Digital image processing and analysis

14. Active shape models

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In this lecture we shall find out about:

- Statistical shape models
  - What they are
  - What they are used for
- Principal Component Analysis (PCA)
  - What it is
  - How it is used for shape modelling
- Point distribution models
- Active shape models
- Active appearance models

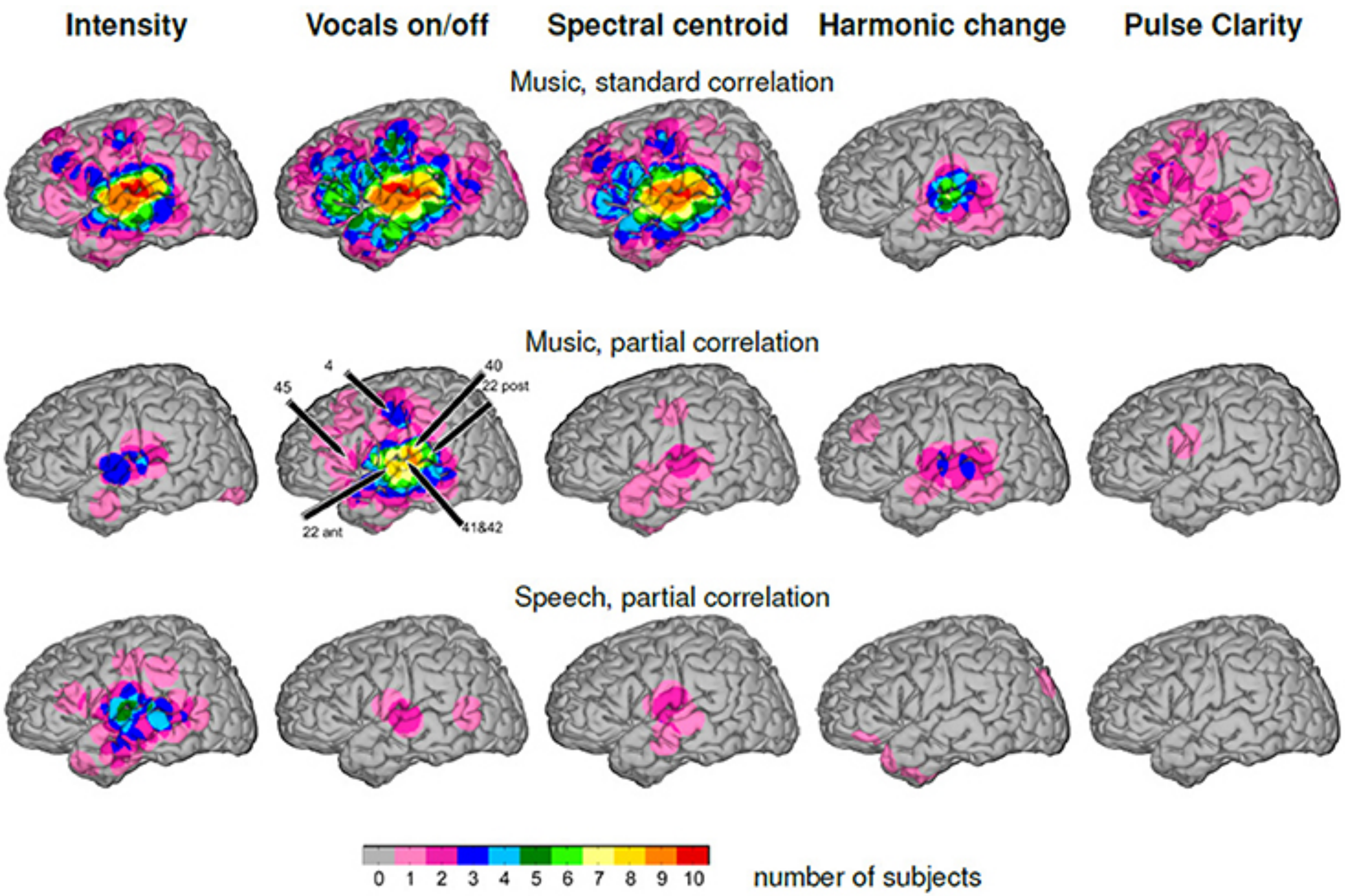
Acknowledgement*: A number of slides come from:*
vision.mas.ecp.fr/Personnel/iasonas/course/shape.pdf
cmp.felk.cvut.cz/cmp/courses/33DZOzima2007/slidy/pointdistributionmodels.pdf

Active shape models
Tracking for animation

https://www.youtube.com/watch?v=DcLJ8aDq74g&feature=related

Active shape models
Face recognition and tracking

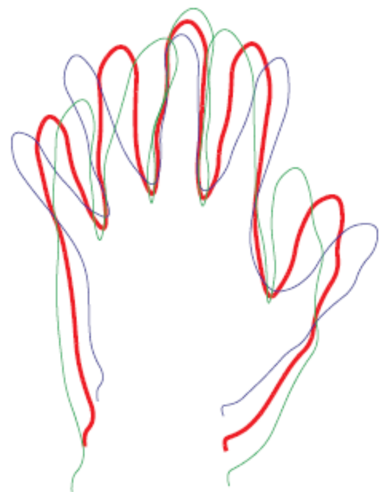
Active shape models
Statistical model of brain activity



Rock Correlation

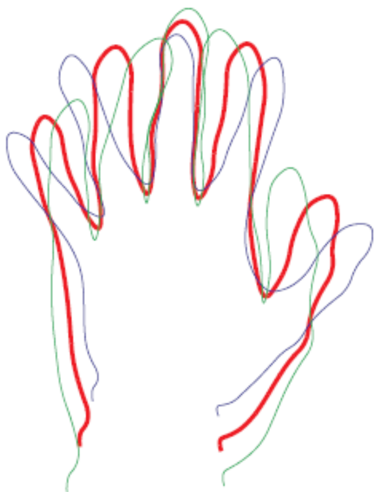
Statistical shape model
What it is

- A model of shape which describes plausible statistical variations in shape of a given class of objects.
- The shape of an object is represented by a set of n points.

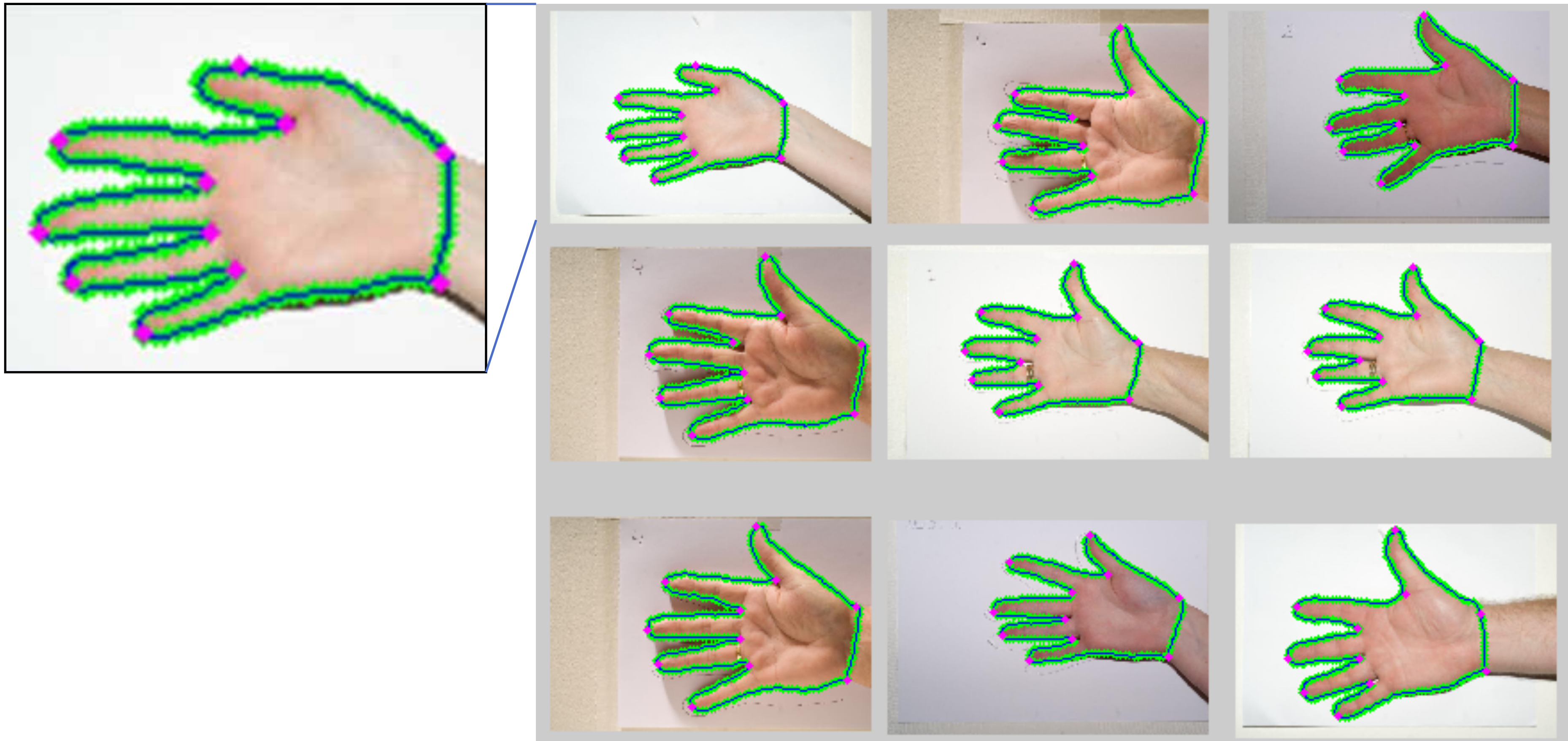


Statistical shape model
What it is for

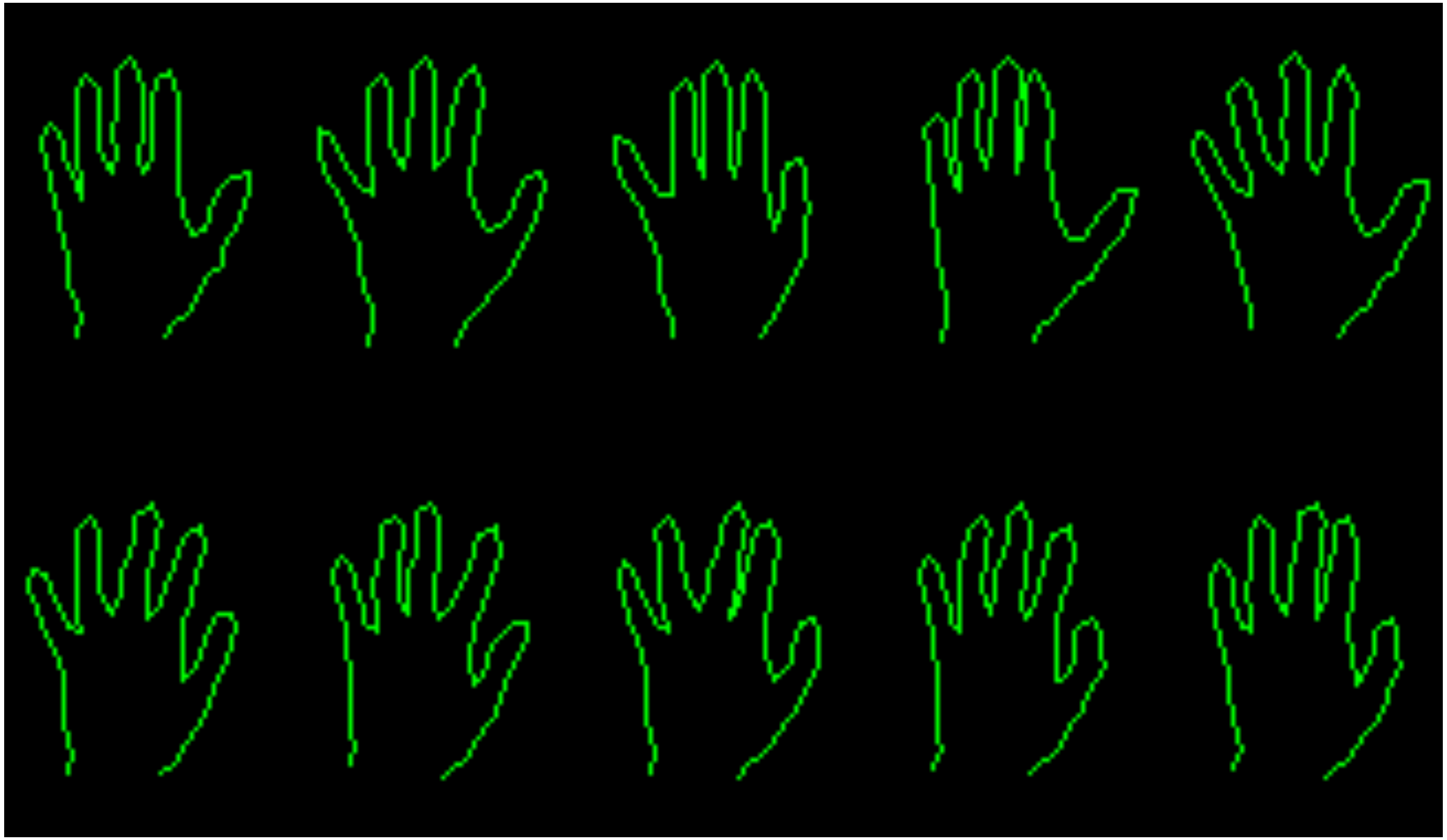
- The aim is to derive models with which to analyse new shapes, and to synthesise shapes similar to those in a training set.
- The training set typically comes from hand annotation of a set of training images.
- By analysing the variations in shape over the training set, a model is built which can mimic these variations.



Statistical shape model
Training set

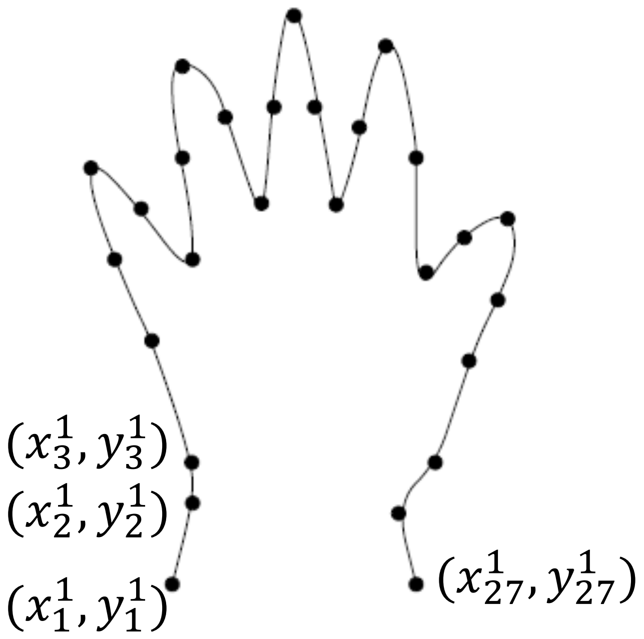


Statistical shape model
Training set



https://personalpages.manchester.ac.uk/staff/timothy.f.cootes/Models/pdms.html

Statistical shape model
Training



Input:

- M training samples
- N points each

$$\mathbf{x}^i = (x_1^i, y_1^i, x_2^i, y_2^i, \ldots, x_N^i, y_N^i)'$$

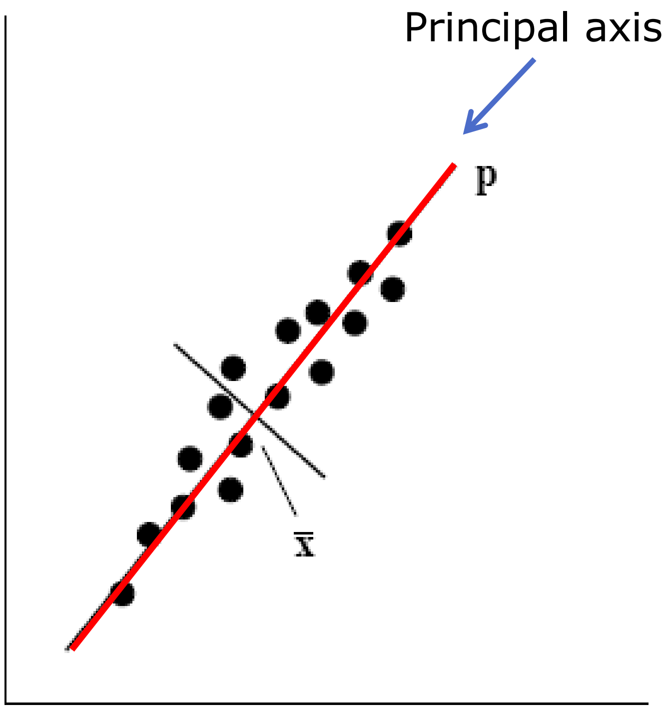
i=1 to M

Procedure:

- Rigidly align all shapes
- Calculate the mean and the covariance matrix
- PCA (eigen analysis) — find principal modes

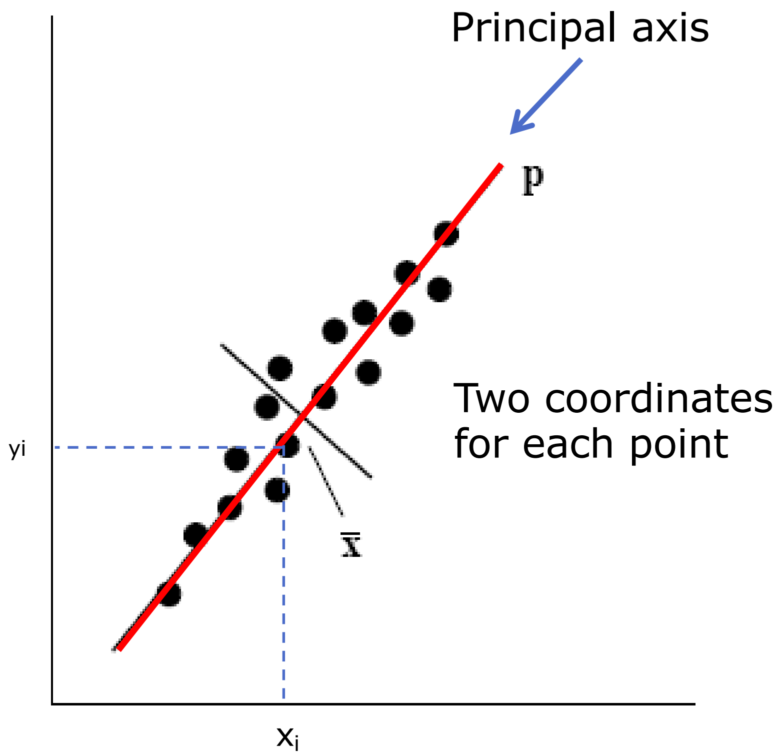
Principal **C**omponent **A**nalysis

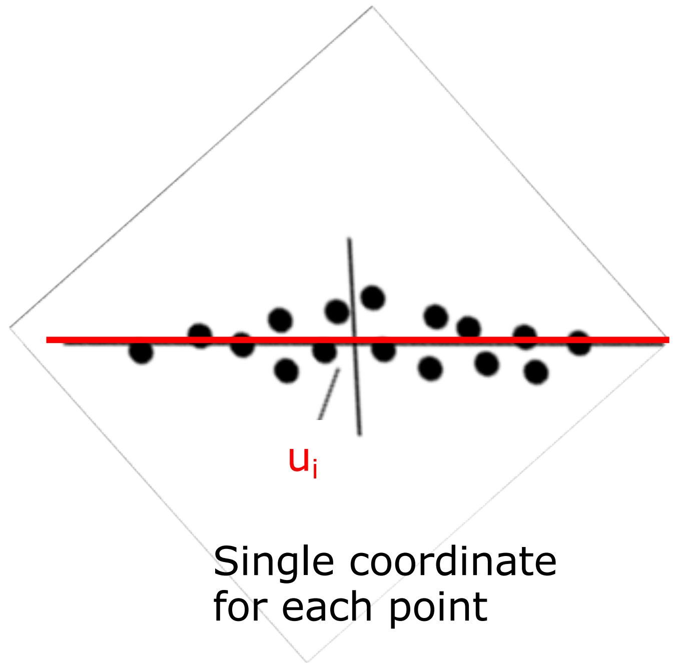
Principal Component Analysis
2-dimensional example



Principal component analysis (PCA) is a statistical technique used to emphasize nature of true variations in a dataset.

Principal Component Analysis
2-dimensional example



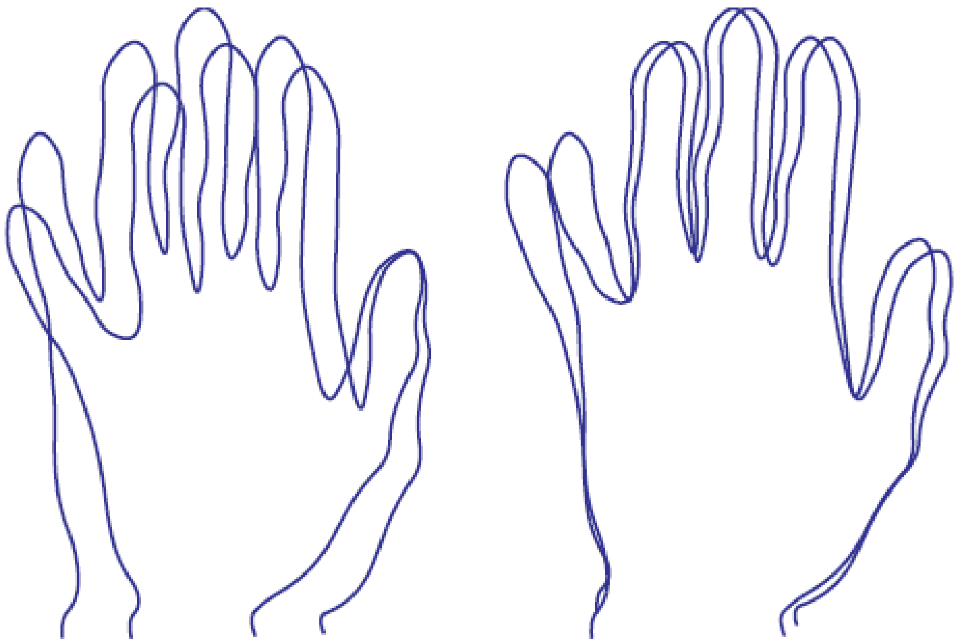


Principal components define a new coordinate system.

After applying PCA any point can be approximated by the nearest point on the principal axis.

In this way dimensionality of the point set is reduced (here from two-dimensional to one-dimensional).

Statistical shape model
Training: align two shapes

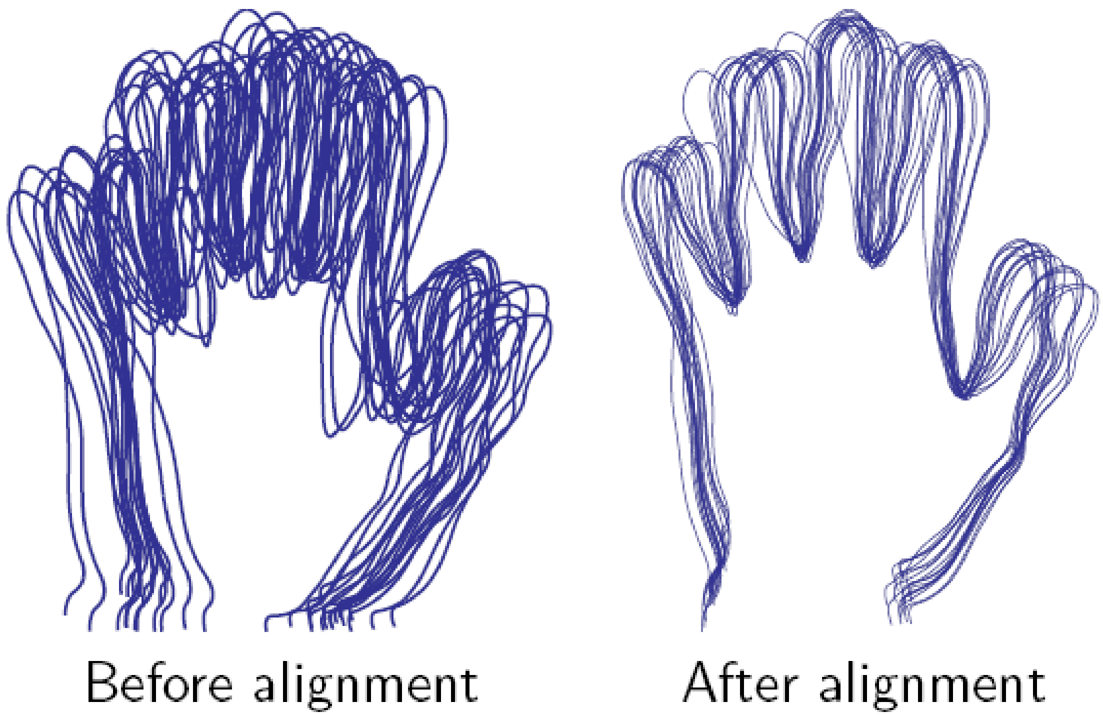


before alignment after alignment

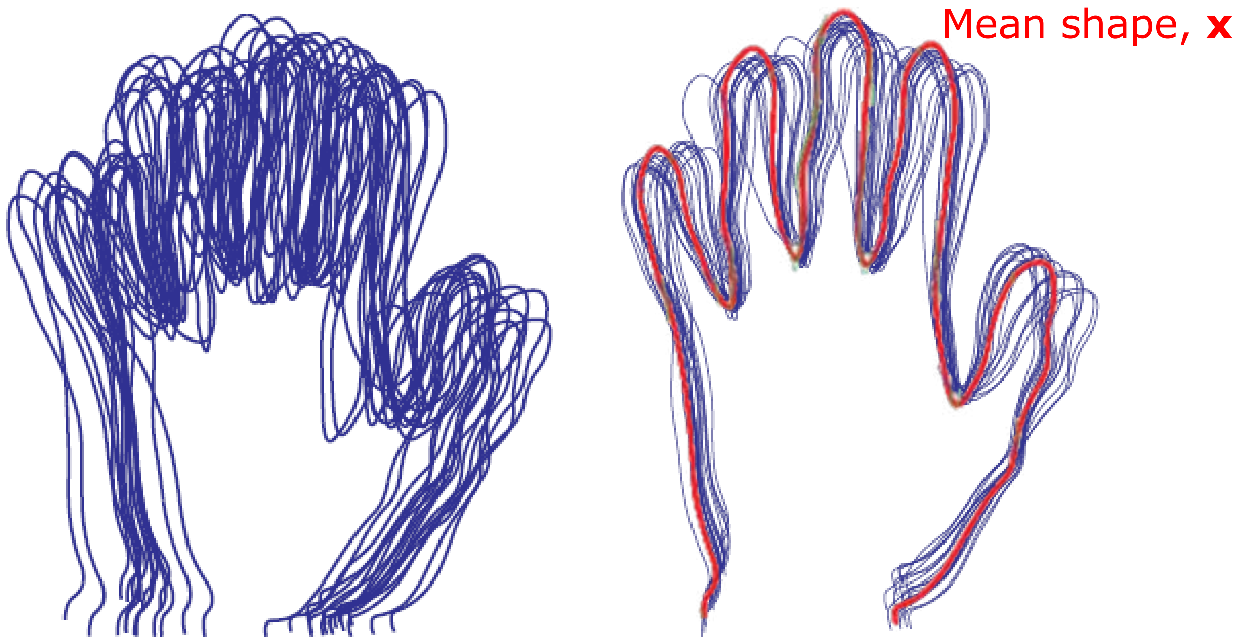
$$\mathbf{x}^{(1)} = (x_1^{(1)}, y_1^{(1)}, x_2^{(1)}, y_2^{(1)}, \ldots, x_N^{(1)}, y_N^{(1)})^T$$
$$\mathbf{x}^{(2)} = (x_1^{(2)}, y_1^{(2)}, x_2^{(2)}, y_2^{(2)}, \ldots, x_N^{(2)}, y_N^{(2)})^T$$

Find a transformation (rotation, translation, scaling) of $\mathbf{x}^{(2)}$

Statistical shape model
Training: align many shapes



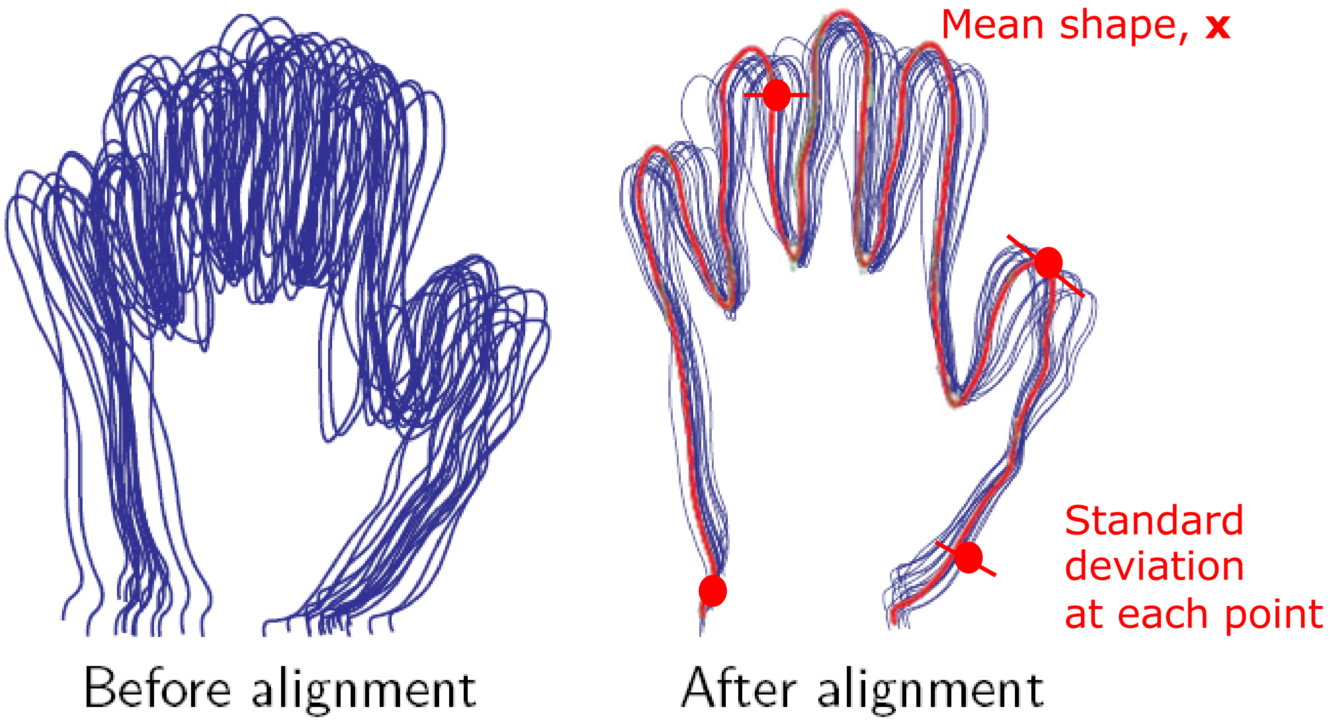
Statistical shape model
Training: align many shapes



Before alignment After alignment

We have obtained M (mutually aligned) boundaries $\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2, \ldots, \hat{\mathbf{x}}^M$ and the mean $\bar{\mathbf{x}}$.

Statistical shape model
Training: align many shapes



We have obtained M (mutually aligned) boundaries $\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2, \dots, \hat{\mathbf{x}}^M$ and the mean $\bar{\mathbf{x}}$.

Principal Component Analysis

Principal components $\mathbf{p_i}$ define a new coordinate system.

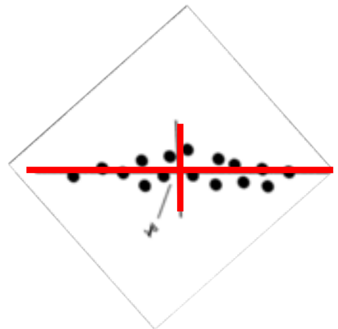
<u>Any</u> shape vector $\mathbf{x}$ can be represented as

Parameter defining the magnitude of variation along each principal axis

Point distribution model

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}\,\mathbf{b}$$

Matrix of principal components, one per row, arranged in the order of the largest variation



This includes shapes not originally in the training dataset

Point distribution model
Generating new shapes

- **Input:** M non-aligned boundaries $\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^M$.
- **Output:** mean $\bar{\mathbf{x}}$ and dimension- reduced matrix $\mathbf{P}_K$
- **New shape generation:**

$$\widetilde{\mathbf{x}} = \bar{\mathbf{x}} + P_K\, \mathbf{b}_K$$

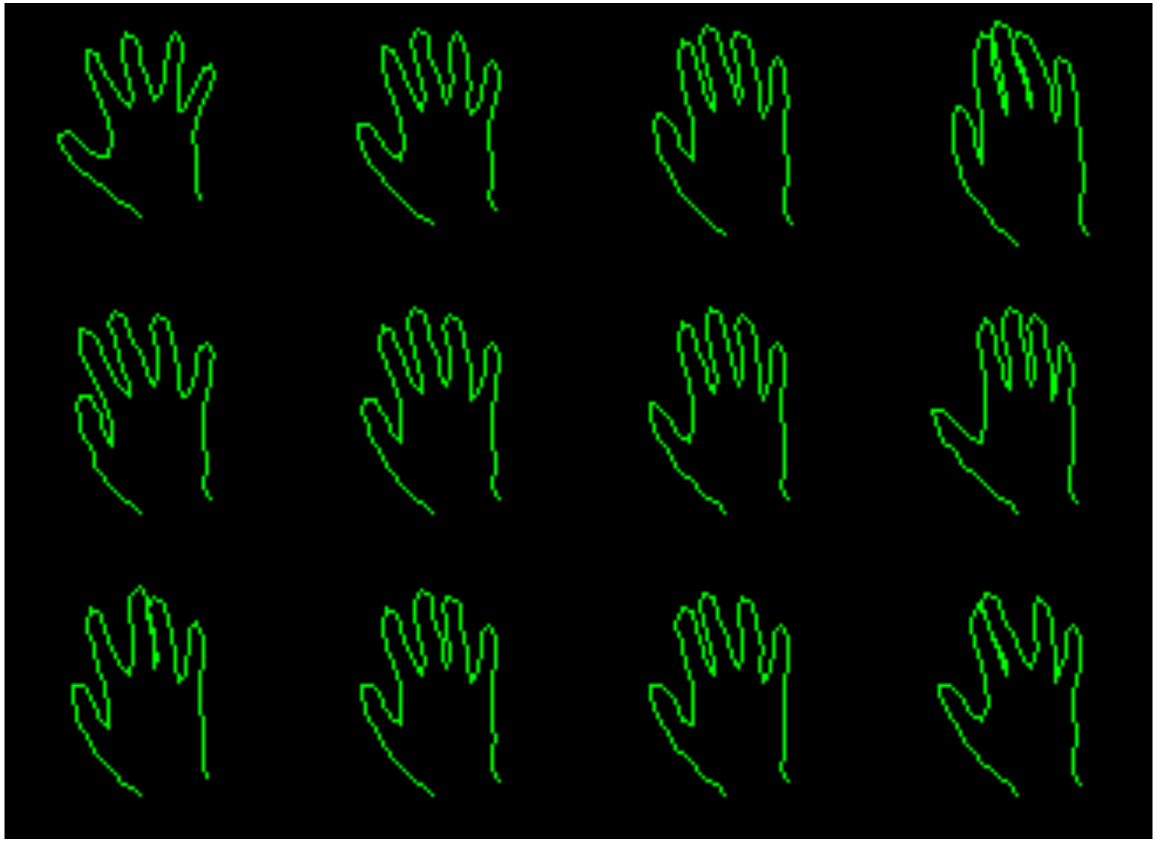
For "well-behaved' shapes

$$-3\sqrt{\lambda_i} \leq b_i \leq 3\sqrt{\lambda_i}$$

K (the number of reduced dimensions chosen)
defines the number of **modes of variation**

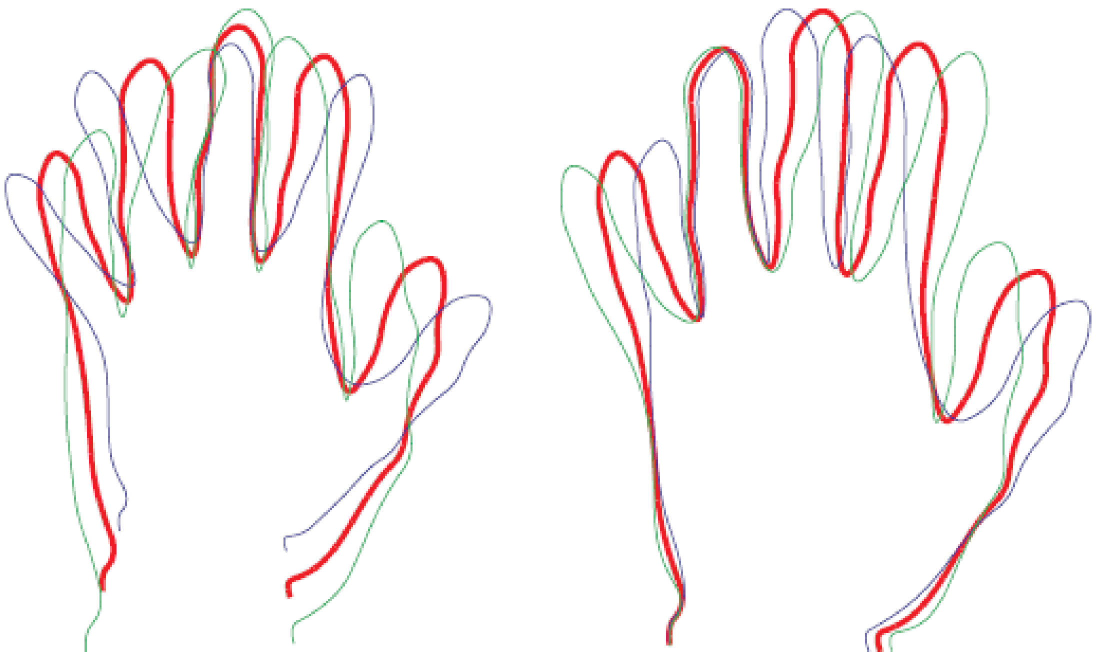
Point distribution model
Modes of variation

Here the number of modes of variation, K = 3.
By varying the first three parameters of the shape vector, b, one at a time, we can demonstrate some of the modes of variation allowed by the model:



(Each row obtained by varying on parameter and fixing others at zero)

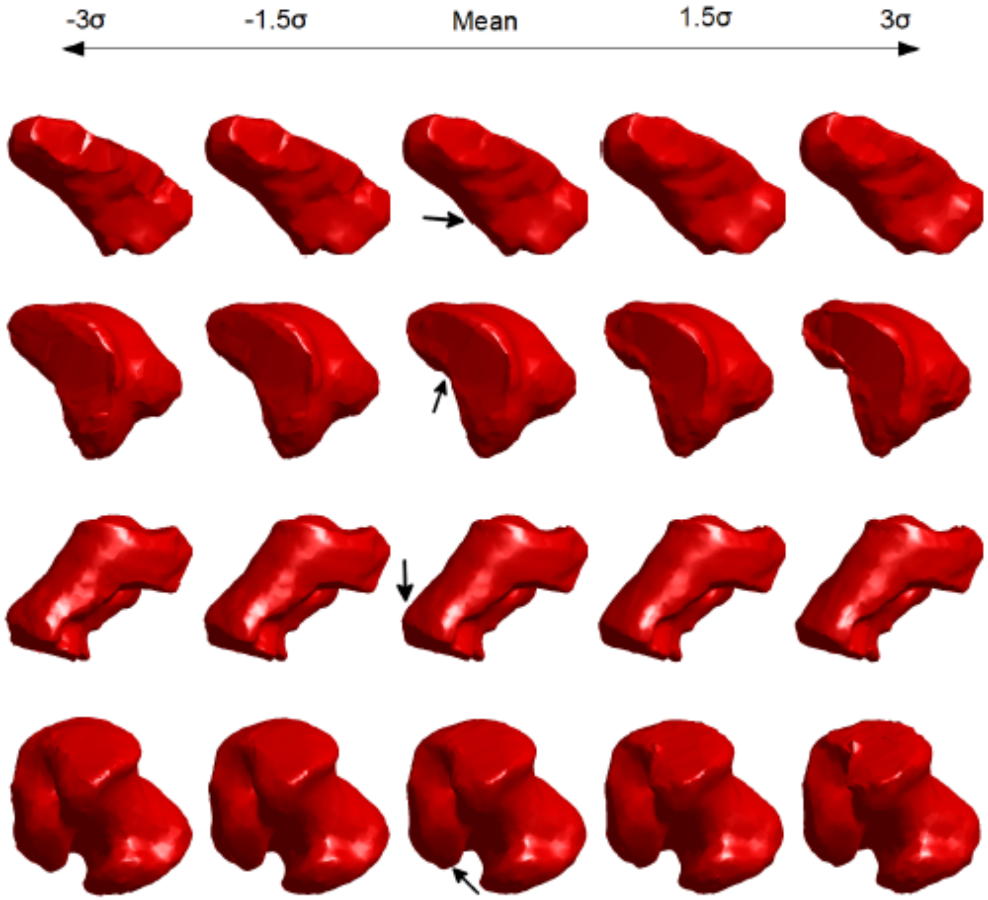
Point distribution model
Generating new shapes



First mode   Second mode

The mean shape is in red, the shape corresponding to $-3\sqrt{\lambda}$ in blue and the shape corresponding to $+3\sqrt{\lambda}$ in green.

Point distribution model
Modes of variation



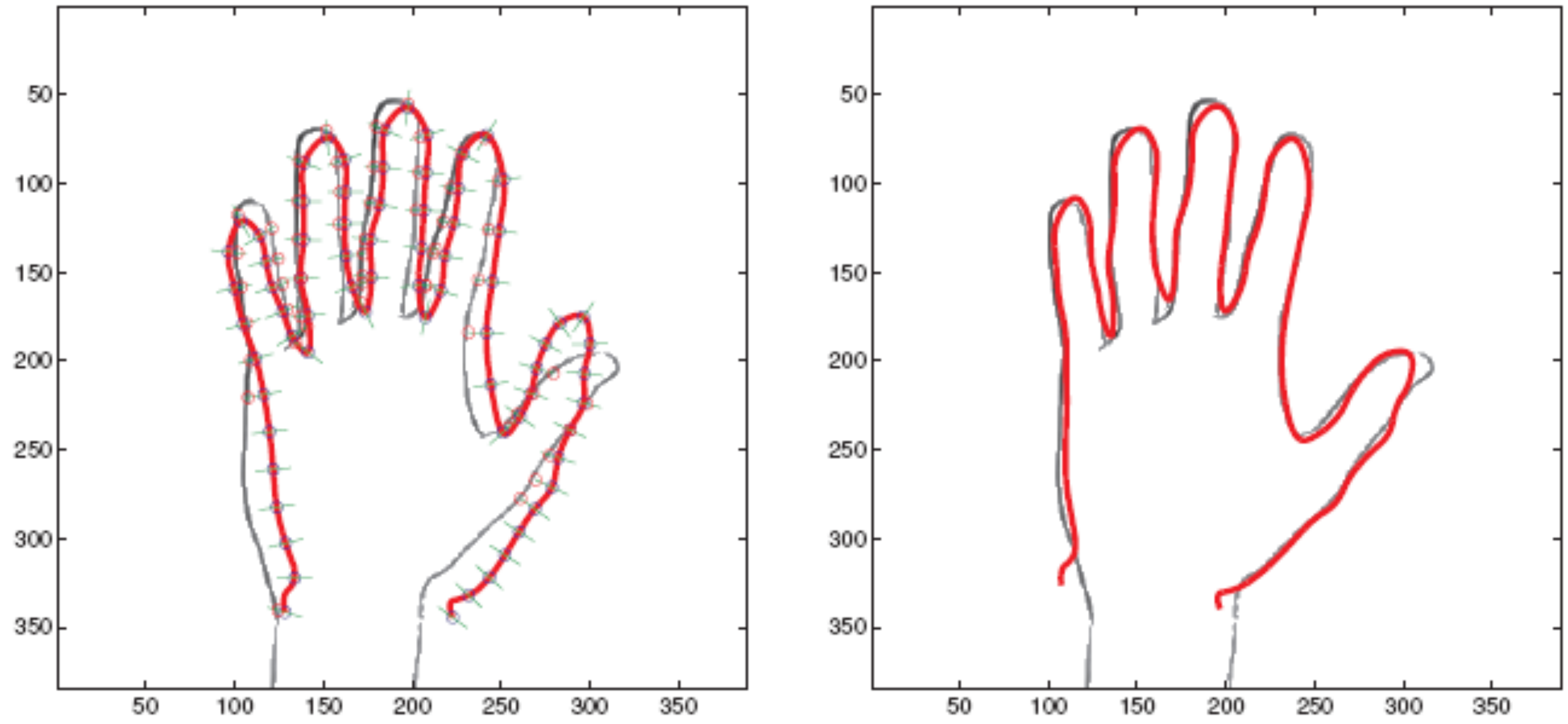
Our research on rheumatoid arthritis.
We generate a 3D statistical model of each bone in the hand.
You can see how shapes vary in four different bones.

Point distribution model
Fitting model to a given shape

- Calculate an edge map of the image
- For each landmark $\mathbf{p}_i$ we find a line normal to the shape contour.
- New position $\mathbf{p}'_i$ is the maximum of the edge map on the line.

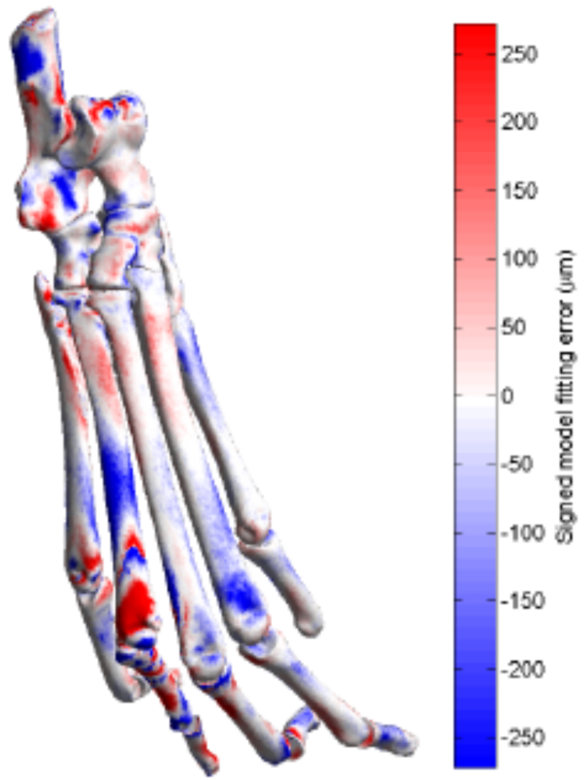
Point distribution model
Fitting model to a given shape



First iteration

Final postition

Point distribution model
Fitting model to a given shape



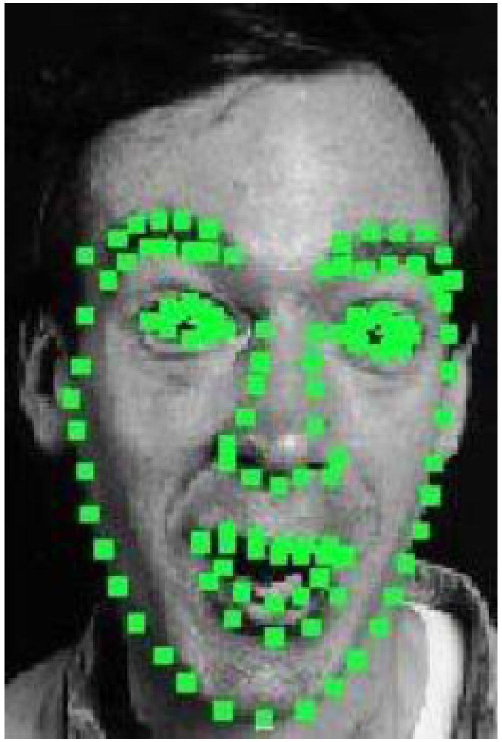
Our research on rheumatoid arthritis.
We generated a 3D statistical model of a normal hand, then fitted the model to a diseased hand and computed local differences

Active shape models
What they are

- Active shape models (ASMs) are statistical models of the shape of objects which iteratively deform to fit to an example of the object in a new image.
- The shapes are constrained by the Point Distribution Model to vary only in ways seen in a training set of labelled examples.
- The shape of an object is represented by a set of points (placed at the locations corresponding to those in the shape model).
- The ASM algorithm aims to match the model to a new image.

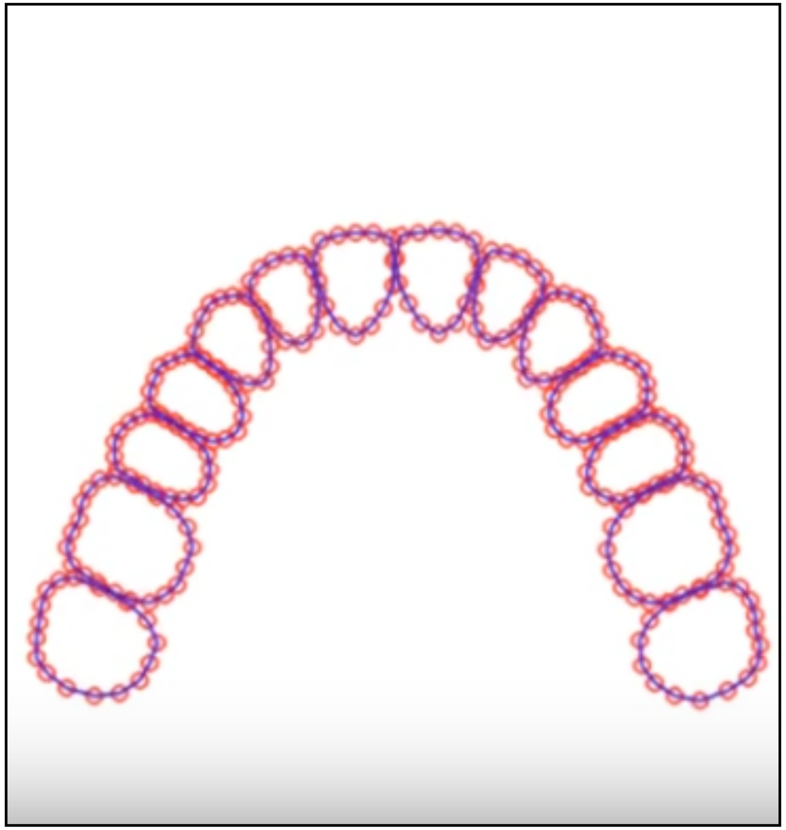
Developed by Tim Cootes and Chris Taylor in 1995.

Active shape models
Examples



T. F. Cootes, C. J. Taylor, D. H. Cooper, and J. Graham. Training models of shape **from sets of examples.** BMVC 1992

Active shape models
Examples



https://www.youtube.com/watch?v=XFvZSWZIubM

Active appearance models
What they are

- An Active Appearance Model (AAM) contains a statistical model of the ***shape*** and ***grey-level appearance of the object*** of interest.
- AAM uses a statistical model, similar to that of shape, to represent the ***intensity variation*** across a region.
- Given a set of training images, labelled with landmark points, image warping is used to deform each image so that the object has the mean shape.
- After that a statistical model of the grey-levels across the object is built.

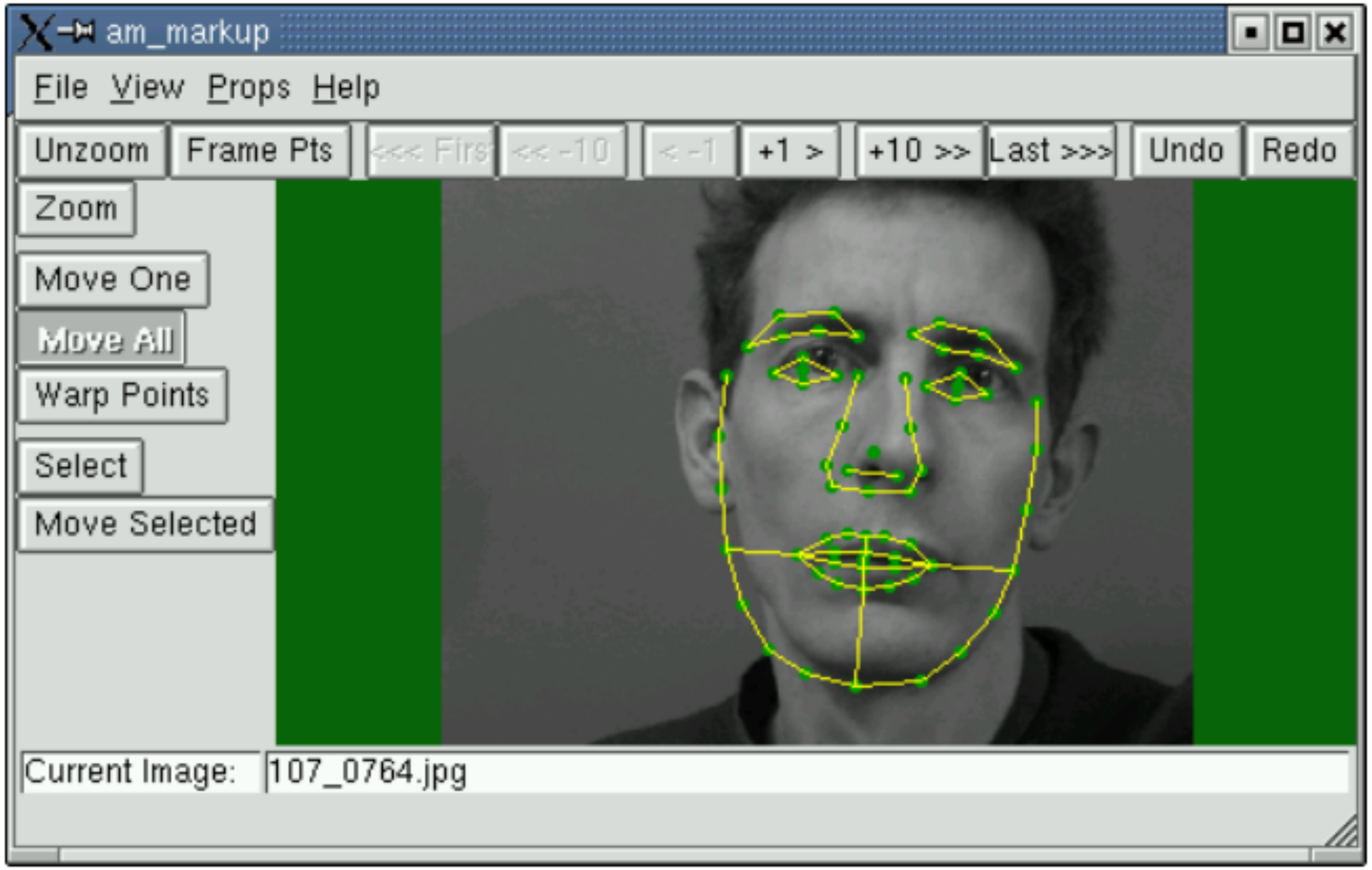
Active appearance models
How they are generated

- **Input**
  - M training samples
  - MxN pixels each
  - $x^i = \left(x^i_{(1,1)}, x^i_{(1,2)}, \dots, x^i_{(1,M)}, x^i_{(2,1)}, x^i_{(1,1)}, \dots, x^i_{(M,N)}\right)$

Procedure:

- Rigidly align all shapes
- Calculate the mean and the covariance matrix
- PCA (eigen analysis) — find principal modes

Active appearance models
How they are generated



Download Tools and Data from:

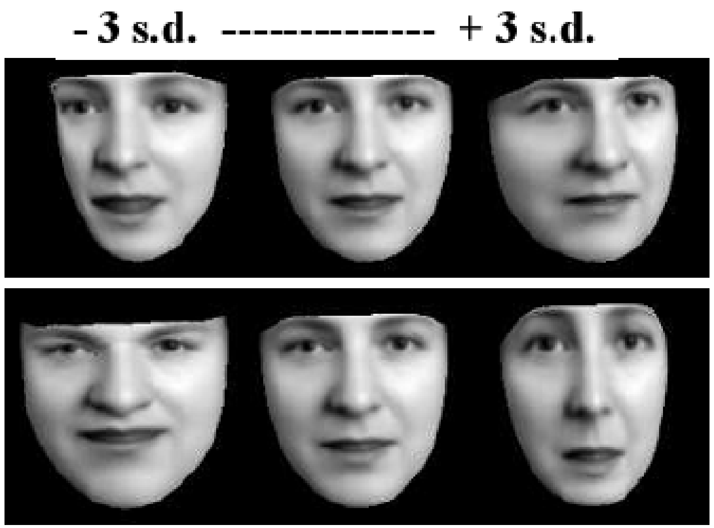
Modelling and Search Software https://personalpages.manchester.ac.uk/staff/timothy.f.cootes/software/am_tools_doc/

Active appearance models
Training

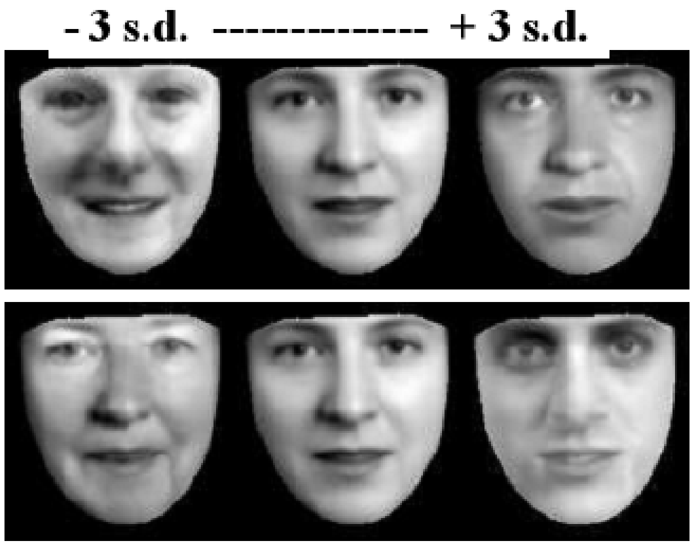
- Training images
- $x_1, \ldots, x_N$

T.F. Cootes, G. J. Edwards, and C. J. Taylor. Active appearance models, 1998
T. F. Cootes, G. V. Wheeler, K. N. Walker, C. J. Taylor: View-based active appearance models.2002,

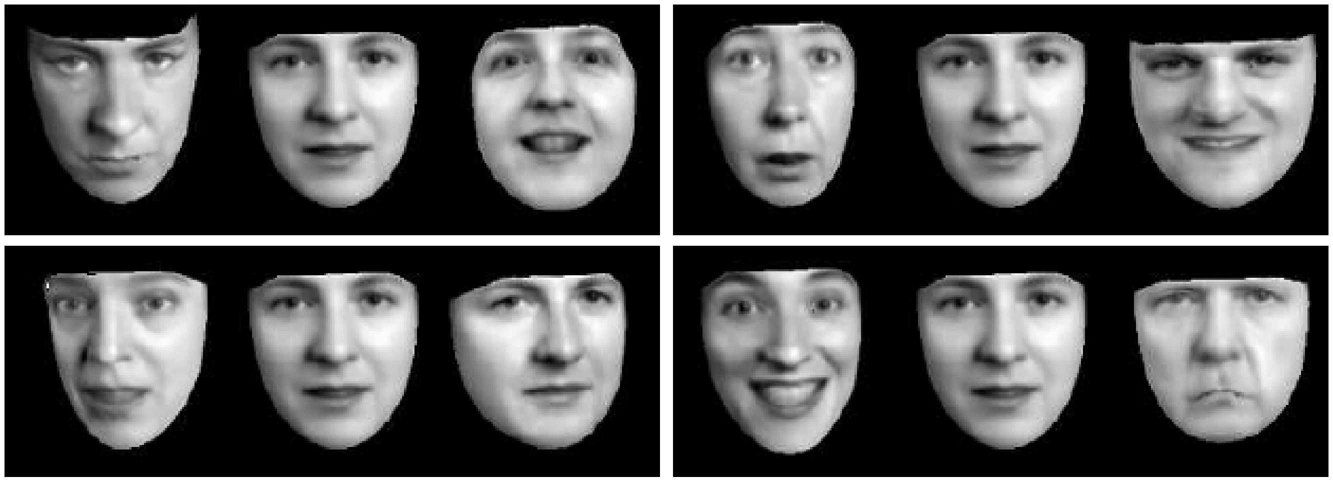
Active Appearance models
Modes of variation



First two modes of shape variation



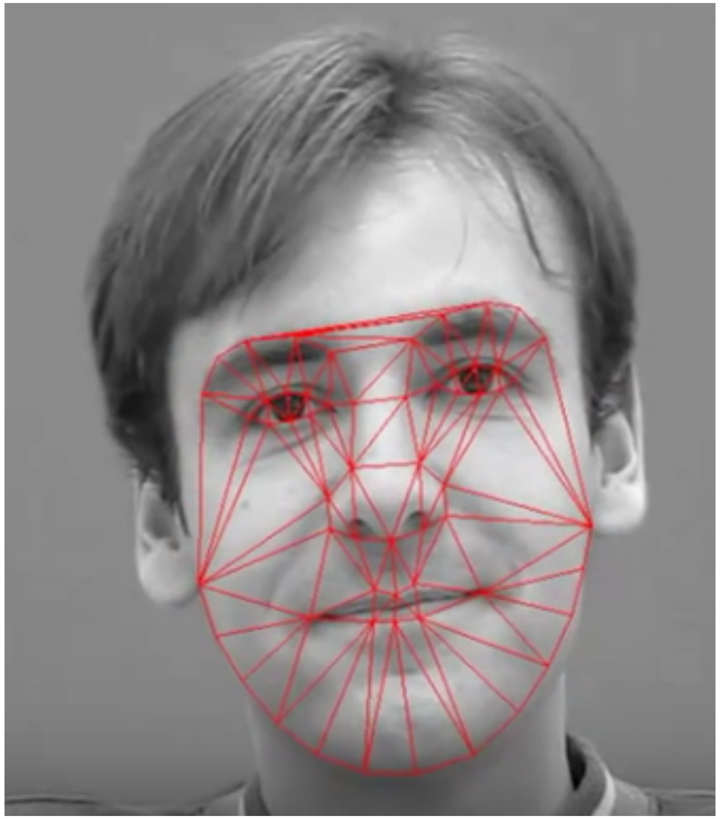
First two modes of gray-level variation



First four modes of appearance variation

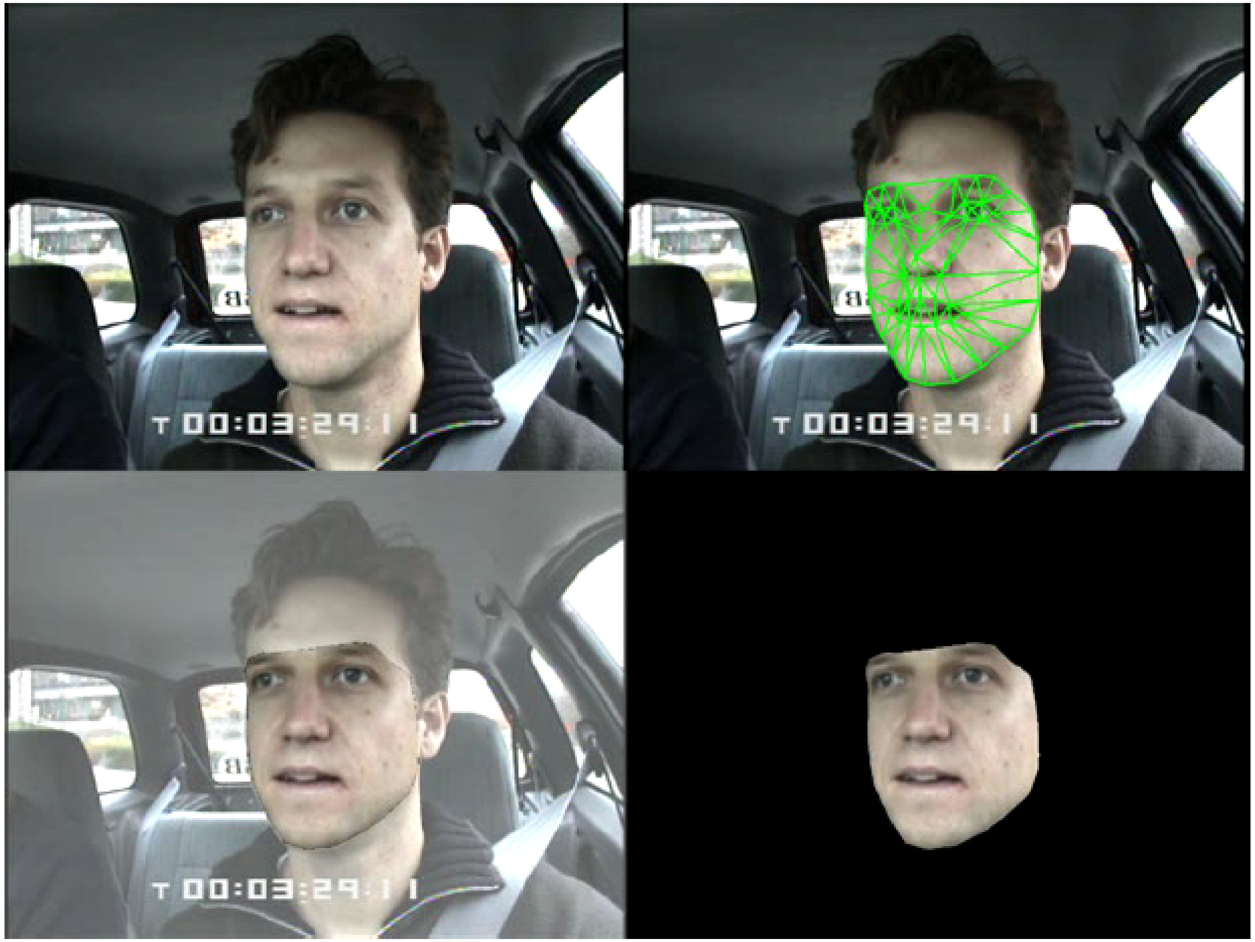
Active appearance model
Search results

Active shape models
Face tracking



https://www.youtube.com/watch?v=I3YsqHCQB4k

Active appearance model
Face tracking



CMU group: I. Matthews, S. Baker, R. Gross
(230 Frames per second, 2004)

Active shape models
Tracking for animation

https://www.youtube.com/watch?v=w_Z7YUyCEGE

In this lecture we have covered:

- Statistical shape models
  - What they are
  - What they are used for
- Principal Component Analysis (PCA)
  - What it is
  - How it is used for shape modelling
- Point distribution models
- Active shape models
- Active appearance models

Next lecture:

- Advanced topics part 2: Beyond colour – multispectral imaging
    - Multispectral imaging: why colour does not always tell the full story
    - Multispectral imaging: general principles
    - How to acquire multispectral images
    - Analysis
    - Applications

Further reading and experimentation

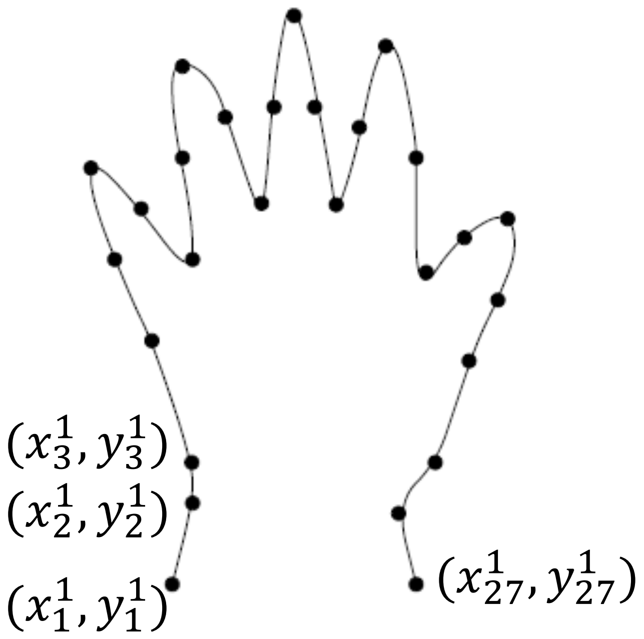
- **Principal Component Analysis**
- https://georgemdallas.wordpress.com/2013/10/30/principal-component-analysis-4-dummies-eigenvectors-eigenvalues-and-dimension-reduction/

- **Overview of many topics related to this lecture**
- http://personalpages.manchester.ac.uk/staff/Timothy.F.Cootes/tfc_research.html

- **Point distribution model**
- http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/COOTES/pdms.html

Statistical shape model
Mathematical details

Statistical shape model
Training



Input:

- M training samples
- N points each

$$\mathbf{x}^i = (x_1^i, y_1^i, x_2^i, y_2^i, \ldots, x_N^i, y_N^i)'$$

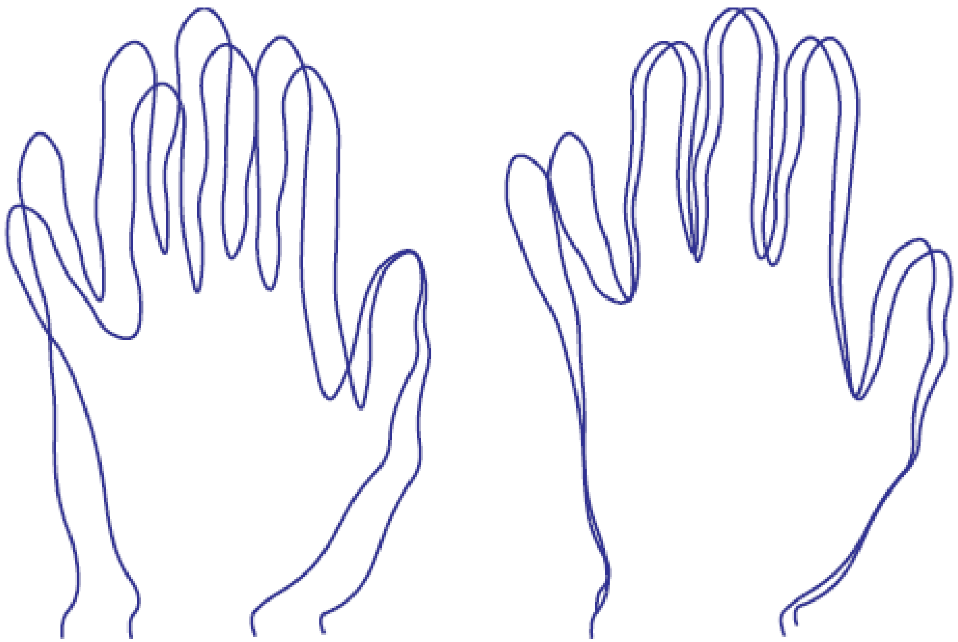
i=1 to M

Procedure:

- Rigidly align all shapes
- Calculate the mean and the covariance matrix
- PCA (eigen analysis) — find principal modes

Principal **C**omponent **A**nalysis

Statistical shape model
Training: align two shapes

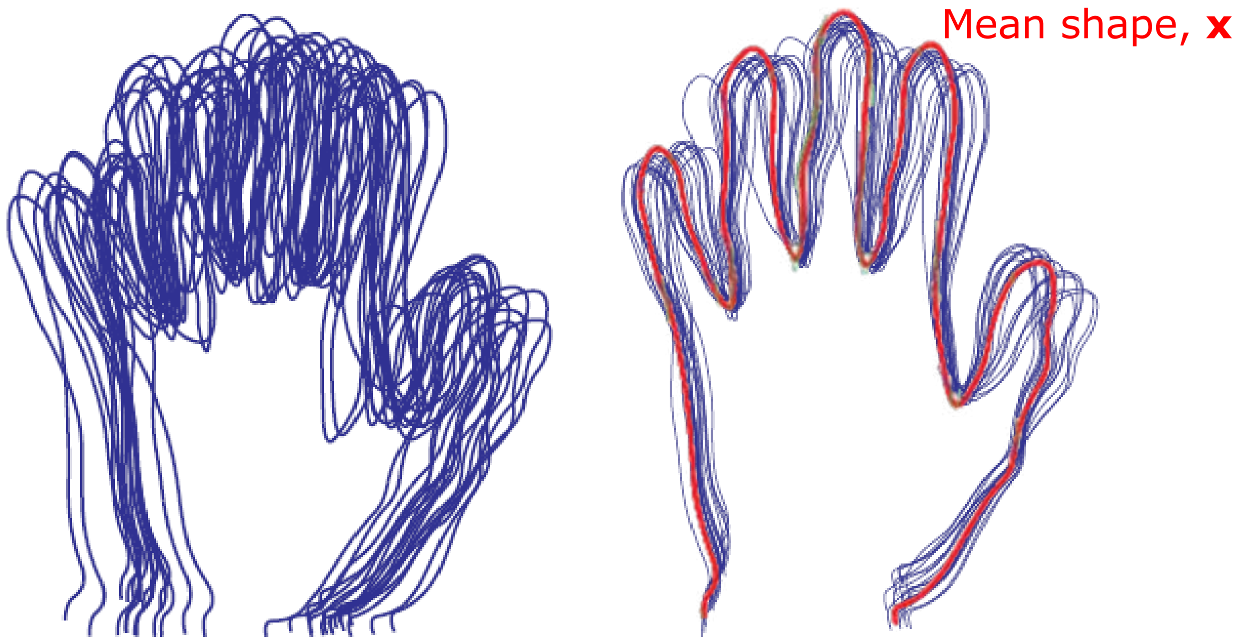


before alignment   after alignment

$$\mathbf{x}^{(1)} = (x_1^{(1)}, y_1^{(1)}, x_2^{(1)}, y_2^{(1)}, \ldots, x_N^{(1)}, y_N^{(1)})^T$$
$$\mathbf{x}^{(2)} = (x_1^{(2)}, y_1^{(2)}, x_2^{(2)}, y_2^{(2)}, \ldots, x_N^{(2)}, y_N^{(2)})^T$$

Find a transformation (rotation, translation, scaling) of $\mathbf{x}^{(2)}$

Statistical shape model
Training: align many shapes



Before alignment　　　After alignment

We have obtained M (mutually aligned) boundaries $\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2, \ldots, \hat{\mathbf{x}}^M$ and the mean $\bar{\mathbf{x}}$.

- Align each $\mathbf{x}^i$ with $\mathbf{x}^1$, for $i = 2, 3, \ldots, M$, obtaining $\{\mathbf{x}^1, \hat{\mathbf{x}}^2, \hat{\mathbf{x}}^3, \ldots, \hat{\mathbf{x}}^M\}$.
- Calculate the mean $\bar{\mathbf{x}} = [\bar{x}_1, \bar{y}_1, \bar{x}_2, \bar{y}_2, \ldots, \bar{x}_N, \bar{y}_N]$ of the aligned shapes $\{\mathbf{x}^1, \hat{\mathbf{x}}^2, \hat{\mathbf{x}}^3, \ldots, \hat{\mathbf{x}}^M\}$.

$$\bar{x}_j = \frac{1}{M}\sum_{i=1}^{M} \hat{x}_j^i \qquad \text{and} \qquad \bar{y}_j = \frac{1}{M}\sum_{i=1}^{M} \hat{y}_j^i \ .$$

Statistical shape model
Training: align many shapes

We have M boundaries $\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2, \ldots, \hat{\mathbf{x}}^M$ and the mean $\bar{\mathbf{x}}$.

- Variation from the mean for each training shape

$$\delta \mathbf{x}^i = \hat{\mathbf{x}}^i - \bar{\mathbf{x}} \, .$$

- Covariance matrix $\mathbf{S}$ $(2N \times 2N)$

$$\mathbf{S} = \frac{1}{M} \sum_{i=1}^{M} \delta \mathbf{x}^i \, (\delta \mathbf{x}^i)^T$$

Now carry out Principal Component Analysis (PCA)

Principal Component Analysis

- Eigen decomposition

Eigenvalues

Covariance matrix $\rightarrow$ $$\mathbf{S}\mathbf{p}_i = \lambda_i \mathbf{p}_i$$ $\leftarrow$ Principal components

$$P = \left[\mathbf{p}^1 \mathbf{p}^2 \mathbf{p}^3 \ldots \mathbf{p}^{2N}\right]$$

Principal components $\mathbf{p_i}$ define a new coordinate system. The are mutually orthogonal, so any shape vector **x** can be represented as

Point distribution model $\rightarrow$ $$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}\,\mathbf{b}$$

Mean shape

- Eigen decomposition

$$\mathbf{S}\mathbf{p}_i = \lambda_i \mathbf{p}_i$$
$$P = \left[\mathbf{p}^1 \mathbf{p}^2 \mathbf{p}^3 \ldots \mathbf{p}^{2N}\right]$$

We know eigenvalues λ_i are real because **S** is symmetric, positive definite. Eigenvectors (principal components) $\mathbf{p}_i$ are orthogonal, so **P** is a basis and any vector **x** can be represented as

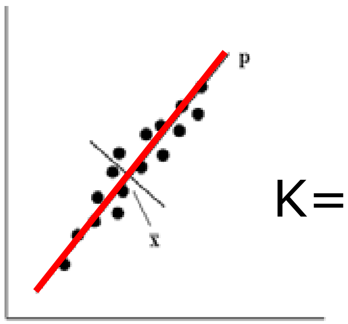
$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}\,\mathbf{b}$$

Point distribution model

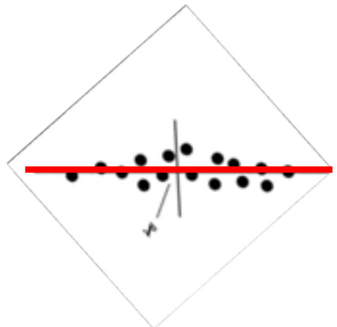
- Order eigenvectors $\mathbf{p}_i$ and eigenvalues λ_i such that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \lambda_{2N}$. Most changes are then described by the first few eigenvectors.
- Consider only K largest eigenvalues.

$$\text{Approximation} \quad \mathbf{x} \approx \bar{\mathbf{x}} + \mathbf{P}_K\, \mathbf{b}_K$$
$$\text{with} \quad \mathbf{P}_K = \left[\mathbf{p}^1 \mathbf{p}^2 \mathbf{p}^3 \ldots \mathbf{p}^K\right]$$
$$\mathbf{b}_t = \left[b_1, b_2, \ldots, b_K\right]^T$$

Choose the smallest K, such that $\sum_{i=1}^{K} \lambda_i \geq \alpha \sum_{i=1}^{N} \lambda_i$.



K=2



K=1

Point distribution model
Generating new shapes

- **Input:** M non-aligned boundaries $\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^M$.
- **Output:** mean $\bar{\mathbf{x}}$ and reduced eigvector matrix $\mathbf{P}_K$
- **New shape generation:**

$$\widetilde{\mathbf{x}} = \bar{\mathbf{x}} + P_K \, \mathbf{b}_K$$

For "well-behaved' shapes

$$-3\sqrt{\lambda_i} \leq b_i \leq 3\sqrt{\lambda_i}$$

K (the number of eigenvalues chosen) defines the number of **modes of variation**