Congestion Game in Teamfight Tactics

Ran Zhang, CID: 02031320
Sanyang Liu, CID: 02040950
Yaxuan Wang, CID: 02051576

December 2023

1 Introduction

Teamfight Tactics (TFT), a popular auto-battler strategy game within the League of Legends universe, has captivated the gaming community with its unique blend of tactical decision-making and strategic depth. In TFT, players assemble teams of champions ('comps'), forming a diversity of synergies (or traits), strategically position them on a grid, and engage in automated battles against opponents. The game's dynamic nature is characterised by an ever-evolving meta-game, where players constantly adapt their strategies based on available champions, items, and the current state of the game. This academic report delves into the complex dynamics of TFT by employing a novel perspective - modelling the game-play process as a congestion game. We aim to address key questions surrounding the TFT congestion game model. How do player decisions impact congestion within the game environment? What are the strategic implications of different comps, and how do they influence each other? The following sections will adapt the concept of congestion game to not only shed light on individual decision-making but also elucidate the intricate interactions between different compositions. On finding the social optimal and the equilibrium of this congestion model, we strive to contribute valuable insights to the burgeoning field of game theory applied to video games.

2 Congestion Game Modelling

2.1 Model Specification

On modelling the TFT game-play process as a congestion model, we simplify the decision-making process of all 8 players into two main aspects, choosing starting strategies and comps. Those who arrive at the endpoint with a lower cost will have a better ranking. Therefore, each player is self-routing. In this section, We shall specify the characteristics of different comps. Usually, players will set their directions to 1 final set of comps and gradually build up in later rounds. We have selected 8 representative comps in $S9$ whose statistics are shown in Table 1.
We shall divide the comps into 4 categories: scaling (Fast 9 and Noxus Darius), mid-speed (Invoker Karma and Ionia Challenger), tempo (Slayer Zed and Kayle Poppy) and burst (Strategist Azir and Bastion Aphelios). Each category has its characteristics. Scaling costs gold the most but has a high Top 4 rate once it is completed. Mid-speed is not very demanding in early strategies and has a moderate gold cost compared to scaling comps. Tempo usually requires players to speed up and dominate in early rounds by re-rolling for 3-star low-cost champions. Burst comps have a weak period before getting appropriate items but are competitive in later games. Note that different comps have different win rates and different difficulties to be fully built which may affect players' preferences and cause congestion. Comps with higher pick rate in Table 1 would often lead to more serious congestion in the game.

Comps	Avg Place	Win Rate (%)	Pick Rate (%)	Top4 Rate (%)	Cost of Comps
Fast 9	4.07	22.2	0.53	56.3	124
Noxus Darius	4.4	12.9	0.44	51.3	94
Invoker Karma	4.41	13.9	0.78	50.6	74
Ionia Challenger	4.64	9.34	0.54	47	65
Slayer Zed	4.77	11.7	0.15	45.2	70
Kayle Poppy	4.89	15.3	0.24	41.3	59
Strategist Azir	4.66	8.12	0.41	47.1	75
Bastion Aphelios	4.37	9.42	0.96	53.1	84

Table 1: Pick/win rate of different comps in S9 [1].

2.2 Congestion in Strategy

In the opening phase, players will decide their early strategies, which play a crucial role in the ultimate victory due to the mechanism of streaks. We shall simplify them into win-streak, lose-streak and no-streak, among which the first two would gain extra gold if achieved successfully. A streak is in general difficult to maintain, especially when duelling with an opponent with the same win/lose-strategy, where one of their streak must cease to an end.

2.3 Congestion in Comps

The congestion lies where all players share the same pool of champions for which the number of each champion is limited. In each round, players will be dealt with five champions and one needs to decide which of them to keep (at a cost of 1-5 golds) and which not. Three identical champions will be merged to one 2-star champion, and 3 identical 2-star champions will be merged into one 3-star, which is of much better quality respectively. Players would aim to have their champions upgraded to 3-star as the game progresses to defeat others. However, the number of each champion in a game is limited (which varies from 10 to 29). Hence it is less likely for several players to use a similar comp and aim to upgrade the same champion. In reality, players would switch to other comps to avoid being mutually deteriorated with others.

2.4 Congestion Game Formation

We first construct the congestion network as the following:

1. **Nodes:** The network has a start node $START$ and a target node END. There are 3 nodes u_i representing starting strategies (win-streak, lose-streak and no-streak) respectively, and 4 nodes v_j representing 4 representative sets of comps (scaling, mid-speed, tempo and burst). The modelled network has 9 nodes in total.

2. **Edges:** For each $i \in \{1, 2, 3\}$, there is a directed edge from the start node $START$ to the node u_i, which we denote as e_i^u. For each $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3, 4\}$, there is a directed edge from u_i to v_j denoted as e_{ij}. There is a directed edge connecting each v_j to END which is defined as e_j^v. There are 19 directed edges in total.

3. **Weights (Cost functions):** Each edge e in the network will be associated with a cost function $c_e(x)$ defined later in Section 3.

Fig. 1 is an illustration of the above congestion network. There are 8 players (or users) in the above network with the same origin $START$ and destination END. In the next section, we aim to define the cost functions $c_e(x)$ for each edge, with domain $\{0, 1, ..., 8\}$ and range $[0, 100]$.

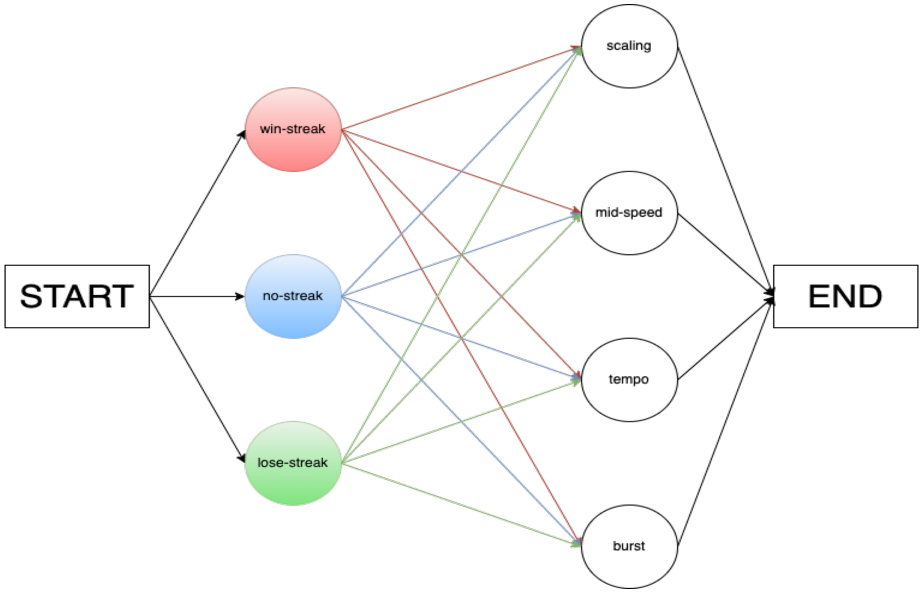

Figure 1: Illustration of the modelled congestion network of TFT.

3 Cost Functions

3.1 Costs of Starting Strategies

We assign the costs of different strategies according to our game experience. The cost in this stage refers to the necessary extra expense required (gold, item, health) to maintain the streak. The cost functions are shown in Fig. 2, where $c_1^u(x)$ represents the cost function of edge e_1^u from the starting point to win-streak and analogously for the other 2 edges.

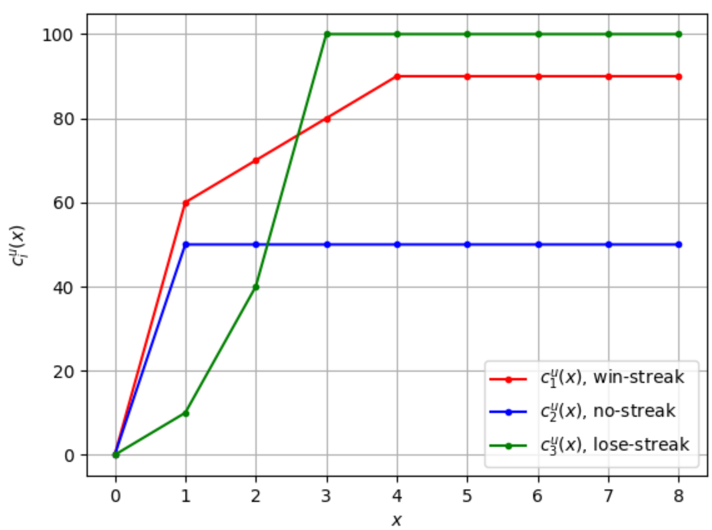

Figure 2: Cost functions $c_i^u(x)$ from the starting point to streak strategies.

3.2 Costs of Comps

We define the cost of a set of comps from a given strategy as $c_{ij} = 100 - s$, where s is the degree of feasibility between the strategies and comps. The value of s is statistically determined by the following factors: avg health, avg gold and final ranks of different comps [1] under each strategy. Notice that there is no congestion in this stage, as each cost function is a constant. We essentially assume that players cannot switch to other comps once determined, while in reality players can (though often disadvantaged) gradually switch to other comps halfway through the game. The table of values of s is shown as in Table 2:

	scaling	mid-speed	tempo	burst
win-streak	90	55	75	30
no-streak	40	80	70	20
lose-streak	0	85	10	100

Table 2: Scores s on the feasibility of transferring to final comps from initial strategies.

3.3 Costs to Endgame

The cost functions of the last stage are mainly determined by two factors: the difficulty to fully build up the desired comps, and the Top 4 rate once this comp is fully built up. Fig. 3 illustrates the cost functions from comps to endgame. For instance, $c_1^v(x)$ is the cost function of edge e_1^v from scaling comps to the end node.

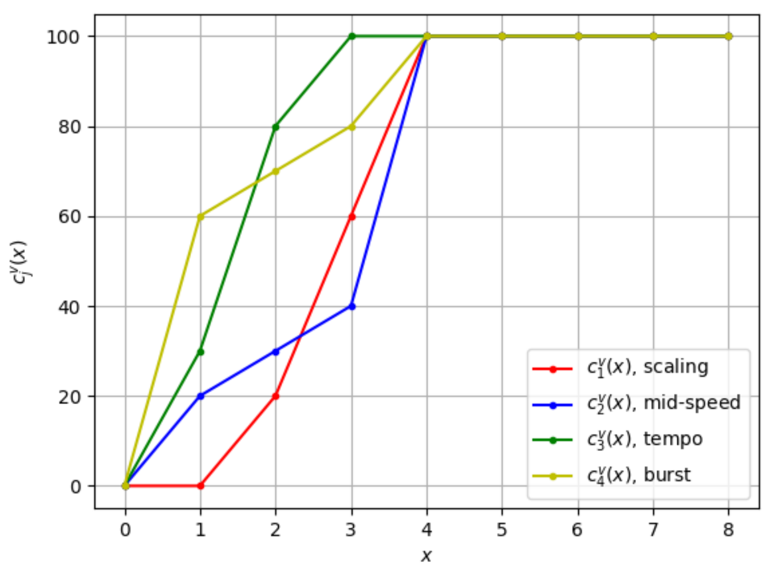

Figure 3: Cost functions $c_j^v(x)$ from comps to endpoint.

The cost $c_j^v(x)$ when $x = 1$ is determined by the Top 4 rate of comps (shown in Table 1). Comps j with higher Top 4 rate will have lower $c_j^v(1)$. The level of congestion is correlated to the difficulty of fully building a set of comps. In Table 3, we calculated the approximated probability of getting desired champions in one re-roll (re-draw five champions at a cost of 2 gold). The figures are obtained based on a TFT mechanism that distinct champions have different probabilities to be drawn, which also varies with the player's level as the game progresses. For example, for the **Fast 9** comp, players would normally re-roll at level 9, with the need of 3 "Bel'Veth"s and 'Ahri's each at a probability of 0.1.

Fast 9 (9)		**Noxus Darius (7)**		**Invoker Karma (8)**	
Bel'Veth (3)	0.1	Darius (9)	0.15	Karma (9)	0.15
Ahri (3)	0.1	Katarina (9)	0.15	Shen (3)	0.1
Slayer Zed (6)		**Kayle Poppy (4)**		**Bastion Aphelios (8)**	
Zed (9)	0.18	Kayle (9)	0.24	Aphelios (3)	0.1
Katarina (9)	0.12	Poppy (9)	0.24	Sejuani (3)	0.1
Ionia Challenger (8)		**Strategist Azir (8)**			
Kai'sa (3)	0.09	Azir (3)	0.075		
Shen (3)	0.09	Nasus (3)	0.075		
Yasuo (3)	0.09	Jarvan IV (3)	0.075		
		Lux (3)	0.075		

Table 3: Probability and number of champions needed for comps.

We estimate the number of re-rolls needed by random walk simulation. For example, to estimate the number of re-rolls in **Fast 9**, we set the initial state of the system at $(0, 0)$. In each iteration, the system has a probability of $p_0 = 0.8$ to stay, $p_1 = 0.1$ to move rightward by one unit and $p_2 = 0.1$ to move upward by one unit. We calculate the expected number of iterations that the system reaches the region $[3, +\infty) \times [3, +\infty)$ by 10^6 simulations. Detailed simulation process can be found in [2]. Intuitively, if it takes a larger number of re-rolls to complete one certain comp, the player would be severely disadvantaged and stand less chance to win. In practice, the probability of one card appearing in one re-roll is proportional to the champions available in the pool, which decreases significantly if there is a congestion. For example, for scaling comps, the probability of rolling a desired champion is already low (as there are only 10 'Ahri's in total). If another player is joining this route, it has a steeper increase in cost than the mid-speed ones, as shown in Fig. 3. For all 4 types of comps, it is generally very hard to compete for the Top 4 once more than 3 players are choosing the same set of comps.

4 Solutions

In this section, we will take a deeper look at the above congestion game model and solve it systematically by finding the social optimum and equilibrium.

4.1 Social Optimum

Suppose there are x_i players choosing the edge e_i^u for each i and y_{ij} players choosing the edge e_{ij}. To find the social optimum of the game, it is equivalent to solve the following linear constrained minimisation problem on the average cost per user:

$$
\begin{aligned}
\min_{x_i, y_{ij}} \quad & \frac{1}{8}\left(\sum_{i=1}^{3} x_i\, c_i^u(x_i) + \sum_{i=1}^{3}\sum_{j=1}^{4} y_{ij}\, c_{ij} + \sum_{j=1}^{4} z_j\, c_j^v(z_j)\right) \\
\text{s.t.} \quad & \sum_{i=1}^{3} x_i = 8, \\
& \sum_{j=1}^{4} y_{ij} = x_i,\ \forall i \in \{1,2,3\} \\
& \sum_{i=1}^{3} y_{ij} = z_j,\ \forall j \in \{1,2,3,4\} \\
& x_i \in \mathbb{N}_0,\ y_{ij} \in \mathbb{N}_0
\end{aligned}
$$

We solve the problem by complete induction on x_i and y_{ij} which has at most 75582 cases.
On solving the minimisation problem, the social optimum is $x_1 = 2$, $x_2 = 4$, $x_3 = 2$, $y_{11} = 2$, $y_{22} = 3$, $y_{23} = 1$, $y_{34} = 2$, with the optimal value 107.5. This means that the social optimum lies where there are 2 players choosing win-streak strategy with scaling comps; 4 players choosing no-streak strategy among which 3 of them plays mid-speed comps and 1 of them plays tempo; 2 players choosing lose-streak strategy with burst comps. The average cost per player is 107.5. The costs for the win-streak players are 100, and the costs for all other players are 110.

4.2 Equilibrium

To find the equilibrium of the game, it is equivalent to finding the minimum of the global flow function. We define the problem as the following:

$$
\begin{aligned}
\min_{x_i, y_{ij}} \quad & \sum_{i=1}^{3}\sum_{k=1}^{x_i} c_i^u(k) + \sum_{i=1}^{3}\sum_{j=1}^{4} y_{ij}\, c_{ij} + \sum_{j=1}^{4}\sum_{l=1}^{z_j} c_j^v(l) \\
\text{s.t.} \quad & \sum_{i=1}^{3} x_i = 8, \\
& \sum_{j=1}^{4} y_{ij} = x_i,\ \forall i \in \{1,2,3\} \\
& \sum_{i=1}^{3} y_{ij} = z_j,\ \forall j \in \{1,2,3,4\} \\
& x_i \in \mathbb{N}_0,\ y_{ij} \in \mathbb{N}_0
\end{aligned}
$$

Notice that all cost functions are piecewise linear, so the objective function is a linear combination of piecewise linear functions. This problem can be reformulated into several integer linear programming problems. We solve it again by complete induction and obtain the exactly same unique solution as the social optimal in Section 4.1. The algorithm used to obtain both results can be found in the Git-hub group repository [2]. Notice that the Price of Anarchy, $POA = \frac{107.5}{107.5} = 1$, since the unique equilibrium aligns with the social optimum. This means that the congestion model is perfectly ideal and 'efficient', even if players deceptively pursue their self-interest.

5 Conclusion and Further Refinement

In unravelling the intricate dynamics of Teamfight Tactics through the lens of a congestion game model, the key finding of our analysis is that the social optimum aligns with the unique equilibrium of the modelled congestion game. This solution coincides with the actual TFT gaming environment, where the main-stream strategies and compositions are employed in a manner closely aligned with the distribution outlined in our solution.

Nevertheless, further improvements can still be made to our model. In reality, players exhibit dynamic behaviours, often transitioning between different compositions and employing diverse strategies. Our model, which assumes a static adherence to a single strategy, may benefit from enhancements that capture the fluidity of players' decision-making. Incorporating the ability for players to switch between compositions and considering measures for the gradual buildup of desired compositions (through which further potential congestion might occur) could refine the model's accuracy and applicability.

The cost functions are defined experientially in the current model, which could be further improved by gathering the statistics of the winning rate under the given distribution of strategies employed. The richness of the TFT game-play experience lies in the vast array of available compositions, and several other mechanisms, introducing a layer of complexity beyond the scope of our current model. Future endeavours could involve refining the model to accommodate this complexity by increasing the volume of the database of compositions, and adopting other techniques on scoring comps such as the Analytic Hierarchy Process (AHP), which would provide a more detailed and realistic representation of the TFT strategic landscape.

References

[1] *Meta Trends LoLCHESS.GG.* `https://lolchess.gg/decks?sort=top&patch=1314&revision=0&dt=0` 2023.8.2.

[2] GameTheory-CW. `https://github.com/iivvyy-w/GameTheory-CW/tree/main/CW2` 2023.