Logical representation

- 1. Language, concrete rules, deals with propositions, no ambiguity.
- 2. Drawing a conclusion
- 3. Lays down some important communication rules.
- 4. Consist of precisely defined syntax and semantics (support inference)
- 5. Each knowledge can be translated into logic syntax and semantics.

Syntax	semantics
How can you construct sentences.	interprets
which symbol we can use in knowledge representation	meaning

Propositional logic

- 1. Declarative statement
- 2. Either true or false or both,
- 3. Logical and mathematical form

Propositional logic is a branch of mathematics that studies the logical relationships between propositions (or statements, sentences, assertions) taken as a whole, and connected via logical connectives.



Example

- 1.it is Sunday,
- 2. The sun rises from west.
- 3.3+3=7
- 4. 5 is a prime number.
- 4. Boolean logic.
- 5. Tautology (always true), contradiction (always false).
- 6. Questions, commands and opinions never be a propositional logic.

Some sentences that do not have a truth value or may have more than one truth value are not propositions. For Example,

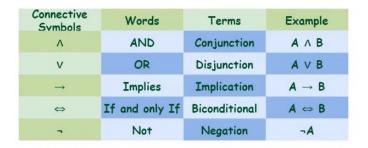
- 1. What time is it?
- 2. Go out and Play
- 3. x + 1 = 2

Propositions constructed using one or more propositions are called compound propositions.

Logical connectives (operators).

- 1. Negations.
- 2. Conjunctions.
- 3. Disjunctions.
- 4. Implications.
- 5. Bidirectional.





Truth table of propositional logic

Negation

P	Negation of p
Т	F
F	Т

Conjunction

P	q	P^q
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

Disjunction

P	q	Pva
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

Properties of operators

- Commutativity.
 Associativity.
- 3. Identity element.
- 4. Distributive.
- 5. De Morgan's law
- 6.double negation elimination.

Precedence of Connectives in Propositional Logic

When evaluating **compound propositions** with multiple logical connectives, it's important to follow a specific **order of precedence** to ensure accurate results. Similar to arithmetic operations, **logical operators** are evaluated in a defined sequence, from highest to lowest precedence.

Order of Precedence

- 1. NOT (\neg) Negation has the **highest precedence** and is evaluated first.
- **2.** AND (\wedge) Conjunction is evaluated next, after negations are resolved.
- 3. OR (\vee) Disjunction comes after AND operations.
- **4. IF-THEN** (\rightarrow) Implication is evaluated after OR.
- 5. IF AND ONLY IF (\leftrightarrow) Biconditional has the **lowest precedence**.

Example: Precedence in Action

Consider the following logical expression:

$$\neg P \lor (Q \land R)$$

- **Step 1:** Evaluate ¬**P** (Negation).
- Step 2: Evaluate $\mathbf{Q} \wedge \mathbf{R}$ (AND).
- Step 3: Evaluate $\neg P \lor (Q \land R)$ (OR).

The final result depends on the proper evaluation order, ensuring the correct outcome.

Logical Equivalence in Propositional Logic

- 1. De Morgan's Laws:
 - These laws show how **negations of conjunctions** and **disjunctions** behave:

$$\neg (P \land Q) \equiv (\neg P \lor \neg Q)$$

$$\neg (P \lor Q) \equiv (\neg P \land \neg Q)$$

- 2. Double Negation:
 - Negating a negation gives the original proposition:

$$\neg(\neg P)\equiv P$$

- 3. Implication and Disjunction:
 - An implication can be rewritten as:

$$P \rightarrow Q \equiv \neg P \lor Q$$

Tautologies and Contradictions

• **Tautology:** A tautology is a statement that is **always true**, no matter the truth values of its individual propositions.

• Example: P∨¬P≡True

• Contradiction: A contradiction is a statement that is always false.

• **Example:** P∧¬P≡False

In propositional logic, **logical operators** follow specific properties that allow us to **manipulate and simplify logical expressions**. Understanding these properties is essential for building efficient AI systems that rely on logical reasoning.

De Morgan's Laws

These laws describe how **negations** distribute over **AND** (\land) and **OR** (\lor) operations:

First Law:

•
$$\neg (P \land Q) \equiv (\neg P \lor \neg Q)$$

This means that the negation of a conjunction is equivalent to the disjunction of the negated propositions.

Second Law:

•
$$\neg (P \lor Q) \equiv (\neg P \land \neg Q)$$

This means that the negation of a disjunction is equivalent to the conjunction of the negated propositions.

2. Commutative Property

This property states that the **order of the propositions** does not affect the result of **AND** (\wedge) and **OR** (\vee) operations:

- AND:
 - $P \land Q \equiv Q \land P$
- OR:
 - P∨O≡O∨P

3. Associative Property

This property allows us to **group propositions** in any order when using **AND** or **OR** operations:

- AND:
 - $(P \land Q) \land R \equiv P \land (Q \land R)$

- OR:
 - $(P \lor Q) \lor R \equiv P \lor (Q \lor R)$

4. Distributive Property

This property states that **AND distributes over OR**, and vice versa:

- AND over OR:
 - $P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)P$
- OR over AND:
 - $P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$

Applications of Propositional Logic in Al

1. Knowledge Representation in Expert Systems:

• Represents **rules and facts** to solve domain-specific problems (e.g., medical diagnosis systems).

2. Reasoning and Decision-Making:

• AI agents use logical rules to make **decisions** (e.g., robot vacuum cleaners deciding when to start cleaning).

3. Natural Language Processing (NLP):

 Helps analyze text and respond logically (e.g., chatbots understanding weather-related queries).

4. Game-Playing AI:

• Uses logic to **make strategic moves** (e.g., deciding checkmate in chess).

Limitations of Propositional Logic

1. Inability to Handle Complex Relationships

• Propositional logic cannot represent **relationships between multiple objects** or deal with hierarchies of information.

2. No Handling of Uncertainty

• It works only with **true or false** values and cannot deal with **probabilities or uncertain outcomes**, limiting its use in real-world applications involving incomplete data.

3. Limited Expressiveness

• It cannot represent **time-based sequences** or dynamic events, which are crucial in some AI systems like speech recognition and robotics.

4. Scalability Issues

