in MATLAB[2].

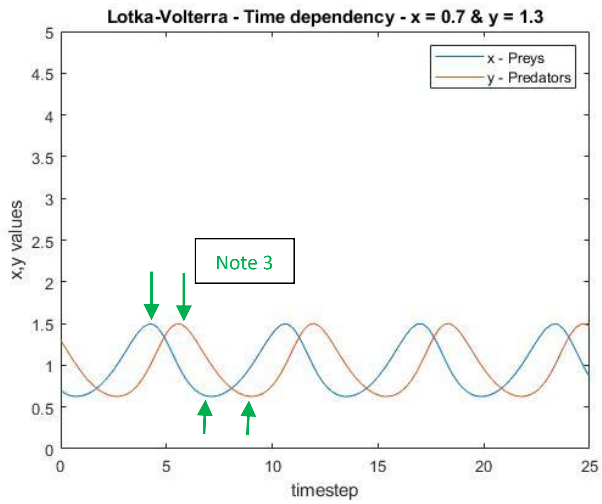
[1] Lotka–Volterra equations (https://en.wikipedia.org/wiki/Lotka–Volterra_equations)

Time dependency:

Table 1a: Time dependency of preys and predators and their observations

	#1: When the number of preys are way less than the predators, we can a sudden decline in the life of predators. - Possible verdict: Since less preys, predators go hungry and die or shift from the region. Similarly, we can see a rise in the predators as the number of preys also increase.
	#2: A similar case can be observed when x = 0.5 and y = 1.5. It can be said that the life cycle of predators and prey are directly proportional to each other. And for the matter of face, both the elements in the system can coexists on a single steady state.

[2] MATLAB – Ode45 Solver (https://www.mathworks.com/help/matlab/ref/ode45.html)

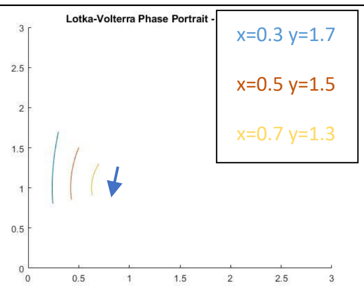


#3
Another interesting observation is the peak of the x and y values.
It can be seen that as the number of prey and predators are make close to each other, the peak and the least values changes and is limited depending on the initial values._

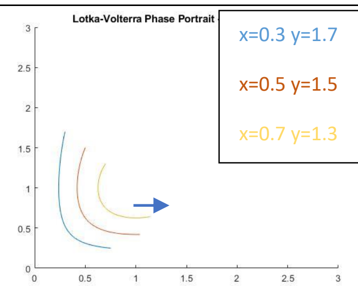
- Verdict: More the difference, higher amplitude graph is observed.
- And vice versa.

Phase portrait:

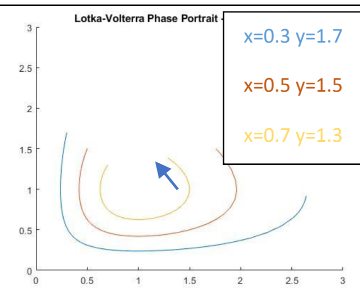
Table 1b: Phase portrait representation of x and y values



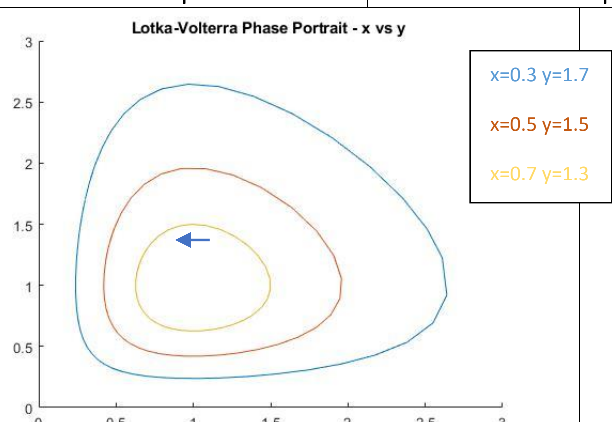
Time step = 1



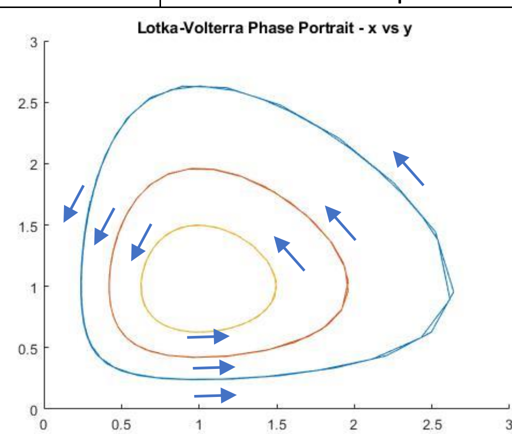
Time step = 3



Time step = 5



Time step = 7



Time step = 7+n (14 in figure)_

Observations:

- Because of the oscillations between preys and the predators (as observed in table 3a), we get a spiral but continuous loop of the competition.
- Neither of the prey / predator gets extinct and they depend on each other balancing presence.
- As can be seen in Table 3b, the graphs follows the same path continuously irrespective of the future time steps.
- Another thing that can be observed is, as any of the prey/predator starts dominating with the population they tend to lose it because of the extinction of the other. Reasons like lack of food can result in to increase mortality rate, illness and shift from the region.
- Similar for the preys, after the number of predators seems less in a region they gradually tend to dominate that place because of its resources and safety.
- This way, the Lotka-Volterra equations allow this competition between prey and predators to never end.