Image registration

Acknowledgements

- The slides in this talk are based on the following sources:
  - Image Registration and Fusion. Professor Michael Brady, Department of Engineering Science, Oxford University.
  - Image Registration. John Ashburner, Functional Imaging Laboratory, UCL.
  - Image Registration: A Review. Xenios Papademetris, Department of Diagnostic Radiology, Yale School of Medicine.
  - Medical Image Registration. Eren Turgay, University of Kentucky.

Image registration

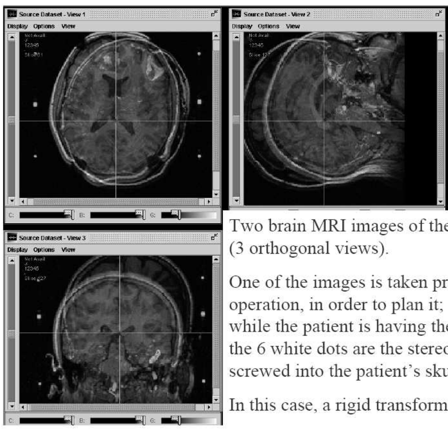
- ***Geometric*** (and ***Photometric)*** alignment of one image with another
- Implemented as the process of estimating an optimal transformation between two images.
- Sometimes also known as “Spatial Normalization” (SPM)
- Images may be of same or different types (MR, CT, visible, fluorescence, ...)

Examples of image registration

- Aligning an image taken prior to an operation, to help plan the procedure, with one taken during the operation (for example to avoid use of a stereotactic frame)
- Aligning an image taken now with one taken on a previous occasion (monitor the progression of disease, discover the fact of a disease)
- Aligning two images of different sorts of the same patient (data fusion)

Examples of image registration

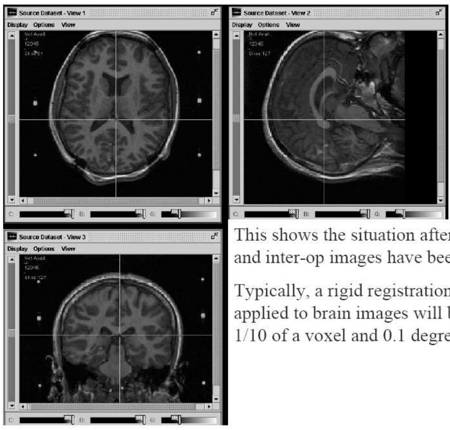
- Aligning the images from two different patients;
- Aligning the images of a subject to an atlas, or, constructing such an atlas from the images of several subjects;
- Aligning the images of patients and aligning those of normals to develop a statistical model of variation associated with a disease;
- Aligning the images from many thousands of subjects around the world as part of a clinical/drug trial



Two brain MRI images of the same patient (3 orthogonal views).

One of the images is taken prior to the operation, in order to plan it; the second while the patient is having the operation: the 6 white dots are the stereotactic frame screwed into the patient’s skull.

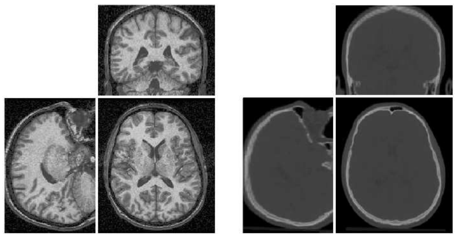
In this case, a rigid transform suffices



This shows the situation after the pre-op and inter-op images have been aligned.

Typically, a rigid registration algorithm applied to brain images will be accurate to 1/10 of a voxel and 0.1 degrees of rotation

Example: rigid CT/MR registration



Components of registration

- The registration problem can be formulated as:

$$T = \arg\min \sum_k sim(I(\mathbf{x}_k), J(T(\mathbf{x}_k)) \downarrow)$$

- What entities do we match? Features, intensities, ...
- What class of transforms? Rigid, affine, spline warps, ...
- What similarity criterion to use? Normalised cross-correlation, ...
- What search algorithm to find the minimum T?
- What interpolation method to use? Bilinear, spline, ...

Reference and target datasets

- Feature images (e.g. edge images)
- Landmarks / control points
- Raw intensities
- Combinations of the above

Control Points

<u>Intrinsic</u>
Markers within the Image

<u>Extrinsic</u>
Manually or Automatically selected

Feature matching

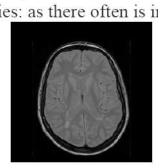
- Finds the best match between a set of features, such as surfaces and corners
- Features are extracted in a preprocessing step that may or may not be automated
- Common features are assumed to have spatial locality

Image intensities

Simplest similarity criterion: conservation of intensity

$$\sum_{i,j} p_{i,j}(i-j)^2$$

This works well in the simplest case; but it is relatively ineffective, even if there is a functional dependence between intensities: as there often is in medical images of different types:



Same anatomy but left is T_1 weighted, right is T_2 weighted

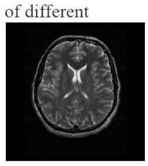
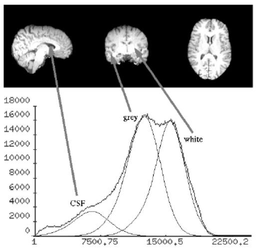


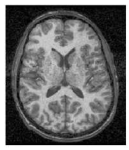
Image intensities

Image histogram

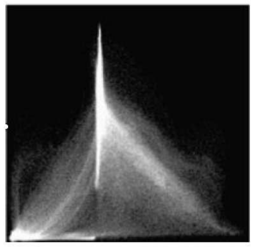


the histogram is the (discrete approximation to the) probability that a pixel has intensity i:

$$p_i = \frac{1}{N} | \{k : I(\mathbf{x}_k) = i\} |$$



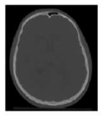
MRI image



the Joint Histogram

Note that the huge peak in the CT histogram corresponds to the intensity range spanning WM, GM, CSF, since these cannot be distinguished on the basis of x-ray attenuation

Given a transform T, the concept is extended to that of ***joint histogram***, the probability that, under T, intensity i is paired with j :



CT image

$$p_{i,j}(T) = | \{k : I(\mathbf{x}_k) = i \text{ and } J(T(\mathbf{x}_k)) \downarrow = j\} |$$

Feature types

Feature Spaces and Their Attributes
RAW INTENSITY - most information
EDGES - intrinsic structure, less sensitive to noise Edges [Nack 77] Contours [Medioni 84] Surfaces [Pelizzari 89]
SALIENT FEATURES - intrinsic structure, accurate positioning Points of locally maximum curvature on contour lines [Kanal 81] Centers of windows having locally maximum variances [Moravec 81] Centers of gravity of closed boundary regions [Goshtasby 86] Line intersections [Stockman 82] Fourier descriptors [Kuhl 82]
STATISTICAL FEATURES - use of all information, good for rigid transformations, assumptions concerning spatial scattering Moment invariants [Goshtasby 85] Centroid/principal axes [Rosenfeld 82]
HIGHER LEVEL FEATURES - uses relations and other higher level information, good for inexact and local matching Structural features: graphs of subpattern configurations [Mohr 90] Syntactic features: grammars composed from patterns [Bunke 90] Semantic networks: scene regions and their relations [Faugeras 81]
MATCHING AGAINST MODELS - accurate intrinsic structure, noise in one image only Anatomic atlas [Dann 89] Geographic map [Maitre 87] Object model [Terzopoulos 87]

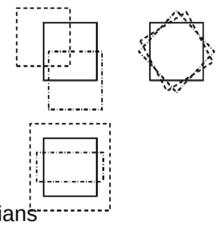
Transformation Model

- Rigid
- Affine
- Piecewise affine
- Non-rigid or elastic

Rigid Transformation Model

- Used for within-subject registration when there is no distortion
- Composed of 3 rotations and 3 translations
- Linear – can be represented as a 4x4 matrix

2D Rigid Transforms



- Translations by t_x and t_y
  - $x_1 = x_0 + t_x$
  - $y_1 = y_0 + t_y$
- Rotation around the origin by Θ radians
  - $x_1 = \cos(\Theta)\, x_0 + \sin(\Theta)\, y_0$
  - $y_1 = -\sin(\Theta)\, x_0 + \cos(\Theta)\, y_0$
- Zooms by s_x and s_y
  - $x_1 = s_x\, x_0$
  - $y_1 = s_y\, y_0$

3D Rigid-body Transformations

- A 3D rigid body transform is defined by:
  - 3 translations - in X, Y & Z directions
  - 3 rotations - about X, Y & Z axes
- The order of the operations matters

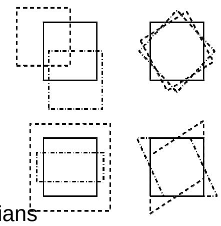
$$
\begin{pmatrix} 1 & 0 & 0 & X_{trans} \\ 0 & 1 & 0 & Y_{trans} \\ 0 & 0 & 1 & Z_{trans} \\ 0 & 0 & 0 & 1 \end{pmatrix}
\times
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi & 0 \\ 0 & -\sin\Phi & \cos\Phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\times
\begin{pmatrix} \cos\Theta & 0 & \sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\Theta & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\times
\begin{pmatrix} \cos\Omega & \sin\Omega & 0 & 0 \\ -\sin\Omega & \cos\Omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Translations — Pitch about x axis — Roll about y axis — Yaw about z axis

Affine Transformation Model

- Used for within-subject registration when there is global gross-overall distortion
- More typically used as a crude approximation to fully non-rigid transformation.
- Composed of 3 rotation, 3 translations, 3 stretches and 3 shears.
- Also a linear transformation – can be represented as a 4x4 matrix

2D Affine Transforms



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- Zooms by s_x and s_y
  - $x_1 = s_x\, x_0$
  - $y_1 = s_y\, y_0$

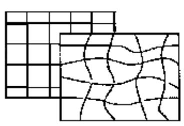
* Shear
  * $x_1 = x_0 + h\, y_0$
  * $y_1 = y_0$

Piecewise Affine Transformation Model

- Simple extension to fully non-rigid transformation
  - Typically use different affine transformation for different parts of the image
- Strictly speaking non-linear

Non-rigid (elastic) transformation model

- Needed for inter-subject registration and distortion correction
- Non-linear i.e. no matrix representation
- Many different parameterizations e.g.
  - Spline parameterizations (b-splines, thin-plate splines)
  - General diffeomorphisms (e.g. fluid models)
  - Truncated basis function expansion methods (Fourier parameterizations)



Spline warps

- Original image is modeled as a thin sheet subjected to a double bending
- Selected points (landmarks) are independently displaced
- Displacements have both *x* and *y* components, so 2 independent warps are computed
- A smoothness criterion is used to minimize the bending energy required based on a physical model

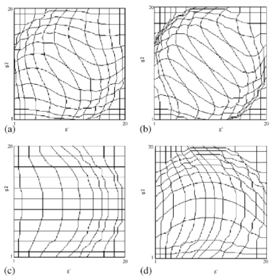
Elastic models

- Model the original image as an elastic body acted upon by two types of forces
- External forces drive deformation
- Internal forces provide constraints

Viscous fluid model

- Problem: elastic deformations do not allow for severe localized extremes
- Solution: model the image as a viscous fluid whose internal forces relax as the image deforms over time
- Risk: increased possibility of mis-registration

Non-linear registration examples



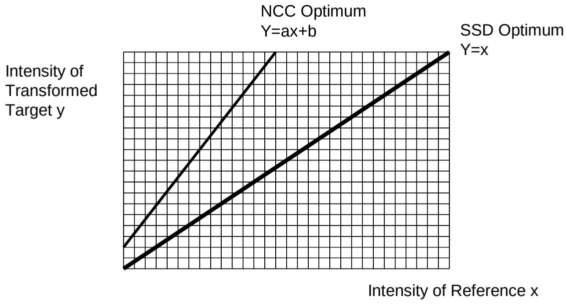
Similarity Metrics

- Feature-based Methods
  - Distance between corresponding points
  - Similarity metric between feature values
    - Similar curvature, etc
- See also http://www.cs.princeton.edu/~bjbrown/iccv05_course/iccv05_icp_gr.pdf

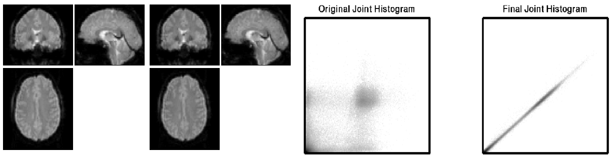
Similarity metrics (objective functions)

- Intensity-based Methods
  - Mean Squared Difference
    - Only valid for same modality with properly normalized intensities
  - Joint histogram – Wood's metric
  - Mutual Information
    - More general metric which maximizes the clustering of the joint histogram.
  - Normalized Cross-Correlation
    - Allows for linear relationship between the intensities of the two images

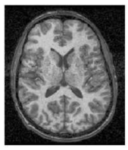
The Joint Histogram



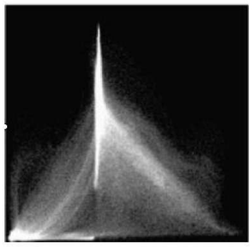
Mean-squared Difference



- Minimising mean-squared difference works for intra-modal registration (realignment)
- Simple relationship between intensities in one image, versus those in the other
  - Assumes normally distributed differences



MRI image



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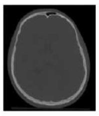
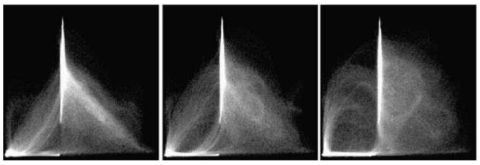


CT image

$$p_{i,j}(T) = |\{k : I(\mathbf{x}_k) = i \text{ and } J(T(\mathbf{x}_k))\downarrow = j\}|$$

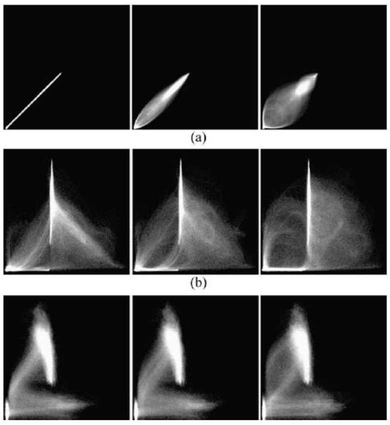
Roger Woods' heuristic observation



Images perfectly aligned | 2mm displacement of one image to the side | 5mm displacement of one image to the side

Heuristic observation is that when the images are aligned, the joint histogram appears "sharpest" : "Woods' criterion"

Why this should be the case is still not certain!



Top: MR-MR (head)

Middle: MR-CT

Bottom: MR-PET

Left: aligned

Middle: 2mm translation

Right: 5mm translation

Heuristic observation is that when the images are aligned, the joint histogram appears "sharpest"

Registration based on Woods' criterion

Recall the registration problem: find $T = \arg\min \sum_k sim(I(\mathbf{x}_k), J(T(\mathbf{x}_k))\downarrow)$

How?

Initialise T … perhaps mobilising application-specific knowledge

Compute the joint histogram of I and $J(T)$

How?

Is this the sharpest? Are the images as aligned with the current T most similar?

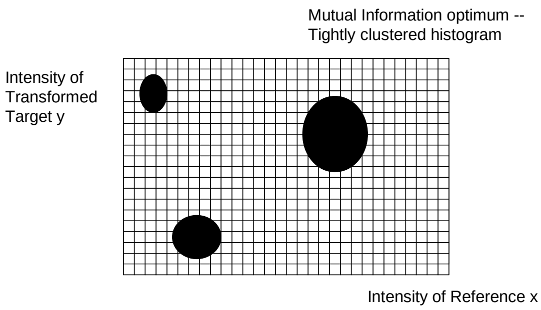
Likely to be a local minimum unless you start near enough the answer!

Done! (you hope)

How?

Modify T a bit, in a way that seems likely to increase the sharpness of the joint histogram: $T \leftarrow T + \delta T$

Mutual information

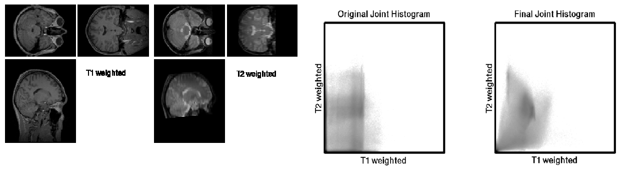


Mutual Information

- Algorithms for maximising mutual information (between intensities) have been the most popular for medical image registration to date.

$$MI(I,J\,|\,T)=\sum_{i,j} p_{i,j}\log\frac{p_{i,j}}{p_i p_j}$$

Mutual Information

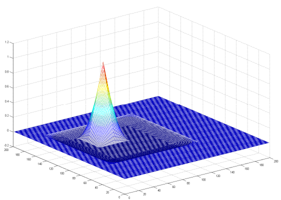


- Used for between-modality registration
- Derived from joint histograms
- Related to entropy

Correlation Based Techniques

Given a two images T & I, 2D normalized correlation function measures the similarity for each translation in an image patch

$$C(u,v)=\frac{\sum_x\sum_y T(x,y)I(x-u,y-v)}{\sqrt{\sum_x\sum_y I^2(x-u,y-v)}}$$



Correlation must be normalized to avoid contributions from local image intensities.

Correlation Theorem

- Fourier transform of the correlation of two images is the product of the Fourier transform of one image and the complex conjugate of the Fourier transform of the other.

Fourier Transform Based Methods

- Phase-Correlation
- Cross power spectrum
- Power cepstrum

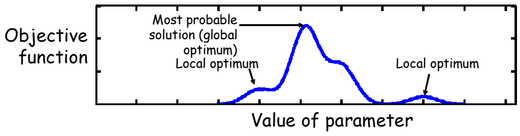
All Fourier based methods are very efficient, only only work in cases of rigid transformation

Similarity Metrics

Similarity Metric	*Advantages*
Normalized cross-correlation function [Rosenfeld 82]	accurate for white noise but not tolerant of local distortions, sharp peak in correlation space difficult to find
Correlation coefficient[Svedlow 76]	similar to above but has absolute measure
Statistical correlation and matched filters[Pratt 78]	if noise can be modeled
Phase-correlation [De Castro 87]	tolerant of frequency dependent noise
Sum of absolute differences of intensity [Barnea 72]	efficient computation, good for finding matches with no local distortions
Sum of absolute differences of contours [Barrow 77]	can be efficiently computed using "chamfer" matching, more robust against local distortions - not as sharply peaked
Contour/surface differences[Pelizzari 89]	for structural registration
Number of sign changes in pointwise intensity difference [Venot 89]	good for dissimilar images
Higher-level metrics: structural matching: tree and graph distances [Mohr 90], syntactic matching: automata [Bunke 90]	optimizes match based on features or relations of interest

Optimisation

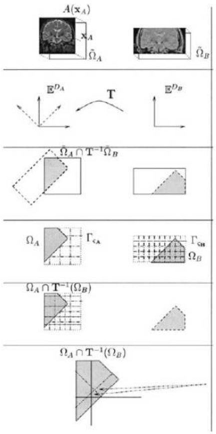
- Optimisation involves finding some “best” parameters according to an “objective function”, which is either minimised or maximised
- The “objective function” is often related to a probability based on some model



Optimization Methods

- Gradient Descent
- Conjugate Gradient Descent
- Multi-resolution search
- Deterministic Annealing

These topics will be covered in more details in the Data Analysis module (CS2)



Two images defined over domains of Euclidean space

T is a transform between the two spaces

T is restricted to the volumes

Then further to the part of a grid inside the volumes

The grid on the overlap has to be resampled

The transformed grids don’t overlap: interpolation is necessary

Transformation

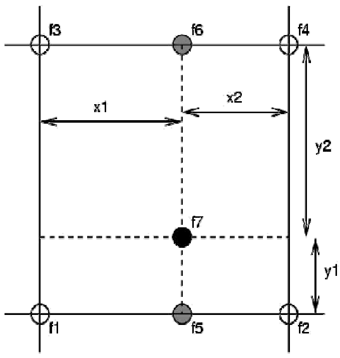
- Images are re-sampled. An example in 2D:

for y_0=1..n_{y0} % loop over rows
    for x_0=1..n_{x0} % loop over pixels in row
        $x_1 = t_x(x_0,y_0,\mathbf{q})$ % transform according to **q**
        $y_1 = t_y(x_0,y_0,\mathbf{q})$
        if $1 \le x_1 \le n_{x1}$ & $1 \le y_1 \le n_{y1}$ then % voxel in range
            $f_1(x_0,y_0) = f_0(x_1,y_1)$ % assign re-sampled value
        end % voxel in range
    end % loop over pixels in row
end % loop over rows

- What happens if x_1 and y_1 are not integers?

Simple Interpolation

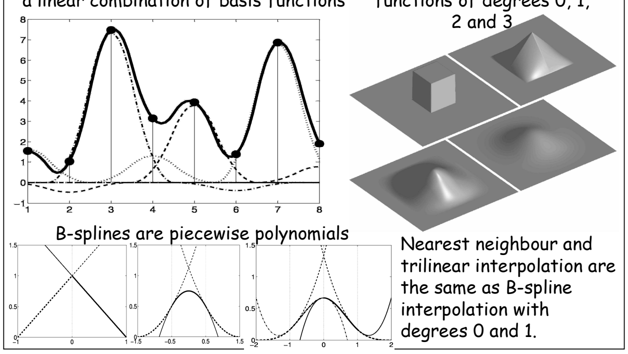
- Nearest neighbour
  - Take the value of the closest voxel
- Tri-linear
  - Just a weighted average of the neighbouring voxels
  - $f_5 = f_1\, x_2 + f_2\, x_1$
  - $f_6 = f_3\, x_2 + f_4\, x_1$
  - $f_7 = f_5\, y_2 + f_6\, y_1$



B-spline Interpolation

A continuous function is represented by a linear combination of basis functions

2D B-spline basis functions of degrees 0, 1, 2 and 3



B-splines are piecewise polynomials

Nearest neighbour and trilinear interpolation are the same as B-spline interpolation with degrees 0 and 1.

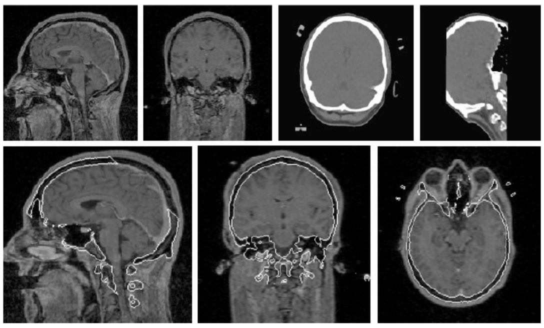
Hierarchical strategies

- 'Coarse to fine', general to specific
- Increasing complexity
  - Data
  - Warp
  - Algorithm/model

Multiresolution

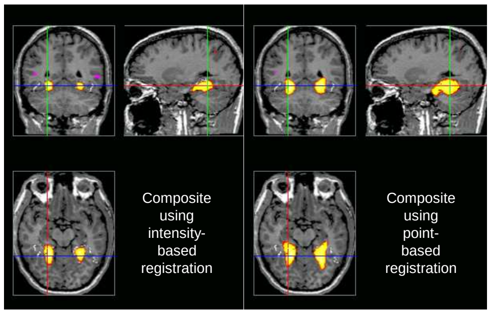
- Most of the optimization methods are applied in a multi-resolution scheme. The following is typical:
  - The registration is first run at a crude resolution e.g. the images are first resampled to 6x6x6 mm
  - The results are used to initialize a second stage where the images are resampled at 3x3x3 mm
  - The process is repeated once more with the images resampled to 1.5x1.5x1.5 mm

Registration by maximising mutual information

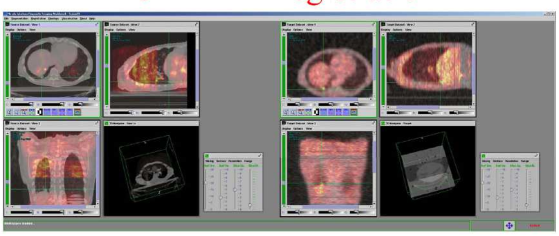


Derek Hill et. al., Physics in Medicine and Biology, 46, 2001

Data fusion: Composite functional maps

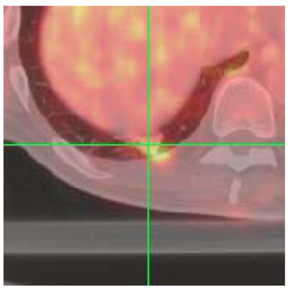


CT – PET registration



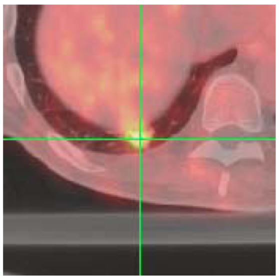
Non-rigid registration is necessary

Rigid registration poor



Is the tumour in the lungs or the stomach?

Non-rigid registration



Looks plausible; but how could you be sure?

Are you prepared to risk your software against getting sued?

SUMMARY: Components of the image registration process

- Reference and target datasets
- Transformation model
- Similarity Criterion
- Optimization Method
- Interpolation method

References

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- Thévenaz et al. *Interpolation revisited.* IEEE Trans. Med. Imaging 19:739-758 (2000).
- Andersson et al. *Modeling geometric deformations in EPI time series.* Neuroimage 13:903-919 (2001).