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On Teaching Logarithms using the Socratic Pedagogy

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Introduction

Instructors of post-secondary remedial mathematics classes often voice similar challenges from students: low attendance, lack of motivation, and negative attitude toward mathematics. These challenges often result in students to low performance on assessments. The effective mathematics teacher must first look at oneself to determine if changes can be made to improve student motivation, attitude toward mathematics, and class attendance. We formed a collaboration to examine our current teaching methods in remedial classes and based on our findings, worked together to research a new pedagogy we could adopt in our remedial classes to improve our students' enjoyment of learning mathematics.

Upon examination, we found we relied too greatly on teaching our remedial students in a lecture-based style. In addition to our students being bored, we determined that teaching mathematics primarily by lecturing may not always be effective for remedial students. Our research began with a search for a teaching method that would employ student thinking, creativity, and understanding. We found the Socratic Pedagogy met these needs and decided to focus our joint efforts on creating lessons using this method to teach our remedial students. We chose the topic of logarithms as we both have found this to be a particularly challenging topic in the past. Our logarithm lesson, taught using the Socratic Pedagogy, was designed with the objective of improving students' overall enjoyment and classroom experience while learning the content.

The Socratic Pedagogy

The Socratic Pedagogy rests on the philosophy that knowledge and understanding are already present within people and that this knowledge could be drawn out using appropriate questioning, in turn advancing that very understanding. The Socratic Pedagogy of teaching focuses on constructing questions instead of answers for the students. This is a system of learning based on inquiry, questioning, exploration, and discovery. This method is responsible, in part, for drawing out ideas and thoughts that help build toward real-world construction of self-determined hypotheses.

> By helping students examine their premonitions and beliefs while at the same time accepting the limitations of human thought, Socrates believed students could improve their reasoning skills and ultimately move toward more rational thinking and ideas more easily supported with logic [3, p.7].

Using the Socratic Pedagogy, teachers use questioning methods to both evaluate and teach mathematics. Students construct both knowledge and understanding, instead of passively receiving knowledge. The ability to understand is an emergent process that is constantly being revised. Learners have some knowledge, and can create new knowledge accordingly by cooperative actions, not solely based on the teachers' effort.

Utilizing the Socratic method of teaching, learning is shared throughout the class; the emphasis of the class becomes questioning rather than answering; and thinking is valued as the essential classroom activity [1]. In lieu of overtly giving students the steps and definitions necessary to understand systems, teachers pose questions designed to lead students to discover how the systems work. There are a few basic tenets to the Socratic Pedagogy. First, the objective of teaching is inquiry. Its purpose is to help modify student argument or thought. Second, its method is dialogue, mainly between teacher and student. The teacher's role is to ask non-leading questions and the students' role is to answer questions using prior experience and/or knowledge [2].

Development of the Lesson

Our research led us to adopt the Socratic Pedagogy as an ideal teaching method to reach our students. We chose logarithms because we both found this to be a topic our students historically have difficulty understanding, particularly our remedial students. Even though we were teaching at different colleges, we were both teaching similar remedial College Algebra courses. Hence, logarithms were chosen as the vehicle to deliver lesson plans using the Socratic Pedagogy.

We employed the Socratic Pedagogy to teach two lessons in logarithms in two College Algebra remedial classes, with a total of 43 students. Subsequent classes were taught using a lecture method. One was in a 2-year urban college with approximately 22,000 students in New York City while the other was in a 4-year suburban Catholic University with approximately 6,200 students in Connecticut. In comparison, the students at each one of these institutions were extremely heterogeneous. We maintained the belief that regardless of the students' backgrounds, their experiences in learning logarithms using the Socratic Pedagogy would be similar.

We collaborated to develop lessons employing the Socratic Pedagogy to teach logarithms and logarithmic properties. A deadline was set for our lesson implementation;

we committed ourselves to a regular schedule of weekly phone conversations, bi-weekly meetings, and regular e-mail correspondence. Throughout the lesson construction phase, the philosophy of the Socratic Pedagogy was foremost in our minds, and numerous versions of our lesson plan were developed, revised and revisited based on our understanding of the Socratic Pedagogy.

Three questions were investigated in response to our lesson plans:

1. Will students enjoy being taught using the Socratic Pedagogy?
2. Will students want to be taught other topics using the Socratic Pedagogy?
3. What benefits will be observed in the classroom, (either during or after), a few lessons using the Socratic Pedagogy?

Our objectives within the context of this study included making mathematics accessible for all learners. We took two days in our College Algebra classrooms to teach logarithms using the Socratic Pedagogy. Lesson plans were composed to teach logarithms as the inverse operations of exponents and to teach logarithmic properties. As a follow up, we each conducted a comprehensive classroom discussion with the goal of anecdotal evidence of searching for improvement in students' overall enjoyment and classroom experience while learning about logarithms.

Underscoring the standards of the Socratic Pedagogy, it was important that our students were taught using questions rather than lecturing at them. We committed to not "tell" the students anything, rather it was our expectation that students would be able to discover the properties by tapping into and extending their current mathematical knowledge, and then explain to us how each one of the properties worked. The students weren't told or asked to memorize anything. Instead, they were lead to discover that "logarithms are like exponents" by using specifically designed questioning by the teacher.

Teaching Logarithms using the Socratic Pedagogy, Two Lesson Plans:

We agreed on some basic strategies:

1. We would not explain how logarithms or their properties worked. This meant as teachers, we were not allowed to "teach" in our standard sense. The teacher is forced to take a back seat to our natural proclivity to tell students the results. Instead, we had to trust our questions were good enough so that our students could define logarithms themselves.

2. Students would be asked explain any and all of their conjectures. Regardless of whether or not their answer was correct, it was expected that any time a student came up with a response, we would ask them to explain it. Thinking would be praised on any level.

3. We would give students a pattern with three examples to follow. We used three examples because we thought that 4 examples would be too leading, and that 2 examples were not enough to establish a pattern. The pattern would then be broken with the fourth example (a completely new problem would be posited), and we would know if the majority of the class understood logarithms.

4. We would use vocabulary in context. We would not write down definitions, or define definitions for our students. Vocabulary would casually be mentioned in the context of our lesson (in this case,

for the different parts of logarithms), and let students pick up an understanding of how to use the vocabulary in this manner.

5. The students would derive generalizations for logarithms. The students would be able to tell us that "logarithms are like exponents", or that "adding in logarithms is the same as multiplication" (assuming the base is the same). This is the fundamental reason we teach using the Socratic Pedagogy.

Next, we include two partial lesson plans for logarithms. The lesson plan requires students to discover and to understand that logarithms can be expressed as exponents.

Lesson Plan #1: Logarithms are Exponent's Inverse

Board Work	*Instructor Questions*	*Possible Student Responses*	*Instructor's Response*
$\log_2 8 = ?$	log base 2 of 8 is equal to what number? Can someone use a 2 and an 8 to find another number?	4 , 28, 16, 2^8 (ask the students to explain each conjecture)	Those are great answers, and all of them use the 2 and the 8, however the answer is actually 3.
$\log_2 8 = 3$ $\log_2 16 = ?$	So, the log base 2 of 8 equals 3. I wonder how we can use all three of these numbers together? Lets' try another, what is the log base 2 of 16?	4, 32, 14, etc.	All of those answers are thoughtful as well. 4 is the answer Hmm..how can we use a 2 a 16 and a 4 to generate a mathematical answer?
$\log_2 8 = 3$ $\log_2 16 = 4$ $\log_2 32 = ?$	(at this point most students should catch on). Raise your hand if you think you know what the log base 2 of 32 is equal to.	5, etc.	So, when the base is a 2, the argument is a 32, the answer is a 5. Ok. Now, we are going to break the pattern. Lets try a difficult one
$\log_3 9 = ?$	Raise your hand if you think you know the answer to log base 3 of 9	2	Correct the answer is 2.

$\log_3 9 = 2$	Hmm…I wonder if someone can raise their hand and explain how logarithms work. Do they look like any other operations we have already seen? Why is log base 3 of 9 equal to 2? Does this work for the log base 2 of 8 equals three as well?	The students may say that they act like exponents. Or because 3 to the second power equals 9. And this works as well for all of the other log questions.	So can we rewrite logarithms as if they were exponents?
$\log_a x = b$	If I wanted to rewrite this general form of logarithms, raise your hand if you can tell me how I could rewrite log base a of x equals b using an exponential equation.	$a^b = x$	

We continued with our lesson, by giving students numerous examples and practice to sharpen their understanding of logarithms. We would place the variable in different places, and have the students practice solving questions about logarithms, similar to ones found in their textbook or homework. It is important to note that many students had prior exposure to logarithms. Students have been taught many, if not all of the topics covered in these courses while in their high school mathematics courses. Prior exposure is not an indicator of their knowledge in these topics, but to cover the possible case that some students did have a solid foundation in logarithms, we asked students who were familiar with logarithms and/or their properties to 'keep their knowledge a secret' for a short period until after the property was discovered by more of their classmates.

If we were teaching logarithms using our traditional lecture format, we would have told the students initially that logarithms and exponents are inverse operations, and that you can generate

$\log_a x = b$ by using the exponential form of $a^b = x$. Instead, we constructed questions we know will help our students discover this fact on their own. We used our knowledge of the mathematics to help pique the student interest in what it is we were teaching. By using the Socratic means of teaching, we create an environment to help our students to look for patterns, establish connections, and understand logarithms themselves. Rather than being passive receivers of our knowledge, our students create ownership to the lesson. *They* discovered the property, *they* found out how it worked, *they* looked to make the connections themselves.

Lesson Plan #2: Addition of Logarithms (when the base is the same) acts like Addition

Board Work	Instructor Question	Possible Student Response	Instructor Response
$\log_2 2 + \log_2 4 =$	Let's discuss some properties of logarithms. Raise your hand if you can tell me what you believe log base 2 of 4 plus log base 2 of 8 is equal to.	3, $\log_2 6$, $\log_4 6$ ask the students to explain each one of these responses	These are all interesting conjectures, lets see if we can prove what this is equal to;
$\log_2 2 + \log_2 4 = \log_2 8$ 1 + 2 = 3	(the students can help with the proof, they know that log base 2 of 2 is 1, and that log base 2 of 4 is 2.) I wonder how we can rewrite the number 3 as a log base 2?	Log base 2 of 8 is equal to 3.	Ok, so log base 2 of 2 plus log base 2 of 4 is equal to log base 2 of 8. I wonder if there is a short cut here?
$\log_2 2 + \log_2 8 =$	Raise your hand if you can explain what you think the answer to log base	$\log_2 16$	Can you prove that as well? (have student prove and explain…

	2 of 2 plus log base 2 of 8 is equal to…		
$\log_2 2 + \log_2 8 = \log_2 16$ 1 + 3 = 4	Ok. So, I wonder if we really have to prove these? I wonder if there is any other way to find log base 2 of 2 plus log base 2 of 8 without proving it?	By multiplying the 2 and the 8?	Hmm…when we multiplied the arguments in our first example, did that work? Does that work for this example? I wonder if it will work for our next example?
$\log_2 2 + \log_2 16 =$	So, raise your hand if you can tell me what the log base 2 of 2 plus the log base 2 of 16 is equal to without proving it.	$\log_2 32$	Ok, so if we were to prove it, would it work?
$\log_2 2 + \log_2 16 = \log_2 32$ 1 + 4 = 5	Is it true that log base 2 of 32 is equal to 5?	Yes	So, maybe we have found a shortcut? Let's try one more example, we are going to break the pattern here…
$\log_3 3 + \log_3 9 =$	Raise your hand if you can tell me what log base 3 of 3 plus log base 3 of 9 is equal to…	$\log_3 27$	Can we prove this one as well?
$\log_3 3 + \log_3 9 = \log_3 27$ 1 + 2 = 3	Ok, so raise your hand if you can explain how addition works in logarithms, assuming the base is the same	You keep the base the same and multiply arguments	So, lets sum up this rule using general algebraic terms;
$\log_a x + \log_a y =$	If we wanted to condense log base a of x plus log base a of y to a single logarithmic expression, raise	$\log_a(xy)$	

	your hand if you can explain what this would equal.		

It is not a difficult task for a teacher to tell students, $\log_a x + \log_a y = \log_a(xy)$. Then, the teacher engineers numerous examples for the students to work out to make sure they understand the property. Instead, we reverse the order when using the Socratic Pedagogy. We ask the students to justify, (and make sense of their justification) that addition of logarithms acts like multiplication. Once again, our students are active participants in their learning. Learning is shared and modified, and thinking and understanding are valued.

Results

As a follow up, we each conducted a comprehensive classroom discussion looking for anecdotal evidence of improvement in students' overall enjoyment and classroom experience while learning about logarithms. In response to our first question, "Will students enjoy being taught using the Socratic pedagogy?" the majority of the responses were positive. Many students stated during the classroom discussion that they really enjoyed thinking about logarithms using the Socratic Pedagogy. Through informal dialogue with our students, we received the following direct quotes:

"I would like to try out more ways to learn with the Socratic method."

"Thought a little easier than some other sections."

"I found the Socratic method helpful because instead of giving answers and rules, we were forced to find the answers for ourselves."

"In high school I never understood how to work with logarithms and now I understand how to."

"I think it was a good lesson."

"I like it, it really made me think."

"I really got it, for the first time, I really enjoyed learning."

There was some negative feedback as well:

"The Socratic method just felt like being teased with information."

"The Socratic method wasn't really a good teaching method for me because I like when things are explained first step by step."

"I personally didn't like the lesson, but I am willing to try and see how it would work using something other than logs."

"I'd rather you tell us the info first."

Negative feedback may derive from lack of comfort with a new type of teaching experience. Students may be resistant to change, relying instead on the old way of being taught. They were not quite convinced the new way of being taught would resonate with them. We do assert, however, if given more exposure to this method of teaching, more students may find the Socratic method of learning to be an enjoyable experience.

In response to our second research question, "Will students want to be taught other topics using the Socratic Pedagogy?" we were both informed from a majority of our students, that they would enjoy learning other topics using this method. One of us was sought out after class from a student who typically tends to be quiet and non-participatory, and was asked if we could teach more lessons using this method.

An unintentional outcome of using the Socratic Pedagogy was that our classes' participation and enthusiasm for learning mathematics grew, in part explaining our results to our third question, "What benefits will be observed in the classroom, (either during or after), a few lessons using the Socratic Pedagogy". Many students who don't normally participate during our regular lectures did during these two classes. We speculate this was possibly due to our students receiving positive feedback for their thinking regardless of the precision of their answer. Therefore, students who normally don't participate because they are scared of giving an incorrect answer, felt safe to make mistakes during these lessons.

After these lessons were taught using the Socratic Pedagogy, numerous benefits were observed. One example was that students would try to convince themselves (through proof or through numerical substitution) that some property worked. As an example, during one of our classes, we asked students, "What is another way we can express $\log x^3$?" One student stated immediately "$3\log x$". When the student was asked to justify his answer, he stated because "$\log x^3 = \log x + \log x + \log x = 3\log x$". This student discovered the property that exponentiation of an argument in logarithms is the same as multiplying the logarithmic expression by the exponent. This property was not included in our initial lesson plans to teach using the Socratic Pedagogy, however a modified version of it will be included the next time we teach using this method.

Future Research and Implications

In extending this research, the primary objective will be to determine if teaching using the Socratic Pedagogy will have any type of effect on how students either understand or retain their understanding of logarithms and their properties. Future research could assist in answering the question: Will teaching using the Socratic Pedagogy impact student understanding of logarithms? It could also help answer the question: Will students who have learned via the Socratic method perform better (either on quizzes or exams) than students who were not taught using this method? In addressing our questions about teaching using the Socratic Pedagogy, it is important to note that we relied on anecdotal evidence from discussions with our students. There is no quantitative data to accompany our research. We look forward to addressing the above questions in future studies using quantitative data and statistical analysis.

Using the Socratic Pedagogy to teach logarithms resulted in numerous benefits for researchers as well. We taught ourselves a new way to understand and learn mathematics. We pushed ourselves out of our comfort zone and challenged ourselves to become better teachers. We felt responsible for our student lack of enthusiasm in our remedial classes, and we changed this by researching new methods. We also gained a renewed enjoyment for lesson planning, choosing to create lesson plans that held onto the ideals of the Socratic Pedagogy but also not losing the rich mathematics at the same time.

Teaching using the Socratic Pedagogy had numerous benefits both for our students and ourselves. There are many topics in remedial college mathematics courses that can be taught using the Socratic Pedagogy. It is our hope that other mathematics educators of remedial students will gain from our study, either by using the above lesson plans or following our process and designing their own plans using the Socratic Pedagogy and implementing those plans in their remedial mathematics classrooms.

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