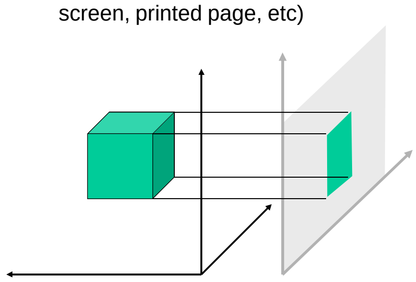
Viewing Transformations

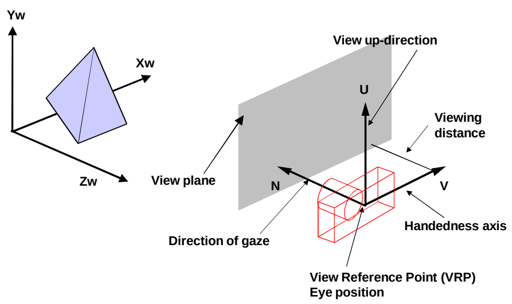
Viewing parameters
Viewing projections
- Parallel
- Parallel orthographic projection
- Parallel oblique projection

Viewing transformations

- Viewing transformations are concerned with projecting 3D objects onto 2D surfaces (display screen, printed page, etc)

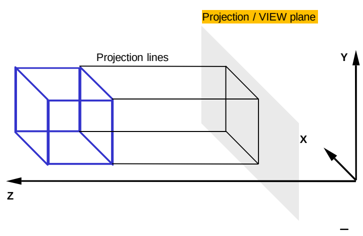


Virtual camera - definitions



Viewing projections

- Parallel
  - centre of projection in the infinity
  - the projection lines are parallel



Parallel orthographic projection

- Projection lines perpendicular to the projection plane
- Transformation matrix

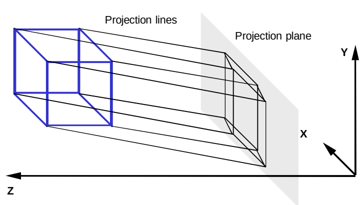
$$P_{par} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x_p = x$

$y_p = y$

Parallel oblique projection

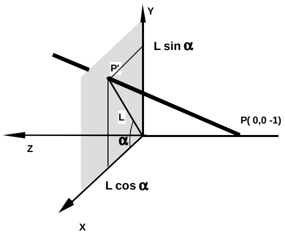
- Projection lines parallel, but not perpendicular to the projection plane



Parallel oblique projection

Matrix

$$P_{obl} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ L\cos\alpha & L\sin\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

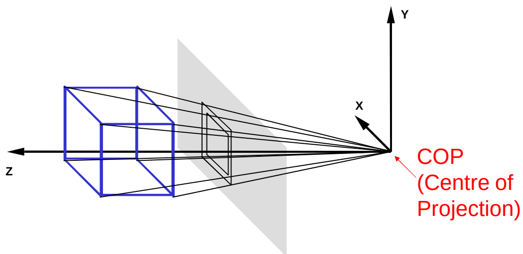


Parallel projections

- Preserve relative dimensions of the objects
- Do not give realistic appearance

Perspective projections

- The projection lines meet in one or more projection centre(s)

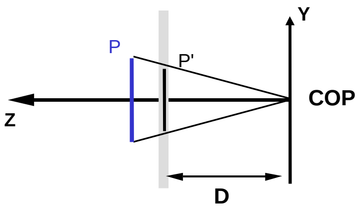


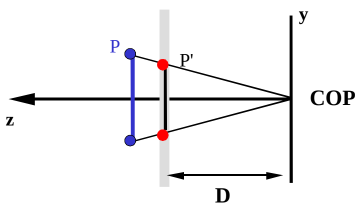
Perspective projections

- COP can be placed at:
  - at the negative part of the Z (N) viewing axis, with the viewing plane at COP
  - at the centre of the viewing coordinate system, with the viewing plane at the positive part of the Z (N) viewing axis

COP at the centre of the viewing coordinate system

- COP is at the centre of the viewing coordinate system ($Z_{COP} = 0$)
- Projection plane is on the positive part of the Z (N) axis
- Projection plane is at distance D from the COP





P = (x, y, z)

P' = (x_p, y_p, D)

$$\frac{y_p}{D} = \frac{y}{z} \qquad \frac{x_p}{D} = \frac{x}{z}$$

$$y_p = \frac{D\,y}{z} = \frac{y}{z/D} \qquad x_p = \frac{D\,x}{z} = \frac{x}{z/D}$$

Transformation matrix

$$P_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & \mathbf{0} \end{bmatrix}$$

$$y_p = \frac{D\,y}{z} = \frac{y}{z/D} \qquad x_p = \frac{D\,x}{z} = \frac{x}{z/D}$$

Example point transformation

Non-homogeneous coordinates

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & \mathbf{0} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{z}{D} \end{bmatrix}$$

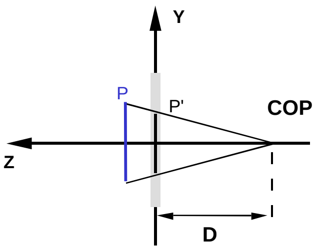
This vector is not in the homogeneous coordinate system.

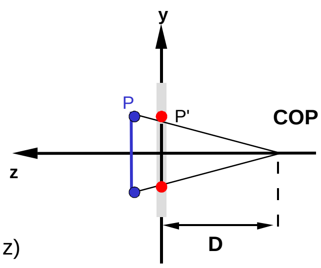
Convert to the homogenous coordinates by dividing all the components of the vector by the last term: $\frac{z}{D}$

$$\begin{bmatrix} D \cdot \frac{x}{z} \\ D \cdot \frac{y}{z} \\ D \\ 1 \end{bmatrix}$$

Projection plane at the centre of the viewing coordinate system

- COP is on the negative part of the z axis ($Z_{COP} = -D$)
- Projection plane is at the centre of the viewing coordinate system
- Projection plane is at distance D from the COP





$P = (x, y, z)$

$P' = (x_p, y_p, 0)$

$$\frac{yp}{D} = \frac{y}{z+D} \qquad \frac{xp}{D} = \frac{x}{z+D}$$

$$yp = \frac{D\,y}{z + D} = \frac{y}{(z/D) + 1} \qquad xp = \frac{D\,x}{z + D} = \frac{x}{(z/D) + 1}$$

Transformation matrix

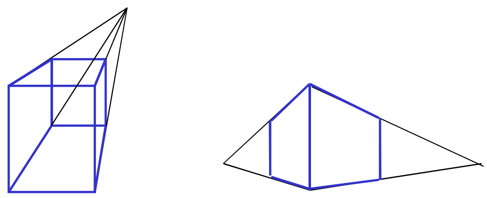
$$P_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/D & \mathbf{1} \end{bmatrix}$$

$$yp = \frac{D\,y}{z + D} = \frac{y}{(z/D) + 1} \qquad xp = \frac{D\,x}{z + D} = \frac{x}{(z/D) + 1}$$

Example point transformation

Perspective projections

- do not preserve relative dimensions
- give realistic appearance



Homework

- Define a cube of edge length 10. Place it such that its front bottom right corner is at location (-3, -2, 12).
- Compute the perspective projection of the cube when the COP is at (0,0, -20) and the projection plane is at the centre of the coordinate system.
- Compute the perspective projection of the cube when the COP is at (0,0, -10) and the projection plane is at the centre of the coordinate system.
- Compute the perspective projection of the cube when the COP is at (0,0, 10) and the projection plane is at the centre of the coordinate system.
- Place the front bottom right corner of the cube at location (-3, 0, 5). Compute the perspective projection of the cube when the COP is at (0,0, -10) and the projection plane is at the centre of the coordinate system.

Next lecture

Animation