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Modeling and model predictive control of a nonlinear hydraulic system

Petr Chalupa*, Jakub Novák

Faculty of Applied Informatics, Tomas Bata University in Zlin, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic

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ABSTRACT

This paper deals with modeling and control of a hydraulic three-tank system. A process of creating a computer model in the MATLAB/Simulink environment is described, and optimal PID (proportional–integral–derivative) and model predictive controllers are proposed. The modeling starts with the creation of an initial mathematical model based on a first-principles approach. Then, the initial model is enhanced to obtain better correspondence with a real-time system, and parameters of the modified system are identified from measurements. The real-time system contains nonlinearities which cannot be neglected, and therefore are identified and included in the final mathematical model. The resulting model is used for a control design. As the real-time system has long time constants, usage of the Simulink model dramatically speeds up the design process. Optimal PID and model predictive controllers (MPC) are proposed and compared. Model predictive controllers for both a linearized model and the nonlinear system are proposed. The techniques described are not limited to one particular modeling problem, but can be used as an illustrative example for the modeling of many technological processes.

1. Introduction

Most current control algorithms are based on a model of a controlled plant [1]. A plant model can also be used to investigate the properties and behavior of the modeled plant without the risk of damage or violating the technological constraints of the real plant. Two basic branches of modeling are used in practice: the black box approach and first-principles modeling.

The black box approach to the modeling [2,3] is based on the analysis of input and output signals of the plant, and knowledge of the physical principles of the modeled plant is not required. On the other hand, a model obtained by the black box approach is generally valid only for the signals that it was calculated from.

The first-principles modeling provides a general model which is in the optimal case valid for a whole range of plant inputs and states. The model is created by analyzing the modeled plant and combining physical laws [4]. On the other hand, there are usually a lot of unknown constants and relations when performing an analysis of a plant. Thus, first-principles modeling is especially suitable for simple controlled plants with a small number of parameters and for obtaining basic information about a controlled plant (range of gain, rank of suitable sample time, etc.).

A combination of the two methods is used in this paper. Basic relations between plant inputs and outputs are derived using mathematical physical analysis, and the model obtained is further improved on the basis of measurements. The relations obtained are used to design a Simulink environment with characteristics as close as possible to those of the real-time system "DTS 200 Three-Tank-System" [5]. The major reason for creating the model of this laboratory equipment is

* Corresponding author. Tel.: +420 57603 5204.
E-mail address: chalupa@fai.utb.cz (P. Chalupa).

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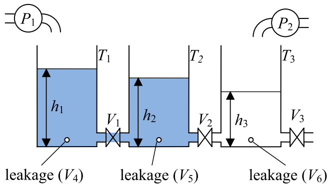

Fig. 1. Scheme of the three-tank system Amira DTS200.

Fig. 2. Amira DTS200 photo.

the big time constants of the plant — and thus time-consuming experiments. The model can dramatically decrease the time needed for controller development because only promising control strategies are applied to the real plant and verified.

The paper is organized as follows. Section 2 presents the modeled system — Amira DTS200. The derivation of an initial ideal model using first-principles modeling and the enhancement of this model on the basis of real-time experiments is carried out in Section 3. Section 4 presents the resulting Simulink model in detail and Section 5 deals with the PID and MPC control of the plant. The results are summarized in Sections 6 and 7.

2. The DTS200 system

The Amira DTS200 system consists of three interconnected cylindrical tanks, two pumps, six valves, pipes, a water reservoir at the bottom, measurement of liquid levels and other elements. The pumps pump water from the bottom reservoir to the top of the left and right tanks. Valve positions are controlled and measured by using electrical signals, which allow precise positioning.

A simplified scheme of the system is shown in Fig. 1. The pump P_1 controls the inflow to tank T_1 while the pump P_2 controls the liquid inflow to tank T_3. There is no pump connected to the middle tank T_2. The characteristic of the flow between tank T_1 and tank T_2 can be affected by valve V_1, the flow between tanks T_2 and T_3 can be affected by the valve V_2 and the outflow of the tank T_3 can be affected by valve V_3. The system also provides the capability of simulating leakage from individual tanks by opening the valves V_4, V_5 and V_6. A photo of the Amira DR300 system is presented in Fig. 2

The pumps are controlled by analogue signals in the range from −10 to 10 V. The heights of the water level are measured using pressure sensors. Each valve is operated by two digital signals which control the motor of a particular valve. The first signal gives the order to start the closing of the valve while the second signal is used for the opening of the valve. If none of the signals is activated, the valve remains in its current position. Each valve also provides three output signals: an analogue voltage signal corresponding to the current position of the valve and two informative logical signals which state that the valve is fully opened or fully closed.

The overall number of inputs to the modeled plant DTS200 is 14:

- Two analogue signals controlling the pumps.
- Twelve digital signals (two for each of the six valves) for the opening/closing of the valves.

The plant provides 21 measurable outputs which can be used as a control feedback or for measurements of plant characteristics:

- Three analogue signals representing level heights in the three tanks.
- Six analogue signals representing the positions of the valves.
- Twelve logical signals (two for each of the six valves) stating that the corresponding valve is fully opened/closed.

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3. Modeling of the plant

This section is focused on the derivation of a mathematical model of the three-tank system and its adaptation to the DTS200 plant using real-time experiments.

3.1. The ideal model

This derivation of an ideal plant model is based on ideal properties of individual components. The ideal flow of a liquid through a pipe can be derived from Bernoulli and continuity equations for ideal liquid.

Since the flow through a valve depends only on the level difference, the valve position and constants representing pipes and cylindrical tanks, the whole mathematical model can be written as follows:

$$\begin{aligned}
\frac{dh_1}{dt} &= q_1 - k_1\sqrt{|h_1 - h_2|}\cdot \mathrm{sign}\,(h_1 - h_2) - k_4\sqrt{h_1} \\
\frac{dh_2}{dt} &= k_1\sqrt{|h_1 - h_2|}\cdot \mathrm{sign}\,(h_1 - h_2) + k_2\sqrt{|h_3 - h_2|}\cdot \mathrm{sign}\,(h_3 - h_2) - k_5\sqrt{h_2} \\
\frac{dh_3}{dt} &= q_2 - k_2\sqrt{|h_3 - h_2|}\cdot \mathrm{sign}\,(h_3 - h_2) - k_3\sqrt{h_3} - k_6\sqrt{h_3}
\end{aligned} \tag{1}$$

where symbols h_1, h_2 and h_3 represent the water level heights in tanks T_1, T_2 and T_3 respectively, k is a parameter representing the valve position and q represents the inflow as a change of water level over time:

$$k_i = v_i \frac{S_{V\max}\sqrt{2g}}{S_T} \quad i = 1, 2, \ldots, 6 \qquad q_i = \frac{q_i'}{S_T} \quad i = 1, 2. \tag{2}$$

The cross-sectional areas of all three tanks are the same and are symbolized by S_T.

This ideal model is successfully used in many control system studies as a demonstration example [6,7]. Although the ideal model is based on simple equations, an analytical solution of the outputs for a given course of inputs is complicated. The problem lies in the nonlinearity of Eqs. (1). Also the computation of a steady state for given constant inputs is a very complicated task and leads to the solution of a higher order polynomial equation.

3.2. The enhanced model

This section describes the enhancement of the initial model and measurement of characteristics of individual parts of the DTS200 system.

Characteristics of the pumps:

The characteristics of the pumps were measured to refine (1). The amount of water pumped within a certain time was measured for different settings of the driving signals u_1 and u_2. The characteristics of the two pumps are similar and close to linear but do not start at a lower bound ($u = -1$ MU), but at least the value of approximately $u = -0.85$ must be applied to the pump to obtain a non-zero output. The maximal pumping of approximately 6 mm/s of tank level rise represents 5.37 l/min. The dynamics of the pumps are very fast comparing to other time constants present in the system and therefore were neglected.

Characteristics of the valves:

As stated in Section 2, each of the plant's six valves is driven by two dedicated logical signals. These signals are used for starting the valve's motor in a closing or opening direction. If no signal is activated, the valve remains in its current position. Each valve provides three output signals. The current valve position is determined by an analogue signal. Higher values of the signal represent a closed valve and lower values represent an opened valve. The other two signals are logical and state that a valve is opened or closed. Valve characteristics are studied in more detail in [8].

Valve flow parameters for valves:

Valve flow parameters k_i, as appeared in (1), were computed from measurements of draining through individual valves which are connected to outflow pipes (V_3, V_4, V_5 and V_6). The draining of a tank to the reservoir situated below the tanks is described by a differential equation:

$$\frac{dh\,(t)}{dt} = -k\sqrt{h\,(t) + h_0} \quad h\,(t) = \frac{k^2}{4}\cdot t^2 - k\sqrt{h\,(0) + h_0}\cdot t + h\,(0) \tag{3}$$

where $h(0)$ is the initial water level and h_0 is the vertical length of the outflow pipe. Due to the mechanical configuration of the plant, the value of h_0 for outflow valves V_3, V_4, V_5, and V_6 cannot be measured directly. But it can be identified from the draining course. The valve was partially closed to different positions at the beginning of the draining experiment and a relation between the valve position and the value of k was achieved.

A similar approach to obtaining values of k can also be used for valves V_1 and V_2 which interconnect tanks T_1 and T_2, and T_2 and T_3, respectively.

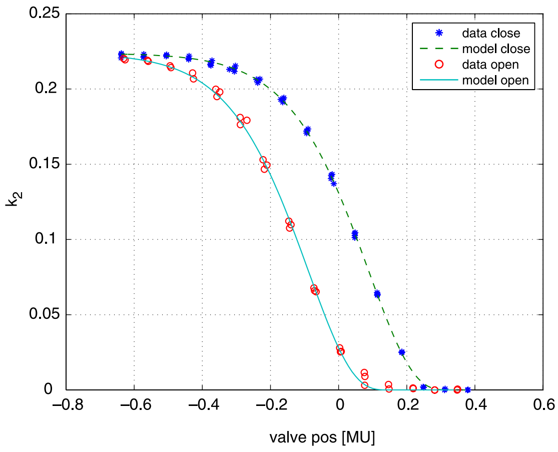

Fig. 3. Model of parameter k for valve V_2.

Valve hysteresis:

Experiments unveiled a hysteresis present in all valves. The value of the valve position itself does not give sufficient information about the current value of parameter k_2. As can be seen from Fig. 3, if the position is 0 MU the value of k_2 can be anywhere in the range 0.03–0.13. In the particular case of using the valve as an actuator, the hysteresis should be taken into account. Otherwise the control process can easily become unstable.

Modeling of valve characteristics:

The course of the relation between the valve position in MU and k is similar to the step responses of the dynamical system and therefore it was modeled in a similar way. A model based on the transfer of a fourth-order aperiodic system produced satisfactory results. Thus the relation between the position and the k border curve was as follows:

$$\begin{aligned} & pos < pos_0 : k = k_{\max}\left[1 - \frac{1}{6} e^{\frac{-b}{a}} \frac{b^3 + 3b^2a + 6ba^2 + 6a^3}{a^3}\right] \\ & b = pos_0 - pos \\ & pos \geq pos_0 : k = 0 \end{aligned} \tag{4}$$

where pos is a valve position in MU, and parameters a and pos_0 were obtained by a nonlinear regression. The regression for valve V_2 is presented in Fig. 3.

Even the enhanced model is based on the ideal liquid assumption. Many more precise models of the flow have been proposed; see e.g. [9,10].

4. The Simulink model

All the models of individual parts of the DTS200 plant were incorporated into a single block in the MATLAB/Simulink environment. The block has the same inputs and outputs as the real plant. Thus it contains all 14 inputs and 21 outputs described in Section 2. The Simulink block of the resulting model of the DTS200 plant is depicted in Fig. 4.

The model is designed as a masked subsystem where only the necessary initial states are to be entered by a user. The initial states are the initial water levels in individual tanks and the initial valve positions and corresponding values of the valve parameters k.

Masked subsystems and subsystems are used also to model individual parts of the plant. For example, the internal structure of a valve state subsystem is presented in Fig. 5. This hierarchical structure is useful for maintaining lucidity.

A detailed description of the Simulink model can be found in [11].

5. The PID and model predictive control

The control experiment was carried out to verify the Simulink model and to find out how the controllers cope with nonlinearities in the plant.

The three-tank system control has been studied by a number of authors but in most cases pumps have been used as actuators [12–15].

The experiment configuration considered in this paper was as follows: There was a constant inflow to tank T_1 produced by pump P_1. The valve V_1 between tanks T_1 and T_2 was opened to a constant position. Valves V_4 (leakage from tank T_1)

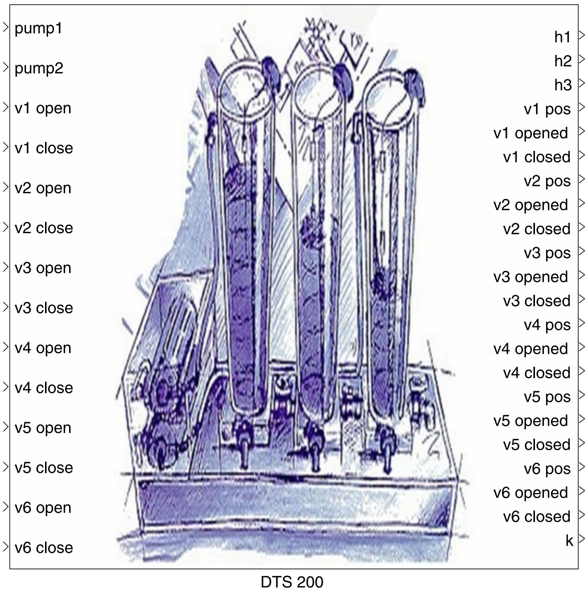

Fig. 4. Block of the Simulink model of DTS200.

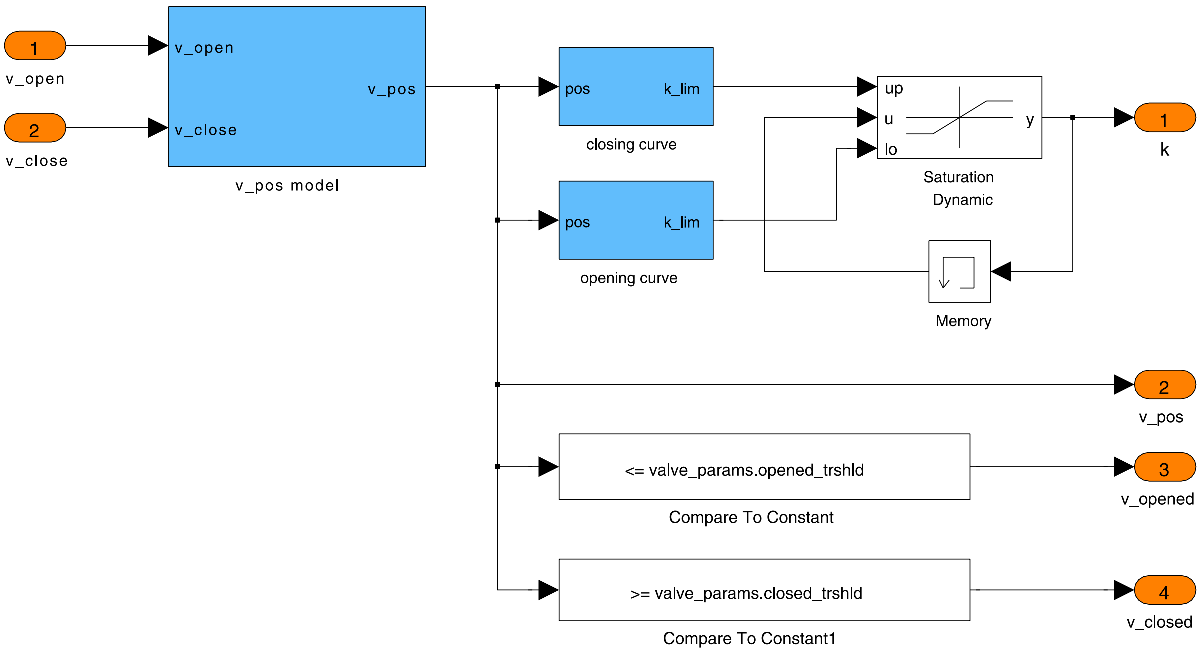

Fig. 5. Internal structure of the valve state subsystem.

and V_2 (interconnection of tanks T_2 and T_3) were closed. The goal of the control was to drive the water level in tank T_2 using valve V_5 as an actuator. It should be noted that the characteristic of valve V_5 is nonlinear and contains a large hysteresis.

The static characteristic of the system is presented in Fig. 6. It can be seen that the closed or a slightly opened valve position leads to saturation of the water level at the maximum. A security circuit is activated when the water level in the tank T_1 reaches the level of approximately 600 mm and stops the pump P_1 to prevent overflowing of the tank. When the water level decreases, the pumping is re-enabled. On the other hand, the valve being opened to more than 50% leads to emptying of the tank. Only a range of approximately 25%–50% leads to the desired steady states. But a large hysteresis can be observed in this range and the envelope is very steep.

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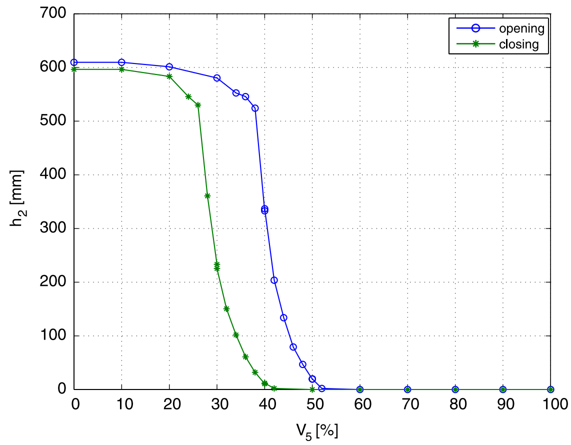

Fig. 6. Static characteristics of the controlled system.

The plant was in a steady state at the beginning of all experiments. The valve V_5 was partly opened and the initial water levels were as follows:

$$\begin{aligned} h_1(0) &= 235 \text{ mm} \\ y(0) = h_2(0) &= 151 \text{ mm} \\ u(0) &= 35\%. \end{aligned} \tag{5}$$

The valve position was controlled by a simple internal controller connected to the valve drive. This internal control loop is driven by the required valve position signal. This control signal is a percentage, where 0% corresponds to a fully closed valve (i.e., a step change of the $v5$ *closed* signal) and 100% represents a fully opened valve (i.e., a step change of the $v5$ *opened* signal). A sample time of $T_s = 0.1$ s was used for measurements, but all the controllers used a sample period of $T_c = 5$ s.

The quadratic criterion was used to compare individual courses from the control error point of view:

$$J_e = \sum_{i=1}^{N} e^2(i) = \sum_{i=1}^{N} [w(i) - y(i)]^2 . \tag{6}$$

Usually control signal differences are penalized or evaluated. But in the case of DTS200 the crucial problem lies in starting and stopping the valve motor, because often starting of the valve decreases its durability. The displacement of the valve is not as important as the fact that the valve motor had to be started. Valve starts were measured using the following criterion:

$$J_{u,starts} = \sum_{i=1}^{N-1} [v5\_open(i+1) - v5\_open(i) > 0] + \sum_{i=1}^{N-1} [v5\_close(i+1) - v5\_close(i) > 0] . \tag{7}$$

It is also important to measure the total time for which the valve motor was running. Thus a criterion was proposed to measure the time for which signal $v5$ *open* or $v5$ *close* was enabled:

$$J_{u,running} = T_0 \sum_{i=1}^{N} [v5\_open(i) > 0] + T_0 \sum_{i=1}^{N} [v5\_close(i) > 0] \tag{8}$$

where T_0 is the sample time for the valve signals.

5.1. The PID control

First of all the plant was controlled by a classical PID controller. Controller parameters were tuned to minimize criterion (6). A MATLAB function *fminsearch* was used for this task. This function uses the Nelder–Mead Simplex Method to find the criterion minimum [16]. Resulting courses are depicted in Fig. 7.

It can be seen that even though the valve position (control signal) changes smoothly, the output is not so smooth. This corresponds to overcoming of valve's hysteresis. Even more sophisticated methods of PID tuning have been proposed by different authors; see for example [17].

5.2. The model predictive control using the linear model

A linear model of the system was identified by applying a random signal to its input, and parameters of the linearized model were used in the MPC controller. The Simulink control scheme is presented in Fig. 8.

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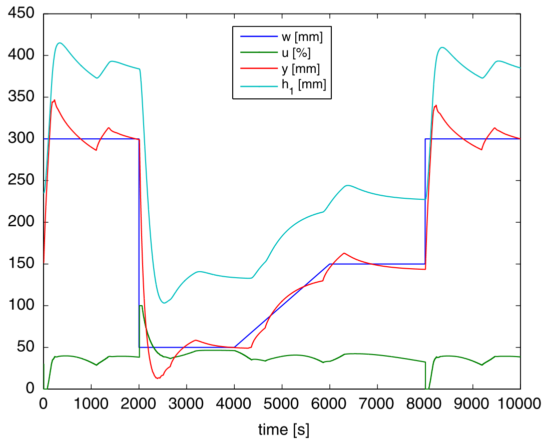

Fig. 7. Optimal PID control of DTS200.

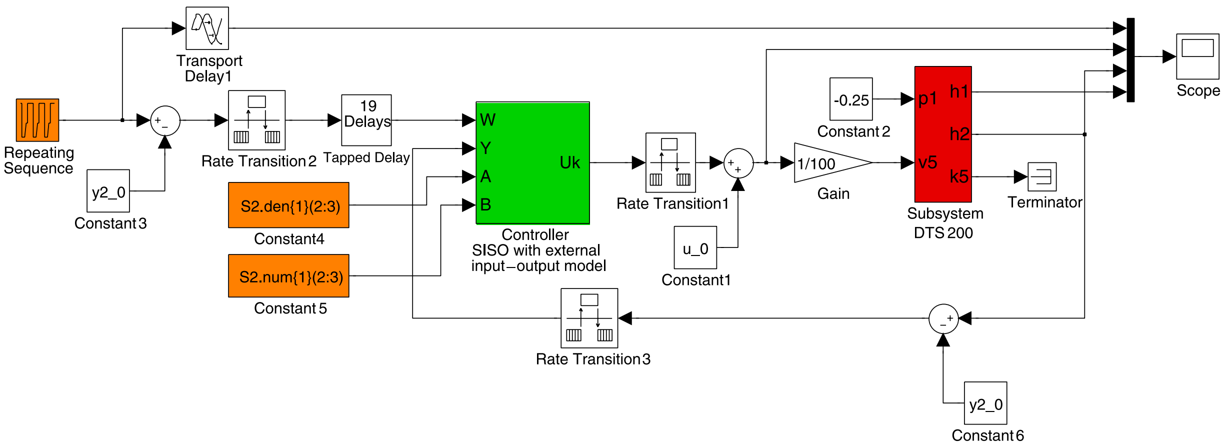

Fig. 8. MPC scheme.

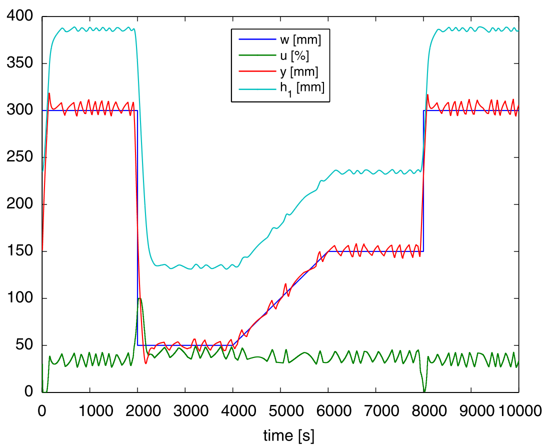

Fig. 9. Control courses using MPC.

A controller from the STuMPCoL library [18] was used to perform the control task. A quadratic criterion with both control and a prediction horizon equal to 20 samples was used to compute the control signals, and a receding horizon strategy was used [19]. The resulting control courses are presented in Fig. 9.

It was observed that the MPC copes better with crossing of the hysteresis, but on the other hand the control signal was oscillating around steady states.

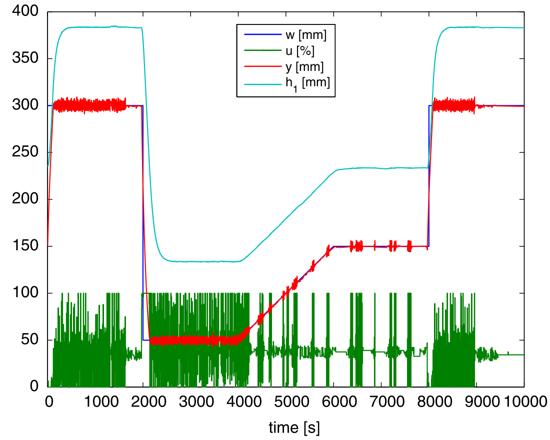

Fig. 10. Nonlinear MPC using pattern search with the sample time 5 s.

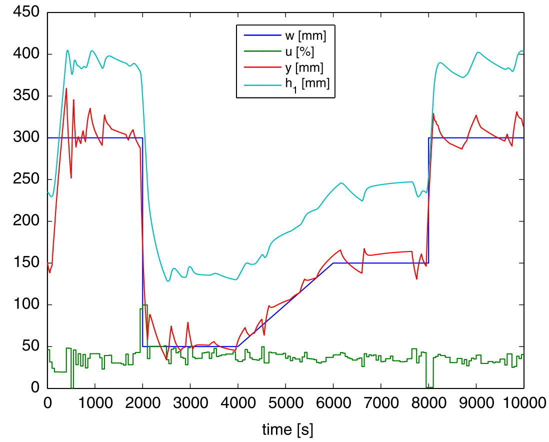

Fig. 11. Nonlinear MPC using pattern search with the sample time 50 s.

5.3. The nonlinear model predictive control

Besides the control approaches based on a linear model of the system, which were presented in previous subsections, a nonlinear predictive control was also applied to the system. The controller was based on solving a nonlinear optimization problem using the Global Optimization Toolbox of MATLAB [20]. The nonlinear predictive controllers presented in this section had the same input and output properties as the MPC presented in Section 5.2: 20 samples of the future reference signal (w) and the current output value of the system (y) were provided to the controller and the aim of the controller was to find the optimal course of the control sequence (20 samples) for minimizing criteria (6)–(8). A great advantage of the nonlinear MPC is the possibility of choosing an arbitrary optimization criterion, including nonlinear ones like (7) and (8). On the other hand, the computational expense is far higher than in the case of MPC using a quadratic or a linear criterion and a linear model of the controlled system, and reaching the global optimum is not guaranteed.

The current state of the nonlinear model of the DTS200 plant was provided to the controller and the controller could apply different control sequences to find the optimal one. This optimization process is highly nonlinear, and derivative-free methods from the Global Optimization Toolbox were used to cope with it.

The receding horizon principle was applied to the control sequence. Just the first sample of the control sequence was applied to the system, and the optimization was repeated in the next sample time step.

The first controller was designed to minimize just the control error criterion (6) without penalizing the course of the control signal. The controller worked with the small sample time of 5 s, and the pattern search method [21] was used to find the optimal control course. Control results are presented in Fig. 10.

The tracking performance of the controller is quite good, but as the control signal is not penalized the control valve position is often changing from totally closed to the totally opened position.

The second nonlinear model predictive controller also used the pattern search method, but control signal criteria (6) and (8) were taken into account and the sample time was increased 10 times to 50 s. The resulting control courses are presented in Fig. 11.

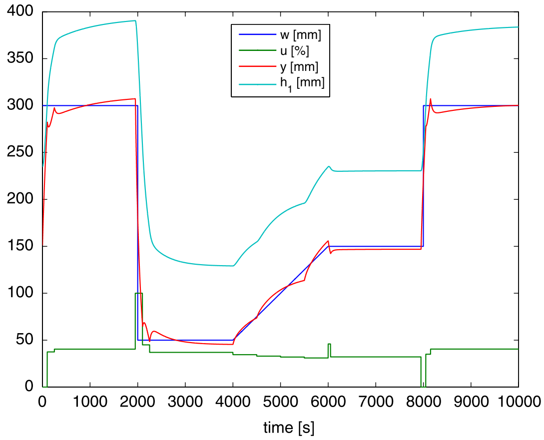

Fig. 12. Nonlinear MPC using global search with the sample time 50 s.

Table 1
Control criteria.

Controller	J_e (mm^2)	$J_{u\ start}$ [1]	$J_{u\ running}$ (s)
PID (Fig. 7)	617.8	3118	1572.1
MPC linear model (Fig. 9)	209.7	5396	2718.2
Nonlinear MPC1 (Fig. 10)	208.1	4037	5087.6
Nonlinear MPC2 (Fig. 11)	619.1	986	554.8
Nonlinear MPC3 (Fig. 12)	202.4	13	25.8

The tracking performance of the controller is not as good as in the previous case, but the control signal is much smoother and the valve does not change its position so often.

The third presented setting of the controller is based on similar settings to the second nonlinear MPC, but it uses the GlobalSearch method [22] and far more iterations were allowed in each sample time. The resulting control sequence was steady for long time periods while preserving good tracking performance. The results are presented in Fig. 12.

6. Comparison of control courses

The values of criteria defined by Eqs. (6)–(8) are summed up in Table 1. The MPC was more accurate, but it also has slightly higher actuator demands.

7. Conclusion

The paper presents a Simulink model of a hydraulic system. The Amira DTS200 three-tank system was considered, but the techniques used could be easily generalized to a wide set of hydraulic systems. Despite the simplicity of the ideal model, the real-time system contains several nonlinearities which incorporate complexity into the system. The hysteresis of the valves plays an especially big role in the plant behavior.

PID and MPC controllers were designed to control the system and verified the usability of the model for the controller design. The best control performance was reached using the nonlinear MPC with the global search optimization method. Further improvement of control courses can be achieved by fine-tuning of the nonlinear controller and incorporating other global optimization methods.

Acknowledgment

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