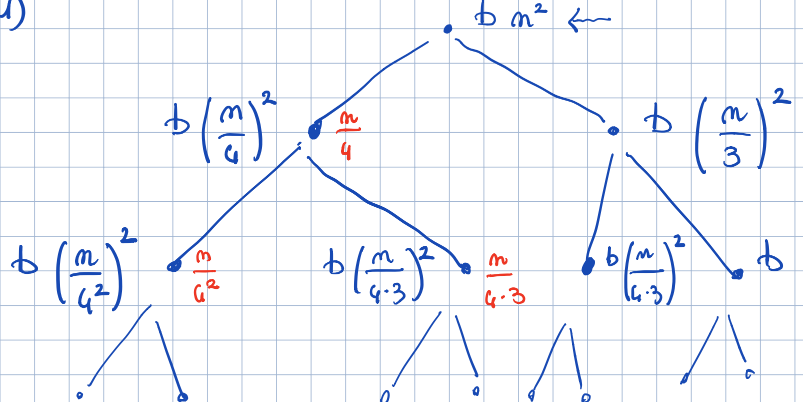
1) $T(n) = \begin{cases} \cancel{\Theta(1)}\ a & n = 1 \\ T\left(\frac{n}{4}\right) + T\left(\frac{n}{3}\right) + \cancel{\Theta(n^2)}\ b \cdot n^2 & n > 1 \end{cases}$

$T(n) = \Omega(n^2)$

2) $T(n) = \begin{cases} \cancel{\Theta(1)}\ a & n = 1 \\ 2T\left(\frac{n}{2}\right) + \cancel{\Theta(n)}\ b \cdot n & n > 1 \end{cases}$

1)



$\rightsquigarrow\ b \cdot n^2$

$b\, n^2\left(\left(\frac{1}{4^2}\right) + \left(\frac{1}{3^2}\right)\right)$

$\rightsquigarrow\ *$

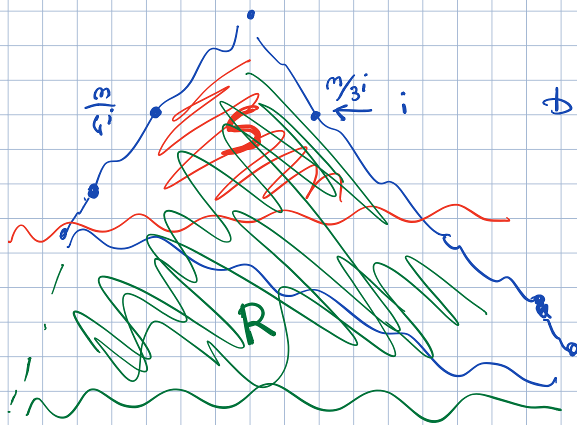
$*\ b\, n^2\left(\left(\frac{1}{4^2}\right)^2 + 2\,\frac{1}{4^2\cdot 3^2} + \left(\frac{1}{3^2}\right)^2\right)$

$= b\, n^2\left(\frac{1}{4^2} + \frac{1}{3^2}\right)^2$

$\vdots$

i-esimo livello

$b\, n^2\left(\frac{1}{4^2} + \frac{1}{3^2}\right)^i$



$b m^2 \left(\frac{1}{4^2}+\frac{1}{3^2}\right)^i$

$\frac{m}{3^y}=1 \quad y=\log_3 m$

$\frac{m}{4^x}=1 \quad x=\log_4 m$

$S \leq T(m) \leq R$

R

$$T(m) \leq \overbrace{\sum_{i=0}^{\log_3 m -1} b m^2 \left(\frac{1}{4^2}+\frac{1}{3^2}\right)^i + a\cdot 2^{\log_3 m}}^{R}$$

foglie

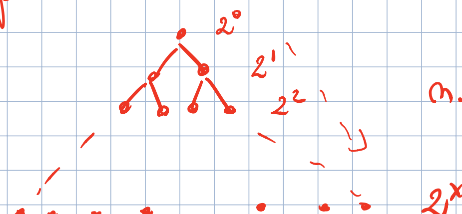
$$\boxed{x^{\log_y z} = z^{\log_y x}}$$

$$T(m) \leq b m^2 \underbrace{\sum_{\dots}^{\dots} \underbrace{\left(\frac{1}{4^2}+\frac{1}{3^2}\right)}_{<1}^i}_{d \text{ costante}} + a\cdot m^{\log_3 2}$$

$$\leq b'\cdot m^2 + a\cdot m^{\log_3 2} = O(m^2) \rightsquigarrow \Theta(m^2)$$

Usando S $\quad T(m) \geq \dots = \Omega(m^2)$

Albero binario completo



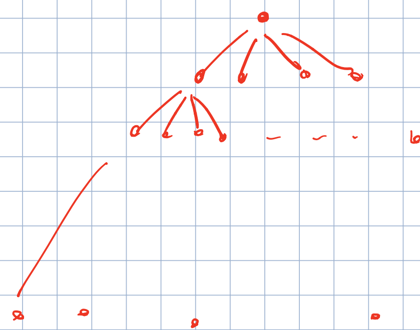
$m.i = \sum_{i=0}^{x-1} 2^i = 2^x - 1$

x altezza dell'albero

= lunghezza del più lungo cammino radice - foglia

$x \rightsquigarrow$

Albero k-ario completo



$k^0 = 1$

k^1

k^2

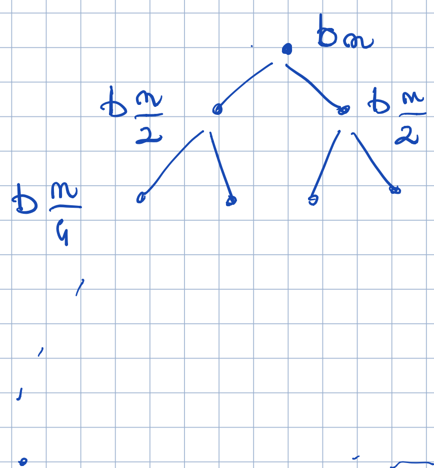
k^i

nodi interni

$$\sum_{i=0}^{x-1} k^i = \frac{k^x - 1}{k - 1}$$

k^x foglie

n. n.



$\rightsquigarrow bn$

$\rightsquigarrow 2b\frac{n}{2} = b\cdot n$

$\rightsquigarrow 2^2 b\frac{n}{2^2} = bn$

$\vdots$

$\cancel{2^i}\, b\frac{n}{\cancel{2^i}} = bn$

$\frac{n}{2^x} = 1 \quad \rightsquigarrow 2^x \cdot a = 2^{\log_2 n}\cdot a$

$\frac{n}{2^x} = 1 \qquad x = \log_2 n$

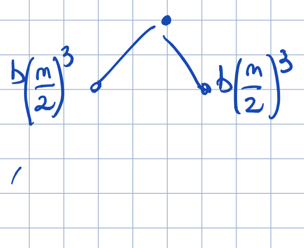
$bn + bn + \dots + bn$

$$T(n) = \sum_{i=0}^{\log_2 n - 1} b\cdot n + a\cdot \cancel{2^{\log_2 n}}^{\,n}$$

$$= bn\cdot\log_2 n + a\cdot n = \Theta(n\log n)$$

Caso base $n = 2 \qquad \frac{n}{2^x} = 2 \qquad x = \log_2 n - 1$

$$T(n) = \begin{cases} \cancel{\Theta(1)}^{\,a} & n=1 \\ 2\,T\left(\frac{n}{2}\right) + \cancel{\Theta(n^3)}^{\,bn^3} & n>1 \end{cases}$$



$$b\,n^3$$

$$2\,b\left(\frac{n}{2}\right)^3$$

$$\vdots$$

$$2^i\,b\left(\frac{n}{2^i}\right)^3$$

$$\cdots \quad a\cdot 2^{\log_2 n}$$

$$a^x\cdot b^x = (a\cdot b)^x$$

$$\frac{n}{2^x}=1 \qquad x=\log_2 n$$

$$2^i\left(\frac{1}{2^i}\right)^3 = 2^i\cdot\left(\frac{1}{2^3}\right)^i = \left(\frac{2}{2^3}\right)^i$$

$$T(n) = \sum_{i=0}^{\log_2 n-1} 2^i\,b\left(\frac{n}{2^i}\right)^3 + a\cdot \cancel{2^{\log_2 n}}^{\,n}$$

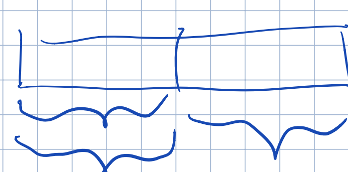
$$= b\,n^3 \sum_{i=0}^{\log_2 n-1}\left(\frac{2}{2^3}\right)^i + a\cdot n$$

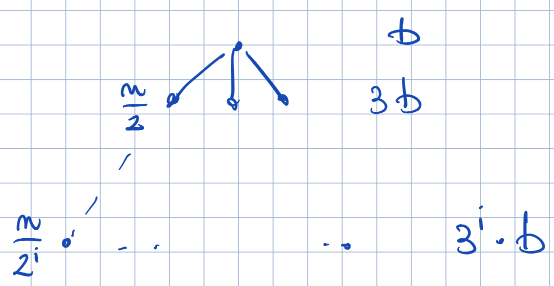
$$= b'\,n^3 + a\cdot n = \Theta(n^3)$$

$$T(n) = \begin{cases} \cancel{\Theta(1)}^{\,a} & n=1 \\ 3\,T\left(\frac{n}{2}\right) + \cancel{\Theta(1)}^{\,b} & n>1 \end{cases}$$

$$\xi$$

$$2$$





$\frac{n}{2^x} = 1$ $\qquad 3^x \cdot a = 3^{\log_2 n} \cdot a$

$$T(n) = \sum_{i=0}^{\log_2 n - 1} 3^i b + 3^{\log_2 n} \cdot a$$

$$= b \frac{3^{\log_2 n} - 1}{3 - 1} + a \cdot n^{\log_2 3}$$

$$= b' \cdot n^{\log_2 3} - \ldots + a \cdot n^{\log_2 3} = \Theta\left(n^{\log_2 3}\right)$$

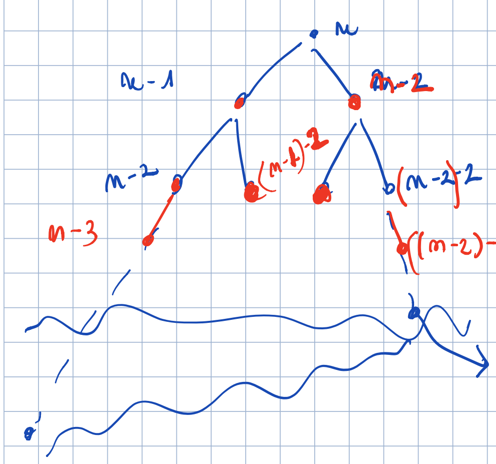
$$T(n) = \begin{cases} \\ a T\left(\frac{n}{b}\right) + \Theta(n^k) \end{cases}$$

Esempio Fibonacci

$$F(n) = \begin{cases} 0 & n=0 \\ 1 & n=1 \\ F(n-1)+F(n-2) & n>1 \end{cases}$$

```
Fibonacci (n) {
        if (n=0) return 0
        if (n=1) return 1
        return F(n-1) + F(n-2)
}
```

$$T(n) = \begin{cases} \overset{a}{\cancel{\Theta}}(1) & n \leq 1 \\ T(n-1) + T(n-2) + \overset{b}{\cancel{\Theta}}(1) & n > 1 \end{cases}$$



b

$2b$

$2^i b$

$n - 2x \leq 1 \Rightarrow \frac{n}{2} \approx x$

$$T(n) \geq \sum_{i=0}^{n/2-1} b 2^i = b \frac{2^{n/2}-1}{2-1} = \Omega(2^{n/2})$$

```
Fibonacci (n) {
    ~~Fib~~ ← ~~new-array (n+1)~~
x   ~~Fib~~[0] ← 0
y   ~~Fib~~[1] ← 1
    for (i ← 2 to n) {
      z  ~~Fib~~[i] ← Fib[i-1] + Fib[i-2]
    }    x ← y
         y ← z
    return Fib[n]
}
```

$T(n) = \Theta(n)$ ~~Non~~ InPlace

InPlace

n. n.

Sostituzione di Variabile

$$T(n) = \begin{cases} \Theta(1) & n \leq 2 \\ 2T(\sqrt{n}) + \Theta(\log n) & n > 2 \end{cases}$$

$b \cdot \log n$

$n \overset{def}{=} 2^m$ nuova variabile $\Rightarrow$ $m = \log_2 n$

$$T(2^m) = 2T(2^{\frac{m}{2}}) + b \cdot \log 2^m$$

$T(2^m) = S(m)$ nuova equazione

$$S(m) = 2\,S\left(\frac{m}{2}\right) + b\,\overset{m}{\cancel{\log 2^m}}$$

$$S(m) = \Theta(m \log m)$$

$$T(2^m) = \Theta(m \log m)$$

$$T(n) = \Theta(\log n \cdot \log\log n) \quad \Leftarrow$$