Digital image processing and analysis

10. Post-processing: mathematical morphology

Professor Ela Claridge
School of Computer Science

Previous lecture:

- What is image segmentation
- Principles of the region-based segmentation
- Global methods: pixel classification
  - Thresholding
  - Multi-object classification
  - Feature spaces
- Local methods
  - Bottom up: region growing
  - Top down: image partitioning

In this lecture we shall find out about:

- Post-processing methods
  - Mathematical morphology – basic operations
- Feature extraction
  - Mathematical morphology – combined operations

Segmentation
Post-processing

- Segmentation often produces objects with irregular boundaries and spurious “holes”, and “noise” in the background.
- These can be corrected with various post-processing methods.

Mathematical morphology

- **Morphology** (from the Greek and meaning "study of shape") is concerned with study of form and shape.
- **Mathematical morphology** is a theory and technique for the analysis and processing of geometrical structures.
- In image processing the technique is applied to shapes in **binary** images.
- The image shapes are modified using a "brush" of a pre-defined shape called a ***structuring element*** (SE).

Mathematical morphology

- Two principal operations of mathematical morphology are **dilation** and **erosion**.
- Dilation (expansion)
  - adding a "layer" of pixels to the periphery of objects
  - the object will grow larger, close objects will be merged, holes will be closed
  - Dilation of image I is denoted as $I \oplus SE$
- Erosion (shrinking)
  - removing a "layer" of pixels all round an object
  - the object will get thinner, if it is already thin it will break into several sections
  - Erosion of image I is denoted as $I \ominus SE$

Mathematical morphology
… is like potato stamping

Choose the shape (SE)

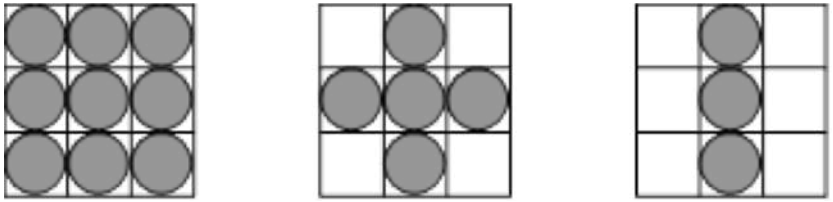
Choose the colour
(background or foreground)

Choose the location in the image

Apply the stamp

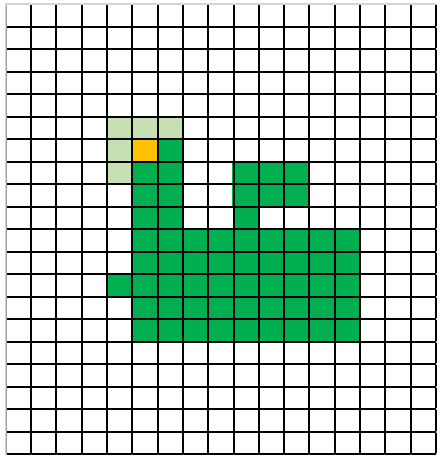
Mathematical morphology

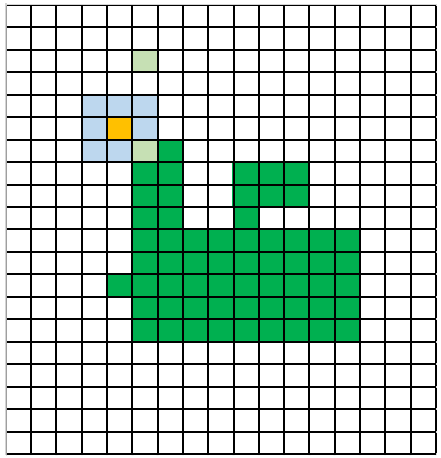
- Operations of mathematical morphology are defined in terms of interactions of two sets of points. One set (usually a large one) corresponds to an image I; the other (usually much smaller) is a ***structuring element*** SE.
- A structuring element can be thought of as a "potato stamp" with which an image is "overprinted" in a number of specific ways, depending on the morphological operation.
- Examples of typical structuring elements (grey dots indicate "active" members of the structuring element set):

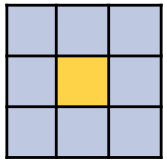


Mathematical morphology

Next three slides demonstrate the process of dilation and erosion







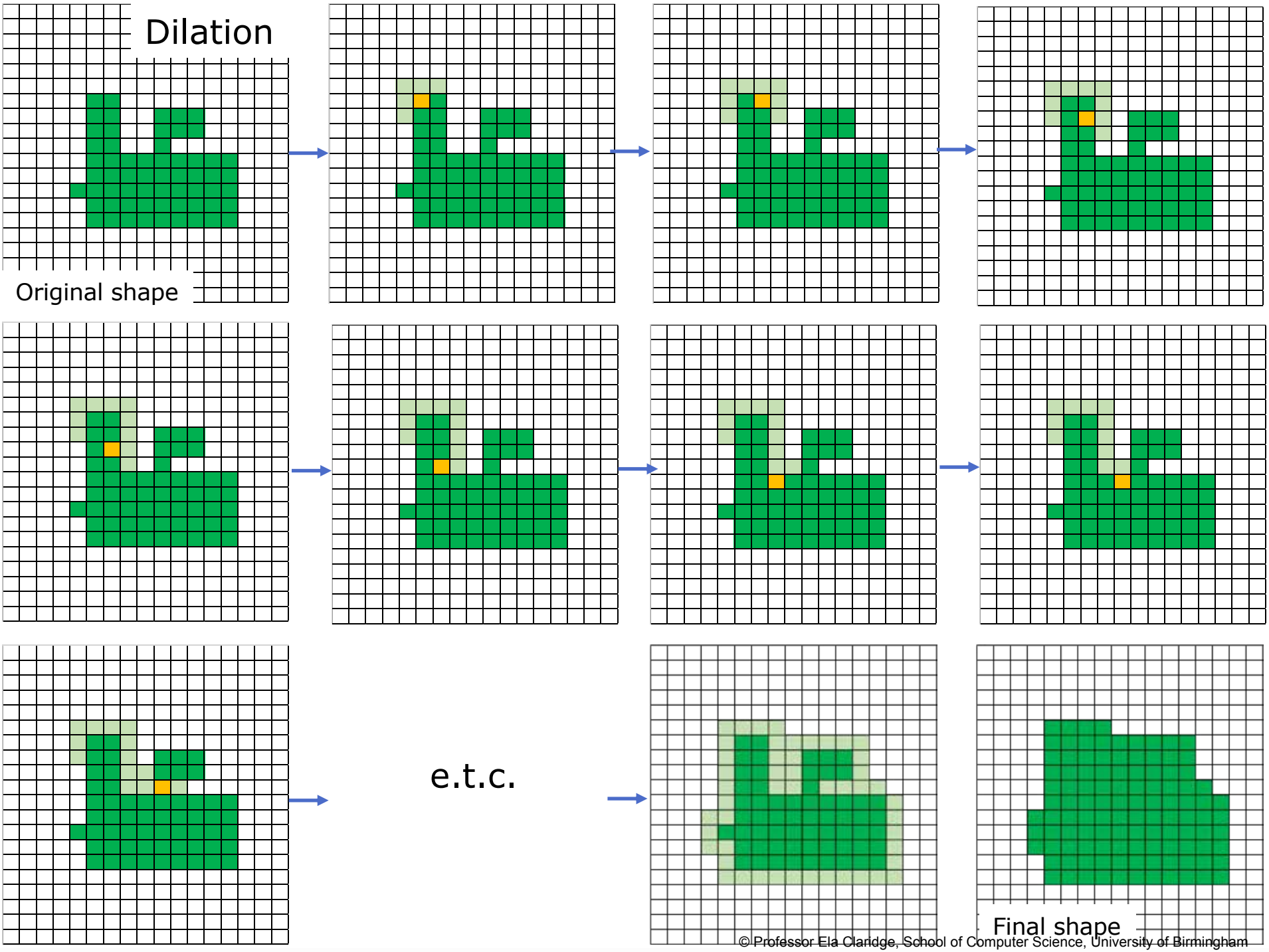
Structuring element (SE)

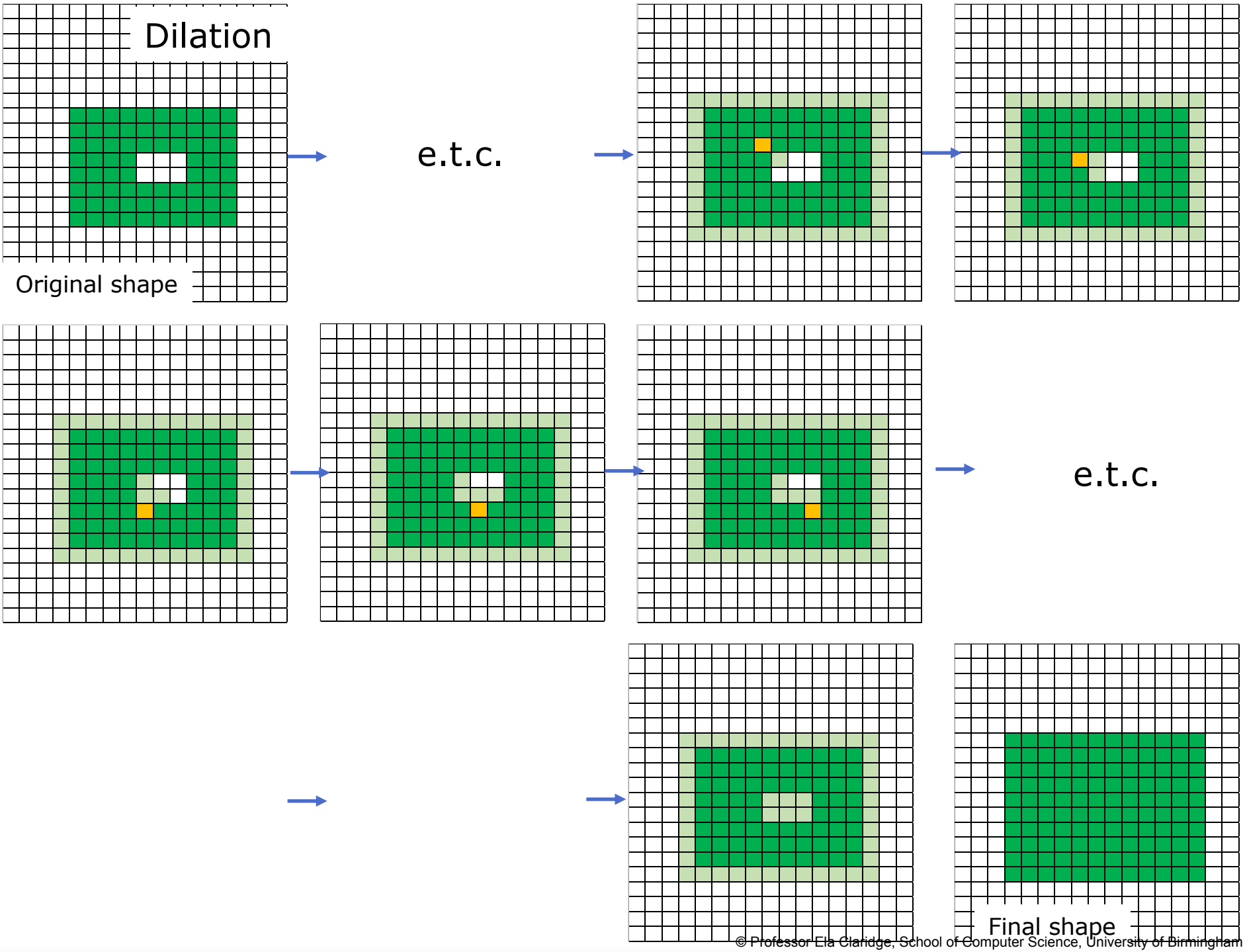
Object

Centre of SE

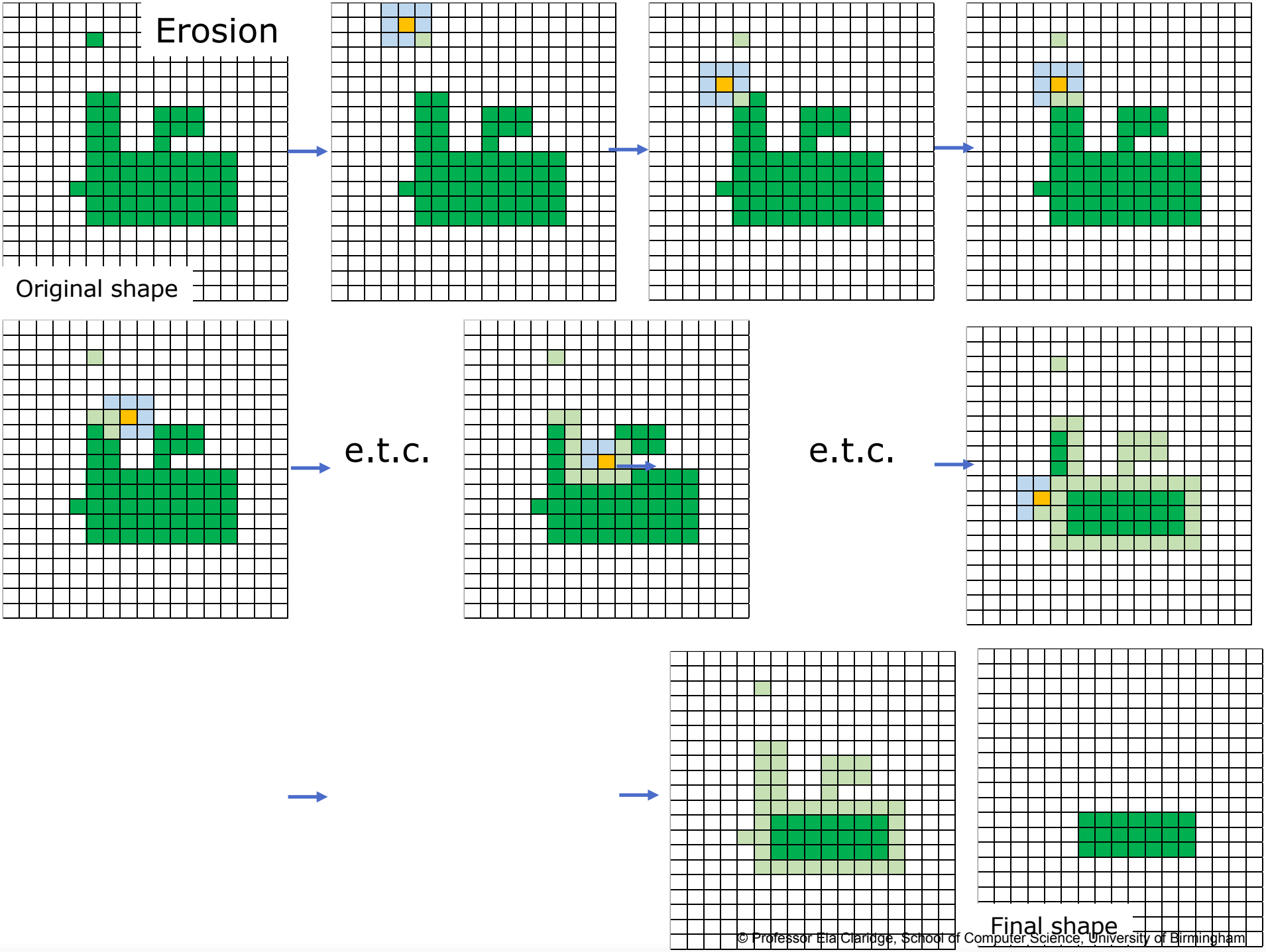
Coverage of SE

Object pixels affected by dilation or erosion



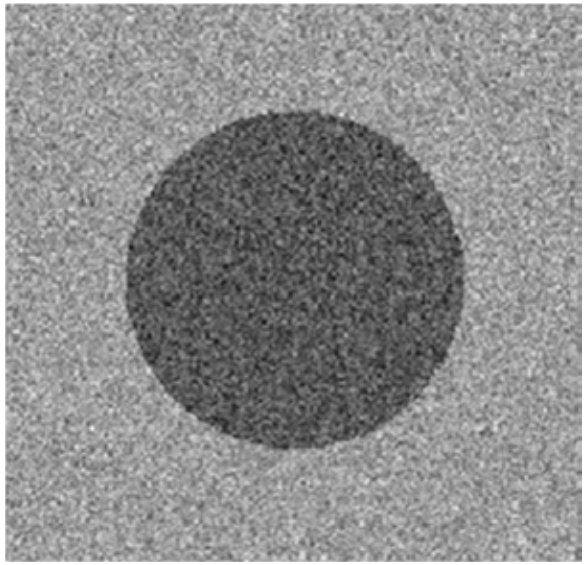


Erosion



Mathematical morphology
Post-processing

Objective: segment the bright disk from a noisy image



Image

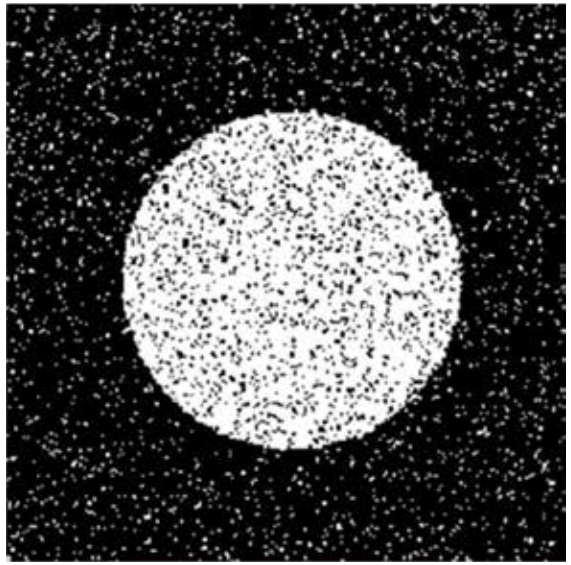
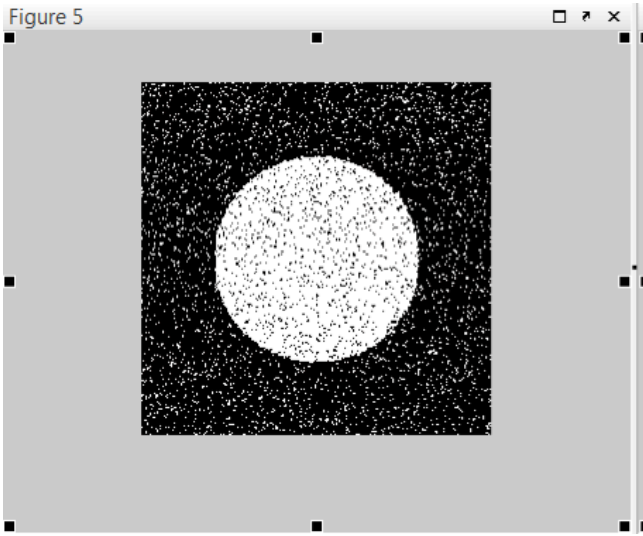
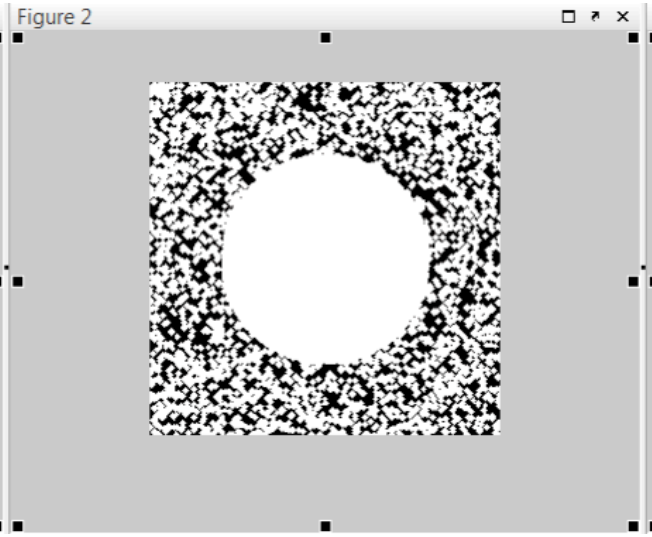


Image after thresholding. Noise present in both the object and the background

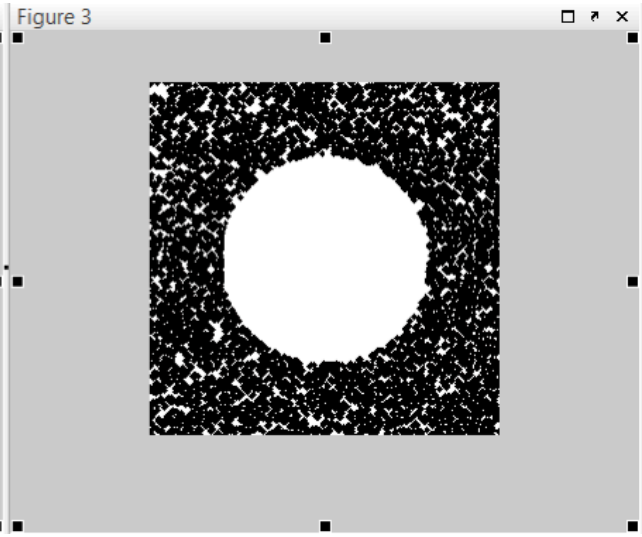
Mathematical morphology
Post-processing



Thresholded image



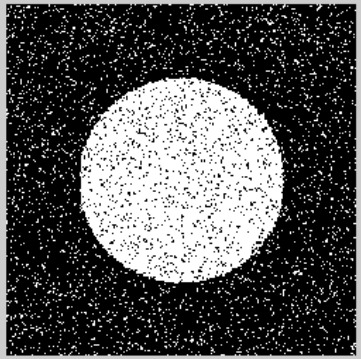
After dilation.
The "holes" in the object disappeared, but noise in the background increased.



After erosion.
The noise in the background decreased, but holes in the object did not re-appear.

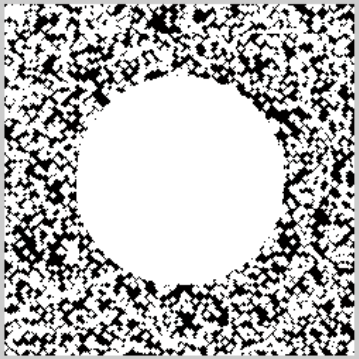
Mathematical morphology
Post-processing

Figure 5



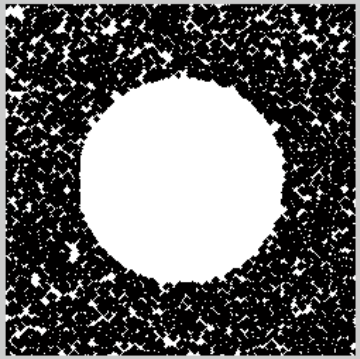
After thresholding

Figure 2



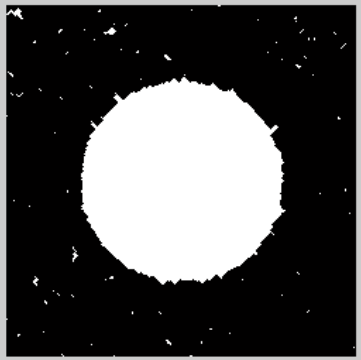
After dilation

Figure 3



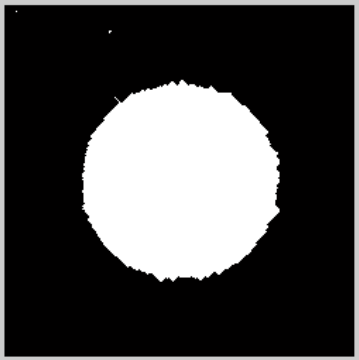
After erosion

Figure 4



After next erosion

Figure 6



After next erosion

Problem?
Change of size

Mathematical morphology
Combined operations

- Dilation and erosion can be used in combinations. Two important methods are:

- ***Opening:*** erosion followed by dilation by the same amount

$$(I \ominus SE) \oplus SE$$

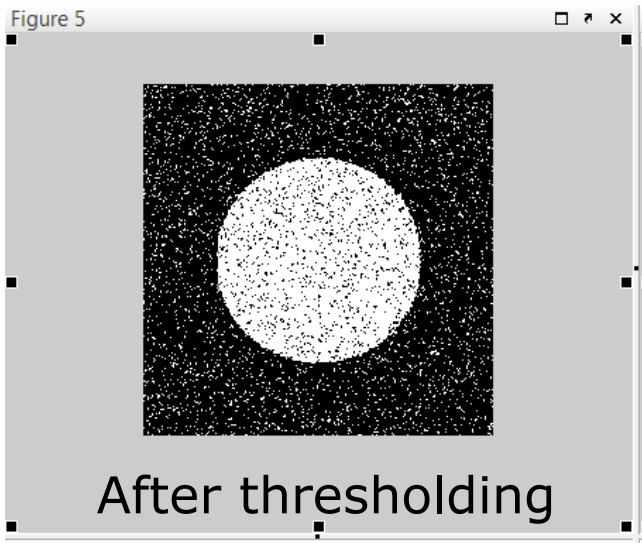
  - Useful for smoothing peripheries and removing small features.

- ***Closing:*** dilation followed by erosion by the same amount

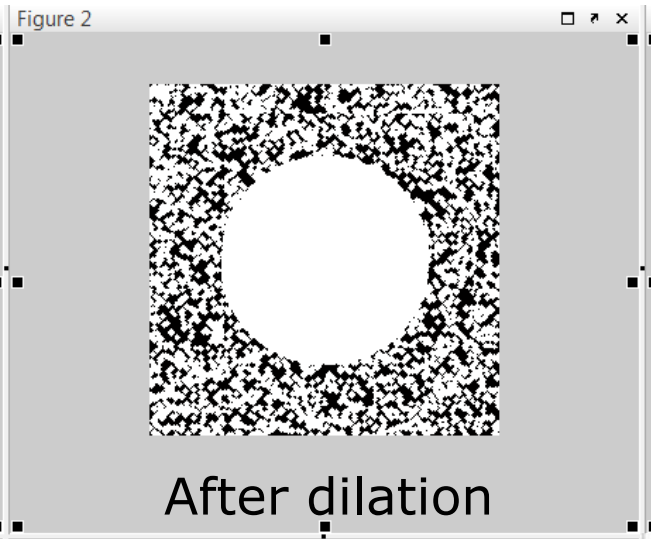
$$(I \oplus SE) \ominus SE$$

  - Useful for filling small holes and cracks in objects.

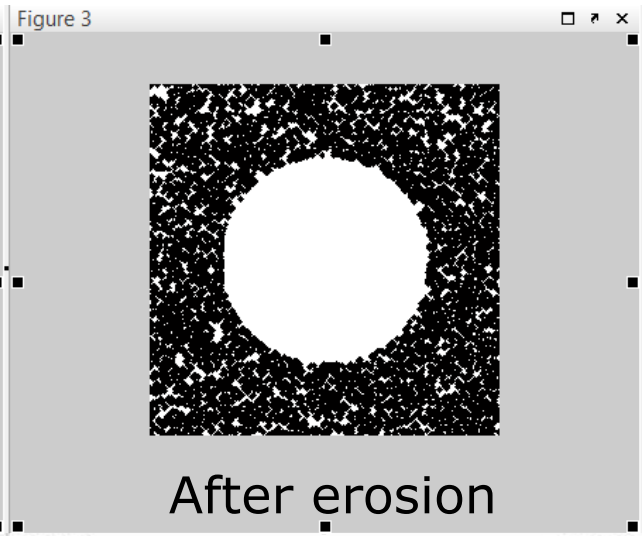
Mathematical morphology
Combined operations



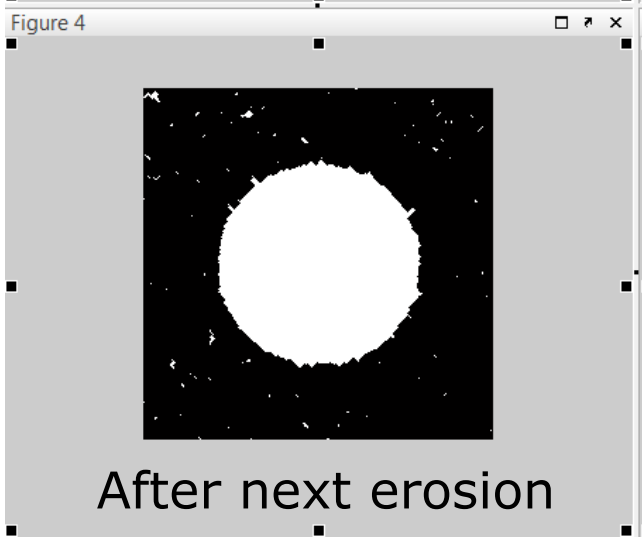
After thresholding



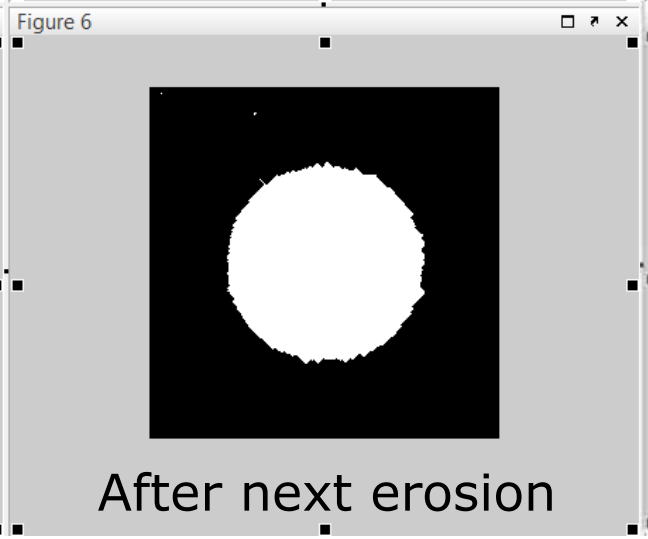
After dilation



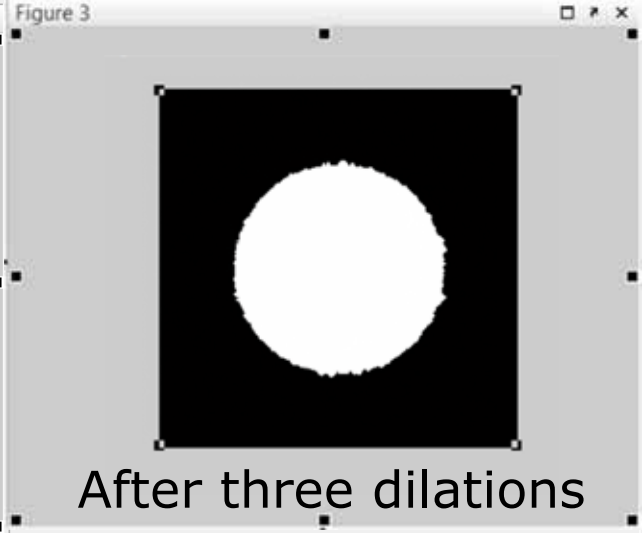
After erosion



After next erosion



After next erosion



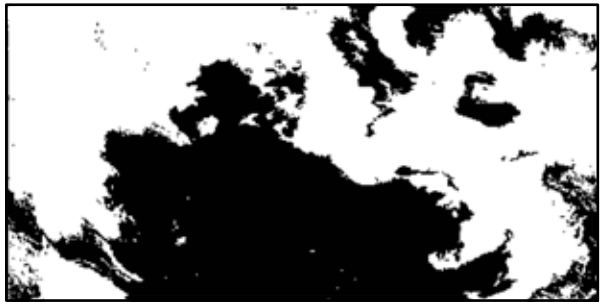
After three dilations

Mathematical morphology
Combined operations

- Other useful applications of morphological operations include:
  - Boundary extraction: $(A \oplus B) - A$
  - Hole filling
  - Distance transform
  - Skeletonisation
  - … many others … - see Further Reading

Mathematical morphology
Boundary extraction

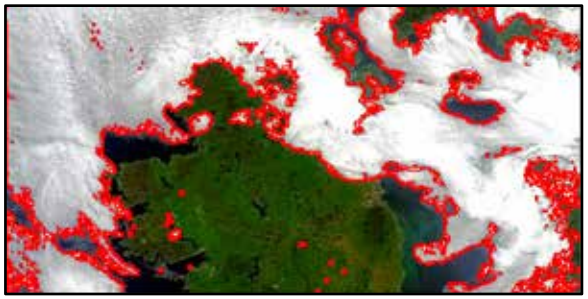
Original image I_0



Thresholded image I_1

After closing and dilation I_2

Boundaries:
result of subtraction I_2-I_1

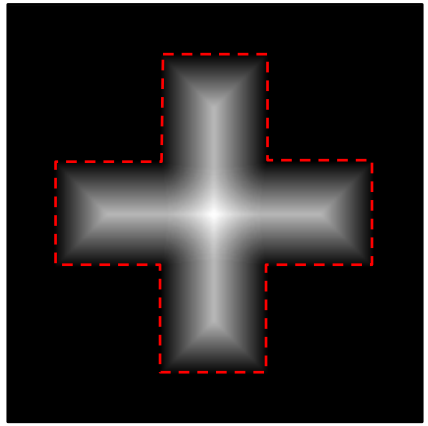


Original image I_0, red overlay shows boundaries

Mathematical morphology
Hole filling

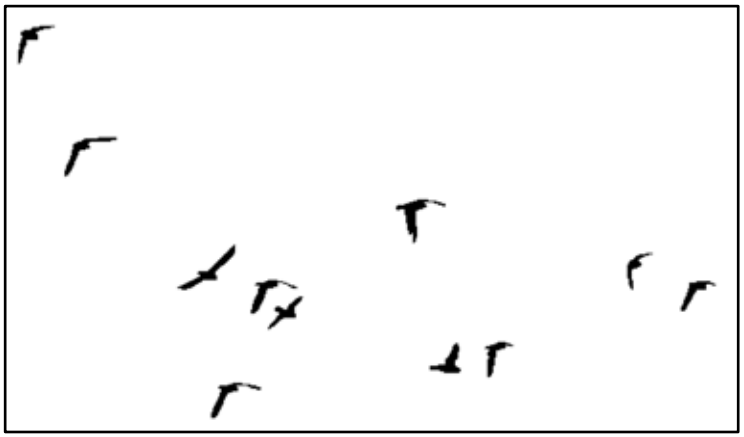
Mathematical morphology
Distance transform

Binary image

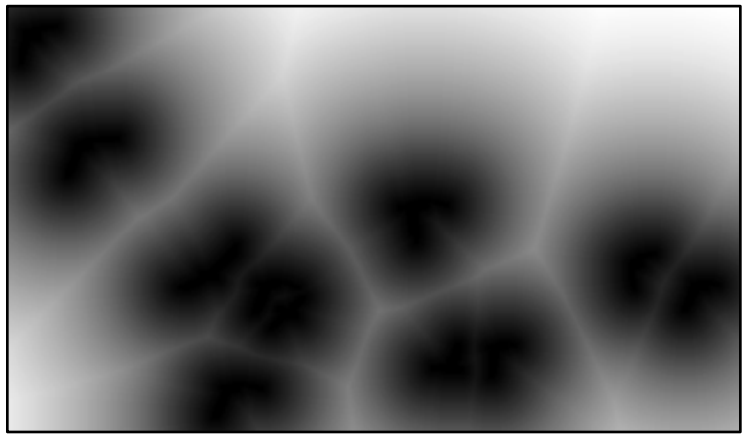


Distance transform:
Pixel value represents a distance from the nearest boundary point.
(This is an intrinsic image)

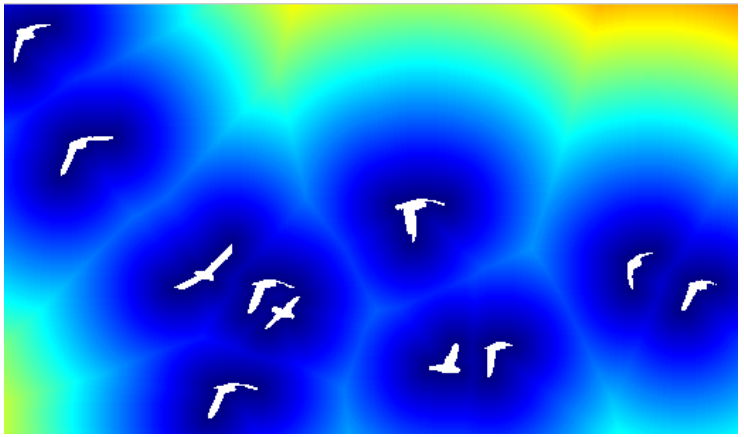
Mathematical morphology
Distance transform



Binary image
(object = sky)



Distance transform



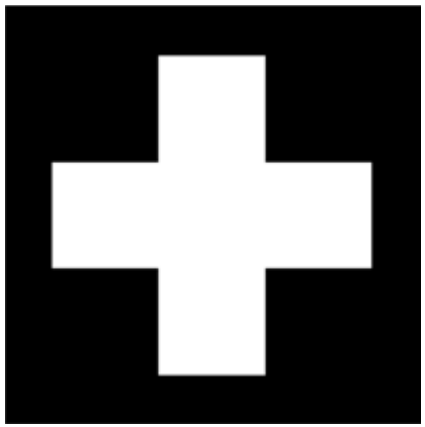
Distance transform
(false colours, birds superimposed)

Mathematical morphology
Skeletonisation

Mathematical morphology
Skeletonisation

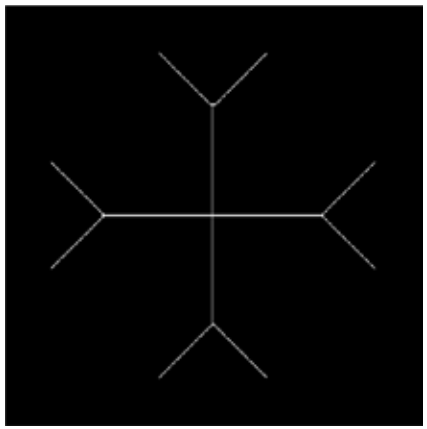
- **Skeleton**: a set of pixels equidistant from at least two pixels on the boundary of the object.

- *"Imagine the object to be burning up by a fire which advances at a constant rate from the boundary. The places where two lines of fire meet form the medial axis." [Prof. Charlene Tsai]*

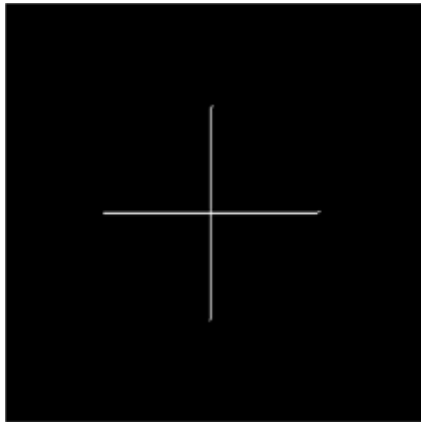


Binary image

Distance transform

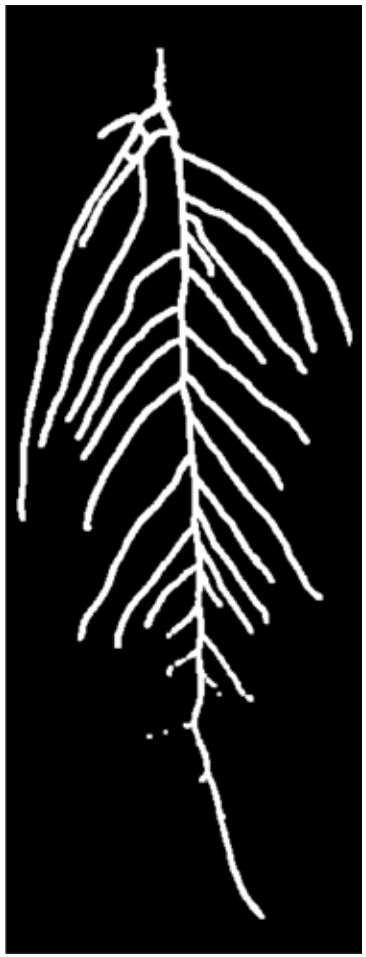


Skeleton

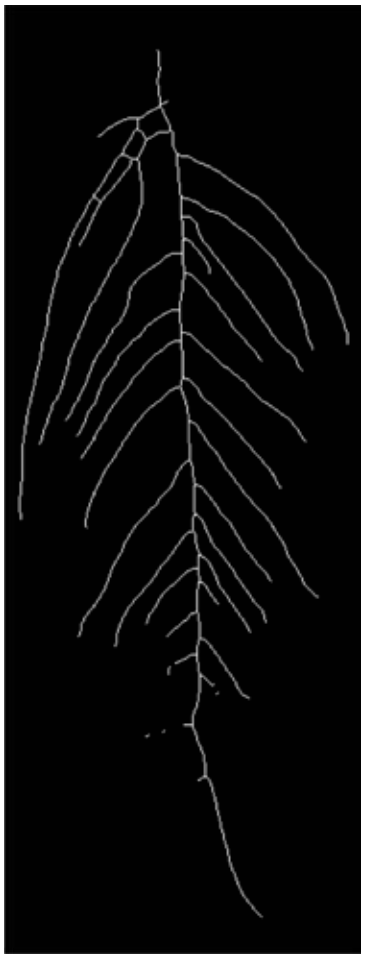


Skeleton truncated (FIJI version)

Mathematical morphology
Skeletonisation



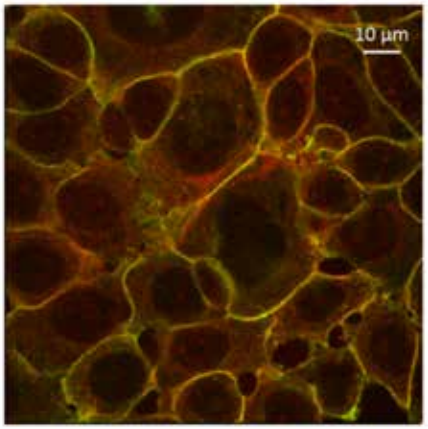
Binary image

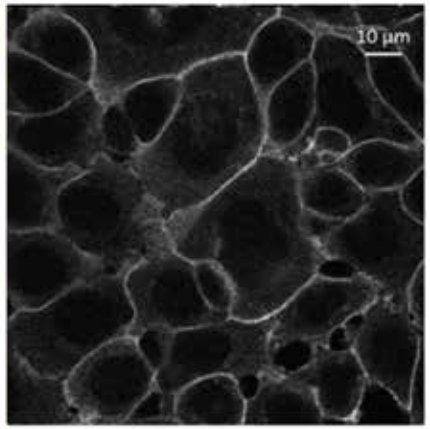


Skeleton

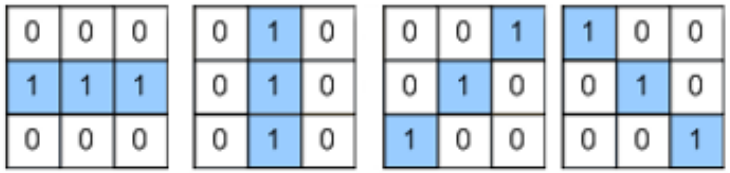
Mathematical morphology
Applications: cell segmentation

Colour image



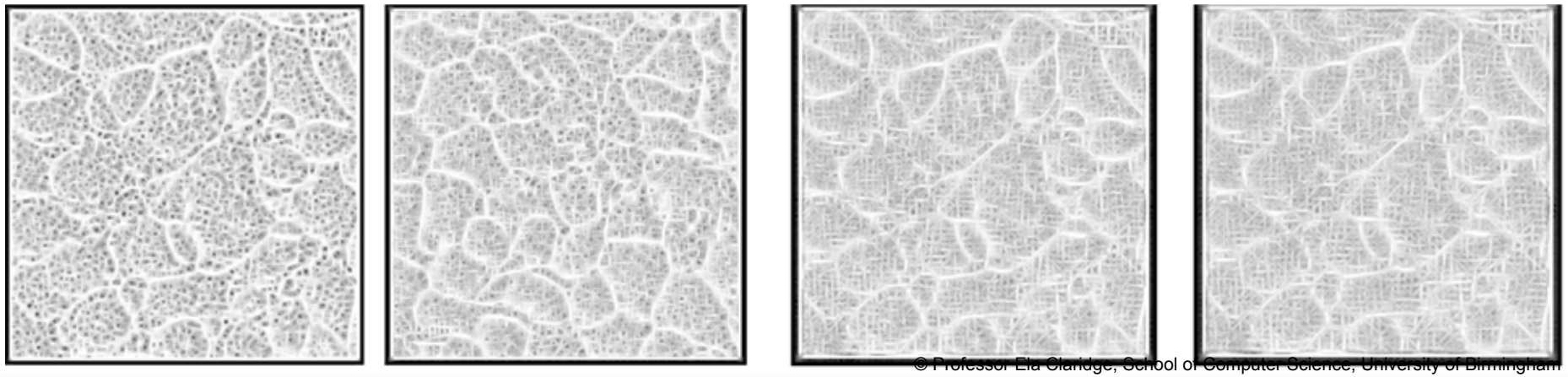


Green channel

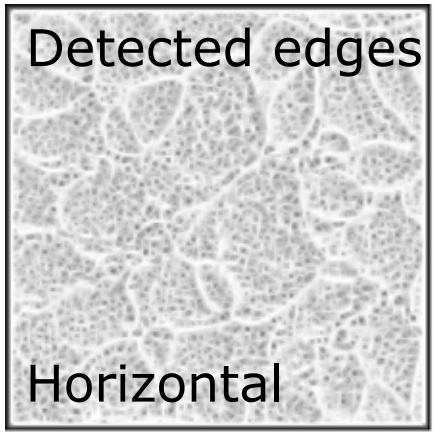


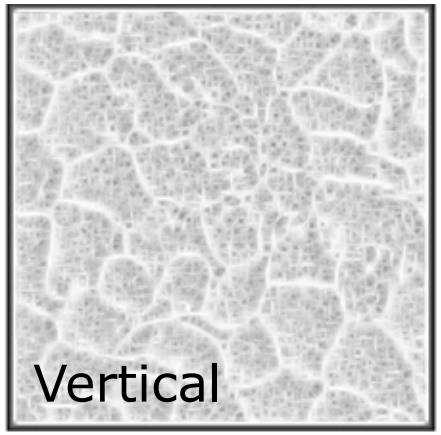
Directional edge detection filters (bright on dark)

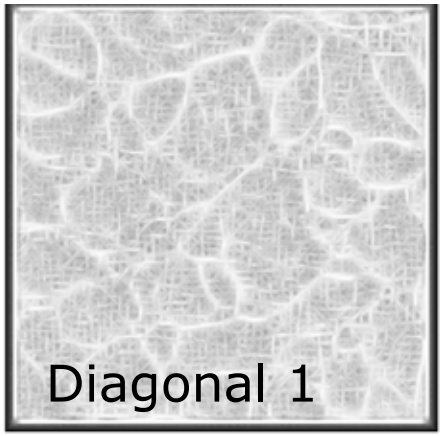
Detected edges

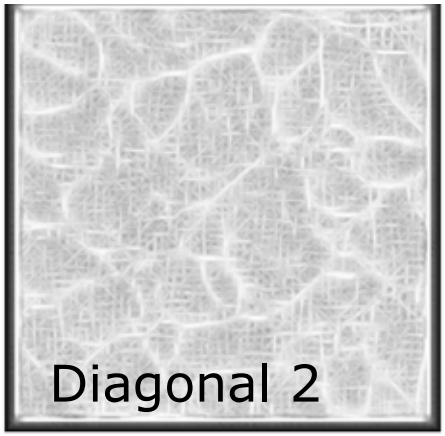


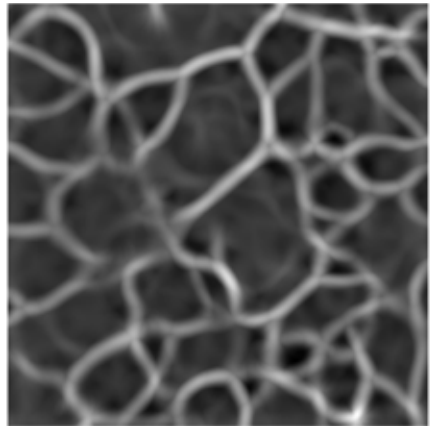
Mathematical morphology
Applications: cell segmentation

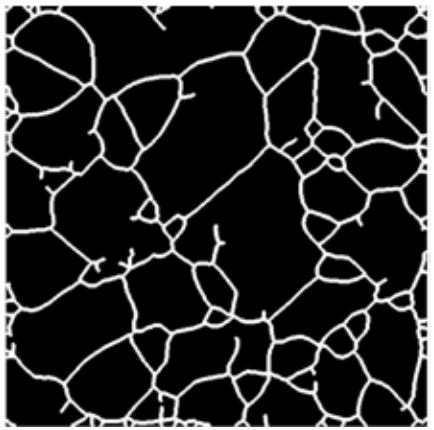


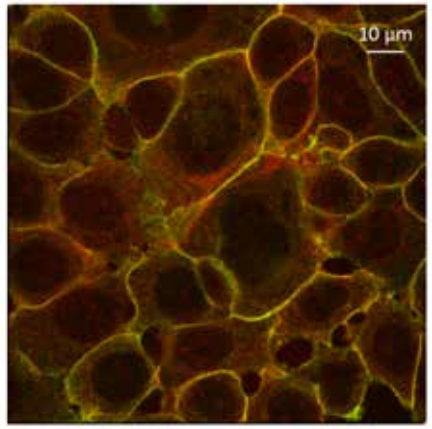












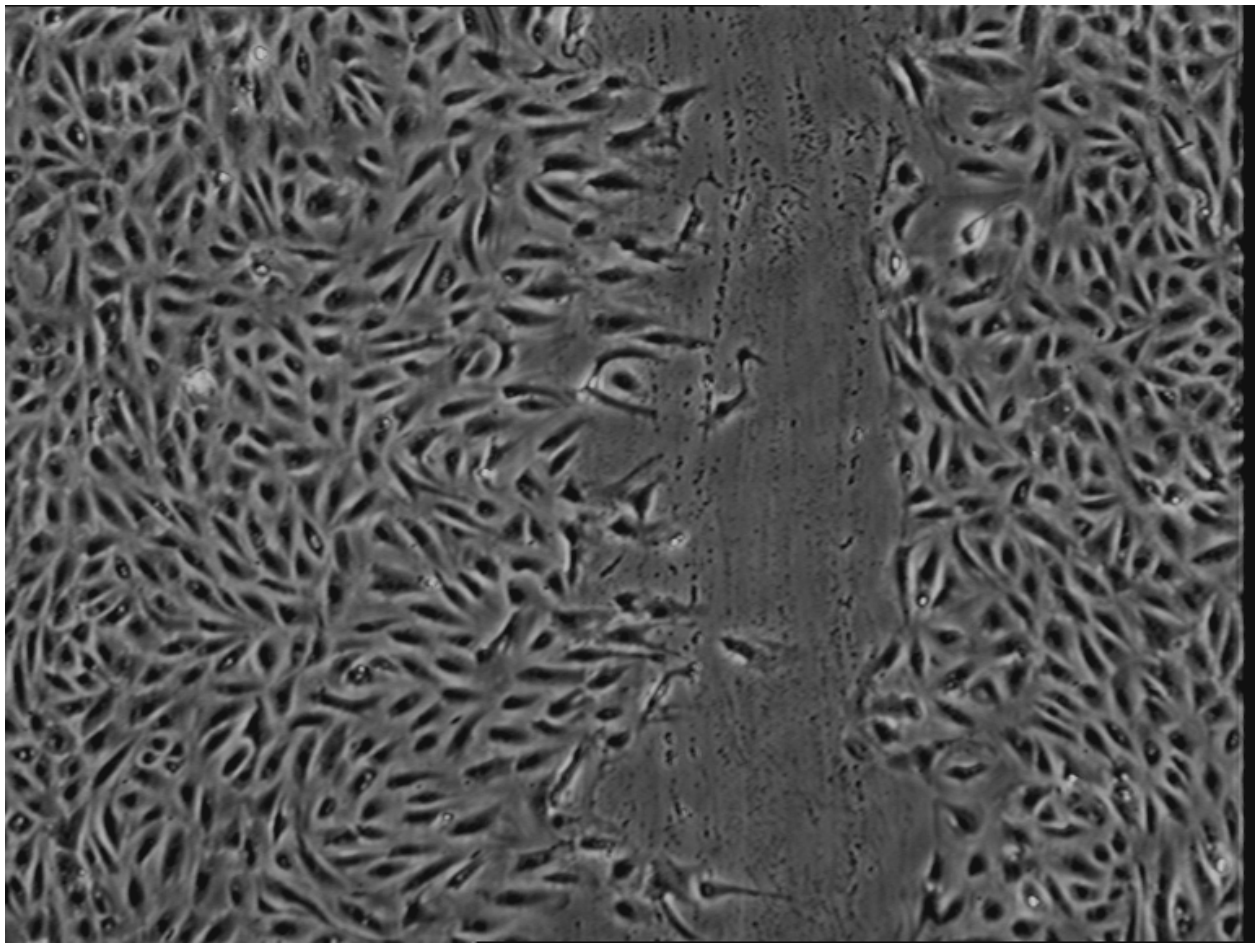
Max of the four directions, inverted, Gaussian smoothed

Thresholded

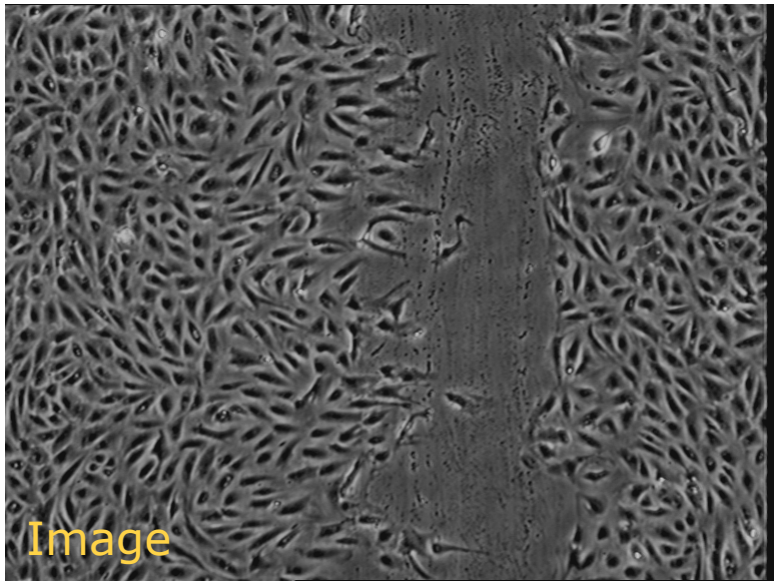
Eroded, skeletonised

Compare with the original colour image

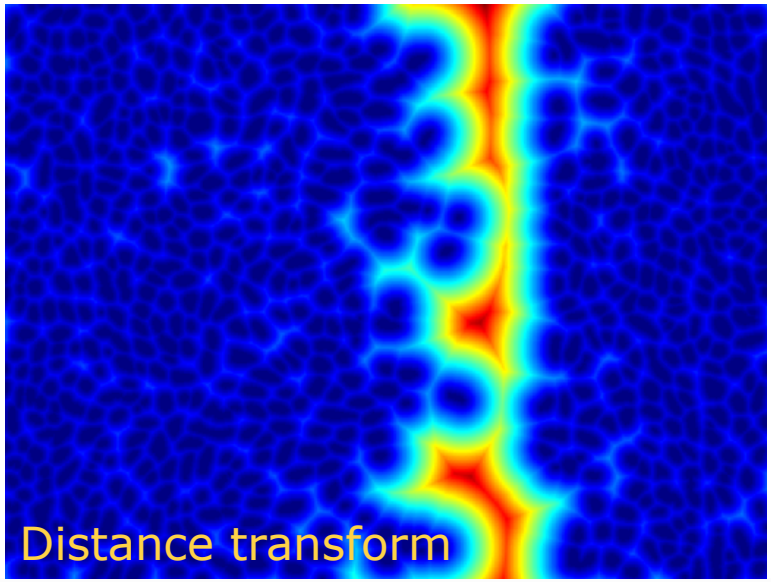
Mathematical morphology
Applications: cell segmentation

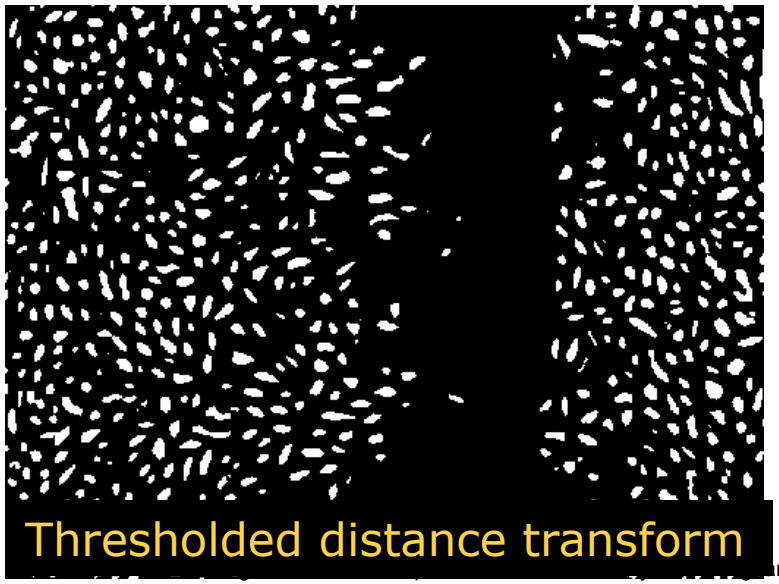


Mathematical morphology
Applications: cell segmentation

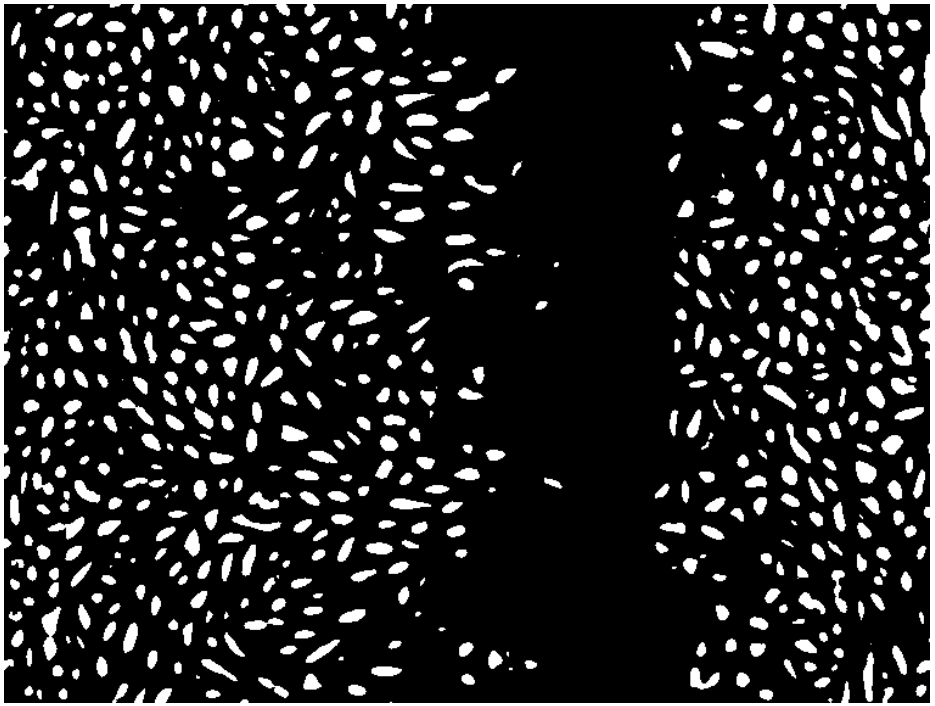


Cell interiors are at a small distance from the edges

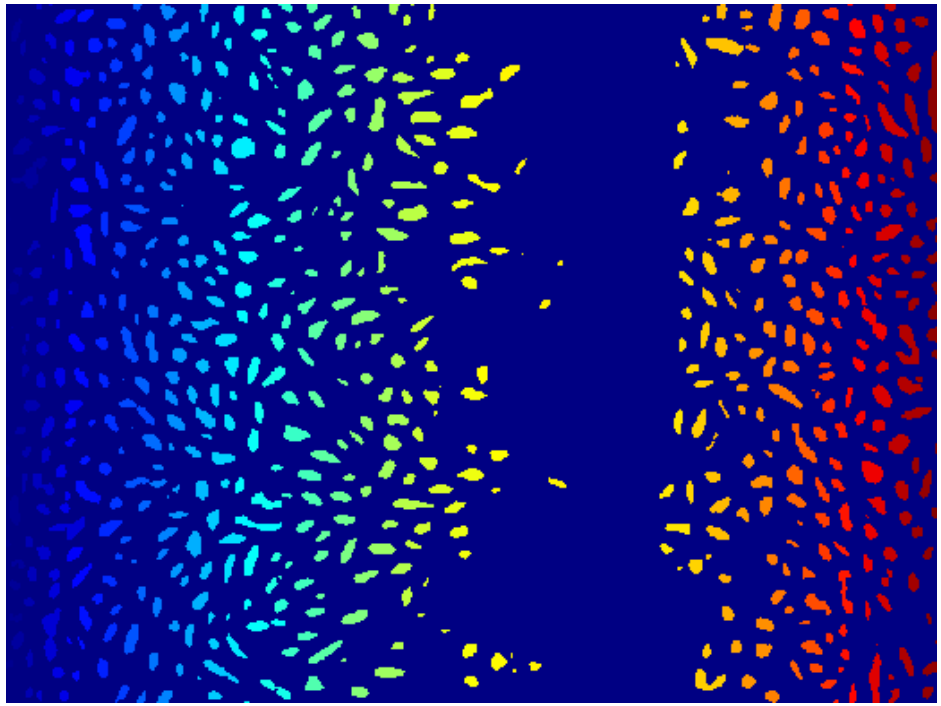




Mathematical morphology
Applications: cell segmentation



Thresholded distance transform



Labeled image (682 cells)
(see next lecture)

Mathematical morphology
Grey level morphology

- Morphological operations can also be applied to grey level images
- Grey level dilation – equivalent to max filter
  - for every pixel, replace the image area overlapped by the structuring element with the ***maximum*** grey level value within this area.
- Grey level erosion – equivalent to min filter
  - for every pixel, replace the image area overlapped by the structuring element with the ***minimum*** grey level value within this area

In this lecture we have covered:

- Post-processing methods
  - Mathematical morphology – basic operations
- Feature extraction
  - Mathematical morphology – combined operations

Next lecture:

- How to get coordinates of the object boundaries
  - Hand-on-the-wall walk around the object
- How to count objects in a segmented image
- How to measure objects
  - Area
  - Boundary / Perimeter
- How to measure object locations
  - Bounding box
  - Centroid

Further reading and experimentation

- **Book chapters**:
- Gonzalez, R.C. & Woods, R.E. Digital Image Processing, Addison-Wesley (various editions), 8.4.
- Sonka, M. Hlavac, V. Boyle, R. (various editions) Image Processing, Analysis and Machine Vision, Chapman & Hall Computing, chapter 10.
- Umbaugh, S.E. Computer vision and image processing : a practical approach using CVIPtools , Prentice Hall International (various editions), 2.4.6.

- *HIPR2 resources*
- **Mathematical morphology**
- http://homepages.inf.ed.ac.uk/rbf/HIPR2/morops.htm
- **Distance transform**
- https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm
- **Skeletonisation**
- http://homepages.inf.ed.ac.uk/rbf/HIPR2/skeleton.htm
- **Pixel labelling**
- http://homepages.inf.ed.ac.uk/rbf/HIPR2/label.htm