Cálculo de L_λ e do erro ($\epsilon(L_\lambda)$)

$$\begin{aligned} L_\lambda &= 4\pi d^2 F_\lambda & (1) \\ \epsilon(L_\lambda) &= 4\pi d^2 \epsilon(F_\lambda) & (2) \end{aligned}$$

Cálculo de $L_{\mathrm{H}\alpha}^{int}$ e $\tau_{\mathrm{V}}^{\mathrm{NEB}}$:

$$\begin{aligned}
L_\lambda^{obs} &= L_\lambda^{int} e^{-\tau_\lambda} & (3) \\
L_\lambda^{obs} &= L_\lambda^{int} e^{-(\frac{\tau_\lambda}{\tau_{\mathrm{V}}})\tau_{\mathrm{V}}} & (4) \\
\frac{\tau_\lambda}{\tau_{\mathrm{V}}} &= q_\lambda & (5) \\
L_\lambda^{obs} &= L_\lambda^{int} e^{-q_\lambda \tau_{\mathrm{V}}} & (6) \\
\frac{L_\lambda^{obs}}{L_{\lambda'}^{obs}} &= \frac{L_\lambda^{int} e^{-q_\lambda \tau_{\mathrm{V}}}}{L_{\lambda'}^{int} e^{-q_{\lambda'} \tau_{\mathrm{V}}}} & (7) \\
\ln\left(\frac{L_\lambda^{obs}}{L_{\lambda'}^{obs}}\right) &= \tau_{\mathrm{V}}(q_{\lambda'} - q_\lambda) \ln\left(\frac{L_\lambda^{int}}{L_{\lambda'}^{int}}\right) & (8) \\
\tau_{\mathrm{V}} &= \frac{1}{(q_{\lambda'} - q_\lambda)}\left[\ln\left(\frac{L_\lambda^{obs}}{L_{\lambda'}^{obs}}\right) - \ln\left(\frac{L_\lambda^{int}}{L_{\lambda'}^{int}}\right)\right] & (9) \\
\tau_{\mathrm{V}} &= \frac{1}{(q_{\lambda'} - q_\lambda)}\left[\ln\left(\frac{F_\lambda^{obs}}{F_{\lambda'}^{obs}}\right) - \ln\left(\frac{F_\lambda^{int}}{F_{\lambda'}^{int}}\right)\right] & (10) \\
\tau_{\mathrm{V}}^{\mathrm{NEB}} &= \frac{1}{(q_{\mathrm{H}\beta} - q_{\mathrm{H}\alpha})} \ln\left(\frac{F_{\mathrm{H}\alpha}^{obs}/F_{\mathrm{H}\beta}^{obs}}{F_{\mathrm{H}\alpha}^{int}/F_{\mathrm{H}\beta}^{int}}\right) & (11) \\
L_{\mathrm{H}\alpha}^{int} &= L_{\mathrm{H}\alpha}^{obs} e^{(q_{\mathrm{H}\alpha}\tau_{\mathrm{V}}^{\mathrm{NEB}})} & (12)
\end{aligned}$$

Propagação de erro no cálculo de $L_{\mathrm{H}\alpha}^{int}$:

$$\begin{aligned}
L_{\mathrm{H}\alpha}^{int} &\equiv L_{\mathrm{H}\alpha}^{int}(L_{\mathrm{H}\alpha}^{obs}, \tau_{\mathrm{V}}^{\mathrm{NEB}}) \equiv L_{\mathrm{H}\alpha}^{int}(L_{\mathrm{H}\alpha}^{obs}, L_{\mathrm{H}\beta}^{obs}) & (13) \\
\epsilon(L_{\mathrm{H}\alpha}^{int}) &= \sqrt{\left(\frac{\partial L_{\mathrm{H}\alpha}^{int}}{\partial L_{\mathrm{H}\alpha}^{obs}}\right)^2 \epsilon(L_{\mathrm{H}\alpha}^{obs})^2 + \left(\frac{\partial L_{\mathrm{H}\alpha}^{int}}{\partial L_{\mathrm{H}\beta}^{obs}}\right)^2 \epsilon(L_{\mathrm{H}\beta}^{obs})^2} & (14) \\
\frac{\partial L_{\mathrm{H}\alpha}^{int}}{\partial L_{\mathrm{H}\alpha}^{obs}} &= e^{(q_{\mathrm{H}\alpha}\tau_{\mathrm{V}}^{\mathrm{NEB}})} & (15) \\
\frac{\partial L_{\mathrm{H}\alpha}^{int}}{\partial L_{\mathrm{H}\beta}^{obs}} &= \left(\frac{\partial L_{\mathrm{H}\alpha}^{int}}{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}}\right)\left(\frac{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}}{\partial L_{\mathrm{H}\beta}^{obs}}\right) & (16) \\
\frac{\partial L_{\mathrm{H}\alpha}^{int}}{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}} &= L_{\mathrm{H}\alpha}^{obs} q_{\mathrm{H}\alpha} e^{(q_{\mathrm{H}\alpha}\tau_{\mathrm{V}}^{\mathrm{NEB}})} & (17) \\
\frac{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}}{\partial L_{\mathrm{H}\beta}^{obs}} &= -\frac{1}{L_{\mathrm{H}\beta}^{obs}(q_{\mathrm{H}\beta} - q_{\mathrm{H}\alpha})} & (18) \\
\frac{\partial L_{\mathrm{H}\alpha}^{int}}{\partial L_{\mathrm{H}\beta}^{obs}} &= -\left(\frac{q_{\mathrm{H}\alpha}}{q_{\mathrm{H}\beta} - q_{\mathrm{H}\alpha}}\right)\left(\frac{L_{\mathrm{H}\alpha}^{obs}}{L_{\mathrm{H}\beta}^{obs}}\right) & (19) \\
\epsilon(L_{\mathrm{H}\alpha}^{int}) &= e^{(q_{\mathrm{H}\alpha}\tau_{\mathrm{V}}^{\mathrm{NEB}})} \sqrt{\epsilon(L_{\mathrm{H}\alpha}^{obs})^2 + \left(\frac{q_{\mathrm{H}\alpha}}{q_{\mathrm{H}\beta} - q_{\mathrm{H}\alpha}}\right)^2 \left(\frac{L_{\mathrm{H}\alpha}^{obs}}{L_{\mathrm{H}\beta}^{obs}}\right)^2 \epsilon(L_{\mathrm{H}\beta}^{obs})^2} & (20)
\end{aligned}$$

Propagação de erro no cálculo de $\tau_{\mathrm{V}}^{\mathrm{NEB}}$:

$$\tau_{\mathrm{V}}^{\mathrm{NEB}} \equiv \tau_{\mathrm{V}}^{\mathrm{NEB}}(L_{\mathrm{H}\alpha}^{obs}, L_{\mathrm{H}\beta}^{obs}) \tag{21}$$

$$\epsilon(\tau_{\mathrm{V}}^{\mathrm{NEB}}) = \sqrt{\left(\frac{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}}{\partial L_{\mathrm{H}\alpha}^{obs}}\right)^2 \epsilon(L_{\mathrm{H}\alpha}^{obs})^2 + \left(\frac{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}}{\partial L_{\mathrm{H}\beta}^{obs}}\right)^2 \epsilon(L_{\mathrm{H}\beta}^{obs})^2} \tag{22}$$

$$\frac{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}}{\partial L_{\mathrm{H}\alpha}^{obs}} = \frac{1}{L_{\mathrm{H}\alpha}^{obs}(q_{\mathrm{H}\beta} - q_{\mathrm{H}\alpha})} \tag{23}$$

$$\frac{\partial \tau_{\mathrm{V}}^{\mathrm{NEB}}}{\partial L_{\mathrm{H}\beta}^{obs}} = -\frac{1}{L_{\mathrm{H}\beta}^{obs}(q_{\mathrm{H}\beta} - q_{\mathrm{H}\alpha})} \tag{24}$$

$$\epsilon(\tau_{\mathrm{V}}^{\mathrm{NEB}}) = \frac{1}{(q_{\mathrm{H}\beta} - q_{\mathrm{H}\alpha})} \sqrt{\left(\frac{\epsilon(L_{\mathrm{H}\alpha}^{obs})}{L_{\mathrm{H}\alpha}^{obs}}\right)^2 + \left(\frac{\epsilon(L_{\mathrm{H}\beta}^{obs})}{L_{\mathrm{H}\beta}^{obs}}\right)^2} \tag{25}$$

Propagação do erro no cálculo de $F_{\mathrm{H}\alpha}^{obs}/F_{\mathrm{H}\beta}^{obs}$:

$$F_{Balmer}^{obs} = \frac{F_{\mathrm{H}\alpha}^{obs}}{F_{\mathrm{H}\beta}^{obs}} \tag{26}$$

$$F_{Balmer}^{obs} \equiv F_{Balmer}^{obs}(F_{\mathrm{H}\alpha}^{obs}, F_{\mathrm{H}\beta}^{obs}) \tag{27}$$

$$\epsilon(F_{Balmer}^{obs}) = \sqrt{\left(\frac{\partial F_{Balmer}^{obs}}{\partial F_{\mathrm{H}\alpha}^{obs}}\right)^2 \epsilon(F_{\mathrm{H}\alpha}^{obs})^2 + \left(\frac{\partial F_{Balmer}^{obs}}{\partial F_{\mathrm{H}\beta}^{obs}}\right)^2 \epsilon(F_{\mathrm{H}\beta}^{obs})^2} \tag{28}$$

$$\frac{\partial F_{Balmer}^{obs}}{\partial F_{\mathrm{H}\alpha}^{obs}} = \frac{1}{F_{\mathrm{H}\beta}^{obs}} \tag{29}$$

$$\frac{\partial F_{Balmer}^{obs}}{\partial F_{\mathrm{H}\beta}^{obs}} = -\frac{F_{\mathrm{H}\alpha}^{obs}}{F_{\mathrm{H}\beta}^{obs\,2}} \tag{30}$$

$$\epsilon(F_{Balmer}^{obs}) = \frac{1}{F_{\mathrm{H}\beta}^{obs}} \sqrt{\epsilon(F_{\mathrm{H}\alpha}^{obs})^2 + \left(\frac{F_{\mathrm{H}\alpha}^{obs}}{F_{\mathrm{H}\beta}^{obs}}\right)^2 \epsilon(F_{\mathrm{H}\beta}^{obs})^2} \tag{31}$$

Cálculo de O_3N_2 e $\log Z_{neb}$:

$$O_3N_2 = \frac{F_{[\mathrm{O\,III}]}^{int}}{F_{[\mathrm{N\,II}]}^{int}} = \frac{F_{[\mathrm{O\,III}]}^{obs}}{F_{[\mathrm{N\,II}]}^{obs}} e^{\tau_{\mathrm{V}}^{\mathrm{NEB}}(q_{[\mathrm{O\,III}]} - q_{[\mathrm{N\,II}]})} \tag{32}$$

$$\log Z_{neb} = -0.14 - 0.25 \log O_3N_2 \tag{33}$$

$$\tag{34}$$

Propagação no erro de O_3N_2 e $\log Z_{neb}$:

$$O_3N_2 \equiv O_3N_2(F^{obs}_{[\mathrm{O\,III}]}, F^{obs}_{[\mathrm{N\,II}]}, F^{obs}_{\mathrm{H}\alpha}, F^{obs}_{\mathrm{H}\beta}) \tag{35}$$

$$\frac{\partial O_3N_2}{\partial F^{obs}_{[\mathrm{O\,III}]}} = e^{\tau_V^{\mathrm{NEB}}(q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]})}\left(\frac{1}{F^{obs}_{[\mathrm{N\,II}]}}\right) \tag{36}$$

$$\frac{\partial O_3N_2}{\partial F^{obs}_{[\mathrm{N\,II}]}} = -e^{\tau_V^{\mathrm{NEB}}(q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]})}\left(\frac{F^{obs}_{[\mathrm{O\,III}]}}{{F^{obs}_{[\mathrm{N\,II}]}}^2}\right) \tag{37}$$

$$\frac{\partial O_3N_2}{\partial F^{obs}_{\mathrm{H}\alpha}} = e^{\tau_V^{\mathrm{NEB}}(q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]})}\left(\frac{q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]}}{q_{\mathrm{H}\beta}-q_{\mathrm{H}\alpha}}\right)\left(\frac{F^{obs}_{[\mathrm{O\,III}]}}{F^{obs}_{[\mathrm{N\,II}]}F^{obs}_{\mathrm{H}\alpha}}\right) \tag{38}$$

$$\frac{\partial O_3N_2}{\partial F^{obs}_{\mathrm{H}\beta}} = -e^{\tau_V^{\mathrm{NEB}}(q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]})}\left(\frac{q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]}}{q_{\mathrm{H}\beta}-q_{\mathrm{H}\alpha}}\right)\left(\frac{F^{obs}_{[\mathrm{O\,III}]}}{F^{obs}_{[\mathrm{N\,II}]}F^{obs}_{\mathrm{H}\beta}}\right) \tag{39}$$

$$\epsilon(O_3N_2) = \frac{e^{\tau_V^{\mathrm{NEB}}(q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]})}}{F^{obs}_{[\mathrm{N\,II}]}}\sqrt{\epsilon(F^{obs}_{[\mathrm{O\,III}]})^2+\left(\frac{F^{obs}_{[\mathrm{O\,III}]}}{F^{obs}_{[\mathrm{N\,II}]}}\right)^2\epsilon(F^{obs}_{[\mathrm{N\,II}]})^2+\ldots}$$
$$\overline{\ldots+\left(\frac{q_{[\mathrm{O\,III}]}-q_{[\mathrm{N\,II}]}}{q_{\mathrm{H}\beta}-q_{\mathrm{H}\alpha}}\right)^2\left[\left(\frac{F^{obs}_{[\mathrm{O\,III}]}}{F^{obs}_{\mathrm{H}\alpha}}\right)^2+\left(\frac{F^{obs}_{[\mathrm{O\,III}]}}{F^{obs}_{\mathrm{H}\beta}}\right)^2\right]} \tag{40}$$

$$\epsilon(\log Z_{neb}) = \sqrt{\frac{\partial \log Z_{neb}}{\partial O_3N_2}^2\epsilon(O_3N_2)^2} \tag{41}$$

$$\frac{\partial \log Z_{neb}}{\partial O_3N_2} = \frac{0.25}{\ln(10)}\frac{1}{O_3N_2} \tag{42}$$

$$= \frac{0.25}{\ln(10)}\left(\frac{\epsilon(O_3N_2)}{O_3N_2}\right) \tag{43}$$