SFR(Ha) calibration

Eduardo Alberto Duarte Lacerda

June 2019

1 Current SFR calibration using Hα luminosity

We want to calibrate the Hα luminosity, L(Hα), as an SFR (ψ) indicator (e.g. Kennicutt, 1998) using a linear relation

$$\psi_{\mathrm{H}\alpha} = k \times \mathrm{L}(\mathrm{H}\alpha). \tag{1}$$

Our job is find that k. We will dive in such endeavour through the nature of the Hα photons.

The total amount of l-light[1], we receive from stars formed t years ago is

$$\mathrm{d}\Lambda = l(t)\,\mathrm{dM}(t). \tag{2}$$

Integrating Eq. 2 over the Universe time ($T_U \sim 14$Gyr), we would see, today, a total of

$$\begin{aligned} \mathrm{dM}(t) &= \psi(t)\,\mathrm{d}t & (3)\\ \Lambda = \Lambda(t = T_U) &= \int_0^{T_U} l(t)\,\mathrm{dM}(t) & (4)\\ &= \int_0^{T_U} l(t)\,\psi(t)\,\mathrm{d}t & (5) \end{aligned}$$

l-light. From the case B hydrogen recombination one out of each 2.206 ionizing photons produces an Hα photon. So the intrinsic Hα luminosity can be theorically calculated as

$$\mathrm{L}(\mathrm{H}\alpha) = h\nu_{\mathrm{H}\alpha}\frac{\mathrm{Q}_H}{2.206} \tag{6}$$

[1] $l(t)$ can be any function which describes the time-evolution of any generic radiative source per unit formed mass (thus IMF-dependent) of an SSP.

Where Q_H is the rate of H-ionizing photons. Just to remember, we assume here that no ionizing radiation escapes the nebula, L(Hα) has been corrected for extinction and that dust does not eat much of the $h\nu < 13.6$ eV photons. We know that dQ_H can be written as Eq. 2. Integrating it as Eq. 5 we have:

$$Q_H = \int dQ_H = \int q_H(t)\, dM(t) \tag{7}$$

$$Q_H(t = T) = \int_0^T q_H(t)\, \psi(t)\, dt \tag{8}$$

In these equations above, q_H is the H-ionizing photon rate per unit formed mass. We can use it as our kind of l-light in Eq. 5 considering all photons that can ionize hydrogen (h$\nu \geq 13.6$ eV or $\lambda \leq 912$) and write it as follows:

$$q_H(t) = \int_0^{912} \frac{l_\lambda\, \lambda}{hc} d\lambda, \tag{9}$$

where l_λ is the luminosity per unit formed mass per wavelength in solar units [$L_\odot/M_\odot$] for an SSP[2]. With this we still need to analyze how the integration of q_H evolves in time in order to obtain an SFR (ψ). Integrating q_H from today to T_U we have the number of H-ionizing photons produced by our l-light:

$$\mathcal{N}_H = \int_0^{T_U} q_H(t)\, dt \tag{10}$$

Using Bruzual & Charlot (2003) SSP models we can see the evolution of $\mathcal{N}_H$ in time in the Figs. 1, 2, 3 and 4. This figure shows us the evolution of $\mathcal{N}_H$ in time in absolute values (left-upper panel) and in relatively the total $\mathcal{N}_H$ (right-upper panel). In Cid Fernandes et al. (2011, Fig. 2b) we can see the time evolution of q_H over all ages and metalicities[3]. The same plot is reproduced here at the bottom panel of Fig. 1. It's easy to twig that the number of H-ionizing photons rapidly converges to its maximum close to $t = 10$ Myr. For an SFR which is constant over that time-scale ($\psi(t) \rightarrow \psi$) the Eq. 8 converges to

$$Q_H = \psi\, \mathcal{N}_H(t = 10\mathrm{Myr}, \mathrm{IMF}, Z_\star). \tag{11}$$

[2]One can notice that I do not wrote an explicit dependency on Z, IMF and isochrones, on l_λ (hence q_H all and his products), but exists those exists.

[3]In that study the http://starlight.ufsc.brSEAGal/ group used 'Padova 1994' (Bertelli et al., 1994) stellar tracks with an IMF from Chabrier (2003)

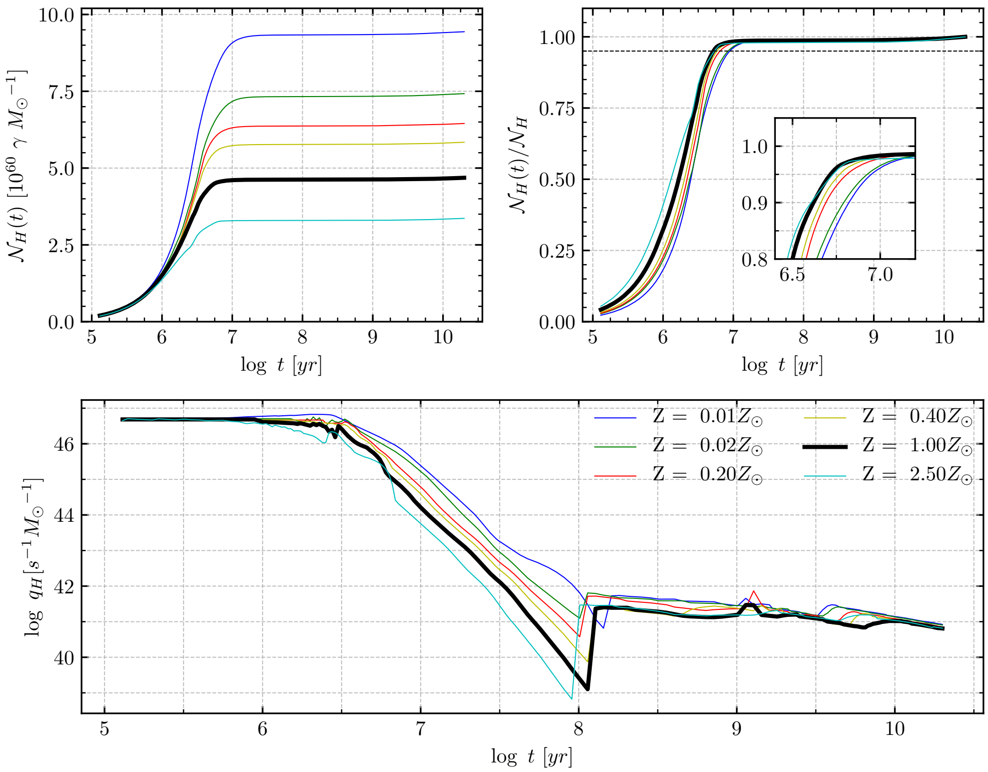

Figure 1: Upper left panel: The time-evolution of the number of the photons ($\mathcal{N}_H$) for six metallicities (from 0.01 $Z_\odot$ to 2.5 $Z_\odot$) that compose our SSP models. The solar metallicity is drawn as a thick black line. Upper right panel: The same from upper left panel but normalized by the total value of $\mathcal{N}_H$. The black dashed line shows 95% of the total $\mathcal{N}_H$. A zoom is also provided for a better view of the region around 10 Myr. Bottom panel: Evolution of the H-ionizing photon rate per unit formed mass also for the same six metalicities. For this version of figure we use 'Padova 1994' stellar tracks (Bertelli et al., 1994) with a Chabrier extinction law.

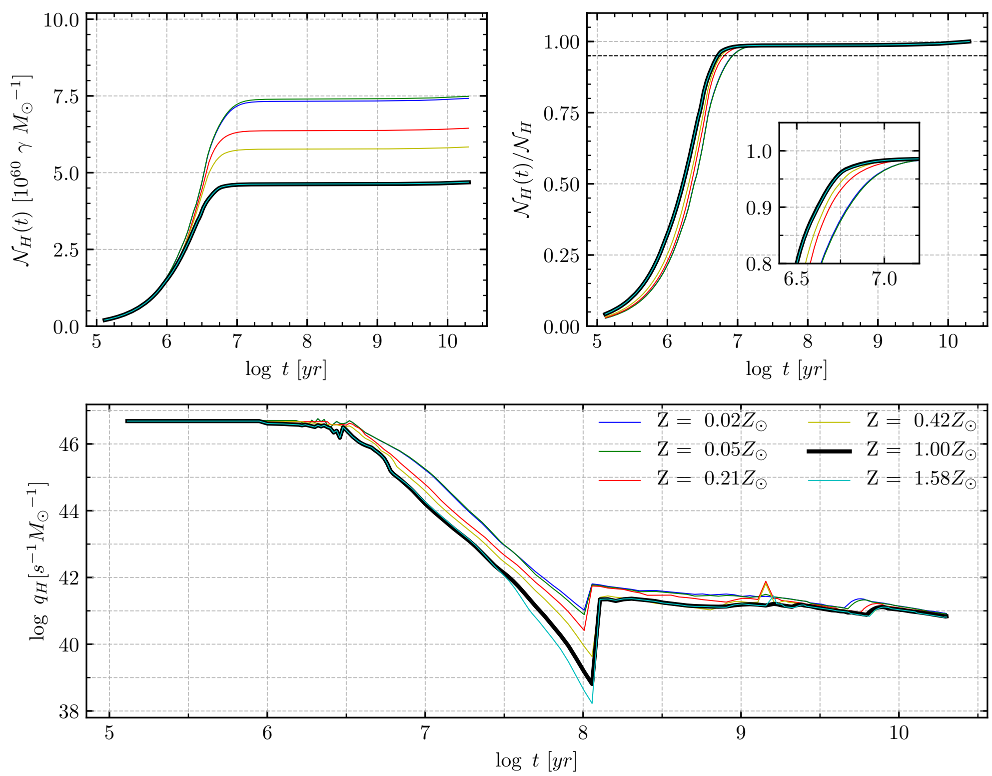

Figure 2: The same as Fig. 1 but with stellar tracks from (Girardi et al., 2000) (a.k.a. Padova 2000) and also a Chabrier (2003) IMF. The metallicities for this configuration goes from 0.02 $Z_\odot$ to 1.58 $Z_\odot$.

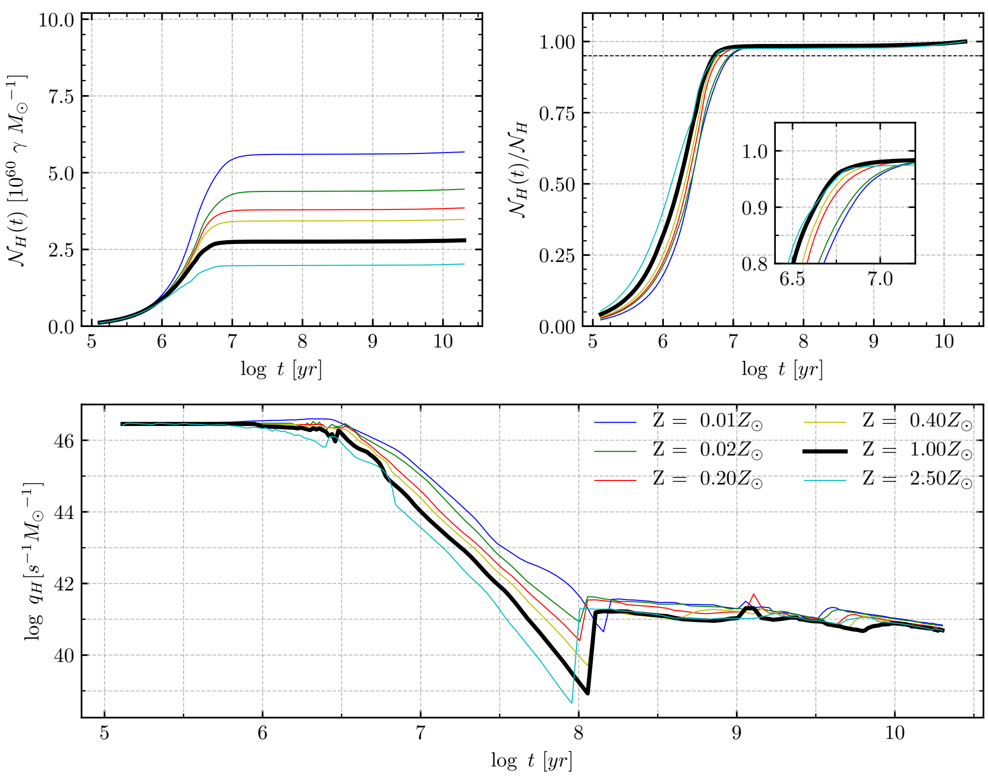

Figure 3: The same as Fig. 1 but with a Salpeter (1955) IMF.

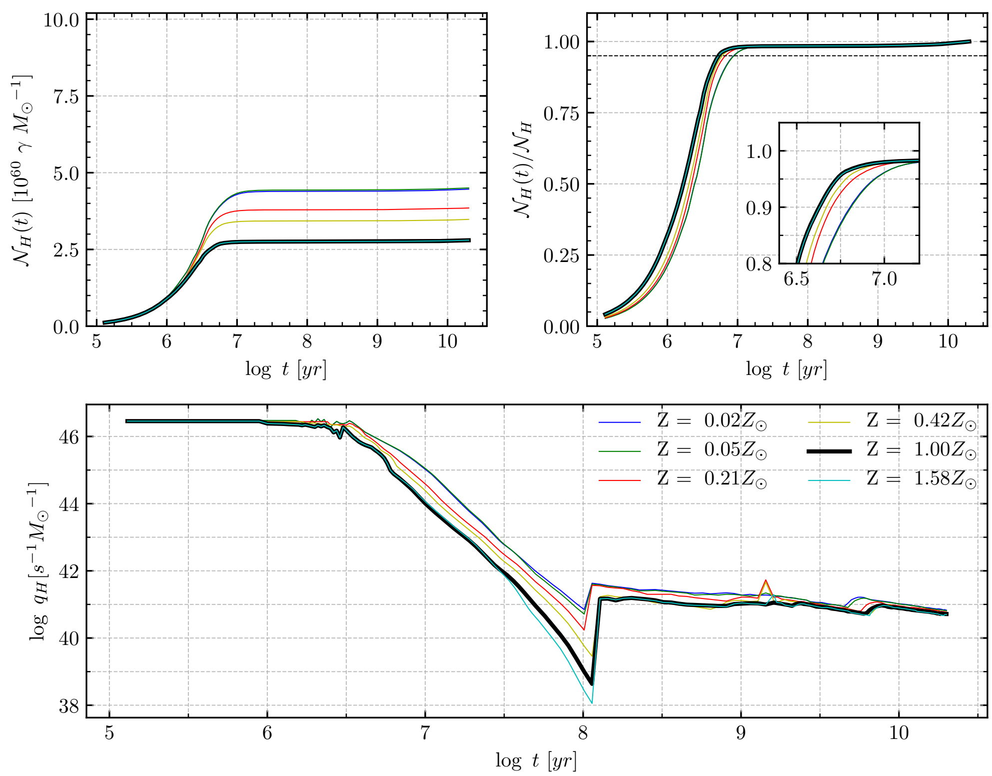

Figure 4: The same as Fig. 2 but with a Salpeter (1955) IMF.

Replacing Eq. 11 in Eq. 6, we are now able to write:

$$\psi_{\mathrm{H}\alpha} = \frac{2.206}{\mathcal{N}_H \, h\nu_{\mathrm{H}\alpha}} \times \mathrm{L}(\mathrm{H}\alpha) \tag{12}$$

This method gives us a recent SFR. Recent because we use the value of N_H at $t = 10$ Myr. For the last step, solving Eq. 12 we found the value for k in Eq. 1 ($\psi \equiv$ SFR) for solar metallicity, Padova (2000) stellar tracks and an IMF from Salpeter (thick black line in Fig. 4):

$$\mathrm{SFR}_{\mathrm{H}\alpha}[\mathrm{M}_\odot\ \mathrm{yr}^{-1}] = 3.21 \left(\frac{\mathrm{L}(\mathrm{H}\alpha)}{10^8 \mathrm{L}_\odot}\right) = 8.38 \times 10^{-42}\ \mathrm{L}(\mathrm{H}\alpha)\ [\mathrm{ergs\ s}^{-1}] \tag{13}$$

In Table 1 we show the value of k for each set of models and metallicity, the fraction of ionizing photons at $t = 10$ Myr and the age where the fraction of ionizing photons is 95, 98 and 99 per cent. Fig. 5 brings the distribution of k in function of metallicity for all models. For the IMF of Salpeter (1955), the values of k changes in metallicity from a factor of ~ 1.5 for Padova2000 to ~ 3 for Padova1999 configuration. Changing to a Chabrier (2003) IMF, the values of k varies in a factor of ~ 2 for Padova2000 for almost 10 for the Padova1994 isochrones. We have to take in account that for the Padova2000 models, the metallicity range from 0.02 $\mathrm{Z}_\odot$ to ~ 1.6 $\mathrm{Z}_\odot$, when Padova1994 models goes from 0.005 $\mathrm{Z}_\odot$ to 2.5 $\mathrm{Z}_\odot$. The Kennicutt (1998) calibration for solar metallicity (7.9×10^{-42} ergs s^{-1}) was calculated with other SSP models and with a Salpeter (1955) IMF.

References

Bertelli G., Bressan A., Chiosi C., Fagotto F., Nasi E., 1994, , https://ui.adsabs.harvard.edu/abs/1994A275

Bruzual G., Charlot S., 2003, http://dx.doi.org/10.1046/j.1365-8711.2003.06897.x , http://adsabs.harvard.edu/abs/2003MNRAS.344.1000B 344, 1000

Chabrier G., 2003, http://dx.doi.org/10.1086/376392 , http://adsabs.harvard.edu/abs/2003PASP..115..763C 115, 763

Cid Fernandes R., Stasińska G., Mateus A., Vale Asari N., 2011, http://dx.doi.org/10.1111/j.1365-2966.2011.18244.x , http://adsabs.harvard.edu/abs/2011MNRAS.413.1687C 413, 1687

Table 1: List of values of k for each model and metallicity. Together we add the fraction of ionizing photons at $t = 10$ Myr, $N_H(10Myr)/N_H^{tot}$) and the ages where the fraction of ionizing photons is 95, t_{95}, 98, t_{98} and 99 per cent, t_{99}.

Set of models	Z	k [$M_\odot$ yr^{-1}]	k [10^{-42} ergs s^{-1}]	$N_H(10Myr)/N_H^{tot}$	t_{95} [Myr]	t_{98} [Myr]	t_{99} [Gyr]
Padova1994.chab	0.0001	0.97	2.54	0.9615	8.71	13.80	0.81
Padova1994.chab	0.0004	1.23	3.21	0.9657	8.32	13.18	2.75
Padova1994.chab	0.0040	1.40	3.65	0.9783	6.31	10.47	2.20
Padova1994.chab	0.0080	1.54	4.02	0.9813	5.75	9.12	2.20
Padova1994.chab	0.0200	1.91	5.00	0.9834	5.01	7.94	2.75
Padova1994.chab	0.0500	2.68	7.00	0.9775	5.25	508.80	7.75
Padova1994.salp	0.0001	1.62	4.24	0.9574	9.12	15.85	3.25
Padova1994.salp	0.0004	2.05	5.36	0.9616	8.71	15.85	5.00
Padova1994.salp	0.0040	2.34	6.13	0.9749	6.61	12.59	5.00
Padova1994.salp	0.0080	2.59	6.76	0.9783	6.03	10.47	5.25
Padova1994.salp	0.0200	3.21	8.39	0.9806	5.25	9.55	5.50
Padova1994.salp	0.0500	4.47	11.68	0.9734	5.50	2600.00	9.50
Padova2000.chab	0.0004	1.23	3.21	0.9660	8.32	13.18	2.40
Padova2000.chab	0.0010	1.22	3.18	0.9652	8.32	13.18	1.90
Padova2000.chab	0.0040	1.40	3.65	0.9786	6.31	10.47	1.90
Padova2000.chab	0.0080	1.54	4.02	0.9818	5.75	9.12	1.80
Padova2000.chab	0.0190	1.91	5.00	0.9830	5.01	7.94	3.75
Padova2000.chab	0.0300	1.91	4.99	0.9829	5.01	8.32	3.25
Padova2000.salp	0.0004	2.05	5.36	0.9618	8.71	15.85	5.50
Padova2000.salp	0.0010	2.03	5.32	0.9612	8.71	15.14	5.00
Padova2000.salp	0.0040	2.34	6.13	0.9753	6.61	12.02	4.75
Padova2000.salp	0.0080	2.59	6.76	0.9788	5.75	10.47	4.25
Padova2000.salp	0.0190	3.21	8.39	0.9800	5.25	9.55	6.75
Padova2000.salp	0.0300	3.21	8.38	0.9800	5.25	9.55	6.25

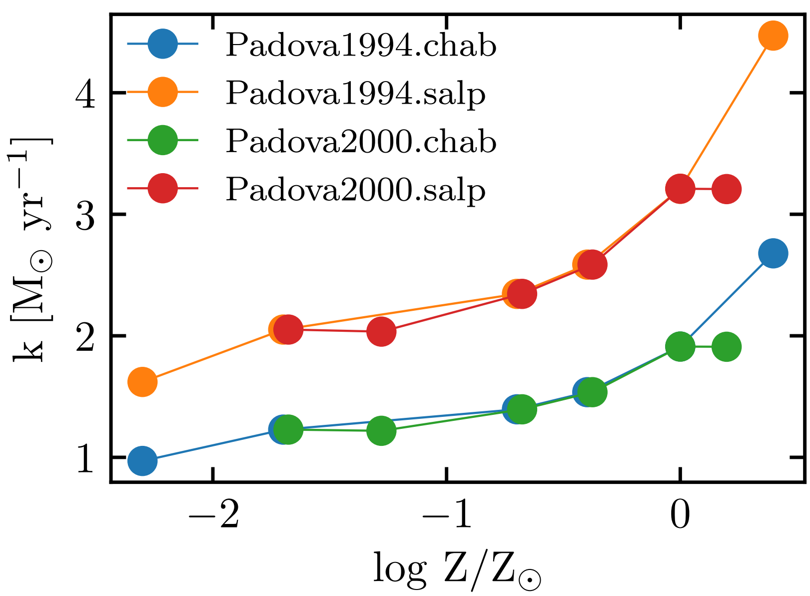

Figure 5: Values of k in function of metallicity, stellar tracks and IMF.

Girardi L., Bressan A., Bertelli G., Chiosi C., 2000, http://dx.doi.org/10.1051/aas:2000126 , http://adsabs.harvard.edu/abs/2000A

Kennicutt Jr. R. C., 1998, http://dx.doi.org/10.1146/annurev.astro.36.1.189 , http://adsabs.harvard.edu/abs/1998ARA

Osterbrock D. E., Ferland G. J., 2006, Astrophysics of gaseous nebulae and active galactic nuclei

Salpeter E. E., 1955, http://dx.doi.org/10.1086/145971 , http://adsabs.harvard.edu/abs/1955ApJ...121..161S 121, 161

A The creation rate of Hα photons

Consider a volume element ΔV in an H II region. The number of $p + e \rightarrow \mathrm{H}^0$ recombinations per unit time is $n_p n_e \alpha_B(\mathrm{H}^0)\Delta V$, where n_e and n_p are the proton and electron densities, and $\alpha_B(\mathrm{H}^0)$ is the case B total recombination coefficient of H. From Table 2.7 in Osterbrock & Ferland (2006) $\alpha_B(\mathrm{H}^0) = 2.59 \times 10^{-13}$ recombinations $\times\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ (for a temperature of 10000 K). During the recombination cascade some of these re-married electrons will go through $n = 3 \rightarrow 2$ transitions, producing Hα. Let $\alpha^{eff}_{\mathrm{H}\alpha}(\mathrm{H}^0)$ be the coefficient which counts only these recombinations. $\alpha^{eff}_{\mathrm{H}\alpha}(\mathrm{H}^0)$ is not explicitly given in Osterbrock & Ferland's book, but they do provide (in Table 4.7) the coefficient for Hβ, $\alpha^{eff}_{\mathrm{H}\beta}(\mathrm{H}^0) = 3.03 \times 10^{-14}$, as well as the ratio of volume emissivities, the famous $j_{\mathrm{H}\alpha}/j_{\mathrm{H}\beta} = 2.87$. Using

$$\frac{\alpha^{eff}_{\mathrm{H}\alpha}(\mathrm{H}^0)}{\alpha^{eff}_{\mathrm{H}\beta}(\mathrm{H}^0)} = \frac{j_{\mathrm{H}\alpha}/h\nu_{\mathrm{H}\alpha}}{j_{\mathrm{H}\beta}/h\nu_{\mathrm{H}\beta}}$$

(where the ratio of photon energies appears because j measures energy while α measures a number of recombinations), we finally obtain $\alpha^{eff}_{\mathrm{H}\alpha}(\mathrm{H}^0) = 1.17 \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$, or $0.453 \times \alpha_B(\mathrm{H}^0)$. In other words, one out of every $1/0.453 = 2.206$ recombinations produces an Hα photon. Since in equilibrium each photoionization is balanced by a recombination, and as long as no $h\nu > 13.6$ eV radiation escapes the nebula, we can also say that one out of every 2.206 ionizing photons produces on Hα photon.