THE APPENDIX

3-dimensional transformations

Translation

$$T = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

about Z axis

$$R_z = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & 0 \\ \sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

about X axis

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

about Y axis

$$R_y = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi & 0 & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

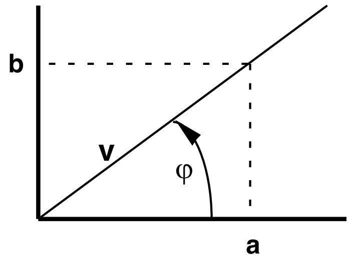
Combined rotation matrix

$$R_{xyz} = \begin{bmatrix} \frac{V_x}{|\mathbf{V}|} & \frac{V_y}{|\mathbf{V}|} & \frac{V_z}{|\mathbf{V}|} & 0 \\ \frac{U_x}{|\mathbf{U}|} & \frac{U_y}{|\mathbf{U}|} & \frac{U_z}{|\mathbf{U}|} & 0 \\ \frac{N_x}{|\mathbf{N}|} & \frac{N_y}{|\mathbf{N}|} & \frac{N_z}{|\mathbf{N}|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective projections

$$P_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & \mathbf{0} \end{bmatrix} \qquad P_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/D & \mathbf{1} \end{bmatrix}$$

Basic trigonometric definitions



$$\cos \varphi = \frac{a}{v}$$

$$\sin \varphi = \frac{b}{v}$$

$$v = \sqrt{a^2 + b^2}$$

$\sin 0^\circ = 0 \qquad \sin 90^\circ = 1$

$\cos 0^\circ = 1 \qquad \cos 90^\circ = 0$

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$\sin(-\varphi) = -\sin(\varphi)$

$\cos(-\varphi) = \cos(\varphi)$

Bézier curves

Bézier function *P(u)* in parametric form:

$$P(u) = \sum_{k=0}^{n} p_k B_{kn}(u)$$

Blending functions:

$$B_{kn}(u) = C(n, k)\, u^k (1 - u)^{n-k}$$

where

$$C(n, k) = \frac{n!}{k!\,(n-k)!} \qquad n! = n \cdot (n-1) \cdot \ldots \cdot 1, \qquad 0! = 1$$

Dot product

$$\bar{\mathbf{a}} \cdot \bar{\mathbf{b}} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$$

$$\bar{\mathbf{a}} \cdot \bar{\mathbf{b}} = |\bar{\mathbf{a}}| \cdot |\bar{\mathbf{b}}| \cdot \cos \varphi$$

Cross product

$$\bar{\mathbf{a}} \times \bar{\mathbf{b}} = \begin{vmatrix} \overline{\mathbf{1_x}} & \overline{\mathbf{1_y}} & \overline{\mathbf{1_z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \overline{\mathbf{1_x}}\,(a_y \cdot b_z - a_z \cdot b_y) +$$

$$\overline{\mathbf{1_y}}\,(a_z \cdot b_x - a_x \cdot b_z) + \overline{\mathbf{1_z}}\,(a_x \cdot b_y - a_y \cdot b_x)$$

$$|\bar{\mathbf{a}} \times \bar{\mathbf{b}}| = |\bar{\mathbf{a}} \cdot \bar{\mathbf{b}} \cdot \sin \varphi|$$

END OF APPENDIX