5 Point Estimation

Exercise 5.1
Assume that $x_1, \ldots, x_n$ are n samples generated by a normal distribution with the unknown mean m and the known variance σ^2. The normal probability density function (pdf) with mean m and variance σ^2 is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}, \forall x \in \mathbb{R}.$$

1. Calculate the maximum likelihood estimate $\hat{m}$ of the parameter m.
2. Calculate the bias and the variance of $\hat{m}$.
3. Calculate the Cramer-Rao bound for any estimator of m.
4. Is $\hat{m}$ an efficient estimator ?

Exercise 5.2
This work must be done with R.

1. Simulate $M = 1000$ sequences of $n = 4$ random samples from an exponential distribution with parameter $\mu = 3$.
2. Computes M times the maximum likelihood estimate $\hat{\mu}$ (one estimate $\hat{\mu}_i$ per sequence).
3. Plot the empirical probability density function $\hat{f}(\hat{\mu})$ of $\hat{\mu}$.
4. Study the behavior (shape, location, spreading) of $\hat{f}(\hat{\mu})$ when n is increasing from $n = 4$ to $n = 1000$. What can you conclude on the quality of $\hat{\mu}$?
5. Plot the bias of $\hat{\mu}$ as a function of n for $n = 4$ to $n = 1000$. Plot also in an other figure the empirical variance of $\hat{\mu}$, together with the Cramer-Rao bound, as a function of n.

Exercise 5.3
Assume that $k_1, \ldots, k_n$ are n samples generated by a Poisson distribution with the unknown mean λ. The Poisson probability mass function (pmf) with mean λ is

$$\Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \forall k \in \{0, 1, 2, 3, \ldots\}.$$

1. Calculate the maximum likelihood estimate $\hat{\lambda}$ of the parameter λ.
2. Calculate the bias and the variance of $\hat{\lambda}$.

Exercise 5.4
Assume that $x_1, \ldots, x_n$ are n samples generated by a normal distribution with zero mean and the unknown variance σ^2.

1. Calculate the maximum likelihood estimate $\widehat{\sigma^2}$ of the parameter σ^2.
2. Calculate the bias of $\widehat{\sigma^2}$.