Composite transformations

General properties of matrix multiplication
Combining transformations
Composite transformations
Implementation

Revision: 2D transformations in homogeneous coordinates

- Point (vertex) representation

$$\mathbf{P} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Transformation matrix – general form

$$T = \begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix}$$

Revision: 2D transformation matrices

- Translation $T = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix}$
- Scaling $S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Rotation $R = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Revision: 2D transformation matrices

- Manipulating 2D vertices via matrix multiplication

$$\mathbf{P}' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

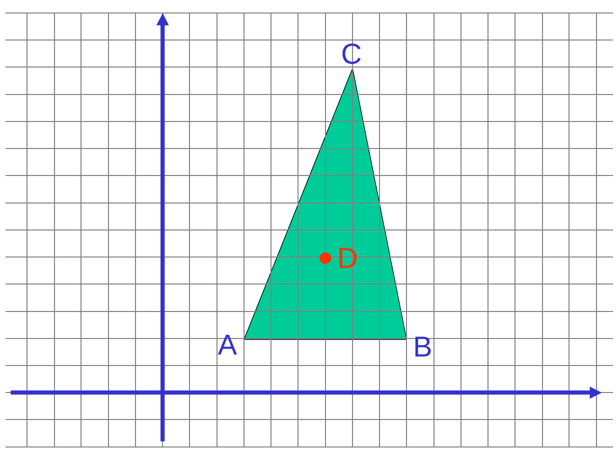
Problem 1: combining transformations

Given is a triangle with homogeneous vertices

$A = [3\ \ 2\ \ 1]^T$
$B = [9\ \ 2\ \ 1]^T$
$C = [7\ 10\ \ 1]^T$

Rotate the triangle by 90º about point D

$D = [6\ \ 5\ \ 1]^T$



Solution 1 (2D)

A detailed solution is given in the “Combined transformation exercise” on the module’s web page

Composite transformation

- A complex transformation achieved by combining a sequence of basic transformations
- Example: rotation about a fixed point
  - translate the object so that the axis / centre of rotation coincides with one of the axes / centre of the coordinate system
  - perform the specified rotation
  - translate the object so that the axis / centre of rotation is moved back to its original position

Transformations can be combined by matrix multiplication

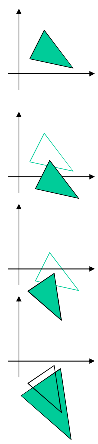
General properties of matrix multiplication

- Associative
  A · B · C = (A · B) · C = A · (B · C)
- Non-commutative
  A · B ≠ B · A

Combining transformations

- If transformation matrices T1 and T2 are to be applied to vertex P in order
  first T1
  then T2
- the composite matrix M will be defined as:
  M = T2 ·(T1 · **P)**

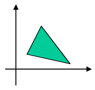
Models of implementation



1. Carry out operations directly on the points
   - P’ = Translate(P, Tx, Ty, Tz)
   - P’’ = $\text{Rotate}_{\text{Axis}}$ (P’, Angle)
   - P’’’ = Scale(P’’, Sx, Sx, Sz)

Models of implementation

2. First create a single combined transformation matrix M, then transform the points:



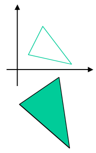
M1 = CreateTranslationMatrix(Tx, Ty, Tz)

M2 = CreateRotationMatrix(Axis, Angle)

M3 = CreateScalingMatrix(Sx, Sy, Sz)

M = CombineTransformations(M1,M2,M3)

P' = TransformPoints(P, M)



Problem 2: changing coordinate system (1)

A triangle with homogeneous vertices

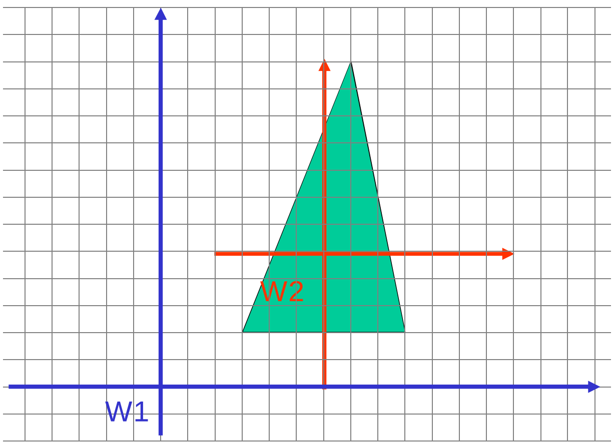
A = $[3\ \ 2\ \ 1]^T$

B = $[9\ \ 2\ \ 1]^T$

C = $[7\ 10\ \ 1]^T$

is defined in a 2D coordinate system W1.

Using matrix operations, compute the coordinates of the triangle in another 2D coordinate system, W2, the centre of which is placed at point D = (6, 5) in the coordinate system W1.



Problem 2: outline solution

The transformation which aligns the axes of the W2 system with the axes of the W1 system will also change the coordinates of the object from the W1 system to the W2 system.

Here it is translation by vector –D = [-6 -5]

Implementation:
- Define the appropriate translation matrix T
- Transform the triangle vertices

Verify graphically

Problem 3 (2D)

A triangle with homogeneous vertices

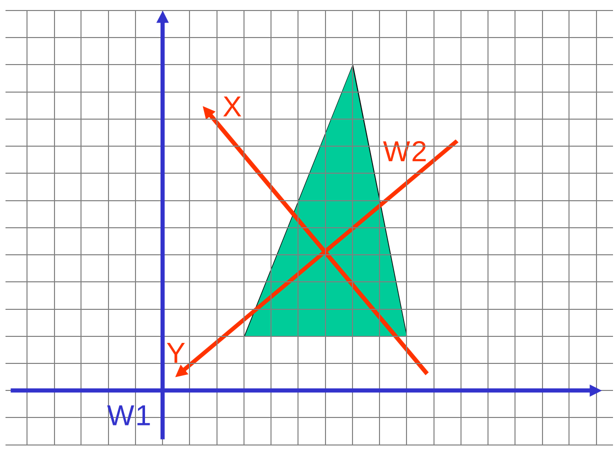
A = $[3\ \ 2\ \ 1]^T$

B = $[9\ \ 2\ \ 1]^T$

C = $[7\ 10\ \ 1]^T$

is defined in a 2D coordinate system W1.

Using matrix operations, compute the coordinates of the triangle in another 2D coordinate system, W2, the centre of which is placed at point D = (6, 5) in the coordinate system W1 and the Y axis of which points towards the centre of W1.



Problem 3: outline solution

The transformation which aligns the axes of the W2 system with the axes of the W1 system will also change the coordinates of the object from the W1 system to the W2 system.

We need two transformations:
translation by vector –D = [-6 -5]
rotation by the angle which aligns
the axes of the two systems

Implementation:
- Define the appropriate translation matrix T
- Define the appropriate rotation matrix R
  hint: do not compute the angle,
  get the rotation matrix directly from the W2 axes
  in the same way as we did in 3D

Transform the triangle vertices

Verify graphically

Homework

Compute numerical solutions to problems 2 and 3

Next lecture

Viewing transformations