

CAN WE FIGURE THIS OUT TOGETHER? SURFACING BELIEFS THROUGH COLLABORATIVE PLANNING

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Abstract

In this study, self-study methodology is utilized to investigate the impact of collaborative planning on the authors' development as mathematics teacher educators (MTEs). The authors, two early-career MTEs, met regularly to plan instruction and journaled frequently about the experience. Upon completion of the course, meeting transcripts and journals were analyzed to determine beliefs that surfaced during the planning. The experience of developing a midterm exam is shared to demonstrate how beliefs about the development of positive relationships and environments, characteristics of effective mathematics teaching, and preservice teachers' development of mathematical knowledge for teaching guided the collaborative planning process. Both authors showed development as MTEs, and features of their relationship that made collaborative planning successful are discussed.

Keywords: self-study, teacher beliefs, mathematics teacher educators, collaborative planning

Mathematics teacher education is not a simple endeavor, as evidenced by the length and complexity of the Association of Mathematics Teacher Educators' (AMTE) Standards for Preparing Teachers of Mathematics (2017). One way mathematics teacher educators (MTEs) can reflect on and improve their practice is through collaborative course planning with a colleague. In this study, we investigate the impact of collaborative planning on our development as MTEs. Although each of us has more than a decade of experience working in elementary and middle school classrooms and significant training and experience working with preservice teachers (PTs) during our masters' and doctoral degrees, we both faced significant challenges when we began teaching at our current institution. For both of us, this was our first tenure-track job; Rebecca began shortly before the onset of the pandemic, and Katie began shortly after. Despite having a supportive department and supportive mathematics education colleagues, we often felt overwhelmed in our first few years. Finding ourselves in similar situations and having complementary teaching goals and philosophies, we quickly became a source of motivation and support for one another. During the spring quarter in 2022, we were scheduled to teach different sections of the same course and decided to collaboratively plan the course. Self-study methodology provided a means for us to identify how we developed as MTEs through collaborative planning. In this study, we use our beliefs about teaching and learning mathematics as a lens to examine how we made decisions in our collaborative planning relationship. We developed our skills and knowledge as MTEs and supported each other through our critical friendship.

Background

Preparing to teach mathematics requires PTs to develop a wide range of related knowledge and skills. This includes knowledge of mathematical content, pedagogy and practices, students as learners, and the social contexts of mathematics teaching and learning

(AMTE, 2017). In addition to providing standards to guide the education and preparation of mathematics teachers, the Association of Mathematics Teacher Educators also offer several assumptions that underlie the standards document. Notably, MTEs “must be committed to improving their effectiveness in preparing future teachers of mathematics” (AMTE, 2017, p. 3). One way to begin to reflect on and improve effectiveness is through interrogating beliefs about teaching and learning mathematics and its connection to actions taken in the classroom.

Research on teacher beliefs has been an active area for many years, although explorations of MTE’s beliefs specifically are only beginning to emerge. While synthesizing the state of research on beliefs, Pajares (1992) acknowledged that a variety of conceptions of beliefs exist and argued that understanding teachers’ beliefs about content areas was essential for better understanding student learning. Beliefs are not readily visible, and inferring beliefs includes considering ways a person provides evidence of their beliefs, which could include behaviors, belief statements, or other artifacts (Rockeach, 1968). Beliefs are complex, interrelated, and never held in isolation. Thus, considering the relation among beliefs can be helpful for understanding them (Green, 1971).

Research on mathematics teachers’ beliefs has explored their beliefs in a variety of ways. Classroom teachers’ beliefs about the effectiveness of inquiry-based teaching, along with knowledge and attitudes, have been found to influence instructional practices related to inquiry-based teaching (Wilkins, 2008). Beswick and Callingham (2014) found that MTEs’ and PTs’ beliefs differ significantly, especially in assumptions about what confuses children and how children learn mathematics. MTEs held beliefs that were more learner-focused than PTs’. As former classroom teachers now teaching PTs, we recognize the potential for our mathematics teaching experience and our responsibilities as MTEs to result in changes in our own beliefs.

The mathematics education literature has illustrated the complexity of relationships between beliefs and practices and has identified misalignments (Phillip, 2007). Authors acknowledge teachers can act in ways that are in contrast with their beliefs because of factors such as limited time or lack of resources (Cross & Hong, 2012; Herbel-Eisenman et al., 2006). Speer (2005) offers that part of the misalignment may be related more to methods used for data collection and a “lack of shared understanding between teachers and researchers of the meaning of terms used to describe beliefs and practices” (p. 361). In other words, the collection of data about beliefs by outside observers makes it difficult to support claims about those beliefs. Although research exists on MTEs’ beliefs more broadly (Lovin et al., 2012), the connection between MTEs’ beliefs and practices has not received as much attention. MTEs’ beliefs about teaching and learning mediate instructional strategies, materials, and content used in the teaching of mathematics content courses for PTs (Li & Superfine, 2016). MTEs’ goals and purposes when designing mathematics content courses for PTs also include affective ideas. Specifically, Appova and Taylor (2017) found expert MTEs provided PTs with experiences meant to develop “orientations toward teaching the subject,” which is “teachers’ knowledge and beliefs about the purposes and goals for teaching [the subject] at a particular grade level” (Magnusson et al., 1999). Li and Superfine (2016) found MTEs valued providing positive experiences, those that supported PTs’ positive mathematical identities while learning mathematics. The connection among beliefs, practices, and instructional decisions for MTEs has implications for their courses and PTs’ experiences.

These existing reports focus on MTEs’ professional development and factors that influenced course decisions, whereas our article illustrates the complexity and contingent decision-making of MTEs as informed by beliefs and through collaborative planning. In this study, we analyze our collaborative planning relationship through the lens of our beliefs about teaching mathematics. Our inquiry was guided by the following questions: *How do we enact our beliefs about teaching through our practice? How does collaborative planning support or inhibit*

such enactment? How does collaborative planning support our development as MTE's?

Context

We are two White, middle class, cisgender women who teach at a predominantly White institution in the Pacific Northwest. Both of us have K–8 classroom teaching experience. Rebecca taught kindergarten, first grade, and fifth grade and also worked as an instructional coach in the Southeast. Katie taught middle school mathematics for 10 years in a K–8 school and later worked as a mathematics instructional coach in the Midwest. We both have experience teaching courses and facilitating professional development for in-service teachers as well as teaching methods and content courses for PTs. We met when Katie was hired to her current position, a year after Rebecca began her position at the same institution. This was the first tenure-track appointment for both of us. We became fast friends, despite our first year of interaction being entirely online because of the COVID-19 pandemic. Initially, we found solidarity in our shared experiences, particularly related to being new faculty who had relocated from afar; our colleagues had collegial relationships, institutional knowledge, and support systems in the area, but we did not. Our similar philosophies and complementary personalities soon deepened our friendship.

Because of the pandemic, our institution, which operates on a quarter system, was fully online for over a year. The final quarter of Rebecca's first year—spring of 2020—was entirely online, and all three quarters of Katie's first year (Rebecca's second) were also entirely online. During Katie's first year, we spoke often about leading with empathy and centering our PTs' physical, social, and emotional well-being. We faced many challenges that year. Teaching online brought a need for drastic changes, from the way we incorporated manipulatives to the way we expected PTs to engage with course content. We felt empowered to make decisions we may have hesitated to make prior to the pandemic; for example, we decided not to give exams, an activity which had historically been included in the course across all instructors prior to the pandemic.

Every degree-seeking student at our institution is required to take two courses that meet the general undergraduate requirement (GUR) for quantitative and symbolic reasoning. PTs take a series of three mathematics education courses that blend mathematics content and pedagogical methods. The first two courses in our sequence of mathematics education courses meet the university's GUR requirement. Thus, these courses are expected to maintain a high level of rigor related to their mathematics content. One strategy for demonstrating such rigor is through the inclusion of exams that assess mathematics content knowledge as well as pedagogical knowledge.

Rebecca taught the first course in the sequence for two quarters prior to the onset of the pandemic and incorporated slightly modified versions of exams that had been developed by other instructors of the course. During the last quarter of her first year, which was entirely online because of the pandemic, Rebecca chose to replace traditional exams with assessments that allowed for collaboration and open use of resources, such as group projects or essay responses to multistep tasks. She continued such assessments throughout online teaching, rather than using proctoring software to attempt to administer traditional exams. Katie, who did not have experience with any of the courses prior to the pandemic, also eschewed traditional exams during her year of online teaching. Traditional written exams had not been an integral part of Katie's teaching in previous settings, so this decision felt more natural to her than to Rebecca. Katie would not have included traditional exams unless explicitly required to even if her first year of teaching these courses had been face-to-face.

At the beginning of Katie's second year (Rebecca's third), we both eagerly returned to face-to-face teaching. We had not anticipated the significant challenges of returning to in-person

instruction. One notable challenge involved continuing to provide flexibility to PTs with attendance, participation, and due dates as we navigated facilitating cooperative learning and discussion-focused courses. Rebecca also felt pressure to return to the status quo—administering traditional exams as a way of demonstrating the mathematical rigor of the courses—despite questioning whether this was the most effective or meaningful form of assessment. Katie had not used exams in her courses in the previous two face-to-face quarters that year, unaware this was an unspoken norm.

This study began during the spring quarter of 2022, when we found ourselves teaching different sections of the same course, the third and last in the series of blended elementary content and methods courses for preservice K–8 teachers. Both of us had previously taught the course; however, spring 2022 was our first experience teaching it in an online format. Rebecca taught the course in a face-to-face setting in the fall of 2021 and reflected on the difficulty of transitioning from an online modality to teaching in-person. Katie had not yet taught the course in a face-to-face setting. In an effort to improve our practices, specifically related to developing conceptual understanding and assessment, we decided to collaboratively plan the course. We met for several hours prior to the beginning of the quarter and outlined the overall trajectory of the course. We refined major assignments and planned weekly homework. Once the course began, we met for several hours weekly (via Zoom) to discuss what was working well and what needed modification. During the meetings, we asked each other critical questions about the purpose of each activity, overall course goals, and pedagogical decisions. Although we collaboratively planned and used the materials we created, our willingness to question and challenge each other marked this partnership as a critical friendship (Costa & Kalick, 1993).

Our quarter of collaboration concluded with each of us identifying improvements in our teaching. We began this collaborative planning journey wanting to find ways to learn about its impact on our development as MTEs. We decided to use self-study as a mechanism for thinking critically about our experiences, successes, and challenges throughout the process of collaborative planning. Specifically, we sought to answer two research questions:

1. How do we enact our beliefs about teaching through our practice? How does collaborative planning support or inhibit such enactment?
2. How does collaborative planning support our development as MTE's?

Methodology

Self-study methodology is focused on the study of one's own practices for the sake of improvement (LaBoskey, 2004). Mathematics educators are increasingly using self-based methodologies to study their practice, although it has not yet met with mainstream acceptance (Suazo-Flores et al., 2019). MTEs have used self-study to examine their transition from secondary mathematics teacher to elementary MTE (Simpson, 2019), to examine the development of mathematical knowledge for teaching teachers (Masingila et al., 2017), and to examine the alignment between beliefs and practices (Bahr et al., 2018). Self-study has also been used to research and better understand MTEs' specialized knowledge through the collective interrogation of beliefs (Lovin et al., 2012). Notable parallels exist between the recent developments in teacher education and increases in the use of self-study: acknowledging the authority of practice, a shift in focus from theory to practice, recognizing the uniqueness of situations and their context, and an increased emphasis on collective learning (Korthagen & Lunenberg, 2007). With our study, we expand on the existing literature, interrogating our beliefs about teaching and learning (Lovin et al., 2012) and reflecting on the relationship between our beliefs and practices (Masingila et al., 2017) for the purpose of improving our own practice and also to contribute to the broadening field of such studies.

We were focused on improving our practice, especially in making content delivery more

student-centered and through the incorporation of meaningful assessments. Aware that critical reflection and examination would be an important part of our collaboration, we recorded our meetings and journaled about the process in anticipation of possibly engaging in self-study in the future. During the quarter, we were preoccupied with making decisions together and carrying them out in our respective courses. Once the quarter concluded, we turned to analyzing our experiences, looking for insights into the experience of planning itself. All collaborative planning meetings were recorded, and transcripts were automatically generated via Zoom. At the conclusion of the quarter, we watched each meeting and refined the transcripts. Thus, the meeting recordings and transcripts provided evidence of our collaboration as well as shifts in our practice.

Reflective journaling was used for several purposes in the self-study. Our reflective journals provided data, supported analysis, and prompted critical friend discussions. We reflected on the experience of watching the meetings through journaling, noting interactions we believed were significant in influencing our teaching or characterizing our collaborative relationship. Notable interactions included decision-making, disagreements, and discussion of our beliefs about effective teaching and the goals and purposes of the course. For example, after watching one meeting, Katie noted in her reflective journal that our discussion about one of the course assignments—a curriculum analysis project—was particularly meaningful:

Our discussion around the curriculum analysis project, the components that are included and the way it builds, aligns with our beliefs of wanting to convince our PTs to teach in ways similar to how we model. I think “pulling back the curtain” on our planning and seeing that it is more than finding a fun task is something that students don’t see enough of [Katie, reflective journal on September 7 about March 25 meeting].

We also sought evidence of our beliefs about teaching during the semester of our collaborative planning. To understand which beliefs were guiding our practice, in the quarter following our collaborative planning, we each wrote a philosophy of teaching, outlining our beliefs about teaching and learning mathematics. We then revisited teaching and diversity statements we had written years ago when initially entering the job market, before meeting each other. Each of us then journaled about our teaching and diversity statements, noting whether or how our philosophies had changed between beginning to work at this institution and the conclusion of the quarter in which we planned collaboratively. Next, we read each other’s statements, wrote about our impressions in our reflective journals, and distilled each other’s teaching statements down to primary beliefs using evidence from the statements. Each of us identified evidence of approximately 10 beliefs in the other’s teaching statements. We member-checked these lists of beliefs by reading what our partner had distilled from our teaching statements and confirming alignment with our teaching philosophy. We then combined these distilled beliefs, leading to one list that included beliefs about teaching in general, teaching mathematics specifically, and teaching mathematics and pedagogy to preservice elementary/middle school teachers. Reflective journaling about our teaching philosophies and beliefs made us more conscious of our own and each other’s beliefs. Our reflections often focused on similarities we noticed in our teaching statements. For example, Rebecca reflected on Katie’s journal,

We are so much alike, especially in what motivates us to teach and what we think is important in our classrooms. This shouldn’t be surprising—that’s why we work so well together. But I am surprised by how clear our similarities seem to be, even in writing, even before we knew each other [July 20, 2022].

We further discussed the connections between our beliefs and teaching actions. Katie commented on Rebecca’s journal, “It is all about actions! I appreciate that you said that explicitly [in your teaching statement]” [July 19, 2022].

Beliefs about teaching mathematics content for PTs can encompass a wide range of ideas. Beliefs influence teachers' behavior in several ways, as "filters for interpretation, frames for defining problems, and guides and standards for actions" (Fives & Buehl, 2012, p. 478). Once we had identified evidence and interpreted beliefs based on our philosophies of teaching, we revisited our meeting transcripts, looking for evidence of the beliefs in our discussions and planning. We used the list of beliefs as a lens to identify which beliefs surfaced during our collaborative planning. Our beliefs included several categories, but in this article, we focus on how three categories of beliefs informed our collaborative planning: the development of positive relationships and environments, characteristics of effective mathematics teaching, and developing mathematical knowledge for teaching (MKT) in our PTs. These beliefs were *central and primary*, meaning they are strongly held and foundational (Thompson, 1992).

We share our findings using a narrative representing the context of the course, the nature of our collaborative planning relationship, and the effect of our collaboration on our teaching practices. Although we situate the findings in one strand of our conversations across the quarter, namely generating and incorporating a meaningful midterm exam, these findings regarding how our beliefs inform our teaching and as an extension our collaborative planning apply more broadly to our decision-making regarding other aspects of the course (e.g., decreasing the amount of lecture in favor of more student-centered activities). We also highlight aspects of our relationship that made this collaborative planning meaningful and beneficial, even in the face of disagreement. Through selected transcripts and analysis, we illustrate how our beliefs informed our planning decisions, how collaborative planning developed our own facility as MTEs, and how our relationship made such development possible.

Findings

During the spring 2022 quarter, we met for more than 40 hours. Our friendship framed our meetings. We always began by checking in on each other, discussing our nonacademic lives or personal interests. We always closed by reflecting more generally on the goals we had set during the meeting and providing words of affirmation and encouragement to each other. Every meeting included validation, flexibility, and laughter. We frequently commented on how happy we were to be collaborating and how we felt our courses were changing for the better, such as when Katie commented, "We're doing great things" [April 6, 2022]. We took the time to learn more about each other, particularly in regard to teaching experience. For example, we watched a video of Katie teaching prior to her PhD. Katie felt the video would be good to share with PTs in both classes. After watching the video, Rebecca commented, "That video of you with your [seventh-grade] students was really fun. I have a teacher crush on you now" [April 13, 2022].

We have experience, both individually and as a pair, working with other collaborators on projects that push us to think deeply and critically about our identities, culture, positionality, and whiteness. Before this study, we were used to asking personal and critical questions of each other, our beliefs, and the ways in which we (and our courses) may cause harm or be inaccessible to students. In a journal, Katie commented about our working relationship:

In most educator groups, I usually find someone that I have a connection with and feel that I can speak openly with. However, oftentimes at some point, they say something that makes me wonder if I can push back on their ideas without offending. Something that sets our group apart is that we all openly acknowledge that we will make mistakes and that we have been raised and worked in systems that are deeply inequitable, racist, ableist, etc. Our goals, decisions, questions, and push backs to each other are deeply rooted in care for each other, care for our students, and wanting to do the "right" thing while acknowledging we will make mistakes. This is a journey not a destination

[September 2, 2022].

Our critical work and mutual respect allowed us to critique each other's ideas and choices in a way that was at times tense yet productive. We trusted each other's professional judgment. Further, we conceptualized critique of each other's practices as care rather than criticism. Questioning pushed us to become better selves and teacher educators. Our mutual care allowed us to have disagreements while still expecting our relationship to remain strong.

Through the story of our midterm exam, we share ways our beliefs and relationship shaped our course through collaborative planning. Before the course began, at Rebecca's request, we decided to give a midterm and final exam. This initial decision led to many more discussions, spanning several tense meetings, as we determined the scope, questions, and facilitation of the midterm exam.

Purpose of the Exam

Rebecca wanted a midterm to hold PTs accountable for their learning. Yet we grappled with how to give a humanizing midterm that would also provide valuable information about PT understanding.

Katie: So maybe let's just think about, what is the purpose for giving an exam? Is it to see what [PTs] can do independently? It is to . . . ?

Rebecca: I'm not gonna lie, this is partly against my philosophy. Part of it is an accountability thing for me. I think that this past quarter, my [PTs] had very little accountability [because of the COVID-19 modifications]. They definitely weren't learning as much because of it, and it frustrated me. So, I think that's part of it. They need an exam. It needs to count as a grade. I don't know . . .

Katie: Okay.

Rebecca: They also need to understand that you are accountable for coming to class and learning things and then demonstrating that learning. If you can't demonstrate that learning, you're not . . . I'm not going to give you an A if you haven't exhibited learning at that level. Which is kind of against everything that I've done for the last two years. I don't know. [March 24, 2022]

Rebecca's desire to use an exam to hold PTs accountable felt in contrast with her beliefs. For several quarters, during pandemic online learning and the transition back to face-to-face courses, we had both avoided formal exams to reduce anxiety and show empathy for the many challenges our PTs faced during a time that was mentally and emotionally exhausting for many educators and students. However, Rebecca was beginning to feel her PTs were less motivated to do course readings, complete assignments on time, and attend class regularly. She felt an exam could be a way to increase engagement and accountability.

Our Beliefs, Relationship, and Development as MTEs. Throughout our discussions about the purpose of the exam, beliefs about positive environments and characteristics of effective teaching surfaced. We wanted our exam to provide evidence of conceptual understanding of the mathematics we were teaching. We also wanted the exam to contribute to positive relationships with PTs rather than erode them. We struggled to reconcile these goals with Rebecca's desire for increased student accountability. Our relationship as friends and colleagues played an important role in these discussions. Katie respected Rebecca's desire to give a midterm, although she did not share the sentiment. Katie repeatedly pushed our conversation in directions that prompted reflection on our beliefs. The challenges related to writing, administering, and grading a midterm were not easily resolved, and engaging in this work has informed our use of exams in future courses.

Structure of the Exam

Rebecca was committed to giving a written exam; Katie was unconvinced this was a good idea. It was surprising to find ourselves taking opposite stances; we are very compatible teaching partners, and to this point had not yet experienced a disagreement to this degree. After an extended discussion of the purpose and attempts to write items that would give us new insight into PT understanding, we still lacked resolution. Yet we had included plans in our syllabi to give a midterm exam, and the scheduled date was rapidly approaching. Toward the end of one conversation Katie expressed her frustration.

Katie: Okay, I don't care. I don't care about this test at all, so I don't know why I'm acting like I do. Let's just make it easy and be done with it. It's just . . . so we're taking a whole day of class. That means it should be semi-long. I don't know. . . . I mean. I am okay doing it, and it's going to be fine.

Rebecca: Okay . . . but let's talk about this. I don't think I can just not have [any exam] because it's already built into my syllabus.

Katie: Right. Correct.

Rebecca: Do you want to blow this up and do something completely different? Like just do something else?

Katie: I could do. . . . I don't know. I'm sorry for being so . . .

Rebecca: Please stop apologizing. I'm not mad at you. Every question that you have, I'm also having. And like, same, Katie, same. I'm not upset with you. It's good that you're pushing me to think about these things [April 25, 2022].

Even through moments of frustration, we recognized our tension was coming from a desire to do the "right" thing. We both wanted our exam to help PTs learn, but we were unsure how it should look. Rebecca reassured Katie that these conversations were productive.

Our Beliefs, Relationship, and Development as MTEs. In deciding how to structure the exam, our beliefs about characteristics of effective teaching surfaced. We wanted our students to understand that teaching mathematics is complex, requiring specialized knowledge, reflective practice, and continuous learning. We ourselves engaged with this complexity as we worked through decisions about the structure for our exam. We also wanted our exam to contribute to learning itself, rather than just measuring it, which reflects another characteristic of effective teaching: that every activity, including assessments, can be an opportunity for learning when designed well. Further, we realized all these characteristics also applied to teaching *about* teaching. As our PTs were developing ideas about effective teaching, we were developing our own MKT teachers. Our relationship contributed to this development. Katie felt comfortable being honest about her frustration, and Rebecca carefully listened and valued Katie's input.

Preparing PTs for the Exam

The PTs had expressed nervousness about the midterm and asked for study guides, extra resources, and support. We approached assessment as formative for both students and teachers. The format and PTs' expectations of a midterm seemed in conflict with such a view of assessment, causing tension for us. Through collaborative planning, we began to craft an exam we felt would support the PTs and provide insight into their conceptual understanding.

Katie: Is [the midterm] a one-time thing or something that they can revise and resubmit?

Rebecca: So, that's a really good question.

Katie: I think that will impact how we write it.

Rebecca: I . . . you know how much I struggle with keeping up with grading.

Katie: I know, same.

Rebecca: I usually do not allow revise and resubmit for that reason, because I don't

have the time. I'm flexible. But I think I would prefer it not be.

Katie: Okay.

Rebecca: But I am super open to the idea of . . . in this tweet I shared with you [from Howie Hua (2018)]. I like the idea that [PTs] have five minutes at the beginning [of the midterm]. No utensils out. Put everything away. You have five minutes to read this [exam]. And now you have five minutes to talk to anybody you want, and then sit down and work. I'm also open book, open notes, everything. I'm totally fine with that. The problem with that is it takes [PTs] longer [if the exam is open book] because they want to find stuff.

Katie: Yeah, I think the balance I like with [PTs] is . . . Open note is easy, because we have journals that we've made. Open book is hard because it's such a dense book.

Rebecca: I'm also thinking about . . . [PTs] need to have an exam question that's got a picture of a Common Core Standard. They should identify parts of the standard because I'm so tired of them not knowing. And they also need to have an exam question that says, "Tell me the difference between a Standard for Mathematical Practice and a Standard for Mathematical Content."

Katie: But if it's open book, they can just look it up. . . .

Rebecca: So, I'm thinking about, what are things [PTs] should not leave this class without knowing? What are those things, and how can I use an exam to emphasize the importance of those things?

Katie: Okay, I agree [March 24, 2022].

We decided to allow PTs to use their notes and any work they had done for the class. We also allowed them to talk to each other about the midterm items for five minutes before beginning to work on it. We decided against including an item about the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) in favor of emphasizing its importance during in-class activities. Although it required more time for grading, we also chose to allow PTs to revise their work and resubmit the midterm. Instead of traditional corrections, we asked PTs to describe what they did not understand on the midterm and provide evidence for how their understanding had changed. Recognizing when their knowledge was insufficient and taking steps to learn mimics the reflective practice undergone by effective teachers.

Our Beliefs, Relationship, and Development as MTEs. In our discussions about how to support and prepare students for the exam, the beliefs that surfaced were predominantly about the development of positive relationships and environments. Both Katie and Rebecca struggled with the tension between a high-stakes test and the work they had done to communicate to their PTs the belief that all students are brilliant and capable of learning mathematics. If they performed poorly on the exam, would PTs question their own capability as doers of mathematics? What role should exams play in classrooms where we are deeply committed to building positive student relationships and learning environments? Rebecca's desire for increased accountability also generated questions about characteristics of effective mathematics teaching and the development of MKT. Does our exam demonstrate characteristics of effective teaching? How do we hold PTs accountable for their learning and prepare them to teach mathematics? Can taking an exam help PTs develop MKT?

Such questions arose often. Although we still do not have clear answers to any of these questions, confronting them allowed us to make decisions and move forward with our planning. We also consistently reflected on our own developing MKT. We wondered how our context working with undergraduate PTs affected the way we thought about such ideas compared to our experiences as K–8 teachers. Doing this work encouraged our own development as MTEs.

Noting the role our relationship played in these discussions is important. Rebecca remained determined to have an exam, and Katie, although she did not feel such a desire herself, was willing to help write it and implement it in her classes. This speaks to our shared commitment to collaborative planning—we worked together to find activities, assignments, and experiences that we both felt good about and actually used with our students. Neither one of us was merely going through the motions. Katie’s frequent questioning of Rebecca, asking her to justify her desire for an exam, also showed a great deal of care. Many of our disagreements arose not because Katie was trying to get her way, but because she could see conflict in Rebecca’s own beliefs. She knew the type of teacher Rebecca wanted to be and was concerned her increased desire for accountability was moving her away from being that type of teacher. Rebecca, in turn, listened to Katie, accepted her scrutiny as a form of care, took her questions seriously, and used them to reflect more deeply.

Developing Exam Items

Deciding which content to emphasize and how to assess it occupied much of our meeting time.

Rebecca: There’s so much that I want them to know . . . but I am not heartbroken if they walk out of my class not really knowing how to calculate sums of interior angles.

Katie: I agree. So I think that’s the answer—we don’t need that [item].

Rebecca: Okay, so . . .

Katie: Our exams communicate our values.

Rebecca: Okay.

Katie: I think it’s important for [PTs] to think about the sum of interior angles. I just don’t care that much if they can do it independently, personally. It’s something you can look up, and they’re going to go through it with us in class, conceptually. [March 24, 2022]

We tried to be intentional in writing exam items that would provide us with new insight into PTs’ thinking, allow for them to share their reasoning, and not overly benefit those who kept “good notes.” Katie felt strongly the exam should give PTs an opportunity to be reflective about their own growth, asking, “Okay, can we have something like . . . reflect on an activity that we’ve done this quarter. Talk about how the activity supported conceptual understanding of an idea. Describe the idea. Or something like that?” [April 25, 2022]. This led to a discussion about how we could ask such a question and also grade it.

Katie: How can we write it in a way where it’s clear enough that we get specific things?

Rebecca: What specific things are you wanting them to say? What does an ideal answer have in it?

Katie: I want them to talk about conceptual understanding and I want to ask about a misconception. Like how does this activity support . . . I don’t know. I just want them to actually be reflective on something. Okay, what if it was like, “How did it support your understanding of the important ideas? Use three relevant vocabulary terms in your description.” I don’t know.

Rebecca: Are we wanting them to think about. . . ? Okay, so it sounds like you’re wanting them to talk about some conceptual understanding, like a big mathematical concept. What if instead of “reflect on something that we’ve done,” what if we asked them to choose a big mathematical idea that we’ve explored so far this quarter and outline a tentative plan for how you would teach this activity to students? What would you do to help students learn this big idea? What misconceptions do you anticipate students might have about this idea? What tools would you use to help students explore this idea?

Katie: I like that better. I wonder if they’re just going to restate an activity we did [in

class]. [April 25, 2022]

Since our exam was going to be open note, we didn't want to ask questions where PTs would just regurgitate something they had heard earlier. Yet we were unsure how to grade such an item. We eventually decided to make it a "bonus" question at the end of the exam.

Rebecca: What if we make the bonus question to reflect on an activity that we've done this quarter that has changed your thinking? Basically, what we were trying to get at before, but it's bonus. So we don't have to . . .

Katie: Yeah, okay. Then we'll see what kind of answers we get to this and it can help us write it for the final in a better way. [April 25, 2022]

We also struggled to develop items for the exam that were fair and aligned to what we had taught in the course but were also interesting, meaningful, and challenging as well as allowed PTs' to demonstrate their understanding.

Katie: Yeah. I mean [the suggested and existing test items] are perfectly fine questions. Giving a test just makes [PTs] feel bad. . . . It doesn't tell me. . . . I don't know.

Rebecca: Okay, so this is bringing up the same . . . what's the point? Why are we doing this?

[Moments later]

Rebecca: It's really hard to write tests.

Katie: How am I going to grade this?

Rebecca: Any of it? Or are you talking about a specific [item]?

Katie: Any of it . . . [April 25, 2022]

In our continued attempts to develop meaningful items, we revisited our beliefs about the purposes of assessment.

Katie: This just feels like finding that sweet spot between a question that's easy because we've actually done it and a question that pushes their thinking and is challenging. Because that's what we should be trying to figure out. I don't want to give a test where everyone gets 100%. What's the point? If it's hard, I don't feel like I've adequately prepared them for it. So there's two things we can do: We can write a challenging exam, but I'd feel compelled to make a review or sample items so they know what to expect. Or we just give them an easy exam and most people get 100% and that's fine, too. We're learning what to do.

Rebecca: Which one of those do you prefer?

Katie: I don't know. I didn't really give many exams [as a middle school teacher]. I'm on board. But this is what I struggle with. This feels high stakes. I don't want people to believe this was too hard or to feel like I didn't prepare them. But if it's not hard, what's the point? Not just hard, but you know, like . . .

Rebecca: Reasonably challenging.

Katie: Thinking in a new way, where they have to show me they can apply their knowledge in a new situation. . . . I don't know. [April 25, 2022]

Recognizing we needed an organizing structure to guide our selection of items, we decided to look at the overall goals for the course. What big mathematical or pedagogical ideas did we want to ask about so that we better understood our PTs' thinking?

Katie: Okay. So, what do we feel like we don't have information from them about? I think categorizing [shapes—related to geometric understanding] is a good thing to ask. . . . I like the measurement question. [April 25, 2022]

In considering big ideas, we landed on a topic—coordinate graphing and transformations—about which Katie felt an exam item would provide needed insight into PTs' thinking as well as a challenge, while still being fair.

Katie: That could be challenging and interesting. I feel like they definitely got the

coordinate [transformation] rules, and I feel like they definitely grappled with symmetry and thought about it in a meaningful way. I loved that class [period]—everyone understood why plus three [in an expression for the y -coordinate] was going to move [the image] up three. Will they have to think about it and resituate themselves again? Yes. I don't want them to memorize it. But I feel like we can get their understanding and push it in a way that's new. I don't know if I feel that way about [any of] the other [topics on the exam]. [April 25, 2022]

Realizing a meaningful exam item could be developed on the topic of transformations pushed Katie to think about other topics for which this could be done. She suggested we revisit another topic—quadrilateral hierarchies—and we began brainstorming items for this topic. This discussion marked a turning point for us in our item development. By focusing on the big ideas of the course and reflecting on the tasks and class periods in which we developed those ideas, we were able to revise existing exam items (and design some new ones) that we both liked.

Our Beliefs, Relationship, and Development as MTEs. In our work to develop exam items, all three categories of beliefs surfaced. We both felt the exam should help PTs develop MKT and sought to include items that reflected pedagogy in addition to content. We also considered how the exam itself communicated our own MKT. Our beliefs about the characteristics of effective mathematics teaching drove our item development. Specifically, the development of conceptual understanding through student-centered learning experiences was consistently in mind as we determined whether items would be meaningful in helping us learn about our PTs' thinking. Further, our belief that effective teaching requires careful planning, with clear goals tied to standards or program outcomes, was a driving force behind our decisions about which items to include. Finally, her beliefs about positive learning environments led Katie to push for an item encouraging PTs to reflect on their own growth.

We both greatly appreciated the answers our PTs gave when they reflected on their own growth. It gave us insight into which tasks PTs felt helped them learn and what conceptual and pedagogical ideas they recognized in the tasks. We have both since used this question on exams in other courses. This experience greatly influenced our development as MTEs, especially our sense of agency. We were able to find a balance between challenge and alignment, make connections to our overall goals for the course, and write an exam that did not feel like it was in conflict with our beliefs. We had accomplished what previously felt impossible.

Reflecting on the Exam

Our discussions after administering and grading the exam reflected our sense of accomplishment.

Rebecca: I think the exam turned out well.

Katie: I do too. [PTs] were very successful. I think it illuminated for me and for them some things they didn't understand. [May 11, 2022]

Our successful implementation of this exam didn't mean we were always in favor of exams, however. We reflected critically on whether we should have a final exam.

Katie: I just feel like finals week of spring quarter is such a hard time. . . . They've been putting their all into everything. They've been doing a good job. They came prepared for the midterm.

Rebecca: I think they're doing well. I think they're taking the class seriously. They're coming to class unless they have a good reason not to, and when they come to class they're engaging well. So, I don't feel like . . . I think the midterm was a good measure of their understanding and it was good for me, too. I do think it was a wake-up call for some of them—they realized that, yes, I am going to hold them accountable for [learning]. I

feel like I got a lot out of the midterm that helped me think critically about the course and [about their understanding]. It's helping me teach them better. I don't know that a final exam would be anything but evaluating. [May 11, 2022]

We had come full circle. Rebecca felt the exam made her PTs more accountable for their engagement and learning. Providing specific feedback to PTs on their exams along with the opportunity to make revisions made the exam itself an opportunity for further learning. We both agreed it gave us valuable information about PTs' understanding and allowed us to make informed decisions as we progressed through the rest of the course. We did not feel like a final exam would afford those opportunities; it would be evaluative only. Thus, we decided not to have one.

Our Beliefs, Relationship, and Development as MTEs. Reflecting on the exam surfaced all three categories of beliefs. We both felt the exam demonstrated effective teaching, especially the potential of assessment to inform instruction. Taking the exam, in combination with discussing the results, considering feedback, and considering purposes of assessment, contributed to PTs growing MKT. Finally, the exam helped us know our students better. Neither of us felt it negatively affected our learning environment or relationships with students. Knowing that we had successfully implemented an exam gave us confidence in our abilities as MTE's. It was our relationship as friends, colleagues, and critical friends that made this development possible.

Discussion

During collaborative planning, we made decisions together. Although there were challenges, our commitment to a shared product allowed for compromise and reflection. Our struggles with the course midterm illustrate challenges and rewards of collaborative planning. We spent 10 hours discussing the midterm, exploring its purpose and structure, deciding how to prepare and support students, considering items to include, and reflecting (after implementation) how it fit into our larger assessment landscape. We experienced great tension as we planned the exam yet were ultimately able to align it with our beliefs. Collaborative planning allowed us to consider curricular decisions, offer alternative ideas, and reach consensus.

Instructional decisions we made collaboratively were motivated by beliefs about the development of positive relationships and environments. Both of us are driven to recognize the humanity of our PTs, attend to their social/emotional needs, and show empathy and flexibility in our practices. We both initially felt a traditional exam worked against these goals. Even Rebecca, who was highly motivated to give an exam, suggested giving the midterm was misaligned with her beliefs. Would giving an exam inevitably lead to some PTs feeling anxious or getting incorrect answers, and thereby cause them to develop negative mathematical identities? Would grading the exam, including marking incorrect answers, mean we were focused on deficits rather than strengths? How could we reconcile the demands of traditional classroom settings—grading practices, assessment practices, expectations for class participation—with our goals for humanizing practice? We do not claim to have answered these questions regarding the practice of giving exams in our future courses, but regarding the midterm in spring 2022, we can address them. Our collaborative planning allowed us to see the inclusion (or not) of humanizing practices is not necessarily binary. We found a way to incorporate a written exam without undermining our beliefs about empathy and relationships. We encouraged PTs' to reflect, posing an assessment item that called for reflection on changes in their understanding. Allowing PTs to make revisions turned the exam from a punitive, deficit-focused experience into an opportunity to continue learning and building strengths in mathematical understanding. Our collaborative planning created a space where we challenged

assumptions, questioned decisions, and continued searching for ways to implement an exam without sacrificing the relationships we were developing with PTs or the positive classroom environment we had strived to build.

While collaborative planning, we were motivated by our beliefs about the characteristics of effective mathematics teaching. We believe (and want our PTs to believe) that teaching mathematics is complex, requiring specialized knowledge, reflective practice, and continuous learning. In discussing the exam (both before and after administration) with PTs, we considered how the exam mirrored our course goals. We wrote items that emphasized conceptual understanding above procedural memorization. Finally, the allowance of resources revisions positioned the exam as an opportunity for learning itself, as opposed to only a measurement of understanding.

Finally, beliefs about the development of PTs' MKT guided the exam development. In line with Appova and Taylor's (2017) findings, we sought to provide opportunities for our PTs to develop knowledge of instructional strategies and students' understanding. Appova and Taylor studied others and inferred MTE thinking regarding learning experiences for PTs; we describe the evolution of our decisions in the case of the midterm. We included items about pedagogy and instructional strategies in addition to mathematical content, and administering the exam provided opportunities to discuss the role of assessment with our PTs.

We have focused on the development of our midterm exam, but our beliefs about positive relationships and environments, characteristics of effective mathematics teaching, and the development of MKT surfaced in our collaborative planning for other aspects of teaching as well. At the start of the quarter, Rebecca specifically stated her goal to reduce time lecturing and increase time for PTs to collaboratively problem solve. Thus, much meeting time was spent developing student-centered instructional tasks that would support PTs' development of conceptual understanding of mathematical ideas. In addition, we spent a great deal of time revising our curriculum analysis project, a hallmark of the course, to better align with a larger range of course goals. We also restructured the project to require students to develop it over time throughout the entire course instead of working on it only during the last few weeks. Throughout our collaborative planning, regardless of the topic, we decided together which content to emphasize, the design of assignments, and how to structure class time.

The ways in which we understood our primary beliefs and enacted those beliefs contribute to the mathematics education literature based on MTEs' beliefs and actions. One way to enact beliefs is through careful planning, keeping your beliefs at the forefront of instructional decision-making. Although Lovin et al. (2012) offered how collaboration among colleagues can allow for clarifying one's own beliefs, we found our collaborative relationship allowed us to use our primary beliefs in a dynamic way. We used our beliefs and our collaborative conversations to navigate difficult decisions. As we collaboratively planned, grounding decisions in our beliefs about teaching and learning, our conversations often focused on how our beliefs were related; this built greater understanding of our beliefs (Green, 1971). Collaborative planning brought forth our beliefs and self-study allowed us to clarify them. We know ourselves better after having done this work.

In the mathematics education literature on teacher beliefs, attention has been paid to alignment between teacher beliefs and practices (Cross & Hong, 2012; Herbel-Eisenman et al., 2006; Phillip, 2007). Rewatching our meetings, we recognized moments of pride about making decisions that supported our beliefs about effective teaching and learning. When we had differing views on elements of the course, such as whether and how to give a midterm, our collaborative planning relationship supported our persistence in conversation. Our self-study allowed us to identify the role of our beliefs in these conversations. Through discussion and perspective-taking of critical friends, we were aware of moments of misalignment. We circled

back across meetings and made decisions that were suitable for both of us and that aligned with our beliefs. Speer (2005) offers that perceived inconsistency between professed and attributed beliefs may actually be related to methods and data collection employed by researchers. In the case of the midterm, what initially appeared as misaligned beliefs to each of us was clarified through sustained collaboration. We achieved shared understanding about each other's beliefs. We found our collaborative planning allowed for opportunities to reflect on our decisions and learn more about relationships between our professed and attributed beliefs.

Misalignment with our beliefs was often the result of feeling compelled to follow institutional or departmental expectations. The expectation to assign and use grades to demonstrate the mathematical rigor of the course pressured us (particularly Rebecca) to return to the status quo of incorporating traditional exams, which was the prepandemic norm. As early-career faculty, we are still interpreting implicit institutional expectations. Collaborative planning helped us recognize we can find harmony between institutional expectations and our beliefs about humanizing practices and effective teaching. We successfully implemented an exam in a way that contributed to PTs' learning.

Collaborative planning became a space for reflection, critical questions, and alternative possibilities. Sharing the course planning, we learned from each other, incorporated new learning, and were intentional in the ways we centered our courses around our PTs' mathematical ideas. We sought feedback from PTs and each other regularly to inform our decisions and revisions, which led to courses that were co-constructed by the MTEs and PTs. Further, we genuinely enjoyed planning together. Although it was time consuming and often rife with difficult discussion, this practice was affirming and supportive. Teaching online during the pandemic had been overwhelming, isolating, and often tedious. During the quarter we planned collaboratively, we found ourselves excited to teach the lessons we created. We were energized by our preparation, intentional decisions, and the intentionality the preparation afforded in our teaching. Planning together allowed us to celebrate teaching. As our careers develop, we will continue to face tensions between institutional expectations and our beliefs. Yet this experience has empowered us. We now know it is possible to reconcile seemingly disparate expectations and we are capable of doing it. We *can* figure this out—together.

Implications

Collaborative planning is not new (Puchner & Taylor, 2006; Vangrieken et al., 2015). However, using self-study as a mechanism to understand this practice allowed us, as new tenure-track MTEs, to move beyond *collaborative planning is beneficial*, to understanding the ways we developed in our roles as MTEs. This collaborative planning experience made us better teachers. On an instructional level, we were more thoughtful and intentional in all of our course decisions. On a personal level, we supported and affirmed each other, developed confidence in our relatively new tenure-track positions, and reignited our joy for teaching after a period of extreme challenge. Collaborative planning also served as a way to deepen our critical friendship. Our shared investment in the course and our trusting relationship allowed us to ask provocative questions and advocate for each other and our PTs, all of which are elements of critical friendship (Costa & Kallick, 1993).

During the spring of 2022, we turned to our beliefs about effective teaching and learning of mathematics when faced with difficult decisions or disagreements. The diversity in our beliefs, and our awareness of shifts or clarification of our beliefs, pushed our thinking and led to course improvements. Perspective-taking and critical friendship resulted in transformative praxis. In the face of our original sentiment of disliking exams, we found a way to write an exam that aligned with our beliefs about teaching and learning mathematics. A significant change in perspective was possible through our collaborative planning. Moments when we felt out of alignment with our beliefs led to reflection. In moments when we had agreement, our beliefs that underpinned

the ideas were similar. The times in which we had disagreements, we often found our beliefs were less similar, yet malleable. We were able to create a midterm exam that emphasized PTs' thinking and offered alternatives to a traditional exam. At the conclusion of the quarter, we recognized we could not, or rather should not, give a final exam because it did not serve the same purpose as our midterm.

Research on teacher beliefs and planning is an area that has received a lot of attention, whereas less is known about MTEs' beliefs. One of the strengths of this study is in its methodology. Regular meetings and reflective journals among critical friends provide unique insight into ways that our beliefs affect our collaboratively planned course. Previous research has included interviews of MTEs, both engaged in dialogue with their colleagues about their beliefs (Lovin et al., 2012) and sharing their perspectives on the design of their courses with researchers (Li & Superfine, 2016), offering insight into how MTEs' beliefs about teaching and learning mathematics can affect their courses. However, our study allowed for insights that interviewing never could. We navigated challenges, thought about important ideas over multiple weeks, and challenged each other regularly. Our meeting conversations were organic discussions, going in whatever directions our ideas and questions led. As such, we have data that captures ways in which our thinking was consistent and ways in which it has changed over time. Many elements from our collaborative planning have continued beyond our one quarter of teaching a shared course. We continue to give midterms, for example. This experience was transformative for both of us, and we hope to repeat such collaborative planning for a different course as soon as circumstances allow. This work resides at the intersection of the deep friendship we have built and the commitment we each have to reflective practice. This overlap cultivated a powerful critical friendship, which will have a sustained impact on our individual teaching and learning.

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