Development of An Eclipse Mapping Routine Using Python for Analysis of *Kepler* Data

NATHAN OLIVER SMITH

Supervisor: Dr. Richard Olenick

A thesis submitted in fulfilment of
the requirements for the degree of
Bachelor of Science

The Department of Physics
Constantin College
The University of Dallas
Irving Texas USA

14 August 2019

CHAPTER 1

Introduction: Algol Binaries and Accretion Disk Formation

1.1 Binary Characteristics

Of the vast number of systems in the cosmos, roughly half contain multiple stars. Including Triple and Quadruple star systems. The most common, however, is the binary star system, which involves two stars orbiting around a common center of mass known as the barycenter. Within this class of star system, the characteristics can vary greatly. For example, the period of the orbit can range from a few minutes, to a few years. In turn, this disparity dramatically impacts orbital dynamics and stellar evolution, creating a variety of binary sub-classes and beautiful stellar fireworks.

1.1.1 Formation

Theoretically, Binaries can form in two ways. The first of which occurs when two stars pass closely together and fall into each others gravitational pull, forming an orbit. While this is not impossible, it is unlikely due to the precise circumstances that need to exist in order for this type of formation to occur. For example, recall that a bound system has a negative total energy. In contrast, the total energy of an unbound system is positive. Gravitational capture is only possible if the system's total energy can shift from positive to negative. Hence, there must be some form of energy loss. Energy loss could have a range of sources, such as internal energy loss -as a result of tidal distortion- or loss due to a third body. For tidal distortion to occur, the stars must pass at precise distance; an extremely close approach can result in merging, while a distant approach will has no chance for the correct energy loss. These circumstances are very rare, and could only occur in the cores of globular clusters and other dense star fields.

Due to the scarcity of circumstance and plethora of observation, gravitational capture cannot be the primary formation process.

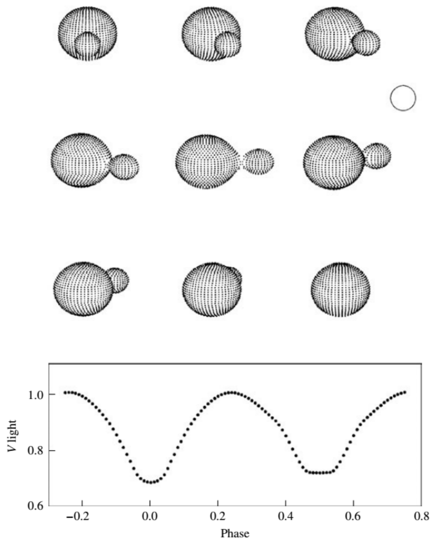
The second, and more likely case, occurs during proto-star formation within molecular clouds. These 'stellar nurseries' are massive clouds of gas that can spontaneously collapse in the right conditions, forming the cores of soon to be main sequence stars. Statistically, gas clouds of this size can have a range of densities at different points. Knowing this fact, consider the basic case for the minimum radius of gravitational collapse -Jeans Length- neglecting magnetic fields, rotation, and turbulence:

$$R_J \propto (T/\rho)^{1/2} \tag{1.1}$$

For the sake of argument, let the temperature " T " be fairly constant throughout the cloud. Given the likely variance of density " ρ " relative to volume, this stellar nursery could certainly satisfy Eq. 1.1 in various spaces and collapse into multiple points at once. This is known as cloud fragmentation. These cloud fragments must also collapse quickly to avoid re-merging. Furthermore, the cloud must be spherically asymmetric and not concentrated about the center. Cloud fragmentation likely occurs much more often than gravitational capture. Furthermore, many Binaries contain pre-main-sequence stars which suggests that binary system and proto-star formation occur at relatively the same time.

1.1.2 Algol Type Eclipsing Binaries

A special subclass of these systems is the Algol Type Eclipsing Binary. Eclipsing means that the stars in the binary system are orbiting such that one star eclipses the other, blocking out a portion or all of its partner's light. Consider Figure 1.1 on the next page. Notice the orbital procession by which the smaller primary star and larger companion eclipse each other. This corresponds to the brightness measurement over time graph in the bottom section of the figure. The dips in the data correlate with the full eclipses of the binary. Furthermore, the term 'Algol Type' refers to two main-sequence stars which are semidetached. Notice in Figure 1.1 how the stars form a tear drop shape as they orbit. A system is 'semidetached' when only the larger



0.41

FIGURE 1.1: Eclipsing Binary [Carroll and Ostlie 2017]

companion star has this shape. This phenomena is due to the gravitational tidal forces from the system's close orbit. The specific shape of the tear drop is governed by the Roche Lobe Surface Equipotiental which is crucial to understanding the nature of these binaries and will be discussed in the next section.

1.1.3 Roche Lobe Equipotiental and Dwarf Novae

The Roche Lobe Equipotiental Surface is incredibly important when understanding Semidetached Eclipsing Binaries because it defines the effective gravitational potential at any point in the system. In Figure 1.2, the lines in the diagram represent where the gravitational potential is equal to some value. In particular, notice the point the 'L points'. These are known as *Lagrange Points*, which define special points in space where the effective gravitational potential is zero. If one was to place an object at one of these points, the object would not fall because the gravity of each star,

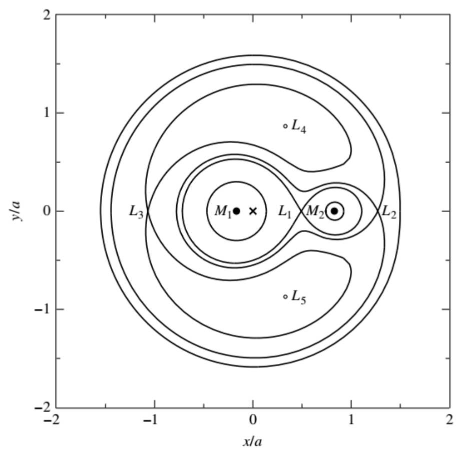


FIGURE 1.2: Roche Lobe [Carroll and Ostlie 2017]

combined with the centrifugal force of the orbit, act
equally. The Roche Lobe Equipotiental is defined as

$$\Phi = -G\left(\frac{M_1}{S_1} + \frac{M_2}{S_2}\right) - \frac{1}{2}\left[\frac{G(M_1+M_2)r}{a^3}\right]^2 \tag{1.2}$$

Where S_1 and S_2 are an objects distance from the binary stars with masses M_1 and M_2. *a* is the semi-major axis, *G* is the gravitational constant, and *r* is the distance of the object from the center of mass of the binary. The characteristics of the first Lagrange point, denoted 'L1' in Figure 1.2, gives matter the ability to flow, transferring mass between stars. The distance from the first Lagrange point to the secondary star satisfies the equation

$$\frac{M_1}{(a-r)^2} = \frac{M_2}{r^2} + \frac{M_1}{a^2} - \frac{r(M_1+M_2)}{a^3} \tag{1.3}$$

Where *r* is the distance from the secondary star to the first Lagrange point. Notice the tear drop shape around the larger star, 'M1'. This is known as the star's 'Roche Lobe' which defines the points where the effective gravitational potential is zero. In other words, where $\Phi = 0$. At certain stages of stellar evolution, this star will expand rapidly, filling its Roche Lobe. In consequence, the outer atmosphere of the star will begin to 'fall off', flowing through the L1 point towards the smaller star, 'M2', and spiral in the same direction of the orbit to form an accretion disk. This process is known as *Mass Transfer*. When a white dwarf acts as the primary star in a system of this type, a *dwarf nova* can occur. Dwarf novae belong to a broader class of binaries called *cataclysmic variables*, which are noted for their periods of outburst which increase their brightness by a factor of 10. These white dwarfs have an average mass of 0.86 $M_\odot$, compared to 0.58 $M_\odot$ for their isolated brethren, and the less massive secondary star is usually a G spectral type.

1.1.4 Mass Transfer and Accretion Disk Characteristics

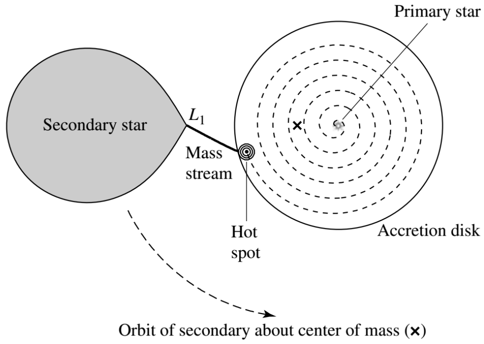


FIGURE 1.3: Mass Transfer Diagram [Carroll and Ostlie 2017]

Luminosity in the accretion disk due to mass transfer is the defining characteristic of dwarf novae. Essentially, mass transfer is the process by which matter becomes gravitationally unbound to a body and becomes bound to a more compact body such as a white dwarf. Due to the rotation of the system, as mass accretes around the white dwarf, a thin disk of hot gas forms within the plane of the orbit. In dwarf novae, this process takes on some surprising qualities. The first dwarf nova was observed in 1885, but it wasn't until 1974 that their precise characteristics were discovered, when astrophysicist Brian Warner argued that the observed periodic outbursts were due to the luminosity of the accretion disk, as opposed to the fusion of accreted hydrogen in classical novae. Modern theory, known as the Tidal Disruption Model, (cite) suggests that dwarf novae outbursts are due to instability in the accretion disk, where the gas reaches a critical temperature and effects the total viscosity, which is the internal friction that converts the directed orbital motion of the gas into random thermal motion. This, in turn, causes a greater influx of mass from the secondary star, compounding on the brightness and illuminating all of the gas orbiting the white dwarf. At certain stages of disk formation, enough orbital motion can be converted into heat to cause the gas to spiral down into the primary white dwarf star (see Figure 1.3). Little is known about the physical mechanism that causes accretion disk outbursts because the typical model of weak particle interactions does not account for the amount of heat generation. Other possibilities, such as turbulence from convection, and random gas motions can provide better models of dwarf novae outburst.

Like a star, a thin accretion disk can be treated as a black-body, emitting a continuous spectra at each radial distance. This provides useful information about the theoretical temperature profile and general structure. Because the disk's mass is substantially less then the mass of the primary or secondary star, the disk is only influenced by the primary white dwarf. Hence, the total kinetic energy can be estimated as:

$$E = -G\frac{M_{primary}M_{disk}}{2r} \tag{1.4}$$

Where *r* is the distance from the white dwarf. As stated before, the directed orbital motion of the disk is being converted into heat loss due to internal friction, causing the gas to fall to the white dwarf surface. If we assume the distance is *steady state*, where it is not changing with time, and that the mass transfer rate $\dot{M}$ is constant, we can use conservation of energy to say that the amount of energy radiated is equal to the amount of gas that moves in and out of an imaginary ring around the disk in some time interval:

$$dE = \frac{dE}{dr}dr = \frac{d}{dr}(-G\frac{M_{primary}M_{disk}}{2r})dr = G\frac{M_{primary}\dot{M}t}{2r^2}dr \tag{1.5}$$

Moreover, because luminosity times time is related to energy:

$$dL_{ring}t = dE = G\frac{M_{primary}\dot{M}t}{2r^2}dr \tag{1.6}$$

Using the Stephan-Boltzmann law we can also say that

$$dL_{ring} = 4r\pi\sigma T^4 dr = G\frac{M_{primary}\dot{M}t}{2r^2}dr \tag{1.7}$$

Solving for temperature, we can find the disk temperature at radius *r*.

$$T(r) = \left(\frac{GM_{primary}\dot{M}}{8\pi\sigma R^3}\right)^{\frac{1}{4}}\left(\frac{R}{r}\right)^{\frac{3}{4}} \tag{1.8}$$

For the purposes of accretion disk imaging, Equation 1.6 can be simplified with the form

$$T \propto \left(\frac{1}{r}\right)^{\frac{3}{4}} \tag{1.9}$$

Eq. 1.9 will be used to compare the temperature profiles of generated disk images. Comparing and contrasting the temperature profile of the reconstructed image gives valuable information about abnormal structures in the disk. On the other hand, consistent residuals could also reveal errors in the code and give insight to future improvements.

1.2 Statement of the Problem and the Usefulness of Eclipse Mapping

The goal of this particular method of accretion disk imaging is to gain insight into the structures and nature of accretion disks by taking advantage of the geometric orientation of dwarf novae to our planet. This method provides an alternative to Spectral Tomography which requires spectral data that is less accessible to many who want to make progress in this field. The method discussed in this paper, Eclipse Mapping, merely requires photometric data, which is easily accessible via Kepler's MAST database. It also requires some a priori knowledge of the system's orbital mechanics which can also be determined through light curve analysis. Furthermore, with the use of python's versatile set of libraries, cloud computing, and big data applications such as Spark, this method can be scaled, automated, and sped up simultaneously to generate results for a vast variety of dwarf novae. These improvements also allow for faster testing of different parameters to generate optimal results. The problem of eclipse mapping can be expressed by asking 3 questions: What is the nature of eclipse mapping? What are the results of optimal and sub-optimal parameters? Finally, how can these processes be scaled and optimized with modern data applications and open source development? (will probably add more to this section but am moving on for now)