Shell Education

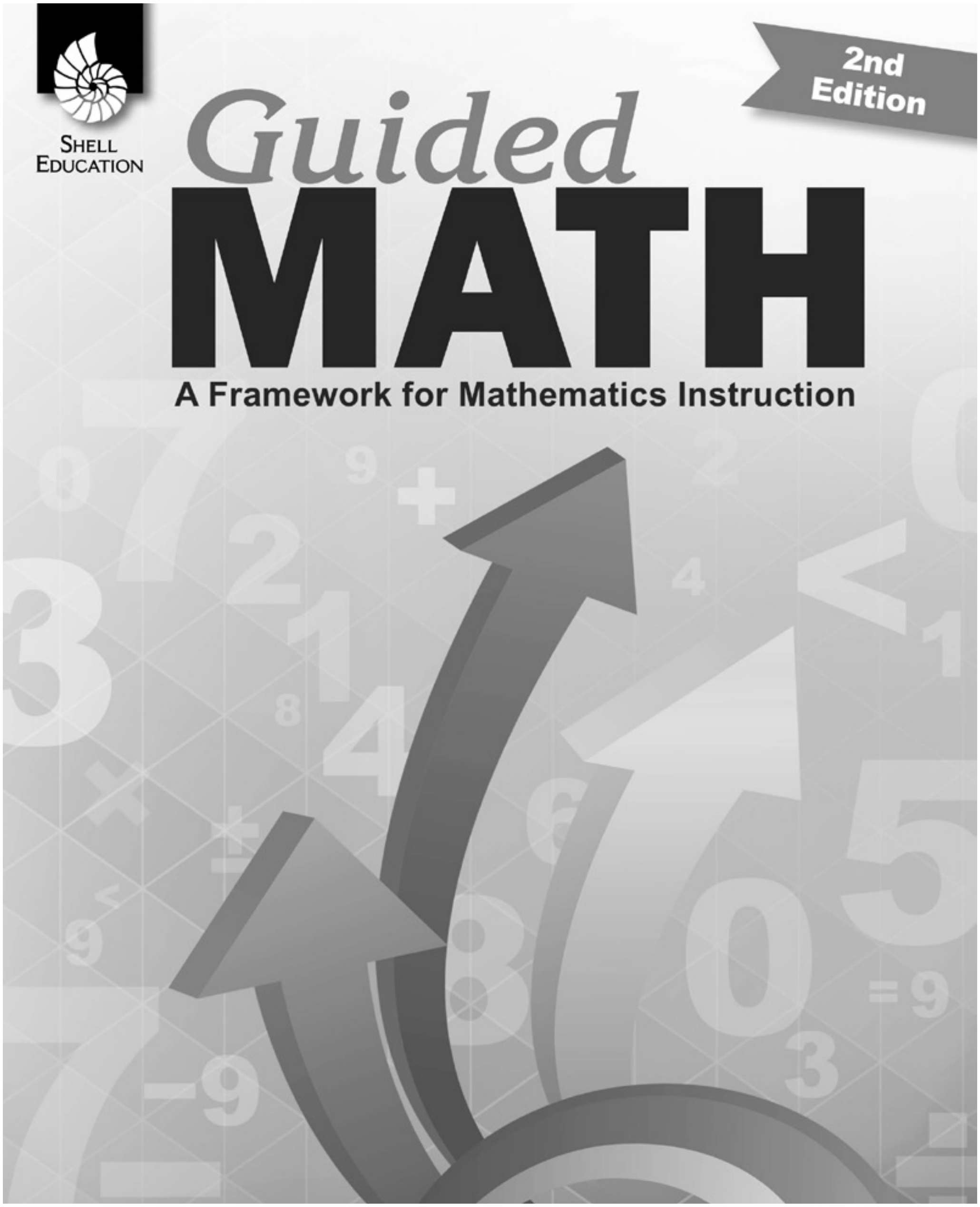
2nd Edition

# Guided MATH

## A Framework for Mathematics Instruction

Laney Sammons

Foreword by Donna Boucher

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## Standards



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# Foreword

In 2005, I was teaching 5th grade math at a school in the suburbs of Houston. I had taught 3rd and 4th grade before moving to 5th grade, so I was familiar with gaps in the mathematical understanding of my students at these grade levels. After several years teaching 5th grade, I realized that with each successive year, the achievement gaps I saw in my students were growing deeper and wider. Once students fell behind, the gaps rarely closed with the traditional, whole-class instructional setting. I was remediating 2nd grade mathematical skills while trying to teach 5th grade concepts and standards. Without a strong foundation, the new concepts made no sense to students. By 5th grade, students who struggle in math have likely been struggling for most of their elementary careers. I experienced the heartbreaking consequences of that failure on a daily basis. Some students tried to mask their failure by acting out, while others just withdrew and accepted defeat.

I realized that I needed to change the way I taught to meet the needs of my students. Clearly, a "one-size-fits-all" type of instruction was not getting students where they needed to be. In fact, this approach was likely to blame for them being in the predicament they were currently in. I searched for books to help me restructure my class to better meet the needs of my students, but to be honest, there just wasn't much available. So, I experimented. While I primarily still used whole-group instruction, I began using small-group lessons as part of my math block. Books on differentiation helped me create tiered tasks for my math centers. Over the next five years, through trial and error, I cobbled together an effective classroom structure. My students flourished. For the first time in years, they were experiencing success!

Five years later, I stepped onto a campus in an instructional coach role. I thought I had dealt with mathematical gaps at my previous school, but the new campus was an eye-opening experience. Kindergarten students had vastly different experiences prior to entering school, and the range of readiness was dramatic. With a large percentage of students functioning below grade level, we had our work cut out for us. The philosophy of our administrators was that students learn best by doing, and hands-on experiences were valued over worksheets and textbooks. Our teachers used Lucy Calkins's Reading and Writing Workshop approach for literacy, so I began working with the teachers to implement a similar approach to math while incorporating what I had done previously. Moving toward small-group lessons was a huge paradigm shift. As a coach, I once again found myself looking for resources to help support their growth and understanding.

Thankfully, that was the year Laney Sammons's original *Guided Math* book was published. My first reaction was a selfish one, wondering where this book had been when I needed it! I immediately recognized how helpful the book would be for our teachers as they made the transition from only using whole-group instruction in mathematics. I kicked off a campus book study, and our journey toward becoming a Guided Math campus began. Each year, more teachers and teaching teams embraced the framework as they witnessed the great math instruction that was taking place in these Guided Math classrooms. It was an exciting and rewarding experience.

The changes in this new edition are exciting! For each of the components of the Guided Math framework, Laney has included new and practical details for implementation. Some of these suggestions include games to allow students to interact with the math word wall, strategies for facilitating math discussions during math stretches, options for different math workshop models, and a 15-day plan for implementing Guided Math. Teachers will find this new edition provides practical information to make the transition to Guided Math feel less daunting. User-friendly features, such as chapter snapshots and reflection opportunities, support teacher learning and are useful for book study groups.

If I had to choose a new favorite chapter, it would be the chapter on small-group lessons. The planning and implementation of small-group lessons is often the biggest hurdle teachers face when moving away from whole-class instruction. This new chapter not only provides additional resources for planning and implementing small-group lessons, but it also reflects a fundamental shift in thinking about the best way to use small-group lessons within the Guided

Math framework. When the first edition was published, small-group lessons were primarily used for differentiation following whole-group instruction. Experience has shown, however, that the most effective use of small-group lessons is to present new concepts. Step-by-step directions for planning small-group lessons are included along with samples of planning tools and examples of full lessons. This is sure to take much of the mystery out of teaching in small groups.

I began blogging in 2012, often citing Laney's work and creating resources to support the Guided Math framework. Each additional book in the Guided Math suite found a place on my bookshelf. Laney's body of work has had a profound impact on me personally and on teachers and students across the country and internationally. I crossed paths with Laney at the National Council of Teachers of Mathematics Annual Conference in 2014. That was the beginning of our personal and professional relationship. Over the past four years, we have collaborated on conferences, presented Guided Math institutes and workshops together, and co-authored a book called *Guided Math Workshop*. Since the publication of the original book, Laney has worked extensively with teachers, schools, and districts implementing Guided Math. The knowledge she gained from those boots-on-the-ground experiences makes this second edition an even more valuable resource for teachers trying to navigate the changing landscape of mathematics instruction. Whether you are replacing your dog-eared original copy or just starting out on your Guided Math journey, this book is a must have!

Donna Boucher
Co-author of *Guided Math Workshop*

# Preface to the Second Edition

Without a doubt, most people join the teaching profession because they care deeply about young people and place great value on education. I am no exception. In 1989, when my younger child entered first grade, I tentatively entered a kindergarten classroom as a rookie teacher. I don't know who was more nervous—the 18 youngsters who were beginning school for the first time or me.

The vast differences among my students became apparent to me very quickly. One of my students did not know what crayons were. She had never held a pencil before. Several children had attended a preschool focused on traditional kindergarten skills. They knew their numbers through 10 and they could write their names, as well as several sight words, beautifully. Another child, before he had even put his jacket in his cubby, curled up on the floor in deep slumber. Some children had a strong sense of story and loved listening to read-alouds. Others were unable to follow and comprehend simple stories. A few students had difficulty getting along with their peers. Almost all of them struggled initially with the idea of doing things together as a class rather than following their individual interests. As I led lessons and tried to establish smooth transitions, it often felt as if I were trying to carry a large load of laundry. One sock would fall to the floor, and as I picked it up, another would drop. And so it was with my students. Just as one student was coaxed back into the group, another would head off to pursue something that had just attracted their attention.

The differences I observed in my kindergarten class that year are mirrored in classrooms throughout the country in elementary, middle, and high schools. Fortunately for me, there was a very talented and experienced full-time teaching assistant in my classroom. She was able to provide support and scaffolding for me as a rookie teacher, and she and I were able to do the same for my "rookie" students. Because we worked with these students in small groups and differentiated instruction for them, they were successful and ready to move on to the first grade.

I've often heard the expression, "Everything I need to know for life I learned in kindergarten." I might change that slightly to say, "Everything I need to know to teach I learned in kindergarten." My years of teaching kindergarten taught me how important it is to find out all you possibly can about your students and to adjust your instruction to meet their ever-changing needs. Rather than becoming frustrated teaching one-size-fits-all lessons, my teaching assistant and I spent lots of time supporting below-level learners, prodding reluctant learners, piquing the curiosity of adventurous learners, and praising the accomplishments of all learners as we guided them toward the next steps of their learning.

In the subsequent years, I moved to other schools and grade levels. Although the needs of the students in each grade I taught were as diverse as those in my first kindergarten class, the support of a full-time teaching assistant was no longer available. I faced the daunting task of trying to help my students achieve academic success while recognizing that they each had unique needs. Whole-class instruction made this extremely difficult. I knew how students' needs could be met with a teaching assistant in the classroom. How could I adapt that knowledge to make it feasible to implement in my classroom without any assistance? My colleagues were facing the same challenges. Together, we devised plans for working with small groups of students while the rest of the class was engaged in independent work. Unit pretesting gave us an idea of our students' proficiencies so we could create need-based groups. Generally, the three small groups we created each worked with the teacher for 20 minutes of the 60-minute class period. The lessons we planned were designed to respond to what we had identified as their needs. We began to feel a little better about addressing the needs of our students. They responded by showing much greater interest and by making gains in their achievement. But we were still novices at this game.

Eventually, my school implemented Guided Reading based on the model developed by Irene Fountas and Gay Su Pinnell (1996, 2001). As we applied this teaching framework to language arts instruction, it became apparent to me that this model was an effective method for differentiating instruction to meet the needs of my learners. And, with some tweaking, I began to apply it to my math instruction, too. This method allowed me to teach each group at an instructional level that maximized its learning, and by using the approach with both language arts and mathematics, it was easy to establish and teach consistent routines and procedures to students. Soon, some of my colleagues and I began using what we now call Guided Math.

Later, as an instructional coach, I worked with our entire staff to make the transition to teaching math using the Guided Math framework. From my communication with teachers across the country, it is obvious that many of them are beginning the process of implementing their own versions of Guided Math. It is my hope that this book will provide some encouragement and guidance to those teachers. There is no one "right" way to use Guided Math. By sharing our ideas, we can help each other implement Guided Math in ways that work best with our teaching styles and in our classrooms.

In the almost 10 years since *Guided Math: A Framework for Mathematics Instruction* was first published, it has been gratifying to see teachers throughout the United States and Canada enthusiastically embrace the framework. Over the years, I have been privileged to work closely with many teachers as they implemented Guided Math in their classrooms and adapted it to make it their own. I am fortunate to have had these grassroots experiences. From them, I have learned much. I have also learned from the many excellent math educators who generously shared their ideas and experiences in articles, books, conference presentations, and on social media. Not surprisingly, my vision for Guided Math has evolved over the years. The time has come to share these ideas with others in a revised second edition of *Guided Math*.

This chart describes some of the revisions you will find in this book:

| |
|---|
| *An expanded grade-level recommendation*<br>As the framework has been implemented in elementary and middle schools, many high school teachers are now discovering it. They recognize its practicality in meeting the learning needs of all students and providing a format that enables greater student engagement. This edition is written for all those who teach mathematics, not just kindergarten through eighth grade teachers. |
| *The inclusion of updated research*<br>Much has been written about mathematics education in the past 10 years. Many new ideas have arisen from continued research on the most effective ways to teach math. Some of those ideas have been incorporated into this book. For those of you who, like me, enjoy keeping abreast of the latest in mathematical education articles and books, look to the References Cited section of this book (pages 366–373) for my most recent research sources. |
| *A variety of ideas for creating an environment of numeracy in the classroom and for planning engaging and instructionally valuable math warm-ups*<br>Additional suggestions for these components of the Guided Math framework are described based on real-life classroom experiences. |
| *An expanded and more in-depth chapter on planning and teaching small-group lessons*<br>Small-group instruction is at the heart of Guided Math. You will find strategies for identifying the gaps in knowledge and skills that students need to be successful in learning new content. While identifying gaps is important, it is even more important for teachers to take effective steps to fill those gaps. The small-group lesson structure allows teachers to plan a single lesson with options for differentiation that address the needs of students who lack the prerequisite knowledge and skills as well as those of students who may require additional challenges. |
| *Specific guidance in establishing a manageable math workshop including designing workstations*<br>An effective math workshop structure is a hallmark of the Guided Math framework. It allows the teacher instructional flexibility as their students work independently. Students may review previously mastered material for greater understanding and retention, work on computational fluency, or engage in mathematical investigations. Working independently in heterogeneous groups, learners explore mathematics together. Chapter 6 on math workshop describes workshop models and workstation tasks. |

*Additional insights on one-on-one math conferences with students*

These student/teacher conversations offer valuable glimpses into students' mathematical thinking and help teachers target specific learning needs. This second edition offers strong guidance in implementing this component of Guided Math.

*Assessment options that support student learning*

To make the most of the Guided Math framework, assessment data is essential before, during, and after new mathematical content is taught. Chapter 8 focuses on multiple methods of assessment to identify learning needs.

*A First Fifteen Days plan for introducing Guided Math in the classroom*

Practical routines and procedures must not only be established but also taught to students prior to implementing Guided Math if its implementation is to be successful. Included in this edition is a 15-day plan that walks teachers through this vital step.

As always, I am eternally grateful to my entire family for their ongoing and enthusiastic support of my work. Although it has been a great inconvenience to him at times, my husband Jack has been understanding and has gone way beyond what might reasonably be expected in assuming additional household responsibilities. My thanks and love to him and my whole family.

For the new ideas in this book, I owe a debt of gratitude to the many math educators from whom I continue to learn so much. To those of you who are in classrooms working with young mathematicians daily, those of you who are coaches who continue to educate yourselves and share new ideas with teachers, and those of you who are administrators who provide support to those teachers who are willing to try new ideas to make their instruction the best it can be: thank you. I am also grateful for the researchers who are studying how to make mathematical education for students challenging, rigorous, and effective. Thank you for sharing your knowledge through articles, books, podcasts, conference presentations, Twitter posts, and any other ways you use to spread the word. Speaking for the many of us who follow your inspiring work: thank you.

—Laney Sammons, 2019

***Teachers continue to search for effective means to teach their students and for ways to adapt instructional methods to accommodate all learners.***

# Chapter 1

## Guided Math: A Framework for Mathematics Instruction

Think back to your school days. Picture your math classes. What do you remember? Many of us recall instructions to get out our math books and open to a specified page. The teacher explains the lesson using the chalkboard or overhead projector. One or two students may be called on to solve problems at the board as the rest of the students practice at their desks. Some of us may remember using manipulatives in our early grades but probably not beyond second grade. Then finally, the teacher assigns problems from the book for classwork and homework. These assignments are later turned in, checked, and graded. Periodically, quizzes are given to check understanding. At the end of the chapter, a test is given. The teacher then moves on to the next chapter.

Was this method successful? For many of us, the answer is yes. The teacher-centered approach provided the instruction we needed. We applied this instruction to problems to be completed, and our understanding increased. If it didn't, we comforted ourselves with the knowledge that some people just don't have mathematical minds. We decided to make the most of our other skills. Many of us simply opted out of math classes as soon as we could. All too often, this was considered good enough. Students either "got it" or they didn't. Their grades indicated how well they "got it." Unfortunately, too many of us didn't "get it."

Mathematical literacy has been, and continues to be, a serious problem in the United States (U.S. Department of Education 2008). In 2007, research indicated that 78 percent of adults could not explain how to compute the interest paid on a loan, 71 percent could not calculate miles per gallon on a trip, and 58 percent could not calculate a 10 percent tip for a lunch bill (Phillips 2007). According to the U.S. Department of Education's National Mathematics Advisory Panel, "there are persistent disparities in mathematics achievement related to race and income—disparities that are not only devastating for the individuals and families but also project poorly for the nation's future, given the youthfulness and high growth rates of the largest minority populations" (2008, 12).

Recent results from the National Assessment of Educational Progress (NAEP) show that only 40 percent of fourth grade students rated as proficient or advanced in 2017, while 20 percent ranked in the lowest level, below basic. The scores for eighth grade students are similar with 34 percent scoring as proficient or advanced and 30 percent scoring in the lowest performance level (National Center for Education Statistics 2004). Additionally, according to the 2015 results of the Programme for International Student Assessment, the United States placed 38 out of 71 countries in math. This ranking is behind countries such as Estonia, Vietnam, and Latvia, and below the average of the 35 members of the Organization of Economic Cooperation and Development that sponsors the test (Pew Research Center 2017).

We must change our mathematics instruction because too many of our students are falling behind. Unfortunately, many teachers are still using the traditional, whole-class instructional method in classrooms. Some teachers recognize the need for change from traditional instructional methods and are making those changes. However, the teacher-centered, large-group instruction model of teaching is still prominent in mathematics classrooms across the nation.

Because of the limitations of this method of instruction, students are often presented with the message that there is a particular way in which mathematics must be done—that there is only one right answer and only one right way to find that answer. The emphasis is on learning a set procedure rather than on conceptual understanding. In his book *The Math Instinct*, Keith Devlin states, “The problem many people have with school arithmetic is that they never get to the meaning stage; it remains forever an abstract game of formal symbols” (2005, 248). As Arthur Hyde (2006) points out, by fourth or fifth grade, students seem to have lost the problem-solving skills they had when they began kindergarten. Lack of conceptual understanding handicaps many students as they face more difficult math challenges in the upper grades.

Rather than inspiring students to understand and make sense of math, current instructional methods too frequently focus on memorization and formalized procedures. This focus on memorization squelches the natural curiosity learners have about mathematics. To improve the quality of mathematics education, Jo Boaler urges educators to equip their students with a mathematical mindset so that they “approach math with the desire to understand it and to think about it, and with the confidence that they can make sense of it” (2016, 34). But unless traditional instructional methods change, teachers will continue to struggle to teach mathematics as a “flexible conceptual subject that is all about thinking and sense-making” (35).

Furthermore, the traditional methods for teaching math offer few options for effectively addressing the diverse learning needs of students. As Jennifer Taylor-Cox so aptly describes: “We aim for the middle and pray for ricochet. We hope the knowledge we impart to the center will bounce around until everyone gets it” (2008, 1). Students who don’t “get it” fall further and further behind as teachers struggle to find the time to help them. Teachers are frustrated trying to meet the needs of those students while continuing to challenge others who master the concepts quickly. Some students complain of being bored while others fail miserably in understanding the content being taught. It is easy for teachers to feel caught in the middle of a tug-of-war game when trying to balance the needs of all learners. With the ever-increasing diversity of students in classrooms today, it has become evident that students’ mathematical success hinges on teachers’ ability to differentiate instruction so that all learners are both supported and challenged as they work to master the required curricular standards (Sammons 2013).

The frustrations felt by educators are only increased by the demands for accountability enacted by state and federal governments. School systems are struggling to eliminate the gaps in achievement between minority and majority students, between special education and general education students, and between students receiving free and reduced lunches and the rest of the student population. It is no longer acceptable to have a portion of our students underachieve in mathematics. Disappointingly, after several years of gaps in the scores of these groups slowly narrowing (according to NAEP assessments), the 2017 results show no further narrowing of these gaps, and they even show slight increases in the gaps since the 2015 assessments (Nation's Report Card n.d.).

Driven by these pressures and their own professional desires to provide quality mathematical education, teachers continue to search for effective means to teach their students and for ways to adapt instructional methods to accommodate all learners. Making this task even more complicated is the fact that students who are slower learners for one concept in mathematics may very well be accelerated learners with other concepts.

As states have upped the ante with the adoption of more demanding math standards based on the standards developed by the National Council of Teachers of Mathematics (2000) and the Common Core State Standards (Common Core State Standards Initiative 2015), teachers have discovered that methods they have used successfully in the past are no longer effective. The demands of the more rigorous curriculum standards call for new ways of teaching.

As I grappled with these frustrations in my own classroom, I gradually developed a model that offers all students opportunities to develop their mathematical skills at increasingly challenging levels of difficulty with the ultimate goal of helping them gain the ability to function independently in the world of mathematics. I learned the importance of establishing and maintaining a classroom framework that is organized to support numeracy, just as teachers have done for literacy for many years.

## Instructional Components of Guided Math

These are the instructional components of the framework:

1. Classroom Environment of Numeracy
2. Math Warm-Ups
3. Whole-Class Instruction
4. Small-Group Lessons
5. Math Workshop
6. Math Conferences
7. Ongoing System of Assessment

Together, these components allow teachers to support each student's efforts at varying levels according to their immediate learning needs, to promote the development of mathematical mindsets, and to provide rigorous instruction.

## Classroom Environment of Numeracy

Environments rich in mathematical opportunities for children are essential if we want children to develop a thorough understanding of mathematics. When students begin to recognize how numbers and problem solving affect their everyday lives, mathematics becomes more meaningful. Because learning is both a social and constructive process, students learn best through active engagement in authentic opportunities to use and extend their number sense.

The creation of a classroom environment supporting numeracy enables students to build on their previously acquired understanding of numbers. An organized mathematical support system for students requires that we encourage learners to use manipulatives, compute, compare, categorize, question, estimate, solve problems, converse, and write about their thinking processes. Ideally, a math-rich classroom environment and engaging activities will help students become increasingly aware of mathematical and problem-solving opportunities throughout their everyday lives, thus putting a "math curse" on students as authors Jon Scieszka and Lane Smith describe facetiously in their children's book of that title (1995).

The establishment of a community of learners is inherent within a classroom supporting the learning of mathematics. Students who feel respected and supported are willing to take risks in problem solving and in exploring mathematical ideas. They openly share their thinking with others and learn that mistakes are valuable learning experiences.

Through their engagement in mathematical dialogue, students construct the meaning of mathematics and develop enduring understandings of the big ideas or concepts while they acquire procedural and computational fluency. To establish a strong and flourishing learning community, teachers need to understand both their students and the mathematical "landscape of learning" through which they hope to guide them (Fosnot and Dolk 2001). With careful planning and ongoing reflection, teachers are able to foster the strong social aspects that are integral for learning, to teach behaviors that promote constructive conversation, to organize the physical aspects of their classrooms for immersing students in an environment of numeracy, and to provide classroom procedures that encourage student participation in all components of the Guided Math framework. Chapter 2 focuses on ways to establish numeracy-rich environments within the classroom.

## Math Warm-Ups

Math warm-ups are designed to take place first thing in the morning in self-contained classrooms or, alternatively, at the beginning of math class in departmentalized settings. These brief, focused math tasks set the tone for students' mathematical learning. Upon their arrival or as math class begins, students are asked to answer a question, consider a mathematical problem, or reflect on something they have previously learned.

Math stretches are one type of warm-up that teachers can easily implement. For example, students may be asked to add to a Number of the Day chart in which the teacher selects and writes a number at the top of a sheet of chart paper. Each student is expected to record the number in a different way on the chart. Ways of expressing the number will vary depending on grade level. Since the ways of writing the number will vary depending on the student, close observation of the chart will provide the teacher with valuable information about the number sense of individual students as well as an overall picture of the level of understanding of the entire class. More ideas regarding math stretches can be found in Chapter 3.

**Number of the Day**

**4**

**K–2 Students**

$3 + 1$ or $6 - 2$

**3–6 Students**

$(5 \times 4) - (2 \times 8)$ or $2^2$

**Secondary Students**

$32 \div (-8 \times 2 \div -4^0) \times 2$

A calendar board may also be incorporated in this component of Guided Math provided that the tasks are not overly repetitious and are completed in a timely manner. These activities usually begin as students observe the calendar and date. From there, the teacher briefly reviews previously covered mathematical skills, previews upcoming skills, provides practice in rote counting skills or math facts, encourages mental math skills, and engages students in problem-solving activities. Often, students sit on the floor around the calendar board, but sometimes, students remain at their desks to participate in these oral or written activities. This daily warm-up encourages students to nimbly move from one area of mathematics to another in a nonthreatening, fast-paced way. The predominately oral nature of these activities promotes conversations about mathematical concepts. This fosters a deeper, more enduring understanding in students. Chapter 3 further explores instructional ideas for mathematical warm-ups.

## Whole-Class Instruction

Many educators today are moving away from the traditional, teacher-directed method of instruction. However, this type of instruction can have a place in today's classroom, providing it isn't the only, or even primary, method of instruction.

Certainly, whole-class instruction requires the least amount of teacher preparation. In its most common form, the teacher introduces the lesson, teaches it as students listen and are questioned, provides a practice activity for students, and either summarizes the lesson or has students summarize it. Traditionally, students remain in their desks and face the teacher, who is at the front of the classroom. Most of us are familiar with this traditional method of teaching from our own time as students. Whole-class instruction remains an option within the Guided Math framework to be used when there is a compelling reason for it. The Guided Math framework offers teachers a variety of instructional methods to use instead of relying solely on whole-class instruction.

Whole-class instruction can be an excellent method for presenting activating strategies, for making literature connections at the beginning of lessons, or for reviewing mastered concepts. With this format, teachers may choose to present mini lessons or model problem-solving strategies, thinking aloud as they do so. Moreover, this component can be used as a time for a "math congress" (Fosnot and Dolk 2001) or for a math huddle, as it is called in the Guided Math framework. In a congress or math huddle, students come together following mathematical investigations to share their discoveries or to discuss their mathematical thinking.

Teaching to the whole class is a very straightforward method of instruction, but it requires a remarkable amount of teacher skill to be done well. Although it often appears that discourse during this kind of instruction is "off the cuff," to be effective, teachers must juggle what they know about their students and the mathematical concepts on their "horizons" to guide conversations with meaningful questions. Even a skillful teacher may be unable to reach some students because of students' lack of attention, boredom, inability to understand the instruction, or their often incorrect confidence that they already know how to do the activity and therefore don't need to listen. Chapter 4 offers suggestions on how to use this instructional component of Guided Math most effectively and when it should be avoided.

## Small-Group Lessons

Small-group lessons designed to teach new mathematical content while also targeting specific student mathematical learning needs are at the heart of the Guided Math framework. Teachers should first assess their students formally or informally and then group them according to their proficiencies in a given skill for small-group lessons. The groups are homogeneous yet fluid. As individual students' levels of understanding change and gaps in prerequisite knowledge and skills vary, so do the groups. This method of mathematics instruction is analogous to Guided Reading instruction as espoused by Irene Fountas and Gay Su Pinnell in their books *Guided Reading: Good First Teaching for All Children* (1996) and *Guiding Readers and Writers Grades 3–6* (2001).

Using Guided Math instruction, teachers can work with small groups whose composition is determined specifically by students' instructional needs. This allows teachers to closely observe student work, monitor student attention, provide strong support for below-level learners, and extend extra challenges for proficient learners.

With small-group lessons, teachers strive to move all students forward in the appropriate grade-level curricular progression while at the same time addressing gaps in knowledge and skills. For students with significant gaps, teachers briefly address those specifically and then move them into the current lesson with scaffolding that challenges them to successfully experience the new content being taught.

Using this model for small-group instruction, teachers have the option of varying the amount of time they spend with each group according to the specific needs of the students. For example, when a teacher is introducing a new concept, one group may quickly grasp the concept or skill and be able to move on to independent practice. Another group may need significantly more time working directly with the teacher in the small-group setting. Rather than boring those students who have already mastered the concept with continued whole-class instruction, this model encourages those students to move on to independent work quickly, freeing time for more intensive instruction with students who struggle with specific concepts.

Not only can the amount of instructional time differ but so can the content of the material addressed and the amount and level of difficulty of the practice work assigned. Guided Math groups offer teachers an efficient way to provide differentiated instruction to meet the needs of diverse learners. Chapter 5 examines, in greater depth, how to establish and effectively use small-group lessons for Guided Math.

## Math Workshop

So, when the teacher meets with small groups or conducts one-on-one conferences with students, what are the other students doing? For small-group instruction and conferencing to be effective, it must be uninterrupted. Students who are not engaged directly with the teacher should have meaningful work to do and know how to follow established and practiced procedures for independent or group work. These students work independently in a math workshop.

As the school year begins, students are taught how to work independently. The teacher establishes expectations and routines for this work during the first few weeks of school. Students learn how to access materials they may need, how to follow rules for working with manipulatives, how to handle any questions they may have, and what to do if they complete their assigned work. Periodically, the teacher may need to revisit these expectations.

Because each instructional minute of the day is so important, it is essential that each student is engaged in meaningful work. Providing something beyond busy work also helps prevent discipline problems because students who are working on challenging tasks are less likely to disrupt the class. At the same time, it is essential that the work be something that students can successfully complete independently. Math workstation tasks might consist of practice working with previously mastered content to deepen students' understanding, number sense activities that will help them improve their computational fluency, or math inquiries or investigations. In Chapter 6, math workshop is described in more detail with suggestions for establishing procedures and routines, planning workstation tasks, and effectively managing this essential component of the Guided Math framework.

**Power Phrase**

***math workstation:*** *a task students complete independently that provides practice, promotes computational fluency, or encourages mathematical curiosity*

## Math Conferences

The Guided Math framework offers teachers valuable opportunities to interact with students and observe communication between students as they work during small-group lessons. Sometimes one-on-one time with students is needed to aid the teacher in assessing student understanding of mathematical skills or concepts, clarifying or correcting student misunderstandings and errors, or extending and refining a student's understanding.

At any time, teachers can confer with individual students. In very much the same way that teachers have used one-on-one reading and writing conferences, these math conferences encourage learners to further their understandings of mathematical concepts. Because most students enjoy supportive individual attention from their teachers, math conferences also strengthen the relationships between students and teachers.

When one-on-one conferences are used effectively, teachers encourage students to reflect on their own understanding. Additionally, these conversations can generate rich information about how to best work with students. Teachers are able to identify specific teaching points for individual learners and for the class as a whole.

In Chapter 7, individual math conferences are covered in greater depth. A basic structure for individual conferences is presented, strategies for successful conferring are described, and methods for recording anecdotal notes are suggested.

## Ongoing System of Assessment

How do teachers determine the mathematical learning needs of their students? How do they know how to group their students for effective small-group lessons? How do they determine individual mathematical learning needs so that they can differentiate instruction in a productive way?

Ongoing and accurate assessment provides teachers with timely information about individual and class needs. In mathematics instruction, a student's level of proficiency can vary drastically from concept to concept. This makes assessing mathematical knowledge and thinking skills more challenging than assessing reading ability, where periodic running records and comprehension questions provide a strong indication of reading level.

Teaching Guided Math entails much more than simply following a textbook chapter by chapter and assigning the same set of problems to the entire class. With instructional time being so valuable, it is important not to waste time teaching what students already know. It is also important to refrain from moving ahead page by page if students are struggling. So, how do Guided Math teachers determine student needs?

A balanced system of assessment offers teachers a complete picture of each child's understanding rather than just a glimpse from the results of a single test. Formative and summative assessments—including observations of students' work, discussions with students, and assessment of their finished products—all give valuable perspectives on students' capabilities and needs. In addition, to maximize student learning, students themselves should be involved in assessing their own work based on established criteria, rubrics, or exemplars. Balanced assessment involves far more than just grades on tests and report cards. Chapter 8 examines the kinds of individual and class assessments that help teachers refine and extend their instruction to meet the needs of each student.

## Guided Math in Practice

What does Guided Math look like in a real classroom? What would others see if they dropped in for a visit? Chapter 9 provides an overview of how the components of this approach come together and are applied. A single classroom teacher can implement Guided Math, but collaboration among teachers makes the process easier and richer. Teachers should consider the various components of Guided Math using the Menu of Instruction (see Figure 1.1 on page 28) as they begin implementation. It may be easier for some teachers to focus first on a few of the components and then gradually add more. Others may prefer to carefully plan and then implement the entire framework at once. There is no one right way to approach implementation.

The very nature of this approach to teaching mathematics allows it to be incorporated flexibly into daily schedules. The constant daily features are the environment of numeracy, math warm-ups, conferencing, and assessment. Teachers in self-contained classrooms may incorporate them throughout the day. During the specific block of time allotted for mathematics instruction, teachers may choose from whole-group instruction, small-group lessons, math workshop, or a combination based on the level of support needed by the class for the mathematical content being taught.

**Figure 1.1—Guided Math Menu of Instruction**

**Daily: Classroom Environment of Numeracy**—A classroom should be a community where students are surrounded by mathematics. This includes real-life math tasks, data analysis, math word walls, measurement tools, mathematical communication, class-created math anchor charts, graphic organizers, calendars, and evidence of problem solving.

**Daily: Math Warm-Ups**—These daily tasks set the tone for math instruction. They may include calendar activities, math stretches, problems of the week, data work, incredible equations, classroom responsibilities, reviews of skills to be maintained, and previews of math content to come.

**Your Choice: Whole-Class Instruction**—This component is effective when students are working at the same level of achievement or when introducing new content with an activating strategy. It may also be used for teacher modeling and think-alouds, read-alouds of math-related literature, math huddles, whole-class math games, reviews of previously mastered skills, setting the stage for math workshop, and administering written assessments.

**Your Choice: Small-Group Lessons**—Students are instructed in small groups whose composition varies based on learning needs. These lessons are excellent ways to introduce new concepts, practice new skills, work with manipulatives, provide intensive and targeted instruction to below-level learners, introduce activities that will later become part of math workshop, conduct informal assessments, and reteach based on student needs. During this component, students can be actively engaged in the mathematical practices, working and communicating with each other as the teacher facilitates their mathematical construction of meaning.

**Your Choice: Math Workshop**—Students are provided with independent workstation tasks to complete individually, in pairs, or in cooperative groups. The work may consist of practicing previously mastered skills for greater understanding and retention, tasks to promote computational fluency, mathematical investigations, math games, math journals or other mathematical communication, or interdisciplinary work.

**Daily: Conferencing**—To enhance learning, teachers confer individually with students, informally assess their understandings, provide opportunities for one-on-one mathematical communications, and determine and deliver teaching points for individual students. Conferences can also help teachers identify previously unknown instructional needs that may become apparent during discussions with students.

**Daily: Assessment**—A balanced assessment system includes ample assessment of learning to inform instruction along with appropriate assessment of learning for each unit.

## Levels of Teacher Support

Levels of teacher support vary according to the instructional approach chosen by the teacher. Figure 1.2 provides an outline of instructional approaches, teacher activities, and support levels that complement the approaches and activities.

**Figure 1.2—Levels of Support for Guided Math Components**

| Instructional Approach | Teacher Activity | Level of Support |
|---|---|---|
| **Whole Group** | Activating strategies<br>Modeling<br>Think-alouds<br>Mini lessons (only when there is a compelling reason for them)<br>Leading students in formulating conjectures or math congresses<br>Directed review | Full support for all students by the teacher specifying the approach to problem solving or guiding the conversation; responsibility on the teacher |
| **Small Group** | Facilitate student exploration of new concepts/skills<br>Extend understanding of previously introduced concepts | Moderate level of support and targeted instruction based on student needs; more responsibility is released to students as the teacher facilitates and provides scaffolding for students who may have gaps in prerequisite knowledge and skills |
| **Math Workshop** | Prepare tasks and materials for workstations | Low level of support; tasks should be those which students can complete without teacher assistance; responsibility is shifted to students |

## Whole-Group Instruction: Full Level of Teacher Support

The ultimate goal of mathematics instruction is to lead students to a comprehensive understanding of mathematical concepts that they can then apply independently throughout their lives. The flexible nature of the Guided Math model allows for the gradual release of responsibility from teacher to students as described by P. David Pearson and Margaret C. Gallagher (1983). With this model, teachers are intentional about providing students with varying levels of instructional support. Ultimately, teachers enable their students to assume full responsibility for acting independently as mathematicians. This model is not the "I, We, You" model. In the "I, We, You" model, teachers simply show students a new procedure (I), lead the class in working together to apply the procedure (We), and then ask students to try it independently (You).

While the Guided Math "gradual release of responsibility" model begins with teachers doing most of the work during the whole-class instruction component of the framework, the focus at this time should not be on showing a set procedure to be replicated later by learners. Instead, teachers might lead students through an activating strategy to help them tap into their background knowledge and stimulate interest in a mathematical concept. Or, through modeling and think-alouds, teachers might share their thought processes as they engage in the mathematical practices. For example, in demonstrating how mathematicians approach problem solving, teachers might opt to model how they identify and clarify the information they have, determine whether or not they need to know more, consider what mathematical knowledge they already have that may help them solve the problem, visualize the problem, or review a variety of strategies to determine which might be most efficient to use.

This kind of modeling is vastly different from demonstrating a particular procedure that students will be expected to use without an understanding of the mathematical concepts. Rather than teachers using their time of maximum support to provide "students with a pre-packaged solution...right away and then...a bunch of (often contrived) problems which that solution can be used to solve" (Bledsoe 2014, n.p.), the time is used to develop their students' mathematical thinking and practices.

For this phase of the model to be most effective, it must be teacher-centered. Students listen, answer questions, and turn and talk with partners when requested. As a result, the teacher has minimal opportunity to monitor comprehension or communicate with most of the class. Therefore, it should be used sparingly and only with specific purposes in mind.

## Small-Group Lessons: Moderate Level of Teacher Support

The next stage of the "gradual release of responsibility" model for Guided Math calls for students to take responsibility for learning concepts and skills as they work in small groups. After determining the prerequisite knowledge and skills that learners need for success with each lesson, teachers identify which, if any, students have gaps in those areas. When introducing new content, teachers can then provide the appropriate scaffolding to fill those gaps. Students receive just enough scaffolding to challenge them to move beyond their current independent capabilities while still holding them responsible for working diligently to construct new mathematical meaning from their learning experiences.

When teaching small-group lessons, it is especially important for teachers to understand the positive impact of productive struggle. Students must be allowed to grapple with the complexities of math. James Hiebert and Douglas Grouws describe mathematical tasks that lead to productive struggle as those that are "within reach but that present enough challenge, so there is something new to figure out" (2007, 388). To make small-group lessons as effective as possible, teachers must recognize the very fine distinction between productive struggle and unproductive frustration when they scaffold the learning of their students. In general, many teachers find it difficult to avoid stepping in to "help" before students really have a chance to wrestle with challenging problems. Too much scaffolding deprives students of valuable learning opportunities. As teachers, we must consider how we can "provide scaffolds, such as purposeful questioning, that honor and build on the thinking of students without removing the demands of the task or doing the thinking for them" (Huinker and Bill 2017, 213).

Small-group lessons are ideal for challenging students with rich, open-ended tasks using what is sometimes referred to as a "You, Y'all, We" structure (Green 2014; Bledsoe 2014; Rawding 2018). With this instructional model (see Figure 1.3), students are first asked to try to make sense of a given task on their own. This is the "You" phase. After students have had time to explore and grapple with the task independently, they work collaboratively on the task with a partner. In this "Y'all" phase, working in pairs, students share their thinking and continue their efforts to make sense of the task. Finally, in the "We" phase, the entire small group comes together to discuss their thinking, the connections they made, and their findings in a math conversation facilitated by the teacher.

**Figure 1.3—Responsibilities in the You, Y'all, We Structure**

| | Teacher Responsibilities | Student Responsibilities |
|---|---|---|
| **You** | • Provide a rich, open-ended task.<br>• Observe students closely as they work.<br>• Confer with students to learn more about their mathematical reasoning.<br>• Ask purposeful questions to spur deeper thinking. | • Individually explore the task.<br>• Confirm understanding of task.<br>• Make math connections.<br>• Consider strategies to solve problems or to construct mathematical meaning. |
| **Y'all** | • Observe pairs of students as they work.<br>• Monitor the working relationships of the student partners to encourage productive collaboration.<br>• Confer with pairs of students to learn about their reasoning.<br>• Ask purposeful questions to spur deeper thinking. | • Collaborate with a partner to further explore the task.<br>• Share mathematical thinking about the task with a partner.<br>• Brainstorm ideas for a solution or understanding.<br>• Jointly devise a way to express how to solve the problem or explain new mathematical insights. |

| | Teacher Responsibilities | Student Responsibilities |
|---|---|---|
| **We** | • Maintain an environment in which students are comfortable sharing their mathematical thinking with others.<br>• Facilitate a math conversation among students regarding the task.<br>• Encourage student reflection.<br>• Ensure the participation of all students.<br>• Ask purposeful questions to spur deeper thinking.<br>• Support students in generating a summary of the ideas shared or mathematical conclusions reached. | • Exhibit active listening.<br>• Show respect for everyone and for the ideas shared.<br>• Share the ideas generated individually and with a partner.<br>• Consider carefully the ideas shared by others.<br>• Express agreement or disagreement with ideas shared by others along with supporting evidence.<br>• Actively collaborate with the group to arrive at a summary or conclusion. |

The small-group format is a setting in which students are comfortable sharing their thinking and taking moderate risks with teacher support. Teachers are able to closely observe and listen to students talk—a valuable informal assessment opportunity that allows for immediate feedback, the identification of misconceptions, the delivery of brief teaching points, and the type of questioning that provokes deeper and more critical thinking by fledgling mathematicians.

## Math Workshop: Low Level of Teacher Support

In the final stage of the Guided Math "gradual release of responsibility" model, students assume complete responsibility for their learning during math workshop. They work independently, individually, or in small groups on math workstation tasks. Before participating in this independent work, they should be very familiar with the procedures and expectations of the teacher and should also be able to carry out the assigned work with no additional guidance. It is during this time that they draw upon the mathematical concepts and skills they have already mastered and engage in tasks to improve computational fluency. Students might be involved in problem solving, investigations, games, math-related reading, mathematical writing, or other tasks to increase their independent mathematical competency.

The "gradual release of responsibility" model challenges students to assume more and more responsibility for their own conceptual understandings and problem-solving skills. They also become adept with mathematical practices and computational fluency. Since learning is rarely a completely linear process, the level of teacher support required for each student may vary from day to day and from lesson to lesson. Guided Math offers teachers an instructional framework that encourages students to gradually assume increasing responsibility as they learn, while at the same time providing scaffolding and support when needed.

## Scheduling Guided Math Components

Most teachers who implement Guided Math in their classrooms feel that it is important to have small-group lessons and math workshop every day. Maintaining a regular routine is beneficial for all students. They know what to expect each day. Usually, three to four small-group lessons are conducted daily as other students work independently at math workstations. It is with this kind of consistency that the Guided Math framework is most effective.

For teachers just beginning implementation or who desire more options for their mathematics instruction, the framework offers flexibility. Figure 1.4 (page 36) shows how the components of Guided Math may be woven together for instruction during the week.

***On Monday***, the entire mathematics period is taught as a whole group as the class begins with an activating strategy. A problem is then presented, which the teacher solves by thinking aloud to explain the thought process—not a procedure—to students. Following the problem-solving activity, the independent work for the week is introduced with a mini lesson.

***On Tuesday***, the class begins with a read-aloud of mathematics-related literature as the whole class gathers to listen and then to discuss the mathematical connections. Following the read-aloud, students begin independent work in math workshop. The tasks were explained on Monday, so students should be ready to begin with little additional direction. For the first 15 minutes of math workshop, the teacher circulates around the classroom, conferring with individual students. For the last 30 minutes, the teacher meets with Guided Math group 1. The lesson is tailored specifically to the needs of these students who have been grouped together because of their similar skills. They may be students who have already mastered what most of the students are currently working on, and they can be given more challenging instruction. Or they may be students who the teacher noticed have a particular problem that can be addressed easily through small-group instruction. Sometimes, they are students who need additional scaffolding and support with the concepts on which the class is working.

***On Wednesday***, the structure is similar to that of Tuesday. The class begins with a mini lesson, but this time, a problem is posed for the class to solve and discuss. Math workshop begins with the teacher initially engaged in conferencing. The teacher meets with Guided Math group 2 after conferencing, which is a group with different needs.

***On Thursday***, there is no mini lesson so that the teacher can meet with two groups. Students begin math workshop immediately. The teacher spends 15 minutes conferencing, followed by 30 minutes with Guided Math group 3. For the last 30 minutes of the class, the teacher meets again with group 1 from Tuesday for additional instruction.

***On Friday***, the schedule is quite different. Students meet for whole-class instruction as they participate in a math huddle. During this time, they share and discuss their observations, problem-solving strategies, conjectures, or representations with the class. The ideas shared are recorded in a chart or graphic organizer that is posted in the room for future reference.

**Figure 1.4—Sample Flexible Guided Math Schedule**

| | Activity | Guided Math Component |
|---|---|---|
| Monday | • activating strategy<br>• problem-solving think-aloud by teacher<br>• explanation of independent work for the week (investigation, paper/pencil practice, games) | Whole Class |
| Tuesday | • mini lesson (read-aloud)<br>• independent math work with teacher conferencing for the first 15 minutes<br>• Guided Math with group 1 | Whole Class<br>Workshop<br>Conferencing<br>Small Group |
| Wednesday | • problem challenge mini lesson<br>• independent math work with teacher conferencing for the first 15 minutes<br>• Guided Math with group 2 | Whole Class<br>Workshop<br>Conferencing<br>Small Group |
| Thursday | • independent math work with teacher conferencing for the first 15 minutes<br>• Guided Math with group 3<br>• Guided Math with group 1 | Workshop<br>Conferencing<br>Small Group |
| Friday | • math huddle (students share their observations, problem-solving strategies, conjectures, representations)<br>• create a chart or graphic organizer to post in the classroom for reference | Whole Class |

This weekly plan is just one example of the kind of flexibility the framework offers teachers. When planning, it is important to consider the curriculum and the students to determine which of the components work best for each day of instruction.

The U.S. Department of Education's National Mathematics Advisory Panel (2008) suggests that research does not support the contention that mathematics instruction should be completely teacher-centered or student-centered. Instead, it should be "informed by high-quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers" (xiv). While small-group lessons are at the heart of Guided Math and are an essential characteristic of the framework, teachers are encouraged to use their professional judgment to structure mathematics instruction to meet the diverse needs of the students in their classes, curricular demands, and even their own teaching styles. Guided Math frees teachers from the one-size-fits-all model and empowers them to determine the best instructional strategies for each student, for the class, and for the concepts being taught each day.

## Review and Reflect

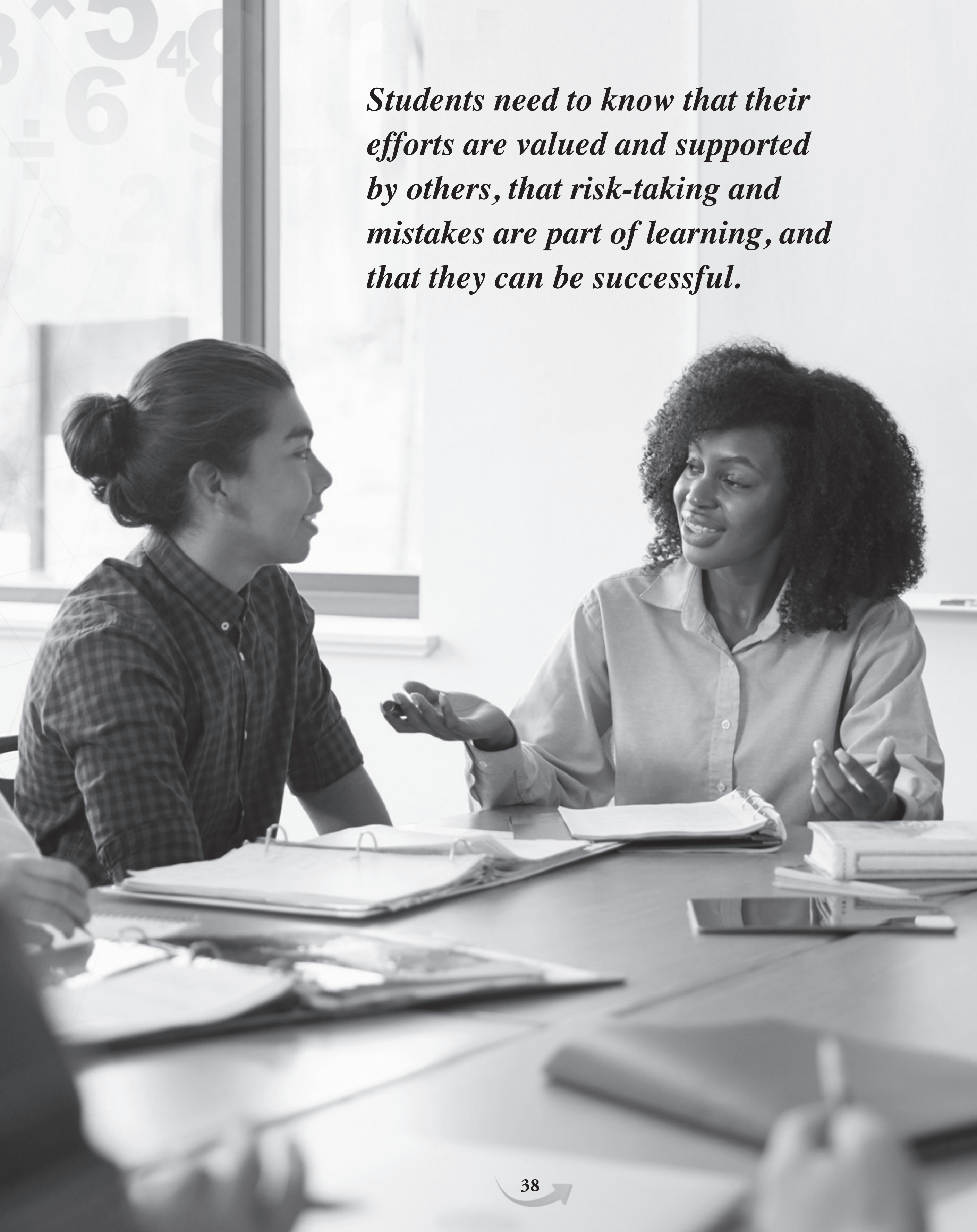
Think of the way you currently teach mathematics.

1. What aspects of your instruction are the most successful?

2. What aspects of your instruction trouble you? Why? In what ways might you wish to change your instruction?

3. Does your math instruction lead your students to a deep conceptual understanding of the math standards that they are learning? If so, what are you doing that contributes to that? If not, how do you think you would like to change your teaching?

*Students need to know that their efforts are valued and supported by others, that risk-taking and mistakes are part of learning, and that they can be successful.*



# Chapter 2

## Creating an Environment of Numeracy

Anyone entering most elementary school classrooms would have no problem determining that reading and writing are being taught there. With overflowing bookshelves, books on desks, poems of the week, alphabet charts, graphic organizers showing story elements, word walls, journals, writing folders, writing centers, vocabulary charts, sets of leveled books, and cozy places to read, it is quite obvious that serious reading and writing are taking place throughout the day.

**Power Word**

***numeracy:*** *the ability to understand and work with numbers*

If only the same were true for mathematics! Certainly, some signs of mathematics instruction may be evident—manipulatives, math books, posted student work, and perhaps a calendar board. However, compared to the environment of literacy that we have worked so diligently to create for our students, we often neglect to do the same for numeracy.

Professional writing about teaching literacy abounds and encourages teachers to create classroom environments of literacy for their students (Fountas and Pinnell 1996; Fountas and Pinnell 2001; Miller 2002; Calkins 2000; Collins 2004). Fountas and Pinnell describe a classroom where "every day, every child in the classroom encounters materials that she can read and that are of interest" (1996, 43). As an underlying theory, they state, "Children learn about written language in an environment that is print rich" (1996, 43). Abundant collections of literature, read-alouds, shared reading, shared writing, interactive writing, writing centers, poetry charts, reading workshop, writing workshop, use of authentic print, and writing for authentic purposes are characteristics of such an environment. Throughout the day, students are immersed in print and conversations about reading and writing, but more than just these attributes contribute to the creation of the rich climate of learning envisioned by these authors.

Teachers strive to organize their classrooms by interweaving these elements and making them accessible to all. That's not an easy task. It requires forethought and planning; no two teachers will do this in exactly the same manner. The organizational structures teachers create reflect their own teaching styles. Not only must the physical arrangement of the room be conducive to this kind of literacy instruction, but procedures that will facilitate it also have to be developed and taught to students as well. When planning, skillful teachers give thoughtful consideration about how to instill a love of reading and writing in their students, as this is essential in building a community of literacy learners. Again, this is no easy task, but it is being done effectively in classrooms every day.

Is the same happening for mathemathics in classrooms? Fortunately, more math teachers at all grade levels are recognizing the benefits of establishing an environment of numeracy similar to the environment of literacy described above. The same techniques can be adapted to mathematics instruction and are equally successful. Now, many more classrooms show vivid evidence of rigorous and exciting mathematical engagement by students. What about the other classrooms?

## Foundational Principles of a Guided Math Classroom

Teachers who have not already done so can begin the process of creating such an environment for their math students by adhering to the foundational principles (see Figure 2.1) upon which the Guided Math framework is built—principles which are very similar to those for literacy instruction or, for that matter, for quality instruction in any content area.

**Figure 2.1—The Foundational Principles of Guided Math**

| |
|---|
| All students can learn mathematics. |
| A numeracy-rich environment promotes mathematical learning. |
| Learning, at its best, is a social process. |
| Learning mathematics is a constructive process. |
| An organized classroom environment supports the learning process. |
| Modeling and think-alouds of mathematical practices by teachers, combined with ample opportunities for collaborative problem solving and purposeful conversations, create a learning environment in which students' mathematical understanding grows. |
| Ultimately, students are responsible for their learning. |

***All children can learn mathematics.*** Although No Child Left Behind brought this principle to the forefront some years ago, it is something that good teachers know instinctively. It is our responsibility to see that *all* students are challenged to achieve in math. The U.S. Department of Education's National Mathematics Advisory Panel (2008) reports that students' beliefs about learning are directly related to their performance in mathematics. Studies have shown that when they believe that their efforts to learn make them smarter, they are more persistent in pursuing their mathematics learning. Carol Dweck's (2006) research confirms what many of us already suspected. Students' mindsets about intelligence affect their learning. Those with a fixed mindset believe their mathematical intelligence is set and nothing they do will change it. For many, this means believing that they just don't have a mathematical mind. Frequently, parents excuse their children's lack of achievement in mathematics as if it were an inherent ability that one either has or doesn't have. Upon encountering this kind of attitude, students are often convinced that no matter how much work they put into learning math, they will never be successful. On the other hand, with a growth mindset, students understand that their basic qualities—including their mathematical capabilities—can be cultivated through effort. Rather than errors being confirmation of a lack of mathematical talent, students see them as learning opportunities and challenges to tackle. To truly meet the mathematical learning needs of our students, we need to create an environment where they appreciate the direct relationship between their efforts and their learning.

***A numeracy-rich environment promotes mathematical learning.*** Borrowing from research about literacy education, where immersion in a literacy-rich environment is considered essential to promote learning, it is equally important that students are immersed in a world of mathematics (Cambourne 1988). As students see numbers and math-related materials throughout the classroom and participate in real-world, meaningful problem-solving opportunities, they begin to see the connection mathematics has to their own lives. Mathematics is no longer solely problems in a textbook, but it becomes a discipline to ponder and wonder about.

***Learning, at its best, is a social process.*** Lev Vygotsky (1978) stressed the importance of children verbally expressing their ideas in the process of reasoning for themselves. Children develop language through their experiences and begin to generalize their ideas through oral communication with a teacher or with fellow students. Reflective conversation and dialogue within a classroom setting are tools. These tools encourage students to engage with the ideas of others and to construct hypotheses, strategies, and concepts (Nichols 2006).

Learning is enhanced as students work with others exploring the same mathematical ideas (Van de Walle and Lovin 2006).

***Learning mathematics is a constructive process.*** Catherine Fosnot and Maarten Dolk (2001) describe mathematical learning in this way: "Children learn to recognize, be intrigued by, and explore patterns as they begin to overlay and interpret experiences, contexts, and phenomena with mathematical questions, tools (tables and charts), and models (the linear Unifix train vs. the circular necklace). They are constructing an understanding of what it really means to be a mathematician—to organize and interpret the world through a mathematical lens. This is the essence of mathematics" (Fosnot and Dolk 2001, 9–10).

***An organized classroom environment supports the learning process.*** Efficient organization of materials, wise use of time, and practical procedures established for students contribute to the effectiveness of the learning environment.

***Modeling and think-alouds of mathematical practices by teachers, combined with ample opportunities for collaborative problem solving and purposeful conversations, create a learning environment in which students' mathematical understanding grows.*** Teachers set the stage for learning in their classrooms. When teachers model how to think like a mathematician to solve both real-life and abstract mathematical problems, students become aware that there is rarely only one correct way to approach solving problems or mathematical exploration. Students begin to recognize the relevance and importance of the mathematical practice skills they are learning. An environment where students feel free to take risks and where mistakes are viewed as opportunities to learn encourages students to investigate, recognize relationships, and form generalizations from their experiences. Effective teachers orchestrate instructional strategies based on the content being learned, the needs of the class, and the needs of individual students, with the ultimate goal of supporting all students as they begin to understand mathematical "big ideas" and grow proficient at organizing and interpreting the world through a mathematical lens (Fosnot and Dolk 2001).

***Ultimately, students are responsible for their learning***. As Marilyn Burns puts it, "You cannot talk a child into learning or tell a child to understand" (2000, 32). That doesn't mean that a teacher is absolved of responsibility. Educators are tasked with establishing the motivation and opportunity for students to learn. Lex Cochran vividly describes a teacher who understood this principle: "And once I had a teacher who understood. He brought with him the beauty of mathematics. He made me create it for myself. He gave me nothing, and it was more than any other teacher has ever dared to give me" (1991, 213–214).

*The foundational principles are adapted from the underlying theories upon which Guided Reading is built, as described by Fountas and Pinnell (1996, 43–44).*

## Building a Classroom Learning Community

There is not a single correct way to teach. However, it is essential teachers provide a challenging and supportive classroom learning community (NCTM 2000). In such a classroom, students understand that they can and, indeed, are expected to engage in making meaning of their world mathematically (Fosnot and Dolk 2001). In this kind of learning community, students are not only given opportunities to learn the "big" ideas of mathematics; they also participate in a carefully supported climate of inquiry where ideas are generated, expressed, and justified, thus creatively exploring mathematical relationships and constructing meaning.

The teacher is no longer the "keeper, dispenser, and tester of knowledge." Instead, being part of the community, the teacher shares in the process, acting as a "model, facilitator, and, at times, a co-learner" (Nichols 2006, 31). The role for students shifts from the traditional classroom model where they sit at their desks, simply listening, answering questions, and receiving information—being filled with knowledge bestowed by the teacher—to one in which students are active participants in their learning.

Of great importance in maintaining a sense of community is respect for all classroom members and the feeling that, as students, they are valued. Students need to know that their efforts are supported by others, that risk-taking and mistakes are part of learning, and that they can be successful. They need to see that mathematics is a creative exploration, a search for patterns and relationships based on mathematical ideas. They need to see that, in this exploration, each member of the community plays an active role and that contributions of differing perspectives often lead to a more complete understanding by all. The understanding of the entire community is more than that of each member on their own. Thus, the importance of the teacher's role in establishing a classroom community cannot be overstated.

In a productive mathematical community, each member is respected and valued. Students follow the lead of their teacher in learning how to treat others. When a teacher has high expectations for all students, they often live up to those expectations, and other students' perceptions of them reflect those expectations. It is unfortunate that in our society we tend to believe that only *some* students are capable of learning mathematics (U.S. Department of Education 2008; NCTM 2000). Even parents often excuse poor achievement by saying that they were "never good in math when they were in school," as if mathematical ability were an inherited talent. Therefore, teachers must convince students, parents, and the community that expectations are high for all students and that effort and achievement in mathematics are linked.

For mathematics education to be truly equitable, skillful teachers realize that high expectations alone will not provide equity for all students, and without equity for all students in the class, the sense of community flounders. In the Guided Math framework, each component offers teachers a myriad of approaches to provide scaffolding and support for all learners.

Communication is at the heart of mathematics—it is essential to clarify thinking, to express ideas, to share with others, to justify processes, and to explore relationships. The NCTM Communication Standard (2000, 60) specifies that instructional programs from pre-K to grade 12 should enable students to do the skills list on the following page.

- to organize and consolidate their mathematical thinking through communication
- to communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- to analyze and evaluate the mathematical thinking and strategies of others
- to use the language of mathematics to express mathematical ideas precisely

The other NCTM process standards (Problem Solving, Reasoning and Proof, Connections, and Representation) also rely heavily on student communication.

Additionally, many states have either adopted the Common Core Math Standards (Common Core State Standards Initiative 2015) or developed state standards very similar to them. Prominent in these standards are the eight Standards for Mathematical Practice (SMPs). The SMPs are consistent for grades K–12. The importance of mathematical communication is implicit in most of the eight standards. It is explicitly addressed in the third and sixth standards in which students must be able to “construct viable arguments and critique the reasoning of others” and “attend to precision.”

Teachers throughout the United States grapple with how they can best provide standards-based instruction on mathematical practices or processes for their students. Until recently, many of the traditional textbooks offered minimal instructional resources to address these standards. Resource materials, when provided, most often presented the standards in isolation. Clearly, though, mathematical practice standards are intended to be taught along with the content standards rather than as stand-alone instruction. When mathematical practices are compartmentalized and taught in isolation, students have difficulty visualizing mathematics as a coherent whole (Hyde 2006). It’s ironic that mathematicians have, for so long, recognized the importance of mathematical communication, yet it has traditionally played such a limited role in mathematics classes in schools, from elementary through high school.

Quality student conversation and dialogue does not occur because the standards demand it or the teacher values it. Highly effective teachers model and give instruction to students on how best to communicate mathematical ideas. They teach students the give-and-take of conversing and clearly define the criteria for quality math talk. What does this type of conversation sound like? What are students' responsibilities when engaged in mathematical conversations? Students learn how to engage with others, to construct hypotheses, to reason, and to justify (Nichols 2006). This accountable talk is not just chitchat. It has a process. It has a purpose. As such, it is part of the means by which students construct meaning in the world of mathematics.

In the book *Making Sense* (Hiebert et al. 1997), the authors list four characteristics of a productive classroom community for mathematics in which communication plays a major role:

1. Ideas are the "currency" of the classroom. All students' ideas have value and can contribute to the learning of the community when shared. Carefully considering the ideas of others is a sign of respect.

2. There is often more than one approach to solving a problem. Students have the freedom to explore alternative methods and then share them with others. Each student, in return, must respect the ideas and approaches of others. This respect for the ideas shared by others allows genuine classroom conversation to occur.

3. Students must understand that it is okay to make mistakes—that errors are opportunities for growth as they are examined and explained. Rather than being something to cover up, mistakes become opportunities to explore erroneous reasoning, and therefore can be used constructively.

4. The correctness or reasonableness of a solution depends on the mathematics itself, rather than the popularity or status of the presenter. Students learn that mathematics makes sense. The focus of discourse is reasoning and logic as it relates to mathematics rather than on satisfying a figure of authority. This understanding frees students to actively explore problem-

solving strategies rather than passively waiting for teacher guidance and boosts confidence in sharing their thinking.

These characteristics are nurtured in classrooms when teachers consider carefully, and then implement, classroom procedures that promote them. This can be accomplished through the establishment of ground rules for discourse and through teacher modeling (Van de Walle and Lovin 2006). In *Thinking Mathematically*, the authors suggest establishing class norms where "students explain their thinking, they listen to one another, and alternative strategies for solving a given problem are valued and discussed" (Carpenter, Franke, and Levi 2003, 35). To encourage accountability in conversation and move construction of meaning to new levels, Maria Nichols (2006) suggests that students be taught that purposeful talk means that they must say something meaningful, listen with intent, and keep the lines of thinking alive (stay focused on the topic being discussed).

Specific practice in the art of conversing can help students perfect their skills in discourse. Often, students are unaware when they either dominate the conversation or are not a part of it at all. Daily talks with parents or peers rarely prepare them for the kinds of dialogue that lead to constructing meaning within a community. Students emulate teachers as they model respectful and worthwhile conversational behavior. Teachers can help students become more aware of who is speaking and of the value of taking turns. Students learn the value of being a listener, reflector, and participant in extending and developing the ideas set forth. As students learn to pause and think before speaking, their comments become more focused. Turning and talking with partners provides students opportunities to practice comments that they may want to share with the group. Sharing one-on-one builds confidence that some students need before they open their thoughts to a larger group. Even turn-and-talk behavior must be taught so that students understand that it is not just each person sharing an idea—they are also expected to be listeners who engage with each other's ideas (Nichols 2006). Effective teachers know that this kind of conversational behavior does not develop on its own. They recognize, in planning, that teaching it is their responsibility.

As students become more practiced in conversing respectfully with purpose, teachers find that it strengthens the sense of community in a classroom. Not surprisingly, an increased sense of community then improves the discourse (Nichols 2006).

## Classroom Arrangement

At the beginning of school every year, teachers typically grapple with how to set up their classrooms. Aesthetically, they want them to be warm and welcoming. They envision their students as they first enter their new school "home." How will they feel? Teachers ponder ways in which they can create an environment that shows their new students how much they are valued.

In addition to the desire to create a welcoming environment, teachers know that the way they set up their rooms can also affect student learning. As desks are set out, bulletin boards are put up, and learning materials are organized and stowed, teachers are concerned with the more subtle messages that students receive about what is valued in learning (NCTM 2000).

Rows of desks facing forward, few available manipulatives, and the absence of student-created and mathematics-related charts send a clear message that they're in a teacher-centered classroom. Students are expected to sit by themselves during lessons. There are no tables or areas available for collaborative discussions or activities. Mathematics instruction becomes a static process instead of an active process.

Other classroom arrangements send different messages altogether to students. In these classrooms, desks are arranged in groups or students sit at tables. There is a large, carpeted space with an easel or whiteboard available. Math manipulatives are organized and are readily accessible. There is a table with seating for six students where small groups meet with the teacher. On the walls are class-created charts of conjectures they have made or of problem-solving strategies. An interactive calendar board may be displayed. Math word walls contain mathematical vocabulary words with definitions and representations. Math-related literature is displayed with student work. Students in these classrooms are clearly creatively engaged in the process of constructing mathematical meaning. In these classrooms, there is evidence of a community of learning at work.

As teachers design their classroom arrangements for Guided Math, they are sensitive to the subtle messages they send and to the effects of the physical layout on not only their students but on the learning process as well. The overall goals in arranging the classroom setting for Guided Math are as follows:

- establish appropriate spaces for each Guided Math component
- create spaces conducive to the social aspects of learning
- facilitate efficient movement within the classroom
- provide ease of access to materials needed by both students and the teacher

Teachers who have already established a classroom arranged to teach Guided Reading have a head start on this process. Their classroom arrangements will most likely work well for Guided Math.

In the Guided Math framework, teachers consider how the classroom arrangement can promote student independence during math work. Greater student independence is possible when there are clearly designated work areas so students know how to find, and then replace, the materials they need as they work. As student capacity to work independently increases, the teacher has more time to address the needs of individual students. During the school year, if it is necessary to modify the arrangement, students can be involved in that process, adding to their sense of ownership and understanding of how the classroom environment affects their work (Fountas and Pinnell 2001).

## Home Areas

Within a Guided Math classroom, students should have their individual home space where they keep their own supplies. This space may be at desks or at tables. Desks should be grouped to encourage student interaction during math class. With this configuration, students may stow their own supplies in their desks. Tables, cubbies, or crates can be provided to hold individual student materials. Either classroom arrangement is designed to provide students easy access to any materials they may need for mathematical exploration and learning.

A Guided Math class begins with math-related warm-up assignments that students are expected to begin immediately upon entering the classroom or as math class begins. In self-contained classrooms, teachers may choose to have students engage in math warm-up tasks as they come into the classroom in the morning, even if math is not the first subject of the day. Students should be thoroughly familiar with these warm-up routines. The tasks to be completed must be clearly described and readily available so that teachers can focus on individual student needs without having to stop what they are doing to give directions as students arrive or as class begins.

## Large-Group Area

Ideally, a part of the classroom is set aside for large-group gatherings or meetings. A carpeted area, if possible, makes a comfortable gathering place for students—although older students may balk at the idea of sitting on the floor. Placement of this area adjacent to an interactive whiteboard or a calendar board allows it to be used for lessons related to warm-ups or calendar lessons as well as for any whole-class lessons. In this gathering area, it is helpful to have an easel with chart paper or a whiteboard that may be used by the teacher and students for mini lessons, modeling, think-alouds, read-alouds, the creation of student-created anchor charts, the listing of steps in problem solving, or the recording of student conjectures. This area should also be supplied with markers, pointers, manipulatives, math tools, and, possibly, math-related books. A hundred chart, number line, or other applicable mathematical graphic representations should be clearly visible for students to reference.

Realistically, not all teachers have classrooms large enough to set aside a separate meeting space. Small classrooms with large classes often make this impossible. If space is an issue, teachers can plan the placement of desks and/or tables in a way that maximizes the inclusiveness of all students during large-group instruction and math conversations. Teachers should never feel compelled to maintain their original room arrangement. If the room layout does not work well or has stopped being effective, it can always be changed to improve learning for students.

## Small-Group Area

An essential space for classrooms designed for Guided Math is an area for small-group lessons. This area usually includes a table that can accommodate up to six students and a teacher. Although a table is preferable, some teachers choose to work with small groups on a carpeted area seated on the floor or at a grouping of desks. Whether working at a table, at desks, or on the floor, the teacher should have an unobstructed view of the rest of the class to maintain order. To maximize both prep time and instructional time with small groups, this meeting area needs to be well-equipped with everything that may be required for lessons. This may include small whiteboards, markers, paper, pencils, erasers, calculators, math tools, work mats, and manipulatives. Since small-group instruction offers ample opportunities for informal assessment, the teacher also needs to have a clipboard or other system for maintaining anecdotal notes or record keeping. In addition, it is useful to have student records readily available, as small-group instruction is tailored to address the needs of individual students. It is important to let students who are called to the group for a lesson know what materials, if any, they need to bring with them so that this valuable instructional time is not wasted by students having to return to their home areas for materials.


***This instructional area is clean and organized.***

## Math Workshop Area

When they work independently in math workshop, students can use space throughout the classroom. Those who are working individually may work at desk, tables, or find space on the floor. Groups of students working collaboratively have the same options. In a well-managed math workshop, students are taught expectations and procedures during the first few weeks of school. The teacher may choose to specify where students will work or allow students to choose their own work spaces. During this independent work time, it is important that students know exactly how to access any materials that they may need and how to return those materials when they are finished. The teacher should also share expectations about student movement around the classroom during workshop time. Again, these procedures should be carefully considered by the teacher and then taught during the first few weeks of school. The procedures may need to be reviewed with students or revised at times throughout the year. Because the teacher is involved in small-group instruction or math conferences with students during workshop time, students who are not with the teacher need to be aware of the behavioral expectations and working responsibly on their own to ensure that the teacher is uninterrupted.

## Organization and Storage of Materials

*Using shelves and tubs keeps instructional materials neat and organized.*

The expression "a place for everything and everything in its place" is especially apt when considering how to organize the many mathematical resources most teachers accumulate over their years of teaching—materials from math professional development; from textbook adoptions; from purchases by the central office, the school, or the grade level; and materials purchased by the teacher. Not only do teachers need to know where their materials are and be able to find them easily when they are to be used, but if students are to be more self-sufficient and independent during mathematics instruction, they also need to be able to access these materials easily. To reiterate one of the Foundational Principles of Guided Math: *an organized classroom environment supports the learning process.*

The first step in organizing mathematics materials is to sort through them and eliminate any that will not be needed. When the materials are culled, they may be sorted according to whether they are for teacher use only or also for student use.

Resources primarily for teacher use are best stored in areas accessible only by the teacher and organized so that they can be readily located when needed. The organizational methods will vary from teacher to teacher. This seems like common sense, but it takes time and effort to organize materials. This time is valuable for teachers. Making the choice to spend time organizing is often difficult, but in the long run, it makes teaching much easier and more effective during the school year. The process of sorting and organizing materials is best done at the beginning or end of the school year. For teachers who are unsure how to organize materials, it is helpful to ask other teachers for suggestions. Using the ideas of others makes it much easier to develop a system of one's own.

Those resources that are to be accessible to students need to be just that. When these materials are well-organized and accessible, students can interact with them independently. This frees the teacher to work with small groups or confer with students. It also makes classroom management easier. Fountas and Pinnell (2001) suggest ways to organize materials for Guided Reading and they work equally well for mathematics materials.

- Place each type of material in a separate container that is appropriate in size and shape.
- Label every container as well as each space in which containers are stored.
- Don't depend on students to arrange materials appropriately on shelves. Instead, have a designated and labeled space for everything.

- Eliminate nonessential materials. An easy test is to determine whether you have used the materials within the last year. If you haven't, get rid of them! Accumulating materials year after year clutters your classroom and interferes with efficient management.

Mathematics materials for students should be placed near areas where students work independently but away from the small-group instruction area. In planning, consider student traffic patterns to prevent traffic jams and the resulting disruptions as students obtain and return materials.

Students also need clearly defined areas for turning in their work, storing math journals, or keeping mathematics portfolios. Some teachers use trays, bins, or baskets. Prior to introducing the organizational structure to students, everything should be well thought out with clearly labeled materials.

## A Numeracy-Rich Environment

Just as literacy-rich environments are essential in teaching reading and writing, so are environments rich in numeracy in teaching mathematics. Mathematical learning is both a social and constructive process (Vygotsky 1978; Steele 1999; Van de Walle and Lovin 2006). Students learn best through active engagement in authentic opportunities to extend their number sense, develop a deep understanding of mathematical concepts, and apply what they have learned about math to evaluate other mathematical ideas and to solve problems. The creation of classroom environments supporting mathematical literacy enables students to build on their previously acquired knowledge of the discipline of mathematics. Providing an organized mathematical support system for students requires that we encourage students to use manipulatives, compute, compare, categorize, question, estimate, solve problems, talk, and write about their thinking processes. A classroom that clearly demonstrates the importance of sharing students' mathematical ideas promotes a culture of mathematical discourse (Ennis and Witeck 2007). Ideally, a mathematically rich classroom environment and engaging tasks will help students become increasingly aware of mathematics and its crucial relationship to their everyday lives.

## Student Calendars or Agendas

Many elementary classrooms have monthly calendars posted on display boards or bulletin boards where daily calendar lessons are conducted in large-group settings. During these lessons, students participate by responding to questions as a group or when called upon by the teacher.

While these lessons do add to students' understanding, for all students to be fully engaged, each student should maintain an individual calendar or agenda. Early in the year, kindergarten students will take out their calendar folders, point to the day of the month, and then mark it off by coloring the square for the day. As the year progresses, they begin to record the numeral in the correct box each day. They may color weekend days in a different color from the weekdays to reinforce those concepts. Some kindergarten teachers have their students create tally charts to indicate the number of days they have been in school.

The complexity of calendar-related tasks increases as students progress through school. This should be reflected in the individual calendar organizers that students use to manage their time and school tasks. Middle and high school teachers have discovered the long-term value of helping students establish the habit of maintaining a calendar for their daily responsibilities. The use of individual calendars or agendas (paper or digital) helps students connect the classroom calendar activity to their own lives in meaningful ways. Practicing these kinds of organizational skills benefits students throughout their lives. Because students so rarely recognize how mathematics affects their daily lives, the role of the teacher is crucial in providing the kinds of activities that increase their awareness (Bamberger and Oberdorf 2007; NCTM 2000). When teachers are explicit about the link between math and the use of calendar/daily organizers, it makes this an activity that fosters students' awareness.

## Manipulatives

Mathematics classrooms should offer a wide selection of manipulatives for students to use as they represent concepts and problems being studied (Ennis and Witeck 2007). Using manipulatives provides a concrete representation that establishes an image of the knowledge or concepts in students' minds (Marzano, Pickering, and Pollock 2001). Research on mathematics education has supported the fact that when manipulatives are used in math classes, students usually outperform other students (Sarama and Clements 2009). According to Douglas Clements (1999, 45), "this benefit holds across grade level, ability level, and topic, given that use of a manipulative 'makes sense' for that topic."

In primary classrooms, concrete manipulatives are almost universally used. More and more upper-grade classrooms are also beginning to use manipulatives regularly. Unfortunately, many older students do not understand that experienced mathematicians often turn to manipulatives to represent mathematical concepts and solve perplexing problems. They mistakenly perceive manipulatives as being appropriate only for very young learners. Because of this common misconception, many older students are reluctant to use them.

Teachers play a critical role in reshaping the attitudes of older students. Middle and high school classrooms where manipulatives are available and used for challenging mathematical problems offer rich learning environments that encourage students to represent their thinking in multiple ways, including using concrete models. Research indicates that many of the digital manipulatives available are just as effective in representing mathematical concepts and problems for learners as tangible manipulatives. It is their manipulability and meaningfulness that make manipulatives so valuable to learning (Clements and McMillen 1996; Clements 1999; Sarama and Clements 2009; Burris 2013). Digital manipulatives are often more readily accepted by older students.

It is important for students to use manipulatives themselves and not only see them used by teachers for demonstration purposes. Some teachers may feel that it is too time consuming to distribute manipulatives or believe that supervising manipulatives is difficult. That is often the case with whole-group instruction. Students may play with the manipulatives, argue with partners, or fail to share, so it is easy to see why some teachers shy away from using manipulatives. However, when students are explicitly taught to use manipulatives appropriately and effectively, they can be one of the most powerful tools in mathematics instruction. The components of the Guided Math framework provide teachers with several instructional formats, in addition to whole-class instruction, in which the use of manipulatives can be more efficiently managed. These will be explored in later chapters.

In a Guided Math classroom, teachers may choose to demonstrate using manipulatives in a whole-class setting or during a small-group lesson. Later, when working with small groups of students, teachers should allow students to use them in a supervised environment. After some experimentation and guided practice, students may work with manipulatives independently during math workshop. Throughout this instructional sequence, the responsibility for using manipulatives is gradually released from teacher to students.

## Concrete-Representational-Abstract Instruction

When effective teachers encourage the use of manipulatives, they also are aware of the importance of students moving from Concrete to Representational to Abstract (CRA) levels of understanding. Frequently, there is a tendency to expect students to progress quickly from working with concrete objects to abstract work with little to no experience with the representational level. The representational level offers an important bridge between the concrete and abstract levels. Unfortunately, if students have not achieved this foundational understanding, working on an abstract level may prove to be very difficult.

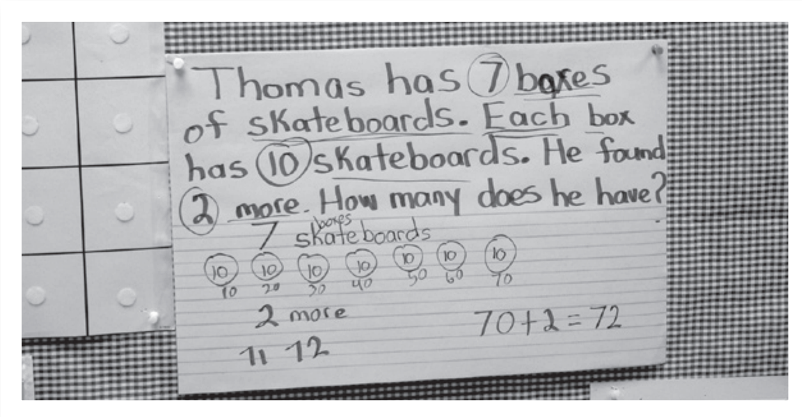
While educators speak of CRA as a progression, it's important to understand that CRA is not always a linear journey. Students may need to move back and forth along the progression, depending upon the mathematical content with which they are working and with their own levels of understanding. Experienced teachers plan their CRA instruction by carefully listening to and observing their students as they work. With this kind of informal assessment, teachers can identify where along the CRA progression students' conceptual understanding falls and then can flexibly adjust instruction to meet their immediate learning needs.

It is important to remember that manipulatives are only a means to an end; their use is not the end goal itself (Clements 1999). These learning tools only enhance students' conceptual mathematical understanding when the objects, whether physical or virtual, are appropriate to represent and illustrate the concepts being learned *and* when students actually link their actions with the manipulatives to the mathematical concepts being examined. Just as students can learn how to perform abstract procedures while lacking mathematical understanding, the same can be true for working with manipulatives. Students may learn how to manipulate objects without ever making the essential mathematical connections. That is why it is imperative that students talk about what they are doing as they work. Students must explain the whys behind their actions.

As students gain experience using different types of manipulatives, they should have access to them so they can decide which tools they will use for representing mathematical ideas or solving problems. They will not always choose wisely, but students learn from their experiences, especially when expected to talk with others about their choices and how well they worked. It is during these conversations that the collective knowledge of the mathematical community of students grows, learning from each other. Students inspire one another to think more deeply about the mathematical concepts they are exploring.

## Problem of the Week

In Guided Math classrooms, teachers take advantage of the natural curiosity of students. Problems of the Week encourage students to explore, investigate, and hypothesize—all of which appeal to their innate inquisitiveness. Students' active engagement in problem solving provides them with opportunities to develop their mathematical skills and understandings (Schackow and O'Connell 2008).



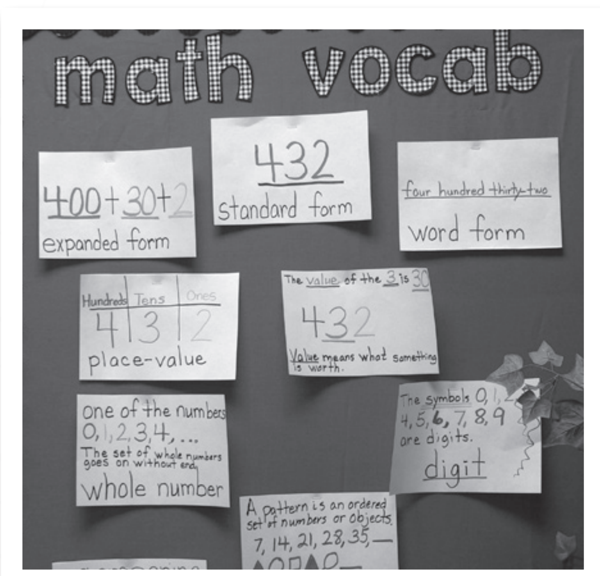

***Students solved this Problem of the Week by drawing a diagram.***

Students or visitors entering a Guided Math classroom encounter ample evidence of problem solving by the class, by groups of students, and by individual students. In most math classes, problems of the day are commonly assigned to students and are often included in mathematics texts. Their use, however, sometimes encroaches on instructional time. Too often, discussion regarding an assigned problem takes much longer than it should. In addition, students often rush to a solution without reading the problem carefully, considering the information given, determining strategies for solution, and then thinking about whether their solutions make sense.

Posing problems of the week challenges students to use what they know about math to solve challenging problems but eliminates the shortcomings of assigning a problem of the day. A problem is posted on Monday and then, when solved, surrounded with various solutions represented in multiple ways to show student problem-solving processes. Each day of the week, students are responsible for only one step in the problem-solving process. For example, on Monday, students may be asked to unpack the problem. On Tuesday, they may need to find out information they have and if they need more information. On Wednesday, students might choose their strategy to solve the problem. On Thursday, they may solve the problem, showing their work. On Friday, students write a solution sentence explaining why their answer makes sense. This highly visible documentation of mathematics written and drawn by students (in early grades, some of it may be done as shared writing—a joint effort of students and teacher) is not intended simply to prove that much learning has occurred, although it does that. The posted work serves as a reminder of successful problem-solving methods and as a jumping off point for further in-depth exploration of problem-solving strategies by students.

## Math Word Wall and Vocabulary Displays

The discipline of mathematics rests upon a foundation of a shared, commonly agreed upon vocabulary. The precision of the vocabulary allows mathematical ideas to be accurately expressed and shared. Upon entering school, students may understand a few mathematical concepts. Even so, the vocabulary for expressing them is often lacking. To engage in precise mathematical communication, it is imperative that students learn a new language—the language of mathematics. In fact, Denisse Thompson and colleagues recommend that teachers "consider every student a mathematics language learner regardless of his or her level of English language proficiency" (2008, 11) and provide appropriate language acquisition support. Research documents the value of explicit vocabulary instruction in increasing comprehension of new content (Marzano, Pickering, and Pollock 2001). "Just as learning a foreign language is easiest when the learner is thoroughly immersed in the language, the same principle holds true for learning mathematical vocabulary" (Sammons 2011, 60) and for learning how to communicate mathematical thinking effectively. In mathematics, as in other content areas, a focus on vocabulary is essential.




*It is essential for all students to develop mathematical vocabulary.*

For years, elementary school teachers have created literacy word walls. Frequently used words are posted where they can be seen easily by students. These word walls are created *with* students as new words are identified; they are not already filled with words as the school year begins. Students are expected to use word walls when they are writing to ensure that their words are spelled correctly. Teachers also use word walls for additional instruction. By playing games, student attention is focused on these words and, more specifically, on letter patterns or other characteristics of the words.

Other types of vocabulary displays are also prominent in many classrooms. These may be word banks related to specific topics or units. They may be words that are used frequently in writing but may be unfamiliar to students. Some teachers encourage students to keep vocabulary notebooks in which they define the new words in their own words, use them in sentences, and draw nonlinguistic representations to show their meanings. Vocabulary knowledge is an important foundation as students construct meaning from reading. Since mathematical vocabulary knowledge is equally important in constructing mathematical meaning, Guided Math teachers in all grade levels borrow many of these techniques to promote the acquisition of mathematical vocabulary. Math word walls provide students a readily accessible resource for checking to see that they are using the correct mathematical terminology and encouraging them to assume responsibility for monitoring their own learning.

## Creating and Effectively Using Math Word Walls

Math word walls in some classrooms are used regularly by students and provide a valuable resource, while in other classrooms, they are rarely used by learners, serving primarily as wall decorations. What accounts for this difference? To make word walls effective resources for students, teachers should consider these suggestions:

- *Carefully choose words to be included on the word wall and add them when they are introduced.* Consult the math standards to identify the essential vocabulary students need to enhance their understanding and encourage them to communicate mathematically with precision. It may be necessary to add words from previous grade levels for review and reinforcement of students' understanding. While it may seem easier to create a word wall of the terms to be learned during the year at the beginning of the year, it is more meaningful to students if words as added as they are introduced and being used in context (Sammons 2013).

- *Display easy-to-read word cards that provide sufficient information to help students understand the meaning of each word.* Most often, the word, its definition, and a representation of the meaning are displayed. It is important that both the definition and representation for each word are accurate and reflect what students need to know about the word. In addition, it is important that the posted words cards are very clearly readable to students anywhere in the classroom. Student-created cards may not meet these criteria. So, instead of having students create cards for the word wall, have them create math word cards for other displays or for their own vocabulary notebooks.
- *Keep in mind that word walls should be more than passive displays; they are most effective when used as instructional tools.* Refer frequently to the word wall during lessons. When words are introduced, have students consult the word wall as they record new words in their math journals or vocabulary notebooks, and have them define the terms in their own words and with their own individual nonlinguistic representations. Check to be sure the words, definitions, and representations are accurate so misconceptions are corrected before they become established. Whenever the mathematical concepts are discussed, focus students' attention to the related terms on the word wall. During mathematics conversations, encourage students to consult the math word wall to find just the right word they need to express their mathematical ideas with precision (O'Connell 2007a). Make the word wall an integral part of mathematics instruction.
- *Explicitly teach students to use the word wall as a learning resource and share the expectation that students use the wall as they work.* Students should be held responsible for spelling word wall words correctly and for using them appropriately when they express their mathematical reasoning in any format—oral, written, or

representational. Take notice when students turn to the word wall during class and praise them for it.

- *Engage students in activities that require them to use the word wall.* Students often need additional motivation to take advantage of this highly visible classroom resource. There are many ways teachers can ensure that students actually use it. For example, ask students to write all the words from the wall that are related to multiplication or any mathematical topic and discuss why they chose those words, or give a definition and challenge students to find the word with that meaning.

One of the most effective ways to directly engage students with the word wall is with games. These games (see Figure 2.2 on page 66) prompt students to consult the word wall and expand their mathematical vocabulary knowledge.

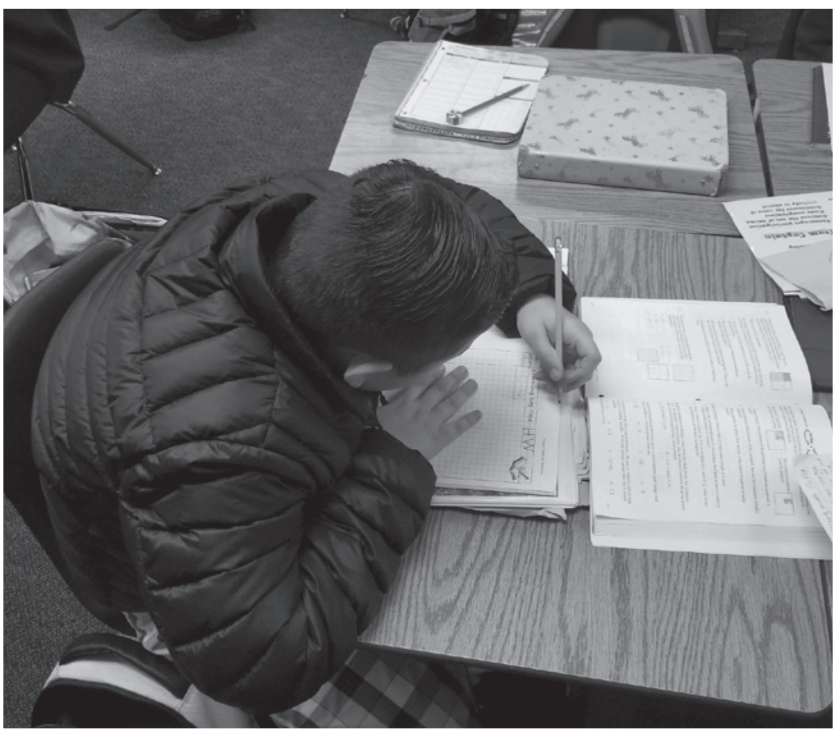


**Figure 2.2—Math Word Wall Games**

<table>
<tr><td>Five Questions</td><td>

- The teacher or a student chooses a word from the word wall.
- Students can ask five yes-or-no questions to try to identify the word. The questions must be related to the meaning of the word rather than addressing things like the beginning letter or number of letters in the word.<br>Ex. *bisector*: Is it related to geometry? Does it have anything to do with angles? Can you draw it with a compass?
- This game may be played by the whole class or with competing teams who take turns asking questions to see which team can first determine the word.

</td></tr>
<tr><td>Word Sort</td><td>

- Using a set of cards containing the words from the word wall, the teacher decides on a way to sort the cards.
- Students remain silent as the teacher holds up cards one by one and places them in the appropriate stack based on the sorting rule.
- As the sorting process continues, students who think they know the rule for the sort put up their hands. When several students have their hands up, the teacher asks the class to share their thinking about the sorting rule.<br>Ex. If the words *triangle*, *rectangle*, *hexagon*, and *pentagon* are in one stack and the other cards are in another pile, a student might believe that the sorting rule is polygons/shapes that are not polygons. Another student might reasonably disagree and think the sort is two-dimensional shapes/not two-dimensional shapes. Yet, if a circle is added to the first pile, it would clarify students' thinking about the rule because a circle is not a polygon.<br>Ex. In a middle or high school class, the terms *variable*, *constant*, *symbol*, and *number* might be placed in the one stack while all other cards are in another pile. Initially, students may use what they know about math to decide that these words are related to both of the terms *expressions* or *equations*. If the word *equal* is then placed in the same pile, however, students would be able to determine that the sorting category was equations rather than expressions.
- This game not only encourages students to consult the word wall when they are uncertain of the meaning of a term and to find inspiration in identifying potential sorting rules, but it also generates thought-provoking discussions among students as they talk about their mathematical reasoning.

</td></tr>
<tr><td>Connection Train</td><td>

- The teacher or a student chooses a word from the wall, writes it on the board or chart paper, and then challenges the class to find another word wall word that is connected mathematically with it.
- When students suggest a word that connects, they must explain the connection.
- The class has the responsibility to decide whether the connection is valid or not—do they agree or disagree? Why or why not? If they agree, the word can be added to the connection train.
- The class then searches for another connection. This continues until they can identify no additional connections.
- As this game proceeds, students may be asked to record the words and the connections in a math journal.<br>Ex. triangle → hypotenuse → line → point

</td></tr>
</table>

Additional word wall games may be found in *Strategies for Implementing Guided Math* (Sammons 2013). Also, a search of literacy word wall games online will provide a wealth of ideas that can easily be adapted for mathematics. The use of games such as these reinforces students' knowledge of the meanings of the words and helps them develop the habit of referring to the word wall when needed.

## Math Journals

The NCTM (National Council of Teachers of Mathematics 2000) mathematics process skills and the Common Core math practices standards (Common Core State Standards Initiative 2015) both emphasize the importance of developing students' ability to communicate mathematically. Since most students do not understand quality mathematical communication, they need to be taught the criteria for communication and be given ample opportunities to practice both oral and written communication (Sammons 2018). Daily use of math journals ensures that students are engaged in ongoing mathematical writing. Math journals also serve as evidence of the importance of written mathematical reasoning in the classroom environment by teachers.

The use of math journals is appropriate for all students. Teachers can readily modify their expectations according to grade level. Younger students might create simple drawings or even use photos taken with digital devices to represent their mathematical understanding and problem-solving attempts. Shared-writing experiences (where students jointly compose the sentences which the teacher records) allow even very young students to create class journals describing their mathematical thinking. Older students may be encouraged to provide representations of their thinking in multiple ways, supplemented with verbal explanations when appropriate. Middle and high school students are capable of much more detailed and extensive mathematical writing in their journals.

Most commonly in math classes, mathematical journal writing is limited to explanations of how students solved problems. Susan

O'Connell (2007a) suggests, however, that math journals also be used by students for the following tasks:

- brainstorm ideas
- record predictions, observations, and conclusions about mathematical explorations
- list questions
- solve and reflect on mathematical problems
- describe concepts
- justify answers
- explain procedures
- summarize the main points of a mathematics lesson
- make connections between mathematical ideas and other content learning
- reflect on learning in mathematics

The regular recording of mathematical understandings, problem-solving strategies, proofs, and conjectures by students serves several very valuable purposes. As with writing in any content area, the actual process of organizing one's thoughts to express them in writing requires one to reflect and clarify thinking. Throughout this process of reflection, uncertainties or even misconceptions may become obvious as ideas that just don't "jive." Planning for writing leads students to monitor and reflect on their own learning, thereby developing their metacognitive abilities. Additionally, by going through the process of thinking and then writing, students have better retention of the concepts.

By reading students' journals, teachers are able to more accurately assess the understanding of their students. The misconceptions and errors they discover in students' writing help them target instruction to correct them. At times, a misconception may simply be discussed with the student in a one-on-one conference. If a number of students seem to share the same misconception, the teacher may choose to form a small group of those students with similar needs and address it in that format. Should the misconception be more widespread, however, it may become a topic for whole-class instruction or for several small-group lessons. Alternatively, the teacher may provide a hands-on exploration by students followed by class "debriefing" with the teacher guiding the discussion in such a way that students themselves discover their errors.

The type of notebook used for math journals varies based on the ages of students, availability of technology, and the desires of the teacher. Increasingly, students are maintaining digital journals in which they write, add photos, draw their own representations, create slide shows, or even add recorded narratives of their thinking. Some teachers prefer three-ring binders in which students have dividers to separate their work. Others prefer inexpensive spiral notebooks. For younger learners, many teachers use the black and white composition books whose bindings are sewn because they tend to hold up well with daily use.

Usually, students are required to begin each entry with the date. With daily use, the journal provides an accurate record of learning that students can consult later to refresh their memories about concepts or strategies. Also, it may be used for graphic organizers or the recording of conjectures developed during class math discussions. To make the journal information more easily accessible by students, some teachers have students write their daily entries starting from the front of the journal. When the class creates conjectures or identifies problem-solving strategies, however, students record them beginning at the back of the journal. Students can refer to these pages throughout the year, when needed, to refresh their memories.

Students should be able to access their math journals when they are involved in whole-class instruction, small-group lessons, or math workshop. While paper journals may be stored in students' desks, it often works well to have them stowed in a basket or bin where they can be retrieved without disturbing the work of others. Wherever they are kept, the organizational system should be clear to all students. They should know that after use, all math journals are to be returned to a designated area. When journals are stored together in one place, it makes it easier for teachers to check them daily, if desired, without having to go from desk to desk gathering them.

## Graphic Organizers

Graphic organizers are visual diagrams that show the relationships among ideas. According to Irene Fountas and Gay Su Pinnell (2001), the use of graphic organizers can help students with these skills:

- see how ideas are organized or organize their own ideas
- use a concrete representation to understand abstract ideas
- arrange information so it is easier to recall
- understand the hierarchy of ideas (from larger to smaller)
- understand the interrelationship of complex ideas

These organizers can also be used to bridge the connection between a student's prior knowledge, what the student is currently learning, and what the student can apply and transfer to mathematical problem solving in the future (Thompson and Thompson 2005).

Graphic organizers enable students to examine big ideas through the use of diagrams or charts in such a way that relationships and patterns become apparent. Because of the importance of being able to recognize relationships and patterns in mathematical thinking, the use of graphic organizers effectively enhances student understanding of mathematical concepts. The NCTM Standards (2000) state that instructional programs from pre-kindergarten through grade twelve should enable students to "create and use representations to organize, record, and communicate mathematical ideas" (67) and "use representations to model and interpret physical, social, and mathematical phenomena" (70). Graphic organizers assist students by providing a way to represent ideas and communicate their mathematical thinking.

Modeling and teacher think-alouds are great ways to introduce a new graphic organizer. The teacher shares their thinking while completing the sections of the graphic organizer. Once the structure is familiar to students, the teacher leads the class or a small group of students in completing the same type of graphic organizer on chart paper or an interactive whiteboard but with different content. Students may also complete individual copies in their math journals. When students are comfortable working with the graphic organizer, they can be asked to complete ones with the same format independently or as a group project. Both chart-size graphic organizers and smaller, individual graphic organizers may be displayed to remind students of the format of the organizers and how they are used, as well as to provide mathematical content students can refer to whenever necessary.

## Example Graphic Organizers

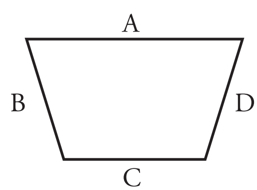
Many of the graphic organizers used in language arts instruction are easily adapted for use with mathematics. A modified Frayer model (Figure 2.3) may be used to promote vocabulary knowledge. Typically, in a Frayer model, the vocabulary word goes in the center. In the top-left corner is the definition of the word, in the top-right corner are characteristics of the word, in the bottom-left corner are examples of the word, and in the bottom right corner are non-examples. With the modified version, students draw a nonlinguistic representation in the upper right-hand corner.

**Figure 2.3—Modified Frayer Model**

**Definition**

the distance around a closed plane figure

**Representation**




$A + B + C + D = P$

**Perimeter**

**Examples**

- the length of a fence around a soccer field
- the total distance around a garden

**Non-Examples**

- the height of a person
- the amount of water in a pool

Other graphic organizers highlight important mathematical relationships as well:

- A graphic organizer for analogies may be used to highlight relationships between mathematical vocabulary words. For example, *sum : addition :: product : multiplication*.
- Compare-and-contrast graphic organizers, such as Venn diagrams, help students analyze the relationships between numbers by having them list ways in which they are similar and ways in which they differ. Focusing on these relationships elevates student understanding beyond simple computational proficiency.
- A diagram illustrating steps in a mathematical process serves as a reinforcement of understanding and later as a reference when posted or recorded in a math journal.

It is important that students see various graphic organizers in use and also be given the choice of which to use in different situations. Unless they have this responsibility, it is difficult for a teacher to determine whether students really understand the meaning of these organizing structures or if they are only copying what they have seen modeled (Ennis and Witeck 2007).

## Class-Made Anchor Charts

Classrooms should visibly reflect the learning that occurs within them. In effective reading and writing instruction, classroom anchor charts are developed to brainstorm ideas, record strategies, create checklists, organize or categorize information, record class stories, and in general, document what has been going on during language arts instruction. The value of these charts is in the thinking and conversations that led to their creation and also in their later use as references by students when posted in the classroom. It is not uncommon for students to look to where a chart had been hung to trigger their memories of the content of the chart long after the chart has been taken down.

> **Power Phrase**
>
> **anchor chart:** *a class-created chart that reflects learning and is used as reference*

In mathematics, class-generated anchor charts serve the same purposes. Mathematical charts document problem-solving strategies, provide organizational formats for information, present data collected in ways that make it easy to interpret, display lists of conjectures students have formulated based on their mathematical observations, record steps in a computational process, show the methods used to solve a particular problem, provide illustrations of concepts, or in other ways, reflect classroom discourses. According to Debbie Miller, these anchor charts "make our thinking permanent and visible, and so allow us to make connections from one strategy to another, clarify a point, build on earlier learning, and simply remember a specific lesson" (2002, 57). Students working independently can refer to these charts to find examples of the way mathematical ideas are expressed (Hoyt 2002, 244).

Whether these charts are a result of teacher modeling, shared writing, or interactive writing, students share the pen with teacher guidance; the charts serve as models of mathematical communication. "The application of anchor charts is quite literally limitless. The use of anchor charts is a powerful way to help students solidify learning and connect math content to large math concepts" (Brown 2014, 54). Although the use of anchor charts originated in elementary schools, their value extends across grade levels.

One of the most important aspects of these charts, however, is that they be created in the classroom with student participation rather than being commercial charts or ones made by the teacher without student input. These charts truly reflect and enhance the learning that happens in the classroom.

## Tools for Measuring

Often, instruments for measuring appear only when a unit involving that kind of measurement is being taught and are used only during mathematics or maybe science. Students see these tools as pertaining solely to a specific unit of study rather than as practical objects used in everyday life. When tools for measuring are displayed in the classroom and used on a regular basis for daily routines and problem solving, students begin to value their utility and understand their importance.

Thermometers in the classroom may be used regularly to check the temperature indoors rather than being used solely during a weather unit to check outdoor temperatures. Teachers might ask a student to check the thermostat or a thermometer in the room if it seems uncomfortable so that students gain a better sense of comfortable room temperatures. Measuring cups and spoons used during cooking activities or to measure food for classroom pets help students connect them to daily use in their homes.

With older students, teachers can connect measurement to science lab experiences and then to its application in the real world. Meter sticks or rulers are useful when students are involved in creating classroom displays or projects. Student investigations and problems created by teachers that include measurement encourage the regular use of these tools and help students develop proficiency in using them.

Additionally, an understanding of units of measure enhances the ability of students in all grade levels to visualize what they read. If, for example, a character is described as 4 feet (1.2 meters) tall, few students actually have a sense of how tall that is unless they get a yardstick and measure it for themselves. If they read that the winning pumpkin at the fair weighed 100 pounds (45 kilograms), most students have no concept of how heavy that is unless they are able to experiment by weighing items.

Visualization is a skill that is especially important in reading nonfiction expository texts, which often rely on the concepts of size, weight, length, distance, and time (Harvey and Goudvis 2000). Teachers should encourage students to pay close attention to descriptive text details and then use tools, if necessary, to help their students improve their abilities to visualize. Because visualization helps them engage more deeply with the text (Keene and Zimmermann 1997), these students become better at reading more complex text. In turn, the visualization strategies practiced in reading also enhance mathematical problem-solving capabilities by improving students' abilities to create mental images of problems they are asked to solve (Hyde 2006).

In self-contained classrooms, science experiments help students use measuring tools in meaningful ways and allow them to experience firsthand the relationship between the two disciplines. Measurement also plays a significant role in helping students understand social studies. How can a student have a true sense of what the conditions must have been like on the *Amistad* unless they have a sense of the dimensions of the tiny ship into which so many slaves were packed for an ocean voyage lasting for weeks? When students participate in many "virtual" experiences through the internet and videos, a sense of scale is frequently missing. What better time to encourage the use of measurement! In this way, students can practice measurement skills, develop a sense of scale, and more fully understand the content of other disciplines.

## Math-Related Literature

Literature related to mathematical concepts has enormous power in classrooms. The lure of quality narratives or nonfiction spurs students to make vivid connections to their own lives and to the world around them. Students who are immersed in literary content of many genres are highly engaged by the text and illustrations as mathematical concepts are identified and explored. According to Susan O'Connell, using children's literature "provides a fun and engaging way to hear about and talk about math ideas. Through story events, students can see math in action" (2007a, 26). Phyllis Whitin and David J. Whitin (2000) describe three ways in which literature may be used in mathematical instruction:

- to examine mathematical patterns
- to promote an understanding of large numbers
- to explore the meaning of mathematical vocabulary

Although the use of math-related literature is more common in elementary classrooms, it can be effective with students of any age. Teachers of secondary students find that it sparks students' interest and broadens their perspective of mathematics. Math-related literature for older students includes texts such as *What's Your Angle, Pythagoras?* by Julie Ellis, *Counting on Frank* by Rod Clement, and *Math Curse* by Jon Scieszka that all infuse the mathematical ideas they address with delightful humor. Another math-themed book, *The Man Who Counted* by Malba Tahan, intrigues students with tales of a man who uses his mathematical powers in solving a series of puzzles. In addition to texts written specifically to address mathematics, teachers can find math connections in almost any text. For example, the reading of Norton Juster's *The Phantom Tollbooth* or *Gulliver's Travels* by Jonathan Swift presents an opportunity for students to explore the concepts of measurement, ratio, and proportion. While math teachers may not have time to read an entire book with students, they may give a brief summary of parts of the book and then choose certain sections to read aloud to the class.

An initial read-aloud by the teacher—to older students as well as younger—supports the understanding of the text by the entire class, even those students who may be unable to read it independently, and builds a sense of community. Reading comprehension strategies, such as making connections, asking questions, visualizing, inferring and predicting, determining importance, and synthesizing employed before, during, and after the reading, support students in building a true understanding of the text (Keene and Zimmermann 1997; Harvey and Goudvis 2000). Use of these strategies during read-alouds models ways in which they are useful in mathematical problem-solving situations. The strategies help students as they endeavor to clearly understand the circumstances in those problems.

Once students clearly understand the story or information from the text, teachers can guide the class to focus on the mathematical relevance of the material using carefully crafted questions. Follow-up instruction may challenge students to find the mathematics in a story (O'Connell 2007a) or provide opportunities for students to investigate issues or solve problems from or related to the text. In many cases, students are better able to understand mathematical concepts and communicate their understandings when filtered through the lens of a story. Even after instruction based on a particular book has ended, the book should remain on display and be available for students to reread and refer to later.

By focusing on the relationship between mathematics and literature, students begin to spot mathematical connections in other stories and books. Experienced teachers are quick to encourage students to explore these connections as they emerge. Frequently, the questions or interests of one student are contagious and motivate the entire class to investigate the questions raised.

## Math Books or Digital Presentations by Student Authors

As students practice their mathematical communication skills and interact with math-related literature, a logical extension is to encourage them to create their own math-related books or digital presentations. Throughout the year, teachers feel the time crunch and realize that there are never enough minutes in the day to teach everything required—writing about math during writing workshop or writing during math class optimizes the use of instructional time in self-contained classrooms. The creation of student-authored math books or presentations is a worthwhile use of time in any math class. The thinking processes students engage in as they write require that they review what they know about the math concepts, organize their ideas concisely, consider how to express their ideas with precision using appropriate mathematical terms, and finally, record those ideas so they can be clearly understood by others.

Student-created books may be simple mini books of mathematical vocabulary or explanations of mathematical concepts (O'Connell 2007a). Brightly colored paper foldables, such as accordion books, flipbooks, or tri-fold books, can be constructed by students. The process of creating these 3-D, interactive graphic organizers is highly motivating and engages students in organizing and communicating mathematical ideas. For examples of the many foldables students can create, consult Dinah Zike's *Big Book of Math* (2003). Students are also often eager to create digital editions of their books that can be displayed and read on computers, digital devices, or interactive whiteboards. Opportunities to express their ideas using apps to produce slide shows, comic strips, or videoed talks similar to TED talks motivate student authors.

Conversely, students may choose to complete complex narrative or informational texts. Students often enjoy creating math-related texts modeled on texts that have been read to them during read-alouds. When their work is based on mentor texts, students are challenged as they emulate the ideas and writing styles found in these familiar books. Their interpretations of these trade books often offer authentic perspectives to which other students relate.

Before expecting students to create books independently, the teacher should first lead the class in composing a class book, possibly based on the format of a favorite story they have read and discussed. In primary grades, the process begins with shared writing. Students compose the sentences and content while the teacher acts as a scribe. Later, it may involve interactive writing in which students work together, guided by the teacher. At times in the interactive-writing process, with teacher guidance, students actually record selected words from the text they are writing. Older students may collaborate to come up with an overall plan for the book with the responsibility for writing individual pages assigned to different students or groups of students. In any of these scenarios, students illustrate the work. Since the illustrations themselves are most often representations of mathematical concepts, this activity reinforces student understanding of what they're learning. Digital resources are also available for students to create mathematical illustrations and representations for their books.

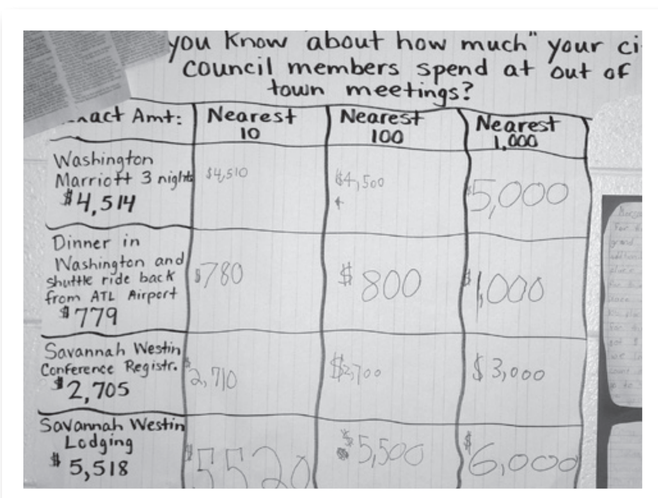
Although they are writing math books, students should still be expected to follow a writing process. They should plan their writing, write drafts, revise, edit, and publish their work. Because they have been exposed to math-related writing from a variety of genres, students will have many models on which to base their own writing. In conferencing with students, teachers are able to address both writing and mathematical teaching points.

Everyone enjoys having their work appreciated. Students' published pieces should be prominently displayed to motivate others to become authors. In addition, teachers may arrange for student authors to read their books in other classrooms. Many schools have classes who are "reading buddies." Reading buddies may write math-focused books for each other to read. Older students enjoy creating books to share with younger partners, especially when they feel they are helping them learn. Knowing that other students will be reading their books also motivates students to create quality products. Teachers working together may develop additional ways to recognize student authors.

## Connecting Math to Other Content Areas

Implicit throughout this chapter is the notion that students learn mathematics best when they see it as an integral part of their world, both in school and at home. Without a doubt, mathematics plays an enormous role in our lives. In fact, NCTM describes the development of mathematical ideas and the use of mathematics in other disciplines as being intertwined (NCTM 2000). As the interrelatedness of mathematical ideas to other content areas becomes obvious to students, they also become aware of the utility of mathematics.

So, why is it that mathematics is often isolated as a subject in our schools? When we help our students become aware of the role mathematics plays throughout the curriculum, they develop a much more expansive and complex understanding. This occurs when teachers explicitly demonstrate the links between mathematics and the other disciplines they teach.




*In this cross-curricular activity, students learn that rounding is an important skill.*

As a teacher, I had a large map in the front of my classroom. Any time places were mentioned, in whatever subject areas we were studying, I pulled the map down and we located that place. It became second nature to my students and me. They came to understand the importance of place. If, for some reason, I neglected to refer to the map, my students reminded me. The same should be true for mathematical connections. When teachers develop a "radar" awareness of mathematical connections as they teach, they will find that the connections abound. Moreover, students will develop the same keen awareness. When students' awareness increases, so does their interest. Soon, they begin to notice the math in their world beyond the classroom. As their interest increases, so does their curiosity about the mathematical connections they discover. Mathematics becomes more than algorithms, more than a set of procedures to follow, more than a set of memorized facts—it becomes a discipline that makes sense and is a real part of their world.

## Chapter Snapshot

An abundance of evidence of mathematical links in the classroom encourages students to see mathematics as an integral part of their lives. As they observe the seamless connection of mathematics to the everyday world, they develop an expanding curiosity about it. This curiosity spurs their motivation to investigate mathematical relationships more deeply. Teachers play a major role in creating this environment. As teachers stimulate, focus, and facilitate learning, students not only learn how to apply what they know about math to solve mathematical problems, but also begin to understand why and how those mathematical procedures work, allowing them at first to glimpse, and then eventually grasp, the complex patterns and relationships in the world of mathematics.

## Review and Reflect

1. Look back at the Foundational Principles of Guided Math (page 41). Which do you think are the two most important principles? Why? How does your classroom reflect those principles?

2. Do you think your students feel that they are members of a mathematical learning community? If so, how did you establish that feeling of community? If not, what can you do to create it?

3. Within your classroom, what would they see that would indicate the importance of mathematics?

***Although these activities are short in duration, they should be carefully planned, based on the standards being taught and the needs of students.***

# Chapter 3

## Guided Math Warm-Ups

Each and every school day begins with students stepping over the thresholds of our classrooms wondering what the day will bring. Some of these students enter with a sense of excitement and wonder. A few look to the teacher, eager to begin. Others straggle in slowly, lugging their backpacks and lagging as they unpack their belongings and settle in. Several students boisterously entertain other students with tales of their morning adventures. Perhaps one student wears an expression that clearly indicates the difficult morning they have experienced. As teachers, we are responsible for these students. No matter how they feel as they arrive, they are here to learn, and we are here to teach them. As we plan our lessons, we ask ourselves how to stimulate their curiosity, motivate them, guide them, and challenge them—all of them, without exception.

The routine we establish for students as they enter our classrooms sets the tone for the day in self-contained classrooms or in math classes where instruction is departmentalized. Students need to know exactly what they are expected to do upon entering the classroom. It is important to create and teach procedures for unpacking supplies, turning in homework, using the restroom, and, most importantly, beginning to work.

In a classroom focused on mathematics, this valuable time also serves as a math "warm-up." Having just arrived, students need time to shift gears from family conversations, riding on the school bus, walking from class to class, and chatting with friends, to readying themselves for math class. So, as soon as students complete their "housekeeping" chores, they should begin mini math activities that touch a range of mathematical concepts. Some activities should require students to review concepts and skills already covered and mastered. Some might relate to mathematical concepts being explored currently. Some might give students a preview or taste of concepts to be introduced or extended. Although these activities are short in duration, they should be carefully planned, based on the standards being taught and the needs of students. The teacher is well aware of individual student needs because of ongoing formative formal and informal assessments.

Participating in a variety of brief mathematical activities during these first 5–10 minutes leads students to make subtle mental shifts into the world of mathematical awareness and learning. The activities may vary from day to day, but whatever the activities, students should clearly know the expectations regarding behavior and academic work. A math warm-up usually includes one or two tasks to be completed quickly by each student and which is later discussed by the whole class. These can include maintaining a math current-events board, carrying out real-life, math-related classroom responsibilities, completing calendar instruction, completing a math stretch, working on a problem of the week, or reviewing previously mastered concepts and skills.

With the daily repetition of a variety of similar tasks and the accompanying class discussions, students begin to recognize patterns and make generalizations from these examples. Students develop personal understandings of mathematics concepts when they connect their experiences and background knowledge to real-life examples and explanations provided by their teachers. Additionally, students are able to "conditionalize" what they know. For example, rather than rote memorization of an algorithm, students understand the conditions under which it can be used appropriately as a result of their conceptual understandings (Hyde 2006).

## Math Stretches

Athletes understand the wisdom in warming up prior to long, strenuous workouts. They know that by beginning with stretching and a slightly slower pace for a short time, they can maximize their performances over the long haul. In some ways, the same is true for our students. As we present them with one or two mathematical stretching activities, they begin to draw on their prior knowledge and bring this knowledge into their working memories where it can be easily accessed for extending their understanding of mathematics. Students come to look forward to these "stretches," often wondering what they will be asked to do as they arrive. The nonthreatening nature of the tasks encourages all students to participate independently and with confidence.

Math stretches promote the mathematical literacy of students of all ages. More than simple computation, mathematical literacy is the ability to properly execute procedures. It requires a knowledge base and the competence and confidence to apply this knowledge in the practical world (Minton 2007, 4). Math stretches foster the mathematical literacy of students by offering challenging opportunities for students to make mathematical connections, engage in authentic mathematical communication, and make use of appropriate mathematical vocabulary (Sammons 2010; Sammons and Windham 2011; Sammons and Dase 2011). Additionally, because these instructional tasks allow for the brief ongoing review of previously mastered concepts and skills, they are ideal vehicles for providing *distributed practice*. While the initial practice (*massed practice*) is essential for mastery, continued practice at less frequent intervals (*distributed practice*) leads to deeper conceptual understanding and increases the probability that it will become stored in the permanent memory (Marzano, Pickering, and Pollock 2001; Marzano 2004).

Because all students take part in the activity, they are usually eager participants in follow-up class conversations—the math huddles. Because of the brevity of a huddle discussion, teachers need to closely examine student responses to quickly identify the most important point of focus. Trying to address too many topics at once diminishes the value of the math conversations. Although, it is often effective for teachers to be flexible and follow the lead of students. What's important is that teachers consciously exercise their professional judgment in determining how to best facilitate students' discussion of the stretch.

The teacher should facilitate the learning dialogue to:

- establish a respectful ambience
- encourage student participation
- model the appropriate use of mathematical vocabulary
- guide students to recognize patterns and relationships that might be observed in the event
- focus attention on the relationship of the task to mathematical "big ideas"

The brief duration of the discussion makes the role of the teacher critical even though students do the majority of the talking. With experience, teachers learn to minimize their own talk, encourage their students' talk, and become adept at recognizing student comments and observations that will lead the class to construct mathematical meaning. Teachers also become skilled in asking highly focused and open-ended questions to prompt students to think more deeply and share their ideas with others.

Math stretching tasks will vary according to students' grade levels, the mathematical standards being addressed, and students' learning needs. In planning appropriate stretches, teachers should take all these into account. It is also important to focus beyond the activity itself and to have in mind exactly what students should take away from the experience.

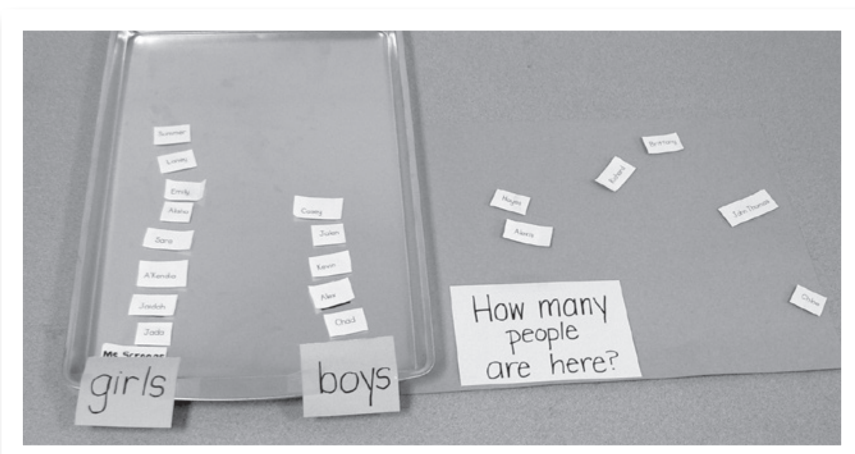
- Is the goal simply skill practice?
- Will the stretch help students to identify important patterns that will lead them to make conjectures?
- Will it serve to help teachers in identifying potential misconceptions of their students?
- Was it created with a specific instructional focus in mind?

A math stretch should be a relatively simple task that does not take much time to complete. Students must be able to work on it independently with only a minimal amount of direction by the teacher. Also, it is essential that a stretch is constructed in a way so that it has a sufficient number of unique options for student responses to ensure a variety of answers. Students have to be taught that their contribution to the stretch must be different from responses that are already posted unless it is a data collection stretch similar to those described below. Therefore, a simple computation task or a problem with a single answer would be inappropriate for a stretch. The following examples can be adapted to work in any classroom.

## Data Collection and Analysis Math Stretches

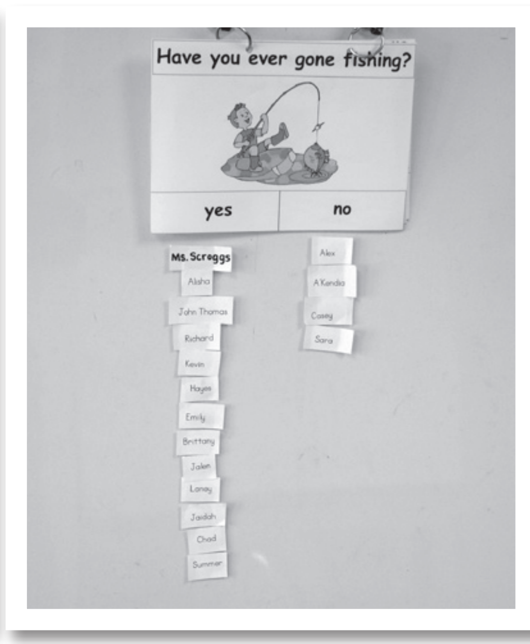
The National Council of Teachers of Mathematics placed increased emphasis on data analysis in their mathematics standards. The standard states that students in elementary school should be able to do the following:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- select and use appropriate statistical methods to analyze data
- develop and evaluate inferences and predictions that are based on data (2000, 48)




***This data-collection activity can also be used as a way of taking attendance.***

Students have enormous curiosity about the world around them, especially their immediate world. Teachers can take advantage of this by posing questions for students to respond to as they arrive at the classroom, thus providing opportunities for data collection and analysis. For younger students, the questions can be quite simple. In a kindergarten class, it may be, "How did you get to school today?" For older students, the questions presented are more complex and should reflect the interests of students. They may be asked to choose a favorite sport, band, app, entertainer, author, or movie from several choices. For students who are interested in sports, the question might involve selecting which team they think will win a major sports event. Students are excited to be asked questions that they feel confident answering and are eager to take part in the data collection.

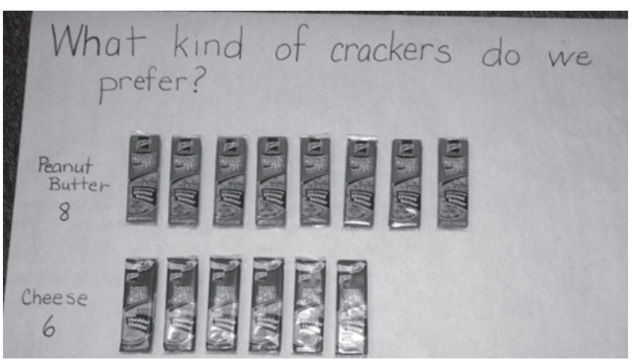

*Using clearly formulated questions enables this type of math stretch to be implemented in a short amount of time.*

Initially, the teacher creates the questions. Because this stretch is designed to be brief, the answer choices should be limited to no more than four. As students become more aware of the data-collection process, the teacher can request student input in generating questions. Students who are invested in the process of gathering data because they helped choose the question and are interested in the topic, find meaning in their subsequent analysis of collecting data rather than seeing it as a mindless task of generating a series of numbers solely to discern patterns and tendencies (Ennis and Witeck 2007).

For this math stretch, students are provided with a method for responding and a class graph is created to represent their responses. Depending on the ages of the students, their previous experiences, and the subject of the data being gathered, the graph may be a "real" graph, a pictograph, or a symbolic graph.

### "Real" Graph

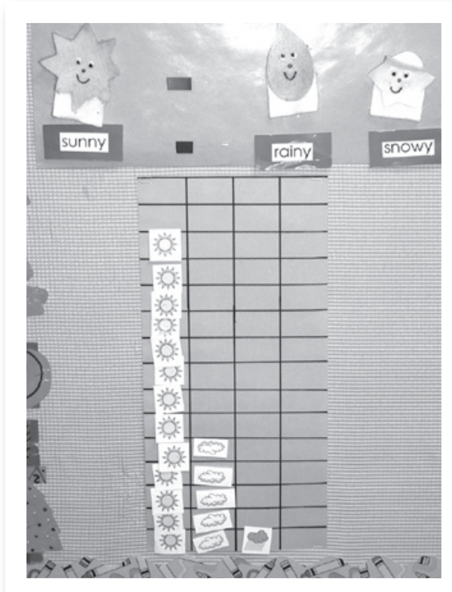
A "real" graph uses the actual objects being graphed. For example, if students were asked which snack they prefer, cheese crackers or peanut butter crackers, the teacher might provide a basket of each. To respond, each student would choose the package of their choice and place it on a grid provided by the teacher, creating a visual representation of the preferences. Or, students might be asked to indicate whether they are wearing shoes with laces or shoes without laces by placing one of their shoes on the grid. Obviously, the use of "real" graphs is limited. However, it does provide a vivid display of data that especially attracts the interest of younger students.




***This real graph enables students to actually see representations of the data collected.***

## Pictograph

A pictograph uses drawings of the objects being graphed. They might be computer-generated pictures provided by the teacher, die-cut shapes, or pictures drawn by students themselves. On interactive whiteboards, teachers are able to design a graphing task displayed on the board where students can drag pictorial images to the graph. Since time is of the essence during warm-ups, teachers usually provide the pictorial representations for students rather than having students create their own. If student-created pictures are used, teachers can ask students to make them the previous day in math workshop.




*This pictograph displays the daily weather.*

## Symbolic Graph

A symbolic graph uses linking cubes, clothespins, sticky notes, tallies, squares, circles, or other marks to represent the objects being graphed. This is often the graph of choice for math stretches with young learners. These graphs can be composed of a variety of materials and be quickly completed by an entire class.


*It is important to collect data that is meaningful to students' lives.*

Before any type of graphic representation is used for a math stretch, it should have been already introduced to the class. Students should be familiar with it and able to work independently in its construction. To ensure that data collection and representation proceed smoothly, teachers should follow these steps:

1. Carefully consider the question being posed to be sure it is clearly stated and to anticipate any confusion students may have.
2. Prepare the graphing format in advance and decide what groups of data will be represented and how.
3. Have all materials available and clearly labeled for students as they enter the classroom.

As students add their responses to the graph, they become curious about their classmates' answers and are drawn to the graphic representation the class is creating. Students will want to analyze it immediately, and they will excitedly watch to see their classmates' responses. At first, they often see this as a game with winners and losers. A well-focused math huddle discussion about the data gathered corrects this misconception.

When the data display is complete, the teacher brings students together for an analysis of it. It is this aspect of the lesson that is the most valuable in helping students truly understand data collection. Teachers must keep in mind the observations and inferences about the data that they want students to make and skillfully use questioning to scaffold student efforts to interpret the data. This math huddle is most effective when these observations come from students' own ideas, often as a result of skillful questions posed by the teacher, rather than from their being explicitly explained by the teacher.

Naturally, the content and structure of these discussions will vary depending on the experience and needs of the class. Consistent with the instructional strategy by which teachers gradually release responsibility to students, these discussions may move from teacher think-alouds where the teacher support is strong, to independent student analysis, both oral and written, where students shoulder the responsibility for analysis. (See Figure 3.1 on page 94.)

When data collection stretches are first introduced to students, teachers can think aloud to share their interpretations of the data collected. They discuss why the data being gathered is important and what they hope to learn from it. In addition, in their comments they can reflect on the kind of graphic display used and how effective they think it is in presenting the data clearly in a way that makes analysis easy. Teachers describe what they observe and what they can infer from these observations. Throughout the think-aloud, they model the use of relevant mathematical vocabulary. The think-aloud process allows students to hear the way mathematicians approach the task of analyzing collected data. As they talk about their analysis of the data, teachers often choose to record the most significant aspects on chart paper. Later, students can turn to the chart as a reference. Hearing a teacher talk about their analysis of the data is important because it gives students a glimpse into the thinking that goes into the process and allows them to hear how mathematical reasoning can be clearly communicated. It is unreasonable to expect students to skillfully analyze data and then describe their analysis unless they know what is expected.

Once students learn what data analysis looks and sounds like, the teacher's role gradually changes. Rather than using a think-aloud, the teacher begins to guide the student discussion of the data using carefully crafted questions. Students are encouraged not only to share the facts from the data collected, but also to begin thinking about the inferences they can make. At times, the teacher may rephrase a student observation or comment to use the appropriate mathematical vocabulary or state a concept more clearly. Teacher questioning leads students to reflect on the meaning of the data and the effectiveness of the graph or chart used. The teacher should record important student observations and inferences on chart paper for future reference. This kind of brief math huddle provides teachers considerable insight into students' understandings and misunderstandings that can be used to guide future instruction (O'Connell 2007a).

Teachers gradually become only facilitators of the discussion as students become more proficient at analyzing data and sharing their thoughts. They may offer some broad, open-ended questions to initiate the discussion or move it along if it stalls. They also ensure that the talk remains focused, respectful, and inclusive.

Eventually, students should be able to record their own thoughts regarding the data collected in their math journals prior to the class discussion. Writing ideas in their journals prior to the discussion helps them to formalize their thoughts, set them down on paper so they can actually see and consider them, and, as a consequence, revise them if needed (O'Connell 2007a). This task directly reflects the NCTM Communication Standard stating that all students should be able to "organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely" (2000, 60). This discourse promotes these standards for mathematical practice as students share their reflections and listen to others. Having students write about their thinking does not replace the need for a discussion.

At times, the teacher may opt to have students engage in a math huddle independently while monitoring the conversation. Students are encouraged to consult what they have written in their journals as they talk about what the data shows. The independent conversation may prompt some students to change their thinking or expand on their original ideas. As their thinking evolves, they can revise what they have written in their journals. Students now become the primary participants in the discussion, but the teacher is still involved. Even with an independent discussion, it is best that the teacher is present to quickly correct misconceptions and informally assess individual and class understanding of data analysis.

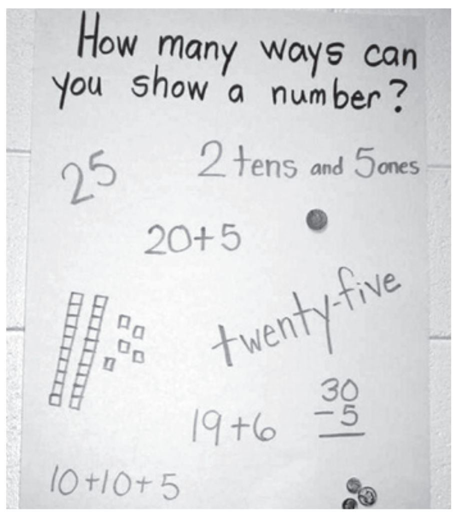
**Figure 3.1—Types of Discussions**

| Discussion Type | Teacher Role | Level of Support |
|---|---|---|
| Think-Aloud | • discusses the reason for choosing the particular graphic representation and reflects on its effectiveness<br>• shares observations and inferences she or he has made<br>• models the use of relevant mathematical vocabulary<br>• records analysis on chart paper for future reference | High level of teacher support |
| Guided | • uses carefully crafted questions to guide discussion of observations and inferences from the data<br>• occasionally rephrases student comments to model relevant mathematical vocabulary and concepts<br>• helps students consider the appropriateness of the graphic representation<br>• records analysis on chart paper for future reference | Moderate to high level of teacher support |
| Facilitated | • poses open-ended questions to stimulate discussion by students<br>• ensures that discussion stays focused on observations, inferences, and choices of graphing representation<br>• analysis may be recorded on chart paper by students or the teacher or in student math journals | Low to moderate level of teacher support |
| Independent | • recording of anecdotal notes about observations, inferences, and choice of graphing representation made by students in math journals before discussion<br>• discussion of journal reflections primarily by students with teacher support, if necessary, to focus discussion or correct misconceptions | Low level of teacher support |

In a variation of this activity for secondary students, a data-collection question may be posed, but students are asked to create their own individual graphs based on this data. The discussion generated would include not only an analysis of the data, but also a discussion of the types of graphs students chose to make (bar, line, circle, etc.), why they chose them, and how well they displayed the data from the question. Some teachers create multi-day stretch tasks beginning with collection of data on one day and then continuing from day to day with different tasks related to the data. Stretch questions using the gathered data for other days might include: What question can you answer with this data? What did you learn from this data? For what other data might this type of graph be appropriate?

## Number of the Day Math Stretches

Promoting number sense and an understanding of number systems is a foundation of mathematics instruction. The NCTM standards state that students in grades K–12 should be able to "understand numbers, ways of representing numbers, relationships among numbers, and number systems" (2000, 32). A math stretch using a Number of the Day chart addresses these standards and is flexible enough to be adapted for use in classrooms at any grade level.




***All students should be able to represent numbers in a variety of ways.***

When a Number of the Day chart is used, the teacher writes a numeral on a sheet of chart paper, a whiteboard, or an interactive whiteboard. Each student records a different representation of that number.

- In kindergarten, the numeral might be 5. On the chart, one student might draw five circles, while another student might draw five dots. A student who is beginning to understand how to compose numbers may write 3 + 2. Another student may write 5.
- In a third grade classroom, a chart might have the numeral 48. Alternative representations might include $4 \times 12$, 40 + 8, 4 tens and 8 ones, 10 + 10 + 10 + 10 + 8, 4 dozen, or 24 + 24.
- A fifth grade teacher may post the fraction $\frac{5}{20}$. Student responses may be $\frac{1}{4}$, .25, $\frac{4}{20} + \frac{1}{20}$, $5 \times \frac{1}{20}$, and $1 \div 4$.
- In a middle or high school math class, the stretch might ask students to represent a number in different ways. Depending on the students, representations might include 100 + 25, 102 + 23, or -5 + 130.

With Number of the Day stretches, students can self-differentiate their responses based on their number knowledge and their motivations. Each student's representation reflects their understanding and provides a quick way for teachers to assess student knowledge. Watching as the chart evolves, students often become very curious about what others are recording. Impromptu math conversations arise between students and spur mathematical interest as they share their thinking with their peers. Some of the best learning occurs as students learn from each other.

When every student has participated in this math stretch, the class gathers to examine the responses and determine their validity. Students are asked to explain the reasoning behind their representations of the given number. Manipulatives should be readily available for use by students to model their reasoning. The use of manipulatives in the discussion may serve as a scaffold for students who are still approaching the understanding of a particular representation. The combination of explanation by a peer, hands-on modeling, and skillful teacher questioning often leads to "Aha!" moments for students who would not be able to make this conceptual leap independently. By focusing on multiple representations of the same number, students begin to recognize patterns and relationships from which they can draw generalizations or conjectures. Students engaged in these conversations are steeped in mathematical ways of thinking, examining “big ideas” and deciding how best to express them (Carpenter, Franke, and Levi 2003). Students’ understanding of numbers becomes highly visible (Hattie 2009, 2012) to teachers during these discussions. Building upon what they observe and hear, teachers are able to target specific students’ needs.

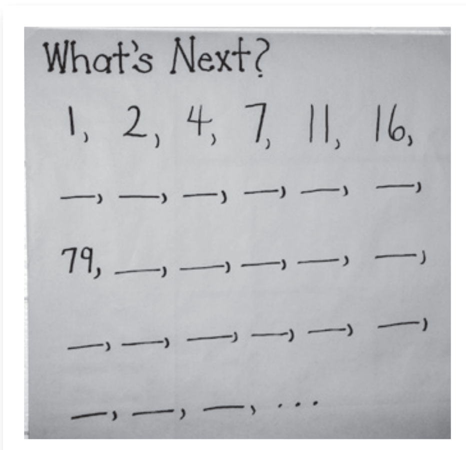
## What’s Next? Math Stretches

Since recognizing patterns and relationships is an integral aspect of understanding mathematics (NCTM 2000), a What’s Next? stretch can be a valuable learning experience for students as they warm up. The teacher creates a What’s Next? chart by beginning a pattern on chart paper, followed by a blank space for each student in the class. As students arrive in the classroom, they study the pattern to decide what comes next. Students take turns filling in the next step in the pattern in the first blank space and put their initials by it. If a student thinks the answer of the previous student was incorrect, they can go to the student to discuss it, but may not change it. The only one who may change the answer is the student who recorded it. This may, of course, become a group discussion as other students join in.

The responsibility for correctly extending the pattern rests squarely on the shoulders of students. In their desire to correctly continue the pattern, they are motivated to work together in arriving at a solution with which they can all be satisfied. Since students must be well-versed in knowing how to respectfully discuss ideas on which they may not agree, this math stretch should not be presented to the class until a sense of community is firmly established. Ultimately, in a group discussion facilitated by the teacher, the class comes to a consensus on not only what is next but also on why it comes next.

The value of this math stretch lies as much in the conversations that flourish as students exchange ideas, as in the exploration of patterns. Students working on their own tend to work with blinders on, locked into thinking about a problem in only one way. Discussing the problem with others leads to multiple approaches. Peer interaction is especially effective in promoting reflection because the comments and differences in thinking of classmates are most likely within a range that spurs genuine intellectual conflict (Hiebert et al. 1997).

Although teachers may be tempted to correct mistaken attempts at continuing the pattern, to do so would prevent the genuine reflective conversations that are generated by students themselves in their search for common ground. Mistakes are a natural part of the learning process and should play an integral role as students struggle to comb out meaning from their mathematical experiences. The internal conflict created by discussion among students concerning misconceptions and errors leads them to reevaluate their thinking and clarify their ideas. They become more willing to tackle challenging problems as they discover that errors are not perceived as failures but instead are seen as valued attempts at problem solving and as learning opportunities.




*Understanding number patterns is an essential concept for all students.*

While many teachers consider the search for patterns to be important only in primary grades, the standards for mathematical practice of the Common Core State Standards explain that "mathematically proficient students look closely to discern a pattern or structure" (Common Core State Standards Initiative 2015).

## How Did I Use Math Last Night? Math Stretches

Students view mathematics in many ways. To some, it seems as if it is a game or puzzle to be figured out. Some students view mathematics as a set of arbitrary rules and procedures they must memorize. Others see it as a dreaded chore to be avoided whenever possible. To almost all students, it is a subject that is associated with textbooks or workbooks. Few fully comprehend the vital role mathematics plays in our daily lives. This particular math stretch encourages students to become more aware of the mathematics in their own lives and encourages them to recognize connections between the mathematics they encounter in the classroom and their real-life experiences.

Teachers have long recognized the value of making connections to improve reading comprehension. By making connections, students draw upon the background knowledge or schema they already possess as they encounter new ideas, melding them together to construct meaning from what they read. During reading instruction, teachers explicitly teach students how "to access and use their prior knowledge and experiences to better understand what they read" (Harvey and Goudvis 2000, 21). Students learn that there are three types of contextual connections they make as they read: text-to-self, text-to-text, and text-to-world.

In text-to-self connections, readers identify situations or links to their own lives in the text. This makes it easier for them to relate to the characters in a story, visualize a scene, or in other ways connect to their reading. Text-to-text connections occur when students are able to see similarities between the text they are currently reading and another text they have read. The more reading experiences students have, the more they have to draw upon in making these connections. With text-to-world connections, students relate what they are reading to what they know about the world in general.

While one might think that these connections would occur spontaneously as students read, that's not always the case. Often, students who begin school with a rich literary background easily make these connections as they read or are read to. For these children, bedtime brought the promise of snuggling with a family member as books were read, often repeatedly reading the same story. Not only were these books shared, but they were also the starting point for conversations linking favorite stories to experiences in their own lives. Talking about the pictures in the book, the relationships between characters, or other aspects of the text expanded the horizons of these children and encouraged them to recognize connections that helped them more fully understand what they were reading.

Conversely, many children, especially those who come from poverty, come to school with minimal exposure to reading. Because they have had so few literary experiences, relating to written materials can be difficult for them. They often fail to draw upon their own experiences or emotions to help them understand the text they are reading. Or, they do not have a broad range of diverse experiences from which to draw connections. Frequently, the very students who, if engaged in a conversation on the playground during recess, would readily share connection after connection, work so hard to read each word that it doesn't even occur to them to make connections when reading. They struggle to decode word after word, rarely pausing to construct meaning from what they have read. By explicitly teaching the strategy of making connections, teachers open the door to a deep trove of personal, internal resources that students may access as they read. This improves students' reading comprehension, and as understanding increases, so does motivation and excitement about reading.

The How Did I Use Math Last Night? math stretch prompts students to apply these reading strategies to mathematics. For homework the night before, students are asked to record a way in which either they or family members were involved in a mathematical activity. Young students may be encouraged to draw a picture if they are not able to write words to describe it (see Figure 3.2 on page 101). Older students will write about it, perhaps in multiple ways (see Figure 3.3 on page 102). They might use words, number sentences, or other representations. When they arrive in the classroom, they will add their examples of how mathematics is used in their daily lives to a chart, bulletin board, or other display. The variety of responses is unlimited:

- telling time to know when to go to bed
- setting the temperature on the oven
- measuring ingredients to cook dinner
- figuring out gas mileage
- balancing a checking account
- determining how much is owed on an overdue book at the library
- counting out silverware to set the table

As students see and understand the responses made by others, they will make more mathematical connections to their own real-life experiences.

**Figure 3.2—Primary Responses for How Did My Family Use Math Last Night? Stretch**

## How Did My Family Use Math Last Night?

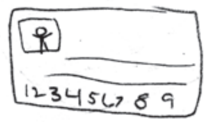
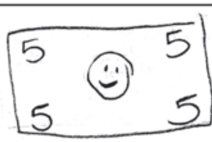
| | |
|---|---|
| Looked at the temperature T.R |  B.A. |
|  V.C. | My t-ball team scored 9 points. PG |
| Daddy bawt milk and got change back. L.R | My mom looked at the sped limit sign so she knew how fast to go. S.W. |
| Read a book and looked at the page numbers ML | Shared candies with three of my friends AC |
| My sister did her math homework. J.F |  O.G. |
|  DM |  RQ |

**Figure 3.3—Secondary Responses for How Did I Use Math Last Night? Stretch**

## How Did I Use Math Last Night?


Teachers may choose to vary this stretch to address specific math concepts. See Figure 3.4 (page 103) for a classroom chart of a stretch entitled How Did You Use Fractions This Weekend?

In *Comprehending Math: Adapting Reading Strategies to Teach Mathematics, K–6* (2006), Arthur Hyde modifies the three types of making connections used in reading to adapt them to math comprehension. The mathematical connections he sets forth are math-to-self, math-to-math, and math-to-world.

With the How Did I Use Math Last Night? math stretch, math-to-self, and math-to-world connections are emphasized as students reflect on and share their mathematical experiences at home and in their environments. Just as with reading instruction, effective mathematics teachers explicitly teach this comprehension strategy so that student understanding is enriched by an awareness of the context that surrounds the concepts being studied (Sammons 2011).

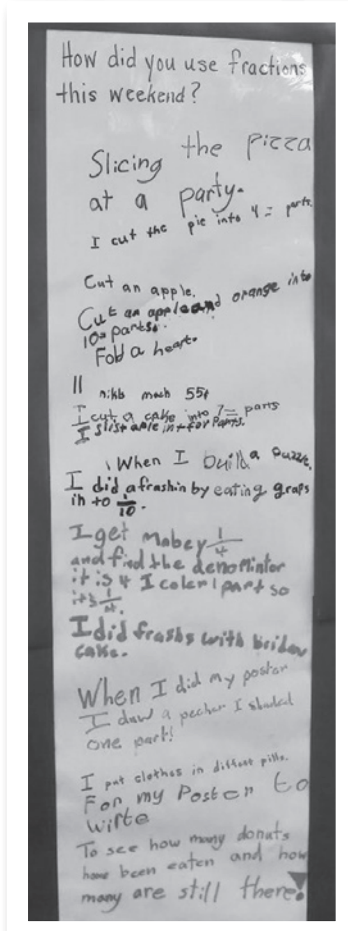



Figure 3.4—A Chart for How Did You Use Fractions This Weekend? Stretch

When it is time to discuss this exercise, students are usually eager to share their math-to-self and math-to-world connections. Later, these displays can be referred to during mathematics lessons to "build bridges from the new to the known" (Harvey and Goudvis 2000, 67). In addition, in this stretching activity, teachers are given insight into the mathematical experiences of their students at home and in the community. These experiences, along with their experiences at school, can provide meaningful contexts for future mathematical tasks (NCTM 2000). As students become adept at making connections, their conceptual understanding increases, and with it, their ability to apply these concepts in new, unfamiliar contexts increases too.

Some teachers may choose to make this a regular weekly stretch so that students form the habit of looking for mathematical connections in their home lives. Other teachers may use it when new concepts are being introduced as a way of helping students tap into their prior knowledge and experiences (Sammons 2011). In either case, these stretches provide opportunities to explicitly teach the strategy of making connections while at the same time actually having students apply the strategy, thus addressing the NCTM Connections Standard (2000) which states that students will be able to "recognize and apply mathematics in contexts outside of mathematics."

## _____ Makes Me Think Of… Math Stretches

As a mini activating strategy for the introduction of a new concept, as a review of a previously studied concept, or as a reflective exercise related to mathematical concepts currently being studied, students are asked to record a word or words that they think of when they are given a mathematical concept or word. For example, if the class is beginning a unit on fractions, the teacher writes, “Fractions make me think of…” on a large sheet of chart paper or on a whiteboard. Each student writes a word or phrase that comes to mind as they think of fractions. It should be something that has not already been recorded by another student. (See Figures 3.5 and 3.6 on pages 105–106.) In kindergarten or early first grade, students may draw simple pictures and label them, using their “best guess” spelling. It is important that these young students be taught exactly what a *simple* drawing is through teacher modeling prior to introducing this stretch. Otherwise, this activity becomes much too time consuming to use as a brief warm-up.

For each word posted during this stretch, students make another connection: math-to-self, math-to-math, and math-to-world. In the ensuing large-group discussion, as students share what they have written and explain why they chose these words, teachers highlight the value of making these connections and encourage students to use this strategy whenever they are engaged in mathematical work. This stretch directly addresses the NCTM Connections Standard (2000) as it encourages students to recognize connections among mathematical ideas and understand how they interconnect.

The student contributions to this chart also enable teachers to gauge their background knowledge and identify any misconceptions they may have. Lessons can then be adapted to meet the specific needs of students. The discussion may also lead some students to tap into previously unconsidered prior knowledge that was sparked by the comments of other students. The student-generated word list can remain posted in the classroom. Students can continue to add words as they think of them. If students decide that a word they posted is not really related to the given word, they may cross it out or erase it.

**Figure 3.5—Sample Elementary ______ Makes Me Think Of… Stretch**

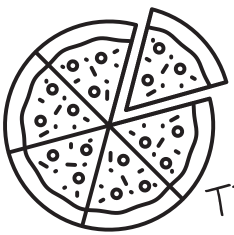
## Percentage Makes Me Think Of…

**Finish this sentence above using words, numbers, or pictures. Add your initials.**

stories on the news about elections
AM

100
MC

$\frac{1}{2} = 50\%$
CM


TR

fractions!
PS


40% chance of rain
SP

coupons at the market
LB

graphs
JA

$\frac{25}{100} = 25\%$
SL

baseball and batting averages
OM

IR

**Figure 3.6—Sample Upper Grade _____ Makes Me Think Of… Stretch**

## Integers Makes Me Think Of…

**Finish this sentence using words, numbers, or pictures. Add your initials.**

| | |
|---|---|
| AL  | ZM<br>-5 -4 -3 -2 -1 0 1 2 3 4 5 |
| ER<br>bounced check | DB<br>below sea level |
| KL<br>over and<br>under par  | MH<br>football<br>yardage  |
| GL<br>charge of electrons | NW<br>the stock market |
| BJ<br>in the hole in a card game | BS<br>subtraction |
| EW<br>stock market gains and losses | DD<br>debt |

## I Notice, I Wonder Math Stretches

Mathematicians benefit from observing and wondering about the world around them. Students are encouraged to engage in these practices with this stretch. Teachers display a picture or post a brief scenario from which students can draw math-related observations or wonderings on chart paper or on an interactive whiteboard. Along with the picture or scenario is a T-chart with one column labeled *I Notice* and one column labeled *I Wonder.* Students examine the picture or read the scenario and then add their own responses in one of the two columns. Of course, they may read what others have responded, but their own addition to the stretch should be unique. This stretch is appropriate for students of any age. Teachers choose pictures or scenarios that will spur students' mathematical thinking. Students' responses are self-differentiated because what each student notices or wonders about is often determined by their level of mathematical understanding. The student postings provide a rich source for mathematical conversations during the math huddle that expand the perspectives of students as they become aware of what others are noticing and wondering. As an alternative approach to this stretch, a picture or scenario may be posted, but on the first day, students are only asked to post what they notice. The next day, the stretch continues with the same picture or scenario, but this time, students respond with what they wonder.

## Planning Math Stretches

These math stretches are only a few examples of the kinds of activities teachers can use to motivate their students to begin thinking mathematically. Teachers may adapt many other activities to create brief math stretches as well. Many teachers choose to work collaboratively in planning their math stretches. When creating math stretches, it is important to keep the following characteristics in mind:

- They are very brief.
- They have enough possible responses so that each student can have a unique response, except for data collection stretches.
- They can be completed by students independently.
- They prompt students to think mathematically.
- They generate mathematical communication.

Some teachers find it convenient to repeatedly use the same five stretches each week, assigning one regularly for each day of the week. Students come to know the schedule and what to expect as they enter the classroom. An example of a weekly plan of the math stretches for a third grade class is shown in Figure 3.7 (page 108).

**Figure 3.7—Math Stretch Plan for a Week**

| Day of the Week | Math Stretch | Topic |
|---|---|---|
| Monday | Number of the Day | 180 |
| Tuesday | What's Next? | 1, 3, 9, 27, … |
| Wednesday | How Did My Family Use Math Last Night? | Real-life mathematical connections |
| Thursday | ______ Makes Me Think Of… | Multiplication |
| Friday | Data Collection | Where would you rather go for a field trip?<br>• science museum<br>• planetarium<br>• aquarium |

In this weekly plan, students know that they will encounter a Number of the Day math stretch every Monday, only the number will change from week to week. On Tuesdays, they will always work on a What's Next? math stretch and so on throughout the week. Having a regular and familiar routine for math stretches allows students to know what they will be expected to do each day and be prepared for it. This also can make planning easier for the teacher.

Teachers who prefer more flexibility in planning pick and choose stretching exercises to best meet their instructional goals. Whether used to preview a new concept, explore current areas of study, or review concepts examined earlier in the year, teachers have flexibility in designing and assigning math stretches, always keeping in mind that the goal is to provide a quick, mathematical "mind focus" as students begin class. Additional math stretch tasks may be found in *Daily Math Stretches: Building Conceptual Understanding* (Sammons 2010; Sammons and Windham 2011; Sammons and Dase 2011) books. Although the tasks are grouped according to the grade level bands (K–2, 3–5, and 6–8), most can be adapted for use with students of any age.

## Mathematical Current Events

For many years, teachers have incorporated current events into their instruction. Even with the instructional demands imposed by testing mandates, most educators recognize the value of students receiving a well-rounded education. For most, well-rounded means that students learn about what is going on in the world in addition to the state curriculum. Textbooks are a poor resource for teaching current events because they can be out of date as soon as they are printed. To keep current about world news, teachers and students turn to the media. Teachers in the past frequently shared articles from newspapers or magazines with their students or required students to bring in articles of interest. Some classes subscribed to weekly news magazines. Today, teachers and students more often turn to the internet to follow the news from around the world. Unfortunately, current events instruction is rarely included in disciplines outside of social studies or science.

Although math is seldom a focal point of current-events instruction, math certainly plays a major role in much of the news:

- In politics, polls rely heavily on numbers.
- Understanding the state of the economy requires knowledge of math—unemployment percentages, inflation rates, or the rising and falling of the stock market.
- Reports from wars include the number of casualties, costs of the conflict, or related statistics.
- Reporting on the effects of a drought, journalists rely on mathematical calculations to accompany their descriptions of conditions in the affected areas.
- Storm coverage includes precipitation amounts, flood levels, wind velocity, monetary damages, and number of people affected.

- Sports coverage provides win/loss statistics, records set, averages, and salary negotiations.
- Tables and graphs have become an integral part of news reports because they organize data in ways that are easily assimilated by the public.

Our students live in a world where current events swirl around them unceasingly. Some students are much more aware of what's going on in the world than others. Some students are only focused on current events relevant to their own particular interests. Some students are almost completely oblivious to world events. Despite their levels of awareness, though, mathematical connections are rarely, if ever, recognized. When current events are discussed in mathematics classes, students become increasingly aware of the ever-present relationship of mathematics to the world all around them. It becomes more meaningful and relevant to students, and so they begin to notice the immediate impact it has on their lives. These connections offer teachers rich opportunities to incorporate real-life mathematical contexts into class investigations and problem-solving activities.

Mathematical current events are fairly easy to incorporate into instruction. Initially, teachers may post math-related news on a bulletin board and discuss it with the class. The teacher may choose to explain, through a think-aloud, the math connection and how it helps the public understand the subject of the news article. The first few current events shared should be diverse and illustrate some of the various ways in which mathematics connects to current events. As students begin to appreciate the role of mathematics in the news, they may be encouraged to bring in news items to be posted and discussed by the class. Lavishly commending students for contributing articles to the current-events board usually motivates others to do the same. If not, extra credit may be given for news brought in and shared, or a homework assignment might require finding a news article and writing about its mathematical connections.

As current events are posted, discussed, and analyzed for mathematical relevance, teachers can also help their students discover how math is sometimes used to distort the truth. Students learn how important it is to evaluate the validity of all news reports when they discover how numbers can be manipulated and skewed. A teacher-guided check for mathematical accuracy in news items often brings to light incorrect data or misleading analyses and leads students to read the news, particularly on the internet, more thoughtfully. This kind of critical assessment is especially important in light of the recent proliferation of *fake news,* both in social media and in more traditional new sources. As students learn how to assess the validity of news items, especially using what they know about mathematics, they gain a valuable life skill.

## Math-Related Classroom Responsibilities

One way to bring real-life mathematical experiences to students is by involving them in classroom responsibilities that require the use of math. Turning the responsibility for these tasks over to students gives them practice with daily problem solving while building a sense of community as students work together. These tasks also have relevance to students because they can see clearly how these tasks affect the class.

In most classrooms, there are many jobs that need to be done. Turning some of these over to designated students frees the teacher to work directly with students needing additional support. It also gives the student helpers valuable mathematical experiences with authentic tasks. Initially, it takes time to identify math-related jobs that are appropriate for students to handle and to teach students how to perform them. Patience in teaching students these tasks in the first few weeks of school pays off in the long run.

One or two students may be given responsibility for taking daily attendance. Depending on their grade level, they may only record the number of students present and the number absent. Older students may add the two numbers together to check their totals, or they may be asked to compute the percentage of students present and absent. Figuring absentee percentages is especially useful if the class sets monthly goals for attendance. Students may also be given the responsibility of determining the best way to display this data so that the entire class can follow its progress toward meeting the attendance goal. This activity gives students authentic practice in computation and data analysis while motivating students in the class to attend school regularly.

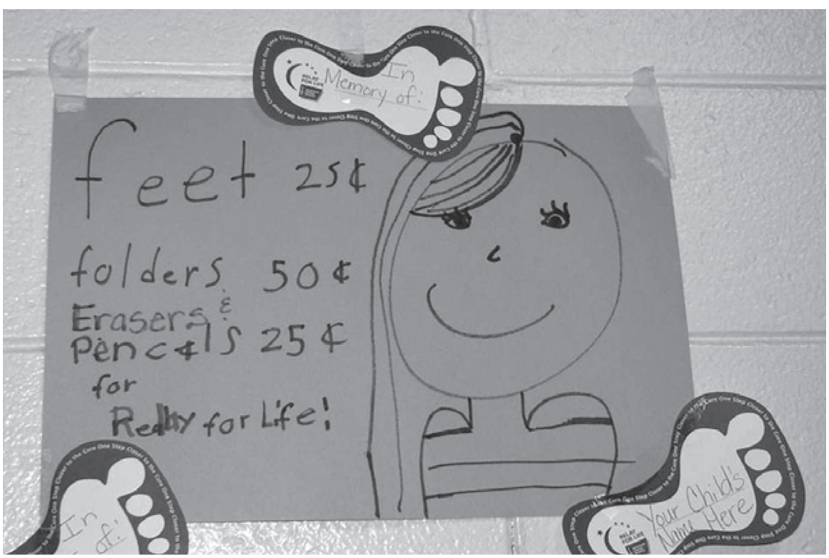
As schools strive to maximize student achievement, many are seeking ways for students to assume greater responsibility for their own learning. Some schools encourage teachers to show students how to set their own personal goals and then self-monitor their progress toward reaching their goals. After students confer with their teachers to set their goals, they create and maintain personal, foldable booklets using graphs to document their progress. These booklets may be used to record data related to math-fact fluency or mastery of standards, for example. To vividly show their progress, students graph, often in bright colors, their data for each goal. When students analyze their personal data, it becomes obvious how mathematics can assist them in their efforts to meet their goals. When students set and then assume responsibility for using data to track their own progress, they learn beneficial life skills and discover the utility of mathematics.

**Power Word**

***mathematize:*** *to think about a situation in mathematical terms*

Additional math-related tasks abound for students in classrooms. If snacks are part of the school day, student helpers may use attendance numbers to determine how many snacks will be needed that day. Fosnot and Dolk (2001) describe a preschool program in which class helpers were asked not just to determine how many snacks were needed, but whether there were enough snacks to serve one to each classmate who was present. The helpers were given sheets containing copies of the snack item on hand in the quantity available. In addition, paper, pencils, and linking cubes were available to these helpers. The actual snack items themselves were not provided because then the helpers would simply have given them out to see if there were enough. Instead, the helpers developed various strategies using the materials available to model or draw the problem mathematically. Because their task dealt with snacks and students, not just numbers, it was very real and meaningful to them. When the helpers arrived at answers, the teacher followed up with strategic questioning, prompting students to communicate the thinking behind their "mathematizing" and supporting the growth of mathematical understanding in these preschool students. This task took students far beyond mere rote counting.

Teachers who are attuned to finding mathematics in day-to-day routines will have no problem involving students of all ages in worthwhile, authentic jobs where they can practice their skills and deepen their understanding of how embedded mathematics is in our lives. When students assume responsibility for math-related tasks, they become increasingly adept at applying their mathematical prowess to everyday problems and grow confident in their understanding of the mathematical concepts involved. Mathematics assumes an importance to them because it affects their own classrooms and classmates rather than being meaningless numbers and word problems from the pages of their math books. Involving students in these classroom jobs aligns directly with the NCTM Connections Standard (2000), which states that students should be able to "recognize and apply mathematics in contexts outside of mathematics" (64). Students spend a large part of each day at school. This provides a meaningful context for students to recognize and apply mathematics.




***Students sell school supplies to raise funds for the American Cancer Society's Relay for Life.***

## Daily Task/Calendar Board

Visitors to elementary school classrooms worldwide find versions of task/calendar boards in various colors, styles, and designs in classroom after classroom. It is not surprising because the calendar board is one of the most versatile teaching tools in elementary classrooms. In fact, because of its utility, teachers of upper grade students may choose to adapt the concept to create a daily task board as a math warm-up for students.

This interactive bulletin board usually includes a calendar, number lines, number charts, some kind of modeling system for place value, and a variety of components that are designed to be introduced at different times during the school year to focus on a variety of mathematical concepts. These components are usually displayed on a brightly colored, inviting bulletin board in front of an open area where the class can gather. The elements of the calendar board vary based on grade level and the needs of students. In upper grades, these boards may not include calendars, but instead the visual elements that align with the math content being learned and simple tasks designed to preview upcoming content, practice what is currently being learned, and review what has already been learned.

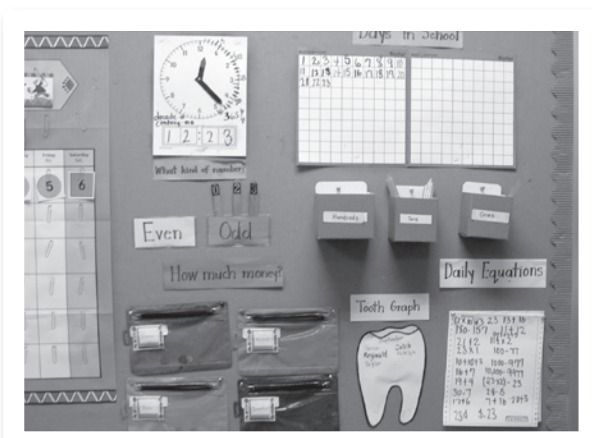
Commercial daily calendar programs are available but are not necessary. The commercial programs provide an array of pre-planned and prepared activities that change from month to month and may also include materials ready for use. If commercially produced programs are used, however, teachers are advised to be selective in deciding which of the components to use and when to use them. Calendar board instruction should be designed to meet the instructional needs of students. Teachers who keep that in mind can choose which of the components meet the needs of their students and use them, not necessarily in the order suggested by the program. However, using ready-made programs can be a valuable time saver for teachers, and as long as the ready-made programs are used to support the established instructional goals rather than guide them, they can be very effective.

The task/calendar board allows teachers to offer consistent, daily learning opportunities covering a range of mathematical concepts.

During this 10- to 15-minute daily instructional segment, students are actively engaged in a continuous learning process as they interact with its elements. The consistency of the daily instruction helps students incrementally build understanding of mathematical "big ideas," challenges them to notice patterns and relationships, and encourages them to share their mathematical discoveries in class discussions. Daily use by teachers has the following benefits:

- gives students support in learning mathematics incrementally as they develop understanding over time
- provides visual models to help students recognize mathematical relationships
- fosters the growth of mathematical language acquisition and promotes student reasoning ability through mathematical conversations
- promotes algebraic thinking
- allows teachers to informally observe their students' mathematical understanding and then adapt instruction to meet students' needs (Gillespie and Kanter 2005, 4)

The task/calendar board is an excellent vehicle for previewing concepts, reviewing previously mastered concepts as maintenance, and engaging students in ongoing, daily practice with concepts that are difficult to master.




***sample calendar board***

## Previewing Content

Providing some prior experience with mathematical concepts that are prerequisites for new content gives students scaffolding for new learning, thereby increasing their success when they eventually work with these concepts. This instructional strategy of previewing is effective for all students, but it is especially effective with below-level learners who frequently bring less background knowledge to the learning experience. Research shows that the more academically oriented experiences students have, the more opportunities they have to store those experiences as academic background knowledge upon which they can draw (Marzano 2004). Giving students opportunities to preview new math concepts and skills enhances their overall achievement. Previewing activates students' prior knowledge and builds additional academic background knowledge with brief, introductory tasks linked to the concepts to come. This bridges the gap between what they know and what they need to know to successfully learn and understand these concepts (Marzano, Pickering, and Pollock 2001). Task/calendar board instruction provides an ideal structure for engaging students in brief tasks a week or two before the actual unit of study when the concepts will be addressed. Furthermore, relevant vocabulary can be presented and revisited daily, leading up to the actual introduction of the concepts. When students finally encounter these concepts in greater depth, they are able to confidently make connections to their recently accumulated experiences and build on those foundations.

> **Power Phrases**
>
> ***massed practice:*** *practice sessions placed very close to each other during the mathematical instruction on the standards being taught that lead to mastery*
>
> **distributed practice:** *practice sessions that are at less frequent intervals that lead to deeper conceptual understanding and retention*

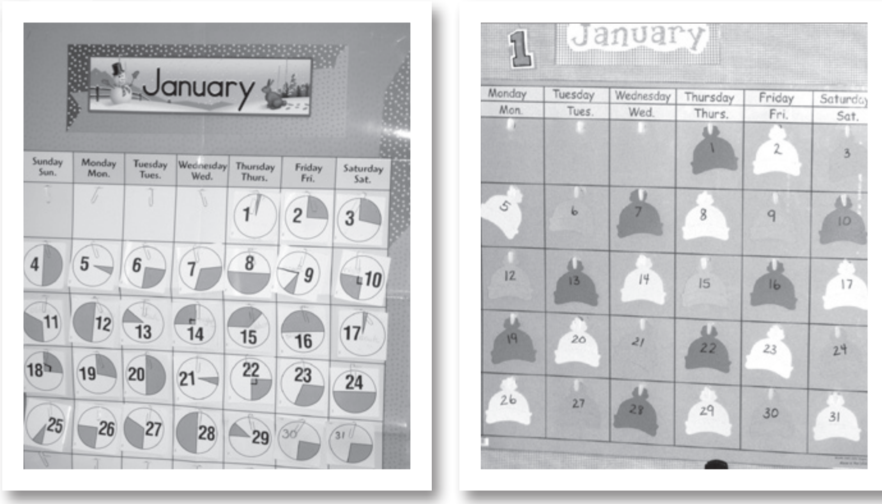
Once concepts have been learned, students need to continue to work with them to maintain and strengthen their understandings. The initial practice with these concepts during the unit of study is considered *massed practice*—practice sessions placed very close to each other during the mathematical instruction on the standards being taught.

Massed practices lead to student mastery, but for deeper conceptual understanding, student work with these concepts needs to be continued at less frequent intervals (Marzano, Pickering, and Pollock 2001). This *distributed practice*, as it is known, is often neglected until the weeks before high-stakes testing when teachers provide intense review of the entire year's curriculum. As an alternative, the task/calendar board offers teachers a time to briefly revisit previously taught concepts and skills on a regular basis. This helps students maintain understanding and competency while freeing the regular mathematics instruction time for teaching other mathematical standards. Rather than frantic review prior to testing, the ongoing revisiting promotes deeper understanding and increases the ability of students to apply their knowledge to new or different contexts.

## Ongoing Practice

Additionally, the task/calendar board provides opportunities for year-long ongoing practice of skills with which students traditionally have difficulty—what might be called grade level *hot spots*. For example, second grade students often struggle with finding missing addends. Therefore, a teacher may decide that on the task/calendar board *every* Tuesday, they will include a problem or task requiring them to determine missing addends, working together as a group. By the end of the year, students will have thought about and practiced the skill at least once a week. That ongoing practice makes an enormous difference. With experience, teachers are able to identify five concepts that are the most difficult for students to master so they can be included in the daily task/calendar board activities. These brief activities scaffold and strengthen the conceptual understanding of these *hot spots* throughout the year.

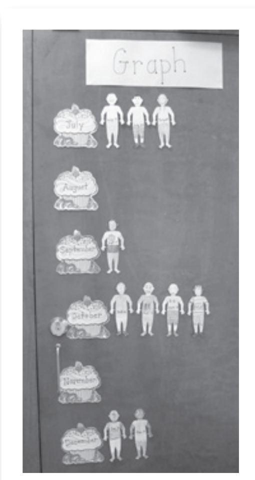
The variety of elements that can be included on a task/calendar board lesson is limited only by a teacher's creativity. Young students may use the calendar to determine the date and the passage of time. In doing so, they often identify patterns the teacher creates with the daily calendar pieces. Depending on the grade level, the pieces may repeat a pattern of simple shapes and colors or can have much more complex patterns related to mathematical concepts being studied. For these students, the seasons and weather may also be discussed.




*Calendar pieces can display complex concepts, such as angle measure, or simpler concepts, such as color patterns.*

## Graphing Practice

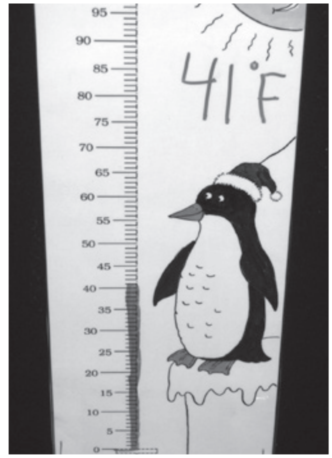
Graphing activities related to real-life data collections are frequently included in task/calendar boards. Throughout the year, students are called upon to compile different kinds of graphs and tables. As a group, they analyze the data to determine trends and patterns. Topics for graphing activities can be chosen by teachers or students. Elementary classes typically compile graphs that track birthdays or weather patterns. A class may even decide to assist the lunchroom staff by compiling data on favorite lunch selections. Secondary students may collect and compile data regarding issues of interest specific to them, sometimes related directly to their schools. For example, a current topic of concern is the start time for high schools. Students may analyze data about students' preferred starting times. Using authentic scenarios helps students understand how graphing and mathematics are part of their daily lives.




*This graph displays students' birthdays.*

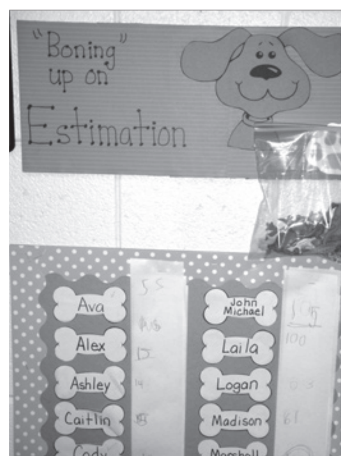
## Measurement Activities

Task/calendar board elements often include measurement activities. They may involve determining the outdoor or indoor temperature, sorting canned food by weight, finding the circumference of pumpkins during the fall, or determining the heights of students. Many upper grade students are woefully lacking in a sense of size of the measurement units. Incorporating brief activities that call upon them to interact with these units in multiple ways develops their abilities to visualize them in other contexts. Teachers can creatively connect measurement opportunities to other content-area units of study throughout the year, so that they are not limited solely to the few weeks of the measurement unit. These activities support the NCTM Measurement Standards, which state that students should be able to understand the measurable attributes of objects and use appropriate tools to determine measurements (2000).




*This measurement activity combines mathematics and science.*

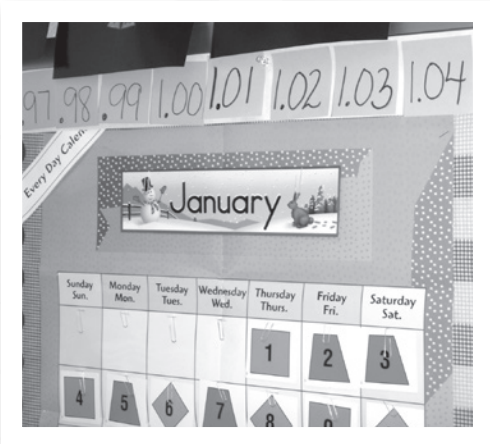
To hone students' estimation skills, the teacher may provide weekly estimation challenges. A jar or bag of objects is shown to the class, and students estimate how many objects are in it, or an object may be shown to the class for students to estimate its length or weight. The ability to accurately estimate quantities or measurements is an important skill that adults use in many aspects of their lives. The repetition of these activities throughout the year challenges students to develop greater proficiency at these skills than the experiences they get during a unit lasting a week or two.



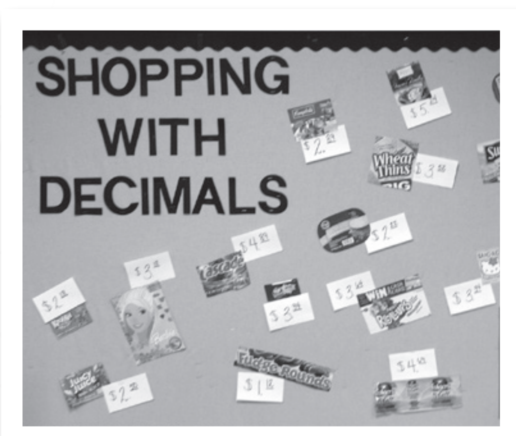

*Estimation can be difficult, so it should be practiced often.*

## Number Lines and Place Value

In the primary grades, number lines on the task/calendar board are created by adding a number for each day of school. Skip counting can be practiced as a group as number patterns are identified. Prediction activities are often included. At the beginning of the year,




***On this number line, each day is represented as 0.01.***




***This activity provides students with an understanding of the relevance of mathematics in real life.***

students might predict how far around the room the number line will extend by the end of the school year and be given opportunities to adjust their predictions periodically during the year.

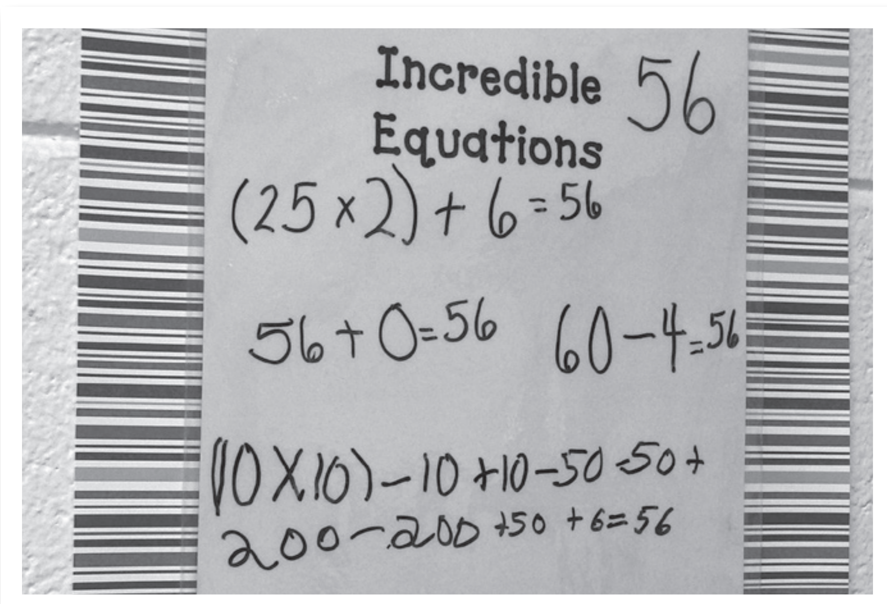
The task/calendar board also provides opportunities to address place value. In primary classrooms, straws may represent ones and then be bundled to become tens or hundreds. In upper grade classrooms, students work with place value for decimals, numbers of much greater magnitudes, or numbers of much less magnitude. This element of the task/calendar board, along with the previous two elements, addresses the NCTM Number and Operations Standard and the Representation Standard (2000).

## Working with Money

Money activities are often included in the task/calendar board during the year. These may range from simple identification of coins and their values to complex problems involving calculations of the total costs of items, the amount of change someone will receive, or calculating interest or taxes. With older students, this may be linked with their work experience and reinforce personal finance skills. Although technology has made tracking money in jobs much less demanding, this skill is still essential.

## Incredible Equations

To develop students' number sense, the task/calendar board may include an incredible equations component similar to the number of the day chart used during the math stretch. For incredible equations, the teacher provides a number. The number may come from the day's date or the number of days they have been in school. Students suggest equations that equal that number. Some may be quite complex. Because of the nature of this task, students work at their own levels of understanding, but they may be exposed to the more complex thinking of their classmates. Students may create these equations individually or work together in groups. Students often enjoy choosing the one they think is the "most incredible" after several equations have been recorded. This element of competition and fun motivates students in their explorations as they compose and decompose numbers. These activities support student understanding of the NCTM Number and Operations Standard (2000).



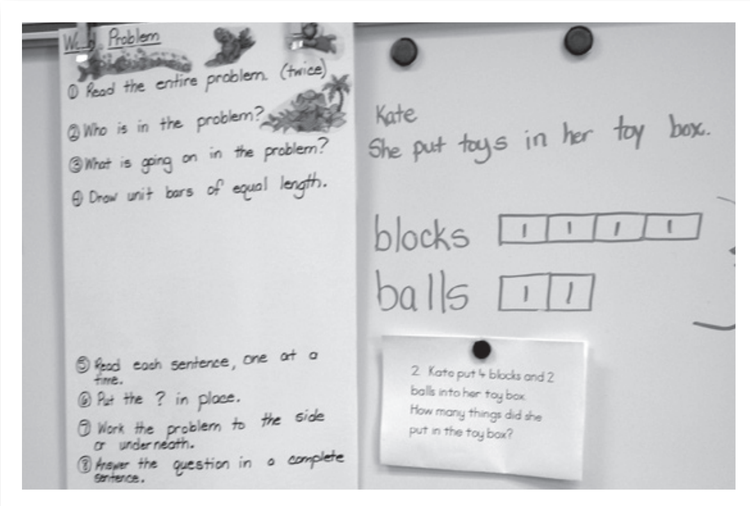

*This type of activity naturally allows for differentiation and gives students a chance to respond at their own levels.*

## Problem of the Day

Daily problems are commonly a component of the task/calendar board. The teacher facilitates the problem-solving efforts as students tackle problems or conundrums, often presented in unfamiliar contexts to which they can apply mathematical skills they have already mastered. By presenting these in contexts that students may not be used to seeing, teachers prepare their students to be flexible and creative in problem solving. This helps avoid the panic that sometimes occurs if students encounter problem-solving contexts with which they are not familiar. These ongoing challenges align with the NCTM Problem Solving Standard (2000). See Figure 3.8 for examples of daily problems.

**Figure 3.8—Examples of Problems of the Day**

| Grade Range | Daily Problem |
| --- | --- |
| K–2 | How many cubes do you think you can hold in one hand? Make a prediction and then find out. Work with a partner to write an addition sentence to show how many cubes you and your partner could hold altogether, each of you using only one hand. |
| 3–5 | How many three-digit numbers can you think of whose digits have a product of 60 when multiplied? |
| 6–8 | The lengths of two sides of a quadrilateral are even numbers. The lengths of the other sides are odd numbers. Will the perimeter of the quadrilateral be an even or an odd number? Explain how you know. |
| 9–12 | A diver climbs the high-dive platform and then dives into the pool. Draw a brief sketch of a graph to show his elevation over time. Explain why your graph looks the way it does. |




*Daily problems allow for daily practice of problem-solving strategies.*

Teachers should use the task/calendar board elements described here selectively. Many teachers choose to include a few of these elements on a regular basis in their task/calendar board every day. Some tasks are addressed just once or twice a week. Others may be used only a few times a year. The flexibility of this instructional tool allows teachers to adjust the instruction based on the needs of the class. A word of warning, though—because of the many options available with a task/calendar board, teachers are sometimes tempted to devote too much time to this warm-up, which cuts into their instructional time. When using a task/calendar board, teachers should decide how much time they will allocate to it and then adhere strictly to that time limit.

## Planning for Math Warm-Ups

Although the amount of time allotted to math warm-ups is relatively brief, these routine events can have a major impact on the attitudes of students in the mathematics lesson to follow. Careful planning helps students transition from the hustle and bustle prior to class to a more focused mathematical mindset. Encouraged to dip into their wells of mathematical knowledge in these brief, focused activities, students are nudged toward readiness for the mathematics lesson to come.

The consistent structure of the math warm-up components makes planning relatively simple for teachers. Teachers can choose elements from the array of components that support the learning needs of their students and address the standards they are teaching. The consistent structure also facilitates classroom management as students are taught the basic procedures for these elements at the beginning of the year. Upon entering the classroom each day, they are well aware of the expectations and can assume responsibility for carrying out the expected tasks with little direction from the teacher. This allows the teacher to focus on working with those students who need extra support.

## Review and Reflect

1. Think about how your students enter your classroom and begin working. Is there a mathematical connection that prompts them to begin thinking mathematically? Does it involve more than an activity sheet?

2. Why is it important to help students recognize the links between math and their own lives? What are you doing in your classroom to help students make this connection? How can you make the link even stronger?

## Chapter Snapshot

It is important for students to prepare their minds for their math classes. Brief math warm-ups help students to transition to the demands of a mathematical classroom environment. Many types of tasks may be used to spur this transition, including math stretches, mathematical current events, math-related classroom responsibilities, and task/calendar boards.

Math stretches can be designed to address any math concept or skill being studied, depending on grade level and students' level of understanding. The inclusion of all students in the math stretch generates a shared mathematical learning experience for the class community. This is important because students can refer to this shared experience during discussion in a math huddle, and this builds their background knowledge on the concepts discussed.

Daily task/calendar board activities can be used to reinforce mathematical concepts that need to be covered throughout the school year. Depending on the grade level, concepts such as place value, fractions, operations, and problem solving can be covered during this instructional time. Daily task/calendar time also provides an opportunity to have a math huddle, where the daily math stretch is discussed. The math huddle discourse incorporated into math warm-ups challenges students to expand their thinking and affords teachers valuable insights into students' mathematical understanding.

***Whole-class instruction provides teachers with a quick method of presenting the information to all students.***

# Chapter 4

## Whole-Class Instruction in a Guided Math Class

Chapter 1 begins with a description of a traditional math class in which working with the whole class at once was the primary mode of instruction used for every lesson. This instructional approach has been used in schools for a long time. But is it the most effective method? Do we need to move away from this traditional instructional approach? If yes, why should we change?

Teachers who have been in education over the years know well the cyclical nature of what are regarded as the best teaching practices. Teachers are caught in the crosswinds as educational trends come and go and then come again. We frantically work to adapt our classroom teaching methods to what must certainly be the "right" way to teach, the most effective way of reaching all students, especially those who need extra support or more challenging tasks.

In our passion to reach all students, we grasp at the hope offered by these new approaches. As we embrace the new, we often recognize the value of some of the methods we have used in the past, which are being abandoned in light of the current advice of our educational leaders. Being pushed in one direction and then pulled in another, teachers can become frustrated and unsure of their capabilities. It is no wonder that teachers may be hesitant to abandon their teaching methods for those currently being touted as "most effective."

Fortunately, many teachers trust their own instincts and experience, regularly reflecting on the efficacy of their teaching practices. These educators continually study new educational research in search of ways to refine and improve their teaching methods. Joining in professional learning communities with their colleagues, they try new approaches in their classrooms, adapt them as needed, if effective, and integrate them into their overall instructional approach without necessarily abandoning methods that have proven to be effective in the past. Their focus is on using teaching methods that work in situations where they work best.

Before discarding methods entirely in our efforts to improve our instruction, we should first evaluate their effectiveness in a variety of contexts. While whole-class instruction may not be the most effective approach for all lessons, there are times when it can be used effectively.

## The Advantages of Whole-Class Instruction

As we know, whole-class instruction was the mainstay in schools of the past and is still used in many mathematics classes today. Any instructional method that has been used over such a long period of time by so many teachers must have some advantages, and it does. Whole-class instruction provides teachers with a quick method of presenting information to all students. Everyone is given the same information and essentially engages in the same activity, at the same time.

Lesson planning is straightforward. Teachers are only required to plan a single lesson at one instructional level. All students are expected to complete the same assignments. In contrast to having to plan for workstations, independent student activities, and small-group lessons for several groups, planning for whole-class instruction is streamlined.

When teachers aim to capture the interest of an entire class and tap their prior knowledge at the beginning of a unit of study or when a new concept is introduced, whole-class instruction may be a good choice. Activating strategy activities will stimulate student interest about upcoming topics of study and give the teacher valuable insights about what their students already know or about misconceptions they may have. As a kickoff for a new instructional unit or even when introducing a new concept or skill with an activating strategy, whole-class instruction is effective.

In Chapter 2, the importance of building a classroom learning community was emphasized. One of the ways to establish a strong sense of community among students is through their participation in common experiences. Common experiences are generated when students work together as one in a class. Even when students have been working together in smaller groups rather than as a whole class, having them come back together for reflection and conversation about the experience adds to a feeling of being one community. Moreover, these discussions also serve to deepen their mathematical understanding and improve their communication skills (Sammons 2018). According to Hiebert and colleagues, "Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in math" (1997, 6). The reflection and reasoning they do as they talk about shared mathematical experiences explicitly support both the NCTM Process Standards of Problem Solving, Reasoning and Proof, Communication, Connections, and Representation (2000) and the CCSS Standards for Mathematical Practice (2015).

Whole-class instruction allows teachers to spend a greater amount of time engaged in direct teaching to each and every student in the class—although it may not be the most productive use of time. Teachers can choose to provide direct instruction to the whole class for the entire math period, if desired. In contrast, when working directly with small groups of students during small-group lessons, the teacher works with each group of students for only a portion of the class period. However, teachers often feel that they can accomplish more in a shorter period of time working with students in a small group because this instruction is more effective. When deciding whether or when to use whole-class instruction, teachers have to carefully weigh the trade-off between increased effectiveness or greater amount of time of direct instruction.

There may be times a teacher chooses to use whole-class instruction because the entire class is working at approximately the same level. Since students' needs are similar in these situations, the instruction may not need to be differentiated. Most often, when this occurs, it is for a review of previously mastered concepts or skills—perhaps just to ensure they are maintained, prepare for a summative assessment, or perhaps practice for high-stakes testing. These types of general reviews usually consist of undifferentiated, quick overviews, which also serve to identify students who may need additional support for mastery.

## The Challenges of Whole-Class Instruction

Despite the advantages described here, many teachers find whole-class instruction to be frustrating as they endeavor to meet diverse student needs. It is exceedingly rare that all students in a class are at the same level of competency at the same time. Even experienced teachers often find that whole-class instruction lessons go over the heads of some students, leading to passivity and lack of attention, while failing to challenge others, leading to boredom. At best, only a fraction of the class is fully engaged at any time.

No matter how proficient a teacher may be, it is almost impossible to ensure that all students are attentive and academically engaged through whole-class instruction. Anyone who has ever been in a classroom with students knows firsthand how easily distracted they can be. Even when most students are actively involved in a lesson, some of them miss key instructional content while intently examining something in their desks, gazing out the window, or trying to gain the attention of nearby students.

The NCTM standards (2000) and the CCSS math standards (2015) both include mathematics process/practices standards that are consistent for all students from kindergarten to grade twelve. For students to meet these standards, they need lots of practice being actively engaged in solving challenging problems, thinking critically about the math they are learning, and talking with others about their mathematical experiences. These experiences include what they are doing, why they are doing it, and most importantly, what they are learning from their work. This talk is most productive when it occurs as students work and reflect. Unfortunately, with whole-class instruction, inclusive student communication is limited. Classroom talk tends to be dominated by the teacher as they inform and then question students. Even when students are part of a conversation, the very nature of whole-class instruction precludes everyone from taking an active part in the math talk. There simply isn't enough time for each student to have a turn. Students may have some opportunities to turn and talk with partners, but true conversation between students where mathematical ideas are exchanged, reflected on, and debated, does not happen often during whole-class instruction. As Irene Fountas and Gay Su Pinnell write, large-group instruction "tends to marginalize those students who need more interaction and closer contact with the teacher…" (2001, 219). While teachers may encourage class discussion of shared mathematical experiences in a whole-class setting, other modes of instruction offer better opportunities for students to develop these skills. For example, as students work on mathematical tasks in small-group lessons, teachers are better able to facilitate discussions to ensure that every student actively participates.

Research indicates that giving students timely and specific descriptive feedback has a significant positive impact on learning. Researcher John Hattie emphatically states, "The most powerful single modification that enhances achievement is feedback. The simplest prescription for improving education must be 'dollops of feedback'" (1992, 9). Yet, with whole-class instruction, the "dollop" of descriptive feedback teachers are able to provide to their students is extremely limited.

Even when students are working independently on practice problems while their teacher circulates around the room to check their work, the limited class time available results in feedback to students that tends to be quite brief, if any is given at all. Moreover, to maximize its impact on student achievement, Marzano, Pickering, and Pollock (2001) recommend that not only should students receive specific feedback, but they should also be given time and encouragement to continue working on the task so they can apply what they learned from the feedback. With whole-class instruction, the amount of time for individual feedback and for students to apply it to improve their work is often not sufficient to maximize student achievement.

Another drawback to whole-class instruction is the fact that there are always some students who do not quite understand a lesson and then diligently work to complete assigned tasks based on misconceptions. With large class sizes and the time constraints of whole-class instruction, a teacher may not be able to get to these students during the independent practice time. When the teacher does confer with them about their misconceptions, it may be difficult for them to modify their thinking once they have already practiced their work based on their misconceptions. If students' work is not checked while they work, their misconceptions remain and can become deeply ingrained.

Closely related to these issues is effective assessment. In recent years, teachers have become increasingly aware of the importance of both summative and formative assessments. Summative assessment, which is more evaluative in nature, allows teachers to see exactly what students have achieved after a unit or units of study. This is the predominant type of assessment so many of us experienced during our school years. Summative assessment is designed to be an assessment *of* learning after instruction. Formative assessment is an ongoing assessment used to inform daily instruction and make it more effective. As such, formative assessment is assessment *for* learning. These day-to-day, often informal, assessments provide immediate, accurate evidence of student learning that teachers use to make crucial instructional decisions every day.

The use of formative assessments has been shown to trigger remarkable gains in student learning. They "provide a continuous flow of evidence of student mastery of classroom-level learning targets that lead over time to attainment of the desired achievement standards" (Stiggins 2004, 22). When teachers work with students in small groups or confer individually with them, they learn what their students are thinking mathematically and can more accurately gauge their level of learning. This permits teachers to adjust instruction accordingly. Moreover, students can play a role in the formative assessment process by reflecting on their own learning with the use of rubrics or other assessment tools. Not only does this student self-reflection benefit teachers in planning their instruction, but it also benefits students by helping them develop the habit of reflecting on their own learning. This time of reflection is an important life skill. As students learn to determine their own levels of mastery, teachers can show them how to set reasonable, yet challenging learning goals and how to monitor their progress toward reaching those goals.

Teachers gather some of the most valuable formative assessment information as they closely observe and listen to students work. This is difficult to do when teaching in a whole-class format. When using whole-class instruction, teachers tend to rely more heavily on short paper-and-pencil formative assessments. These, for the most part, assess outcomes rather than the thought processes leading to these outcomes. While these assessments may provide a glimpse into the level of student learning, they fail to provide information about the complexities underlying students' understanding and mastery. Without this additional information, the value of these assessments for planning instruction is lessened.

## Choosing When to Use Whole-Class Instruction

Whole-class instruction is one component available to teachers when teaching with the Guided Math framework. Teachers who are knowledgeable about both its advantages and challenges make use of it selectively for appropriate instructional tasks. While whole-class instruction is not ideal for regular, extended daily use, there are situations when addressing the entire class at once is not only satisfactory but also very effective. It is important to consider the lesson content, the purpose of the instruction, and the needs of students when planning what type of instruction to use so that each lesson best meets those needs. Whole-class instruction works well for the following kinds of mathematical activities:

- involving students in activating strategies
- making math connections to literature
- setting the stage for math workshop
- conducting math huddle conversations
- providing practice and review work
- administering formal testing or assessments
- teaching mini lessons (occasionally)

## Activating Strategies

Research consistently shows that activating prior knowledge is critical to students' learning (Marzano, Pickering, and Pollock 2001). Just as reading comprehension involves both conscious and unconscious strategies to access, use, and modify the prior knowledge of the reader, those involved in mathematical activities are engaged in a similar, complex process as they seek to understand mathematical concepts and solve problems. The connections students make with their own individual sets of knowledge make it possible for them to engage in higher-level comprehension strategies (Fountas and Pinnell 2001).

In addition to activating students' prior knowledge, teachers enhance learning by helping students anticipate new knowledge being introduced. These cognitive activating strategies are not summaries or overviews but instead act as scaffolds to bridge the gap between the prior knowledge of the learners and what they need to know to be successful with the new concepts. Previewing related, previously learned key vocabulary is one way of preparing students for concepts to come. Some teachers present a concept map of the upcoming unit or build a foundation for learning by creating a new experience for students that acts as a hook to stimulate their interest (Thompson and Thompson 2005). These motivational peeks at "coming attractions" can be achieved through a variety of techniques.

## KWL Charts

One of the most frequently used activating strategies is a KWL chart (see Figure 4.1). This type of chart has students recall knowledge they already have about a given topic, consider what they would like to learn about it, and then reflect on what they have learned once the topic is taught. Not only does this chart tap into students' background knowledge, but it also gives direction to student learning by having them list what they want to learn. Teachers discover the current knowledge and possible misconceptions of their students as the class completes their KWL charts and can use this information to plan instruction.

The chart itself consists of three columns. The first column is generally titled "Things I Know." In many cases, what students think they know may not actually be true, so it may be best to label it *K—Things I Think I Know*. Throughout the unit, the class may return to the chart and revise it as they find errors in what they thought they knew. The second column is titled *W—Things I Want to Know*, and the third column is titled *L—Things I Learned*.

**Figure 4.1—KWL Chart**

| **K**<br>**Things I Think I Know** | **W**<br>**Things I Want to Know** | **L**<br>**Things I Learned** |
|---|---|---|
| | | |

To use this chart as an activating strategy, the teacher introduces the upcoming unit and encourages students to think about what they already know (the link to prior knowledge). The person recording (teacher or student if working with older students) enters responses in the left-hand column. Students may also have their own individual copies to record responses. Secondary students may work independently or in groups to complete the organizer prior to a class KWL chart discussion.

Then, the class brainstorms questions they have about the topic which serves as a hook to motivate learning while the recorder adds them to the chart. The chart should hang in the classroom as a reference and to remind students of what they want to learn as the unit of study progresses. At various times during the unit, the class may add questions or begin to fill in the third column with things they have learned. Later, the chart can serve as a review tool when the unit has ended.

In his book *Comprehending Math: Adapting Reading Strategies to Teach Mathematics, Grades K–6* (2006), Arthur Hyde adapts the KWL chart specifically for use in problem solving. In many ways, an activating strategy serves the same purposes when trying to understand a mathematical problem as it does in preparation for a new unit. The first column in Hyde's chart becomes, *What do you know for sure?* Here, students may add prior knowledge they have that may be connected to the problem as well as facts from the problem itself. The second column becomes, *What are you trying to find out?* In this column, students record what they need to figure out or find to solve the problem. Answering this question focuses students on the relevant issues in the problem. The third column becomes, *Are there any special conditions in the problem*? This is often the most difficult for students to determine. Using this chart helps students blend their own knowledge with what they learn from the problem. Then, the chart highlights and clarifies the task in solving the problem.

## Anticipation Guides

An anticipation guide (see Figure 4.2 on page 137) is a set of questions about concepts in an upcoming unit. Students are asked to answer these questions based on their background knowledge and experiences before the new unit begins. The questions are crafted to prompt students to make connections and wonder about the content in the upcoming unit. If students have not worked with anticipation guides previously, teachers should reassure them that they are not expected to be able to answer all the questions, but to do the best they can. Initially, highly motivated students may be upset if they don't know the answers. By explaining the purpose of the guide, student concerns may be alleviated. At the conclusion of the unit, students enjoy returning to the anticipation guide to see how much they have learned. Furthermore, it can help students identify any confusion or misunderstandings that remain so that they can be addressed.

**Figure 4.2—Anticipation Guide**

**Anticipation Guide for Decimal and Percent Concepts**

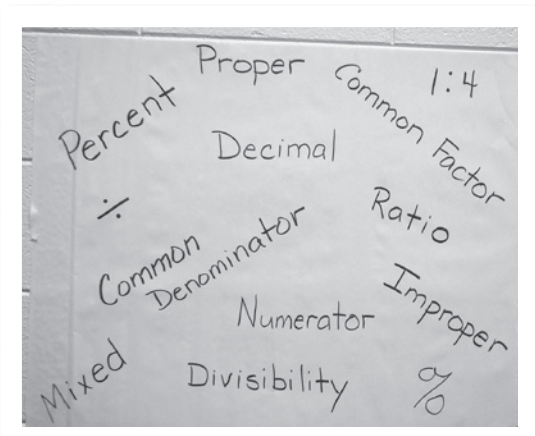
Decide whether each statement is true or false, and mark your answer in the column marked *Before*. Once the unit of study is finished, read each statement again. Decide whether each statement is true or false, and mark your answer in the column marked *After*. Did any of your choices change?

| Before | After | Statement |
|---|---|---|
| | | **1.** Multiplication and division of two numbers will produce the same digits, regardless of the position of the decimal point. |
| | | **2.** The position to the left of the decimal point is the position of the tenths. |
| | | **3.** The term percent is simply another name for hundredths. |
| | | **4.** $\frac{3}{4}$ is equal to 0.75. |
| | | **5.** If a book originally costs \$10.00 and is on sale at 25% off, it will now cost \$8.00. |
| | | **6.** The sum of 0.25 and 0.25 is 0.5. |

To prepare an anticipation guide, the teacher chooses the most important concepts to be presented in the new unit. Then, the teacher creates up to 10 statements related to those concepts. Some of the statements will be true, while others will be false. To generate student interest, the statements may be controversial or contrary to what a student might believe at present. To introduce the unit, students are asked to complete the anticipation guide either individually or in small groups. As they try to determine whether the statements are true or false, students should reflect on what they already know and become aware of their uncertainties or gaps in that knowledge. Teachers remind students that throughout the unit, they should look for information to determine the validity of the statements. The guides are saved, and as the unit ends, students complete them again. Following the second completion of the guides, students discuss their answers, explaining their choices. If students still have some incorrect answers, teachers can go back over the content that was not completely understood with any students who may need additional help—not with the entire class unless the lack of understanding was widespread. This provides scaffolding to support the learning of those students who may need additional instruction while challenging students who have already mastered the concepts to engage in work that is more appropriate.

## Word Splashes

A word splash previews unfamiliar vocabulary from an upcoming unit while building links to prior knowledge and creating hooks to motivate student learning. To create a word splash, teachers review the standards included in the upcoming unit of study, identifying relevant and catchy vocabulary. Some of the vocabulary may be words students have previously learned, and some may be unfamiliar to students. Once the vocabulary is chosen, the words are presented to students all at once, "splashed" across a chart or interactive whiteboard.




*This word splash previews vocabulary for converting fractions, decimals, and percents.*

As a class, students read the words and brainstorm ideas about how the words are related. They do this by using their prior knowledge of the words and what they know about the coming unit of study.

If students do not have sufficient content knowledge they need to be able to understand the meaning of a word, teachers may choose to explain the definitions of new vocabulary. However, introducing the word's meaning can be postponed until the word's meaning will make sense to students. For example, if students will be learning about finding the perimeter of a rectangle, the definitions of relevant vocabulary words, such as width or length, might be discussed. Students understand that a rectangle has sides—two of which may be considered the width and the opposite sides, the length. However, because students are still unfamiliar with the concept of perimeter, it is best to wait to introduce the actual word *perimeter* until students learn about the concept.

During a word splash, students try to figure out how the vocabulary words are related. They may ask themselves, "Why did the teacher choose those particular words for the word splash?" Students should monitor the accuracy of their predictions as the mathematical unit progresses by reviewing the word splash. This review serves as motivation to pay close attention to the new concepts being introduced.

Toward the end of the unit, students may be asked to create their own word splashes as a way of summarizing the content of the unit. The words they choose to include allow teachers to informally assess student understanding of the new concepts. With this specific information about their students' thinking, teachers can modify instruction to ensure that every student is successful.

KWL charts, anticipation guides, and word splashes are a few of the possible activities that can be used at the beginning of a unit with the whole class because little or no differentiation is needed with these activities. The purposes—previewing vocabulary, linking to prior knowledge, and hooking student interest—lend themselves well to whole-class instruction.

## Mathematical Connections to Literature

People of all ages are naturally drawn to good stories. Stories make ordinary situations come alive. They make fantasy seem real. Many teachers use stories in their classrooms to inspire, motivate, and teach students. So, it's not at all surprising that literature is very effective when used as a vehicle for teaching mathematical concepts to students.

The use of engaging literature to teach mathematical concepts is not a new idea. For many years, teachers have garnered student interest by linking a mathematical concept to an event in an engaging piece of literature. Through the story situations and the actions of the characters populating the stories, students learn about mathematical concepts and their applications (O'Connell 2007a).

Because students become thoroughly engaged in stories, math problems or concepts related to stories are almost as relevant to them as mathematical links to their real lives. And, in some cases, the contexts are much more entertaining. One of the most well-known stories used to teach subtraction of money in primary grades is *Alexander Who Used to Be Rich Last Sunday* by Judith Viorst. Young students identify with poor Alexander whose money slowly disappears during the week through a combination of unwise expenditures and misfortunes. Depending on the grade level and capability of students, they can either use coins to replicate Alexander's steady loss of money or they can subtract as the story proceeds. What student has not been in a similar situation? Viorst surely intended this book to reflect common childhood experiences in an entertaining way rather than to teach mathematics. Yet, its use in math class certainly prompts students' interest in learning about money.

For older students, books like Shelley Pearsall's *All of the Above* pique students' curiosity about mathematics. Based on a true story, this book tells of four inner city students who go on a quest to build the world's largest tetrahedron. After learning about the quest described in the text, students may well set out to take on a similar challenge.

Another math-related book that appeals to the sense of curiosity of older students is *The Number Devil* by Hans Enzensberger. In this tale, a math-hating boy encounters a Number Devil who leads him in an exploration of infinite numbers, prime numbers, Fibonacci numbers, and more. Sharing this fantastical tale with students broadens their mathematical horizons and inspires their sense of wonder.

Teachers can take advantage of students' affinity for stories by establishing connections between the stories and the mathematics they are learning. Just as math is found in almost all aspects of our lives, mathematical links can be found in most pieces of literature. Capitalizing on these links, teachers in self-contained classrooms can teach mathematics during language arts lessons or language arts during mathematics lessons. Other teachers make these links by injecting a bit of literature into their math instruction. The integration of disciplines reinforces the message that mathematics is an integral part of our lives, not an isolated subject from a textbook. In addition, linking math to contexts explored in literature strongly supports the NCTM Connection Standard (2000) that states students should be able to "recognize and apply mathematics in contexts outside of mathematics."

To help students recognize links between literature and math, teachers can use a think-aloud strategy as they read or discuss a text, detailing mathematical connections and questions that occur to them. Over time, the responsibility for finding these mathematical connections can be turned over to students, who are asked to find the math in what they read. Students discover how prevalent math is in all areas of their lives. According to Arthur Hyde, "It is the responsibility of the teacher to help students see and experience the interrelation of mathematical topics, the relationships between mathematics and other subjects, and the way mathematics is embedded in the students' world" (2006, 135). The use of literature is an effective method with which teachers can promote understanding of those relationships.

By reading mathematics-related literature, listening to teachers talk about the math links they notice, and then describing their own mathematical connections, students become more fluent with new mathematical vocabulary. Providing opportunities for students to hear and use newly introduced mathematical language in authentic contexts is an optimal way to promote the academic vocabulary development of students (O'Connell 2007a). Participation in these kinds of conversations supports the NCTM Communication Standard (2000, 60), which states that students should be able to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others." In addition, their experiences decoding math-related text strengthens their reading comprehension—an essential skill for many problem-solving tasks.

As the value of this math-language arts linkage became apparent to authors and mathematics educators, many began writing literature for students specifically designed to teach mathematical concepts, as well as to entertain—several examples were mentioned earlier in this chapter. These authors supply teachers with rich sources of instructional materials that involve students in mathematical sense-making and problem-solving activities in a nonthreatening environment (Whitin and Whitin 2000).

Whole-class instruction is often an efficient way to share rich, mathematics-related literature with students. It is also an effective format for teachers to describe their own individual mathematical connections as a book is read or discussed. Follow-up discussions and tasks may be completed as a whole group, in small groups, or independently by students as part of math workshop.

## Setting the Stage for Math Workshop

A visitor to a class engaged in a well-planned math workshop may be forgiven for failing to understand the considerable effort that goes into putting the structures and procedures in place that make it all possible. Any list of effective uses of whole-class instruction should include the essential process of creating a classroom community, teaching procedures, and the ongoing need to provide directions—all necessary for setting the stage for math workshop. A detailed plan for setting the stage during the first 15 days of the school year is suggested in Chapter 9.

During the first few weeks of school every year, students learn the basic procedures that make math workshop possible. Most often, teachers choose to work with the whole class in creating and teaching these procedures. Going through this process is a shared student experience, which leads the class to a true sense of community. As the class comes together, students begin to respect the rules established and to function effectively. It is difficult to envision this process of community building with any mode of instruction other than whole class.

Although the initial community-building phase is completed within a few weeks, it is usually necessary to revisit the process throughout the year. You may need to remind students of the procedures whenever they begin to get lax about following the established guidelines. Teachers may choose to call the class together to regroup and re-establish its sense of community. Having the class meet as a large group can also be used to address specific problems that may arise. The class discusses these problems and then works together to find solutions.

Whenever students are engaged in independent mathematical work, whether working in groups, pairs, or individually, their directions must be very clear. If all students are to be involved in the same work, it works best to give directions to the whole class at one time. Group work is valuable for many purposes, but a teacher's time is wasted if it is spent repeating the same instructions with group after group. Using whole-class instruction as an introduction to math workshop ensures a smooth transition into these independent activities.

## Math Huddle Conversations

To maximize the conceptual understanding that students gain by engaging in mathematical tasks and investigations, they should talk about the experiences and their understanding of math in constructive conversations (Sammons 2018). Meeting together in a math huddle, students "communicate their ideas, solutions, problems, proofs, and conjectures with one another" (Fosnot and Dolk 2001, 27).

In contrast to the traditional model of instruction, in which students simply receive the knowledge and wisdom dispensed by their teachers, students are responsible for expressing their ideas, listening thoughtfully to each other, and justifying their mathematical thinking based on their experiences when they gather for a math huddle conversation. These student interactions encourage students to "propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills" (NCTM 2000, 21) while building conceptual understanding and extending their learning.

When students are actively involved in constructing mathematical ideas, they are usually eager to share their thinking. The new mathematical vocabulary they are learning is of value because it lets them express their thoughts clearly. Eventually, students' use of this kind of language actually furthers their mathematical thought (Fosnot and Dolk 2001). The very process of reflecting, orally expressing their thoughts, and receiving feedback on those ideas helps students fine-tune their mathematical understanding.

Engaging in math huddle conversations spurs students to extend their thinking as they revise, refine, and test mathematical conjectures (Schultz-Ferrell, Hammond, and Robles 2007). Students are encouraged to listen intently and respectfully question fellow classmates about their thinking. Teachers act as facilitators, maintaining the focus of the discourse, encouraging participation by all, and ensuring that the conversation is respectful.

As students share their observations, reflections, and conjectures, misconceptions quite often become apparent to teachers—and sometimes become evident to students themselves as they talk. Closely examining the misconceptions and errors together often clarifies understanding for all students and leads them to recognize that making mistakes is integral to the process of solving problems and developing a deeper, more complex understanding of mathematics. A healthy classroom culture encourages students to view mistakes as an important part of learning and think of misconceptions as learning opportunities. Students who understand this become more willing to take risks and share ideas without fear of ridicule or censure (Hiebert et al. 1997).

## Practice and Review Sessions

Throughout the academic year, students are involved in the process of learning, practicing, and reviewing mathematical concepts and skills. Ongoing opportunities for practice and review allow students to refine, extend, and retain what they have learned. While practice and review can occur in any of the Guided Math components, whole-class instruction is very effective for this purpose.

In traditional classroom settings, practice and review opportunities have been paper-and-pencil tasks. If the high-stakes state assessments students take are still paper-and-pencil tests, students need opportunities to practice using the same format that they will face during the "big" tests. Paper-and-pencil practice tests are efficiently administered in a whole-class setting.

Many teachers successfully use technology to review and accurately monitor student understanding through the use of "clickers" or classroom-response systems. During whole-class review sessions, each student indicates their response to questions using a device or their personal cell phone. The answers are tallied and sometimes presented in graphic formats. Teachers are able to determine the overall level of understanding of the class and address misconceptions. In addition, these systems, when assigned to specific students, allow the teacher to monitor the responses of each student individually. Using this data, teachers can effectively target their instruction for these students during small-group lessons.

More productive for review and practice, no doubt, are activities that motivate and engage students of any age through games, music, or physical movement. While some games are more appropriate for independent work during math workshop, many allow a class to be divided into groups to compete. Competing in mathematical-related games motivates students to repeatedly practice mathematical skills, something that would normally be considered a chore. Work becomes play. That may be why Robert Marzano and Debra Pickering (2005) include student participation in games as step six in their six-step plan for building academic vocabulary. However, games are not only effective in building vocabulary but also for building mathematical knowledge.

Teachers can create many different types of games based on the specific skills students are learning or have learned. One of these academic games can be played similar to Jeopardy®. Teachers prepare answers, and students must supply the questions.

Another game, True or Not?, challenges teams of students to determine which problem solutions are correct and which are not. Teams of students are given the same set of problems that have already been solved. The solutions (some correct, some incorrect) are given. Teachers can choose to show the work leading to each solution or not. Teams have a set amount of time to work together to find out which solutions are correct. For the incorrectly solved problems, teams must be able to explain where the error or errors occurred. Points can be assigned for the number of solutions that teams were able to determine were correct or not during the allotted time.

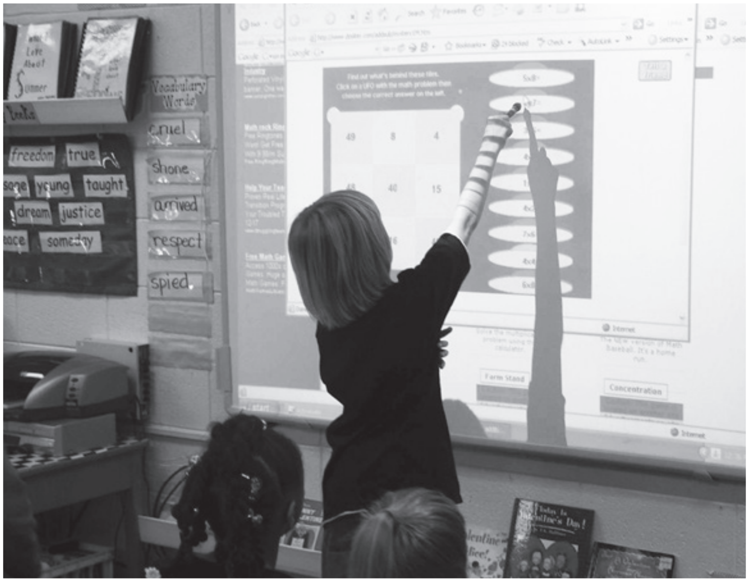
With the game A Little Help from My Friends, each student is given a set of problems to solve. Students are allowed to walk around the room to complete assigned problems of other students. When they complete a problem for another student, they discuss the solution process with the student and then initial it. Each student can complete and initial only one problem on any one student's sheet, but each student can work with multiple students during the game. This practice activity encourages students to discuss their problem-solving approaches with others as well as discover diverse ways to solve problems.

For younger learners, groups of four students can compete to see which group can correctly write the facts in a fact family when they are given the numbers in the family.

To reinforce mental math skills, classes can play Around the World. Students are seated at their desks. Two students who sit side by side stand and are given a problem to solve. The first one to answer correctly moves on to the next desk and competes against that student. The other student sits down at the desk where they are. The student who is the first to answer correctly continues to move from desk to desk, competing with student after student until they are defeated or until all students have played.

To play the Fly Swatter game, the class is divided into two teams. One representative from each team comes to the front of the room. Each child is given a (clean) fly swatter. Answers and non-answers to a series of questions are posted in front of the competitors. When given a question, the first student to swat the correct answer earns a point for their team. The content of the questions is determined by teachers to reinforce or review math skills and knowledge based on the needs of their classes.

Many online games make effective whole-class activities.




***This student is playing a game that reviews mathematical concepts being taught in class.***

Songs that teach mathematical concepts are readily available. Some may even accompany a math textbook series. Younger students love to join in singing simple songs accompanied by movement. Older students enjoy raps accompanied by dance moves, which teach the basic math facts or reinforce mathematical concepts. The humor that is incorporated in some of these songs is a welcome break for students whose classes don't always afford them levity as they learn. Secondary teachers can find grade-appropriate math songs with a simple online search. As always, it is important for teachers to carefully screen these to be sure they are appropriate before sharing them with students. The use of music and dance to reinforce mathematical understanding is uniquely suited to whole-class instruction where students can participate, interact with their classmates, and enjoy these activities.

## Testing and Assessments

An essential component of mathematics instruction is assessment. Summative assessments serve evaluative purposes to determine how well students have mastered the standards. Formative assessment data provides teachers with the information they need to tailor their instruction to meet the needs of their classes and their individual students. Of course, using a variety of assessment methods offers a more accurate view of student achievement than repeatedly using the same type of assessment. Assessment as an essential component of Guided Math will be addressed more thoroughly in Chapter 8.

One method of assessment, which is used perhaps more frequently than any other, is the paper-and-pencil test—multiple-choice questions, constructed-response questions, or other question variations. Unless a student requires a specific accommodation that cannot be met in a whole-class setting, testing of this kind can effectively and efficiently be administered whole class. Because the time available for small-group instruction is limited, it should be used for activities that have the most value and impact. It is better to avoid using limited small-group instructional time if the teacher's primary role is to be monitoring students who are taking a test. An exception to that, however, is when very young students, with little or no prior experience, are taking paper-and-pencil tests for the first few times. Under those circumstances, the small-group format allows teachers to provide support for learners as they become familiar with the testing format and mechanics (e.g., reading the text aloud, monitoring to be sure students are recording their responses in the correct place, keeping their attention focused).

Most states and districts today use computers for testing. While the ways in which this kind of testing is administered vary greatly from school to school, it is important that students have practice taking tests in whatever system is being used. In most instances, whole-class instruction will have little or no place in practice for computerized testing.

## Mini Lessons

Language arts teachers often present a brief whole-class mini lesson at the beginning of a class to introduce new ideas or later in the class to review the learning that took place. However, this practice is not recommended for math instruction. Math students need to be actively interacting with math in many ways (e.g., mathematical explorations, problem-solving tasks, mathematical conversations) to help them develop greater conceptual understanding and computational fluency. Because mini lessons are primarily teacher directed, teachers should be cautious in using them routinely. Whole-class mini lessons are best used only when there is a compelling reason to do so.

As teachers, we know that our students come to our classes with very diverse learning needs. Some must overcome considerable gaps in their background knowledge to learn new concepts or skills as they are introduced. Additionally, there are some students who may understand what is to be taught or who will very quickly master it with little instruction. The same one-size-fits-all mini lesson may be too difficult for some students while it's too easy for others. During the same lesson, inattentive students may completely miss the point of even the

most skillfully presented whole-class instruction. Sometimes, there are students who are confused but reluctant to ask questions in front of the whole class. These students may remain silent and uncertain about the focus of the lesson. In these very common circumstances, how effective is a whole-class mini lesson?

If not in a mini lesson presented to the whole class, how do teachers present new mathematical content to their students? Small-group lessons offer a more effective instructional format for meeting the learning needs of all students. With the small-group format, teachers can more easily monitor and redirect student attention immediately when it wanders. Gaps in essential prerequisite knowledge and skills can be addressed as lessons begin for groups of students when needed. Teachers can respond promptly to looks of confusion and correct students' misconceptions. Those students who quickly master a lesson can be given additional challenges. Moreover, because teachers can so closely watch and listen to students working together during a small-group lesson, they have a much more accurate sense of what their students know and don't know yet. Teachers can spontaneously adjust their lessons when new learning needs become evident. This instructional flexibility renders small-group lessons not only more effective, but also more efficient time wise than whole-class mini lessons for introducing new concepts and skills. Because their learning needs are attended to so swiftly, students tend to develop an understanding more quickly.

Another significant drawback to using whole-class mini lessons is the amount of time they take. Teachers who use whole-class mini lessons regularly find that mini lessons take more time to teach than they expect. Although they may plan to spend no more than 10 minutes on a lesson, frequently the lessons run much longer—leaving little time for math workshop and small-group lessons. Faced with these time constraints, teachers often find that they can only meet with a single small group during a class period.

For that reason, when used, mini lessons should be kept very brief—no more than 10 minutes. Teachers sometimes use a timer to ensure that the mini lesson does not extend into instructional time designated for math workshop. The mini lessons must be carefully planned and succinct. Teachers should clearly determine a single, specific teaching point for the lesson and then carefully plan how to present it in those few minutes. In fact, the "architecture" of writing, as illustrated by Lucy Calkins (2000), offers an excellent framework that can be applied to mathematics mini lessons (Figure 4.3).

**Figure 4.3—Architecture of a Mathematics Mini Lesson**

| | |
|---|---|
| Connection | • with yesterday's lesson<br>• with the ongoing unit of study<br>• with students' schoolwork either in math or another subject area<br>• with an experience outside of school |
| Teaching Point | • present verbally<br>• demonstrate or model |
| Active Engagement | • students try out a skill or strategy<br>• students act like researchers as they watch a demonstration<br>• students plan their work out loud<br>• students imagine trying a skill or strategy |
| Link to Ongoing Student Work | • students turn to their own work and apply the teaching point |

*(Adapted from* Growing Readers: Units of Study in the Primary Classroom *by Kathy Collins, 2004.)*

## Connection

Calkins (2000) suggests that teachers begin mini lessons by making a connection to what students have learned in earlier lessons or to students' real-life experiences. By tapping into their prior knowledge, teachers generate student interest and prepare them for the new ideas to be presented.

## Teaching Point

The teacher states the teaching point very clearly by saying, "Today, I am going to teach you…" More than just telling, though, the teacher demonstrates and models the strategies or concepts that they want students to learn. The teacher alerts students to particularly important aspects of the demonstration or modeling by saying, "Please pay close attention to…" or "Notice how I…" Teachers think aloud, describing their thought process and the strategies they may be using. Explicitly demonstrating or modeling the use of mathematical strategies and mathematical language gives students a broad foundation upon which they can base their independent work later in math workshop. It is important to avoid teaching a procedure for which students have no conceptual understanding or for solving a particular type of problem without understanding the problem and the math involved. Instead, the teaching point should provide students with strategies and model mathematical practices that can be applied in many situations.

## Active Engagement

Before students are expected to apply the teaching point in their independent work, they are given an opportunity to try it out in very brief guided practice. This gives the teacher a way to determine how well students understand the teaching point and to reinforce student learning through guided practice (Pearson and Gallagher 1983).

It is important that teachers devise ways to provide active engagement for students that do not require large amounts of time. Teachers may use “turn-and-talk” where a student turns to a specified partner or to the student seated closest to them as a way of encouraging active engagement by their students. During this time, students may be asked to restate the teaching point in their own words, describe what they noticed in the teacher demonstration, tell how they will apply the demonstrated strategy, or use individual whiteboards to show how a procedure or strategy is used.

## Link to Ongoing Work

The final component of the mini lesson is to explicitly link the teaching point to ongoing student work. Before the conclusion of the mini lesson, students are reminded that the teaching point is something that they, as mathematicians, should remember and use whenever appropriate, whether engaged in mathematics at school or at home. Students are also reminded that thoughtful mathematicians consider the possible strategies they may use and then apply only those that will help them solve the problems they are working on. The link is simply a quick, to-the-point reminder for students to connect the teaching point to their future mathematical explorations and problem-solving activities.

Figure 4.4 (see pages 154–155) is an example of what an elementary teacher might say in a mini lesson teaching students how to use drawings to help visualize a problem. The application of this reading comprehension strategy to mathematical problem solving is appropriate for any grade level, although the problems to be solved will vary from grade to grade.

**Figure 4.4—Elementary Sample Mini Lesson**

| Step | Teacher Talk |
|---|---|
| Connection | Mathematicians, you are getting to be such good problem solvers. It's exciting to see how you approach new problems. We talked earlier about how we can make connections to things in math problems. As we have worked together in small groups to solve problems, I have heard you talk about connections you have made. Today, I am going to teach you another strategy that mathematicians use when they are trying to solve a problem. |
| Teaching Point | When mathematicians have to solve problems, they know how important it is to be able to picture or visualize them. If I can actually picture what is happening in a problem, it makes it easier for me to solve. It's kind of like a movie in my mind. When I have a problem to solve, sometimes I draw a picture of it.<br><br>Today, I brought cookies for the class to celebrate the completion of our class book. The recipe I used made three dozen cookies. Will I have enough cookies for everyone to have at least one? Will there be leftovers? If so, what should I do with them?<br><br>Let's see what I know about the problem so I can picture it. I know that a dozen is 12. So, I'm going to draw 12 cookies. (*Draw 12 circles on the board to represent a dozen cookies.*) Notice how I take the information from the problem and draw it. Now I've drawn one dozen circles to represent the dozen cookies, but my recipe made three dozen. I'm going to draw two dozen more cookies. (*Draw 24 more circles on the board to represent two dozen more cookies.*) To help me picture how many I have altogether, I'm going to circle groups of 10 so I can figure out exactly how many I have. (*Circle groups of 10 circles.*) Okay, that's three groups of 10 and six ones—36. We have 20 students in our class—and me, of course. That makes 21 people. So, I should have enough cookies since 36 is greater than 21. But, I also wanted to know how many would be left. I'm going to draw happy faces to show the cookies that we will eat. The number 21 is 2 tens and 1 one. Let's see—I'm drawing a big happy face on 2 tens. (*Draw happy faces on two groups of 10 circles.*) Then, I'm drawing a happy face on 1 one. (*Draw a happy face on one additional circle.*) So, I have 1 ten and 5 ones left. That's 15.<br><br>Did you notice how I drew a picture to show me what the problem was about? Drawing a picture really helped me see what was going on in that problem. When you need to understand a problem, it helps to try to visualize it. Drawing a picture is one way of being able to "see" what's going on. |

| | |
|---|---|
| Active Engagement | Now, I want you to try to help me solve the rest of my problem. What shall we do with the leftover cookies? We found that 15 cookies will be left. Please use your whiteboards. Draw the cookies that will be left. Show what you think we should do with them. (*Give students time to draw the cookies and show what they will do with them.*) Now, turn and talk to your partner. Tell your partner what you would do with the leftover cookies. (*Give students time to discuss the question with their partners while listening closely to their comments.*)<br>"You drew the leftover cookies to show exactly how many there were and then showed what you would do with them. I heard many interesting ideas for what to do with the cookies. By picturing what was going on in the problem, it was easier to solve it." |
| Link | So, mathematicians, remember that when you are trying to solve a problem, whether it is one here at school or one you face at home, it often helps to visualize it by drawing a picture of what's happening. As you work today, I am going to be looking to see who is using this strategy for problem solving. |

Figure 4.5 (see pages 156–157) teaches students the strategy of asking questions to understand a problem. The application of this reading comprehension strategy to mathematical problem solving is appropriate for any grade level, although the problems to be solved will vary from grade to grade.

**Figure 4.5—Secondary Sample Mini Lesson**

<table>
<tr><th>Step</th><th>Teacher Talk</th></tr>
<tr><td>Connection</td><td>In the last few days, I have enjoyed watching you work as mathematicians to solve problems. We have discussed how to identify the important information we are given in a problem that we need to solve it.</td></tr>
<tr><td>Teaching Point</td><td>Most times when mathematicians have to solve real-world problems, nobody has written out the problem with the information needed to solve it. When that happens, mathematicians ask questions to determine the information they will need. Today, I am going to teach you about that strategy and how to use it.<br><br>Here's a problem that I have with my summer travel plans. I am going to visit my son in Canada. I have to budget for travel expenses. I need to know how much to save.<br><br>At first when I thought about the problem, I was overwhelmed. I didn't know where to start. After thinking for a few minutes, I decided to make a list of questions to help me approach the problem. Here are some of the questions I came up with:<br>• How much time do I have available for travel? This makes a big difference in how I will choose travel. Do I have time to drive? Take the train? Or should I fly because it takes less time?<br>• What is the difference in price with driving, taking the train, or flying? I wonder which is more affordable if time is not an issue.<br>• Is it less expensive to travel some days than others? If my schedule is flexible, can I save money by planning to travel at a particular time or day?<br>• Is the cost or the time spent more important to me as I budget for the trip? I have to make this decision before I can choose how to travel.<br>• How can I find out what I need to know to compare the methods of travel? What resources do I have to help me compare these methods of travel?<br><br>As you can see, there is quite a lot to think about! Yet, if I don't consider these questions, finding a solution is difficult.</td></tr>
</table>

| | |
|---|---|
| Active Engagement | Now, I want you to think of some other questions that might be important for me to ask so that I can calculate how much money I will need to budget for the trip. I only shared with you a few of the questions I had. Now, turn and talk to your partner and come up with other questions you think I might need to answer. (Give students time to discuss this with their partners while listening to their conversations.)<br>In listening to you, I heard many important questions. Some of these are: Where am I going to stay? How long will I be there? Do I need to pay for meals? Am I planning to do any sightseeing that may cost money? By asking more questions about the problem, it will make it easier for me to solve it. |
| Link | So, mathematicians, remember that when you are trying to solve any problem either at school or away from school, it helps to ask questions to be sure you have all the information you need. As you work on solving problems today, remember to use this strategy if you are not sure you have the information you need. |

## Tips for Effective Mini Lessons

To make the most of mini lessons, Kathy Collins (2004) makes several recommendations. These recommendations have been slightly adapted to fit mathematical mini lessons.

***Limit student talk***. Mathematical discourse is an integral part of mathematics instruction, but the mini lesson is not the place for it. The amount of student talk should be guided and controlled to maintain focus and keep the lesson short.

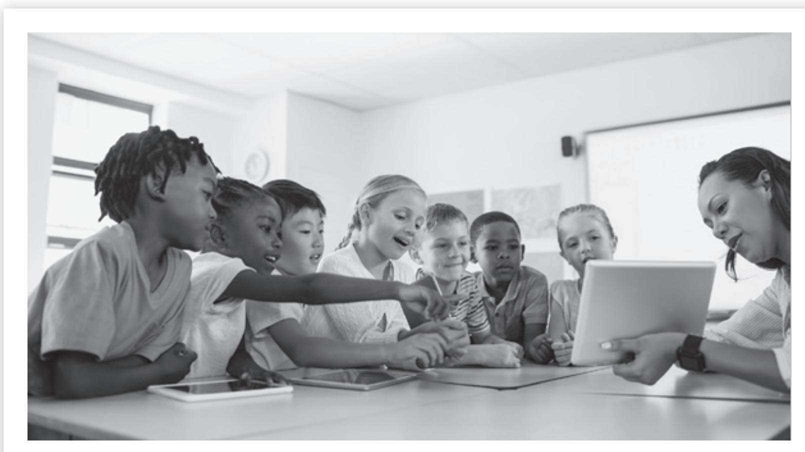
***Keep the connection brief***. Avoid the temptation to ask questions to make the mathematical connection. Frequently, in trying to access prior knowledge and reminding the class of previous lessons, teachers will question students. Rather than drawing out the mini lesson with extensive questioning, the teacher can simply remind the class of what has already been learned. This way, the teacher can draw students' attention to the connection they want to highlight.

***State the teaching point simply and reiterate it***. Avoid over-explaining the mathematics teaching point but repeat it often during the mini lesson. Many mathematical concepts will take students a while to understand deeply.

***Demonstrate the mathematics teaching point***. Show, model, and use think-alouds to help students understand the teaching point.

***Use a familiar context for problem solving***. If the context is familiar to students, they can concentrate on the mathematics teaching point itself instead of having to concentrate on understanding the context of the problem.

***Match the active engagement to the mathematics teaching point***. The goal of the active engagement component is to involve students in applying the mathematics teaching point.



## Chapter Snapshot

When planning mathematics instruction, the primary focus should always be the standards to be learned and the needs of the students as identified through both informal and formal assessments. As teachers keep both the curriculum and student needs in mind, the flexibility of the Guided Math framework allows them to pick and choose between the components of Guided Math, matching their instructional goals with what they believe will be the most effective and efficient modes of instruction.

Because whole-class instruction is largely teacher-directed, all students are engaged in essentially the same work. For lessons that require little differentiation, whole-class instruction is an option to be considered—perhaps for an entire class period or only a portion of the period. Activating strategies, math huddle conversations, practice-and-review sessions, and testing are appropriate for working with the whole class. Mini lessons should be used only if there are compelling reasons for presenting them with the whole class rather than with small groups.

## Review and Reflect

1. Think back to the previous week of mathematics instruction in your classroom. How much of the instruction was whole class? Why did you choose to use that instructional method?

2. In which situations do you use whole-class instruction most frequently? How effective is it in those situations?

3. Are there any ways in which you plan to change the way you use whole-class instruction? If so, what changes will you make?

***By flexibly grouping students based on their immediate strengths and needs, teachers can tailor their teaching to provide the specific instruction that fills existing gaps in students' knowledge and challenges all learners.***

# Chapter 5

## Teaching Small-Group Lessons

In her book *Teaching with Intention*, Debbie Miller writes of creating "the luscious feeling of endless time" in classrooms. While recognizing the enormous pressures teachers face daily, she warns of "getting done" taking precedence over "doing," and of "finishing" becoming more important than "figuring out."

> When kids are given time to puzzle through something that's challenging (with just enough support from their teachers to be successful), they're not only learning about the task at hand, they're learning about who they are and how they go about figuring things out. They're developing those can-do, let-me-have-at-it attitudes that we want so much for them. (Miller 2008, 106)

Although Miller writes about reading instruction, teachers want these same things for their students as they learn mathematics. In spite of time constraints, which often lead teachers to emphasize procedural fluency over conceptual understanding and use worksheets rather than problem-solving activities, we want more for our students. We want to give them that "luscious feeling of endless time" as they ponder problems and share ideas. We want to have time to work with students who require additional support to scaffold their learning. We want to challenge those students who demonstrate their understanding quickly and are ready to move on. We want each and every student to feel challenged, yet supported, in their mathematical learning.

How can we achieve this? Of the hours students spend in school each day, only a limited amount of time is scheduled for mathematics instruction. Even when we know the value of experiential learning, grappling with challenging problems, having to draw upon what they already know about math, recognizing patterns, and constructing mathematical meaning, we are tempted to simply tell our students what they need to know so we can cover it all. How can we possibly provide that "luscious feeling of endless time"?

It may not be possible to give our students the gift of more time, but the instructional formats we choose affect the instructional flow of our math classes. As teaching professionals, we are responsible for shaping our instruction, selecting the parts of textbook or math resource lessons that will most effectively lead our students to understand the mathematical content they are learning, and being confident in our judgment when we decide that not all parts of pre-written lessons are essential. When we assume the responsibility for designing focused small-group lessons to address the needs of students, with the goal of teaching them to truly understand the math with which they are working, our frantic search for more time in the classroom begins to ease. Our focus shifts from fitting it all in to teaching for meaning.

When working with small groups, a teacher's goal is to lead students to develop conceptual understanding and to have "toolboxes" of effective strategies they can draw upon to navigate independently in the world of mathematics. The teacher works with a small group of students who have similar instructional needs. The composition of these groups is fluid and flexible and may change daily. The architecture of a small-group lesson is similar to a mini lesson (page 151).

- The lesson begins with a connection to what students have been learning.
- The teacher then explicitly states the teaching point for students.
- Students engage in tasks designed to allow them to explore the concepts connected to the teaching point.
- The lesson concludes with a link and reflections by students.

Student work should nudge them beyond their existing knowledge and skills, requiring them to stretch their thinking. The teacher plays the role of a facilitator by providing enough scaffolding to move students to a higher level of competence in understanding and skill where they will be able to work independently. Sometimes, the tasks themselves require students to work through processes that increase their understanding. Other times, the questions posed by the teacher lead students to consider aspects of their tasks not previously considered. The teacher can facilitate interactions between students in the group to move those students to a higher level of understanding. Occasionally, the teacher provides direct and explicit instruction. Most importantly, students are active participants working with and exploring mathematics with an abundance of math talk throughout the small-group lesson. Central to this instructional approach is the belief that students *learn* mathematics by *doing* mathematics.

## The Advantages of Small-Group Lessons

Many teachers who use Guided Reading successfully with their students are discovering the advantages of small-group lessons. By flexibly grouping students based on their immediate strengths and needs, teachers can tailor their teaching to provide the specific instruction that fills existing gaps in students' knowledge and challenges all learners. Without the stress of whole-class instruction, teachers can essentially slow down and savor the feeling of "endless time" as they work with their students. Instruction is focused, materials are easily managed, conversation flows freely, and student efforts are more readily monitored. Students receive the support that they need to expand their conceptual understandings and improve their procedural fluency because their instruction is differentiated to support their learning. As Irene Fountas and Gay Su Pinnell describe it, "In the comfort and safety of a small group, students learn how to work with others, how to attend to shared information, and how to ask questions or ask for help" (2001, 18). These abilities are as valuable in mathematics as they are in reading.

In fact, in her writings on differentiation, Carol Ann Tomlinson (2000) suggests that teachers can challenge all learners by providing instruction at varied levels of difficulty, with scaffolding based on needs, and with time variations by using multiple instructional groups. In small groups, the process, the product, and the content of learning may vary. Even the learning environment can be adjusted. While some students may need the structure of working at a table, another group may do well in less formal ways, perhaps working with manipulatives on the floor. As teachers learn to recognize the learning styles of their students, they can adjust instruction to maximize learning.

Because of the flexibility and fluid nature of small groups in Guided Math, the frequency with which a teacher meets with a group and the amount of time spent when they meet may vary. The teacher determines the length of time needed for each group depending upon the instructional tasks to be completed and needs of students in each group. At times, a group meeting may simply consist of a brief pulse-check to be sure the students are on task and understand the mathematical concepts. At other times, a group may require more intensive work with the teacher. This may be especially true if its members are struggling with new concepts or problem solving. By meeting with small groups, teachers consider all aspects of the instructional goals for each group to design customized plans of instruction. Teachers are able to teach at the point of need of each group, prodding students forward along the continuum of mathematical understanding.

Communication is an integral part of all small-group mathematics lessons. The intimacy of the small group encourages all students, even those students who may be reluctant to do so in a larger group, to share their thoughts. Teachers can challenge students with systematic and progressive instructional tasks, model the use of relevant vocabulary terms, pose thought-provoking questions to focus and extend thinking, listen intently as students describe their thoughts and defend their ideas, and teach students how to consider and respectfully question the thinking of their peers. The reflection required by students to verbally describe their mathematical thinking leads them to mentally organize their ideas and internalize them. The questions and comments of their peers serve to deepen and improve their conceptual understanding and mathematical agility. By carefully listening to students during these verbal exchanges and observing them as they work, teachers gain invaluable insights into their students' mathematical understanding. They learn when to move ahead with instruction, when more teaching is needed, and what specific teaching points need to be addressed for these learners. These are all essential for teachers as they plan future effective instruction to extend learning.

The close working relationships that develop between students and teacher or their peers in a small instructional group allow students to feel supported in taking risks as they expand their problem-solving abilities. Lev Vygotsky (1978) described a "zone of proximal development," as the distance between the actual problem-solving ability of a learner based on what they can do independently and the potential ability under the guidance of an adult or with the collaboration of a peer. Fountas and Pinnell (2001) refer to it as "the learning zone." Students who are supported by their teacher and by their peers learn to move beyond their independent capabilities. With practice, they gradually assume greater responsibility for learning new mathematical concepts, tackling challenging problems, and acquiring new strategies and skills. The social nature of learning is highly supported in a small-group setting where opportunities for communication and collaboration abound.

One of the advantages of working with a small group of students is the ability to easily monitor student behavior. With six students sitting around a small table, a teacher can easily spot when a student's attention strays and quickly regain it. In contrast, with a class of 20 or more students, inevitably there are several students whose attention is not focused on the lesson being taught. Although teachers strive to present engaging lessons and use multiple strategies to capture the attention of all students, if a teacher has to pause to regain the attention of some students, the attention of others is interrupted. Moreover, no matter how diligent the teacher, some students will remain distracted. Working with small groups enhances the teacher's ability to maintain students' attention.

As a part of traditional whole-class mathematics lessons, students are asked to complete several problems independently, practicing skills that have been taught during that lesson or a previous lesson. The teacher usually circulates around the room checking student work. For those students whose work is checked at the beginning of this process, this process works well. If those students have misconceptions, they are quickly identified and corrected. They benefit from teacher feedback. However, for students whose work is not checked until much later in the class session, this procedure can create difficulties. Some students who are working problems incorrectly might complete all their work before the teacher reaches them and provides feedback. During this time, they continue their work relying upon mathematical misconceptions. These misunderstandings are reinforced with this practice. Often, by the time the teacher reaches them, these misconceptions have become ingrained. Correcting these misconceptions is now much more difficult and requires greater attention from the teacher. Even more unfortunate are students whose work is not checked at all by the teacher during the lesson because of lack of time. Those students may never receive timely corrective feedback. It is only when their work is turned in that their errors are discovered. In the meantime, they may have diligently completed a homework assignment further ingraining their misconceptions. When their errors are finally noticed and addressed, they may prove to be very difficult to correct.

Teachers who work with small groups of students, on the other hand, find it much easier to monitor the work of their students. If errors are made due to misconceptions or even carelessness, the teacher notices immediately. If only one student is confused, the teacher can help clarify the misunderstanding individually with the student. However, if the teacher notices that many students are having the same problem, the teacher can reteach or clarify the concept for those students immediately. Sometimes, it is worthwhile to pause before stepping in to correct students' work. Given time, students often discover their errors. The process of recognizing and correcting errors can be a meaningful learning experience for students.

An essential component of successful mathematics instruction is effective assessment. Assessments may be formal or informal, summative or formative. To maximize student learning, what is important is that teachers engage in continuous, ongoing assessment. Much of the most valuable formative assessment data gathered by teachers results from observing students and talking with them about their mathematical thinking as they work. As experienced teachers are well aware, a correct solution is not always a valid indication of understanding. When students are able to explain their answers and the processes they used to obtain them, teachers can rest assured that students have a solid understanding of the concepts. By intently listening to and observing students working with mathematical problems, teachers gain a wealth of information about their students and their capabilities. Fortunately, the small-group setting is ideal for this kind of assessment.

## The Challenges of Small-Group Lessons

If small-group lessons offer teachers so many advantages, why aren't all mathematics teachers routinely using small groups? Sometimes, teachers feel more comfortable teaching as they were taught, especially if they are under stress. To teachers who have never experienced working with small groups for mathematics or had chances to visit other classrooms to see it in practice, planning for small groups may seem overwhelming. These teachers may wonder how to group students, how to plan lessons for small groups, and how to monitor and provide valuable, challenging work for the rest of their class as they are working with only a few students at a time.

Small-group lessons may require more extensive planning by teachers, as does any plan for differentiated instruction designed to address the complex learning needs of students. When planning for small-group lessons, teachers have the options to vary the content of the lesson, the way in which it is presented, or the product that students produce, while still maintaining a focus on the standards being taught.

At times, teachers may choose to teach very different lessons for different groups of students. More commonly though, teachers create a single lesson with multiple paths for differentiation. In addition to planning for the basic lesson, each plan should also clearly identify the prerequisite knowledge and skills students need to be successful with the lesson. For students with gaps in essential knowledge and skills, the lesson includes specific instructional strategies to target those gaps and allow them to move on to the current lesson with support. For those students who need an additional challenge, the lesson plan also includes tasks designed to meet their needs.

As teachers know only too well, students very often surprise their teachers. Teachers should have preplanned lesson options for differentiation while working with small, homogeneous groups of students. Mid-lesson, a teacher may discover previously unknown gaps in students' conceptual understanding or skills or may be pleasantly surprised to find that a group grasps the lesson content more quickly than anticipated and would benefit from more challenging tasks. Because a variety of options have been planned and prepared for every lesson, teachers can quickly differentiate the lesson to meet unexpected learning needs of students.

When developing plans for small-group lessons with instructional options, it is important for teachers to examine carefully the standards they are teaching and the progression of learning leading to the current lesson. What do students need to understand and be able to do to fully engage in a lesson that addresses the standards? What should students have already learned? How can gaps in students' prior knowledge be most efficiently remedied? In what ways can students be prompted to think more deeply and complexly about what they are learning when they demonstrate a basic understanding of the standard? Spending time thinking about these questions prior to writing lessons make the planning of differentiation options much easier.

Teachers should know their students' current mathematical understandings to make the most of their small-group lesson time. Teachers' use of ongoing, deliberate, and specific assessment, both informal and formal, provides important information about their individual students as well as an overview of the class as a whole. Teachers then apply this knowledge to gather students together in groups to provide instruction that gives students opportunities to move forward in their journeys to mathematical competency. Whether the necessity for ongoing assessment and more complex planning is a challenge perhaps depends on the perspective of the teacher. It is, however, a consideration for teachers who are thinking about implementing small-group instruction in their classrooms.

One consequence of small-group lessons is that each student receives less direct instruction from the teacher. In some classrooms, teachers spend all their instructional time working with the entire class, as a whole. When teachers work with small groups, the remainder of the class must be engaged in independent work, usually in math workshop. Although the independent work assigned should always be meaningful, it is important to keep in mind that these students are working independently. Direct teacher instruction is limited to the time when students are with the teacher in small groups. How much students learn as they work independently depends largely on the value and rigor of the tasks planned and how well the teacher has established and taught procedures that allow this work to proceed uninterrupted.

To teach effective small-group lessons, teachers have to plan independent work that both helps students increase their mathematical understanding and can be completed accurately without additional assistance from the teacher. Even with meaningful workstation tasks for students, the routines and procedures for math workshop must be well-planned, taught, and practiced. Before implementing group work, teachers should devote at least a few weeks to teaching students how math workshop should function and what the expectations are for their behavior during that time. (See Chapter 6 for more information about teaching routines and procedures for math workshop.) In classrooms where teachers already use Guided Reading, many of these procedures will overlap and may be taught at the same time.

## Effective Uses of Small-Group Lessons

Small-group lessons are a component of Guided Math that offer teachers an enormous amount of flexibility in meeting the needs of students and maximizing the impact of their teaching. Instruction is targeted precisely to meet the learning needs of students in a group. Working with only a small group of learners at a time enables teachers to provide 15 to 20 minutes of "high-quality, intensive instruction that is appropriate for every member of the group" (Fountas and Pinnell 2001, 217). The fluid nature of each group guarantees that students receive instruction that meets their immediate needs.

Although these lessons may be used effectively by teachers for many purposes, there are some uses for which working with small groups of students is particularly effective:

- teaching new concepts and skills
- differentiating instruction
- teaching with manipulatives
- assessing student learning
- supporting mathematical practices

### Teaching New Concepts and Skills

As I began using Guided Math in my classroom, I used whole-class instruction to introduce new content. I learned very quickly, however, the folly of that approach. Even with carefully planned whole-class instruction, learners who required less up-front instruction and needed additional challenge appeared bored and disengaged while learners who lacked essential background knowledge were frustrated and unengaged. Consequently, I had to spend a considerable amount of time in small-group lessons reteaching what I had already taught during the whole-class instruction to those students. In spite of knowing better, I found that I was doing more *telling* students what I wanted them to know rather than *facilitating* their learning. I began to wonder why I was devoting valuable instructional time to presenting new content in a whole-class format even though it was not adequately addressing my students' unique learning needs. I knew there had to be a better way.

With that motivation, my teaching began to change. I began to rely much more heavily on small-group lessons. Presenting new content to students in a small-group setting allowed me to differentiate my lessons based on what I knew about the learning needs of the students in each group, while also responding immediately to students' questions, misconceptions, looks of confusion, or nods of understanding. Watching students work and hearing them discuss what they were learning gave me insightful, real-time formative assessment information. Students understood concepts and skills more quickly and in greater depth because of their hands-on engagement in math tasks that called upon them to notice, wonder, and communicate with one another—and with me. I experienced a deep satisfaction as I observed my students' interest in mathematics blossom. In addition, my students began to assume more responsibility for their own learning as they reflected upon what they were learning and as I encouraged them to monitor their own progress toward their learning goals. I discovered that small-group lessons are perhaps the most powerful instructional tool for introducing new concepts and skills to students. In fact, based on my experience, if I could only use this instructional format for one purpose, it would be for teaching new mathematical ideas.

I was working with a smaller group of students. However, that alone was not responsible for the positive impact I saw in my students. This instructional format made other changes possible as well. Initially, my small-group lessons were very much like my whole-class instruction, only with fewer students. Yet, over time, I discovered how much more I could do. Small-group instruction allowed me to more actively engage my students with lessons that required them to do most of the work and most of the thinking. I learned that the small-group setting is a more conducive format for teachers who want to function as facilitators of student learning rather than as the sole dispensers of knowledge. With small-group lessons, teaching more readily progressed beyond the traditional trajectory of telling, showing, and then assigning practice. My students began to assume greater responsibility for constructing their own mathematical meaning from carefully structured mathematical experiences, often open-ended tasks, designed and facilitated by me.

The most important roles for the teacher in small-group lessons are planning meaningful tasks, observing students as they struggle with new ideas or problems, listening to them talk about their mathematical reasoning, asking probing questions, encouraging reflection, and monitoring students' learning to ensure that they come away from lessons with what they need to know and be able to do. Making this kind of shift in instruction requires a tremendous amount of thought and practice, and it does not happen overnight. The change is well worth the effort required.

It is not surprising that teaching small groups of students has a positive impact on students. Most teachers have longed for smaller classes. Every teacher has experienced the frustration of having some students quickly grasp the concepts they are teaching and not being able to challenge those students because of a commitment to students who need more support to master the standards.

**Power Word**

***differentiation:*** *the process of tailoring or altering instruction to meet student needs*

When teachers work with small groups of students to teach challenging, new concepts and skills, they have the flexibility to adjust their teaching methods to meet student learning needs. Students who have already demonstrated strong foundational understanding leading up to the new lessons may need only brief introductions before moving on to independent work. For students who come to a lesson with minimal background knowledge, the lesson may be lengthier and initially focus on the concrete aspects of the concepts to provide scaffolding for their learning. For students who continue to struggle with concepts, additional work with the group can support their learning and ensure that the essential mathematical standards are mastered.

## Differentiating Instruction

Carol Ann Tomlinson (2000) describes differentiated instruction as a philosophy of teaching and learning rather than as an instructional strategy to be used when and if a teacher has time. This philosophy is based on the belief that even students who are the same age have differences in their readiness to learn, their interests, their styles of learning, their experiences, and their life circumstances. These differences are significant enough to affect what students need to learn, the pace at which they need to learn it, and the support they will need to learn it well.

So often, however, we begin our instruction aiming toward the middle and praying for ricochet as Jennifer Taylor-Cox (2008) so aptly describes. Then, when we discover some students are struggling with the concepts we are teaching, we double down and focus our attention on their needs with the ultimate goal of boosting each and every one of them over the bar of conceptual understanding. In the world of high-stakes testing and ever-increasing demands for teacher accountability, a laser-like focus on students who need additional support is understandable and laudable. Without a doubt, the learning of all students is an overarching goal of teachers.

With so much instructional time targeting students who require the most support, however, middle- and high-achieving students sometimes languish. Typically, these students are given additional practice of skills already mastered, are asked to peer tutor other students, or are encouraged to spend time reading when they complete their assignments, rather than being challenged to think deeply about more complex mathematical concepts. For years, teachers have been urged to provide enrichment for high-achieving students but have been given conflicting messages about exactly how they should support them.

Working with small groups of students makes it possible for teachers to effectively vary their lessons and tasks to target the diverse learning needs of all students. Successful differentiation depends heavily on timely assessment information to identify whether students have gaps in essential prerequisite knowledge and skills or perhaps already possess an understanding of the lesson focus. Relying on this formative assessment information, teachers are able to work with small groups of students with similar instructional needs. Placing students into static high, middle, and low groups defeats the purpose of differentiation. By design, differentiation is meant to address *identified* and *current* needs relating to the lesson-specific content. The composition of small groups for instruction should be flexible, but homogeneous, varying from day to day based on comparable learning needs.

In schools everywhere, dedicated teachers individually or in professional learning groups engage in ongoing reflection on their teaching practices and search for ways in which they can more effectively meet the learning needs of all students without neglecting the needs of any. By being able to tailor lessons to make them specific to the needs of small groups of students, teachers avoid the frustrations inherent in the exclusive use of whole-class instruction. It allows the core of what students learn to remain constant while varying the "how" (Tomlinson 2000).

When mathematics lessons are most effective, all students are engaged in learning the current standards being taught. Too often, though, well-intentioned efforts to differentiate lessons to satisfy diverse learning needs seriously short-change below-level learners. Teachers may be reluctant to introduce new mathematical content to students who lack the prerequisite knowledge and skills they need to be successful with it. If new content is not introduced until students' gaps in their background knowledge are filled, they will fall further and further behind. Yet, if new concepts and skills are taught without addressing students' lack of necessary foundational knowledge on which the new content builds, the consequence is equally dire. When that happens, students too often fail to understand the lesson and, as a result, become increasingly frustrated and discouraged.

Effective small-group lessons, however, specify differentiation options that directly address any gaps in foundational knowledge and skills students may have *and* then immediately move them into the new lesson content with scaffolded support. These lessons are designed to attend to the gaps students may have that hinder learning, while at the same time allow these students to begin learning the same content as the rest of the class.

Lessons for students who lack sufficient foundational knowledge begin with brief, specific, and highly focused remedial instruction, but only for those who need it. Following that brief remediation, these students are introduced to the new content of the current lesson, but with scaffolded support from the teacher. Because the gaps being addressed are prerequisites for the lesson, in most instances, students will be continuously revisiting and drawing upon the foundational knowledge that they may have lacked as they work with the basic lesson and the new content that the class is learning. The goal of this kind of differentiation is to help learners build the foundation they need to succeed and then offer the support that allows them to continue to fill the gaps they had as they master the new content.

## Teaching with Manipulatives

Students retain knowledge and skills better when they are actively involved in the learning process. Research shows that 90 percent of what we both say and do is retained, compared to only 50 percent of what we hear and see (Thompson and Thompson 2005). To promote long-term knowledge and understanding, engaging students in hands-on learning with ample opportunities for discussion is essential. Teaching students at all grade levels to use manipulatives and making manipulatives available provides opportunities for active learning. When accompanied with meaningful student conversation facilitated by teachers, conceptual understanding and long-term learning result.

Hiebert et al. (1997, 53–54) list the following ways in which students can employ manipulatives as tools for learning:

- providing a record of mathematical activity
- providing a way of communicating mathematical ideas
- providing an aid to thinking

Each of these ways supports the mathematical process standards recommended by NCTM (2000): problem solving, reasoning and proof, communication, connections, and representation. In addition to the list above, manipulatives are also valuable for representing mathematical concepts, strategies, and problem scenarios—which also directly supports NCTM's process standards.

The act of creating concrete representations of mathematical concepts establishes an image of that knowledge in students' minds (Marzano, Pickering, and Pollock 2001). When students visualize and then manipulate aspects of the mathematical ideas they are exploring, they gain deeper understanding of the concept (Ennis and Witeck 2007). Not surprisingly, their use also aligns directly with the CCSS mathematical practices (2015).

Manipulatives are only effective, however, when students understand what they represent. For learners to use them in ways that lead to increased understanding, they must construct meaning for the manipulatives by examining them closely, trying them out in different contexts, and listening to the ideas of others. The symbolic meaning that may seem obvious to adults could be lacking for students. Deep conceptual understanding emerges when students have opportunities to actively use manipulatives following teacher modeling and think-alouds. As students interact with manipulatives, conversation among students and between students and teachers helps reinforce their understanding of what these objects represent mathematically.

Once students begin to understand the manipulatives themselves, these hands-on objects become a "testing ground for emerging ideas" (Van de Walle and Lovin 2006, 8), giving learners something to ponder, explore, discuss, and use to model their mathematical ideas. Students' understanding of the mathematical meaning of the manipulatives continues to evolve as they use them.

Establishing an environment where students use manipulatives effectively to build understanding is sometimes a challenge for teachers. Although most elementary classrooms have collections of manipulatives available, this is not always the case in secondary classrooms. However, if manipulatives are not provided with textbooks, school systems will often provide them. Secondary teachers may find that their access to manipulatives is limited. Moreover, older students are sometimes reluctant to make use of them. To them, manipulatives may be seen as something used only by young students or students who require additional support. It is important for all students to realize that mathematicians frequently rely on the use of manipulatives for problem solving and the representation of mathematical concepts.

Teachers may hesitate to use manipulatives in their math classes if they are unsure of the management techniques needed to support their effective use. With whole-class instruction, the management of manipulatives tends to be daunting. Because instructional time is so precious, some teachers view the time spent distributing and collecting manipulatives as wasted time. Once these objects are in students' hands, monitoring their use can also be difficult. If students fail to understand the mathematical meaning of the manipulatives and how they can be used to solve problems or construct mathematical understanding, manipulatives are nothing more than toys. And, that's the way these students will use them. If instruction is interrupted repeatedly to remind students to use manipulatives productively, important teaching time is lost. Considering these challenges, it is understandable why whole-class instruction tends to limit the use of manipulatives in math classes.

When teachers work with small groups, however, many of these hindrances are eliminated. Fewer manipulatives are needed for a lesson. Rather than distributing them to an entire class, the teacher has the manipulatives ready for use at the table where small groups meet. Teachers can easily monitor students' attention as they model the appropriate use of manipulatives and the value of these tools in solving problems. As a result, students in small-group lessons are not only more focused because of the close monitoring by the teacher, but they can more closely observe the manipulative use being modeled by the teacher because of their proximity. As students assume greater responsibility for working with these objects, the teacher is able to easily interact with students to refine or extend understanding. Conversations that arise among students in the small group setting allow them to learn from each other as they examine mathematical problems, explore methods of solution, and identify questions that arise as they work with manipulatives.

As teachers provide meaningful and productive small-group work with manipulatives, there are several instructional rules of thumb for teachers suggested by John Van de Walle and LouAnn Lovin:

1. Introduce new models by showing how they can represent the ideas for which they are intended.
2. Allow students (in most instances) to select freely from available models to use in solving problems.
3. Encourage the use of models when you believe it would be helpful to students having difficulty (2006, 9–10).

Working with students in small groups as opposed to whole-class instruction makes it easier for teachers to adhere to these guidelines.

According to Van de Walle and Lovin, the most widespread error that teachers make in using manipulatives is designing lessons in which they provide students the exact directions for using a particular manipulative to solve a problem (2006). Although manipulatives are used by students in these lessons, this practice encourages rote learning as students blindly follow set procedures without linking them to mathematical ideas. Assessing students' conceptual understanding in this type of lesson is impossible because students are only following the directions of their teacher rather than relying on their own mathematical reasoning. Teachers cannot rely on the simple physicality of manipulatives to automatically convey mathematical meaning to students. Rather, students "need teachers who can reflect on their students' representations for mathematical ideas and help them develop increasing sophisticated and mathematical representations" (Clements 1999, 47).

Unfortunately, when whole-class instruction is the norm, teachers tend to resort to cookie-cutter lessons in which little individualization is encouraged. To get through the lesson quickly with a large group, it seems easier to model one way and to clearly lay out the problem-solving steps students should follow. Teachers may erroneously believe that students who follow their directions to determine answers actually understand the mathematical concepts and then are frustrated when their students are unable to transfer this rote learning to other situations.

Teaching small-group lessons makes it possible for teachers to afford their students more leeway in choosing which manipulatives they will use and flexibility in applying alternative problem-solving methods. A variety of manipulative materials should be made available to students as they explore various problem-solving strategies and wrestle with perplexing mathematical concepts. Students should have many opportunities to decide for themselves which manipulatives they will use. During lessons with small groups of students, teachers can observe their attempts at model making and problem solving. Students may not always make wise choices, but they will learn from these experiences—especially when encouraged to talk with other students about what they tried. By sharing their successes and missteps, the entire group benefits from the experiences of its individuals.

Manipulatives play a valuable role in developing students' understanding of abstract mathematical concepts and skills and in problem solving, but it is possible for them to be overused by both teachers and students. When the use of manipulatives is no longer required to build understanding or problem-solving capabilities, it is counterproductive to require their use. Rebecca Ambrose warns, "we certainly do not want to insist that all children use manipulatives when many are capable of using more sophisticated strategies. We do not want to send the message that manipulatives are the favored problem-solving technique" (2002, 20). Indeed, some students are reluctant to make the transition from using concrete objects to more representational or abstract methods. When that happens, teachers have to gently wean them from their use of manipulatives.

## Assessing Student Learning

Assessment is critical for effective teaching. Only when we are clear about what we want our students to learn and can identify exactly where students are on that learning trajectory can we plan lessons that truly meet each students' needs. But, knowing where students are cognitively isn't always easy because where they are changes constantly.

Teachers who rely primarily on end-of-unit tests to evaluate the learning of their students are essentially flying blind when planning instruction. With no idea of the extent of their students' prior knowledge, lessons are the same for one and all. With no ongoing formative assessment on which to rely, teachers either assume that all students have learned exactly what was taught in each lesson, or they decide that teaching is their job and learning is the responsibility of their students. If they have "taught" the mathematical content, they have fulfilled their obligation. And after the unit test, they will find out if students have fulfilled theirs.

This is a rather harsh view of teachers. Fortunately, most teachers are deeply committed to student learning and feel a much greater responsibility to their students than this description implies. In addition to chapter tests, teachers monitor daily work and homework assignments. They give quizzes and assign projects. These are all examples of formative assessments that provide timely evidence of student learning. Although the information gained from these assessments is timelier than that from unit tests, there are still gaps between the instruction and the assessment.

Ideally, to plan and deliver instruction effectively, teachers should continuously monitor students' performance for evidence of learning and identify their specific learning needs. What they learn as they observe, listen to, and confer with students is another type of formative assessment, but one that is conducted informally. It permits teachers to adjust their instruction "on the run," so they can promptly respond to identified learning needs. It is the data from these formative assessments that are most effective in informing teaching decisions. And, it is most easily gathered in the setting of a small-group lesson.

As teachers meet with small groups of learners, they provide timely and descriptive feedback based on that data to let their students know precisely where they are in relation to where they need to be academically and exactly what they need to do to reach their learning goals (Stiggins 2005). This is a powerful way to help focus student learning. Linking assessment and learning allows students to become more aware of the "how" of learning and also leads to greater student ownership and investment in learning (Davies 2000). Because teachers interact so closely with their students in small-group lessons, assessment and feedback of this kind are possible. With experience, the processes of teaching and assessing informally become interwoven, one affecting the other, leading to a flexible but durable fabric of instruction.

## Teaching the Mathematical Practices

The discipline of mathematics is complex. Students must master a myriad of skills and develop a deep conceptual understanding of crucial mathematical content and practices to become competent mathematicians. Yet, while it is commonly expected that teachers should focus on teaching content, that emphasis is often at the expense of teaching the mathematical practices and processes.

The content standards constitute only a portion of the standards developed by both NCTM and CCSS. The remaining standards address the essential processes and practices mathematicians use. (See Figure 5.1.)

**Figure 5.1—Essential Processes and Practices**

| |
|---|
| NCTM (2000) addresses problem solving, mathematical reasoning, communicating mathematically, making mathematical connections, and representing mathematical ideas accurately with their process standards. |
| The eight standards for mathematical practice specified by CCSS (2015) focus on problem solving, abstract and quantitative reasoning, constructing viable arguments and critiquing the reasoning of others, modeling with mathematics, using appropriate tools strategically, attending to precision, looking for and using structure, and looking for and expressing regularity in repeated reasoning. |

The process and practice standards are an integral part of the discipline, and unfortunately they have often been neglected. Instead, procedural competency, demonstrated through computation, was stressed in mathematics education. In fact, for some people, mathematical skills and computation have become synonymous.

In recent years, concerns about the mathematical competency of our students have increased as comparisons of their performances to those of young people in other countries have been publicized. Students from the United States no longer rank near the top in these comparisons based on international tests. As educators have begun to examine exemplary mathematical instructional practices, it has become apparent that teaching mathematical processes and practices should be a critical component of mathematics education. It is crucial that students have opportunities to cultivate both content and process skills to ensure that they develop deep conceptual understanding and the ability to apply their mathematical knowledge.

In spite of the strong connection between the content and process/practice standards, the process/practice standards have often been bypassed as teachers struggle to cover the amount of content required by state curriculum standards. Additionally, because teachers tend to teach the same ways in which they were taught, many are unsure how to teach these standards. But, with the increased focus on these standards, teachers are discovering how to integrate content and process skills into their lessons.

The kinds of learning experiences educators offer students clearly determine the development of their mathematical understanding. Students who are actively engaged in rich problem-solving tasks supplemented with focused discourse begin to grasp the mathematical practices even as they develop conceptual understanding and hone their procedural fluency. Because quality mathematical lessons in which content and process are entwined consist of more than simple show-and-tell teaching with an assigned worksheet, teachers often find them difficult to execute in a whole-class setting.

Working with small groups of students with similar learning needs allows teachers the flexibility that they need to address the process and practice skills as well as content. Teachers can challenge students to solve problems while offering support through carefully crafted questions. Students in these groups can be encouraged to collaborate and talk about their mathematical ideas as they work together—to question, to reason, and to reflect. Manipulatives are easily managed as students create models to represent their mathematical ideas. As students share the strategies they are using, teachers help them translate their ideas into mathematically appropriate communication and then guide them as they construct meaning and extend their understanding.

Teachers who are searching for ways to incorporate more complex learning activities and tasks into their instructional plans often find that in small groups, they can try what they would be hesitant to attempt with the whole class. With small-group lessons, teachers discover that student interest and excitement about mathematics increases and with it, student motivation.

## Forming Small Groups for Lessons

In all classrooms, students possess wide ranges of experience, background knowledge, and skills. Furthermore, after entering a classroom, students progress at varying rates (Fountas and Pinnell 2017). So, flexible grouping for instructional purposes makes sense to meet students' diverse and ever-changing needs.

> **Power Phrase**
>
> ***flexible grouping:*** *the process of creating student groups based on instructional needs determined by continual formative assessment*

### Differentiating Instruction

In the past, most groupings of students were based on data from lagging indicators including previous grades, standardized tests from the prior year, and even recommendations from past teachers. Only the general ability of students was considered. Once placed into a group at the beginning of the school year, a student remained in the same group throughout the year. Consequently, since instruction was based on progressing through a set sequence of materials, there was little, if any, opportunity for students to catch up with more-advanced groups. More than likely, students remained in the same groups year after year. Students in higher groups who might have difficulty with a particular concept were moved along at the same pace and with the same instruction as those students who mastered it quickly. If they didn't get it, so be it—maybe they would next year. Conversely, students in the lower groups who quickly mastered some areas of the curriculum were still forced to go through additional instruction until the other students mastered it or until it was time to move on.

In the past, minority students and students from economically disadvantaged homes were frequently assigned to the low groups for a variety of reasons. The rigidity of traditional grouping guaranteed that these children remained there. Students in these groups were often inundated with skill and drill worksheets in misguided attempts to help them "catch up," while students in the higher groups were challenged to explore mathematical concepts and learn through mathematical games. Falling further behind, too many of these students lost confidence in their mathematical abilities, grew to dislike mathematics, ceased trying to succeed, and, in some cases, dropped out of school as soon as they could.

Flexible grouping to address students' learning needs differs from the traditional grouping model that was just described. With the Guided Math framework, group composition is fluid and flexible, varying from day to day to accommodate the evolving mathematical learning needs of students. Because of this, teachers sometimes struggle in determining exactly how to group their students.

Forming Guided Reading groups is a more clear-cut process than forming Guided Math groups. Teachers assess reading levels by performing running records as their students read aloud. This formative assessment indicates the reading level of each student. For regular Guided Reading lessons, those levels are used to determine grouping. Any time a teacher observes changes in reading levels, students may be moved from one group to another. Teachers may also group students together for a lesson or two to address a specific need.

For mathematics, however, there is no single assessment to help teachers determine mathematics levels. There is no mathematics leveling system that corresponds to the multiple reading leveling systems available to teachers. Although there are overarching concepts in mathematics, the capability of a student may vary from domain to domain. So, how are teachers to identify similar mathematical needs for student grouping?

One answer is to use available data to form the groups. Since groups are established based on common needs, those needs provide an instructional compass for teachers. This is very different from the traditional grouping of students where they were grouped together in high, middle, and low groups. Without specific data by which they could navigate, teachers often blindly followed the sequence of the instructional materials they were given without varying their instruction to meet the specific learning needs of their students.

Teachers today are fortunate to have an array of student data available to guide them as they make these important instructional decisions. The data is only a benefit to teachers, however, when it targets the relevant curriculum areas, is available in a timely manner, and is used effectively by teachers. Looking at lagging indicators of student success, such as grades from the previous year or standardized state assessments, generally supplies only broad overviews of the achievement levels of individual students. The data is too broad and is certainly not timely. While it is useful for discerning overall trends, it is of little value in identifying instructional targets for daily lessons.

On the other hand, the data from formative assessments provide information about students' proficiency with specific standards and elements. For example, according to NCTM standards (2000), all students in grades 3–5 should "recognize and generate equivalent forms of commonly used fractions, decimals, and percents." Let's say that a student named Nina demonstrates that she can recognize equivalent fractions. Her teacher has observed her working with a variety of representations successfully. In addition, her verbal explanation of her work indicates that she has a clear understanding of this portion of the standard. It would be a disservice to Nina to place her into a group in which students were still struggling with that concept.

Later, when Nina is given a brief written formative assessment, she has trouble generating equivalent fractions. Clearly, she needs some extra instructional support to meet that part of the standard. Nina's learning is best supported by her inclusion into a group of students who share her individual needs. A teacher planning instruction for this group would have no difficulty identifying an appropriate teaching point based on the data from these formative assessments.

Nina's next step in meeting this standard is learning how to generate equivalent fractions, not just recognize them. When planning instruction, the teacher would want to challenge the group with work that was just beyond what they are capable of doing independently. With teacher scaffolding to boost conceptual understanding, students stretch to achieve greater proficiency. The laser sharp accuracy of this kind of instruction efficiently keeps students on track in their learning, with no instructional gaps or unnecessary repetitions of instruction.

To identify the next steps in learning for students in small groups, teachers need to look first at the curriculum standards and then use the data from a variety of formative assessments, both informal and formal. Using this data, teachers identify the highest level of understanding with which the group of students has demonstrated and then focuses instruction on the next sequential or logical teaching point. Throughout this teaching process, the composition of the groups changes as students' needs change. Some students may have an "aha" moment when a concept suddenly makes sense to them while the remainder of the group is still working on understanding. Those students should then be moved to a group whose needs more closely align with theirs. Conversely, while the rest of the group has mastered a standard, one or two students may still need additional support. These students' needs are better served in a group that is continuing to work on that standard. Teacher instruction is most effective when it varies to continuously support students as they move toward mathematical understanding and proficiency.

## Unit Pretests

Some teachers prefer to give an initial assessment at the beginning of a unit to help them discover the areas of strengths and needs of their students. There are two kinds of pretests. One kind of pretest mirrors a final unit test, while another tests the knowledge and skills students need to have already acquired to be successful with the new content. Tests that mirror final unit tests provide very little useful information for grouping. The data from these tests only identifies students who may already have mastered standards to be taught in the upcoming unit. However, pretests that assess students' current prerequisite knowledge and skills offer more valuable information regarding instructional needs. Students who have gaps in these areas can be grouped together to ensure that those gaps are addressed prior to the introduction of new concepts. Once gaps are addressed, teachers continue to provide scaffolded support, if needed, as the new unit is taught. The data from these tests may be used initially to form homogeneous groups of students for the unit. As the unit progresses, however, the composition of the groups will change as students' learning needs evolve.

## Performance with Earlier Sequential Concepts

Often, a new unit consists of concepts that build directly on those taught in an earlier unit. For example, in second grade classes, a study of place value precedes a unit on addition or subtraction with regrouping. Students who do not completely understand place value are likely to need more support to understand how to regroup when adding or subtracting than students who have demonstrated a strong understanding of it. In an algebra class, students first learn about polynomials before they are taught about factoring them. As with the previous example, students who struggle to understand polynomials will most likely require additional scaffolded support when they are learning how to factor them. In these circumstances, teachers may be able to form appropriate groups without pretesting because they are already familiar with their students' needs based on evidence from previous lessons.

## Formative Tests

As students progress through a unit of study, many teachers administer brief paper-and-pencil formative tests to identify how their students are doing with the new concepts. Since Guided Math groups are fluid, they may be adjusted at any time to reflect the instructional needs of individual students based on assessment results. To be effective, these assessments are administered throughout the unit rather than waiting until teachers feel that most students have mastered the concepts being taught. By assessing often during the unit and then immediately acting upon the findings, teachers are able to accurately target their instruction so it includes additional support or additional challenges for students as indicated.

## Performance Tasks

Performance tasks are part of the curriculum in an ever-increasing number of states. Just as the moniker implies, students are expected to demonstrate their competence by being able to perform specified tasks. Increasingly, instructional units based on these standards include a variety of performance tasks for students to complete. They often provide rubrics for students that describe exactly what constitutes exemplary work. As students learn to use rubrics to assess and revise their work, their use offers teachers valuable knowledge to help them identify the next steps in learning for students. By grouping together students who have the same next steps, teachers are able to efficiently meet their students' needs and maximize learning.

## Observations of Student Work

Experienced teachers know that assessment is an ongoing process; it is more than testing and performance tasks. They know that the sooner they detect misconceptions, the easier they are to correct. The sooner they find students have grasped a new concept, the sooner those students can be moved to the next teaching point. By observing students as they work, teachers can fine-tune and focus their instruction. To manage information from observations, some teachers use checklists while others simply record anecdotal notes. Technology offers teachers many ways to document anecdotal notes. Tablets and laptops are convenient to use in the classroom, and various apps make recordkeeping easier for today's teachers. Photographs, audio or video recordings, and notes on digital devices are all being used by teachers to chronicle their observations.

Teachers develop recording systems based on their own preferences, but the importance of recording these observations cannot be overstated. Although it is tempting to think that one will be able to remember what has been observed, more likely than not all but the most obvious observations will be forgotten as the day progresses. Recording observations as they occur provides an accurate overview of student learning and needs. Based on these observations, teachers can plan to have Guided Math groups meet just once to address a misconception or meet for a more extended period to reinforce learning of difficult concepts.

### Mathematical Conversations with Students

Just as effective teachers recognize the wisdom of observing students as they work, they understand the value of talking with students about their work and their mathematical understanding. Because a student arrives at a correct answer does not necessarily mean they understand the mathematics involved. When teachers listen to learners explain what they did to arrive at an answer or explain why they think a conjecture is or is not true, the degree of conceptual understanding of students becomes crystal clear. Once again, an efficient recording system is essential for teachers to be able to effectively adjust their instruction based on this kind of informal assessment. It is virtually impossible to retain an accurate mental record of what has been gleaned from student conversations when they are a regular part of mathematics instruction. When anecdotal notes are clearly recorded in a timely manner, teachers can then draw upon them in grouping students appropriately.

### Benchmark Tests

Some school districts conduct benchmark testing to measure student progress over a grading period or a year. The data from these assessments is certainly more current than that from the previous year's state standardized tests, yet it comes a little too late to use when grouping students for daily instruction. Despite these limitations, the data does allow teachers to identify students who appear to have some gaps in their learning. A teacher would be remiss if they failed to use this information to remedy those gaps. By grouping students according to the documented gaps and addressing those areas of need, teachers support students in mastering curriculum goals.

## Organizing for Small-Group Lessons

An essential element in effective small-group instruction is a well-organized work area. Teachers with limited lesson time want to make the most of it. When the meeting space is clearly defined and the lesson materials are prepared and readily available at the beginning of each math class, group instruction can begin promptly. Even good teachers with excellent lessons may squander valuable time if they have to clear off the tables to be used, gather the resources needed, and remind students to return to their desks for pencils or paper during the time scheduled for small-group lessons. Giving thought to organizational details as lessons are planned leads to the efficient use of this precious instructional time.

Most often, teachers meeting with groups choose to use small tables that will accommodate up to six students and the teacher. Some teachers prefer to meet with students on carpeted areas of the floor or at groupings of desks. The choice of meeting space is up to the teacher, but it should be carefully considered and designated before small-group lessons begin. The location of these work areas should also allow teachers clear views of students who are working independently so that their activities can be monitored. If a co-teacher or volunteer is in the room during mathematics instruction, the classroom teacher may consider establishing a second area where students can meet to work in small groups. Whatever areas teachers designate for work with small groups, they should be uncluttered and ready for use as class begins.

Areas designated for small-group lessons should be equipped with the materials teachers and students most often use to explore mathematical concepts and solve problems. Although students may have supplies at their desks, the goal is to maximize the instructional time for each group. Even responsible students have been known to forget to bring needed materials to work areas. If the materials are already there for student use, time is not wasted while students return to their desks or find other students from whom they can borrow pencils. The list on page 191 includes suggested items to have available during small-group lessons.


***This small group is meeting on the floor instead of at a table.***

- whiteboard(s) for student use
- chart pad and easel or whiteboard for teacher use
- work mats
- manipulatives
- measuring tools
- calculators
- paper
- pencils or pens
- erasers
- crayons, markers, or colored pencils
- specific materials for the planned lesson

To effectively address individual students' needs, it helps to have ready access to relevant student assessment data—both formal and, perhaps even more importantly, informal data. As teachers work with individual students, they can refer back to anecdotal notes from earlier observations, citing both the student's strengths and the next learning steps needed. Along with the previously recorded data, teachers should keep at hand materials needed to record their observations of current student work. These materials may include checklists to document evidence of students' learning or simple, organized systems to record dated notes about what students can do and understand with additional notations about areas that need more work.

Because small-group instruction occurs during math workshop, students spend some time transitioning from one activity to another. When selecting a model for math workshop, it is important to consider the number of transitions each model requires. Although periodic movement may be beneficial for some students, many teachers find that a model in which students remain at a single workstation throughout math class has many advantages. A variety of math workshop models will be examined in Chapter 6.

Transitions go more smoothly when teachers plan exactly how students can most efficiently move from one learning activity to another with as little disruption as possible. Once a vision of how that should occur is in place, it can be broken down into teachable components so that students understand precisely how the transitions will work. Students need to know the procedures they will follow to make transitions, and they also have to develop the capacity to follow those procedures—that takes practice. At the beginning of the school year or when Guided Math is first implemented, teachers should plan to spend several weeks teaching students these expectations and having students repeatedly practice before beginning work with small groups. Throughout the school year, when needed, teachers can revisit these procedures with their students to ensure swift, orderly transitions.

## Planning Small-Group Math Lessons

Implementing effective small-group lessons depends on teachers knowing exactly what their students know and are able to do. This insight enables them to plan next steps in learning for students to meet their unique needs. The most productive lessons provide just the right levels of challenge for students working together in a group. With lessons that are too easy, students give very little thought to the mathematical content. Those who are bored may choose to avoid doing the assigned work and even sometimes engage in problematic behavior—a situation that is clearly not conducive to learning.

Lessons that are too difficult are equally ineffective. Students often become discouraged and frustrated. The experience of feeling overwhelmed is not conducive to mathematical learning. Some learners simply shut down cognitively. Some try their best and thoughtlessly follow a series of procedures they have been “taught,” hoping for the best. Many students experience a strong sense of inadequacy and develop negative attitudes toward mathematics that may last whole lifetimes.

However, when students experience the “just right” instruction made possible with small-group lessons, the teacher can almost hear an audible sigh of relief from many students. The intimacy of a small group is a perfect setting for students to stretch their mathematical minds. They feel supported and free to take risks. In a supportive environment challenged with well-designed mathematical tasks, students are nudged beyond their current understanding; their mathematical comprehension and skills increase. In these lessons, teachers encourage students to wrestle with problems so that they can feel the satisfaction of independently completing difficult tasks. All the while, teachers gently guide their students' mathematical explorations with thoughtful questions to help them make mathematical sense of their world. As they become puzzled and perhaps intrigued by the mathematical questions they encounter, students "begin the real journey of doing and constructing mathematics" (Fosnot and Dolk 2001, 49). The increase in proficiency and conceptual understanding they achieve establishes a solid foundation for future mathematical learning. With small-group "just right" lessons, teachers support their students in reaching their full potentials.

To make this type of differentiated instruction possible, a small-group lesson plan should provide a menu of alternative options. This lesson should have a basic lesson that is designed to teach the current concepts or skills to be learned, but there should also be options for providing additional challenge and plans for rebuilding gaps in students' prerequisite knowledge and skill. The basic lesson should be designed to teach the current concepts or skills to be learned, but the lesson plan should also include options for providing additional challenge and for rebuilding gaps in students' prerequisite knowledge and skills. The lesson is designed with the overall goal of ensuring that all students master the standards being taught, but with the recognition that some learners will require scaffolded support, while others will be best served with more complex interaction with the concepts and skills being learned. As shown in the flow chart in Figure 5.2 (page 194), lessons can be adapted by teachers to meet students' needs. Even in the midst of instruction, teachers can seamlessly adapt the lesson when they discover students who need greater challenge or additional support in rebuilding prerequisite knowledge and skills. According to NCTM, "equity ... demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (2000, 12). The differentiated instruction made possible with small-group lessons goes a long way in helping teachers provide equitable instruction.

**Figure 5.2—Instructional Flow of Small-Group Lessons**

**Plan and administer brief pre-assessment of pre-requisite knowledge and skills.**

↓

**Use data to form groups with similar needs.**

↓

**Deliver small-group lesson.**

↙ ↓ ↘

| **Rebuilding Group** | | **Standard Group** | | **Challenge Group** | |
|---|---|---|---|---|---|
| address gaps and standard lesson | | standard lesson | | abbreviated standard lesson | |
| More support needed? Return to address gaps, as needed. | Mastered quickly? Provide challenge. | More support needed? Return to address gaps as needed. | Mastered quickly? Provide challenge. | More support needed? Teach full standard lesson or address gaps, as needed. | Mastered quickly? Provide challenge. |

Because of the increased complexity of these lessons, the planning process for creating them is somewhat more demanding than for a typical whole-class lesson. The steps listed in Figure 5.3 (page 195) describe the typical planning process for a lesson. These steps can serve as a lesson plan template to plan small-group lessons with differentiation options.

**Figure 5.3—Planning Lessons for Small Groups**

1. Identify the standards and big ideas to be learned. What are the criteria for success for students in meeting the standards?
2. Determine what knowledge and skills are essential prerequisites for success with the lesson. What should students *already* know and be able to do to be successful with the lesson?
3. Use information from assessments (both formal and informal) to form groups based on student needs. If important assessment data is not available to determine if students have the prerequisite knowledge and skills needed, plan and administer a brief, simple assessment specific to that knowledge and those skills. This may be given the day before as an exit ticket.
4. Create the lesson. How is the lesson *connected* to previous student experiences? What is the specific *teaching point* for the lesson? How will the lesson *actively engage* students? How is the teaching point *linked* to students' future work as mathematicians?
5. Plan tasks to provide *additional challenge* for students who may have already mastered or who very quickly master the standards being taught. How will the lesson engage these students?
6. Plan specific ways to *rebuild prerequisite background knowledge and skills*. How will instruction be scaffolded to fill gaps and then continue to support students with the new content being learned?
7. Identify the *essential mathematical vocabulary* students need to know.

## Step 1—Identify the Standards and Big Ideas

Because the amount of time spent teaching each small-group lesson is considerably less than is typically spent for whole-class instruction, it is imperative that instruction is very focused and efficient. Lessons should clearly target both the standards to be learned and the unique learning needs of students.

The standards are at the forefront of lesson planning. It is important for teachers to have a thorough understanding of the meaning of the standard and to know where it falls in the progression of mathematical learning. Additionally, related to the standards are "big ideas," the central organizing ideas of mathematics principles that are essential to the conceptual understanding of the standards (Fosnot and Dolk 2001). The big ideas are the basic concepts that students should understand and be able to apply appropriately. These are the foundation for mathematical growth.

It is also imperative that students learn how to think mathematically. They should be aware of how mathematicians have generated mathematical ideas throughout history (Van de Walle and Lovin 2006) and how they are applied in real-world contexts. Learning mathematics entails more than simply memorizing set procedures and developing automaticity with math facts. In planning small-group lessons, teachers should have in mind the relationship between the individual standards, the big mathematical ideas students need to understand, and the standards for mathematical practice.

It is helpful for teachers to collaborate as they determine exactly what students need to know and be able to do to demonstrate mastery of the standards and an understanding of the crucial big mathematical ideas to be learned. In schools where collaboration is not the norm, individual teachers bear the responsibility for determining the criteria for success for their students.

- What are the expectations for student work?
- What will be considered evidence of student success?
- How does a student demonstrate proficiency?

Whether the criteria are a result of collaboration or are established by individual teachers, they are the basis for assessing student learning and guiding the lesson plans. Once teachers have a clear vision of what students should know and be able to do, they can plan instruction that will lead students to acquire that knowledge and skill.

## Step 2—Determine the Essential Prerequisite Knowledge and Skills

Knowing the learning trajectory leading to the current content of a lesson is critical to being able to gauge the specific learning needs of students. Lessons are based on assumptions that students have previously acquired certain knowledge and skills that are prerequisites for success with the lesson. Lessons most often do not, and indeed should not, include reteaching these crucial prerequisites. A short summary review of foundational knowledge and skills may prove helpful for some learners, but this review typically falls short of supporting students who have significant gaps and is a waste of time for students for whom it is unnecessary. As a result, students with gaps in the foundational knowledge upon which the lesson is based frequently require additional reteaching. The students for whom the review was not needed missed out on the opportunity for more challenging instruction.

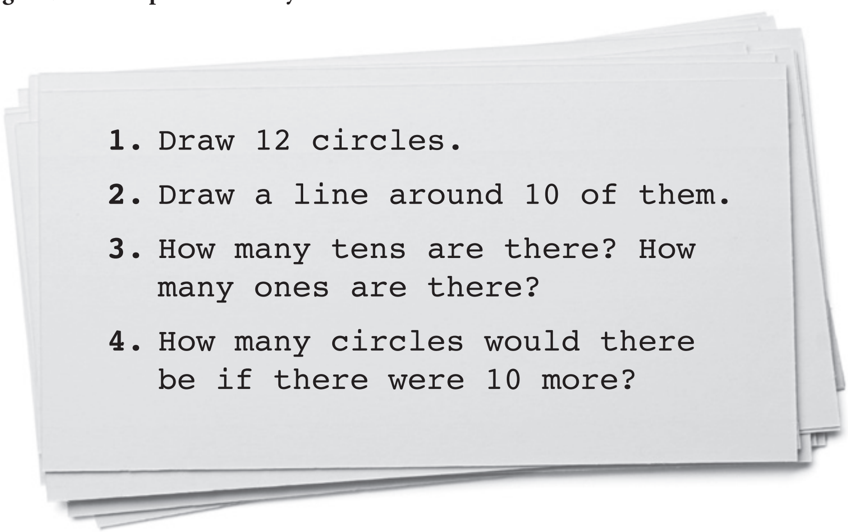
In planning, teachers should call upon their teaching experience and knowledge of the curriculum to determine what students should have already learned leading up to the current lesson. Sometimes, that information is easily established by looking back at the curriculum for the current year. Often, however, teachers may have to examine the curriculum from previous years to clearly discern the learning trajectory leading up to the content of the lesson. Another way to discover more about the learning progression is by conferring with teachers from other grade levels. Developing an understanding of the mathematical learning trajectory helps teachers not only identify potentially impactful gaps students may have, but also gives them valuable guidance regarding what they can do to support students when a lack of essential background knowledge threatens to hinder learning.

## Step 3—Use Assessment Data to Form Groups

Once the lesson standards and big ideas are selected and the prerequisite knowledge and skills identified, teachers look to the available assessment data to form small groups of students with similar needs. The data used for grouping should be as recent as possible and provide information aligned directly with the prerequisite knowledge and skills of the lesson. If data regarding those areas are not available, a very brief but specific assessment can be constructed and administered. Teachers often find it helpful to give this assessment as an exit ticket the day before the lesson is to be taught. These assessments should consist of one to four questions. This makes them easily checked. Students' answers will help teachers determine which of them have gaps in their background knowledge that need to be filled to support their learning. An effective method of administering the assessment is having students record their answers on index cards. Teachers simply collect students' cards, check their responses for accuracy, and then sort the cards into several groups based on similar instructional needs. Each stack of cards then becomes a group for the lesson. This type of assessment has the advantages of specifically targeting the essential foundational knowledge necessary for the lesson, providing timely data, and being quick and convenient to use. While it is not one hundred percent accurate in identifying needs, it offers a valid basis for grouping. If teachers discover during a lesson that students are grouped inappropriately, it is easy enough to move them to other groups.

Figure 5.4 is a sample formative assessment for a lesson on teaching students about place value to identify 10 more than a number. The prerequisites for the lesson may include understanding the concepts of tens and ones. Students would record their answers on index cards that the teacher would collect and sort according to their responses.

**Figure 5.4—Sample Elementary Formative Assessment**



1. Draw 12 circles.
2. Draw a line around 10 of them.
3. How many tens are there? How many ones are there?
4. How many circles would there be if there were 10 more?

The first three parts of this exit ticket assess students' abilities to count objects and their knowledge of tens and ones. Students who are not able to draw 12 circles or draw a line around a group of 10 might be grouped together to strengthen their counting skills or their knowledge of groups of 10 as their lesson began. Students who could do that but could not tell how many tens and ones there were would be placed together for initial support on those concepts at the beginning of their lesson. The fourth question allows the teacher to see whether there are any students who already knew how to use their knowledge of tens and ones to find 10 more. Students who could already identify 10 more would be in a group receiving additional challenges.

Figure 5.5 is a sample formative assessment for a lesson on introducing the addition of integers. The prerequisites for the lesson may include understanding what an integer is and understanding the concept of additive inverses. Students would record their answers on index cards that the teacher would collect and sort according to their responses.

**Figure 5.5—Sample Secondary Formative Assessment**

1. Write 2 numbers that are integers.
2. Write 2 numbers that are not integers.
3. Give an example of two additive inverses. What makes them additive inverses?
4. What is the sum of 5 + (-6)?

Using this simple assessment, students who may not understand what an integer is can be grouped together for a lesson that begins with a review of integers and then provides additional scaffolding to support them as they learn how to add integers. Students who do not understand the concepts of additive inverses or zero pairs can become a group so they can receive support to meet their learning needs. Those students who were able to find the correct sum can also be placed together in a group. Once the teacher confirms that they understand adding integers, additional challenge can be provided for them.

By grouping students in this way, those who will benefit by having their lesson begin with reteaching or review of important foundational knowledge and skills receive that support. Those who do not need the support can move directly to the new content of the lesson. Students who already understand what is to be taught are able to work on tasks that prompt them to examine the content of the lesson in greater depth. If teachers ever discover they have erred in grouping or in assessing students' readiness for the lesson, they have the flexibility to adjust either the lesson or the group composition.

A group may meet together for one lesson or for an entire unit of study depending on students' needs. The ability of teachers to vary the makeup of the groups ensures that each student's instruction matches their needs. Ideally, students work just beyond their independent capability, but within the area where they can be successful with support from a teacher or peers. By working within Lev Vygotsky's (1978) Zone of Proximal Development, learning is maximized. Students who prove to be capable of working on tasks beyond those of their instructional groups are easily moved to more appropriate groups. Similarly, students who begin to struggle may be moved to a group better suited to their learning needs.

## Step 4—Create the Lesson

The structure of a small-group lesson is the same as that for a mini lesson, consisting of a connection, a teaching point, active engagement, and a link. See page 151 for more information on the architecture of a lesson.

**Connection**—Beginning the lesson with a connection to students' previous experiences helps them recognize the relationship between what they will be learning in the new lesson to what they have already learned. A connection may be made to mathematical concepts and skills learned earlier in the week, earlier in the year, or even in a previous grade. Teachers explicitly describe the connection to students rather than asking them to recall and share what they have learned previously. In this way, teachers can relate exactly what connections they want students to recognize. Making this part of the lesson teacher-centered requires less time than calling on students in hopes that one of them will articulate the connection the teacher has in mind. Thus, it leaves more time for student engagement later in the lesson. Students' math talk is important, and there should be ample time for that later in the lesson.

- In introducing the concept of volume, a fifth grade teacher may remind students that they learned how to find the area of plane figures when they were in third grade, but that today they will be working with some of the solid figures they have studied.
- In an algebra lesson focused on teaching students how to use the structure of an expression to identify ways to rewrite it, the teacher might connect the lesson to what students learned about expressions in middle school or in an earlier lesson during the current school year.

**Teaching Point**—The teacher should explicitly share a specific teaching point for each lesson. The teaching point should clearly describe what students will learn in the lesson. This allows students to look for it during the lesson and focuses their attention on its salient points. Because lessons are most effective when students are actively involved in constructing their own mathematical meaning, letting them know up front about the focus of the lesson minimizes the possibility that they will be distracted by unimportant or irrelevant details. While the teaching point alerts students to an overview of what they will be learning, it does not reveal the precise details.

- In the fifth grade lesson about volume, students might build solid figures with cubes. Then, the teacher might tell students that they will "learn how to find the amount of space a solid shape takes up." Students may be asked to listen carefully for new vocabulary terms throughout the lesson and encouraged to think about how this experience connects with other mathematical concepts they have learned.
- In the algebra class, the teaching point of the lesson informs students that they will "learn more about the distributive property and how it applies to writing expressions." They might be asked to reflect during the lesson on whether collecting like

terms is an application of distributive property. Rather than telling them it is, students are given the opportunity to work with problems that will lead them to that conclusion.

**Active Engagement**—Students' active engagement is the most crucial part of a small-group lesson. During active engagement, students explore mathematical ideas. As they work with the problems, they are challenged to talk about what they notice, the meaning of what they notice, what connections they can make, how what they are doing relates to larger mathematical ideas, and how it relates to the real world. Students act as mathematicians engaging directly with the core mathematical content of the lesson.

Although students carry the primary responsibility for *making* meaning, this does not suggest that the role of the teacher is diminished in any way during the active engagement phase. On the contrary, the teachers' responsibilities are far greater than they are in traditional whole-class lessons. Effective engagement of students requires meticulous planning, a laser-like focus on learning goals, careful attention to students' interactions and mathematical work, and a confidence by teachers in their own professional judgment.

- In what ways can the lesson lead students to understand the teaching point?
- What are the students understanding during the lesson?
- What misconceptions might they have?
- What questions will move them to greater clarity and understanding of the mathematical concepts and skills being learned?

**Link**—The link of the lesson serves two purposes. It is an excellent opportunity for teachers to restate what students have learned during the lesson and to remind them that what they have just learned is something that they are expected to use in their future mathematical work. Furthermore, the link is a time of reflection for students. Each student shares something mathematical they learned or thought about during the lesson. Norms for reflection during the link phase make it clear to students that what they share must be something unique to them and cannot be something that someone else has already shared.

Student reflection is critical to the lesson. Knowing they will be called upon to share their reflections tends to increase student attention throughout the lesson. Furthermore, asking students to reflect supports their overall learning. When they reflect, students:

- mentally review and summarize what occurred during the lesson
- consider how well they understand the teaching point
- decide on a thought to share with their group
- determine how to clearly express their thinking
- orally share their idea and listen to others communicate their reflections

These processes, which are crucial in building students' mathematical comprehension, greatly increase the likelihood that students will retain what is learned and that they will be able to successfully apply it in other contexts.

## Step 5—Plan Additional Challenge Tasks

The intent of the teacher when planning for small groups is to present a lesson that is at the right level of difficulty for the learners in the group. All students deserve the opportunity to be appropriately challenged and to learn to their fullest potential. Effective planning of small-group lessons should always include differentiation options for students who may have already mastered the standards or who very quickly understand the standards being taught.

Data from previously administered assessments sometimes reveal students who are ready for more complex tasks designed to deepen the current lesson focus. Yet, even when the data suggests students may be prepared for this kind of challenge, it is important to confirm that they actually have the depth of knowledge and skills needed before assuming they have successfully mastered the standards being taught. In addition to these students, teachers may find that other previously unidentified learners quickly develop an understanding of the concepts and skills and are ready for greater challenges. Thus, it makes sense to include in every small-group lesson a differentiation option that offers additional challenge to those for whom it is appropriate.

What kinds of tasks can teachers provide to offer additional challenge for these students? Two options often considered by teachers include:

- acceleration forward along the curriculum map
- enrichment of the curriculum by having them dig deeper and think more critically

Although accelerating students through the curriculum may initially appear to be the simpler option, it is rarely the best for students. With so little time for students to delve into more complex understanding of the mathematical concepts and skills with which they are dealing, it is indeed a luxury to be able to offer students the time to explore and think more deeply.

Teachers often have difficulty locating quality resources for lesson tasks that spur students to delve more deeply into the concepts and skills addressed in the regular lesson. Marian Small recommends parallel tasks as an effective strategy for providing differentiation. She describes parallel tasks as "sets of two or more related tasks that explore the same big idea but are designed to suit the needs of students at different developmental levels" (2012, 183). Teachers might begin with a task from the active engagement section of the regular lesson and then adapt it so that it remains close in context but meets the needs of more proficient students who may require greater challenge. Students participating in either the regular lesson task or the additional challenge task would still be able to contribute to a whole-class discussion or even engage in workstation talk regarding the topic of the lesson because of their common experiences with these parallel tasks. While Small writes of giving students a choice of which parallel task they will tackle, the provision of assigned parallel tasks affords teachers an effective way to differentiate small-group lessons to flexibly target the identified needs of all students.

If elementary students are learning how shapes in different categories may share attributes, they could complete these parallel tasks:

- **Active Engagement:** Students work in pairs to compare two shapes using a double Venn diagram.
- **Additional Challenge:** Students work in pairs to compare three shapes using a triple Venn diagram.

If secondary students are learning to solve linear equations with one variable, they could complete these parallel tasks:

- **Active Engagement:** Post this equation ($2m + 10 = 40$) where $m$ represents a mystery number. Pairs of students draw a number card which may or may not be the mystery number. Before solving the equation to find out whether or not their number is the mystery number, they consider what they know about numbers. Then, students share whether they think their card contains the mystery number and why or why not. After making their predictions, they prove whether or not it is and explain how they know.
- **Additional Challenge:** Pairs of students work together to create equations with one variable that will be true even if the variable has more than one value. They share their equations with the group and justify their answers.

Assigning parallel tasks allows teachers to provide additional challenges to students directly connected to the lesson. In planning parallel tasks, teachers have to spend some time planning both the content of the lesson and what they want students to gain from the work. These tasks offer challenges for students that are directly aligned to the mathematical content of the overall lesson while requiring minimal teacher time spent searching for alternative differentiation options.

Parallel tasks are only one option for differentiation. Textbooks and other mathematics resources sometimes offer meaningful tasks that can be adapted to provide differentiation that encourages students to delve more deeply into the concepts and skills of the lesson. In addition, there are many online instructional resources available to teachers. It is important to carefully review all material to ensure that it truly addresses the standards being taught. Far too many lessons and tasks fail to support the grade level standards they claim to address.

## Step 6—Plan Strategies to Rebuild Prerequisite Background Knowledge and Skills

To address gaps in essential prerequisite knowledge or skills, small-group lessons should include options for very brief and targeted instruction designed to remedy any existing gaps while also making new content accessible to students. For each prerequisite concept or skill identified for the lesson, a strategy to address it should be included. These remedial strategies should not be full lessons, but instead concisely lead students to a better understanding so that they can proceed to the new content with continued teacher support, if needed.

Figures 5.6 and 5.7 (see page 207) give examples of how to rebuild prerequisite background knowledge and skills. If students are learning that shapes in different categories may share attributes, the prerequisite knowledge and skills may include being able to name and identify shapes and knowing the vocabulary terms that describe their attributes. Students who have gaps in either of these areas will receive remedial instruction as *part* of their lesson. The expectation is that once they receive this targeted instruction at the beginning of the lesson, they will then move into the standard lesson. The instructional activities listed in Figure 5.6 are designed to address the gaps in prerequisite background knowledge for this lesson that students may have.

**Figure 5.6—Sample Elementary Strategies to Rebuild Prerequisite Background Knowledge**

| | |
|---|---|
| *Unable to name and identify the shapes* | Review the shapes and their names with concrete examples. Play "I have (the name of a shape). Who has (the picture of a shape)?" Continue with the current lesson. If needed, make labeled concrete shapes available for use by students. Monitor use of vocabulary terms, reinforcing their appropriate use. |
| *Lack of vocabulary knowledge of the terms that describe attributes of the shapes* | Use concrete objects to identify each attribute. Have students create motions or hand signals to stand for each term. For example, for "face" students might bring a hand, palm facing the face, down in front of their faces. For vertex, raise hands together above the head with fingers pointing up. Continue with the current lesson, providing vocabulary cards with nonlinguistic representations if needed. Closely monitor students' use of appropriate vocabulary. |

For a lesson aimed at teaching students how to solve linear equations with one variable, students should already know what a variable is and understand what expressions and equations are. The instructional activities listed in Figure 5.7 address gaps in prerequisite background knowledge students may have.

**Figure 5.7—Sample Secondary Strategies to Rebuild Prerequisite Background Knowledge**

| | |
|---|---|
| *Does not understand what a variable is* | Review the meaning of variable and discuss why mathematicians developed the concept. Continue with the current lesson while emphasizing the term and monitoring student math talk to be sure they are using it. |
| *Does not understand expressions and equations* | Review the definition of an expression. Use a balance scale to illustrate the meaning of equal and equations. Discuss how expressions and equations differ. Create an anchor chart showing the two for students' future reference. Continue with the current lesson while monitoring students' math talk to be sure the terms are being used and used correctly. |

While a few students will immediately master the prerequisite concepts and skills as a result of this targeted instruction, most will still require scaffolded support throughout the lesson. It is important to remember that prerequisite knowledge and skills are the foundations upon which lessons rest. Consequently, students continue to engage and wrestle with them even as they work to understand the new content. From these ongoing experiences, students' understandings will continue to increase.

To support their efforts, teachers may provide ongoing instructional scaffolding throughout the lesson. This might include using manipulatives, providing vocabulary cards, or offering other supports. Lessons for some students may be adapted to enable them to focus more clearly on the new content. For example, if a lesson contains lengthy word problems or larger numbers that are not essential to understanding the lesson focus, the wording can be simplified or the numbers changed to accommodate the needs of these students. For instance, if a lesson addresses the concept of equivalent fractions, some students may still be working to grasp the concept of fractions. For these students, the lesson might be adapted so that they work with common fractions such as halves, fourths, and thirds while the regular lesson might also include sixths, eighths, twelfths, and sixteenths.

## Step 7—Identify Essential Mathematical Vocabulary

Because the understanding of relevant vocabulary is inherent in comprehending mathematical concepts and in developing the ability to communicate mathematical ideas with precision, it is important for teachers to identify the terms that students need to know for each lesson. To support students' vocabulary acquisition, teachers can use math word walls, talking point cards, vocabulary games, and other activities. Some of the essential vocabulary may be words that students have learned previously, but may have forgotten or for which they need to develop a deeper understanding. Others may be newly introduced terms related to the content addressed in the lesson.

Sometimes, students understand a concept without being able to explicitly name the concept. If that is the case, give students the vocabulary at the beginning of the lesson. Other terms are best taught after students have an opportunity to develop the conceptual knowledge they need to understand them.

- In a lesson teaching the concept of finding the circumference of a circle, the words *radius* and *diameter* may be vocabulary that should be reviewed because some students may have forgotten their meanings. The term *circumference* can be introduced as the lesson begins even though it is unknown to students because it can be easily understood without building any additional conceptual understanding. Students who already know what a circle is can understand and measure the distance around one. The term *pi*, however, should not be introduced until after students have had hands-on experiences measuring the radius, diameter, and circumference of circles. Once they begin to discover the relationship between them, the term can be introduced to students to name the relationship they have noticed. Without this experience, most students would have no prior knowledge to draw upon to help them understand the meaning of *pi*.

Figures 5.8 (pages 210–211) and 5.9 (pages 212–213) show example small-group lesson plans for teacher reference. The elementary small-group lesson plan focuses on helping students identify strategies for comparing fractions with unlike denominators.

The secondary lesson introduces students to adding positive and negative integers. Students work in pairs and then share their findings. Students' findings are recorded on a chart so they can identify patterns and make predictions as to whether the sums of positive numbers, the sums of negative numbers, and the sum of a positive and negative number will be positive, negative, or zero.

**Figure 5.8—Sample Elementary Small-Group Lesson Plan**

## Standard

Compare two fractions with different numerators and denominators.

## Prerequisite Knowledge and Skills

- Understands what the numerator and the denominator of a fraction represent.
- Can identify equivalent fractions.

**Vocabulary**

- fraction
- numerator
- denominator
- greater than
- less than
- equivalent

## Small-Group Lesson

***Connection:*** During the last few weeks, we have been learning about equivalent fractions and have been comparing fractions that have the same denominator.

***Teaching Point:*** Today, we are going to use what we know about fractions to compare fractions with different denominators.

***Active Engagement:*** Have students work in pairs to solve this problem: *Finn and Maddie decided they would run a mile every day to prepare for soccer season. Finn has run $\frac{3}{4}$ of mile today. Maddie has run $\frac{2}{3}$ of a mile. Who has run the greater part of a mile, Finn or Maddie?* Encourage students to use manipulatives if they wish. Listen closely as students work. Question them: Why are you…? How do you know…? Is there another strategy you could use? Does it help you to be able to identify equivalent fractions when you are comparing fractions? If so, why? Listen for students who use $\frac{1}{2}$ as a benchmark. Have students share their solutions and methods of solution. Why do you think it is important to be able to compare fractions?

***Link:*** Remember that mathematicians often use representations of mathematical ideas. In this case, we used fraction strips. Using these to represent fractional parts helped us to visualize, or see, how the sizes of fractions compare. Have students reflect and share something mathematical they either learned or thought about during the lesson.

## Need for Additional Challenge

Have students work in pairs. Ask them to choose 3 fractions between 0 and 1—each with a different numerator and a different denominator. Ask them to place them on a 0 to 1 number line and explain why they placed each fraction where they did. Create a chart of the strategies that can be used to compare fractions.

## Need for Rebuilding Foundational Knowledge

(List Common Gaps and Ways to Address Them)

- *Does not understand what the numerator and denominator of a fraction represent:* Have students fold a piece of paper in half. Ask how many equal parts there are. Have them label each part $\frac{1}{2}$ emphasizing that the numerator tells how many of the parts is represented by the fraction and that the denominator tells how many equal parts the whole is divided. Have students fold a piece of paper into 4 equal parts and have them label each part. Draw a circle bisected so the parts are not equal. Ask students if the circle is divided in half. Why or why not? Be sure students understand that fractional parts are equal and that students know what the numerator and denominator represent. Continue with the standard lesson as soon as students demonstrate understanding.
- *Unable to identify equivalent fractions:* Demonstrate equality using a balance scale. Help students understand that equivalent fractions are just different representations of the same amount. Use fraction strips or other manipulatives to demonstrate equality of fractional parts. Give students $\frac{3}{4}$ and ask them to find as many equivalent fractions as possible using the manipulatives. Ask frequently—how do you know? Continue with the lesson as soon as students demonstrate an understanding.

**Figure 5.9—Sample Secondary Small-Group Lesson Plan**

## Standard

Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.

## Prerequisite Knowledge and Skills

- Know what an integer is
- Understand how positive and negative integers relate to zero
- Know what a zero pair is

## Small-Group Lesson

***Connection:*** We have been learning about integers: positive, negative, and zero.

***Teaching Point:*** Today, we are going to learn about how to add integers.

***Active Engagement:*** Ask students to turn to a partner and tell them what they know about integers. Listen carefully to identify misconceptions and level of knowledge. Tell students: *We are going to use integer tiles to represent the integers we are going to add (yellow for positive, red for negative). Let's add two positive integers.* (Use tiles to model the addition of +2 and +3. No zero pairs. ) *Okay, now let's try another. This time, +2 and -3.* (Use tiles to model. Discuss the zero pairs.) *And now, let's add -2 and -3.* (Use tiles to model.) *Any zero pairs?*

Have students work in pairs. Tell them: *Find the sum of two positive integers, of a positive integer and a negative integer, and of two negative integers. You can choose which integers to add. Record each addition equation on your recording sheet.* Observe and listen as students work. Ask questions: *How do you know? Why did you…? What do you think?*

*Let's share our work.* Record the addition equations on a whiteboard in three columns (+ and +, + and -, - and -). *Do you see any patterns? Can you make a prediction about what happens when we add integers?* (Help students discover that the sum of two positive integers is always positive, of a positive and negative may be either, and of two negative integers is always negative.) *Do you think that will always be true? Why or why not? Can we make a conjecture?*

***Link:*** Remind students that they learned how to use integer tiles to model addition equations as a way of visualizing the operation—just as mathematicians do as they represent mathematical ideas. Challenge them to try to prove their conjectures. Have students reflect and share something mathematical they learned or thought about during the lesson.

## Need for Additional Challenge

Encourage students to add 3 or 4 integers—first using mental math to predict if the sum will be a positive or negative integer or zero.

## Need for Rebuilding Foundational Knowledge

(List Common Gaps and Ways to Address Them)

- *Does not know what an integer is:* Review the definition of an integer. Show number cards one at a time, including numbers that are integers and those that aren't, and have students tell if the number shown is an integer or not. Continue with the lesson, listening carefully to assess student understanding. Throughout the lesson, continue to emphasize the meaning of the term *integer*.
- *Does not understand how positive and negative integers relate to zero:* Use a number line to show how integers may be positive or negative. Give specific real-life numbers, and have students tell whether they would be positive or negative (allowance students receive, money they spend to go to a movie, the level of a river rising, temperatures below zero). Continue with the lesson, listening carefully to assess understanding.
- *Does not know what a zero pair is:* Use the integer tiles to model how a +1 tile and a -1 tile make a zero pair. Use a number line to show that when added, they have a value of zero. Continue with the lesson, listening carefully to assess understanding.

# Preparing for Small-Group Lessons

After lessons are planned, it is time to prepare for them. Having the necessary materials at hand and a lesson schedule that meets students' learning needs are fundamental for effective lessons.

## Gathering Materials

Once the lesson has been planned, the teacher should gather the necessary materials. It is important to anticipate everything that may be needed for instruction. Even the best lessons can be rendered ineffective by lack of materials. Limited time or poor organization sometimes leads to teachers being unprepared and having to improvise as they teach. Although this will certainly happen to every teacher at times during their teaching careers, it should only occur rarely. For effective use of the limited time available for small-group lessons, priority should be given to preparation. Even when the use of manipulatives is not a part of the planned lesson, students may have trouble with abstract aspects of the instruction and require them. If teachers anticipate the supports students may need and have those materials within easy reach, the lesson is uninterrupted.

> **Key Point!**
>
> *All students need time with their teachers during small-group lessons. Students need to learn at their instructional levels, not only their independent levels.*

## Scheduling Small-Group Lessons

The Guided Math framework allows teachers to differentiate instruction in small-group lessons and also permits teachers to vary how frequently they meet with groups. Students who require more support should meet with the teacher for lessons as frequently as possible. Students requiring less support may meet with the teacher briefly and then work independently. Although small-group instruction with these students may occur less often and be of shorter duration, it is, nevertheless, vital. Face time with their teachers is as important to these students as it is to students who require more support.

Too often, teachers have limited quality differentiated lessons for above-level learners. The needs of students who require additional support can easily dominate instructional time. It is important for teachers to be able to achieve an instructional balance and find a way to meet the diverse needs of all students, so they all can reach their potentials. Through Guided Math small-group lessons, teachers can adjust both their lessons and their lesson schedules based on students' immediate learning needs while working with students who may need additional support, providing quality instruction for students who have neither the need nor challenge, and encouraging greater depth of understanding for students who are ready.

Based on the math workshop model and the instructional requirements of their classes, teachers can create a weekly schedule for small-group lessons. Many teachers choose to meet with each of their small groups every day, but that is not essential. Teachers with large classes may find that trying to divide their classes into three groups that meet daily results in groups that are larger than desired. If the class is divided into four groups, some teachers find the lesson time for each group is insufficient. For these larger classes, teachers may choose to divide their classes into four groups and see only two each day (Figure 5.10).

**Figure 5.10—Sample Small-Group Lesson Schedule for Four Groups**

| | |
|---|---|
| **Monday**<br>11:45–12:15 — Group 1<br>12:15–12:45 — Group 2 | **Wednesday**<br>11:45–12:15 — Group 2<br>12:15–12:45 — Group 1 |
| **Tuesday**<br>11:45–12:15 — Group 3<br>12:15–12:45 — Group 4 | **Thursday**<br>11:45–12:15 — Group 4<br>12:15–12:45 — Group 3 |

In the example shown in Figure 5.11 (page 216), Group 1 is struggling with the standards being addressed, so the teacher decides to meet with them four times during this week. Group 2 has a better grasp of the concepts being taught but still needs considerable support. They will meet on Monday, Tuesday, and Wednesday. Since Group 3 is doing pretty well, it will only meet twice during the week. Group 4 has demonstrated mastery of the unit standards, so the teacher is extending their learning. They will meet on Friday to receive instruction extending the grade-level curriculum. The following week, they will work on tasks aligned to this instruction.

With this schedule, the teacher meets with only two groups per day. Depending on the allotted instructional time and the needs of students, it may be possible to meet with additional groups during a class period. This teacher also preferred to meet with groups on consecutive days to provide greater instructional continuity for students, but other teachers may choose to spread out the lessons over the week. For example, another teacher might meet with Group 2 on Monday, Wednesday, and Friday. Developing schedules is up to teachers and what they think will work best.

**Figure 5.11—Weekly Schedule for Math Groups**

| Group | Mon. | Tues. | Wed. | Thu. | Fri. |
|---|---|---|---|---|---|
| **1.** Felisa, Nori, Brad, Ray, Davisha | X | X | X | X | |
| **2.** Ricardo, Marcus, Karina, Tameshia, Portia | X | X | X | | |
| **3.** Dimitri, Tonya, Mary Beth, Carlos, Keon | | | | X | X |
| **4.** Min, Monica, Quin, Lucas, Rosa | | | | | X |

These are only two examples of how small-group instruction may be scheduled. Schedules may vary from week to week according to instructional needs.

Another element to consider is the behavior of students in the groups. With this weekly schedule, both Group 1 and Group 2 are working independently on Friday. If these groups have built up the endurance necessary to work independently for the entire class period, this schedule will work. However, if students in these two groups cannot behave appropriately for that length of time, the teacher may prefer to only have one of these groups working independently for the whole class period on any given day. The planning of small-group lessons allows teachers to create a schedule that best fits the needs of their classes. It may take a few tries and adjustments before a teacher arrives at a good schedule. Even then, it may need adjustments during the year.

Another option for teachers is varying the amount of lesson time spent with different groups. The GUIDE workshop model (see Chapter 6) offers great flexibility for the scheduling of lessons. Because students remain at one workstation for the entire class period, the teacher simply pulls students from their workstations whenever they are needed for lessons. Students work independently in heterogeneous groups, but students pulled for lessons should share comparable instructional needs. These groups are necessarily homogeneous.

In addition to regularly scheduled small-group lessons, teachers may meet briefly with groups as small as two students to correct misconceptions, verify students' understandings, or provide additional instruction. Small-group lessons may be very brief or last as long as the teacher desires. Teachers often use the scheduling flexibility to differentiate instruction, spending more time with students who have greater learning needs while still providing quality instruction to challenge the others. (See Chapter 9, Figure 9.2 and the accompanying text on page 334 for a description of how this can be done.)

Although varying the lesson length is more difficult for teachers using a rotational model for math workshop, it can be done. In some cases, the small-group lessons are not included as a part of the rotation.

Students rotate from workstation to workstation at set intervals. Work with the teacher, however, is not a part of the rotation. Instead, as with the GUIDE model, the teacher pulls students for small-group lessons as needed. (See Figure 6.5—Rotational Models without a Teacher Lesson on page 244.)

## Teaching an Effective Small-Group Math Lesson

There is more to effective small-group lessons than strong planning. Once planned, those lessons must be taught. The instructional strategies listed in Figure 5.12 are among those that are characteristic of robust mathematics instruction and are described in greater detail in this section.

**Figure 5.12—Strategies for Teaching Effective Small-Group Lessons**

1. Focus on strategies students can use to increase their mathematical understanding, apply what they know to new situations, and solve problems.
2. Provide students with a clear understanding of the learning expectations, including setting criteria for success.
3. Encourage students to use a variety of strategies to solve the problem or complete the activity.
4. Scaffold student learning by giving just enough support to move students to the next level of understanding and proficiency.
5. Promote mathematical communication by students.
6. Give students specific, descriptive feedback on their work and encourage students to engage in self-assessment based on the criteria for success.

### Strategy 1—Focus on Strategic Reasoning

In addition to helping students understand the mathematical content addressed in a lesson, one goal of small-group lessons is to help students develop a toolbox of specific strategies they can use as they work to understand mathematical concepts, apply what they know about math to real life situations, and solve authentic problems. Teachers can focus students on strategic reasoning by leading students to:

- connect mathematical concepts to their own lives, to their knowledge of the world, to another aspect of mathematics, or to a similar problem they have solved
- examine the kinds of questions mathematicians might ask about problems as they work on solving them
- reflect on mathematical concepts they have already mastered
- visualize problems as a first step toward solving them
- focus on mathematical vocabulary terms or other vocabulary terms in word problems that may be difficult for students
- pick out important ideas in mathematical concepts and important details in math problems
- practice using a variety of manipulatives
- monitor their own work by checking for errors, revisiting the criteria for success, and thinking about whether their work makes sense
- use multiple representations of mathematical ideas to communicate ideas, solve problems, and interpret mathematical phenomena

## Strategy 2—Provide a Clear Understanding of the Learning Expectations

Teachers should make learning expectations clear to students. Additionally, students should understand the activity or the task to be performed. As they begin work, it should be with knowledge of what constitutes success with the assignment. Teachers may provide rubrics or checklists that detail expectations, or they may show students exemplars of work on similar activities. As instruction shifts away from almost exclusively using paper-and-pencil computation assessments, teachers have an increased responsibility for letting students know what they are expected to achieve and how they will be assessed. When students have this knowledge before they begin work, they are able to monitor their own progress as they work. The more teachers stimulate students to monitor their thinking and their work, the better their mathematical work will be (Hyde 2006).

## Strategy 3—Encourage Students' Use of Multiple Strategies

On traditional math worksheets, word problems always appear at the end. Students are asked to solve the problems almost as an afterthought and perhaps with little instructional support. Students plug in whatever procedures they used for the rest of the page without giving the problems much thought. Little attention is given to teaching students to think strategically.

As mathematics standards have changed, students now encounter more problem-solving challenges and tasks that not only relate to their lives, but also involve multiple areas of mathematics. In today's classrooms, students are expected to develop much deeper conceptual understandings of mathematics. One way in which students develop these understandings is by learning that there are almost always multiple strategies they can use for problem solving.

During small-group lessons, students should be encouraged to choose which strategy or strategies work best, drawing upon strategies already introduced or using what they know to create new strategies. When they are able to devise their own strategies, they call upon and reinforce their mathematical understandings. But, when teachers prescribe the one "correct" way to solve a problem, students quickly stop relying on their own reasoning. Standard algorithms for problem solving then become abstract recipes for solutions that must be memorized but have no real meaning to students. Students benefit from hearing teachers describe their own thinking while solving problems, seeing multiple strategies used by their peers, and experimenting with strategies themselves, so they begin to develop abilities to generalize about mathematical principles (Hyde 2006). Eventually, students develop the ability to mentally envision models and representations of mathematical concepts and move beyond concrete and symbolic to more abstract mathematical knowledge.

## Strategy 4—Scaffold Students' Learning

Teachers can support their students working in small groups with carefully considered scaffolding. Scaffolding originally referred to the way parents interacted with their children as they learned. As children begin to learn something new, they are highly supported by their parents. This adult support is gradually withdrawn as a child becomes more independent. The learning process is, in essence, a joint social achievement involving both the child and the parents. Parents constantly monitor and respond to their children's increased proficiency by gradually releasing responsibility for the task. Children who are supported in this way are working in what was described by Lev Vygotsky (1978) as the Zone of Proximal Development. In small-group lessons, the role of the teacher is similar to that of the parent, but focused instead on learning in a group setting rather than in a one-on-one environment (Myhill, Jones, and Hopper 2006). This type of instructional support should not only be used with younger students. Scaffolding is an instructional strategy that can be used with students of all grade levels.

Derived from the ideas of Owocki (2003) and Myhill, Jones, and Hopper (2006), scaffolding has the following characteristics:

- Scaffolding occurs with assistance. One or more students engage collaboratively with a teacher or another student to acquire new knowledge, skills, or understanding. This is important because learning is a social process.
- Scaffolding involves inter-subjectivity. The participants in the learning activity strive for a common view and adjust to the perspectives and needs of each other—just as with a child and parent.
- Scaffolding is provided with warmth and a responsiveness toward students' needs. The teacher warmly provides just enough support to help the learners be successful while still allowing students to develop a deeper conceptual understanding. Rather than the teacher's role as dispenser of knowledge, it is that of a caring supporter in the learner's quest for knowledge.
- Scaffolding is focused. Instructional support focuses on a particular skill or aspect of understanding so that students concentrate on mastering one particular aspect of a more global concept.

- Scaffolding avoids failure. The instructional tasks are designed to be challenging to students, yet achievable. (It is important to note, however, that this in no way implies that students should never encounter failure—some of which can result in valuable mathematical learning experiences.)
- Scaffolding is temporary. When planning how best to support learning, teachers should also be planning how to gradually remove the support and relinquish responsibility to students. Thus, scaffolding is temporary, yet essential assistance that many learners need to reach "the next step."

After examining current student data regarding student learning, instruction can be adjusted to offer scaffolded support. Teachers who closely monitor their students' achievements and needs are able to accurately respond by relinquishing responsibility to their students when students are ready to assume it. This is not an exact art, however. Just as parents sometimes hold on too long in fear that their child might falter, teachers sometimes do the same. But when scaffolding is most effective, teachers watch for signs that students are ready to assume more independence, and then let them spread their wings and fly.

## Strategy 5—Promote Mathematical Communication

One of the simplest things to do to enhance student understanding is to ensure students have opportunities to reflect on what they are doing and then communicate about their thinking with others (Hiebert et al. 1997). Because learning is naturally a social endeavor, it increases and becomes richer when ideas are shared. Students taught in small groups tend to be more comfortable joining mathematical conversations. While some students will have their say in any learning environment, there are others who are introverted, lack confidence, or are unsure about their mathematical ideas and may refrain from speaking in class. Consequently, these students must make sense of what they are learning in isolation (Nichols 2006). Small-group settings are perceived by those shy students to be a safer setting, so they often feel less reluctant to engage in math conversations with their peers. Additionally, the small group size makes it easier for teachers to encourage everyone in the group to express their ideas verbally and to keep track of those students who are reluctant to verbalize their ideas and learning.

In addition to the cognitive benefits of having to reflect and organize their thoughts before they speak, students also tend to retain what they have learned longer when they have to communicate it orally or in writing (Sammons 2018). Thoughtful discussion helps students with the following skills:

- summarize and synthesize information
- make inferences and conjectures
- justify their thinking
- expand their mathematical vocabulary
- confirm and extend their understanding
- discover errors in their thinking
- develop the ability to question the ideas of others in a productive way

Students who work jointly with others to investigate mathematical dilemmas discover that wrestling with challenging mathematical ideas is not just about finding answers, but is really more about growing ideas (Nichols 2006).

During small-group lessons, teachers often rely upon questioning to stimulate and expand their students' thinking. The use of a conversational tone by teachers serves as a model for their students. Following the teacher's lead, students learn to respect the ideas of others, whether or not they agree with them. By using student errors as teachable moments rather than as failures, teachers help students understand that errors are stepping stones in their mathematical growth.

Writing about math is just as important as talking about math. Many teachers ask students to maintain math journals. These journals are where students record their strategies, justifications, understandings, and reflections. Journals may also be used by students to record mathematical vocabulary—frequently with definitions in students' own words, nonlinguistic representations, and examples and non-examples.

If students of any age are to produce quality writing, they first need to know the characteristics of quality writing. Teachers should model writing the kind of personal mathematical reflection they expect to see, thinking aloud as they do so. Next, students can work together to compose pieces of shared writing in which students jointly contribute to written reflections on their math work. This may be recorded on chart paper by the teacher or a designated student to be displayed in the classroom as an exemplar for future reference. Students should have several opportunities to experience this kind of writing before being asked to complete their own mathematical reflections in their journals independently.

If math journals are not used, students should regularly be asked to include written explanations or reflections of their work as part of their daily math assignments. As with written reflections in math journals, the writing process should be modeled first before students are asked to do so independently.

## Strategy 6—Give Feedback

Everyone receives feedback during their daily life. As teachers, we receive positive feedback when a student exclaims, "Oh, now I get it!" On the other hand, the feedback is not so positive when we check class work and discover that too many of our students failed to "get it." Whether positive or negative, the feedback we receive affects and directs our next steps. And, so it does with our students.

Effective feedback lets us know exactly what we need to do to achieve success in whatever we are working on. Simply telling a baseball player that they have struck out too many times doesn't improve their performance. Batting coaches know that they need to give very specific feedback on batting stance, the speed of the swing, or how the bat is held to help a player improve their batting average. With specific and descriptive feedback, a baseball player can make corrections and, with practice, improve.

Descriptive feedback lets students know about their learning. It includes the "do more of this" (things done well) and the "do less of this" (things not done well) kinds of feedback (Davies 2000). In a small group of students, teachers can provide effective feedback based on the criteria for success that has been established to students as they work. Instead of waiting until a task is completed and then evaluating it, teachers can immediately focus students on areas in which they are not meeting the criteria so that those areas can be improved before the work is turned in to be evaluated. In addition, in small groups, feedback offered to one student may be overheard by other members of the group who can benefit from it.

Of course, students need ample feedback about not only what they can do to improve their work but also regarding what they are doing well. Knowing exactly what they are doing well enables them to replicate those things in future work. Positive recognition of students' efforts also increases their motivation to continue what they are already doing well. According to Anne Davies (2000, 13), the most productive descriptive feedback meets these criteria:

- comes both during and after the learning
- is easily understood and relates directly to the learning
- is specific, so performance can improve
- involves choice on the part of the learner as to the type of feedback and how to receive it
- is part of an ongoing conversation about the learning
- is in comparison to models, exemplars, samples, or descriptions
- is about the performance or the work—not the person

When students begin to understand how descriptive feedback is based on specific criteria, they can begin to self-assess their work and also give feedback to their peers. Not only does this practice improve students' learning in the classroom, but when students learn how to monitor their work, they are developing a skill that they can use throughout their lives.

# The Value of Teacher Reflection

The effectiveness of future instruction is enhanced when teachers take a moment to reflect at the end of each lesson. At this time, they record observations from the lesson, select the most appropriate next steps in learning for groups or for individual students, identify students who require additional help, and modify group composition, if necessary. See Figure 5.13 for a suggested reflection process.

**Figure 5.13—Teacher Reflection Process after a Lesson**

1. Record and organize informal assessment information based on observations and conversations.
2. Select the next steps in learning for the group, as well as for individual students.
3. Identify students who need support with specific concepts, and plan how to reteach the concept when needed.
4. Adjust the composition of the groups, when appropriate.

## Maintain Anecdotal Notes and Records of Informal Assessments

When teachers are working with small groups of students, they can listen to student conversations, observe their work, and in general, get a feel for the levels of student mathematical understanding. During or after each lesson, anecdotal notes based on these observations should be recorded for later reference when planning lessons or during parent conferences. There are a variety of recording systems teachers can use to maintain the notes they make during their work with small groups.

Here are a few suggestions for keeping records:

- Create a spiral notebook that has a section for each student and mark the pages by divider tabs. During instruction, open to the section for a student, record the date, and make notes of specific observations of the student's work that day.
- Fill a three-ring binder with notebook paper and a divider for each student. Then, use sticky notes to jot down observations during small-group work and attach them to pages in the correct

sections of the notebook. Or, rather than using sticky notes, record the notes directly on the pages in the notebook.

- Create a recipe file box with one divider for each student. Take notes on small notecards of observations made during small-group work and file them accordingly. Use separate note cards for each student.
- Use an app specifically designed for recording notes. One example of an app designed for teacher use is *Confer*. It is structured for classroom use so that teachers can easily add anecdotal notes; sort students by the content of the notes, tags, or date of last note recorded; and create small groups. Other, more generic note-taking apps can also be used by teachers, but they usually require more upfront organization. These include *One Note, Evernote*, and *Note Shelf*. If digital apps are used for note taking, it is essential that they be secure. Access to student data should only be available to those who are authorized to view it.

Whatever system a teacher chooses to use, it is important to maintain these records in a timely manner. The specific observations made by teachers and the feedback students are given during a lesson are valuable sources of data. Anecdotal notes from small-group lessons should be saved so that they can be easily accessed and can be used to guide future instruction. These notes also provide documentation of progress, identify students who need additional interventions, and can be shared with parents during conferences.

## Select the Next Steps for Learning

While carefully considering the anecdotal notes from each group's lesson, teachers should reflect on what learning was demonstrated by the students in the group and any misconceptions or areas in which additional work is needed. Obviously, if students were very successful with the activity or task, the teacher should provide additional challenges by assigning tasks that lead students to more deeply engage with the content or to accelerate students to a more advanced area of instruction, based on the curriculum sequence established by the teacher, school, or district.

If many students in a group are struggling with their work, it is time to go back and present the content in a different way or break it down into smaller, more manageable components. Sometimes, additional time exploring the content is sufficient for students to really make sense of it. When anecdotal notes indicate obvious areas of misunderstanding, the teacher can focus instruction during follow-up lessons.

Each small group has its own unique needs—needs that may not become apparent until the end of a unit with a traditional method of instruction. But, because learning needs are more promptly identified in small-group lessons, they can be immediately addressed. This allows all students to move forward rather than having some learners lagging behind while the class continues its one-size-fits-all quickstep pace as is too often the case with whole-class instruction.

## Identify Students Who Need Additional Support

In spite of the strengths of small-group lessons, some students will still need additional support. Teachers are often torn between the desire to move these students forward even without the foundational understandings essential for the mathematical concepts and skills to come and the recognition that these students really need to build those foundations first, if they are to be successful.

It is a dilemma that all teachers face. Clearly, students who lack the fundamental mathematical foundation upon which future instruction rests are unlikely to be successful without remediation. But, it is equally clear that if these students are not introduced to the concepts and skills mandated by the grade-level curriculum until all their learning gaps are filled, they will continue to fall further behind their peers. Unfortunately, there is no cure-all to completely remedy this situation. But, the small-group lesson plan described in this chapter provides a structure that allows teachers to identify gaps in essential foundational knowledge and skills, act to fill them, and then scaffold the learning of these students as they are introduced to the new content that is the focus of the class lesson.

While these lesson plans are effective in addressing the needs of most students, some learners may require even more intensive support to be successful. In some school systems, these students receive "double dips" of mathematics instruction. This can be achieved in several ways. If students participate in enrichment classes, for example, they can receive additional mathematics lessons during that time with either their homeroom teachers or with other designated teachers. Another option is to have students go to other classrooms for additional instruction during mathematics time in those classrooms. This is effective only if they do not miss reading or math instruction in their regular classrooms. The drawback with these plans is students miss other important content areas, such as science, social studies, or art. Knowing the importance of mathematics for academic success in middle school and high school, some schools consider it a priority and make these difficult decisions about student schedules.

Within the classroom, teachers can also address this challenge by pulling students for individual mini lessons whenever possible—sometimes, first thing in the morning or as the rest of the class works independently. Although this tends to be hit-or-miss instruction, it is extra instruction, and every bit helps.

Of course, when students' needs are well documented, some will qualify for special programs. Small-group lessons allow teachers to become aware of specific areas of need early. Consistent documentation helps these students obtain additional assistance, if they qualify for it.

Some schools support students with the use of peer tutors, tutors from higher grade levels, or adult tutors. The instruction offered by tutors may not be of the same quality that teachers provide, but it often helps. Additionally, sometimes strong relationships develop in these tutoring sessions that boost the confidence of students as their tutors also become their mentors.

Teachers should be cautious with the use of tutors, however. Students, whether peers or from another grade level, are not trained. Even those who are eager to assist other students may not prove to be very effective. Despite good intentions, many student tutors do not understand why another student is struggling with a specific concept or skill. Too often, they just want to tell them exactly what to do, or worse, do it for them. Moreover, when working with a peer, the student who needs extra support may feel inadequate or stop trying. Teachers should also ask themselves whether tutoring is the most beneficial use of time for student tutors. Students who have the potential to dive deeper into the mathematics they are learning are often asked to assume the role of peer tutor. While there are undoubtedly some benefits gained from tutoring, is tutoring really the best use of these students' time?

Any time adults are used as tutors, they should be well trained before being paired with a student. Many adults are unfamiliar with the way math is now being taught. While they do their best to support the learning of students with whom they are working, they may not understand the concepts and skills being taught or might teach in the way they learned.

## Adjust the Composition of the Groups

As teachers review their notes from small-group lessons, it may become apparent that a student has suddenly achieved a level of understanding that goes beyond that of the other students in the group, or conversely, that a student needs more support than others. When this is noted, it is wise to wait until at least after the next lesson to make a change. If it appears that the performance of these students remains consistent, then it is time to move the student to a more appropriate group. Students, as well as teachers, should recognize that the composition of the groups is fluid and changes based on need. Students move to new groups as their academic needs change. If there are no appropriate groups available, teachers can either try to vary instruction within a group or work individually with those students whenever possible.

## A Small-Group Lesson Scenario

Let's return to our hypothetical student Nina, whose next learning steps were described earlier in the chapter. She was able to recognize equivalent fractions, but she was not able to generate them. The teacher grouped Nina with several other students who have the same instructional needs. The teacher designed a lesson to scaffold their learning so they will learn to generate equivalent fractions.

Nina's teacher begins by making a **connection**. She reminds students that they have been working with fractions. Then, the teacher shares the **teaching point**: they will be learning how to find equivalent fractions. The teacher points to the vocabulary card posted behind the small-group table that has the words *equivalent fractions* on it, along with the definition. Students read it aloud.

Moving into the **active engagement** section of the lesson, the teacher uses 12 two-color counters, turning four of them so the red side is up and the other eight so the yellow side is up. Initially, she puts out the four red counters.

Thinking aloud, the teacher says, *"Let's see—I have 4 red counters. I wonder what fractional part of all my 12 counters that is? How can I answer that question? To begin, I know I have 12 counters, so each of them is* $\frac{1}{12}$*. Since four of them are red,* $\frac{4}{12}$ *are red. That means* $\frac{8}{12}$ *are yellow. I didn't even have to count them because I already know that 4 and 8 added together equals 12! I wonder if* $\frac{4}{12}$ *is equivalent to another fraction. Maybe I can separate the counters into equal groups. What if I start with the 4 I have? The reds make one group of 4. Can I make groups of 4 with the other counters?"*

The teacher groups the yellow counters into groups of four.

*"Well, I have 2 groups of 4 yellow counters. Altogether, there are 3 groups now. If I want to use the groups to find the fractional parts, one of the 3 groups is red or* $\frac{1}{3}$ *of the counters are red. And, 2 of the 3 groups are yellow—so* $\frac{2}{3}$ *of the counters are yellow. I still have 12 counters. I still have 4 reds and 8 yellows. In the first instance, the reds make up* $\frac{4}{12}$*. Then, when in groups of 4, they make up* $\frac{1}{3}$*. That means* $\frac{4}{12}$ *and* $\frac{1}{3}$ *must be equivalent fractions!"*

The teacher monitors the expressions of students to be sure they are following her reasoning before continuing.

*"Can you help me now? I found that $\frac{4}{12}$ and $\frac{1}{3}$ are equivalent. Do you remember what fractional part of the whole 8 was? Turn and talk to a partner to share your thinking."*

The teacher listens carefully as students express their ideas.

*"As I listened to you talk, I heard you all say that 8 yellow counters make up $\frac{8}{12}$ of the counters. Look at the groups of 4. Using those groups, can you name another fraction that is equivalent to $\frac{8}{12}$? Turn and talk to your partner again and explain your thinking."*

As they worked in pairs, all students respond with the correct answer, $\frac{2}{3}$, and were able to explain how they knew.

*"Can you think of another way we could divide the 12 counters into equal groups?"*

One of the students suggests groups of 5. They try it and find it does not work because some are left over. Another student suggests groups of 2. They divide the counters into groups of 2 successfully.

*"How many groups are there now?"*

Students count and answer *6*.

*"Can you name another fraction that is equivalent to $\frac{4}{12}$ and $\frac{1}{3}$?"*

One student volunteers the answer $\frac{2}{6}$.

*"How did you know that fraction is equivalent to $\frac{1}{3}$ and $\frac{4}{12}$?"*

The student correctly answers that there are still 12 counters and 4 of them are still red. Even though they are grouped differently, the fractions are equivalent if they are representations for the same amount or quantity. The teacher asks the other students to restate that in their own words.

Then, the teacher gives each pair of students 16 linking cubes, 4 of which are red and 12 of which are blue, and asks students to tell what fractional part the blues represent. The students do this easily. The teacher makes note of this on the recording sheet.

Next, the teacher asks the students to name a fraction that is equivalent to $\frac{12}{16}$. Immediately, two groups of students begin to divide their cubes into equal groups.

A third pair of students seems stuck and doesn't know how to proceed. The teacher asks them to describe the previous demonstration with the counters. As they recall the process, one of them suddenly thinks of a strategy for finding an equivalent fraction. He shares it with his partner. "Let's divide them into groups of 2." The partner responds, "Why groups of 2?" "I chose groups of 2 because I think the counters will divide evenly into groups of 2."

They begin working. The teacher makes notes about these two students to remind herself that they weren't quite ready to work independently.

One pair of students very quickly divides the cubes into 4 groups of 4. As the teacher listens to the pair's discussion, they name $\frac{3}{4}$ as an equivalent fraction to $\frac{12}{16}$. The teacher asks that pair to find another equivalent fraction and then to record in their math journals what they learned.

A second pair of students tries to divide the cubes into groups of 8. They divide them successfully, but then struggle with what to do since one group was composed of both red and blue cubes. The teacher listens for a few minutes as the students discuss their dilemma. When they seem stuck, the teacher asks them to think about the work with the counters.

*"What did you notice about the groups? How did the groups with the counters differ from your groups of cubes?"*

After thinking for a few minutes, one of the students answers, "The groups of counters were either all red or all yellow."

After waiting for a few minutes to give the pair a chance to think of what its next step would be, the teacher asks, *"What do you think you could do to find the equivalent fraction?"*

One of them says, "Oh, I guess the groups have to be all one color or the other, right?" They go on to create groups of 4 in which the groups contain only one color, and they are able to name an equivalent fraction correctly. They record in their math journal what they did to find equivalent fractions.

The group who struggled at first finished creating groups of 2 where each group was composed of only a single color. They surprised the teacher by being able to name the equivalent fraction $\frac{6}{8}$. They were also asked to record their work in their math journals.

When students finish writing, the teacher asks Nina to share what they had done during the lesson. She begins to explain but is more focused on the grouping than on making the connection to finding an equivalent fraction. Another of the students begins to explain his understanding by verbally going through the process from the beginning again. The teacher brings their attention back to the definition of an equivalent fraction. Eventually, one student is able to explain why she knew the fractions were equivalent—because they are different names for the same amount.

To **link** the lesson to the mathematical work of students, the teacher reminds them that they worked like mathematicians today using what they already know to understand how to find equivalent fractions and then asks them to reflect on the lesson. Each student shares something mathematical from the lesson that they learned or thought about.

The teacher is pleased that they made some progress with this lesson, but she feels that the group needs a little more experience creating equivalent fractions with concrete objects. The teacher decides to provide a variety of manipulatives in the next lesson and allow students to choose which they will use. The teacher will observe what they do if they choose more than two different color objects with which to work. The teacher places anecdotal notes in each student's file. Upon reflecting on the lesson, the teacher decides that these students still seemed to have similar instructional needs, so no changes in grouping will be necessary before the next lesson.

## Chapter Snapshot

Teaching students in small groups can be enormously beneficial for both teachers and students. The intimate setting allows teachers to target students' instructional needs and is conducive to exploration, conversation, and discovery. When teachers meet with small groups, students appreciate the attention they receive and clamor to be included. Teachers learn about their students—how they think, how they express themselves, how they work together, and how they learn. Knowing this, teachers can effectively plan and scaffold the learning of their students. Teaching and learning go hand in hand.

## Review and Reflect

1. How does using flexible, needs-based grouping affect student learning? How can it affect teaching strategies employed by teachers?

2. Do you use Guided Reading in your classroom? If so, how can you adapt it to accommodate mathematics instruction? What about it is easily modified for teaching mathematics? What may be more difficult to adapt?

3. What data do you have that can guide you as you create small groups of students for Guided Math instruction?

***Workstation tasks provide students with ongoing practice of previously mastered concepts and skills, promote their computational fluency, and encourage mathematical curiosity and inquiry.***

# Chapter 6

## Supporting Guided Math Workshop

In sharp contrast to teacher-directed instruction, the math workshop format shifts much of the responsibility for learning to students. Math workshop has its origins in the math centers of the 1970s. Students in primary grades explored math concepts using familiar objects to build a strong understanding of the patterns of mathematics. Students learned at centers with tasks designed to give them ample opportunities to explore a wide range of math concepts. Teachers relied on *Mathematics Their Way*, created and written by Mary Baratta-Lorton (1976), for guidance in planning the center activities that built sequentially throughout the year.

Giving students foundational experiences working with concrete objects eased their transitions to symbolic and abstract representation. As Baratta-Lorton reminded teachers, "Abstract symbolization is only used to label a concept which the child already grasps, never as a 'material' from which a child is taught a concept" (1976, xiv). As teachers recognized the truth of her statement, the use of math centers in primary grades proliferated. When students grew older, however, their instruction most often became more traditional and more abstract. While some children were able to make the transition to more abstract thought with success, others struggled with little opportunity to return to the concrete or representational (pictorial) to strengthen their understandings.

Coincidental with the increased use of math centers was the implementation of writing workshops in many classrooms (Calkins 1994; Fletcher and Portalupi 2001). These workshops included mini lessons and time for students to write independently as teachers conferred with individual writers. Writing workshops motivated students to begin taking more responsibility for their writing. It was not long before innovative teachers began using a similar workshop structure for reading instruction. Soon, teachers realized that by teaching students to work independently, it allowed them opportunities to work with small groups of students and also taught students how to work with minimal supervision, which is an important life skill.

With the adoption of rigorous math standards in states across the country, there has been a reemergence of interest in instructional approaches that both deepen students' understanding and also bolster the ability of students to work independently. Teachers regularly using writing and reading workshops in their classrooms naturally turned to the workshop model to enhance their mathematics instruction. Now, this approach is no longer limited to the primary grades. Instead of math centers, students work in math workstations. This shift in terminology is clearly intended to emphasize the expectation that students are engaged in mathematical *work.* Teachers in elementary, middle, and high schools are implementing math workshop to enable them to conduct small-group lessons with their students.

## Math Workshop in the Classroom

Math workshop is a versatile structure that accommodates a vast array of learning activities. Its flexibility is one of its greatest advantages. Students work individually, in pairs, or other groupings at workstations depending upon the task or tasks to be completed. Workstation tasks provide students with ongoing practice of previously mastered concepts and skills, promote their computational fluency, and encourage mathematical curiosity and inquiry. Careful selection of tasks by teachers ensures that students are able to complete them accurately and independently. Students are challenged, but not to the point of unproductive frustration. Before beginning to work independently in math workshop, it is essential that students learn and practice the routines and procedures to make this component of Guided Math function smoothly. The increased responsibility that students assume for their work makes it possible for teachers to extend their teaching roles beyond whole-class instruction. See Figure 6.1 for a description of the roles of students and teachers during math workshop.

**Figure 6.1—Student and Teacher Roles During Math Workshop**

| Students | Teachers |
|---|---|
| • assume responsibility for learning and behavior<br>• function as mathematicians<br>• communicate mathematically with peers<br>• increase ability to work cooperatively with peers<br>• review and practice previously mastered concepts and skills<br>• improve computational fluency | • teach small-group lessons<br>• conduct math conferences<br>• informally assess learning through observations<br>• facilitate mathematical learning through questioning |

*(Adapted from Sammons and Boucher 2017a, 11.)*

## The Roles of Students in Math Workshop

Student engagement in math workshop leads them to assume greater responsibility for their own learning and behavior as they work independently with few teacher reminders to keep them on task. Over time, their endurance builds, and they are able to sustain independent work for longer periods of time. Students understand that they are accountable for their work. They develop essential life skills. These skills are precisely the kinds of skills that employers seek but are not explicit in curriculum standards.

Working independently, students are compelled to think like mathematicians, relying on their own mathematical knowledge, skills, and judgment. They discuss their thinking with others, approach problem solving strategically, and call upon their mathematical knowledge for application in new contexts. Students are expected to work collaboratively at math workstations, sharing responsibilities with others.

Furthermore, the workstation tasks themselves address important areas of learning. Tasks often provide valuable opportunities for students to revisit concepts and skills they have learned previously. What Marzano, Pickering, and Pollock (2001) describe as distributed practice, helps students gain a deeper understanding of mathematical content and leads to better retention. Other tasks afford students practice to improve their computational proficiency.

### The Role of Teachers in Math Workshop

Aside from the intrinsic benefits for students, math workshop is the component of Guided Math that makes small-group lessons possible. Because small-group lessons are the essence of the Guided Math framework, a well-managed math workshop is crucial. Without it, most teachers would forsake small-group instruction altogether. Even when teachers are not teaching small-group lessons, math workshop is a valuable time for informally assessing students through math conferences or by observing and listening to them work. What teachers learn about students' strengths and needs guides future instruction, making the instruction much more effective. Furthermore, when conversations occur between teachers and students who are working on math workstation tasks, the talk can spark students' curiosity. Teachers' interest in their students' thoughts about the discipline encourages learners to think more deeply and critically about their mathematical work.

## Implementing Math Workshop

Planning an effective math workshop is a complex endeavor for teachers. Because many workshop model options are available, choosing one may be difficult when teachers are getting started. Similar to dining at a fine restaurant where the entrees on the menu are all enticing, each with its own unique appeal, math workshop options are many and varied. Choices must be made. For teachers beginning to implement math workshop, it is comforting to know their decision is never final. The most effective Guided Math teachers always strive to improve their instruction. If something is not working quite the way they hoped, these teachers don't hesitate to revise, adapt, or try something else.

## Selecting a Math Workshop Model

One of the first decisions a teacher has to make when planning to implement Guided Math is how math workshop will operate. In evaluating workshop models described in this chapter, the following factors are important to consider:

- *How much student movement does each model require*? Using a rotational model obviously involves considerably more student movement than the GUIDE model. Classroom sizes and teacher preferences vary and will influence teachers' choices for the amount of student movement.
- *What flexibility does each model offer teachers for conducting small-group lessons?* Teachers who desire a great deal of flexibility will want to avoid a rotational model where small-group lessons with the teacher is one of the stations.
- *What is the likely student noise level for each model?* Students working in pairs tend to work more quietly than larger groups of students do. Models in which students choose where they will work and are able to change stations at will may also have greater noise levels.
- *How much workstation task planning is required for each model*? If students work in each station each day, the tasks must be changed more frequently than in a model where students work in a station only once a week.

Additionally, the following factors are important for teachers in choosing a model:

- Will staff or volunteer support be available in the classroom during math workshop?
- How much classroom space is available for workstations and storage of materials?

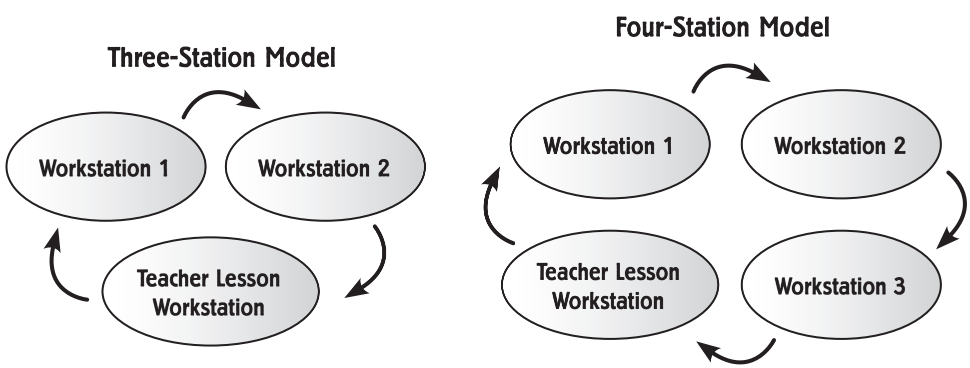
- How does each model align with the teacher's preferred classroom management style?
- What technology is available for student use during math workshop?

## Rotational Models

The rotational model is probably the most commonly used model, although not necessarily the optimal model for math workshop. Most often, students rotate between three or four workstations on a regular schedule. They will typically work in each workstation during one class period.

A teacher station is often included in the rotation. When a teacher station is part of the rotation, students work in homogeneous groups. Because the small-group lesson is incorporated into the rotation and lessons are planned to address specific students' learning, the composition of the groups changes from day to day as students' learning needs evolve. Figure 6.2 shows examples of the rotational model with a teacher station included in the rotation.

**Figure 6.2—Rotational Models with Teacher Lesson Workstation**




To facilitate the smooth transition of students with this model, a management board similar to those in Figures 6.3 and 6.4 (page 243) should be displayed.

Figure 6.3—Sample Management Board (with Teacher Lesson Workstation)

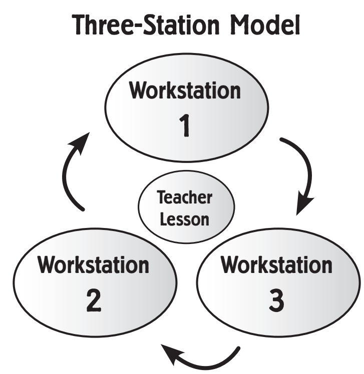
| Red Team | Blue Team | Yellow Team |
|---|---|---|
| Tom, Carlos, Jasmine, Min, Sadie, and Assad | Finn, Sandra, Fatima, Maria, Joseph, and Lois | Sara, Pedro, Mimi, Matt, Krista, and Ravi |
| Teacher Lesson Workstation | Workstation 2 | Workstation 1 |
| Workstation 1 | Teacher Lesson Workstation | Workstation 2 |
| Workstation 2 | Workstation 1 | Teacher Lesson Workstation |

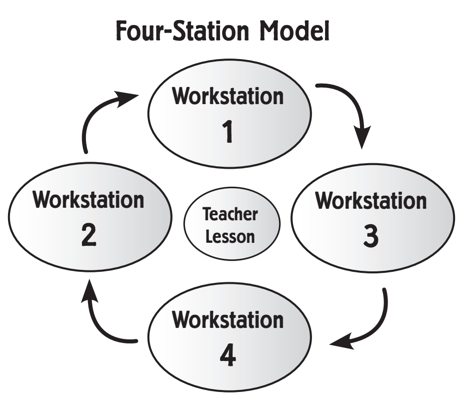
Figure 6.4—Sample Management Board (with Teacher Lesson Workstation)

| Red Team | Blue Team | Yellow Team | Green Team |
|---|---|---|---|
| Tom, Carlos, Jasmine, Min, Sadie, and Assad | Finn, Sandra, Fatima, Maria, Joseph, and Lois | Sara, Pedro, Mimi, Matt, Krista, and Ravi | Ash, Dempsey, Griff, Alice, Yoshi, and Carmen |
| Workstation 1 | Teacher Lesson Workstation | Workstation 3 | Workstation 2 |
| Workstation 2 | Workstation 1 | Teacher Lesson Workstation | Workstation 3 |
| Workstation 3 | Workstation 2 | Workstation 1 | Teacher Lesson Workstation |
| Teacher Lesson Workstation | Workstation 3 | Workstation 2 | Workstation 1 |

However, some teachers prefer to teach small-group lessons that are separate from the rotation. Instead, they pull students as needed from their workstations for lessons while the remaining students continue to rotate from station to station. With this model, the rotating group may be heterogeneous because the teacher is pulling specific students for each small-group lesson based on their needs. The rotating groups can be composed of a mixture of students with varying strengths and needs. Figure 6.5 shows examples of the rotational model when the teacher lesson is not part of the rotation.

**Figure 6.5—Rotational Models without a Teacher Lesson**




A sample management board for rotations that do not include a teacher lesson workstation is shown in Figure 6.6 (page 245).

**Figure 6.6—Sample Management Board (without Teacher Lesson Workstation)**

| Red Team | Blue Team | Yellow Team | Green Team |
|---|---|---|---|
| Tom, Carlos, Jasmine, Min, Sadie, and Assad | Finn, Sandra, Fatima, Maria, Joseph, and Lois | Sara, Pedro, Mimi, Matt, Krista, and Ravi | Ash, Dempsey, Griff, Alice, Yoshi, and Carmen |
| Workstation 1 | Workstation 4 | Workstation 3 | Workstation 2 |
| Workstation 2 | Workstation 1 | Workstation 4 | Workstation 3 |
| Workstation 3 | Workstation 2 | Workstation 1 | Workstation 4 |
| Workstation 4 | Workstation 3 | Workstation 2 | Workstation 1 |

Some teachers choose a rotational model where students work and rotate from workstation to workstation in pairs. Teachers call students to work with them when needed for lessons. Because students work in pairs, it is important that the pairs of students who work together are in the same small group. This will avoid the scenario where one student is left without a partner while the other is working with the teacher.

When students are grouped in pairs, it often works well to have only two rotations during the class period. Workstations in this model contain numerous tasks to engage students. The sample management board in Figure 6.7 (page 246) is for students working in pairs where there are five workstations. In this model, pairs of students only work in two workstations during a class period.

**Figure 6.7—Sample Management Board (with Student Pairs)**

| Students | | First Workstation | Second Workstation |
|---|---|---|---|
| Anuska | Michael | 1 | 2 |
| Tito | Meg | 2 | 3 |
| Chika | Juan | 3 | 4 |
| Skyler | Marta | 4 | 5 |
| Dylan | Shiro | 5 | 1 |

## Workstations with Student Choice

Teachers sometimes implement a workshop model that allows students to choose their workstations. A well-organized management system is especially important with these models to ensure that students understand the options and know where they will be working.

With student choice, a checklist or something similar should be created to help students keep track of workstations already completed. This helps students develop responsibility for completing each station. An example of a workstation checklist is shown in Figure 6.8.

**Figure 6.8—Workstation Checklist**

Name:________________________ Date:___________

**Math Workstations Checklist**

**Directions:** Complete at least five workstations during the week. Draw an *X* next to each workstation when you have completed the task.

☐ Workstation 1: Math Literature

☐ Workstation 2: Computational Fluency

☐ Workstation 3: Problem Solving

☐ Workstation 4: Place Value

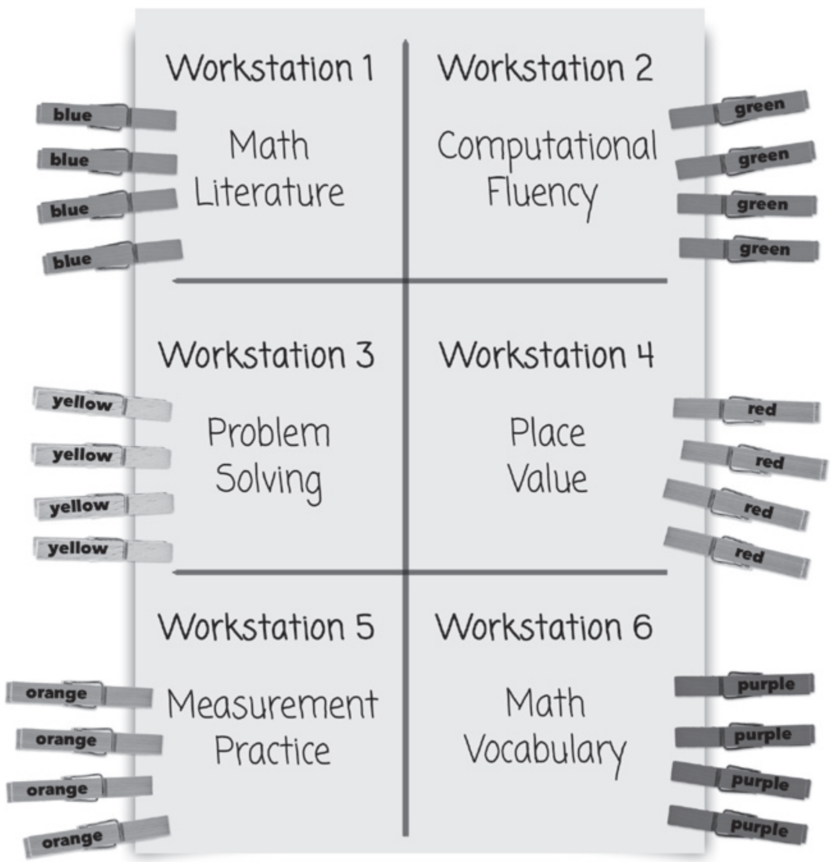
☐ Workstation 5: Measurement Practice

☐ Workstation 6: Math Vocabulary

With student choice, management boards can also regulate the number of students working at each workstation. See the examples (Figures 6.9, 6.10, and 6.11) that use colored clothespins, gingerbread men with Velcro dots, or pocket charts to indicate when a workstation is full.

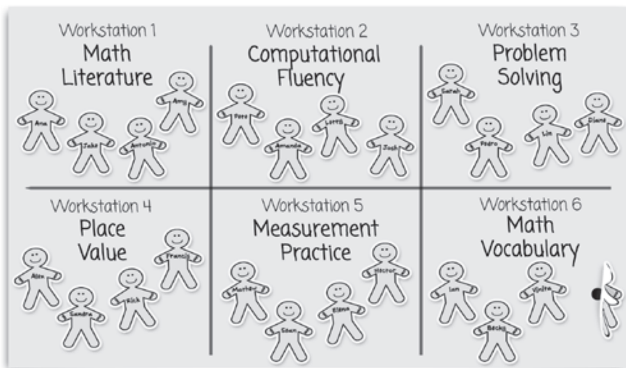
The management board in Figure 6.9 indicates the number of students a station can accommodate with color-coded clothespins. Each student takes a clothespin from the station where they will work and attaches it to their clothing. Students know the station is full when there are no more clothespins.

**Figure 6.9—Workstation Management Board (for Student Choice with Clothespins)**



Another way to let students know whether a workstation is full is with the use of laminated figures. Each student has their own figure backed with a Velcro dot. The number of Velcro dots on the board at each station indicates how many students may work there. Students add their figures to the station where they are working. If there are no dots available, student know they will have to choose another station. See Figure 6.10 for an example.

**Figure 6.10—Workstation Management Board (for Student Choice with Velcro Dots)**




Pocket charts can also be used effectively when multiple workstations are offered and students choose where to work. In Figure 6.11, each workstation label specifies how many students may work there. Students place their name cards beside the stations in which they are working.

**Figure 6.11—Workstation Management Board (for Student Choice with Pocket Chart)**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Workstation 1 | Math Literature (6 max.) | Jasmine | Ennis | Jiro | Ashia | Tonya | Raj |
| Workstation 2 | Computational Fluency (4 max.) | Patricio | Christy | Megan | Jamar | | |
| Workstation 3 | Problem Solving (4 max.) | Keke | Hua | Finn | Amanda | | |
| Workstation 4 | Place Value (4 max.) | Sadie | Scott | Jibri | Thomas | | |
| Workstation 5 | Measurement (4 max.) | Jacob | Kayla | Mason | Miguel | | |
| Workstation 6 | Math Vocabulary (5 max.) | Emanuel | Allie | Vincent | Brittany | Omar | |

## The GUIDE Workshop Model

The GUIDE workshop model offers teachers the greatest instructional flexibility and ease of implementation. With this model, there are five workstations. Each includes a number of tasks. The tasks may be required, optional, or a combination of the two. Students work in only one workstation each day. There is no rotation or movement of students from one station to another with this model. By the end of a week, students will have worked in all five workstations.

The GUIDE model includes the following five workstations:

**Games for Mathematicians:**
Games to help students maintain previously mastered concepts and skills which improve computational fluency

**Using What We Know:**
Problems to be solved, other challenge tasks, or mathematical investigations that require students to draw upon their existing mathematical knowledge and skills

**Independent Math Work:**
Materials and problems used to teach previously mastered content, providing ongoing review and practice

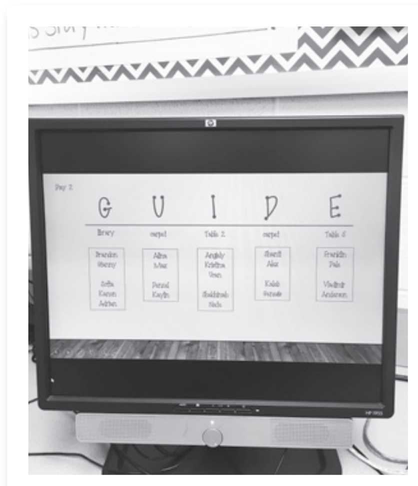
**Developing Fluency:**
Tasks or games which help students develop number sense, computational strategies, and mental math skills

**Expressing Mathematical Ideas:**
Math journals, mathematical vocabulary reinforcement, and other tasks requiring students to communicate mathematically

It should be noted that the tasks in all the stations focus on providing ongoing practice and review of previously mastered concepts and skills, mathematical investigations, or the development of students' computational proficiency.

Using this model, teachers pull students from workstations for small-group lessons. The composition of the groups for lessons can vary based on the lesson content and students' needs. In addition, the length of each small-group lesson can also be flexible and determined by need because lessons are not synchronized with a rotational schedule. Small-group lessons may address the current content and skills being learned or provide remediation for students who have gaps in their background knowledge. Because teachers pull small homogeneous groups for lessons, the groups working in GUIDE workstations may be heterogeneous. This encourages students of varying mathematical strengths to interact and work collaboratively during math workshop.

Teachers should rely on their knowledge of their students when deciding how to group them for GUIDE workstations. Generally, teacher-assigned groups tend to be more effective, but some teachers prefer to allow student choice. For teacher-assigned grouping, it is best to create five evenly balanced groups whose students work well together. An ideal group would include some students who are natural leaders, some who follow directions well, combined with some who prefer to follow the lead of others or who may benefit from peer support as they work.




## Workshop Model Comparison

Teachers can use the chart in Figure 6.12 to compare the models described throughout this chapter. Teachers throughout the years have used all the models successfully to implement Guided Math.

**Figure 6.12—Comparison of Math Workshop Models**

| | Rotation Model with Teacher Lesson Station | Rotation Model without Teacher Lesson Station | Multiple Workstations with Student Choice | GUIDE |
|---|---|---|---|---|
| **Frequency of Models** | each station every day | each station every day | students choose workstations (some may be required) | each station once a week or every five days |
| **Number of Workstations** | three to four | three to four | multiple | five |
| **Task Time Required** | 15–20 minutes | 15–20 minutes | times vary | entire math workshop block |
| **Task Type** | compulsory and/ or choice tasks | compulsory and/ or choice tasks | choice tasks, some may be compulsory | compulsory and/ or choice tasks |
| **Frequency of Task Changes** | daily or weekly | daily or weekly | usually weekly, but at teacher's discretion | only when appropriate |
| **Student Rotation** | between stations at set times | between stations at set times | by choice | remain in one workstation |
| **Grouping of Students** | homogeneous | heterogeneous | heterogeneous | heterogeneous |
| **Small-Group Lessons** | teacher leads with each group rotating daily | teacher leads with homogeneous groups | teacher leads with homogeneous groups | teacher leads with homogeneous groups |
| **Lesson Length** | same amount of time for each small group | vary according to need | vary according to need | vary according to need |

## Deciding How Much Choice Students Will Have

With reading and writing instruction, teachers have learned the value of giving students some choice in what they write and what they read. When students are given choices, they take more ownership of their learning experiences. The math workshop format offers opportunities for teachers to provide task choices for their students—however, choice is only an option, not a characteristic of math workshop. Giving students choices during math workshop is one way to differentiate instruction. Students may make their choices based on their interests, their learning styles, or even their levels of proficiency.

Allowing students some degree of choice has a positive effect on their general well-being, their behavior and values, and their academic achievement (Kohn 1993). How much student choice is optimal, however, has never been determined. Some teachers are more comfortable offering their students instructional options than others. With math workshop, teachers decide how much choice to offer students. Here are some questions to think about when determining how much choice to give students:

- In which math workstation will students work? Are workstations assigned? Or can students choose where to work?
- With whom will students work? Will students be assigned to work individually or with other students? Or do students choose with whom they work?
- Which tasks will be completed? Are all tasks mandatory? Or are some tasks optional?
- What is the method for which a task is completed (e.g., journal entry, creation of a *PowerPoint*® presentation, explanation of work documented on a tablet, physical model)?
- Which materials are used to complete a task (e.g., base ten blocks, virtual manipulatives, diagrams)?
- May a student visit more than one math workstation during a given time period? Can students move from one workstation to another if they complete the work? May a student move if they would rather work at another workstation?

In making decisions regarding student choice, teachers should consider their classroom management style, their students' work habits, and their vision for an effective math workshop. If students are offered choices, teaching them how to responsibly manage making choices must be a part of teaching math workshop routines and procedures. Teachers who are unsure about how much choice to allow students can try it out by offering students a choice to see how well it works. If students respond well, the kinds of choices offered can be expanded.

## Deciding If Students Work Individually, in Pairs, or in Groups

In making this determination, it is important to recognize the fact that the larger the group of students working together, the greater the noise level could be, and the greater the chance that they will drift off task. Yet, that is only one factor to consider when making this decision.

Learning is a social process. While students working individually may be quieter, students need opportunities to learn how to collaborate with their peers, share their mathematical thinking with others, and learn from their fellow mathematicians (Vygotsky 1978). Moreover, mathematics learning increases as students work collaboratively, tackling new math concepts and skills (Van de Walle and Lovin 2006). When students work together, in pairs or in small groups, the classroom truly becomes a mathematical learning community with students sharing ideas and talking together about math. The reflective conversations that result lead young mathematicians to fully engage with the ideas of others by constructing hypotheses, considering strategies, and understanding concepts (Nichols 2006).

When deciding how students will work during math workshop, teachers may choose to offer students opportunities to work in more than one of these settings. Even when *groups* rotate to workstations together, teachers may choose to have them complete tasks *individually* or in *pairs*. Although there are some advantages to having students engage individually in independent work at times, it is advisable to provide ample opportunities for students to work closely with other learners—either in pairs or in small groups.

# The Nuts and Bolts of Organizing Math Workshop

Once teachers determine what model they will use for math workshop, it is time to organize the classroom and workshop materials to create an environment conducive to independent learning. Teachers have to make decisions about room arrangement, workstation storage containers, and workstation locations.

## Arranging the Classroom

Regardless of which workshop model is implemented, the classroom arrangement shares several characteristics. It is quite unlike that of a traditional classroom with desks in rows facing the front of the classroom. Instead, for math workshop, areas for students to work and talk about math together are essential—so tables or desks arranged in groups are the norm.

For small-group lessons, teachers need a table or area in which a group of students can comfortably gather with their math materials. The small-group lesson area should also allow the teacher a clear view of the rest of the classroom so students who are working independently can be monitored. The small-group area should be away from areas in which the work of other students may interrupt the lessons.

In planning the room arrangement, consideration should also be given to how students access workstation materials. The goal is making them easily accessible to students with as little commotion as possible. See Figure 6.13 (page 255) for strategies for planning a classroom layout for math workshop as well as Figures 6.14 (page 255) and 6.15 (page 256) for sample classroom arrangements for GUIDE.

**Figure 6.13—Strategies for Arranging a Classroom for Math Workshop**

- Create areas where student groups of various sizes can work comfortably.
- Design work spaces to accommodate different kinds of activities, (e.g., games, quiet individual work, use of technology, small-group lessons).
- Take into consideration the fact that some workstations noise levels will vary by providing some spaces specifically for quiet work and some for noisier tasks.
- Plan for efficient traffic flow during transitions.
- Use classroom furnishings creatively for multiple purposes, (e.g., a file cabinet as a room divider with a dry erase board hung on its side for student use, a bulletin board as an interactive game board).
- Place workstation materials so they are readily accessible to students.
- Make the classroom space physically inviting and comfortable for mathematical collaboration among students.
- Realize that room arrangements can be changed whenever a better idea occurs to a teacher.

**Figure 6.14—Sample Elementary Room Arrangement for GUIDE**

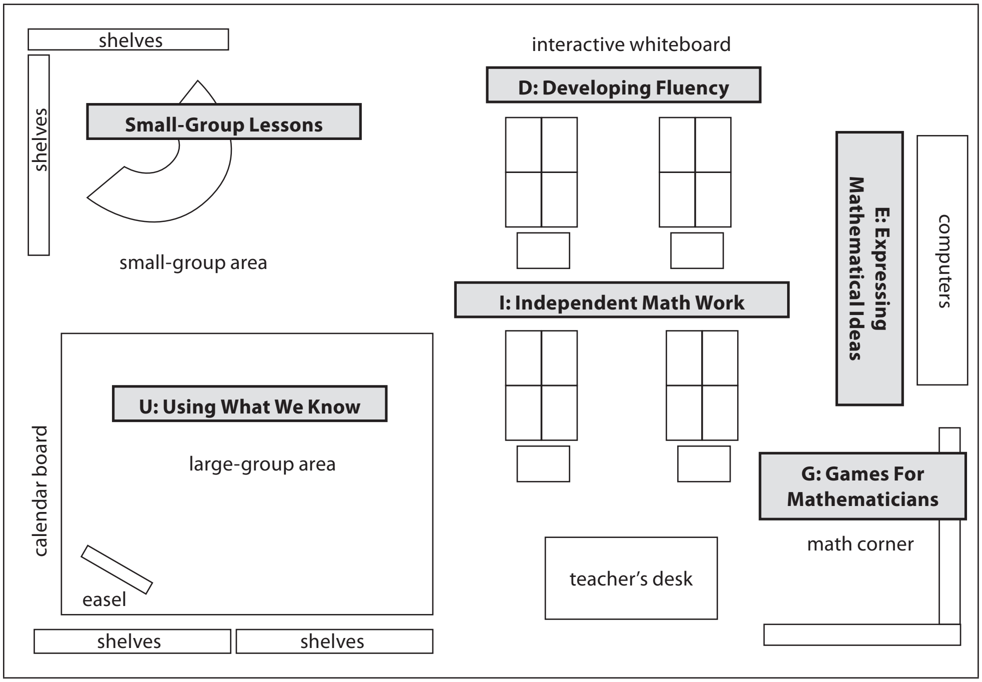
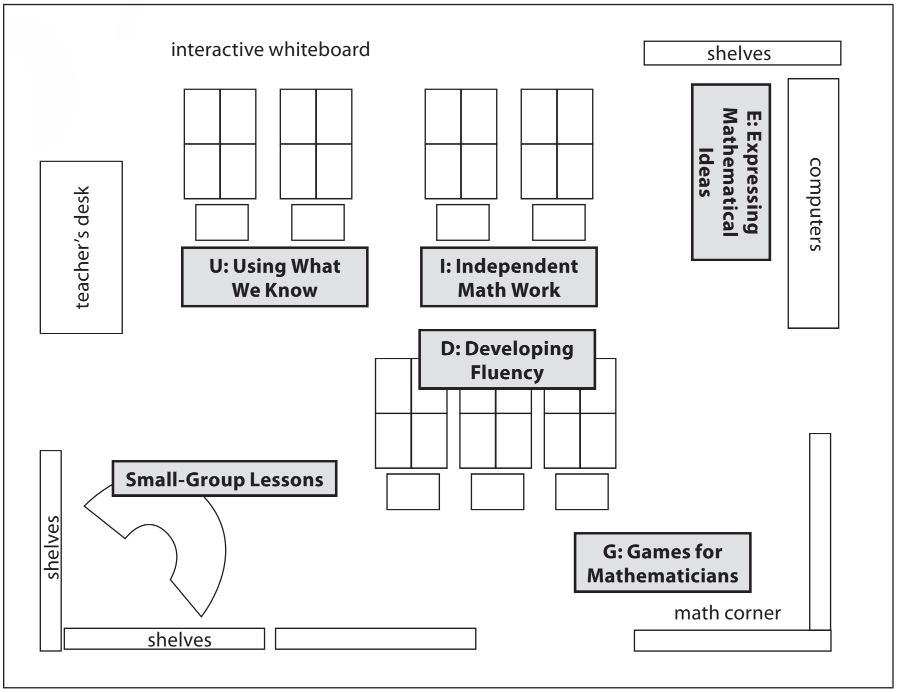


Figure 6.15—Sample Secondary Room Arrangement for GUIDE




## Organizing Math Workstations

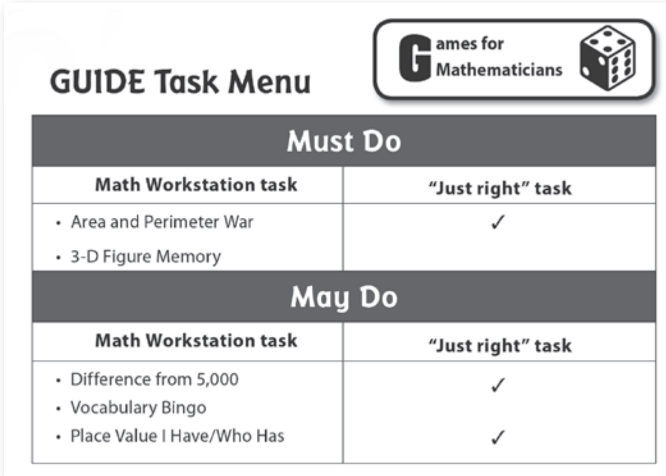
Most teachers find it convenient to use the clear plastic storage boxes that can be found at many big box stores to organize math materials. Because they are clear, it is easy to see what is inside without having to open them. They are also usually designed so that they are stackable—a big plus in most classrooms. But, many other containers can also be used for workstation storage. Sometimes teachers use baskets, plastic totes, sets of drawers on wheels, plastic crates, or even sets of cardboard boxes. In choosing the containers for workstation storage, it is helpful for teachers to also consider where they will be housed in the classroom. To some extent, the choice of a container may be dependent on the available space in the classroom where the stations are to be stored.

When the kind of containers to be used and an area for the storage of the stations have both been determined, it is important to label them. Labeling the containers and the place where each workstation container is stored increases the likelihood that students will replace them correctly when they clean up after math workshop. Teachers should teach students how to efficiently retrieve workstations as math workshop begins and then how to clean up their work areas and return the workstation containers to their proper places.


***This plastic workstation container is labeled with* U *for Using What We Know.***

The workstation containers hold the student tasks for math workshop. It is helpful to attach a menu of workstation tasks (see Figure 6.16 on page 258) for the workstation to the inside of the lid of the container, if possible. If the container does not have a lid, the menu can be laminated or placed in a plastic sleeve and included with the tasks. The workstation menu should not only list the tasks for the station, but should also indicate whether each task is required and whether there are differentiation options for it. For students who may have difficulty reading the menu of tasks, teachers may equip stations with recordings of the menus and task directions on digital devices.

Figure 6.16—Sample Elementary GUIDE Workstation Task Menu


GUIDE Task Menu

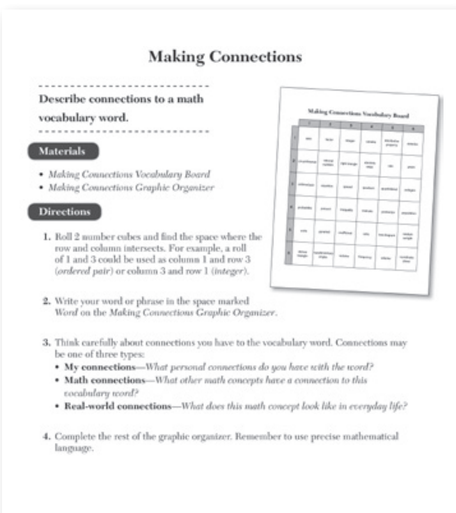
Games for Mathematicians

| Must Do | |
|---|---|
| Math Workstation task | "Just right" task |
| • Area and Perimeter War | ✓ |
| • 3-D Figure Memory | |

| May Do | |
|---|---|
| Math Workstation task | "Just right" task |
| • Difference from 5,000 | ✓ |
| • Vocabulary Bingo | |
| • Place Value I Have/Who Has | ✓ |

Each workstation task should have the directions for student reference, a list of materials, the physical materials required for the task, and a Talking Points card.

Figure 6.17—Sample Secondary Student Directions for a Workstation Task


Making Connections

Describe connections to a math vocabulary word.

Materials

- *Making Connections Vocabulary Board*
- *Making Connections Graphic Organizer*

Directions

1. Roll 2 number cubes and find the space where the row and column intersects. For example, a roll of 1 and 3 could be used as column 1 and row 3 (*ordered pair*) or column 3 and row 1 (*integer*).
2. Write your word or phrase in the space marked *Word* on the *Making Connections Graphic Organizer*.
3. Think carefully about connections you have to the vocabulary word. Connections may be one of three types:
   - **My connections**—*What personal connections do you have with the word?*
   - **Math connections**—*What other math concepts have a connection to this vocabulary word?*
   - **Real-world connections**—*What does this math concept look like in everyday life?*
4. Complete the rest of the graphic organizer. Remember to use precise mathematical language.

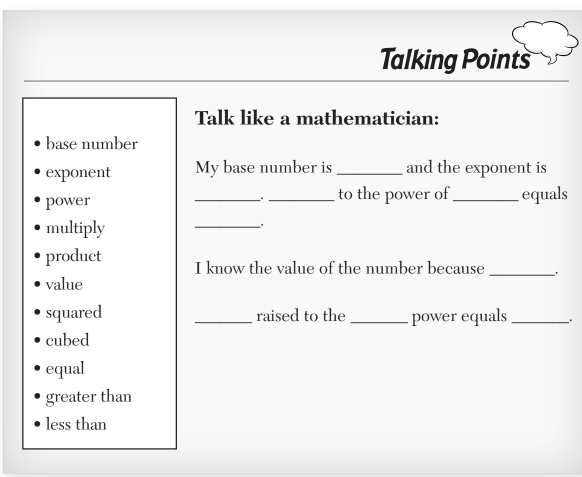
*Student directions* (Figure 6.17) should be clearly stated in student-friendly language. Although tasks should always be introduced to students prior to their being included in a workstation, having the directions for students to refer to when needed makes math workshop operate much more efficiently. When students are unsure of what to do, they can simply refer to the directions.

*The list of materials* is useful during cleanup so that students can be sure that all materials are packed up and stowed when they are finished with the task. The list of materials is incorporated into the student workstation task directions.

*The materials for the task* should be included in the container, if possible. There are times, however, when the materials required are too large for the container or are readily available to students elsewhere (paper, pencils, etc.). If so, that should be clearly indicated on the list of materials with directions for where they can be accessed.

*Talking Points cards* support students as they discuss their mathematical work with others. Each card contains the important mathematical vocabulary terms related to a task. It may also include nonlinguistic representations of the terms and sentence starters to guide students in their initial efforts at math conversations. Of course, just providing Talking Points cards to students without teaching them how to use the cards does little good. Students should understand that they are expected to use the Talking Points cards for every task. Prior to beginning math workshop, students should be taught to use the Talking Points cards through teacher modeling, student role-playing, and student practice. (See Figure 6.18 for an example card.)

**Figure 6.18—Sample Secondary Talking Points Card**



*Talking Points*

- base number
- exponent
- power
- multiply
- product
- value
- squared
- cubed
- equal
- greater than
- less than

**Talk like a mathematician:**

My base number is _______ and the exponent is _______. _______ to the power of _______ equals _______.

I know the value of the number because _______.

______ raised to the ______ power equals ______.

## Managing Math Workshop

Strong classroom management is essential for effective implementation of math workshop.

> Effective management practices springboard student learning and allow teachers to extend their mathematics instruction in a myriad of ways—informal assessment of mathematics proficiency through observation and conversation as students work, the encouragement of deeper student thinking through well-crafted questions, one-on-one conferences with students, and differentiated small-group lessons. (Sammons and Boucher 2017a, 49)

Before students are expected to be responsible for independently managing their time and behavior during the workshop framework, teachers must reflect on exactly how they want their students to behave during math workshop. These behaviors must be explicitly taught. By establishing and teaching guidelines that help the class function independently, teachers help students learn self-management skills.

These self-management skills are the building blocks for individual student achievement and are also the foundation for creating a genuine learning community in which students can work together. Irene Fountas and Gay Su Pinnell (2001, 88) list the following principles upon which a learning community is based:

- All members are trusted with rights and responsibilities.
- All members take responsibility for their own learning and for helping others to learn.
- All members take responsibility for managing their time and activities productively.
- All members learn self-management as part of the curriculum delivered by the teachers.
- All members understand that keeping materials in order helps everyone learn.

These principles reflect a common-sense approach to managing math workshop, and workshop rules or guidelines should promote them. Most teachers would agree with these principles. To have a successful math workshop, however, students must also understand and accept these principles. It is not always easy to know how to teach the principles to students in a meaningful way. Sometimes, it helps teachers to see what other teachers do in their classrooms. Whenever possible, teachers should visit each other's classrooms to see how they are organized for math workshop. It is helpful for teachers to discuss their workshop rules and expectations with each other, especially at the beginning of the school year.

## Routines and Procedures

The responsibility for creating a well-organized math workshop rests squarely on the shoulders of teachers. It is the product of their careful reflection, planning, and preparation. Teachers must create a vision of how their students will work when engaged in independent activities. Students can only work well at math workstations when they understand what "working well" entails. Unless teachers take time to carefully consider exactly how they would like their students to behave during math workshop, there is no way they can convey those expectations to their students. And, if teachers don't explicitly share their behavioral expectations, how are students supposed to know what those expectations are?

Students must learn how to work independently and understand that they are not to interrupt the teacher's work or the work of other students. Math workshop runs smoothly when teachers anticipate common problems that may occur and plan how those problems can be solved by students without interrupting others. Otherwise, math workshop can be chaotic.

Routines and procedures will vary from classroom to classroom, from teacher to teacher. Even for veteran Guided Math teachers, the routines and procedures may change from year to year. It is often helpful to compare ideas for routines and procedures with other teachers, especially for teachers who are new to Guided Math. Teachers should consider exactly how they would like students to behave as math workshop begins each day, in its midst, and as it ends. Effective routines and procedures provide guidance to students in handling the following situations (Sammons 2013):

- transitions (e.g., orderly traffic patterns, signal to change stations if a rotation method is used)
- paper management (e.g., how to turn in completed work, what to do with unfinished papers)
- work area cleanup
- appropriate noise levels
- access to materials that are not in the workstation containers
- emergencies (e.g., what is considered an emergency and what to do if one occurs)
- questions about tasks (e.g., unclear directions, tasks that are too difficult, materials missing from tasks)
- sharing/working with others
- proper use of manipulatives, task materials, and math tools
- options for students if all tasks at workstations are completed
- technology failures
- interpersonal squabbles

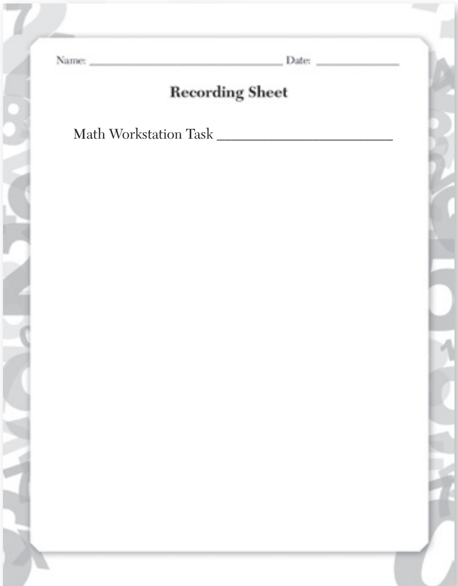
Once a vision for math workshop is formulated, teachers often find it worthwhile to ask students to brainstorm behaviors that positively or adversely affect a collaborative workshop environment. The solicitation of their ideas helps students feel that they are valuable, contributing members of the class community. Teachers, drawing upon their own visions and those of their students, can then guide the class in developing and recording workshop routines and procedures for independent work. An anchor chart with these procedures stated in a simple, positive manner can be posted in the classroom for future reference.

Math workshop runs smoothly when students, by following the guidelines, begin to acquire the self-discipline to monitor their own behavior and work. Discussions about behavioral and academic expectations along with teacher modeling and student role-playing help students visualize what they "look" like. In Chapter 9, a *First Fifteen Day* plan describes how teachers can prepare students to work productively in math workshop. Once students learn the routines and procedures, consistency in enforcing them is crucial. At any time during the year, if students begin to become lax in following these procedures, teachers should not hesitate to revisit them or revise them to fit the evolving needs of the class.

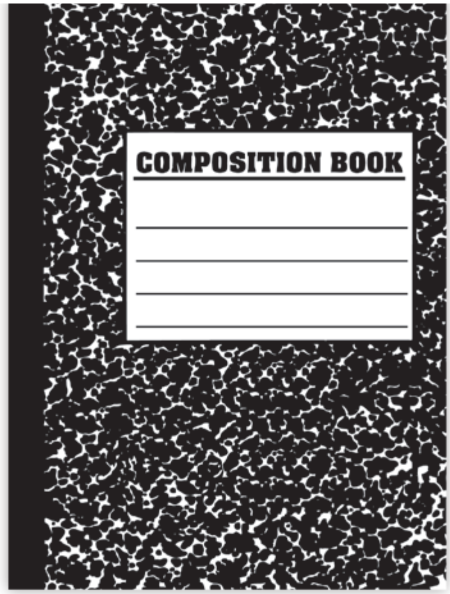
The first few weeks of math workshop may not go as smoothly as teachers would like. In spite of thoughtful preparation and careful teaching of procedures, some bugs may still arise. Teachers should try to anticipate problems that may be predictable and have students practice what they will do if they occur. If unpredictable problems arise, it is often helpful for teachers to step back and take time to observe their own classes. Have students engage in math workshop activities, but rather than working with small groups or conferring with students, teachers can just observe what is happening in their classrooms as students work. Where are problems arising? More often than not, as teachers act as observers, they can recognize patterns in the problems and are able to find deeper solutions (Calkins, Hartman, and White 2005). Sometimes, solutions may be as simple as rearranging the classroom to create easier access to materials or changing the composition of groups working independently.

## Student Accountability During Math Workshop

Even though students are working independently during math workshop, they are still accountable for quality work. In fact, one of the benefits of math workshop is that students learn to assume greater responsibility for their own work and develop better work habits. The most common method of holding students accountable is having students document their work on a recording sheet or in a math journal (see page 264). Even for games, students may record what occurs during each turn. For instance, in a game of Multiplication War, students would document each turn by writing the multiplication problem (based on the number cards they drew) and its product. They can record with an inequality or an equation how their product compared to that of their partner's product.




*Recording Sheet*




*Math Journal*

The written work from math workshop should not require extensive checking by teachers. Usually, when the journals or recording sheets are checked, teachers check quickly to ensure the work was completed accurately and that students completed a sufficient amount of work. The recording by students does not need to be graded and sent home to parents, although a math journal is sometimes shared during parent conferences. It is important, however, that misconceptions, errors, lack of attention, and incomplete work are addressed. Holding students accountable has little value unless there is consistent follow-through by teachers.

Another way in which teachers can encourage accountability is by helping their students "view themselves as mathematicians and recognize their responsibility to learn the craft of mathematicians during math workshop" (Sammons and Boucher 2017a, 52). Creating this kind of learning culture sets the stage for serious academic pursuits by students in which they realize that those "who do the work do the learning" (Hoffer 2012, 52). With such a culture, students' work ethics improve, and they develop the soft skills (e.g., curiosity, persistence, collaboration) that are fundamental to success in the twenty-first century (Claxton, Costa, and Kallick 2016).

Asking students to self-assess can also increase their accountability. Their self-assessments may focus on mathematical work they complete, work behaviors, or on both. The value of student self-assessment on achievement has been well-documented (McTighe and O'Connor 2005; Davies 2000; Black and Wiliam 1998), so it is an especially effective method to ensure student accountability during math workshop.

Of course, self-assessment requires students to reflect on their workshop performance. To prompt student reflection on their behavior and work products, teachers can ask students questions such as these (Sammons and Boucher 2017a, 53):

- What part(s) of your work show evidence that you have met the criteria for quality work?
- What part(s) of your work can be improved? What will you do to improve it?
- How do you rate your understanding of the math you are working on? Why?
- What can you do to extend your learning?
- What are your next steps in learning?

When called upon to not only act as mathematicians, but to also actively reflect on their roles as learners, students become more motivated to be conscientious, independent learners.

## Math Workstation Tasks

Math workstation tasks are the essence of the workshop experience for students. Careful planning ensures that they are at a "just right" level of difficulty for students. This means tasks can be completed independently with accuracy but are not so easy that students are unchallenged (Sammons and Boucher 2017a). Tasks presenting minimal challenge have little educational value and often lead to bored students who stray off task. Yet, tasks that are too difficult are equally problematic. Students frustrated by tasks that are beyond their proficiency levels often interrupt teachers during small-group lessons to ask for help, complete the tasks incorrectly, or become completely disengaged.

Tasks, when well aligned with students' levels of proficiency, reinforce and propel students' mathematical learning (Sammons and Boucher 2017a). Tasks should meet these requirements:

- provide practice, review, or maintenance of previously mastered concepts or skills
- require students to complete work from a previous lesson—provided that students have already demonstrated understanding
- promote computational fluency
- encourage mathematical exploration
- require the completion of a product to represent mathematical thinking

These tasks offer students ongoing distributed practice opportunities. They lead students to deeper understanding and increased ability to apply mathematical knowledge and skills in new contexts, greater proficiency with mathematical computation, and increased retention of the mathematical understanding.

For the sake of efficiency and effectiveness, math workstation tasks should not require a great deal of teacher prep time. Whenever possible, they should be made up of simple materials readily available to most teachers. For example, number cards, dominos, and dice can be used in a multitude of ways to reinforce a wide variety of mathematical content. Games with simple procedures and those for which students already know how to play make the introduction of tasks to students go more smoothly than those with greater complexity. Especially useful are simple games that can be adapted to teach a variety of content. For instance, most students know how to play the card game *War*. This game can be easily adapted to give students practice with a wide range of math content. For example, students might be asked to draw two cards. Then, using their two cards, students have to find their sum, their product, or their value if one card is a base number and the other an exponent. To find who wins each round, students compare their sum, product, or other value to their partners.

Teachers who adopt the GUIDE model for math workshop find that unlike the math centers used in the past, the workstation tasks do not have to be changed daily or even weekly. Because students only work with these tasks once a week, a task may remain in a workstation so long as it is of value in addressing learning needs and students have not become bored with it. Of course, teachers may always replace tasks at any time.

# Sample Elementary Workstation Task

## Where's the Math

**Materials**

- a collection of pictures, each one numbered *(These may be collected from calendars, magazines, the Internet, or school photographs.)*
- recording sheet or math journal

### Overview

Students examine photographs or other pictures to make mathematical connections to everyday life.

### Objectives

Recognizes mathematical connections in real-life situations.

### Procedure

1. Students work with partners to select a picture and record its number on the recording sheet or in their journals.
2. Students examine the picture and then list all the mathematical connections they see in the picture.
3. Students repeat this with another picture until they have completed the task with at least four pictures.

### Differentiation

This task is self-differentiating. The mathematics that students observe will be determined by their mathematical knowledge.

- For below-level learners, teachers can provide questions to guide students' observations: Do you see any shapes? Do you notice any numbers? Is there more than one of anything? Is anyone in the picture doing anything for which they need math? Do you see any fractional parts?
- For above-level learners, the student should create two mathematical story problems for each picture. These problems should be based on the math they noticed in the picture.

Name: ____________________ Date: ____________

## Recording Sheet

Where's the Math?

Picture Number ______________________________

Math connections I noticed: __________________

## Where's the Math

**Materials**

- pictures
- *Where's the Math? Recording Sheet*

**Directions**

1. Work with a partner.
2. Choose a picture. Write the number of the picture on your recording sheet.
3. Talk with your partner. What math do you see in the picture? What math connections can you make?
4. In your journal or on your recording sheet, list all the math you noticed.
5. Choose another picture and repeat.
6. Repeat againfor 2 more pictures.

## Talking Points

**Vocabulary**

- connection
- notice
- observe

(look at the Math Word Wall for more math words.)

**Talk like a mathematician:**

In this picture, I noticed ________.

This picture reminds me of ________.

I made a connection between _______ and _______ because _______.

In everyday life, math is important because _______

# Secondary Elementary Workstation Task

## Where's the Math

### Overview

Students play the card game of War turning over two cards each. The first card represents the base number and the second card represents the exponent.

**Materials**

- a deck of cards (1-10)
- *Exponent War Recording Sheet* or math journal
- calculator (optional)

### Objectives

Develop proficiency in calculating the value of numbers with exponents.

### Procedure

1. Students work in pairs. The cards are divided evenly between the two players and dealt face down.
2. Players each turn up two of their cards at the same time and find the value. The first card is a base number and the second card is an exponent for that number. The player who has the greater value takes all cards drawn and places them face down at the bottom of his or her deck of cards. If the value is equal, they each play another two cards and determine the value. The player with the greater value takes all cards played in the round. If the value is the same again, the procedure is repeated until there is a winner for the round. The numbers drawn are recorded on the recording sheet or in the math journal (72 = 49, 33 = 27, 72 > 33).
3. The game ends when one player has all the cards.

### Differentiation

- For below-level learners, the number cards can be limited to 1 to 5 or numbers appropriate for the students' level of proficiency. Calculators maybe used.
- For above-level learners, students decide which of the cards will be the base number and which will be the exponent.

Name: ______________________ Date: ______________

## Recording Sheet

### Exponent War

Round 1: ________________________________________

Round 2: ________________________________________

Round 3: ________________________________________

Round 4: ________________________________________

Round 5: ________________________________________

Round 6: ________________________________________

## Exponent War

**Materials**

- a deck of number cards
- Exponent War recording sheet or math journal
- calculator (optional)

**Directions**

1. Work with a partner. Divide the cards evenly, dealing them face down.
2. Turn up two of cards at the same time. Find the value if the first card is a base number and the second an exponent for that number. The player who has the greater value takes all cards drawn and places them face down at the bottom of his or her deck of cards. If the value is equal, play another two cards and determine the value. The player with the greater value takes all cards played in the round. If the value is the same again, the procedure is repeated until there is a winner for the round. The numbers drawn are recorded on the recording sheet or in the math journal (72 = 49, 33 = 27, 72 > 33).
3. The game ends when one player has all the cards.

## *Talking Points*

- base number
- exponent
- power
- multiply
- product
- value
- squared
- cubed
- equal
- greater than
- less than

**Talk like a mathematician:**

My base number is _______ and the exponent is _______. _______ to the power of _______ equals _______.

I know the value of the number because _______.

______ raised to the ______ power equals ______.

## Digital Devices and Math Workstations

With the availability of a range of digital devices in math classrooms, math workstation tasks often involve their use by students. Many mathematics programs and apps are available. While these may be appealing to students and very easy to use as math workstations, it is important to carefully assess their value before assigning them as workstation tasks. Typically, the programs and apps most often used emphasize skill drills. Programs that allow students to engage in the creation of presentations or explanations of their problem solving, however, are valuable in helping students become more adept with the mathematical practices set forth in CCSS math standards (2015). Furthermore, there are now many online instructional videos or programs that can be assigned for students to view. Teachers should verify that programs and apps align well with standards, can be easily understood and used by students, provide differentiation options, and offer support if students require it. In addition, to hold students accountable for their work on digital devices, it is advantageous that they include a method for teachers to track and review student work.

Flipped instruction in math workshop is another way that technology may be used effectively. Typically, in flipped classrooms, students are assigned an instructional video to watch for homework. The video focuses on the standards to be taught the following day. Then, the teacher addresses any questions students may have as they practice what was taught in the video the night before. One problem with flipped instruction is that some students do not or cannot view the video the night before and come to class unprepared. In some classrooms where math workshop is implemented, teachers can assign the instructional video as an independent workstation task. Then, students are prepared for the next day's lesson.

Managing digital devices is an important aspect for teachers to consider. These devices are appealing to students, so it is important to establish clear guidelines for their use. When planning for the use of digital devices during math workshop, teachers should consider the questions listed on the following page.

- How will the devices be distributed and collected?
- Who will work with the devices, and when will they work with them?
- How will students know what programs or apps to use?
- What should students do if they do not understand the directions for the program or app?
- What are the behavioral expectations for student use of the devices?
- How will students be held accountable for their work using the devices?
- How should technology problems be handled?
- What are the consequences for improper use of devices?

## Co-Teachers and Teaching Assistants

Mathematics instruction is a shared responsibility for some teachers. In many classrooms, co-teachers or teaching assistants are valuable resources during math workshop. These professionals play supportive roles in maintaining atmospheres conducive to learning and in supplementing the instruction of the mathematics teacher.

Students with special needs are joining many of today's classrooms, so co-teachers may spend math class supporting the instructional needs of both special education students and general education students. There are many models for co-teaching. As co-teachers become accustomed to working together, they soon establish their own instructional styles.

Because of its flexibility, the math workshop structure accommodates a wide range of co-teaching models. In one possible scenario, both teachers meet with small groups while the remaining students work independently. Another possible configuration is for one teacher to conduct small-group lessons while the other confers with students. Alternatively, one teacher may teach whole-class instruction as the other teacher works with a small group of learners who need specific support. Or, both teachers might be engaged in conferring with students as the rest of the class works independently. These are only a few examples.

Co-teachers need time to plan together for productive collaboration to occur. Co-teachers who work together over time discover each other's strengths. They can explore a variety of instructional models to discover which most effectively meets their students' needs and mesh best with their teaching styles. Only when co-teachers have time to discuss concerns about their students and construct a common vision regarding daily instruction are they able to work together, flexibly serving the ever-changing needs of their students.

In addition to co-teachers, teaching assistants or paraprofessionals may be assigned to classrooms. For teachers who are fortunate enough to have this assistance, the math workshop framework becomes even more effective. Teaching assistants can closely supervise the independent work of students to be sure they are on task and have the materials they need. By conferring with students about their work, they can monitor student understanding and correct misconceptions. Teaching assistants should write brief notes about what they observe as they work with students to share with teachers. These observations can help the teacher in planning additional mathematics instruction for students. Teachers maximize the advantages of having an assistant when they take time to share the instructional goals and expectations for students during math workshop.

## Chapter Snapshot

An effectively implemented math workshop benefits students by teaching them how to work independently, encouraging them to assume responsibility for their learning, and holding them accountable for their mathematical work. Workstation tasks positively affect students' learning with tasks that provide ongoing review, opportunities to apply the math they are learning, and practice computational fluency. At the same time, math workshop frees teachers to conduct small-group lessons and to confer with students when needed. A well-organized and effectively managed math workshop enables successful implementation of the Guided Math framework. Donna Boucher and I provide additional detailed information about the implementation of Guided Math Workshop along with a selection of workstation tasks in our Guided Math Workshop books published by Shell Educational Publishing.

## Review and Reflect

1. What are some of the ways that you can organize your classroom to support math workshop?

2. How can implementing math workshop promote the learning goals you have for your class?

***These conversations may appear to be simple "chats," but when done well, the teacher is learning about the students' work, what students understand or need help with, and what the next steps in learning should be for individual students and the class as a whole.***

# Chapter 7

## Math Conferences

According to John Hattie, "when teaching and learning are visible, there is a greater likelihood of students reaching higher levels of achievement" (2012, 18). In his study of research on what works in schools, Hattie found that what makes a difference are "…teachers seeing learning through the eyes of students and students seeing teaching as the key to their ongoing learning" (14). Effective teaching, in other words, rests on a foundation of understanding shared by students and teachers—an understanding of exactly what is being taught and what is being learned. That is hardly a surprising finding. Common sense tells us that "student achievement is greatest when teachers strive to find out what their students know and are learning and then adjust their teaching to meet student needs, and when students assume the responsibility for both knowing what their learning goals are and for monitoring their progress toward meeting them" (Sammons 2014, 10). Based on this premise, math conferences between teachers and students are an important component of the Guided Math framework aimed at making both teaching and learning visible.

In a Guided Math classroom, a visitor for math workshop may see the teacher move from student to student, seeming to chat informally with each student before moving on to the next. Although it may not be obvious to the visitor, the teacher is involved in an extremely effective instructional tool—conferring. These conversations may appear to be simple "chats," but when done well, the teacher is learning about the students' work, what students understand or need help with, and what the next steps in learning should be for individual students and the class as a whole. Furthermore, students are focusing on what is being taught and reflecting on their learning. Conferring is a fundamental piece of Guided Math—occurring most often, but not always, during math workshop.

## What Are Math Conferences?

"Math conferences are one-on-one *conversations* with students about their mathematical work, as one mathematician talking to another" (Sammons 2014, 16). In many ways, conferring is the heart and soul of teaching. As teachers confer with students, they sit alongside them at their level and listen intently to their words, trying to follow their reasoning and probing to determine the extent of their understanding. According to Lucy Calkins, Amanda Hartman, and Zoe White, conferring "gives us an endless resource of teaching wisdom, an endless source of accountability, a system of checks and balances. And, it gives us laughter and human connection—the understanding of our children that gives spirit to our teaching" (2005, 6).

Similar to conversations between colleagues, math conferences share these characteristics (Anderson 2000):

- Conferences have a purpose.
- Conferences have a predictable structure.
- Lines of thinking are pursued with students.
- Teachers and students have conversational roles.
- Students are shown that teachers care about them.

Richard Allington (2012) describes the ability of literacy students to move beyond simple word recall and recitation to comprehension as *thoughtful literacy.* Teachers conversing with students about their mathematical thinking cultivates that same kind of mathematical understanding in students. This might be termed *thoughtful numeracy.* Their mathematical comprehension increases as they reflect and confer with their teachers. Consequently, their abilities to manage the practical mathematical demands of everyday life are enhanced.

Conferring with students as they work gives teachers the timeliest assessments possible. Rather than waiting to see the results of their students' work, teachers immediately discover students' needs as they interact with mathematics. Teachers can then address these needs during the conferences, in small-group lessons, or with whole-class instruction. When teachers use what they learn from conferring with students in planning instruction, their teaching becomes more focused and powerful.

## The Differences between Helping and Conferring

Teachers who confer with students do considerably more than just "help" them. Wendy Hoffer observes that often teachers who converse with their students about their mathematical work are "trapped in the typical pattern of helping that leaves students dependent and awaiting our approval" (2012, 143). But, with effective math conferences, "teachers spur students to think and communicate about their mathematical thinking by listening respectfully and asking open-ended, thought-provoking questions" (Sammons 2014, 186) with the aim of expanding and deepening students' knowledge and skills. Conferences are not just helping students arrive at a correct answer. Figure 7.1 (page 278) describes the essential differences between helping and conferring.

**Figure 7.1—Differences between Helping and Conferring**

| | Helping | Conferring |
|---|---|---|
| **Focus** | • getting answers | • teacher learning about student thinking<br>• student communicating, learning, and reflecting to deepen mathematical thinking and skill |
| **Expectations for the student** | • listening to the teacher<br>• asking questions if needed | • sharing thinking<br>• pondering current or new ideas<br>• making mathematical connections<br>• trying new strategies<br>• extending mathematical understanding and skill |
| **Expectations for the teacher** | • telling or showing students how to improve their work<br>• pointing out errors<br>• reteaching | • active listening<br>• prompting deeper thinking by asking questions<br>• sharing feedback<br>• identifying and teaching next step in learning |
| **Goal** | • students get correct answers | • students think more deeply about mathematics<br>• deeper understanding of mathematical concepts<br>• teacher gains deeper understanding of students' mathematical capabilities |
| **Inferred beliefs** | • Teachers are the dispensers of mathematical knowledge and how-to. | • Students are mathematicians with interesting ideas to share and the capacity to solve problems. |

## Conferring During Math Workshop

While teachers confer with individual students during math workshop, they can't consistently monitor the work of the rest of the class. Therefore, the tasks assigned to the class should be those that students complete independently. So, what happens when they cannot complete the task independently? Teachers should anticipate this situation. When teachers discuss this problem and possible solutions before it occurs, students are prepared to handle difficulties that may arise. Calkins, Hartman, and White (2005) suggest telling students in writing workshop that each student is a writing teacher. Students can turn to each other for help when they have questions. In math workshop, students can also turn to their peers for assistance. Teachers should also have procedures established for students who may hurry through their work and complete a task before the workshop is over. In any event, students should know what to work on next.

There may be times when students need some assistance with their independent work. How can teachers justify not giving that help when needed? When weighing the value of immediately helping individual students who are engaged with workstation tasks compared to the value of uninterrupted math conferences, it is important to keep in mind the benefits of the targeted teaching that conferring allows teachers to offer students. When conferences are interrupted, their value is diminished. To avoid this, teachers can provide students with guidelines for handling any problems that may arise. Later, the teacher can arrange to meet with any students who needed assistance and specifically address those needs. The value of conferences with students far outweighs the on-the-run direction a teacher may be able to give if interrupted during a conference. When students feel they can interrupt teachers as they confer or meet with small groups, it becomes impossible to effectively use these instructional components.

Moreover, students who learn to work independently become more self-sufficient. It is important that students know strategies they can employ to handle problems, apart from immediately turning to their teachers. When they lack a toolbox of strategies for independent work, students may exhibit a "learned helplessness." They immediately ask for help rather than considering the options available and trying to work through the obstacles they face. Their helplessness may be due to a lack of confidence. Too often, though, students are "spoon-fed" and walked through each step of problem solving—in both mathematical and real-life challenges. They fail to develop the endurance to work independently to solve problems. Consequently, these students never experience the satisfaction of solving a problem with which they have struggled. They quickly become frustrated if they have to struggle. By establishing procedures that lead students to assume greater responsibility for their learning, teachers help them grow to be more confident and proficient learners.

In addition to those students who crave constant teacher support and attention as they work, there will inevitably be students who have trouble getting started or staying involved in their work. Sometimes students have needs that require them to be out of their work spaces or those whose goals are finishing first without regard to the quality of their work. These difficulties will be encountered, but they are predictable. And because they are predictable, teachers can plan on how to handle them before they occur, or at least before they occur repeatedly.

Teachers who are experienced using math workshop find that after students begin their workstation tasks, it is helpful to spend a few minutes surveying the room to be sure all students are successfully settling in with their assigned tasks. Often, just being sure that everyone is engaged initially prevents problems later in the workshop. Students may need a brief reminder of expectations as they begin. Taking time to set the proper tone in the classroom helps ensure uninterrupted time once teachers begin to confer with students.

Teachers may choose to target students who appear to be having a hard time getting started for their initial conferences. Classroom management is often easier for teachers if they crisscross the room as they confer, going from one side of the room to the other with each conference, rather than from one student to that student's neighbor. Just the proximity of the teacher keeps some students on task when they are working independently.

If getting materials is a problem for students, teachers can rearrange material storage so that they are readily available. If a particular student is away from their work excessively, teachers should try to discover the reason and remedy it. If a student has a constant need to obtain additional materials, partner him or her with a more responsible student who will be the only one in that group permitted to get up and get materials.

Math conferences with students are an effective way to address problem behaviors, especially if they occur immediately after the behavior is observed. For example, if a student rushes through their work, either not completing it or not doing quality work, a teacher may choose to tackle the problem with an individual conference. If the same problem behavior is occurring with several students, however, the teacher might choose to meet with them as a small group to reteach and reinforce correct procedures and expectations. Frequent follow-up conferences are sometimes necessary with students to ensure a long-term solution. Teachers should continue to closely monitor the work of these students to be sure the problems are remedied. In some instances, students may be assigned to work with groups or have clearly designated activities to work on if they believe that they have completed the assigned independent tasks. Once they build the self-discipline and stamina for sustained quality work, these supports can be removed.

## The Overall Structure of a Math Conference

When teachers first begin to confer with their students, they are sometimes unsure of exactly what they should do. They know that conferring involves conversing with students individually, but they wonder precisely how to help their students become more proficient mathematicians. The traditional style of discourse in classrooms is not one normally found in our interactions with our friends and associates. In schools, teachers traditionally ask questions, call on students, and then let them know if the answer is correct or not. Unfortunately, this pattern tends to reinforce the notion that all knowledge resides with the teacher to be dispensed to students. With this type of discourse, the teacher plays largely an evaluative role, and students recognize this very quickly.

When teachers confer with students, however, the pattern shifts to a sharing of knowledge between students and teachers. Teachers probe to find out what students are thinking, understanding, and wondering. Rather than being afraid to express any confusion they may have, students come to view teachers as partners in their learning journeys. Errors are recognized as valuable steps in these journeys. Carefully crafted questions from teachers encourage students to think more deeply and extend their understanding. Specific, descriptive feedback by teachers lets students know what they have done well and what they may want to reconsider. Most importantly though, students have opportunities to return to their work and apply what they have learned in the math conference.

Calkins, Hartman, and White (2005) propose an architecture for writing conferences that can easily be adapted for math conferences. The structure guides teachers as they confer with their students, so they can discover what students are thinking mathematically and then identify what steps to take to help them progress in their understanding. Following a structure gives purpose to what otherwise may be chatting without focus. Naturally, teachers carefully consider the information they gain through these conversations and based on their specific knowledge of the curriculum, determine specific teaching points for each student conference.

When the conference structure described by Calkins, Hartman, and White (2005) is adapted for math conferences (Sammons 2014), it includes the following steps:

**Research Student Understanding**

- Observe the work of the student closely.
- Listen carefully to the student's description of their work and thinking.
- Search for evidence of strengths and needs.
- Ask questions to clearly understand student's thinking.

**Decide What Is Needed**

- Decide what the student is doing well as a mathematician and offer an authentic compliment linked directly to the standards.
- Decide on a teaching point based on the researched findings.
- Decide how to teach the teaching point.

**Teach to Student Needs**

- Teach the teaching point.
- Monitor and assess student understanding of the teaching point.
- Provide scaffolding, if needed, to ensure student proficiency.

**Link to the Future**

- Summarize the teaching point.
- Express the expectation that the student will remember and use what they have learned.
- Ask the student to share a reflection on what they learned.

*(Adapted from Calkins, Hartman, and White 2005 and Sammons 2014)*

When conferring, the teacher initiates the conversation with a broad question that encourages the student to explain their mathematical thinking and work. As the student shares their thinking about the work, the teacher observes the student work and listens intently. When the teacher has a good grasp of what the student is attempting, the focus of the conversation shifts to the teacher as they begin to teach. The teaching addresses the unique mathematical needs of that student, which the teacher identified during the brief conversation with the student at the beginning of the conference. Toward the end of the conference, the teacher and student continue to interact. The student restates the mathematical teaching point, and the teacher urges the student to always remember to use what they have learned.

***The teacher provides this student with feedback specific to her progress on the activity.***

## Research Student Understanding

The goal of a conference is to move a student from what they can *almost* do independently to what they *can* do independently. This is often a very fine distinction that is not obvious in a finished work product. Only by talking with students about their work, questioning, encouraging, and most importantly, listening can teachers discern this distinction. Conferring with students also makes it obvious that students do not all progress at the same rate. Nor do they respond the same way to the instruction provided. Conferring with students one-on-one allows teachers to take advantage of individual students' "teachable moments" that organically occur during these conversations.

As students begin working on tasks during math workshop, the teacher may have already chosen some students for conferring. If not, a glance around the room to observe the work of students and a few minutes of listening to their discussions may help a teacher decide who will benefit the most from a conference. These students may not always be those who need extra support. Sometimes, students who quickly demonstrate a deep understanding of a concept and need additional challenges or those who are choosing unique ways of interacting with mathematics may benefit from a conference.

With whomever the teacher chooses to confer, the crucial first step of a conference is research—finding out what the student is doing with the assigned task and what their understanding is about the applicable mathematical concepts. Although the conference itself is very brief, this is a time for teachers to slow down. The research phase is primarily a time for "deep" listening (Fletcher and Portalupi 2001), making it possible for teachers to view their students' work products through the students' own lenses. Learning how students perceive their work informs teachers as they select the most important next learning steps for individual students. For many teachers, this may be the most challenging aspect of conferring. From a brief conversation with a student, how can one be confident that the teaching point chosen is the best one? Perhaps it helps to know that there is rarely one right teaching point lesson, and if there is one, it will usually be quite obvious. Different teachers conferring with the same students may decide on different teaching points for very valid reasons based on what they discovered during the research phase. Each of the teaching points may be of value in leading the student to greater understanding.

Teachers who are just beginning the process of conferring should not let concerns over determining the next steps in learning prevent them from carrying out conferences with their students. Only by getting started can they hone their research skills. As teachers gain experience conferring, their confidence grows.

During the research phase of a conference, teachers search for evidence of the mathematical understanding a student has acquired and of the student's ability to apply that understanding appropriately. Entering conferences, teachers already possess bodies of knowledge upon which to draw. It is helpful to pause prior to initiating a conference with a student to reflect on what is already known about the student. Teachers may refer to anecdotal notes or other assessment data to refresh their recollections about the student's previous mathematical work. Being aware of the student's past mathematical achievement allows teachers to more accurately discern the student's growth in mathematical understanding.

- Where has she demonstrated mastery?  In what areas has he struggled?

Teachers may also recall the learning styles of students with whom they are going to confer. This recall only lasts for a moment, but by reflecting on the known information about students prior to conferences, teachers can more easily understand what is currently going on with students as they confer.

- Do they work best with visual representations?
- Do kinesthetic activities support their learning most effectively?
- Do they work well with other students, or are they better working on their own?
- What do they tend to do if they don't understand problems?

After taking a moment to consider what is already known about a student, a teacher may want to simply observe the student at work for another moment. This observation may only take a minute or so, but it helps prepare a teacher for a successful conference.

- Does the student work confidently or seem confused?
- Is the student working productively or doing everything but what they should be working on?
- What strategies does the student appear to be employing to deal with the assigned task?
- Are those strategies being used properly?
- What might the student be overlooking?

When a teacher joins a student to begin conferring, they see the work of the student, whether the work is using manipulatives, drawing illustrations, evaluating expressions, solving problems, or explaining their mathematical thinking through writing. The teacher scans the work prior to beginning the discussion without picking it up. This initial view of the student's work often gives the teacher a direction to pursue as they conduct research.

To obtain a more in-depth understanding, the teacher can ask general questions about the work of the student or more specific, probing questions. These questions are based upon the information the teacher gathered while reflecting and observing to learn more about what the student is trying to do, is able to do, and is not quite doing yet (Calkins, Hartman, and White 2005). Students cannot always put into words the understanding they possess or accurately describe how they are applying what they know. As teachers talk with students, they may restate the students' ideas while using appropriate mathematical vocabulary and model useful forms of mathematical communication. When teachers listen to their students during conferences, they should be especially careful to differentiate between the authentic ideas of students and those that are automatic answers to leading questions.

Calkins, Hartman, and White (2005) warn about problems that teachers may face when conducting research during a conference:

*The research phase takes up too much of the time devoted to a conference.* When teachers first begin to confer with students, they may devote excessive amounts of time to research. There is so much that can be learned with each conference. But, it is important to balance the amount of time spent on each step of the conference to make the conference as effective as possible. Gradually, teachers learn to take in and assess the many aspects of their students' work almost as second nature, allowing students ample time to talk but guiding the discussion away from areas that will not extend their knowledge of their students' levels of understanding.

*The research is not used to determine the next steps in learning.* This problem is by far the more serious. To be sure, there will be times for all teachers when the research does not dictate a teaching point. Sometimes, it may be difficult to determine exactly what a student needs most. When there is a lack of alignment between research and teaching point, it is indicative of a failure to comprehend the purpose of conferences. The value of conferring with students is in the teacher's ability to pinpoint instructional needs and then to teach short, individualized lessons to target those needs. It is good practice for all teachers to occasionally examine their conferences to be sure the teaching is well matched to the research findings.

## Decide What Is Needed

The decide phase of a conference occurs almost simultaneously with the research phase. As teachers conduct research, they are also gauging students' progress, the strategies they use, and overall understanding compared to the students' learning goals. Armed with that knowledge, teachers then determine how they can most effectively extend their students' mathematical proficiencies. Teachers have three responsibilities during this phase:

1. Identify what students are doing well. Give them genuine and specific compliments.
2. Decide what to teach students to move them forward.
3. Focus on how to best use the few minutes left of the conference to teach those points to students.

Just as informal assessment is instrumental in helping teachers determine where their students are on their progression toward established instructional goals, it can also be used to help students appreciate how much they have achieved. In conferences, teachers should make a practice of noticing the specific ways in which their students have been successful so they can give authentic compliments based on these accomplishments. The compliments not only confirm to students that they are progressing but spotlight the things that they should continue to do in the future. By focusing compliments on particular activities or achievements, students are motivated to repeat them.

Just as teachers are sometimes reluctant to have someone come into their classrooms to critique their teaching, students may feel vulnerable when teachers approach them for conferences. Some students know they have not been on task, but most of them have been doing their best. Students' work should be challenging. As a result, students may be using strategies they are unsure about or struggling to understand a problem. Especially in those kinds of situations, having a teacher approach to chat about their work may be intimidating. In addition, most students have noticed that teachers tend to focus on those things that are incorrect or need improvement first to be sure a student's work is corrected or improved.

In classrooms where a strong sense of mathematical community has been established and where students understand the value of taking risks as they learn, students tend to feel less intimidated. Yet, even in these environments, it is important to understand the value of recognizing students' achievements, not just their errors, as the conference progresses. When given an authentic compliment on their work first, students are more open to later teaching points offered by the teacher.

The compliments teachers give can be the entries into the teaching points of the conferences. After a teacher identifies the compliment they will give a student, the focus turns to choosing a teaching point. The more thoroughly teachers know the curriculum, the easier it is to break the essential mathematical knowledge and skills students need into a series of teaching points which progressively build on one another. When the conference research reveals a student's learning need according to this progression, the teacher can identify and deliver an appropriate teaching point that will help lead the student to achieve mastery. This process requires considerable reflection, especially for new teachers.

It is often difficult to determine exactly what a student needs next based on a few minutes of research at the beginning of a conference. As tempting as it may be to leap to a teaching point without completing the research phase, it is worthwhile to wait patiently to fully understand the student's level of understanding before committing to a next step in learning. At best, the teaching points provide students with strategies or increased understandings that will remain with them and will be something they will use as they engage in mathematical thinking and problem solving in the future. Teachers should take comfort in the fact that there is no one absolutely correct teaching point for any particular conference. Using what they know about the student and the standards, teachers do their best to decide and teach. If the teaching proves to be ineffective, there will be many other opportunities to reteach and address the needs of students.

The final component of the decide phase is to choose *how* to teach the next step during the brief conference. Given the limited amount of time available to teach, deciding what teaching method will most effectively convey the message of the lesson is important. Just telling a student what they need to know is not sufficient.

## Teach to Student Needs

After completing the research and determining the teaching point, the teacher must teach it. Drawing on transcriptions of hundreds of writing conferences, Calkins, Hartman, and White (2005) identified three teaching methods that are often used effectively by teachers:

- guided practice
- demonstration
- explaining and showing an example

These three methods work as well for teaching mathematics as they do for writing. In the deciding phase, teachers will have chosen one of these methods to deliver the teaching point.

### Guided Practice

Students learn best when they are actively involved. What students *do* remains with them longer than what they *hear*. When teachers use guided practice to share a teaching point, they have the student practice what is being taught. As a student tries a new strategy or approach, the teacher is right there to coach the student. This ensures that the student begins to use the new knowledge immediately and is using it correctly. With the guided practice method, teachers can scaffold the learning experiences for their students—nudging them toward effective independent application of their new learning.

For example, Marisol is a student who is gathering data about which of three field trip possibilities students in her class prefer. The task is meaningful because the data is going to be used to help Marisol's teacher plan the next field trip. After Marisol works on the project for a while, her teacher notices that she seems to be stuck and so decides to confer with her. Marisol explains that she has gone from student to student, asking them for their field trip preferences. She has neatly listed each student's name and field trip choice. At this point, she is unsure of how to organize the information to find out the top preference of the class.

In an earlier mini lesson, the teacher taught the class how to use a tally chart to organize data. The teacher reminds Marisol that the class has learned several ways to record data so it can be more easily interpreted and asks her to think back on what she learned. With that reminder, Marisol recalls and names several methods of recording data. When the teacher asks what might work best for displaying her data, Marisol recalls that the class used a tally chart to find out the class's favorite flavor of ice cream. She decides to create a tally chart. When asked why she is using a tally chart, Marisol explains that finding a favorite flavor ice cream and finding a favorite choice for a field trip are very much the same, so she thinks it will work well. She begins creating the chart using the information she has gathered. The teacher observes Marisol's work as she draws a chart and begins to record the data with tallies.

When the teacher notices that Marisol records the fifth tally as a vertical line rather than a diagonal line, the teacher decides to wait to see if Marisol will notice how her chart differs from the posted chart the class completed and will self-correct. When Marisol continues without self-correcting, the teacher reminds Marisol that mathematicians who work with tallies always record every fifth mark as a diagonal line going across the previous four vertical lines to create obvious groups of five, making it easier to count how many tallies there are altogether. The teacher watches as Marisol changes her tally recording and remains long enough to see Marisol record the next fifth tally correctly. Before concluding the conference, the teacher asks Marisol to always remember that the use of tally charts is one way in which mathematicians organize data so it can be easily understood. She may not always choose to make a tally chart, but it is one option she should consider. The teacher also reminds Marisol that a simple procedure like having the fifth tally mark cross diagonally over the other four makes it much easier to count the total number of tallies.

In this example of guided practice, the teaching point was effectively taught. The teacher reminded Marisol of the work the class had done, which prompted her to reflect and decide to create a tally chart to display her data. The teacher observed Marisol's work and coached her through her initial effort at independently creating and using a tally chart. The teacher was right beside her, supporting her while she created the chart to display her findings. When she began to record tallies incorrectly, the teacher stepped in to coach her and help her record tallies properly.

This is just one example of how a teacher may use guided practice to teach the next steps in learning during conferences. With this method, students are asked to try what they are learning while teachers provide coaching to support their initial efforts. The focus is on the work of students, however, so teacher talk is minimal.

**Demonstration**

Learning from demonstrations seems to be hardwired into our systems. Watch young siblings and you will see the younger child constantly mimics the older sibling. Although the younger child may not be conscious of learning, there is little doubt that demonstrations by the older child teach the younger one. (Sometimes, unfortunately, they teach things we'd rather younger children not learn.)

Demonstration is almost as effective when used in conferring with students. In math conferences, teachers may choose to model strategies or processes, thinking aloud as they do. When modeling, teachers break down the teaching point into achievable steps and explain the reasoning behind each of the steps. By demonstrating processes or strategies, teachers help students see how they are applied. This helps students as they attempt to replicate them. Demonstrations should always conclude with an emphasis on what students should have noticed or will remember from the lessons. Then, teachers encourage students to always use what they learned from these lessons when they work as mathematicians.

Jamari is struggling to solve a written problem. Through research at the beginning of a conference, his teacher finds that Jamari can easily read the problem and knows exactly what he is being asked to find. But, Jamari seems to be jumping directly into problem solving without taking time to identify the relevant facts in the problem. The teacher decides to use a demonstration to teach Jamari how to reread a problem carefully to identify the important facts before trying to solve it.

The teacher explains, “Whenever I have a problem to solve, first, I think about what it is I need to find out. You’ve already done that with this problem. Next, I want to be sure I know what facts are important to know so I can solve it. I’m going to read the problem again carefully and underline the facts. Then, I’m going to go back and be sure everything I underlined is needed for this problem.”

The teacher goes through this process, underlines the relevant facts, and then asks, “Did you notice how I read the problem carefully, underlined the facts, and then went back to recheck them before I tried to actually solve the problem?” Jamari is then asked to reflect and share what he just learned.

Finally, the teacher reminds Jamari this is a strategy he can use whenever he has a mathematics problem to solve and needs to identify the information he is given. In this conference, the demonstration allows Jamari to see the process the teacher goes through as they explicitly explain what they are doing. The teacher will follow up with Jamari in the next few days to see if he is able to apply this lesson independently.

**Explaining and Showing an Example**

In some situations, a teacher may choose to teach a next step by referring a student to an example. The walls of a Guided Math classroom usually contain anchor charts showing procedures or strategies the class has discussed, used, and recorded for future reference. In addition, as it is solved, a Problem of the Day or a Problem of the Week may also be documented and displayed. With many examples of strategies and procedures available for students’ reference, there are times during conferences when the teacher chooses to explain what it is that the student needs to know and then uses one of these charts.

Jasmine is working to solve a percentage problem using representations or drawings. Each day during the previous week, the class tackled similar problems in their math warm-ups. Their problem-solving procedures and strategies were recorded on chart paper and displayed in the classroom. During the research phase of this conference, the teacher discovers that Jasmine is unclear about how to use a representation or drawing for the problem. The teacher accompanies Jasmine to the charts the class created the previous week and reviews the strategies they used. Jasmine is encouraged to use the charts as examples as she solves her problem. As the conference ends, the teacher summarizes the methods used and reminds Jasmine that the charts in the classroom are there to help her when she is stumped by a problem. The teacher also reminds Jasmine that mathematicians take advantage of the information they have available to them to solve tough problems.

The examples to which teachers refer students may not always be class charts. Students may be encouraged to see what another student or group of students are doing. Students might be guided to examples in their textbooks. They might even be asked to reread pieces of mathematics-related literature to find problem-solving strategies.

## Link to the Future

The purpose of conferences is to help students acquire strategies and processes that become part of their mathematical repertoires. To reinforce the teaching points of conferences, teachers conclude by specifically restating what they hope students have learned and reminding them to apply it in their future mathematical work whenever it is relevant. While it may seem obvious to teachers that students are expected to actually use what was taught, students often need an explicit verbal link to fully recognize their responsibility to apply the new knowledge and skills in other circumstances. To reinforce their understanding, teachers should ask students to briefly explain what they have learned and to consider how to use it in the future. This reflective process serves to strengthen the mathematical foundation upon which students' conceptual understanding rests.

## Keeping Records of Conferences

What teachers learn as they confer with students can be as valuable as what their students learn. Focused conferences are rich opportunities for teachers to gather the missing pieces of the instructional puzzle. Conferring with students gives teachers comprehensive information about what students already know and what they do not yet sufficiently understand. Within the conferences themselves, some of these needs can be addressed promptly. Teachers can also use this data to tailor later instruction in the other components of the Guided Math framework. To do this effectively, teachers must have a method of recording what they learn so that they can refer back to it later when planning lessons. There are many different ways for teachers to keep records of what they learn from conferring.

### Math Conference Checklist

On a checklist form, such as the one shown in Figure 7.2, students' names and instructional goals for a unit are listed. During conferences, a teacher can indicate whether or not each student has mastered each goal (see Appendix A).

**Figure 7.2—Math Conference Checklist**

| | Students | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Math Goals | | | | | | | | | | | | | | | | | | | | |
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## Math Conference Recording Form

**Figure 7.3—Math Conference Recording Form**

| Student | Date | Research | Compliment | Teaching Point |
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On the boxes on a recording form, such as the one shown in Figure 7.3, the teacher lists the names of students as they confer with them and then records what was observed during the research phase, what compliment was given, and finally, what the teaching point of the conference was. (See Appendix A).

## Sticky Note Organizer

**Figure 7.4—Sticky Note Organizer**

| | |
|---|---|
| | |

Sticky notes are a convenient way to record conference notes. Using one note per student, teachers take note of research information, compliment, and teaching point. Teachers add the sticky notes to students' files at the end of the day. See Figure 7.4 for an example of an organizer for sticky notes (See Appendix A).

## Conference Notes on Mailing Labels

Pages of mailing labels may be preprinted with blank areas for student name, date, and categories to make note taking simpler. Teachers simply add their notes as they confer with each student. At the end of a day, teachers can add these labels to students' files. See Figure 7.5 for sample notes using labels.

**Figure 7.5—Sample Conference Notes on Mailing Labels**

| | |
|---|---|
| Name *Maurice*<br>Date *January 10*<br>Research<br>*Tells time accurately to the half hour*<br>*Has quarter hour —15, but not 45*<br>Compliment<br>*Identifying the hour and minute hands, using hour hand to determine hour*<br>Teaching Point<br>*Reviewed that 5 minutes between each number, practicing skip counting by fives to determine minutes after the hour* | Name<br>Date<br>Research<br>Compliment<br>Teaching Point |
| Name<br>Date<br>Research<br>Compliment<br>Teaching Point | Name<br>Date<br>Research<br>Compliment<br>Teaching Point |
| Name<br>Date<br>Research<br>Compliment<br>Teaching Point | Name<br>Date<br>Research<br>Compliment<br>Teaching Point |

## Student Flipcharts

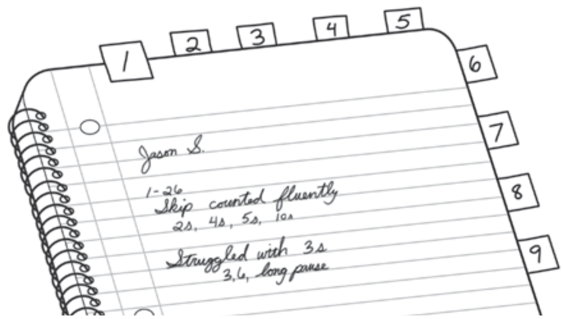
Tape five-by-eight-inch index cards onto a clipboard, beginning at the bottom. (See Figure 7.6.) Students' names are written on the visible lower edge of each card. Conference notes for individual students are recorded on cards. When a card is filled, remove it, file it in the student's file, and replace it with a new card.

**Figure 7.6—Sample Student Flipchart**

### Sectioned Notebooks

A practical method for recording notes is with a thick spiral notebook divided with tabs into sections of five or so pages each, one section for each student. (See Figure 7.7.) Record notes in the appropriate student sections. It is easy to track the learning progress of each student by referring to the notes in their section of the notebook.

**Figure 7.7—Sample Sectioned Notebook**




### Electronic Notes

With the availability of digital devices, many teachers are maintaining conference notes electronically. Programs such as *Confer, Evernote,* and *Noteshelf* offer sophisticated ways to maintain teacher notes, images of student work, and recordings of students as they explain their thinking.

## Using Conference Notes to Plan Instruction

However notes are recorded, the records must be used to inform instruction. They should not be recorded, filed, and forgotten. When used wisely, these valuable notes effectively guide instruction for students. Calkins, Hartman, and White (2005) suggest that conference notes help teachers with the following steps:

***Plan for future conferences***—If teachers notice that many students are struggling with the same concept in a unit, they may want to confer with other students to see if they are also struggling. Or, if students seem well beyond where a teacher thought they would be, it may be worthwhile to confer with additional students to see if perhaps instruction could move ahead a little more quickly than originally planned.

***Recognize the strengths of their students***—When teachers identify students who have mastered the standards, they can modify the instruction for these students and allow them to move on.

***Discover future teaching options***—Conference conversations suggest lessons from which students can benefit that may not be possible to address in that conference. The lessons may benefit the whole class or a small group of students.

***Broaden the scope of conferences***—As teachers review their conference notes, they may decide to confer with students based on previously noted concerns.

***Follow up on conference teaching points***—Teachers who have records can follow up in subsequent conferences to confirm student understanding.

## Strategies for Effective Math Conferences

While each math conference is unique to the student with whom the teacher confers and to the mathematics in which the student is engaged, there are some guidelines to make conferences more productive. Conferences are of most value when teachers do the following:

***Keep in mind the difference between helping and conferring.*** Conferences are most valuable in helping teachers identify what students know, can do, and what else they need to learn. The focus is on the learner communicating their mathematical thinking with the teacher—not the teacher stepping in to correct errors.

***Communicate with students as fellow mathematicians.*** Teachers should be seated beside students and speak in a conversational tone, letting students know how interested they are in learning more about their mathematical ideas.

***Make conferences predictable.*** Students are more comfortable and willing to share their thinking when they know what to expect. Math talk flows more freely when students and teachers can both predict the structure of the conference even with the content varying from conference to conference (Anderson 2000).

***Listen actively to students as they talk.*** It is understandable that teachers feel rushed and distracted by the responsibility of monitoring the rest of the class as they confer. Students often sense the anxiety and distraction of their teachers. Conferences are more productive when teachers take a deep breath, focus on the student, and make an effort to listen actively. The teacher should "not only hear the words that another person is saying, but more importantly, try to understand the complete message being sent (Mind Tools, under "About Active Listening").

***Ask questions to reveal and extend student thinking.*** "Questioning shapes conferences—and, consequently, determines what both teachers and students learn" (Sammons 2014, 192). There is no standard formula for high quality questions that teachers can plug into a conference conversation. Instead, Lucy Calkins states, "when questions grow out of our emerging understanding of the [student], they are alive and fresh and powerful" (Calkins 1994, 225). Math conference questions are best when they arise from an understanding of the student. The best questions provide surprises for both teacher and student. The student "finds himself speaking about information he hardly knew he possessed. The teacher may have had only an inkling" (Graves 2003, 107) of what the student knew.

***Encourage students to do most of the talking***. While many students have become accustomed to the expectation that they will listen as teachers talk—only being required to speak in response to teachers' questions—teachers should gently prod students into becoming more verbal during math conferences. When teachers become comfortable with silence, students realize that they are the ones who are expected to talk and that teachers will wait to allow them time to think before speaking. According to Graves (2003, 99), students will use silence productively "when the conference is predictable, when the setting is right, and when they believe you think they have something worthwhile to say."

***Strive to understand and respond to students' thinking***. It is not always easy to understand the mathematical thinking of students as they express their reasoning during math conferences, yet good questions encourage students to share what they are thinking and reflect on what they are doing. In addition to considering what students are saying, it is also important that teachers are attuned to students' nonverbal cues. Their body language and tone of voice may sometimes convey more than their words.

***Build on student strengths and interests***. Conferences are most effective when teachers build on students' strengths and interests. Graves observes that "a teacher who looks for potential finds listening and observing an exciting venture" (2003, 100). Teachers who look for and recognize students' "mathematical curiosity, knowledge, skills, and perseverance rather than solely for mastery of a specific standard or use of a particular strategy, these mathematical conversations can indeed be astonishing" (Sammons 2014, 203).

***Teach just one thing***. Teachers new to conferring will often recognize several potential teaching points when conferring. However, it is important to focus on a single teaching point during a conference. Limiting what is taught allows teachers to keep the conference brief and to maintain a laser-like focus on the teaching point.

***Avoid power struggles***. Power struggles are no-win situations (Sammons 2014). This statement in no way suggests that teachers give up their instructional authority, but it does recognize the ineffectiveness of battling with students when they can find ways to work toward shared goals (Heibert et al. 1997). Student misconceptions and errors certainly need to be addressed, but experienced teachers know that it is important to first confirm that there is a misconception. Errors may be a result of carelessness, not misconceptions. When teachers call upon students to share their reasoning, it quickly becomes apparent whether or not there are misconceptions. Power struggles may also occur when teachers require students to use one particular algorithm or procedure. Many mathematics educators recommend allowing opportunities where students can "develop, use, and discuss a variety of methods" (Chapin and Johnson 2006, 44) in their mathematical work. Marilyn Burns suggests that "instruction should focus students on multiple strategies for computing and help them explore the parallels and differences among them" (2007, 187). Math conferences offer teachers and students opportunities to talk about differences in mathematical perspectives without engaging in unproductive power struggles.

***Encourage deeper and more complex thinking***. When students and teachers engage in mathematical conversations, teachers should share their own curiosity and wonder about mathematics. This helps students understand the complexity of the discipline and that it is not just a "stolid subject with rigid rules learned from a book" (Sammons 2014, 209). In addition, students who are engaged in rigorous mathematical challenges need people with whom they are comfortable trying out their thinking. Conferences allow students to do that with their teachers.

***Celebrate mathematical growth***. Because so much is learned about students' mathematical progress during conferences, they are ideal times for teachers to celebrate that progress. Explicitly highlighting the achievement of learning goals is an effective way to celebrate—rather than reward—the success of students (Marzano 2007).

***Incorporate humor and playfulness***. Unfortunately, according to Mink, "mathematics is one of the most feared subjects in school" (2010, 7). Too many students come into math class deeply reticent. When teachers inject a bit of humor into their math conferences, it eases the trepidation and anxiety that inhibits students' mathematical learning.

***Use conferences strategically***. Math conferences are, by definition, conversations between a teacher and a single student. Yet, as teachers are well aware, other students likely eavesdrop on conference conversations. Knowing that, teachers can strategically plan conferences so that they indirectly share teaching points with neighboring students (Anderson 2000; Serravallo and Goldberg 2007).

## Chapter Snapshot

Conferring with students is an important component of Guided Math. The one-on-one interaction between teacher and student provides important information about the student's mathematical understanding as well as possible misconceptions. Conversing with individual students, the teacher has the opportunity to further probe the student's thinking or attempt to address misconceptions as they are identified. Additionally, the information teachers glean from students in conferences informs their decisions about the potential need to reteach specific mathematical content to either a student, a small group, or an entire class.

To have the opportunity to confer individually with students on a regular basis, a strong classroom management system must be established by teachers. To avoid interruptions, students must understand what is expected of them as they work independently.

Keeping accurate records of conference findings is an essential task for teachers. There are many different systems for doing this, but finding a system that works well for a teacher is important. When teachers find a system that works well for them, they are more likely to keep it up to date and maintained. With well-recorded documentation, teachers can use what is learned in conferences. Only with well-recorded documentation can what teachers learn in conferences be used to effectively meet students' mathematical learning needs.

Math conferences are addressed in greater depth in my book *Guided Math Conferences* (2014) for teachers who are interested in more information about this component of Guided Math.

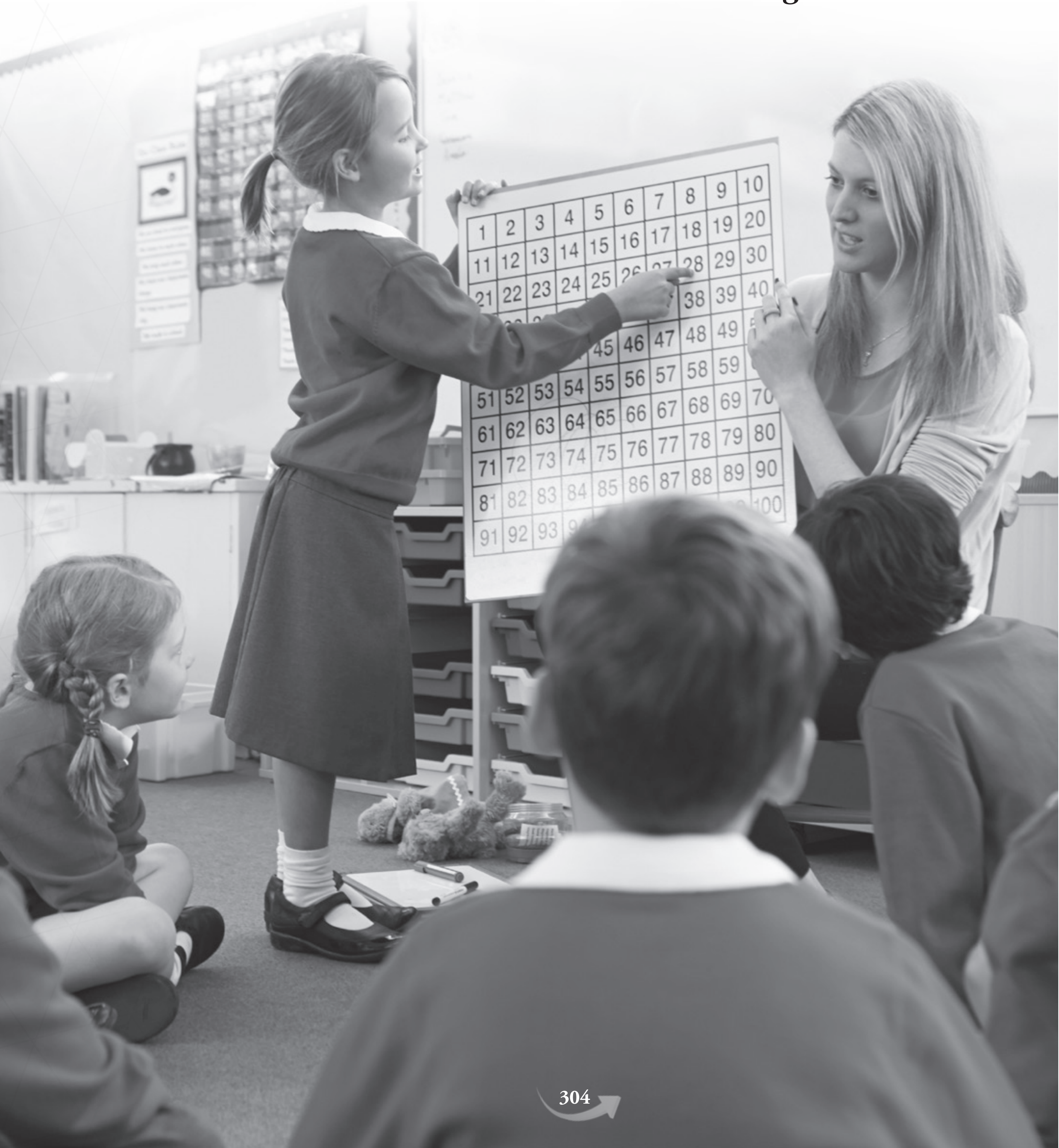
## Review and Reflect

1. In what ways do you probe your students' mathematical thinking? How effective are they?

2. Do you currently conduct math conferences? If so, how frequently? If you are not able to confer as often as you would like, what prevents it?

3. What advantage is there to having a conference structure in mind as you confer with students?

***Effective teaching begins with knowing about students and their mathematical knowledge.***



# Chapter 8

## Guided Math Assessments

In her book *Making Classroom Assessment Work* (2000), Anne Davies beautifully describes assessment as being analogous to the inuksuit of the Native Americans of the North American Arctic region. Inuksuit are manmade stone markers often stacked in the shape of a person and are used for navigation on the tundra. On the harsh landscape of the Arctic tundra, there are few natural landmarks, so inuksuit can make the difference between arriving safely at a destination or not arriving at all. Finding the way through instruction without direction from assessment may not be as dangerous as being lost on the tundra, but without this guidance, its effectiveness is diminished.

Effective teaching begins with knowing about students and their mathematical knowledge. Recent research suggests that teachers' knowledge of both their students' mathematical understanding and misconceptions impacts their instruction "is an important resource for the production of quality instruction and student learning" (Hill and Chin 2018, 1106). Information gathered through assessment not only lets teachers know where they need to begin with their students but also guides their instruction and their students' learning. The more a teacher knows about their students' learning, the more accurately they can tailor their instruction to meet unique student needs. In addition, the more students know about the criteria for quality work and how to assess their own work against these standards, the more likely they are to strive to meet them. The National Council of Teachers of Mathematics put it this way: "Assessment should not merely be done to students; rather, it should also be done for students, to guide and enhance their learning" (2000, 22). In education, the term *assessment* is used in two distinct ways. To use assessment well, teachers must be aware of these differences. Although the two serve different purposes, their roles are complementary.

Teachers constantly search for and gather evidence of their students' learning. Sometimes this is done formally through written assignments and tests and sometimes informally through observations and conversations. When teachers collect evidence of student learning to inform their teaching and enhance student learning, they are conducting formative assessments. Many teachers also teach students how to self-assess their learning and work. Those students who regularly engage in self-assessment consciously monitor their own learning and, as a result, become better students. Additionally, teachers gain new perspectives when they are attentive to students' self-assessments. These assessments are often more accurate than what teachers can gather from simply checking a work product or even observing students as they work. The insights gained are another form of formative assessment and offer additional guidance to teachers as they plan instruction for students. With formative assessment, teachers collect a large amount of information in a variety of ways from a relatively small number of students. Because of the way teachers rely on this assessment data to meet learning needs, formative assessment is truly assessment *for* learning rather than solely *of* learning.

Summative assessment, on the other hand, is a review of the evidence of student learning to decide whether students have learned what they need to know and how well they have learned it. As such, it is more evaluative in nature and consists of judging or placing a value on student achievement. In essence, it is assessment *of* what has been learned. Often, with summative assessments, a small amount of information is collected from a large number of students, as is the case with mandated high-stakes testing. Summative assessments are often valued as forms of accountability. Their data is used for reporting student progress to others and for identifying learning trends (Davies 2000).

## Rationales for Formative and Summative Assessment

Both formative and summative assessments are essential aspects of teaching and learning. Unfortunately, the crucial roles they play daily in enhancing student achievement are sometimes overshadowed by the attention given to well-publicized state-mandated tests. Irene Fountas and Gay Su Pinnell (1996, 73) list the following rationales for systematic assessment:

- continually informing teaching decisions
- systematically assessing students' strengths and knowledge
- finding out what students can do, both independently and with support
- documenting progress for parents and students
- summarizing achievement and learning over a given period—six weeks, a year, or longer
- reporting to administrators, school-board members, and various stakeholders in the community

These rationales focus first on the learning process in the classroom and then on the role of assessment and evaluation for accountability. The first three reasons for assessment listed above apply directly to instruction and student learning. Although informing teaching decisions is first on the list, all three are interdependent and cyclical. Any of them could be said to come first, since teaching and learning are interactive processes. How can teachers make informed teaching decisions without first understanding what their students know and can do? Once a lesson is taught, teachers must find out what their students know and can do to plan the next instructional steps. Teachers plan, teach, assess learning, and adjust their instruction in a cyclical manner. Exemplary teachers even assess and adjust *as* they teach.

Evidence of student learning gathered by teachers is not only used to plan instruction; it is also shared on an ongoing basis with students and parents. Students can improve their performances and increase their learning when they know precisely what they have done well and exactly what they need to improve. The feedback they receive from assessments has the potential to maximize their achievement. Timely, regular reports allow parents to monitor their children's progress. These reports give parents information so they can assist their children at home and reinforce productive work habits. Timely, regular reports to parents also prevent unpleasant surprises when report cards go home or conferences are conducted.

Periodic summative assessments serve a more evaluative purpose. These assessments answer questions such as the following:

- How well did students master the curriculum?
- Are students ready to move on?
- How did the class as a whole perform?
- Does anything need to be retaught before instruction proceeds?

These evaluations inform both students and their parents about student *learning*. They also give teachers an idea of how successful they have been in *teaching* the curriculum. Reflecting on the data from these evaluations helps teachers refine and focus their teaching. When used effectively, summative assessments are important in making learning and teaching visible to everyone involved.

School administrators and school leadership teams often use the data from summative evaluations to make school-wide decisions. Teachers and administrators examine data from these assessments on a regular basis to determine how well students are learning. Rather than waiting until the end of the year to learn how students performed, periodic performance reviews based on data from leading indicators allow schools to fine-tune the interventions they provide for underperforming learners throughout the school year.

The data from high-stakes tests has an enormous impact when it is used by school systems and states to measure the quality of schools. Student success on these tests is often equated with the quality of education provided. There is a widespread belief that the consistent monitoring of learning through standardized testing intensifies and hastens school improvement. There are undoubtedly benefits from public scrutiny of school performance. However, an emphasis on this kind of accountability ignores the consequences of the extensive use of mandated standardized testing. While students whose test scores are good often redouble their efforts and achieve more than they would have otherwise, students who experience repeated academic failure too often simply give up in hopelessness. As Richard Stiggins describes, "Some come to slay the dragon, while others expect to be devoured by it. As a result, high-stakes testing will enhance the learning of some, while discouraging others and causing them to give up" (2002, 758).

Yet, without the effective use of assessment within classrooms to enhance student achievement, teachers may actually hinder the success of the very students that they hope to motivate to succeed. Although teachers may have little say in what testing is mandated, they do decide what kinds of assessments they use in their classrooms and the ways in which they use those assessments to inform their teaching.

## Assessment and Learning in Guided Math

Ongoing formative assessment is essential to inform instruction, especially with Guided Math. Teachers find it difficult to successfully guide students' mathematic understanding without a firm grasp of assessment and how best to use it to adjust instruction. The specificity of information derived from formative assessments is crucial in steering teachers as they make day-to-day instructional decisions. Therefore, teachers must assess more frequently and manage the assessment data effectively. By accumulating, summarizing, analyzing, and reporting assessment results efficiently, teachers have in hand the information they need to make instructional decisions more effectively (Stiggins 2005).

## A Vision for Learning

When beginning a task, most people want to know precisely what needs to be done to be successful. Young artists hone their skills by trying to replicate masterpieces. Musicians listen to the works of virtuosos. Writers devour mentor texts by acclaimed authors. When teachers give their students an idea of what success looks like, they have a much better chance of attaining it. This is especially true with students who lack the experience and background knowledge needed to construct their own visions.

When planning a unit of study, teachers should closely examine the standards, clearly identifying what their students need to learn and how they will know whether students have learned it. Unfortunately, teachers' expectations for student learning are not always shared with students. While some students seem to easily figure out what they need to do to be successful, many students find it difficult to understand what is expected of them. When teachers clearly share these expectations with their students, it is easier for students to focus on important concepts during lessons and monitor their own learning.

Anne Davies lists three steps for teachers to use when describing learning expectations to students (2000, 20):

- *Describe what students need to learn in a language that students and parents will understand*. Summarize learning goals in clear, simple language. This is a more difficult process than it may seem. The goal is a strong alignment of the student-friendly language of the learning expectations with the standards themselves.
- S*hare the description with students and explain how it relates to success in life outside of school.* As a unit of study begins, students should be given a description of what they need to learn and an opportunity to discuss exactly what it entails. When the descriptions are accompanied by samples of what success looks like, students are able to monitor their own learning. If students are shown a set of samples or exemplars that illustrate what development looks like over time, the progression to the learning destination becomes clearer. These exemplars can be used to help develop criteria for success with students, show ways students can represent their learning, assess and provide descriptive feedback, and help parents understand learning expectations.
- *Use the description to guide instruction, formative assessment, and summative evaluations.* Since the descriptions are aligned directly to the standards, instruction should align directly with the descriptions provided to students. Throughout the unit, the assessments and evaluations should be based on the evidence of success described. When students know exactly what the learning expectations are, assessments and evaluations should not surprise them.

## Establishing Criteria for Success with Checklists and Rubrics

Teachers cannot assess student learning without first establishing clear criteria for success. Teachers should carefully examine the standards to understand what should be construed as evidence of success. In what ways will students show how well they know or can do what the standards specify? In the past, most teachers relied almost solely on assessments included with textbooks or created their own assessments that were often based on what had been taught rather than on the standards to be mastered.

With today's emphasis on standards-based learning, teachers determine how they will assess student learning by focusing on the standards before planning for instruction. Many teachers create standards-based checklists or rubrics to assess student learning. Rubrics and checklists can be used for both summative and formative assessments, as well as for self-assessment by students. Students can use checklists or rubrics to monitor, revisit, and revise their work before it is graded. Additionally, students can use these assessment tools to conference with their peers. The benefits of peer conferences are many. In fact, students who engage in peer conferences often gain a better understanding of the criteria while, at the same time, honing their communication skills.

To help students fully understand the criteria for quality work, teachers can involve them in the process of creating checklists and rubrics. By providing a series of work samples showing how student learning progresses over time and discussing the merits of these exemplars with students, teachers help their students discover what is required to meet the standard. After these discussions, students, with teacher guidance, can then participate in determining and listing the criteria to be used later in assessing their work. The criteria may also be the basis to create checklists or rubrics teachers use for assessing work products of students. Additionally, the checklists or rubrics are used by students to guide them in monitoring their own progress.

The established criteria for success also serve as the basis for descriptive feedback. Teachers let students know specifically what it is that they have done well and what may need more work based on these criteria. Effective feedback from teachers prompts students to improve the quality of their work based on these constructive comments.

## Using Checklists

The checklist in Figure 8.1 provides an example of criteria for problem solving. Although the criteria are listed and specific, the checklist does not describe what is required for the problem-solving work to be considered proficient or exemplary in each criterion. It only indicates whether the criteria were met. Though a checklist may be more easily created than a rubric, it may not be as effective when used with students unless the teacher provides a significant amount of very specific feedback to support students as they strive to improve their work. This checklist has a column to indicate if students have met the criteria and another column to indicate that criteria have not yet been met, which clearly communicates to students that more work is expected in order to meet the criteria. After checklists are completed for work products, students are encouraged to revise their work so that it will meet the criteria described.

**Figure 8.1—Problem-Solving Checklist**

| Criteria | Met | Not Yet Met | Comments |
|---|---|---|---|
| **1.** Mathematical representations of the problem are appropriate. | | | |
| **2.** Appropriate strategies are used to solve the problem. | | | |

| Criteria | Met | Not Yet Met | Comments |
|---|---|---|---|
| **3.** Computation is accurate. | | | |
| **4.** Problem-solving process is clearly explained. | | | |
| **5.** Problem solving is extended through recognition of patterns, relationships, or connections to other areas of mathematics or to real-life applications. | | | |

While checklists communicate whether students have met specific criteria in their work product, they do not distinguish levels of quality by measuring how well they met the criteria. Teachers, peers, or students themselves should add comments regarding the quality of the work for each criterion, to let students know how well they met each and focus on ways to improve.

## Using Rubrics

In contrast to a checklist, a rubric is a scoring guide that sets forth precise levels of quality for each criterion. (See Figure 8.2 on pages 315–316.) Rubrics usually list several criteria or domains and then specify gradations of proficiency for each. Sometimes, a scale is included, so a numerical grade may be determined using the rubric. Because it is much more specific than a checklist, it is easier for students to identify precisely how to improve their work. Rubrics developed with student input are often the most effective because students understand them more thoroughly. They are effective tools for student self-assessment and peer review.

According to Heidi Goodrich Andrade (2005), rubrics appeal to both teachers and students for the following reasons:

- *Rubrics are powerful tools for teaching and assessment*. They show students exactly what is expected for quality work and how they can improve their work to meet those expectations.
- *Rubrics lead students to become more thoughtful in judging their own work and the work of others.* With consistent use of rubrics for self- and peer-assessment, students' sense of responsibility for their own work increases.
- *Rubrics decrease the amount of time spent by teachers in assessing the work of students.* Much of the assessment traditionally done by teachers will have already been addressed with self- and peer-assessment by students. If not, teachers can simply highlight an item on the rubric to clearly identify either flaws or strengths evident in student work.
- *Rubrics have an "accordion" nature that makes them ideal for use with heterogeneous groups of students.* The gradations for each criterion can be stretched to accurately describe both superior work and reflect the work of students with special needs.
- *Rubrics are easy to use with students and parents to let them know what students need to do to be successful.*

Rubrics developed and used to assess multiple tasks or assignments allow students to become very familiar with the criteria and the levels of proficiency described because of their repeated use. These assessment tools should be available to students before work begins and should also be posted in the classroom for ongoing reference, if needed, by either teachers or students.

Teachers should model using a rubric by thinking aloud as they describe how they apply the criteria in the rubric to assess the quality of a sample work product. Later, when students work together and are facilitated by a teacher, they practice assessing other exemplars using the same rubric. By seeing the process first modeled and then by working as a group to use the rubric to assess a work sample, students learn how to use this form of assessment independently. Not only do students come to understand a teacher's assessment of their work as conveyed by a rubric, but they also develop the capacity to use a rubric to self-assess their own work and that of their peers.

**Figure 8.2—Problem-Solving Rubric**

| Domain | Exemplary | Proficient | Developing | Emerging |
|---|---|---|---|---|
| **Conceptual Understanding** | Mathematical representations help clarify the problem's meaning.<br>Inferred or hidden information is used.<br>Procedures used will lead to concise and efficient solution. | Mathematical representations are appropriate.<br>All the relevant information is used.<br>Procedures used will lead to a correct solution. | Mathematical representations are inefficient or inaccurate.<br>Some, but not all, of the relevant information is used.<br>Procedures used may lead to a partially correct solution. | Mathematical representations are incorrect.<br>The wrong information is used.<br>Procedures used will not solve the problem. |
| **Reasoning** | Innovative, creative strategies are used to solve the problem.<br>Solution is proved to be correct.<br>Examples and counterexamples are given to support the logic of solution. | Appropriate strategies are used to solve the problem.<br>Each step is justified.<br>Logic of solution is obvious. | Oversimplified strategies are used to solve the problem.<br>Little reasoning is offered that justifies the work.<br>Leaps in logic are hard to follow. | Strategies used are not appropriate.<br>Reasoning does not support the work.<br>Logic is not apparent. |

**Figure 8.2—Problem-Solving Rubric** *(cont.)*

| Domain | Exemplary | Proficient | Developing | Emerging |
|---|---|---|---|---|
| **Computation and Execution** | All aspects of the solution are accurate.<br>Multiple representations verify the solution.<br>Multiple ways to compute the answer are shown. | Computation is accurate.<br>Representations are complete and accurate.<br>Work clearly supports the solution. | Minor computational errors are made.<br>Representations are mostly correct, but not accurately labeled.<br>Evidence for solutions is inconsistent or unclear. | Serious errors in computation lead to incorrect solution.<br>Representations are seriously flawed.<br>No evidence is given of how answer was computed. |
| **Communication** | Explanation is clear and concise.<br>Mathematical vocabulary is used precisely. | Explanation is easy to follow.<br>Mathematical vocabulary is used correctly. | Explanation is not clearly stated.<br>Mathematical vocabulary is used imprecisely. | Little or no explanation for the work is given.<br>Mathematical vocabulary is used incorrectly. |
| **Connections** | A general rule or formula for solving related problems is created.<br>Connection to other disciplines or real-life applications is accurate and realistic. | Recognizes important patterns and relationships.<br>Connection is made to other disciplines or real-life applications. | Recognizes some patterns and relationships.<br>Finds a hint of connections to other disciplines or real-life applications. | Unable to recognize patterns and relationships.<br>Finds no connections to other disciplines or real-life applications. |

*Adapted from a rubric created by the Mathematics and Science Education Center of the Northwest Regional Educational Laboratory* (educationnorthwest.org/sites/default/files/scoregrid.pdf)

When rubrics are well aligned with the mathematical standards, they are not just about assessing a student's work, they are about teaching. Rubrics help guide and focus instruction for teachers. They help students understand the specific learning goals of their assignments so they can concentrate their efforts on those goals. And finally, they allow teachers to provide critiques of student work that are individualized and constructive in a timely manner (Andrade 2005).

## The Value of Descriptive Feedback

Research shows that one of the most effective instructional strategies that teachers employ is providing students with specific and descriptive feedback. Teacher feedback communicates to students how well they are doing relative to the learning objectives (Marzano, Pickering, and Pollock 2001; Hattie 1992). According to the National Council of Teachers of Mathematics, feedback supports "students in setting goals, assuming responsibility for their own learning and becoming more independent learners" (2000, 22). Additionally, feedback helps them understand exactly what is required for quality work products. When feedback is specific to learning targets, it provides scaffolded support for students as they work "to close the gap between where they are now and where we want them to be" (Stiggins 2005, 324).

In most aspects of their lives, people receive ongoing feedback in one form or another and then very often change what they are doing in response to it. Athletes are coached as they train. Their coaches provide specific feedback to maximize performance. The coaching does not occur only at the end of an event, but instead is offered during training so that the athletes are able to adjust and improve their performances before being tested in a competition. Similarly, teachers who provide feedback to their students during their work, rather than after it has been completed, help students maximize their learning.

It is important for teachers to talk with students about their learning—letting them know both what they are doing well and what needs improvement. Students can then use the feedback to adjust what they are doing and how they are doing it. According to Anne Davies in her book *Making Classroom Assessment Work* (2000, 13), the most effective descriptive feedback does the following things:

- comes *during* as well as *after* the learning
- is easily understood
- is related directly to the learning
- is specific, so performance can improve
- involves choice on the part of the learner as to the type of feedback and how to receive it

- is part of an ongoing conversation about learning
- is in comparison to models, exemplars, samples, or descriptions
- is about the performance or the work—not the person

Marzano, Pickering, and Pollock (2001, 96–99) examined numerous studies on the effects of providing feedback and drew the following generalizations to guide its use:

- *Feedback should be "corrective" in nature.* Feedback that only tells students that their answers are correct or incorrect actually has a negative effect on achievement. However, significant positive effects were reported when the feedback lets students know what it is they are doing that is correct or incorrect. Furthermore, when students are asked to continue to work on a task until they are successful, achievement is enhanced.
- *Feedback should be timely.* Timeliness appears to be critical to the effectiveness of feedback. Generally, the greater the delay in receiving feedback, the less improvement there will be in achievement.
- *Feedback should be specific to criterion.* By linking feedback to previously established criteria for success, students know how they can improve their achievement. In contrast, when feedback is normative in nature, such as numerical or letter grades, students merely know how they did in comparison to the performance of other students. They are often at a loss as to how to improve what they are doing. Students may be motivated to improve but may not know how they can be more successful. Feedback based on checklists or rubrics that describe the criteria necessary for a quality performance foster improved student achievement.
- *Students can effectively provide some of their own feedback.* When students are aware of specific expectations for products or performance, they can successfully track their achievement as learning occurs. Releasing this responsibility to students encourages them to become more cognizant of the quality of both their work and performance relative to the established criteria and, consequently, increases their learning.

## Involving Students in the Assessment Process

Fosnot and Dolk eloquently describe what makes student involvement in the assessment process so valuable: "When young mathematicians are hard at work, they are thinking, they become puzzled, they become intrigued; they are learning to see their world through a mathematical lens. Assessment needs to capture the view this lens reveals" (2001, 169).

What teachers learn about students from their work products has its limitations. The ultimate evidence of learning comes from within learners themselves. What exactly do they know, perceive, infer, and understand? Without access to glimpses of the world through their own unique mathematical lenses, teachers may be blind to many of their achievements or their needs. The closest teachers can come to fully understanding their students' mathematical perspectives is through involving students in self-assessment.

When students are involved in the assessment of their own learning, teachers have a more complete vision of the students' achievements. That benefit is important for teaching. But, student participation in the assessment process itself also enhances learning. Students shift from being passively involved to becoming more active learners. According to Davies, students who are involved in shaping and assessing their own learning are more likely to do the following:

- understand what is expected of them
- access prior knowledge
- have some ownership over making it happen
- be able to give themselves descriptive feedback as they are learning
- give the information teachers need to adjust their teaching (2000, 4)

Students who develop greater metacognitive awareness of their own mathematical learning are more likely to tap into their prior experiences and background knowledge to make connections that strengthen their understanding. Becoming more conscious of the "how" of learning, they are motivated to assume more responsibility for their own learning. By assessing their own achievements using criteria in checklists or rubrics and then communicating evidence of their learning to their teachers, students essentially reveal views of the world as seen through their own mathematical lenses. These glimpses of the mathematical perspectives of students provide insights that allow teachers to more effectively meet the individual learning needs of their students. Self-assessment by students, far from being a luxury, is an essential component of formative assessment (Black and Wiliam 1998).

Closely associated with self-assessment is the practice of having students involved in setting their own learning goals. Once students are more aware of the learning process, what they need to learn, and where they are in the learning trajectory, teachers can help them set specific goals for their next learning steps. Identifying their own individual goals increases students' motivation because of their added value for them (Madden 1997). Students feel a sense of ownership, of being in charge of their own learning, monitoring their own successes, and making crucial decisions about how to maximize their achievements (Stiggins 2002).

## Formative Assessment

Formative assessment is intrinsically an assessment for learning, as opposed to simply assessment of what has been learned. It is a systematic process to continuously gather evidence about individual and class learning so teachers can adapt instruction to enhance student achievement. Formative assessment and teaching are complementary processes, one supporting the other. Studies have documented the positive effect of formative assessment in reducing the range of achievement and raising overall achievement (Black and Wiliam 1998). When used effectively, formative assessment narrows the achievement gap.

Teachers rely on the information from formative assessments to advance, rather than just check on, the mathematical learning of their students. According to Richard Stiggins (2002), teachers do this in various ways.

- understanding and articulating, in advance of teaching, the achievement targets that their students are to hit;
- informing students about learning goals, in terms that students understand, from the very beginning of the teaching and learning process;
- becoming assessment literate and thus able to transform their expectations into assessment exercises and scoring procedures that accurately reflect student achievement;
- using classroom assessments to build students' confidence as learners and help them take responsibility for their own learning, in order to lay foundations for lifelong learning;
- translating classroom-assessment results into frequent descriptive feedback (versus judgmental feedback) for students, providing them with specific insights as to how to improve;
- engaging students in regular self-assessment, with standards held constant so that students can watch themselves grow over time and thus feel in charge of their own success; and
- actively involving students in communicating with their teachers and their families about their achievement status and improvement.

Teachers who recognize the value of formative assessment and depend on the information it provides have to know how to assess their students' mathematical knowledge and skills on an ongoing basis to be aware of their achievements and learning needs in real time. Furthermore, they must be able to manage their classrooms in such a way that they can differentiate their teaching to meet the instructional needs they have identified. With traditional whole-class instruction, this is difficult to do. When teachers implement Guided Math in their classrooms, they have a model that allows them to group students flexibly to effectively address the identified instructional next steps for students.

## Using Assessments to Inform Guided Math Groups

Teachers who use Guided Reading in their classrooms know that students are grouped according to their reading levels. These levels are determined by administering running records. Using the results of the running records, an instructional reading level is determined for each student. This is a level that is not too easy, but not so hard that it would be extremely frustrating for the student to read. The instructional level is one at which, with a little support from the teacher, a student can read successfully. With guidance and practice, the student will be able to read at that level independently. As that level becomes too easy for the student, the instructional level is raised.

Unfortunately, in mathematics, there is no comparable overall assessment teachers can use in grouping students for instruction. Instead, teachers group their students for Guided Math small-group lessons based on the data from a variety of assessment methods. Some teachers use pretests for initial grouping when beginning a new unit. If pretests mirror the unit posttests, they supply very little relevant data for grouping. This data merely highlights what students have already mastered, but the data provides no information about gaps in essential foundational knowledge and skills. If unit pretests assess students' prerequisite knowledge and skills for the unit, however, the data may be helpful for grouping students according to their learning needs.

As all teachers know, what students know and can do changes dramatically from week to week. So, the assessment data teachers use to form groups is most valuable when it is timely. Throughout a unit, teachers may administer brief formative assessments to monitor learning and then, based on the results, adjust the grouping as needed. Some teachers create checklists of knowledge and skills based on instructional standards to record real time assessment information. Informal observations of student work and conferences with students may also help teachers create or modify groups based on levels of student learning. Additionally, some computerized programs offer prescriptive assessment data.

To collect the most useful data, teachers should first identify the specific prerequisite knowledge and skills students need to be successful with a lesson to be taught. If teachers already have information from earlier formal assessments or informal observations regarding students' mastery of those essential prerequisite knowledge and skills, that data can be used to form homogeneous groups for small-group lessons. Students who have similar learning needs should be grouped together for small-group lessons.

If relevant assessment information is not available, teachers have the option of creating and administering simple exit tickets that precisely target the prerequisite knowledge and skills students need for an upcoming lesson. Examples of formative assessments tailored to specific lessons are provided in Chapter 5. Having students complete a brief formative assessment the day before a lesson increases the accuracy of the student grouping. During the lesson, teachers address the specific learning needs that are identified during these lessons. Because teachers are so knowledgeable about what their students know and can do, they are able to efficiently target any gaps in essential background knowledge and skills and then introduce new content to students with scaffolded support if needed, as students work through the new mathematical content. With this type of differentiated lesson, gaps in students' background knowledge and skills are efficiently addressed, and students can also move into the current lesson content immediately. Additionally, with the small-group lesson differentiation options, teachers can provide additional challenge for students who are ready to tackle the new content in greater depth.

The goal of flexible grouping in Guided Math is similar to that of Guided Reading. Ideally, students work at instructional levels that are just right for them. It should not be too easy, nor so difficult that it negatively affects learning. Students are challenged to draw upon what they already know about math to extend their mathematical understanding as they wrestle with thought-provoking problems and act as mathematicians. Teachers play an essential role in facilitating student learning. They constantly gauge their students' needs and adjust instruction in response. They recognize and understand the importance of students' engagement in building mathematical understanding and proficiency. And they provide opportunities for ongoing practice because students need to become mathematically proficient.

The composition of some groups may remain fairly constant for several weeks at a time. But for most, the group of students will change frequently. In addition to teaching current lessons, teachers may meet with a group only once or twice to address a specific area of concern. The utility of the Guided Math framework is that it provides such flexibility to teachers to allow them to shape teaching to align with student needs.

When using the Guided Math framework, teachers call upon their professional judgment to determine what their students need, decide how best to group students for lessons, and plan lessons that target their students' needs. Having access to data from timely formative assessments is key to doing this well. In these classrooms, the strong links between teaching, learning, and assessment are particularly evident. Although summative evaluations certainly play an important role in these classrooms, they are not substitutes for consistent and pervasive use of formative assessment. Assessment *for* learning is, without a doubt, a defining characteristic of Guided Math.

## Chapter Snapshot

Effective assessment consists of far more than testing—it is a way to measure students' progress, their understandings and misconceptions, their abilities to solve problems, think critically, and apply their knowledge to new situations. What teachers learn from assessments is used to tailor instruction to meet the specific learning needs of all students. Furthermore, assessment serves not only to hold students accountable but also to hold teachers accountable.

Whether formative or summative assessments are most useful depends upon the type of information teachers require. Formative assessments, administered throughout a unit of study, offer a way to monitor student progress. On the other hand, summative assessments, given at the end of a unit, serve as a gauge of students' total mastery of mathematical content or processes at the end of instruction. Rubrics and checklists are two methods of assessment that allow students and teachers to compare work to specific criteria and can be used for either formative or summative assessment. Regardless of the type of assessment, it is important for students to receive feedback on their progress that is constructive, specific, and timely.

## Review and Reflect

1. Why is assessment essential in a Guided Math classroom? What role does it play in teaching and in learning?

2. Reflect on your classroom assessment. What kinds of assessments do you use in your classroom? Is there a blend of formative assessments and summative evaluations?

***Students who have opportunities to explore mathematical ideas, solve perplexing problems, and generate conjectures truly act as mathematicians.***

# Chapter 9

## Putting Guided Math into Practice

Students entering classrooms at the beginning of each school year seldom think of themselves as mathematicians. All too often, they consider mathematics to be a boring set of rules they must memorize for tests. Rarely do they recognize any mathematical connections to their own daily lives. Mathematics is not seen as a system of relationships and patterns discovered by people who ponder and reflect on their mathematical observations. Nor is it considered a subject of wonder and puzzlement. Indeed, the view of the subject held by most students is very narrow.

By adding rigor and relevance to their mathematical instruction, teachers involve their students in learning experiences in ways that move them beyond this limited view. Teachers stimulate their curiosity about math and open their eyes to the mathematical musings that lead mathematicians to extol on the beauty of the discipline. Students who have opportunities to explore mathematical ideas, solve perplexing problems, and generate conjectures truly act as mathematicians. And, in the process, they learn to appreciate the value of mathematics. Accomplishing this with the traditional, whole-class approach to mathematics is difficult. This may be why so few adults experienced the wonder of mathematics when they were in elementary, middle, or high school.

The desire to inspire students and meet their diverse learning needs motivates teachers to continuously reflect on and adapt their teaching practices. The Guided Math framework offers teachers an alternative to the standard, whole-class model so frequently used for mathematics instruction. With Guided Math, teachers have the option of planning a variety of mathematics lessons to not only target specific instructional needs, but also allow students to participate in investigations and problem-solving activities that lead them to notice and wonder about mathematical patterns and relationships and encourage them to apply what they are learning about mathematical concepts and skills to real-world situations.

## Nurturing Student Mathematicians

When implementing Guided Math, it is important for teachers to keep in mind what leads students to think mathematically. Ideally, math students should do the following:

- enjoy exploring problem solving in a safe environment where errors are seen as learning opportunities
- have chances to try out strategies on a variety of challenging problems
- learn to identify appropriate strategies to use when problem solving
- feel the satisfaction of struggling with and solving difficult problems
- receive specific feedback from teachers and peers on their mathematical work
- participate in conversations in which they use mathematical vocabulary and justify their work
- expand their mathematical understanding through problem-solving tasks and mathematical discourse
- learn to recognize patterns and relationships leading to the development of conjectures
- make mathematical connections

If given these opportunities, students discover that thinking mathematically makes sense. Moreover, students come to believe that *they* are capable of making sense of mathematics (Van de Walle and Lovin 2006). Of course, there is no one simple method for teaching mathematics. Teaching is a complex process. Still, there are some important characteristics of effective mathematics instruction. Teachers nurture and develop their students' mathematical understanding when they do the following:

- understand and teach the mathematics standards
- use ongoing assessment to guide their instruction
- lead their students to construct broader mathematical meaning from their specific tasks
- create a community of learners where mathematics communication, both oral and written, is the norm

In *Comprehending Math: Adapting Reading Strategies to Teach Mathematics K–6*, Arthur Hyde describes the way in which thinking, language, and mathematics are linked—braided together into a "tightly knit entity like a rope that is stronger than the individual strands" (2006, 1–2). Just as reading is closely connected to thinking and language, so is mathematics. Thinking, language, and mathematics are inseparable, mutually supportive, and necessary. In reading, students initially use discrete decoding skills they have learned but then go beyond that to construct meaning by interacting with the text. They draw upon their background knowledge, infer, make connections, ask questions, visualize, determine importance, and synthesize, combining the new information with the old to construct meaning. In working with mathematics, students need to use essentially the same strategies to construct mathematical meaning.

Expanding on the similarities between reading-comprehension strategies and mathematics-comprehension strategies, Hyde (2006, 4) lists eight questions adapted from reading strategies to guide math teachers (see page 330).

1. Are students expected to *construct their own meaning* in mathematics?
2. Are students encouraged to have *ownership of their problem solving*—to choose to use mathematics for purposes they set for themselves?
3. Are students encouraged to do problem solving for *authentic purposes*?
4. Are students encouraged to do *voluntary mathematics*, selecting tasks for information, pleasure, or to fulfill personal goals?
5. How is mathematics instruction *scaffolded*?
6. Does the school help teachers and students build a *rich, mathematically literate environment* or community?
7. Are students encouraged to see the *big picture, important concepts, and vital connections* versus isolated pieces of mathematics?
8. Is *forgiveness* granted to students in mathematics? Is making *mistakes a natural* part of learning? Is doing mathematics seen as a dynamic process that incorporates *planning, drafting, revising, editing,* and *publishing*?

These questions are worth considering as teachers implement Guided Math. The word *teachers* is plural because it is important to have support during times of change. When teachers join together to form learning communities to refine their instructional practices, the process is often much easier and is certainly more effective. Working together, they plan, teach, reflect, share, and then refine. Their ongoing collaboration inspires growth in the teaching dynamics of the entire group.

## Creating a Guided Math Schedule

Teachers have a great deal of flexibility in scheduling the Guided Math components in a math period. Teachers who teach self-contained classes have the added flexibility of being able to incorporate Guided

Math components throughout the day. As students arrive in the morning, they may have a math stretch to complete. Math-related literature may be read to a class during a literacy read-aloud time and then discussed later during the math block. The more that math instruction is integrated into other subject areas, the better. It not only provides a greater amount of time for students to interact with mathematical ideas, but also leads them to recognize the relevance of mathematics in all aspects of life.

Teachers in schools where instruction is departmentalized are still able to exercise flexibility in scheduling the components of the framework. While Guided Math is most effective when math workshop and small-group lessons occur daily, some teachers opt to teach lessons using whole-class instruction on some days. Another option teachers may choose is to assign math warm-up tasks on some days but not others. Teachers may choose to meet with only one small group for a lesson and then confer with students for the rest of the period. Or, they may meet for five minutes with two students and then meet with three larger groups for 15 minutes each.

Teachers should rely upon their professional judgment in planning. Even so, most teachers find it beneficial to have a basic structure for their classes that is used routinely and that students can expect. Teachers of all grade levels can incorporate the components of Guided Math into a single class period. (See sample class schedule in Figure 9.1 on page 332.) It is important to note that most instruction is done in a small-group setting. A whole-class mini lesson is used only when there is a compelling reason to do so. For the majority of the class period, students are working independently in math workstations as the teacher meets with several small groups. The small-group lessons are differentiated to meet their distinct learning needs, filling gaps before moving students into the current lesson or providing additional challenges.

**Figure 9.1—A Sample Guided Math Class Schedule**

<table>
<tr><th>Time Frame</th><th colspan="2">Guided Math Component</th></tr>
<tr><td>Daily</td><td colspan="2">Classroom Environment of Numeracy<br>Nurture a sense of community among student mathematicians. Surround them with math: real-life math tasks, data analysis, math word walls, instruments for measurement, mathematical communication, class-created math charts, graphic organizers, calendars, and evidence of problem solving.</td></tr>
<tr><td>5–15 Minutes</td><td colspan="2">Math Warm-Ups<br>In addition to warm-up activities, choose five hot spot areas and address one of them each day of the week throughout the year.</td></tr>
<tr><td>5 Minutes</td><td colspan="2">Whole-Group Mini Lesson (optional)<br>Use only if there is a compelling reason for using large group instruction (e.g., read-aloud of math-related literature, math word wall work, or activating strategies).</td></tr>
<tr><td>40–55 Minutes</td><td>Small-Group Lessons<br>Group students flexibly based on their immediate learning needs. Differentiate instruction to fill gaps, introduce new concepts, practice, and extend learning. Include time for student reflection and link to future work in each group lesson.</td><td>Math Workshop<br>Students work independently, in pairs, or small groups. Plan math workstation tasks that target skills with which students are secure. Students review concepts and skills from a prior grade or that they have mastered in previous lessons, investigate mathematical ideas, or practice to increase computational fluency.</td></tr>
<tr><td>As Needed</td><td colspan="2">Conferences and Assessment<br>Conduct conferences and assess students as needed, informally or formally. Use assessment results to determine grouping and instruction.</td></tr>
</table>

Teachers may choose to vary the amount of time spent with each small group during a class period (Figure 9.2 on page 334). Students who quickly master a lesson may need less instruction than students who have significant gaps in their essential foundational knowledge and skills. The goal of equity in education is not equal instruction for each student but instead the provision of what each student needs to maximize success. With small-group lessons, teachers can vary the amount of time spent with each group to achieve equity. For instance, if a teacher plans to teach three small-group lessons during a 60-minute class period, it is possible to spend up to 20 minutes with each group for its lesson.

- The teacher may choose to work first with Group 1, whose students, based on assessment data, will need minimal instructional time to understand the topic of the lesson content. The lesson for this group may only require 10 minutes. If the students demonstrate their understanding of the lesson before the 20 minutes has passed, they might be challenged with an additional task to be completed independently or simply asked to return to their workstation tasks.
- The teacher might next choose to meet with Group 2. The assessment data for this group indicates its students do not need additional challenge nor do they need scaffolded support to fill gaps. The instructional time needed for this group to demonstrate their understanding might be a bit longer, perhaps 15 minutes. Once that happens, this group of students can return to their workstations.
- So, of the 60-minute class period, 25 minutes have now elapsed. The teacher then meets with Group 3. This group of students may have some gaps in the prerequisite mathematical knowledge they need to be successful with the content of current lesson. The teacher has 35 minutes remaining in the class period to work with this group, allowing additional time to rebuild their mathematical understanding while introducing new content.

Of course, lessons don't always proceed so smoothly. The teacher chooses to see the groups in this order, so that if the first two groups do not each require a full 20-minute lesson, the teacher will know exactly how much time remains to work with the third group.

The teacher knows that they have up to 20 minutes to spend with each group. If a group needs the entire time, they are given the full 20 minutes. However, if a group does not require the full time allotted, the teacher has the option to add whatever time is left to the lesson(s) of a group(s) that may benefit from that extra time. This kind of variability does oblige teachers to teach efficiently and carefully attend to time. The only time a lesson should exceed the allocated amount of time, in this case 20 minutes, is when an earlier lesson has been accomplished in less than the time allocated.

**Figure 9.2—Varying the Length of Small-Group Lessons**

| | |
|---|---|
| Group 1<br>(solid background knowledge) | 10 minutes |
| Group 2<br>(average background knowledge) | 15 minutes |
| Group 3<br>(significant gaps in knowledge) | 35 minutes |
| **Total** | **60 minutes** |

## Getting Started with Guided Math

A teacher has decided to implement Guided Math in her classroom. She has done the planning and preparation for its implementation. What is next? Teachers may be ready, but what about their students? Guided Math is certainly not what most students have experienced previously. Even students who have been in other Guided Math classrooms may be unaccustomed to the way Guided Math is implemented in their new math classes. Obviously, for Guided Math to work well, students have to understand how it is structured in their new classrooms and how they are expected to behave. In fact, not only must they know what the expectations are, but they must also have time to practice what they should be doing during each of the components, particularly when they are working independently in math workshop.

## Establishing Norms to Nurture a Classroom Mathematical Community

As a school year or semester begins, a class is just a collection of math students who happen to be assigned the same teacher. Some students may know each other well; for others, it is the beginning of new relationships. Teachers who recognize the importance of students' mathematical communication are aware of the responsibility they have in building a sense of community amongst their new students. Only when students trust each other and their teachers are they comfortable sharing their tentative mathematical thinking. It is risky business—especially when students are constructing mathematical meaning from their own unique classroom experiences. Because open and comfortable communication between students is key to stimulating their curiosity and encouraging experiential learning, it is worthwhile for teachers to work closely with their students to develop practical conversational norms. Most students have rarely, if ever, engaged in academic discussions—a far cry from chatting with friends and family members outside of school. Students need specific guidance to help them engage productively in math talk with their peers.

Teachers initiate the process of developing class norms by describing what mathematical conversation is and why it is important. From there, students should brainstorm what kinds of behaviors encourage open communication and what behaviors inhibit it. Some teachers create T-charts to record students' ideas. Next, students can work with teachers to devise a set of norms that promote positive behaviors and limit those that may harm productive discussions. The creation of norms, however, is not all that is needed. Students also need to see productive conversational behaviors modeled, and then they need time to practice them. As they practice, it is important that they receive feedback from the teacher to help them identify what they are doing well and how they can improve.

The norms developed will vary from classroom to classroom, but all norms should address expectations for students' physical behavior, their manner of listening and speaking, their social interaction, and their mathematical thinking. For younger learners, the number of norms should be limited to no more than five and expressed in easily understood language.

1. We share our math ideas.
2. We listen to each other.
3. We think about what others say.
4. We respect others.
5. We learn by sharing ideas with others. (Sammons 2018, 35)

For older students, the number of norms may be increased. See Figure 9.3 for a sample of norms for students in a mathematical community.

**Figure 9.3—Sample Norms for Students in a Mathematical Community**

| | |
|---|---|
| What we do with our bodies | • We show interest by looking at the speakers.<br>• We remain silent and still as we listen. |
| What we do with our minds | • We think carefully about what others are saying.<br>• We use listening comprehension strategies to better understand. |
| How we interact with others | • We allow the speaker to finish before responding.<br>• We share our own ideas when they are relevant.<br>• We respect each other's ideas, even when we disagree.<br>• We are sensitive to the feelings of others. |
| How we express our mathematical thinking | • We use the correct math vocabulary to share our ideas.<br>• We justify our math ideas with evidence. |

*Adapted from Boucher and Sammons 2017; adapted from Sammons 2018, 2017.*

## The First 15 Days

During the first few weeks of school, students should learn what the Guided Math framework is and how it helps them study mathematics. During these first weeks, they begin to collaborate with their peers and develop a robust and productive mathematical learning community. Furthermore, teachers introduce the behavioral expectations for math workshop to their students and give them opportunities to practice those behaviors. A fully functioning math workshop cannot be implemented successfully until students demonstrate an understanding of the expectations, a capacity to meet those expectations, and the ability to work successfully at math workstations.

Although the first weeks of implementation are spent teaching students about Guided Math and their responsibilities, this in no way implies that mathematics is not being taught. Only part of the class period is spent teaching students the routines and procedures during those first 15 days. Most teachers teach a whole-class lesson each day while also setting aside time for students to learn about Guided Math and to develop the capacity to work independently.

As students practice working at workstations, teachers often assign workstation tasks that provide a review of mathematical concepts and skills learned in previous grades. Instead of several weeks of whole-class lessons spent in reviewing these previously taught concepts and skills, the review is accomplished with practice workstation tasks completed by students who are also engaged in building the stamina they need to work independently. In this way, the time students spend practicing how to work at workstations serves as a review of the essential background knowledge for the new mathematical content they will be learning.

Teachers from other grade levels are good sources of ideas for games or other tasks that offer students valuable review and practice of important math concepts and skills. As students work, the teacher should observe their work and provide feedback. This feedback can be given immediately or later during debriefings. During these times, teachers are not teaching small-group lessons or conferring with students, but students should know that teachers are not available for questions. This is a time when students rehearse their workshop behavior. They have to learn how to make decisions independently. What do they do if they finish their work? What if they need additional materials or if they do not understand their task? How can they work productively without interrupting the teacher? The practice specified for the first 15 days gives students the experience they need to become adept at handling problems according to the workshop routines and procedures.

Teachers who are introducing Guided Math, whether it be at the beginning of the school year or whenever they are ready to begin its implementation, can follow this plan for the first 15 days (Sammons and Boucher 2017). Even teachers who decide to ease their way into Guided Math through partial implementation should teach their students the essential routines and procedures for a full fifteen days before expecting their students to work independently in math workstations.

## Week One: Establishing Routines and Procedures

The instructional focus of the first week is teaching students exactly *what* Guided Math is and *why* it is so important to them personally as students. Traditionally, teachers do not share the rationale for instructional strategies. However, it is important to explain the Guided Math framework and why it is important to students. Whole class instruction has many limitations. Students who have mastered the content are often bored. Other students may feel lost or disconnected, especially those with gaps in their background knowledge. Shy or reluctant students may have trouble sharing their ideas with the class (Sammons and Boucher 2017a). Most students are eager to move away from "one-size-fits-all" instruction. During the first week, students learn to appreciate that Guided Math provides "just right" mathematics instruction for them—instruction that targets their immediate learning needs.

After being introduced to Guided Math, the class works to develop a set of routines and procedures. Teachers explicitly teach the Guided Math components, particularly math workshop, and students are encouraged to consider how their behaviors impact those components. Next, teachers brainstorm what math workshop should look like and sound like, as well as how mathematicians work. As a class, students compile a set of routines and procedures they will follow as they work. Of course, teachers should already have in mind clear ideas regarding the behaviors to be addressed. If needed, teachers steer the discussion to ensure the inclusion of the targeted behaviors. The compiled routines and procedures are then recorded on an anchor chart and prominently displayed in the classroom for future reference.

Teachers with more than one math class may choose to develop unique routines with each class or combine the class-generated routines and procedures into a single set for all classes. Even if a teacher prefers to have only one set, students from all classes should be included in developing the routines and procedures. The inclusion of students in this process does more than increase student buy-in and sense of ownership. The thinking and reflection also lead students to a better understanding of the framework, their responsibilities when working within the framework, and their roles as mathematicians.

Figure 9.4 (pages 340–342) offers a sample plan for developing routines and procedures during the first week of Guided Math. For specific lesson ideas for Week 1, see *Guided Math Workshop* (Sammons and Boucher 2017a).

**Figure 9.4—Week 1—Establishing Routines and Procedures**

| **Day 1** | |
|---|---|
| **Focus** | What is math workshop? |
| **Learning Outcomes** | Students describe what math workshop is and why it is important. |
| **Activities** | Class discussion and creation of an anchor chart identifying what math workshop is and why it is important. |
| **Teacher Notes** | Math workshop is important because:<br>• students develop better mathematical understanding and skill<br>• students learn to work independently<br>• students learn to work with partners<br>• teachers can teach small groups and confer one-on-one with students |
| **Day 2** | |
| **Focus** | What does math workshop look like and sound like? |
| **Learning Outcomes** | Students identify how math workshop should look and sound. |
| **Activities** | Review and further clarify what math workshop is and why it is important. Discuss what successful math workshop should look like and sound like. Add to anchor chart. |
| **Teacher Notes** | Math workshop looks like:<br>• students working as mathematicians<br>• students using manipulatives<br>• students writing about math<br>• students playing math games<br>Math workshop sounds like:<br>• students talking about math<br>• students talking at appropriate volumes |

| Day 3 | |
|---|---|
| **Focus** | What do good mathematicians do as they work during math workshop? |
| **Learning Outcomes** | Students know the routines and procedures for math workshop. |
| **Activities** | Review and clarify math workshop anchor chart. Discuss expectations for student behavior during math workshop. Create an anchor chart with no more than six criteria for routines and procedures. Explain to students that they are expected to follow routines and procedures, but that problems may arise. Discuss possible problems that may occur and how to solve those problems independently. |
| **Teacher Notes** | Examples for routines and procedures:<br>• staying on task and in your workspace<br>• cleaning up your workspace when finished<br>• speaking in a soft voice about the math<br>• what to do if you are unsure of your work<br>• using math materials appropriately |
| **Day 4** | |
| **Focus** | What do good mathematicians do as they work during math workshop? |
| **Learning Outcomes** | Students know routines and procedures for independent work during math workshop. |
| **Activities** | Review and clarify what math workshop is and why it is important. Revisit routines and procedures anchor chart. Focus on the first few routines. Examine each routine in detail. |
| **Teacher Notes** | For each routine:<br>• model how it looks and does not look<br>• have students role-play examples<br>• have students role-play nonexamples<br>• have students role-play correct behavior |

**Figure 9.4** *(cont.)*

| **Day 5** | |
|---|---|
| **Focus** | What do good mathematicians do as they work during math workshop? |
| **Learning Outcomes** | Students know routines and procedures for independent work time during math workshop. |
| **Activities** | Review what math workshop is and why it is important. Revisit the routines and procedures anchor chart. Focus on the last few routines. Examine each routine in detail. |
| **Teacher Notes** | For each routine:<br>• model how it looks and does not look<br>• have students role-play examples<br>• have students role-play nonexamples<br>• have students role-play correct behavior |

*(Sammons and Boucher 2017a, 135–137)*

## Week Two: Workstations: The Nuts and Bolts of Math Workshop

When students understand what Guided Math is and have a set of routines and procedures to follow as they work, the focus shifts to practical information about how math workshop functions. Students now learn specifically about designated workspaces, workstation containers, and workstation tasks. Students will still need some review of what they learned during Week 1 and should have opportunities to practice the routines and procedures developed in Week 1.

In Week 2, students learn how to retrieve a workstation and put it away properly. They discover that each workstation contains a menu of tasks, task cards with a list of materials, Talking Points cards, and often, but not always, the materials needed. Students learn to use the workstation menus to find out what tasks the station contains, which must be completed, which tasks are optional, and which have options for differentiation. To ensure that students can work independently, they should also learn how to follow the student instructions provided on the task cards, locate the required task materials, and use the Talking Points cards to support their math talk as they work.

Students learn to work productively and independently during Week 2. During brief periods, students practice the nuts and bolts of math workshop with assigned workstation tasks. Each practice session should be followed with a debrief where students self-assess their work. During this time, teachers offer specific feedback based on their observations. The focus of these discussions is not only on the successful completion of tasks, but more importantly at this point, on students' abilities to work independently and follow the established routines and procedures. With repeated practice sessions of ever-increasing length, students gradually build their stamina for working independently.

Figure 9.5 provides an instructional plan for Week 2 of the first 15 days. Additional information and sample lessons for Week 2 can be found in *Guided Math Workshop* (Sammons and Boucher 2017a).

**Figure 9.5—Week 2—Math Workstations: The Nuts and Bolts of Math Workshop**

| **Day 6** | |
|---|---|
| **Focus** | Where do we work during math workshop? |
| **Learning Outcomes** | Students know where they may work during math workshop. |
| **Activities** | Review routines and procedures. Show students exactly where they will work and how to transition to those workspaces. Have students practice moving to workspaces while following routines and procedures. Then, have students assess how well the transitions worked. What did students do well? What needs more work? Repeat process as needed. |
| **Teacher Notes** | To model the process, have students:<br>• move to their assigned workspaces<br>• return to their seats<br>• debrief and assess how well they did |

**Figure 9.5** *(cont.)*

| **Day 7** | |
|---|---|
| **Focus** | What are math workstations? Where are they stored? |
| **Learning Outcomes** | Students know where math workstations are stored, how they are accessed, and how they are put away. |
| **Activities** | Review routines and procedures and where students work. Introduce where math workstations are stored and accessed. Discuss how students should clean workspaces and return workstations. Provide enough math workstations so students can work in pairs or small groups. Each station should have the same simple task. Have students practice retrieving workstations, moving to workspaces, completing the task, and returning workstations. Then, have students assess how they did. Repeat as needed. |
| **Teacher Notes** | To teach the process:<br>• assign students to groups of four or five<br>• give each group a math workstation<br>• ask students to debrief and assess |
| **Day 8** | |
| **Focus** | How will we know what to do with a math workstation? |
| **Learning Outcomes** | Students will use task menus and student task cards. |
| **Activities** | Review routines and procedures. Show students what they can expect to find when they open their math workstations. Focus on task menus or student task cards. Model how to find task menus and student task cards, and model how to use them to choose and complete the tasks. Using the same workstations from Day 7, ask students to get the workstation, find the task menu and student task cards. Debrief. |

| Day 8 *(cont.)* | |
|---|---|
| **Teacher Notes** | To teach the process:<br>• assign students to groups of four or five<br>• give each group a math workstation<br>• ask students to find the task menu<br>• have each group find the student task card<br>• debrief and assess |
| **Day 9** | |
| **Focus** | What are Talking Points cards? |
| **Learning Outcomes** | Students use Talking Points cards to engage in mathematical talk. |
| **Activities** | Review routines and procedures. Show students what they can expect in their math workstations. Focus on Talking Points cards. Model how to use the Talking Points cards to discuss math tasks. Have students role-play using cards incorrectly. Have students share feedback and then role-play correct usage. Ask students to get their math workstation and move to their workspaces to complete a task. Remind students to use the Talking Points card. Debrief. |
| **Teacher Notes** | To model the process:<br>• assign students to groups of four or five<br>• give each group a Talking Points card<br>• allow time for students to practice<br>• debrief and assess |
| **Day 10** | |
| **Focus** | How do we use math workstations independently? |
| **Learning Outcomes** | Students get math workstations, move to their workspaces, find task menu and student task cards, use Talking Points cards, and return math workstations. |

**Figure 9.5** *(cont.)*

| **Day 10** *(cont.)* | |
|---|---|
| **Activities** | Review and model the process of getting math workstations, using task menu and student task cards, using Talking Points cards, completing the tasks, and then returning workstations when asked. Introduce a signal for completing tasks and cleaning up workspaces. Have students practice the entire process, debrief, and then repeat. Refer to the anchor chart if needed. |
| **Teacher Notes** | To model the process, do the following:<br>• show students a task menu and explain workstation options<br>• show students a student task card and model how to read directions<br>• ask students to share what they observed<br>• have students role-play working through a student task card<br>• solicit feedback from the class<br>• debrief and assess |

*(Sammons and Boucher 2017a, 145–147)*

## Week Three: Thinking Like Mathematicians: Focus on Mathematical Practices

By Week 3, students should be very familiar with the structure of Guided Math and its impact on their mathematical learning experiences. They should also have a clear understanding of what is expected of them as they work independently in math workstations. Most students will still require additional practice before they are ready to fully participate in math workshop. Since students already know the "how to" information, they are primarily in need of practice opportunities. This week is an ideal time for students to focus on thinking and acting like mathematicians as they practice and build stamina.

Whole-class mini lessons during Week 3 address the standards for mathematical practice (e.g., communication and representation, making connections, reasoning and proof, and problem solving). The focus is not on teaching specific skills in isolation without mathematical content.

Rather, these lessons teach students *what* the practices are and *why* they are important to mathematicians. During practice working independently, students then tackle content-rich tasks that involve them in practicing the essential mathematical processes. At the end of this week, students share their reflections about what they have learned during the first 15 days in preparation for the full implementation of math workshop.

See Figure 9.6 for a sample instructional plan for Week 3. Teachers can find more information and sample lessons for Week 3 in *Guided Math Workshop* (Sammons and Boucher 2017a).

**Figure 9.6—Week 3—Thinking Like Mathematicians: Focus on Mathematical Practices**

| **Day 11** | |
|---|---|
| **Focus** | How do mathematicians communicate and represent their mathematical thinking? |
| **Learning Outcomes** | Students understand that mathematicians share their thinking by talking about it, writing about it, and representing it in multiple ways. Students understand they are expected to write about and represent their mathematical thinking daily. |
| **Activities** | Pose a simple problem to be solved together by the class. After a solution is found, model how to talk about the mathematics involved in finding the solution. As a shared writing task, with the teacher as a scribe, have the class describe the problem-solving process. Emphasize the importance of staying focused and using appropriate mathematical vocabulary. Discuss the use of the math word wall. Introduce a math workstation task that requires students to solve a problem and then write about how it was solved. Have students practice the entire math workshop process. During this practice time, students should not interact with the teacher. Observe. End the practice if students are having a difficult time following the routines and procedures. Debrief and assess. Refer back to the routines and procedures anchor chart, if needed. Repeat the practice to build stamina. |

Figure 9.6 *(cont.)*

| **Day 11** *(cont.)* | |
|---|---|
| **Teacher Notes** | To model the process:<br>• solve a problem together as a class<br>• engage in shared writing to describe problem-solving process<br>• discuss how to clearly communicate the problem-solving process<br>• assign a simple problem as a workstation task<br>• have students reflect and assess performance following routines and procedures as they build stamina |
| **Day 12** | |
| **Focus** | How do mathematicians make mathematical connections? |
| **Learning Outcomes** | Students understand that mathematicians expand their thinking by exploring how mathematical concepts connect to other areas of math, to their own experiences, and to the world. |
| **Activities** | Introduce the idea of making connections by linking to literacy comprehension strategies. Choose a math word or concept and think aloud, sharing how you connect it with other math concepts, things in your own life, and the real world. Choose another math concept or word for the class to consider. Create an anchor chart recording student connections to the concept or word. Introduce a math workstation task that requires students to make connections to a math word or concept. Have students practice the math workshop process with that workstation task. Debrief and assess. Practice again with another debrief session. Refer back to routines and procedures anchor chart, if needed. |

| **Day 12** *(cont.)* | |
|---|---|
| **Teacher Notes** | To teach the process:<br>• create an anchor chart to record mathematical connections<br>• model a math-to-self connection by sharing a personal story (add to chart)<br>• model several math-to-math connections by demonstrating how math concepts are related (add to chart)<br>• model a math-to-world connection by sharing a current event (add to chart)<br>• assign a task the requires students to make connections<br>• have students reflect and debrief performance following routines and procedures as they build stamina |
| **Day 13** | |
| **Focus** | How do mathematicians reason and justify their mathematical thinking? |
| **Learning Outcomes** | Students understand that mathematicians justify their mathematical thinking. |
| **Activities** | Ask students why teachers ask them “How do you know?” Discuss why it is important to be able to state a mathematical fact and justify it. Brainstorm synonyms for *justify*. Provide examples of mathematical work, some of which include justification and some of which do not. Have students examine the examples and discuss whether each example includes justification. Introduce a math workstation task that requires students to justify their work. Practice the entire workshop process with that task. With each practice session, increase the time students work independently to build stamina. Do not allow students to interact with the teacher. Observe the process. End the session if students fail to follow routines and procedures. Discuss any problems that arise. Debrief and assess. Repeat the practice. With each practice session, increase the time students work independently to build stamina. |

**Figure 9.6** *(cont.)*

| **Day 13** *(cont.)* | |
|---|---|
| **Teacher Notes** | To teach the process, do the following:<br>• share examples that clearly demonstrate students' mathematical thinking<br>• share examples in which students do not justify their thinking<br>• have students examine the samples to indicate those with and without strong justification<br>• provide a workstation task requiring students to justify their thinking<br>• have students debrief and assess their performance as they build stamina |
| **Day 14** | |
| **Focus** | What process do mathematicians use when solving problems? |
| **Learning Outcomes** | Students understand the most efficient and effective way to logically solve mathematical problems is following a logical process. |
| **Activities** | Introduce the problem-solving process by sharing a personal scenario of solving a problem. Describe being unsure of where to start to solve the problem or what strategies to use. Share a problem-solving graphic organizer. Discuss each step in the problem-solving process. As a class, solve a sample problem using the graphic organizer. Assign a workstation task that requires students to solve a simple problem using a graphic organizer. Practice the entire math workshop process. During these practice times, do not allow students to interact with the teacher. Observe the process. End the practice if students struggle with following the routines and procedures. Discuss any problems that arise as students debrief and assess. Practice again, if needed. With each practice session, increase the time students work independently to increase their stamina. |

| **Day 14** *(cont.)* | |
|---|---|
| **Teacher Notes** | To teach the process, do the following:<br>• solve a problem as a class using a graphic organizer<br>• assign a problem-solving workstation task to practice independent work and build stamina<br>• have students self-reflect and assess their work performance as they build stamina |
| **Day 15** | |
| **Focus** | How are mathematicians accountable for their work? |
| **Learning Outcomes** | Students understand they are accountable for their work during math workshop in the same way mathematicians take responsibility for their work. |
| **Activities** | Review math workshop anchor charts created by the class. Give examples of how mathematicians are accountable for their work. Tell students, "Accountants are people who rely on their computations. Engineers build bridges that must hold the weight of cars and trucks safely. Parents budget for their monthly income." Remind students that they too are accountable for their independent work. Share the ways students will be held accountable (e.g., recording sheets or math journals). Keep the lesson brief so that students have plenty of time to practice independently. Introduce workstation tasks that have accountability built in. Practice the workshop process. Debrief and assess. Address any remaining problems. |
| **Teacher Notes** | To teach the process, do the following:<br>• provide examples of how mathematicians are held accountable for their work<br>• show students a recording sheet and math journal they will use during math workshop<br>• practice the process with a workstation task<br>• have students reflect and assess |

*(Sammons and Boucher 2017a, 153–156)*

# Determining Student Readiness for Math Workshop

Just as students are not all alike, the same is true for classes of students. Although most classes will be ready to begin independent work after three weeks of practice, some classes may be ready in less time and some may require additional practice. As students practice independent work during Week 2 and Week 3, teachers should closely observe their students to assess how well they follow the established routines and procedures and whether they are able to assume responsibility for working independently at workstations. Based on these observations, teachers can tweak the plans to address specific areas where students need additional work.

The task of determining when students are ready to begin Guided Math is complex, and teachers should rely heavily on their professional judgment and their knowledge of their students to make this decision. During the first 15 days, they will have had many opportunities to observe students work. Teachers with prior Guided Math or Guided Reading experience will most likely have a good idea of what to look for. Teachers who need assistance making this decision should consider the following questions when deciding whether students are ready for the responsibility of working on their own (Sammons and Boucher 2017a):

- Do students understand the routines and procedures?
- Do students engage in tasks without undue distractions?
- Do students solve problems without having to ask for assistance from the teacher?
- Do students cooperate with others in their workstations?
- Do students communicate in soft voices about their work with other students in their groups?
- Do students work without distracting others?

It is important for teachers to be realistic in assessing their students' capacity for working independently. If math workshop is started before students are properly prepared and problems ensue, it is discouraging for both teachers and their students. On the other hand, teachers may be too hesitant to begin a fully implemented math workshop, fearing that their students will be unable to work productively without direct teacher supervision. As a result, valuable instructional time is spent teaching and reteaching routines and procedures rather than mathematics. Once teachers are confident that their students understand the workshop routines and procedures and observe their students behaving appropriately *most* of the time during the practice sessions, teachers should feel comfortable beginning math workshop. Doubtless, teachers will sometimes err in their judgment. At worst, the consequence of beginning Guided Math before students are ready is putting it on pause to reteach routines and procedures, if needed, and provide more practice. However, in most situations, problems can be handled effectively without taking that step.

## Addressing Math Workshop Problems

Whenever students are working independently at math workstations, teachers can expect there will be instances where students are off-task for short periods of time or conversations become too loud. Even though students are sometimes off task or inattentive during whole-class lessons, rarely do teachers decide that they cannot teach whole-class lessons because of that. Yet, since math workshop is a new mode of instruction for many teachers, the same student behavioral problems, when they occur during math workshop, are sometimes considered reasons to permanently halt implementation. While math workshop should be a productive time for students, teachers' responses to problems with student behaviors should be commensurate to the types of difficulties encountered. Rather than terminating the use of math workshop whenever a problem arises, teachers should determine how to make it more effective.

When problems occur, teachers should take a step back and reflect on the workshop format they have established. It is often wise to talk with students about what is happening and why. Students usually enjoy both working independently and meeting with teachers in small-group lessons, so they want Guided Math to be successful.

Teachers may want to consider these questions when difficulties arise (Sammons and Boucher 2017a):

- *Is there a particular place where problems arise?* Teachers may find it necessary to rearrange workspaces or refine the procedures to alleviate these problems.
- *Is there a particular time when problems occur?* If certain activities lead to problems, teachers can work to eliminate the source of those problems.
- *Are the routines and procedures themselves causing problems?* Teachers should carefully review these to see where they can be revised to prevent the problems.
- *Do the problems stem from a lack of understanding of the routines and procedures?* It may be necessary to reteach routines and procedures and provide additional practice for students.
- *If students understand the expectations for their behavior, do they fail to follow the routines and procedures?* Perhaps students require additional practice in order to build their stamina. If not, teachers should provide specific feedback to students regarding the problems that are occurring and discuss how they can be resolved. Sometimes, it is necessary to allow logical consequences to occur. If there is a favorite game that leads to behavior problems, it may be removed from the workstation for a period of time. If students move freely about the room to obtain additional materials, that privilege may be revoked either temporarily or permanently.

Teachers should also determine whether the problems that are disrupting Guided Math instruction involve only a few students or a majority of the class. If only one or two students are having problems, it can usually be addressed with them directly. Broader issues affecting much of the class should be discussed with the entire class. Figure 9.7 (page 355) offers suggestions for handling behavior problems.

**Figure 9.7—Suggestions for Solving Math Workshop Behavior Problems**

| Number of Students | Possible Actions |
|---|---|
| Just a few students | • Confer with those students about their behavior. Ask them to assess their behavior and identify how they may improve it.<br>• Try to determine if the workstation tasks are at an appropriate level for students. If not, adjust the task.<br>• Create a nonverbal signal to let these students know when their behavior becomes inappropriate for workstations, so they may self-correct.<br>• Have students work beside you rather than in a workstation for a day or two. |

*(Sammons and Boucher 2017, 133)*

## Teacher Collaboration for Guided Math Implementation

Carol Lyons and Gay Su Pinnell (2001) write that teachers, as do students, learn best when they actively participate with a group of their peers as part of a respected learning community. Because all members of this professional community share the responsibility for learning, they are more comfortable discussing common concerns. They are more willing to experiment with new teaching strategies in a nonthreatening and supportive environment. Additionally, teachers are able to learn by observing each other and reflecting together.

Attempting something new and unfamiliar is rarely easy. So, for many teachers, the kind of collaboration and support described by Lyons and Pinnell is of great value as they begin implementing Guided Math. "Teachers learn best by studying, doing, and reflecting; by collaborating with other teachers, by looking closely at students and their work; and by sharing what they see" (Darling-Hammond 1999, 2). There are many ways this type of collaboration can occur. Teachers from a single grade level may choose to learn, plan, and reflect together. Teachers from multiple grade levels may meet periodically to share their ideas and experiences. Or, academic or instructional coaches may meet with teachers as facilitators, offering support and guidance to those who want to begin Guided Math instruction in their classrooms.

Some schools encourage, or require, professional learning communities to support the implementation of Guided Math.

> By participating in these collaborative study groups, teachers are able to learn about each of the components of Guided Math, plan Guided Math lessons collaboratively, reflect on their own teaching and their students' learning when using the framework, observe their peers as they implement Guided Math, and examine evidence of student learning to gauge the effectiveness of the implementation. (Sammons 2016, 183)

Professional learning communities do not have to be organized by school administrators. Groups of teachers who share a mutual interest in effectively meeting the mathematical learning needs of their students may decide to work together voluntarily. Not surprisingly, groups formed voluntarily are some of the most successful. These teachers enter into the process with the desire to work and learn together. Professional learning groups may choose to meet before or after school, during common planning periods, during additional planning times, or during other times that may be arranged with creative scheduling.

With or without administrative support, most teachers find that working closely with their colleagues eases the implementation process. The activities of these groups may include the following:

- studying professional books and articles
- sharing information after attending professional development opportunities
- brainstorming ideas for classroom design, procedures and routines, warm-up ideas, small-group lessons, and math workstation tasks
- working together to create learning materials and workstation tasks
- sharing instructional materials

- observing each other teach and participating in subsequent debrief sessions
- looking at and assessing student work products as a group
- reflecting on instructional strategies (e.g., what worked well, what did not, what changes could be made)
- reflecting on students' needs and how to meet those needs
- building a sense of collegiality among teachers

When teachers work together toward a common goal, the benefits can be enormous. Louise Stoll, Dean Fink, and Lorna Earl (2003) write "being in a learning community is …a state of mind, is not linear, is bigger than the sum of its parts, and is about learning as a community" (Kindle Locations 1569–1571). Indeed, teachers' efforts become "bigger than the sum of their parts" with their collaboration.

A nine-month plan for professional learning communities that can be used by any group of teachers working collaboratively can be found in my book *Implementing Guided Math: Tools for Educational Leaders* (2016). This plan maps out a gradual, yearlong process in which each component of the framework is carefully studied and gradually put into practice. Included in the plan are recommendations for reading assignments, tasks, and reflections highlighting each of the seven components. In the process of learning about the Guided Math framework, teachers are encouraged to also reflect on their current teaching practices, experiment with new instructional strategies, observe other teachers' attempts at implementation, and consider the impact of their teaching on their students' mathematical achievement.

At times, teachers who share ideas and learn together may become overwhelmed by the multitude of research available on teaching strategies. But, the bottom line for teachers is always student learning, their quest to help students succeed. Regie Routman writes the following about this idea:

> When I suggest that we need to “teach with a sense of urgency,” I’m not talking about teaching prompted by anxiety but rather about making every moment in the classroom count, about ensuring that our instruction engages students and moves them ahead, about using daily evaluation and reflection to make wise teaching decisions. Complacency will not get our students where they need to be. I am relaxed and happy when I am working with students, but I am also mindful of where I need to get them and how little time I have in which to do it. I teach each day with a sense of urgency. Specifically, that means that I am very aware of the students in front of me, the opportunities for teaching and evaluating on the spot, the skills and strategies I need to be teaching, the materials I need, the amount of time available and the optimal contexts and curriculum. (2003, 41–42)

Despite how hurried and harried teachers often become, they always have the needs of their students in the forefront of their minds. Sometimes, to best meet those needs, they have to slow down so they can teach more deeply. When teachers rely on effective instructional frameworks like Guided Math and then plan their lessons based on their own intimate knowledge of students’ learning needs, they can actually “do more instruction, more effectively, in less time” (Routman 2003, 218).

Establishing personal connections with students is at the very heart of teaching. In most classrooms, meaningful relationships between teachers and students create a truly joyful learning environment. Students are more willing to take the risks necessary for deep learning, not just in mathematics but in all subject areas. By slowing down the hectic pace of teaching, teachers discover they have time to listen more closely to their students, understand their students’ learning needs more clearly, provide multiple and varied ways for students to demonstrate their mathematical learning, and then joyfully celebrate success with their students.

## Chapter Snapshot

The Guided Math framework described is only a structure that teachers use to forge their own unique methods for mathematics instruction. The framework is based on research and experience from the classroom, as well as much collaboration and conversation with educators from all grade levels. Only when individual teachers apply it in their classrooms is it fully fleshed out. The teaching style of a teacher, the curriculum being taught, and the strengths and needs of students combine to determine the exact features of the framework as it evolves and is implemented in a classroom.

Teachers planning to implement Guided Math should first reflect on the framework and decide how they will adapt it to make it their own. Only then does the framework become a vehicle for establishing a vibrant mathematics learning community.

## Review and Reflect

1. How will you begin to implement the Guided Math framework in your classroom?

2. How can you create a professional learning community to support you as you modify your mathematics instruction?

# Math Conference Checklist

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Students | | | | | | | | | | | | | | | |
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| | Math Goals | | | | | | | | | | | | | | |

# Math Conference Recording Form

| Student | Date | Research | Compliment | Teaching Point |
|---|---|---|---|---|
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| | | | | |
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# Sticky Note Organizer

| | |
|---|---|
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| | |

Name: ____________________ Date: __________

# Criteria for Problem-Solving Rubric

| Domain | Conceptual Understanding | Reasoning | Computation and Execution | Connections | Communication |
|---|---|---|---|---|---|
| **Exemplary** | Mathematical representations helped clarify the problem's meaning. Inferred or hidden information was used. Procedures used would lead to a concise and efficient solution. | Innovative, creative strategies were used to solve the problem. The solution was proved correct. Examples and counterexamples were given to support the logic of the solution. | All aspects of the solution were accurate. Multiple representations verified the solution. Multiple ways to compute the answer were shown. | A general rule or formula for solving related problems was created. Connection to other disciplines or real-life applications was accurate and realistic. | Explanation was clear and concise. Mathematical vocabulary was used precisely. |
| **Proficient** | Mathematical representation was appropriate. All the relevant information was used. Procedures used would lead to a correct solution. | Appropriate strategies were used to solve the problem. Each step was justified. Logic of the solution was obvious. | Computation was accurate. Representations were complete and accurate. Work clearly supported the solution. | Recognized important patterns and relationships. Connection was made to other disciplines or real-life applications. | Explanation was easy to follow. Mathematical vocabulary was used correctly. |
| **Developing** | Mathematical representation was inefficient or inaccurate. Some, but not all, of the relevant information was used. Procedures used would lead to a partially correct solution. | Oversimplified strategies were used to solve the problem. Little reasoning was offered that justified the work. Leaps in logic were hard to follow. | Minor computational errors were made. Representations were mostly correct, but not accurately labeled. Evidence for solutions was inconsistent or unclear. | Recognized some patterns and relationships. Found a hint of connections to other disciplines or real-life applications. | Explanation was not clearly stated. Mathematical vocabulary was used imprecisely. |
| **Emerging** | Mathematical representations of the problem were incorrect. The wrong information was used. Procedures used would not solve the problem. | Strategies used were not appropriate. Reasoning did not support the work. Logic was not apparent. | Serious errors in computation led to an incorrect solution. Representations were seriously flawed. No evidence was given of how the answer was computed. | Unable to recognize patterns and relationships. Found no connections to other disciplines or real-life applications. | Little or no explanation for the work was given. Mathematical vocabulary was incorrect. |

Name: ______________________________ Date: ______________

# Problem-Solving Checklist

| Criteria for Problem Solving | Met | Not Yet Met | Comments |
|---|---|---|---|
| Mathematical representations of the problem were appropriate. | | | |
| An appropriate strategy was used to solve the problem. | | | |
| Computation was accurate. | | | |
| The problem-solving process was clearly explained. | | | |
| Problem solving was extended through recognition of patterns, relationships, or connections to other areas of mathematics or to real-life applications. | | | |

Name: ______________________________ Date: ______________

# Small-Group Lesson Planning Form

<table>
<tr><td colspan="2">Standards:</td></tr>
<tr><td colspan="2">Prerequisite Knowledge and Skills:</td></tr>
<tr><td rowspan="3">Small-Group Lesson<br><br>Connection:<br><br>Teaching Point:<br><br>Active Engagement:<br><br>Link:</td><td>Additional Challenge</td></tr>
<tr><td>Rebuilding Prerequisite Knowledge and Skills</td></tr>
<tr><td>Essential Vocabulary:</td></tr>
</table>

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