MATHEMATICAL MORPHOLOGY

Post-processing

Pixel classification methods often produce objects with irregular boundaries and spurious 'holes', and 'noise' in the background.

Initial segmentation can be followed by 'noise' removal and boundary smoothing.

- median filtering of the thresholded image
- min/max filtering of the thresholded image
- methods of mathematical morphology

Morphology

study of form and shape

Mathematical morphology

- based on set theory
- sets represent shapes of objects in an image
- sets are members of I^2 space (binary images)

Mathematical notation

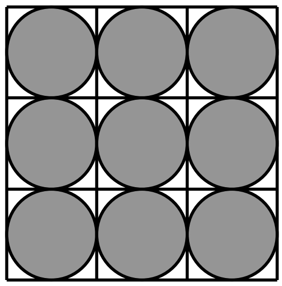
P, S are sets in I^2 space, e.g.

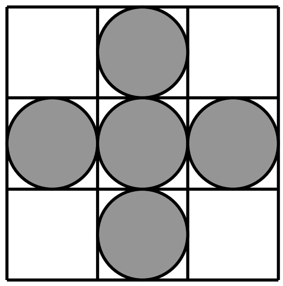
$P = (p_1, p_2)$, $s = (s_1, s_2)$

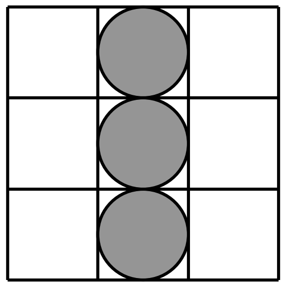
Operations

- Interactions of two sets of points
  - image
  - structuring element

- Examples of structuring elements







Two principal operations of mathematical morphology

- **dilation**
- **erosion**

Dilation

dilation of P (picture) by S (structuring element) defined as:

$$P \oplus S = \{ c \in I^2 \mid c = p + s \text{ for some } p \in P \text{ and } s \in S \}$$

Example:

$P = \{ (0,1), (1,1), (2,1), (2,2), (3,0) \}$
$S = \{ (0,0), (0,1) \}$

$$P \oplus S = \{ (0,1), (1,1), (2,1), (2,2), (3,0), (0,2), (1,2), (2,2), (2,3), (3,1) \}$$

P S $P \oplus S$

Dilation is

- commutative, i.e. $P \oplus S = S \oplus P$
- associative, i.e. $P \oplus (S \oplus T) = (P \oplus S) \oplus T$

The dilation of P by S can be computed as a union of translations of P by the elements of S:

$$P \oplus S = \cup\ (P)_{s \in S}$$

Erosion

erosion of P by S is defined as:

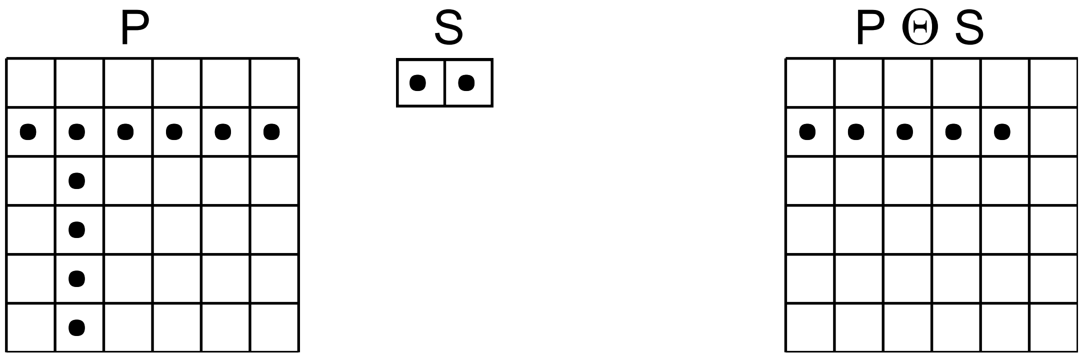
$$P \Theta B = \{ d \in I^2 \mid d + s \in P \text{ for every } s \in S \}$$

Example

P = { (1,0), (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (3,1), (4,1), (5,1) }

S = { (0,0), (0,1) }

$P \,\Theta\, S$ = { (1,0), (1,1), (1,2), (1,3), (1,4) }



The erosion of P by S is the intersection of all translations of P by the points -s where $s \in S$:

$$P \,\Theta\, S = \cap \, P_{-s \in S}$$

Morphological operations applied (normally) to a binary picture (set P)

Intuitive meanings:

Structuring element

- 'template' for morphological operations (set S)
- one-s indicate “active” pixels
- zero-s indicate “passive” pixels

Dilation (expansion)

adding a 'layer' of pixels to the periphery of objects

the object will grow larger, close objects will be merged, holes will be closed

Erosion (shrinking)

removing a 'layer' of pixels all round an object

the object will get thinner, if it is already thin it will break into several sections

Expansion and shrinking can be used in combination
Two important methods are

Opening

erosion followed by dilation by the same amount

$$(P \ominus S) \oplus S$$

useful for smoothing peripheries and removing small features

Closing

dilation followed by erosion by the same amount

$$(P \oplus S) \ominus S$$

filling small holes and cracks in objects

Other useful applications of morphological operations

- boundary extraction
- region filling (a recursive procedure)
- distance transform
- skeletonisation
- thinning and thickening
- approximation to low-pass and high-pass filters

Morphology for grey-level images

Morphological operations can be applied to grey level images

Dilation

replace, for every pixel, image area coinciding with the structuring element with the **maximum** grey level value within this area

Erosion

replace the area with minimum grey level