$$T(n) = \begin{cases} \cancel{\Theta(1)}\ a & n = 1 \\ T\left(\frac{n}{3}\right) + \cancel{\Theta(\log n)}\ b \cdot \log n & n > 1 \end{cases}$$

$$\begin{array}{lll} 0 & n & b \cdot \log n \\ 1 & \frac{n}{3} & b \cdot \log\left(\frac{n}{3}\right) \\ 2 & \frac{n}{3^2} & b \cdot \log\left(\frac{n}{3^2}\right) \\ \vdots & \vdots & \vdots \\ i & \frac{n}{3^i} & b \cdot \log\left(\frac{n}{3^i}\right) \\ \vdots & \vdots & \\ x & \frac{n}{3^x} = 1 & a \end{array}$$

$$\Rightarrow \quad n = 3^x \qquad x = \log_3 n$$

$$T(n) = \sum_{i=0}^{\log_3 n - 1} b \cdot \log\left(\frac{n}{3^i}\right) + a$$

$$= \sum_{i=0}^{\log_3 n - 1} \left(b \log n - b \log 3^i\right) + a$$

$$\sum_{k=0}^{h} k = \frac{(h+1) \cdot h}{2}$$

$$= \sum_{i=0}^{\log_3 n - 1} b \cdot \log n - \sum_{i=0}^{\log_3 n - 1} (b \cdot \log 3) \cdot i + a$$

$$= b \log n \log_3 n - b \log 3 \, \frac{\log_3 n \left(\log_3 n - 1\right)}{2} + a$$

$$= \Theta\left((\log n)^2\right) - \Theta\left((\log n)^2\right) + a$$

$= \Theta((\log n)^2)$

Esercizio

Dato A vettore interi lung n determinare il massimo e il minimo di A

Per ogni soluzione valutare il n.ro di confronti tra elementi del vettore

1ª Soluzione

- Due scansioni con variabile ausiliarie

$$(n-1) + (n-1) = 2n-2$$

- Tutto in un' unica scansione

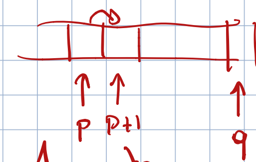
$$1 + 2(n-2) = 1 + 2n - 4 = 2n-3$$ confronti nel caso peggiore

$$2(n-1) = 2n-2$$

2ª Soluzione

- Divide et Impera Mi fermo quando ho solo 2 elem.

```
Min_Max (A, p, q) {
    if (p+1 ≥ q) {
        if (A[p] ≤ A[q]) {
            return (A[p], A[q])
        } else {
            return (A[q], A[p])
        }
    } else {
        r ← (p+q)/2
        (a,b) ← Min_Max (A, p, r)
        (c,d) ← Min_Max (A, r+1, q)
        return (min(a,c), max(b,d))
    }
}
```

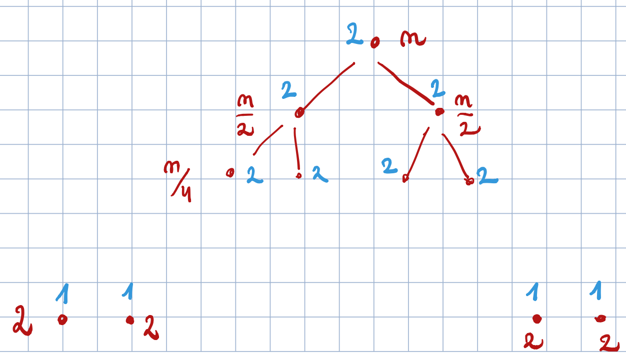


1 confronto

return (min (A[p], A[q]), max (A[p], A[q]))

No 2 confronti!!

$$C(n) = \begin{cases} 1 & n \le 2 \\ 2\,C\left(\frac{n}{2}\right) + 2 & n > 2 \end{cases}$$



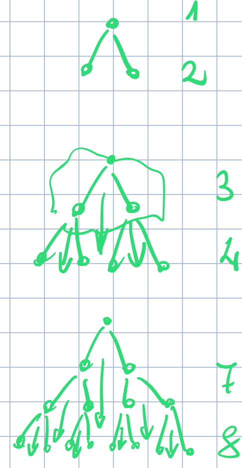
$n = 2^m$

Quanti nodi interni?
$n/2 - 1$

$n/2$ foglie

$$C(n) = \frac{n}{2} \cdot 1 + \left(\frac{n}{2} - 1\right) \cdot 2$$

$$= \frac{3}{2} n - 2$$



Lemma

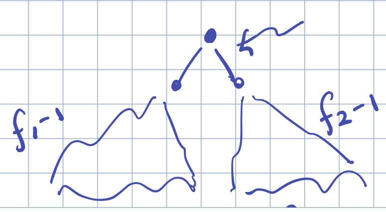
Un albero binario in cui ogni nodo ha 0 o 2 figli con f foglie ha sempre $f-1$ nodi interni

Dim

Per induzione sul n.ro di nodi dell'albero

BASE n.ro nodi $= 1$ • $f=1$ $n.i = 0 = f-1$ ✓

PASSO



$f_1 + f_2$ foglie

$f_1 - 1 + f_2 - 1 + 1$ nodi int.

$= f_1 + f_2 - 1$

f_1 f_2

Prodotto di Interi

$$x \cdot y$$

$$(x_1 x_2 \dots x_n) \cdot (y_1 y_2 \dots y_n)$$

Quante operazioni tra bit?

Algo Naive

$$\begin{array}{r} x_1 x_2 \dots x_n \\ \times \quad y_1 y_2 \dots y_n \\ \hline \end{array}$$

n

n

$\vdots$

n

$O(n^2)$

Algo Divide et Impera

$$(x_1 x_2 \dots x_{\frac{n}{2}} x_{\frac{n}{2}+1} \dots x_n) \cdot (y_1 y_2 \dots y_{\frac{n}{2}} y_{\frac{n}{2}+1} \dots y_n)$$

$$\underbrace{x_1 \cdot 2^{n-1} + x_{\frac{n}{2}} \cdot 2^{n-\frac{n}{2}}}\;\; + x_{n-1} \cdot 2^1 + x_n \cdot 2^0$$

$2^{n/2}$

$x_1 2^{n/2} +$

$$\left[\underbrace{(x_1 x_2 \dots x_{\frac{n}{2}})} \cdot 2^{n/2} + \underbrace{(x_{\frac{n}{2}+1} \dots x_n)}\right] * \left[\underbrace{(y_1 \dots y_{n/2})} 2^{n/2} + \underbrace{(y_{\frac{n}{2}+1} \dots y_n)}\right]$$

$x_a \qquad x_b \qquad y_a \qquad y_b$

$$(x_1 x_2 \dots x_{\frac{n}{2}})(y_1 \dots y_{n/2}) \cdot 2^n + (x_1 \dots x_{n/2})(y_{\frac{n}{2}+1} \dots y_n) \cdot 2^{n/2} +$$

$$+ (x_{\frac{n}{2}+1} \dots x_n)(y_1 \dots y_{\frac{n}{2}}) \cdot 2^{n/2} + (x_{\frac{n}{2}+1} \dots x_n)(y_{\frac{n}{2}+1} \dots y_n)$$

$$\underbrace{x_a \cdot y_a}_{\substack{1\ ch.\\ n/2}}\, 2^n + \overbrace{(\underbrace{x_a \cdot y_b}_{\substack{1\ ch\\ n/2}} + \underbrace{y_a \cdot x_b}_{\substack{1\ ch\\ n/2}})}^{*}\, 2^{n/2} + \underbrace{x_b \cdot y_b}$$

$$O(n) = \begin{cases} 1 & n = 1 \\ 4\, O\left(\frac{n}{2}\right) + \Theta(n) & n > 1 \end{cases}$$

addizioni

Risolvere per esercizio $\Theta(n^2)$

$x_a \cdot y_a \qquad x_a \cdot y_b \qquad x_b \cdot y_a \qquad x_b \cdot y_b$

$$(x_a + x_b) \cdot (y_a + y_b) = x_a y_a + x_a y_b + x_b y_a + x_b y_b$$

$$\underset{*}{x_a y_b + x_b y_a} = \underbrace{(x_a + x_b) \cdot (y_a + y_b)}_{1\ ch\ ric} - \underbrace{x_a y_a}_{1\ ch\ ric} - \underbrace{x_b y_b}_{1\ ch\ ric}$$

$$O(n) = \begin{cases} 1 & n = 1 \\ 3\,O\left(\frac{n}{2}\right) + \Theta(n) & n > 1 \end{cases}$$

$$\Theta\left(n^{\log_2 3}\right)$$

$$T(n) = \begin{cases} \Theta(1) & n = 1 \\ T\left(\frac{n}{4}\right) + T\left(\frac{n}{3}\right) + \Theta(n^2) \end{cases}$$