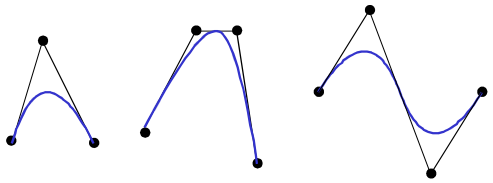
SPLINES

2D Splines
- Bézier curves
- Spline curves

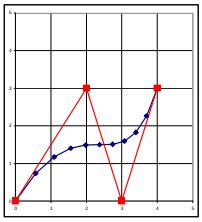
2D Splines

- Smooth curves generated from an input set of user-specified control points (knots)



Raster conversion problem

- Given a set of control points, plot the spline
- A spline curve is usually approximated by a set of short line segments



Control points

Computed points on the spline, inked with short line segments into a smooth curve

Spline defined by parametric equations

P(u) = (x(u), y(u))

- u is the parameter
- x(u) and y(u) are functions of the parameter u which generate x and y coordinates of the curve P

Spline curves can

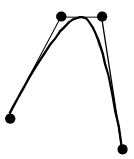
- interpolate control points
- (the curve does not have to pass through them)

or

- approximate control points
- (the curve passes through the points).

Bézier curves

- Bézier curve is generated by forming a set of polynomial functions formed from the coordinates of the control points



Bézier curves: equations

Input

- a set of n+1 control points
- $p_k = (x_k, y_k), k = 0, n$

- Bézier function P(u) in parametric form:

$$P(u) = \sum_{k=0}^{n} p_k \, B_{kn}(u)$$

$$P(u) = \sum_{k=0}^{n} p_k B_{kn}(u) = p_0 B_{0n}(u) + p_1 B_{1n}(u) + \ldots + p_n B_{nn}(u)$$

$$P(u) = \sum_{k=0}^{n} p_k \, B_{kn}(u)$$

p_k – a control point, $p_k = (x_k, y_k)$

$B_{kn}(u)$ is a blending function (a polynomial):

$$B_{kn}(u) = C(n, k) \cdot u^k \cdot (1 - u)^{n - k}$$

function C(n, k) provides values for coefficients:

$$C(n, k) = \frac{n!}{k! \, (n - k)!}$$

$$P(u) = \sum_{k=0}^{n} p_k \, B_{kn}(u)$$

- P(u) in a form showing explicit equations for the individual curve coordinates:

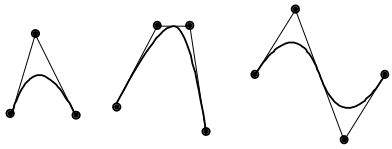
$$x(u) = \sum_{k=0}^{n} x_k \, B_{kn}(u) \qquad y(u) = \sum_{k=0}^{n} y_k \, B_{kn}(u)$$

$$x(u) = \sum_{k=0}^{n} x_k \, B_{kn}(u) = x_0 \, B_{0n}(u) + x_1 \, B_{1n}(u) + \ldots + x_n \, B_{nn}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k \, B_{kn}(u) = y_0 \, B_{0n}(u) + y_1 \, B_{1n}(u) + \ldots + y_n \, B_{nn}(u)$$

$$P(u) = \sum_{k=0}^{n} p_k \, B_{kn}(u)$$

- The function P(u) is a polynomial of degree n (one less than the number of the control points used)
- For example, 3 points generate parabola, 4 points generate a cubic curve etc.



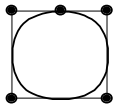
Bézier curves: example

- See a worked-out example of generating a Bézier spline at

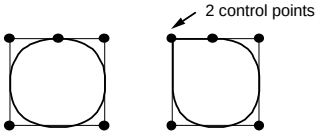
http://www.cs.bham.ac.uk/~exc/Teaching/Graphics/ComputingSpline.pdf

Multiple control points

Condition for closed curves: $p_0 = p_n$



Multiple control points at a same position the curve is "pulled" more towards that position



- In theory Bézier curves can be constructed for any number of points.
- This would lead to polynomials of a very high order which are inefficient to calculate.

Solution

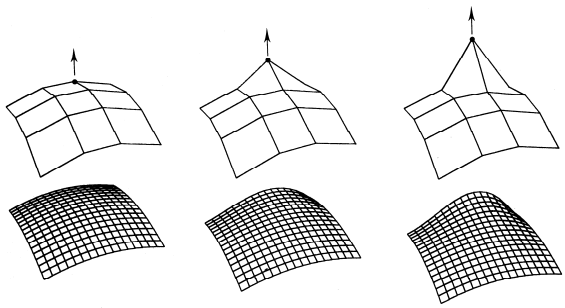
- A set of points is divided into smaller subsets
- Bézier curve specified for each small set
- A very useful property of Bézier curves is that the curve fragments are always smoothly joined

Spline curves

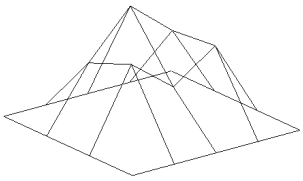
- B-splines are similar to Bézier curves, they differ in the choice of blending functions.
- In a Bézier curve the shape of the curve is influenced by all the control points.
- In B-splines only up to four nearest control points are taken into consideration (i.e. the highest degree of a polynomial is 3, a cubic spline).

3D Bezier patches

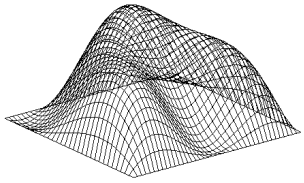
defined on a regular grid



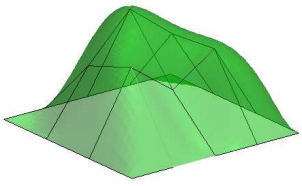
3D Bezier patches



random height map on 5x6 grid

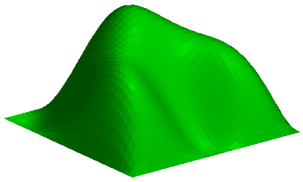


Bezier interpolation (50x60 grid)

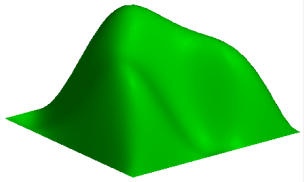


Bezier interpolation (50x60 grid), surface, flat shading

3D Bezier patches



Bezier interpolation surface, flat shading



Bezier interpolation surface, Phong shading

Interactive animation at
http://www.nbb.cornell.edu/neurobio/land/OldStudentProjects/cs490-96to97/anson/BezierPatchApplet/