# Evasion Attacks to Graph Neural Networks via Influence Function

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#### Abstract

Graph neural networks (GNNs) have achieved state-of-the-art performance in many graph-related tasks, e.g., node classification. However, recent works show that GNNs are vulnerable to evasion attacks, i.e., an attacker can slightly perturb the graph structure to fool GNN models. Existing evasion attacks to GNNs have several key drawbacks: 1) they are limited to attack two-layer GNNs; 2) they are not efficient; or/and 3) they need to know GNN model parameters.

We address the above drawbacks in this paper and propose an influence-based evasion attack against GNNs. Specifically, we first introduce two influence functions, i.e., feature-label influence and label influence, that are defined on GNNs and label propagation (LP), respectively. Then, we build a strong connection between GNNs and LP in terms of influence. Next, we reformulate the evasion attack against GNNs to be related to calculating label influence on LP, which is applicable to multi-layer GNNs and does not need to know GNN model. We also propose an efficient algorithm to calculate label influence. Finally, we evaluate our influence-based attack on three benchmark graph datasets. Our experimental results show that, compared to state-of-the-art attack, our attack can achieve comparable attack performance, but has a 5-50x speedup when attacking two-layer GNNs. Moreover, our attack is effective to attack multi-layer GNNs.

# 1 Introduction

Learning with graph data, such as social networks, biological networks, has drawn continuous attention recently. Moreover, graph neural network (GNN) has become the mainstream methodology for representation learning on graphs. GNN was first introduced in [13], which extended conventional neural network to process graph data. Then, various GNN methods have been proposed and achieved state-of-the-art performance in many graph-related tasks such as node classification [10, 18, 23], graph classification [9, 8], link prediction [25], etc.

However, recent works [5, 28, 21] demonstrate that GNNs are also vulnerable to evasion attacks. In evasion attacks, given a target node and a trained GNN model, an attacker can

slightly perturb the graph structure<sup>1</sup> (e.g., add new edges or delete existing edges) to make the GNN model misclassify the target node. Existing attacks can be roughly classified as optimization-based attacks [28, 22, 21] and reinforcement learning (RL)-based attacks [5, 16]. We focus on optimization-based attacks in this paper, as they are more effective. Specifically, optimization-based attacks first formulate the evasion attack as a binary optimization problem, which is challenging to solve, and then propose approximate algorithms to solve a relaxed optimization problem. Although achieving promising attack performance, existing optimization-based attacks have several limitations: First, they are mainly designed to attack two-layer GNNs, and cannot be applied to multi-layer GNNs. Second, they are not efficient, as they involve multiplying GNN model parameters. Third, they need to know GNN model parameters, which is unrealistic in many real-world applications, e.g., when GNN models are confidential due to their commercial value. Thus, the practicability of these attacks could be limited.

In this paper, we aim to address the above limitations. To this end, we propose an optimization-based evasion attack against GNNs based on influence function, which is a completely different perspective from existing works. Specifically, we first introduce two influence functions, i.e., feature-label influence and label influence, defined on GNNs and label propagation (LP) [27], respectively. Then, we prove that our label influence defined for LP is equivalent to feature-label influence for a particular well-known type of GNN, called Graph Convolutional Network (GCN) [10] (and its linearized version Simple Graph Convolutional (SGC) [20]). Based on this connection, we reformulate the evasion attack against GNNs to be related to calculating label influence on LP. As our influences are not specially designed for two-layer GNNs, our attack is applicable to attack multi-layer GNNs. Moreover, label influence can be computed directly and we design an efficient algorithm to compute it. Furthermore, our influence-based attack does not need to know the GNN model parameters, it is thus a more practical attack. Finally, we evaluate our attack against GCN/SGC on three benchmark graph datasets. Compared to the state-of-the-art attack against two-layer GCN/SGC, our attack can achieve comparable attack performance but has a 5-50x speedup. Our attack is also effective to attack multi-layer GCN/SGC. For instance, our attack achieves a 93% attack success rate, when perturbing 4 edges per target node on Cora. Last, our attack shows promising transferability to attack other GNNs.

Our contributions can be summarized as follows:

- We propose an evasion attack to GNNs based on influence function, which is a completely new perspective.
- Our attack is effective, efficient, and practical.
- Our attack has promising transferability.

# 2 Background and Problem Definition

#### Graph Neural Network

Let  $G = (\mathcal{V}, \mathcal{E}, \mathbf{X})$  be a graph, where  $u \in \mathcal{V}$  is a node,  $(u, v) \in \mathcal{E}$  is an edge between u and v, and  $\mathbf{X} = [\mathbf{x}_1; \mathbf{x}_2; \cdots; \mathbf{x}_n] \in \mathbb{R}^{n \times d}$  is the node feature matrix. We denote  $\mathbf{A}$  as the adjacency

<sup>&</sup>lt;sup>1</sup>An attacker can also perturb node features to perform the attack. However, graph structure perturbation is shown to be more effective than node feature perturbation.

matrix, where  $A_{u,v} = 1$ , if  $(u,v) \in \mathcal{E}$  and  $A_{u,v} = 0$ , otherwise; Moreover, we denote  $d_u$  and  $\Gamma_u$  as u's node degree and the neighborhood set of u (including self-loop (u,u)). We consider GNNs for node classification in this paper. In this context, each node  $u \in \mathcal{V}$  has a label  $y_u$  from a label set  $\mathcal{Y} = \{1, 2, \dots, C\}$ . Given a set of  $\mathcal{V}_L \subset \mathcal{V}$  labeled nodes  $\{(\mathbf{x}_u, y_u)\}_{u \in \mathcal{V}_L}$  as the training set, GNN for node classification is to take the graph G and labeled nodes as input and learn a node classifier that maps each node  $u \in \mathcal{V} \setminus \mathcal{V}_L$  to a class  $y \in \mathcal{Y}$ . In this paper, we focus on Graph Convolutional Network (GCN) [10], a widely used type of GNN, and its special case Simple Graph Convolution (SGC) [20].

**GCN.** GCN is motivated by spectral graph convolution [7]. Suppose GCN has K layers. We denote node v's representation in the k-th layer as  $\mathbf{h}_{v}^{(k)}$ , where  $\mathbf{h}_{v}^{(0)} = \mathbf{x}_{v}$ . Then, GCN has the following form to update node representation:

$$\mathbf{h}_v^{(k)} = \text{ReLU}\Big(\mathbf{W}^{(k)}\Big(\sum_{u \in \Gamma_v} d_u^{-1/2} d_v^{-1/2} \mathbf{h}_u^{(k-1)}\Big)\Big). \tag{1}$$

A node v's final representation  $\mathbf{h}_v^{(K)} \in \mathbb{R}^{|\mathcal{Y}|}$  can capture the structural information of all nodes within v's K-hop neighbors. Moreover, the final node representations of training nodes are used for training the node classifier. Specifically, let  $\Theta = \{\mathbf{W}^{(1)}, \mathbf{W}^{(2)}, \cdots, \mathbf{W}^{(K)}\}$  be the model parameters and v's output be  $f_{\Theta}(\mathbf{A})_v = \operatorname{softmax}(\mathbf{h}_v^{(K)}) \in \mathbb{R}^{|\mathcal{Y}|}$ , where  $f_{\Theta}(\mathbf{A})_{v,y}$  indicates the probability of node v being class v. Then, v are learnt by minimizing the cross-entropy loss on the outputs of the training nodes v, i.e.,

$$\Theta^* = \arg\min_{\Theta} - \sum_{v \in \mathcal{V}_L} \ln f_{\Theta}(\mathbf{A})_{v,y}. \tag{2}$$

With the learnt  $\Theta^*$ , we can predict the label for each unlabeled nodes  $u \in \mathcal{V} \setminus \mathcal{V}_L$  as  $\hat{y}_u = \arg \max_i f_{\Theta^*}(\mathbf{A})_{u,y}$ .

**SGC.** SGC is a linearized version of GCN. Specifically, its node representation is as follows:

$$\mathbf{h}_{v}^{(k)} = \mathbf{W}^{(k)} \left( \sum_{u \in \Gamma_{v}} d_{u}^{-1/2} d_{v}^{-1/2} \mathbf{h}_{u}^{(k-1)} \right). \tag{3}$$

SGC has shown to have comparable node classification performance with GCN, but is much more efficient than GCN.

### Label Propagation

Label Propagation (LP) is a conventional semi-supervised node classification method without training. The key idea behind LP is that two nodes having a high similarity (e.g., connected nodes in a graph) are likely to have the same label. Thus, LP iteratively propagates labels among the graph to unlabeled nodes based on node-pair similarity. Let  $\mathbf{y}_v \in \mathbb{R}^{|\mathcal{Y}|}$  be node v's initial label vector (For notation reason, one should note that  $y_v$  is v's categorical label). For instance,  $\mathbf{y}_v$  can be v's one-hot label vector if v is a labeled node, and  $\mathbf{y}_v = \mathbf{0}$ , otherwise. Then, LP is formulated as follows:

$$\mathbf{y}_{v}^{(k)} = \sum_{u \in \Gamma_{v}} d_{u}^{-1/2} d_{v}^{-1/2} \mathbf{y}_{u}^{(k-1)}, \quad \mathbf{y}_{v}^{(0)} = \mathbf{y}_{v}. \tag{4}$$

With K iterations, an unlabeled node u is predicted to be class c, if  $c = \arg\max_{i} y_{u,i}^{(K)}$ .

### **Problem Definition**

We consider targeted evasion attacks<sup>2</sup> to GNNs. Suppose we are given a trained GNN model  $f_{\Theta^*}$  for node classification. We assume v is the target node and c is the target label. We consider an attacker can perturb the graph structure (i.e., add new edges to or delete existing edges from the graph) in order to make  $f_{\Theta^*}$  misclassify the target node v to be the target label c. We call the modified edges by the attacker as attack edges. In particular, we consider a practical direct attack [28], where an attacker can only modify the connection status between v and other nodes in the graph, while cannot modify the connection status among other nodes. We denote the perturbed graph as  $\tilde{G}$  (with the perturbed adjacency matrix  $\tilde{\mathbf{A}}$ ) after the attack and the attack budget as  $\Delta$ , i.e., at most  $\Delta$  edges can be perturbed for the target node. Then, the objective function of targeted evasion attacks to GNNs is formally defined as:

$$\max_{\tilde{\mathbf{A}}_{v}} \left( f_{\Theta^{*}}(\tilde{\mathbf{A}})_{v,c} - f_{\Theta^{*}}(\tilde{\mathbf{A}})_{v,y_{v}} \right) \Leftrightarrow \max_{\tilde{\mathbf{A}}_{v}} \left( [\tilde{\mathbf{h}}_{v}^{(K)}]_{c} - [\tilde{\mathbf{h}}_{v}^{(K)}]_{y_{v}} \right),$$

$$s.t., \quad \sum_{s} |\tilde{A}_{v,s} - A_{v,s}| \leq \Delta, \tag{5}$$

where  $\tilde{\mathbf{h}}_{v}^{(K)}$  is v's representation on the perturbed graph  $\tilde{G}$ .

A target node is called a success to attack the GNN model if the value of the attack's objective function is larger than 0, under the attack budget. Note that Equation (5) is a binary optimization problem and is challenging to solve in practice. [28] proposed an optimization-based attack method, called Nettack, against two-layer GCN. Specifically, Nettack attacked a substitute GNN model (actually SGC) that removed the ReLU activation function in GCN. Nettack has achieved state-of-the-art attack performance. However, it is inefficient and can only attack two-layer GCN/SGC.

# 3 Proposed Influence-based Evasion Attack

In this section, we propose our evasion attack against GNNs via influence function. Specifically, we first define two influence functions associated with GNNs and LP, respectively. Then, we build the strong connection between GNNs (specifically GCN and SGC) and LP via influence. Next, we reformulate the attack's objective function related to final node representation on GNNs to be related to label influence on LP. Finally, we design an efficient algorithm to calculate label influence and implement our attack.

## Connection between GNNs and LP via Influence

We first define two new influence functions. Given two nodes u and v, an influence of u on v indicates how the output (e.g., final node representation in GNNs or estimated node label in LP) of v changes if the input of u is slightly perturbed. Inspired by [11, 24], we define the following feature-label influence on GNN and label influence on LP, respectively.

<sup>&</sup>lt;sup>2</sup>As untargeted attacks are less powerful than targeted attacks, we only consider targeted attacks in this paper for simplicity.

**Feature-label influence.** The feature-label influence of node u on node v associated with u's label  $y_u$  on a K-layer GNN is defined as follows:

$$I_{fl}(v, u; K) = \left\| \left[ \frac{\partial \mathbf{h}_v^{(K)}}{\partial \mathbf{h}_u^{(0)}} \cdot \mathbf{h}_u^{(0)} \right]_{y_u} \right\|_1 = \mathbf{1}_{y_u}^T \cdot \frac{\partial \mathbf{h}_v^{(K)}}{\partial \mathbf{h}_u^{(0)}} \cdot \mathbf{h}_u^{(0)}, \tag{6}$$

where  $\mathbf{1}_{y_u} = [y_1, y_2, \dots, y_n]$  is an indicator vector where  $y_i = 1$  if i = u and  $y_i = 0$ , otherwise.  $\mathbf{h}_u^{(0)} = \mathbf{x}_u$  is u's input feature.

**Label influence.** The label influence of node u on node v after K iterations of label propagation is defined as follows:

$$I_l(v, u; K) = \frac{\partial y_v^{(K)}}{\partial y_u^{(0)}}. (7)$$

Then, we have the following theorem to reveal the connection between GNN and LP in terms of influence.

**Theorem 1.** If the GNN is a GCN/SGC, then:

$$I_{fl}(v, u; K) = C \cdot I_l(v, u; K), \tag{8}$$

where C is constant related to GNN model parameters and u's input feature.

Theorem 1 demonstrates that if a GNN is GCN/SGC, then the feature-label influence on GNN and label-influence on LP are identical.

#### Reformulating Evasion Attacks via Label Influence

Based on our influence functions and Theorem 1, we can reformulate the targeted evasion attack's objective function in Equation (5) to be related to label influence on LP.

First, according to Equation (14) in the Appendix, the target node v's final node representation  $\tilde{\mathbf{h}}_v^{(K)}$  learnt on the perturbed graph can be expressed as follows:

$$\tilde{\mathbf{h}}_{v}^{(K)} = \sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}, \tag{9}$$

where  $\tilde{\mathbf{\Lambda}}_v^{(K)}$  is the node set containing v's neighbors within K-hop on the perturbed graph.

Then, the attack's objective function in Equation (5) is equivalent to the following objective function:

$$\max_{\tilde{\mathbf{A}}_{v}} \left( \left[ \sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)} \right]_{c} - \left[ \sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)} \right]_{y_{v}} \right) 
s.t., \quad \sum_{s} |\tilde{A}_{v,s} - A_{v,s}| \leq \Delta,$$
(10)

Finally, based on the following Assumption 1 and Theorem 1, we reach a theorem that reformulates the targeted evasion attack's objective function via label influence.

**Assumption 1.** Given a target node v and a target label c. We assume that any node u, within the K-hop neighbor of v, has a negligible feature-label influence on v if u is not a label-c node. Formally,

$$\left[\frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}\right]_{c} \approx 0, \quad \forall u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}, \ y_{u} \neq c.$$

$$(11)$$

**Theorem 2.** Let  $\tilde{I}_l(v, u; K)$  be the label influence of node u on the target node v with K iterations of LP after the attack. Then, the attack's objective function in Equation (5) equals to the following objective function on label influence:

$$\max_{\tilde{\mathbf{A}}_{v}} \Big( \sum_{\substack{u \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{u} = c}} \tilde{I}_{l}(v, u; K) - \sum_{\substack{z \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{z} = y_{v}}} \tilde{I}_{l}(v, z; K) \Big),$$

$$s.t., \quad \sum_{s} |\tilde{A}_{v,s} - A_{v,s}| \leq \Delta, \tag{12}$$

where  $\tilde{I}_l(v, u; K)$  is defined as:

$$\tilde{I}_{l}(v, u; K) = \sum_{p=1}^{\tilde{\Psi}_{v \to u}} \prod_{l=K}^{1} \tilde{d}_{v_{p}^{l}}^{-\frac{1}{2}} \tilde{d}_{v_{p}^{l-1}}^{-\frac{1}{2}}$$

$$\tag{13}$$

where  $\tilde{\Psi}_{v \to u}$  is the total number of paths  $[v_p^K, v_p^{K-1}, \cdots, v_p^1, v_p^0]$  of length K+1 from v to u on the perturbed graph  $\tilde{G}$ , where  $v_p^K = v$  and  $v_p^0 = u$ .  $\tilde{d}_u$  is u's degree on the perturbed graph  $\tilde{G}$  and  $\tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_{p-1}^{l-1}}^{-\frac{1}{2}}$  is the normalized weight of the edge  $(v_p^l, v_p^{l-1})$  in path p in  $\tilde{G}$ .

We have the following observations from Theorem 2.

- Our attack does not need to operate on model parameter  $\Theta^*$ , different from existing attacks that involve the multiplication on  $\Theta^*$ . Thus, our attack can be efficient.
- Our attack can be applied to any K-layer GNN, in contrast to existing attacks that can only attack two-layer GNNs.
- The information our our attack needs to know is the labels of the target node v's within K-hop neighbors, i.e., nodes whose labels are  $y_v$  or c. In practice, if these node labels are unknown, we can estimate labels for these nodes via querying the GNN model, and treat the estimated labels as the true labels. Therefore, our attack requires the least knowledge, compared to existing attacks.

Next, we show how to fast calculate the label influence and design our influence-based targeted evasion attack.

#### Efficient Calculation for Label Influence

According to Theorem 2, the attack's goal is to select the minimum set of nodes such that when changing the connection status between the target node v and these selected nodes, the difference between the two label influence terms will be maximized. Observing Equation (12), we note that the two label influence terms are defined on two sets of nodes: a set of nodes have

### Algorithm 1 Our influence-based targeted evasion attack

**Input:** Adjacency matrix **A**, layer K, target node v, target label c, attack budget  $\Delta$ . **Output:** v attacks successfully or not.

```
1: AE \leftarrow 0, success \leftarrow False, \tilde{\mathbf{A}} \leftarrow \mathbf{A};
 2: \mathcal{N}_A, \mathcal{N}_B \leftarrow \text{Find two candidate node sets from } \mathbf{A}.
 3: for a \in \mathcal{N}_A do
            \Delta I_A(a) = d_v^{-\frac{1}{2}} d_a^{-\frac{1}{2}} \cdot LabelInfluence(K-1, a, 1)
 5: end for
 6: for b \in \mathcal{N}_B do
            \Delta I_B(b) = d_v^{-\frac{1}{2}} d_b^{-\frac{1}{2}} \cdot LabelInfluence(K-1,b,1)
 9: while AE < \Delta do
            C_A \leftarrow LabelInfluence(K, v, 1); // d_v \leftarrow d_v + 1
10:
            C_B \leftarrow LabelInfluence(K, v, 1); // d_v \leftarrow d_v - 1
11:
            u^* \leftarrow \operatorname{argsort}(\{C_A + \Delta I_A\} \cup \{C_B - \Delta I_B)\};
12:
13:
            if u^* \in \mathcal{N}_A then
                 \tilde{\mathbf{A}} \leftarrow \mathbf{A} + (v, u^*);
14:
            else
15:
                 \tilde{\mathbf{A}} \leftarrow \mathbf{A} - (v, u^*);
16:
            end if
17:
            AE \leftarrow AE + 1
18:
19: end while
20: if (f_{\Theta^*}(\mathbf{A})_{v,c} - f_{\Theta^*}(\mathbf{A})_{v,y_v}) > 0 then
             success \leftarrow True;
22: end if
23: return success
```

the same label c as the target label, and a set of nodes have the same label  $y_v$  as the target node's label. Intuitively, if we add an edge between v and a label-c node, we can make v be close to label c; and if we remove an edge between v and a label- $y_v$  node, we can make v away from label  $y_v$ . Thus, our idea to solve Equation (12) is as follows. First, we define a candidate set  $\mathcal{N}_A$  which contains label-c nodes that are not connected with v in the clean graph, as well as defining a candidate set  $\mathcal{N}_B$  which contains label- $y_v$  nodes connected with v in the clean graph. We denote  $\mathcal{S}$  as the final selected nodes from  $\mathcal{N}_A$  and  $\mathcal{N}_B$ , and initialize  $\mathcal{S} = \{\}$ . For each node  $u \in \mathcal{N}_A \cup \mathcal{N}_B \setminus \mathcal{S}$ , we change the connection status between v and u and compute the gap between two label influence terms. Next, we record the node  $u^*$  that obtains the largest positive gap. Then, we modify the connection status between v and  $u^*$ , calculate the value of the attack's objective function, and update  $\mathcal{S} = \mathcal{S} \cup \{u^*\}$ . We repeat above steps at most  $\Delta$  times and break if the value of attack's objective function is bigger than 0. Finally, we have the attack edges  $\{(v, u^*), u^* \in \mathcal{S}\}$ .

However, note that when modifying the connection status between v and  $u^*$ , the normalized weight for all edges containing  $u^*$  in all paths  $\tilde{\Psi}_{v\to u}$  in Equation (13) should be recalculated. When the candidate set has a large size or/and the number of recalculated edge weights is large, calculating the exact label influence will have a large computational complexity. Here, we propose an approximate algorithm to efficiently compute the label influences. First, we have the following two observations:

## Algorithm 2 Efficient calculation for label influence via depth first search

```
1: I_{y_v} \leftarrow 0, I_c \leftarrow 0
 2: function LabelInfluence(K, p, s)
           if K = 0 then
 3:
                I_{y_v} \leftarrow I_{y_v} + s \cdot y_{v,y_v};
I_c \leftarrow I_c + s \cdot y_{v,c};
 4:
 5:
 6:
 7:
           end if
           for u \in \Gamma_p do
 8:
                w = d_n^{-\frac{1}{2}} d_u^{-\frac{1}{2}};
 9:
                LabelInfluence(K-1, u, s \cdot w);
10:
           end for
11:
12: end function
13: return I_c - I_{y_i}
```

- When adding an edge between v and  $a \in \mathcal{N}_A$ , we will have new paths  $\{v, a, \dots, u\}$  from v to u passing through a. We denote the nodes in the new paths within v's K-hop neighbors as  $\Delta \mathbf{\Lambda}_v^{(K)}$ , and note that these nodes are within a's (K-1)-hop neighbors in the clean graph, i.e.,  $\Delta \mathbf{\Lambda}_v^{(K)} \subset \mathbf{\Lambda}_a^{(K-1)}$ . Moreover,  $\tilde{\mathbf{\Lambda}}_v^{(K)} = \mathbf{\Lambda}_v^{(K)} \cup \mathbf{\Lambda}_a^{(K-1)}$ .
- When deleting an edge between v and  $b \in \mathcal{N}_B$ , we will remove existing paths  $\{v, b, \cdots, u\}$  from v to u passing through b. For notation simplicity, we also denote the deleted nodes within v's K-hop neighbors as  $\Delta \mathbf{\Lambda}_v^{(K)}$ , and these nodes are within b's (K-1)-hop neighbors in the clean graph, i.e.,  $\Delta \mathbf{\Lambda}_v^{(K)} \subset \mathbf{\Lambda}_b^{(K-1)}$ . Moreover,  $\tilde{\mathbf{\Lambda}}_v^{(K)} = \mathbf{\Lambda}_v^{(K)} \setminus \mathbf{\Lambda}_b^{(K-1)}$ .

Based on the above observations, we can split each label influence term in Equation (12) into two parts: an approximate constant label influence and an approximate label influence defined on the (K-1)-hop neighbors for each node in  $\mathcal{N}_A \cup \mathcal{N}_B$ . We first consider adding an edge between u and  $a \in \mathcal{N}_A$ . Specifically, we have

$$\begin{split} &\sum_{\substack{u \in \Lambda_v^{(K)} \\ y_u = c}} \tilde{I}_l(v, u; K) - \sum_{\substack{z \in \tilde{\Lambda}_v^{(K)} \\ y_z = y_v}} \tilde{I}_l(v, z; K) \\ &= [\sum_{\substack{u \in \Lambda_v^{(K)} \\ y_u = c}} \tilde{I}_l(v, u; K) + \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \subset \Lambda_a^{(K-1)} \\ y_u = c}} \tilde{I}_l(v, z; K)] - [\sum_{\substack{z \in \Lambda_v^{(K)} \\ y_z = y_v}} \tilde{I}_l(v, z; K) + \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \subset \Lambda_a^{(K-1)} \\ y_z = y_v}} \tilde{I}_l(v, z; K)] \\ &= [\sum_{\substack{u \in \Lambda_v^{(K)} \\ y_u = c}} \tilde{I}_l(v, u; K) - \sum_{\substack{z \in \Lambda_v^{(K)} \\ y_z = y_v}} \tilde{I}_l(v, z; K)] + [\sum_{\substack{u \in \Delta \Lambda_v^{(K)} \\ y_u = c}} \tilde{I}_l(v, u; K) - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \tilde{I}_l(v, z; K)] \\ &= [\sum_{\substack{u \in \Lambda_v^{(K)} \\ y_u = c}} \sum_{p=1}^{\psi_{v \to u}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} \right] + \sum_{\substack{u \in \Delta \Lambda_v^{(K)} \\ y_u = c}} \sum_{p=1}^{\psi_{v \to u}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} \right] + \sum_{\substack{u \in \Delta \Lambda_v^{(K)} \\ y_u = c}} \sum_{p=1}^{\psi_{v \to u}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Delta \Lambda_v^{(K)} \\ y_z = y_v}} \sum_{p=1}^{\psi_{v \to z}} \prod_{l = K} \tilde{d}_{v_p^l}^{-\frac{1}{2}} \tilde{d}_{v_p^{l-1}}^{-\frac{1}{2}} \right] + \sum_{\substack{u \in \Delta \Lambda_v^{(K)} \\ y_u = c}} \sum_{\substack{u \in \Delta \Lambda_v^{(K)} \\ y$$

where  $\Psi_{v\to u}$  and  $\Psi_{v\to z}$  are paths in the clean graph. Note that the degree of certain nodes in these paths increases by 1 due to the added edge. Here, we assume all nodes except the target node v do not change the node degree. Therefore  $C_A$  is constant and only need to be

Table 1: Dataset statistics.

Dataset	#Nodes	#Edges	#Features	#Classes
Cora	2,708	5,429	1,433	7
Citeseer	3,327	4,732	3,703	6
Pubmed	19,717	44,338	500	3

calculated once.  $\Psi'_{v\to u}$  and  $\Psi'_{v\to z}$  are paths in the perturbed graph.  $\sum_{u\in\Delta\Lambda_v^{(K)},y_u=c}\tilde{I}_l(v,u;K)$  and  $\sum_{z\in\Delta\Lambda_v^{(K)},y_z=y_v}\tilde{I}_l(v,z;K)$  represent the label influence of the label-c nodes and label- $y_v$  nodes within the (K-1)-hop neighbors of a after adding the edge (v,a), respectively. We can calculate  $\Delta I_A(a)$  for each attack edge (v,a)  $\forall a\in\mathcal{N}_A$ .

Similarly, when removing an edge between u and  $b \in \mathcal{N}_B$ , we replace + with - and assume all nodes' degree do not change, except that the target node's degree decreases by 1. We denote the constant label influence and the variant label influence as  $C_B$  and  $\Delta I_B(b)$ , respectively. We can calculate  $\Delta I_B(b)$  for each attack edge  $(v,b) \ \forall z \in \mathcal{N}_B$ .

Algorithm 1 and Algorithm 2 illustrate how we calculate the label influences and implement our attack, respectively. Figure 8 in the Appendix also shows an example for exact and approximate label influence calculation when K = 2.

# 4 Evaluation

### Experimental setup

**Datasets.** Following existing works [28], we use three benchmark citation graphs (i.e., Cora, Citeseer, and Pubmed) [14] to evaluate our attack. In these graphs, each node represents a documents and each edge indicates a citation between two documents. Each document treats the bag-of-words feature as the node feature vector, and has a label as well. Table 1 shows basic statistics of these citation graphs.

Training nodes and target nodes. We use the training nodes to train GNN models and use the target nodes to evaluate attacks against the trained GNN models. Specifically, we randomly sample 20 nodes from each class as the training nodes; and we randomly sample 100 nodes that are correctly classified by each GNN model as the target nodes. Similar to Nettack [28], for each target node, we choose the predicted label by the GNN model with a second largest probability as the target label.

Compared attacks. We compare our influence-based attack with the state-of-the-art Nettack [28] for attacking two particular GNNs: GCN and SGC. Note that Nettack is only applicable to attack two-layer GCN/SGC. When computing the label influence, our attack needs to know the labels of unlabeled nodes in the graph. When our attack knows the true labels, we denote it as Ours-KL. When the true labels are unknown, our attack first queries the learnt GCN/SGC model to estimate labels for unlabeled nodes and then uses the estimated labels as the true labels. We denote this variant as Ours-UL. As a comparison, we also test our attack that is implemented based on exact label influence calculation, and denote the corresponding two methods with known and unknowns labels as Ours (exact)-KL and Ours (exact)-UL, respectively.

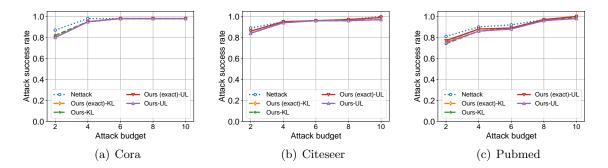


Figure 1: Attack success rate vs. attack budget per target node on a two-layer GCN of all compared attacks on the three graphs.

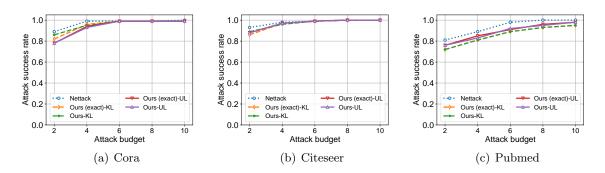


Figure 2: Attack success rate vs. attack budget per target node on a two-layer SGC of all compared attacks on the three graphs.

Evaluation metric. We adopt attack success rate and running time as the metrics to evaluate compared attacks. Given an attack budget  $\Delta$ , attack success rate is the fraction of target nodes that are misclassified to be the target label when the number of attack edges per target node is at most  $\Delta$ . Running time is reported on average across all target nodes.

**Implementation.** We train all GNNs using the publicly available source code. We test Nettack using the open source code<sup>3</sup>. We implement our attack in PyTorch and the source code is available at<sup>4</sup>. All experiments are conducted on a Linux server with 128GB memory and 32 cores.

#### Experimental results

Results on attacking two-layer GCN/SGC. In this experiment, we compare our attacks with Nettack in terms of effectiveness (i.e., attack success rate) and efficiency (i.e., running time) against two-layer GCN/SGC.

Figure 1 and Figure 2 show the attack success rate of all compared attacks vs. attack budget per target node against GCN and SGC on the three graphs, respectively. Moreover, Figure 3 and Figure 4 show the running time of all attacks vs. attack budget per attack node against

<sup>&</sup>lt;sup>3</sup>https://github.com/danielzuegner/nettack

<sup>&</sup>lt;sup>4</sup>shorturl.at/eioqQ

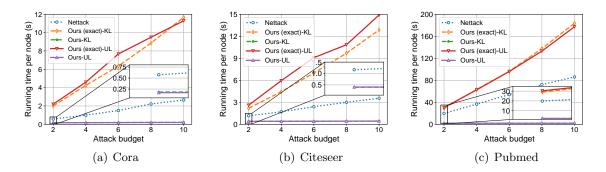


Figure 3: Running time vs. attack budget per target node on two-layer GCN of all compared attacks on the three graphs.

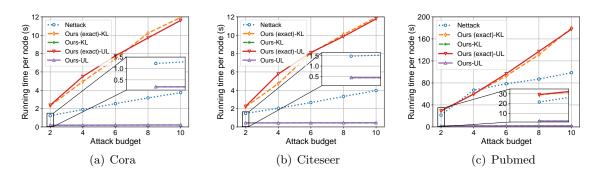


Figure 4: Running time vs. attack budget per target node on two-layer SGC of all compared attacks on the three graphs.

GCN and SGC on the three graphs, respectively. We have the following key observations. 1) Our attacks based on approximate label influence have similar performance with those based on exact label influence, but is much more efficient. Specifically, the difference of the attack success rate between the two is less than 2% in all cases. This shows that our proposed efficient algorithm for label influence calculation is effective enough. Moreover, our attacks based on approximate label influence are 1-2 orders of magnitude more efficient than those based on exact label influence. 2) Our attacks with true labels and with estimated labels have similar performance. Specifically, the difference of the attack success rate between Ours-KL and Our-UL is neglectful, i.e., less than 2% in all cases, and the running time of both Ours-KL and Our-UL are almost the same. One reason is that the trained GNN model has accurate predictions on the unlabeled nodes, and thus most of the estimated labels match the true labels. One should note that Ours-UL knows very limited knowledge about the GNN model and thus it is a very practical attack. 3) Our attacks achieve comparable performance with Nettack. Nettack achieves state-of-the-art performance against two-layer GCN. Our attacks have a slightly lower attack success rate than Nettack when the attack budget is small, e.g., less than 4. When the attack budget is larger than 4, our attacks obtain almost the same performance with Nettack. 4) Our attacks are much more efficient than Nettack. Specifically, our attacks have a 5-50x speedup over Nettack across the three graphs. As the attack budget increases (from 2 to 10) or the graph size increases (from Cora to Pubmed), our attacks achieve better efficiencies than Nettack. The reasons are two-folds. First, Nettack needs to multiply

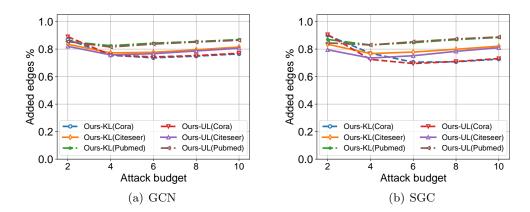


Figure 5: Fraction of added edges among all attack edges generated by our attacks against (a) two-layer GCN and (b) two-layer SGC vs. attack budget per target node on the three graphs.

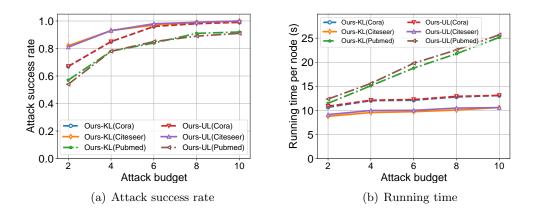


Figure 6: (a) Attack success rate and (b) Running time of our attacks against four-layer GCN on the three graphs.

the GNN model parameters, while our attacks do not. Second, Nettack involves multiplying the node hidden features, while ours is performed by efficiently calculating the label influence. The node hidden features are often high-dimensional, while label influence is related to scalar edge weights.

Analysis of the attack edges. We further analyze the properties of the attack edges. Figure 5(a) and Figure 5(b) show the fraction of the added edges generated by our attacks against two-layer GCN and two-layer SGC, respectively. We have two key observations. First, Ours-KL and Ours-UL generate almost the same fraction of added edges in all attack budgets and all graphs. This again verifies the similar characteristics between Ours-KL and Ours-UL. Second, the fraction of added edges is larger than 0.5 in all cases. This demonstrates that when performing the targeted attack, adding new edges between the target node and the nodes with the target label could be more effective than removing existing edges between the target node and the nodes having the same label as the target node.

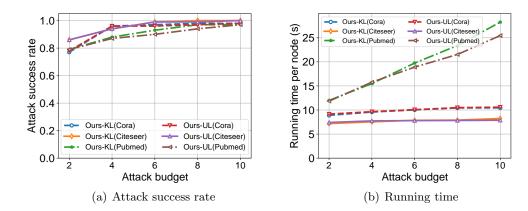


Figure 7: (a) Attack success rate and (b) Running time of our attacks against four-layer SGC on the three graphs.

Table 2: Transferability of our attacks against two-layer GCN to other GNNs on the three graphs. Attack budget per target node is 6.

Dataset	Source	Target				
Cora	GCN	GCN	SGC	GAT	JK-Net	
	No attack	0	0.01	0.03	0.02	
	Ours-KL	0.98	0.82	0.66	0.67	
	Ours-UL	0.98	0.84	0.65	0.70	
Citeseer	GCN	GCN	SGC	GAT	JK-Net	
	No attack	0	0.01	0.01	0.03	
	Ours-KL	0.96	0.78	0.70	0.63	
	Ours-UL	0.96	0.78	0.72	0.63	
Pubmed	GCN	GCN	$\mathbf{SGC}$	GAT	JK-Net	
	No attack	0	0.03	0.04	0.05	
	Ours-KL	0.89	0.80	0.80	0.79	
	Ours-UL	0.88	0.80	0.80	0.77	

Results on attacking multi-layer GNNs. In this experiment, we evaluate our attacks against multi-layer GCN/SGC. Nettack is not applicable in this case. Figure 6(a) and Figure 7(a) in the Appendix show the attack success rate vs. attack budget against four-layer GCN and four-layer SGC on the three graphs, respectively. Similarly, we observe that our attacks with both known label and unknown label are effective and achieve close attack performance. For instance, when the attack budget is 6, our attacks achieve an attack success rate of greater than 90% in almost all cases. Moreover, the difference of the attack success rate between Ours-KL and Ours-UL is less than 2% in all cases.

Moreover, Figure 6(b) and Figure 7(b) in the Appendix show the running time of our attacks vs. attack budget against four-layer GCN and four-layer SGC on the three graphs, respectively. Our attack is efficient. For instance, when the attack budget is 6, it takes our attacks  $\sim 10$ s on average to attack a target node on Cora and Citeseer.

Table 3: Transferability of our attacks against two-layer SGC to other GNNs on the three graphs. Attack budget per target node is 6.

Dataset	Source	Target				
Cora	$\mathbf{SGC}$	SGC	GCN	GAT	JK-Net	
	No attack	0	0.01	0.01	0.02	
	Ours-KL	0.99	0.78	0.73	0.71	
	Ours-UL	0.99	0.75	0.72	0.74	
Citeseer	SGC	SGC	GCN	GAT	JK-Net	
	No attack	0	0.03	0.03	0.04	
	Ours-KL	0.99	0.81	0.79	0.69	
	Ours-UL	0.99	0.84	0.77	0.70	
Pubmed	SGC	SGC	GCN	GAT	JK-Net	
	No attack	0	0.04	0.07	0.07	
	Ours-KL	0.89	0.83	0.76	0.81	
	Ours-UL	0.92	0.83	0.75	0.76	

Transferring our attack to other GNNs. In this experiment, we study the transferability of our attacks, i.e., whether the attack edges generated by our attacks against GCN/SGC can be also effective for other GNNs. Specifically, we use our attacks to generate the attack edges for each target node by attacking the source GNN (GCN or SGC), change the graph structure based on the attack edges, and adopt a target GNN to classify each target node on the perturbed graph. We select two additional representative GNNs, i.e., GAT [18] and JK-Net [24], as the target GNN. If a target node is also misclassified by the target GNN to be the target label, we say the attack edges generated by the source GNN for this target node are transferable.

Table 2 and Table 3 show the attack success rate of transferring of our attacks against two-layer GCN and two-layer SGC to attack other GNNs on the three graphs, where the attack budget per target node is 6. Note that we also show the attack performance for target GNNs without attack, i.e., the prediction error of target GNNs on the target nodes in the clean graph. We have the following observations. First, our attacks against GCN (or SGC) have the best transferability to SGC (or GCN). This is because SGC is a special case of GCN and they share similar model architectures. Second, our attacks are also effective against GAT and JK-Net. Specifically, on all the three graphs, our attacks can increase the classification errors by at least 60% when attacking GAT and JK-Net.

# 5 Conclusion

We propose an influence-based evasion attack against GNNs. Specifically, we first build the connection between GNNs and label propagation via influence function. Next, we reformulate the attack against GNNs to be related to label influence on LP. Then, we design an efficient algorithm to calculate label influences. Our attack is applicable to multi-layer GNNs and does not need to know the GNN model parameters. Finally, we evaluate our attack on three benchmark graph datasets. Experimental results demonstrate that our attack achieves comparable performance against state-of-the-art attack, and has a 5-50x speedup when attacking two-layer GCNs. Our attack is also effective to attack multi-layer GNNs and is transferable to other GNNs.

# 6 Related Work

Adversarial attacks to graph neural networks. Existing attacks to GNNs can be classified as poisoning attacks [28, 5, 29, 22, 16, 26] and evasion attacks [5, 28, 21]. In poisoning attacks, an attacker aims to modify the graph structure during the training process such that the trained GNN model has a low prediction accuracy on the testing nodes. For instance, [29] proposed a poisoning attack, called Metattack, that perturbs the whole graph based on meta-learning. [22] developed a topology poisoning attack based on gradient-based optimization. Evasion attacks can be classified as untargeted attacks and targeted attacks. Given a target node and a trained GNN model, targeted attack means an attacker aims to perturb the graph structure such that the GNN model misclassifies the target node to be a target label, while untargeted attack misclassifies the target node to be an arbitrary label different from the target node's label. For instance, [5] leveraged reinforcement learning techniques to design non-targeted evasion attacks to both graph classification and node classification via modifying the graph structure. [28] proposed a targeted evasion attack, called Nettack, against two-layer GCN and achieved the state-of-the-art attack performance. Specifically, Nettack learns a surrogate linear model of GCN by removing the ReLU activation function and by defining a graph structure preserving perturbation that constrains the difference between the node degree distributions of the graph before and after attack. Our label influence-based attack is a targeted evasion attack.

Adversarial attacks to other graph-based methods. Besides attacking GNNs, other adversarial attacks against graph data include attacking graph-based clustering [4], graph-based collective classification [17, 19], graph embedding [6, 15, 3, 1, 2], community detection [12], etc. For instance, [4] proposed a practical attack against spectral clustering, which is a well-known graph-based clustering method. [19] designed an attack against the collective classification method, called linearized belief propagation, by modifying the graph structure.

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# **Appendix**

#### Proof of Theorem 1

We mainly focus on GCN, as SGC is a special case of GCN and the proof also applies. Our proof is based on the following Lemma:

**Lemma 1.** [24] Given a K-layer GCN, assume that all paths in the computation graph of the GCN model are activated (i.e., via ReLU) with the same probability of success  $\rho$ . Then,

$$\frac{\partial \mathbf{h}_v^{(K)}}{\partial \mathbf{h}_u^{(0)}} = \rho \sum_{p=1}^{\Psi_{v \to u}} \prod_{l=K}^1 a_{v_p^l, v_p^{l-1}} \cdot \mathbf{W}^{(l)}, \tag{14}$$

where  $\Psi_{v \to u}$  is the total number of paths  $[v_p^K, v_p^{K-1}, \cdots, v_p^1, v_p^0]$  of length K+1 from node v to the node u with  $v_p^K = v$  and  $v_p^0 = u$ . For  $l = 1, \cdots, K$ ,  $a_{v_p^l, v_p^{l-1}} = d_{v_p^l}^{-\frac{1}{2}} d_{v_p^{l-1}}^{-\frac{1}{2}}$  is the normalized weight of the edge  $(v_p^l, v_p^{l-1})$  in the path p.  $\Theta = \{\mathbf{W}^{(l)}\}$  is the GCN model parameters.

Via spanning the node representation  $\mathbf{h}_{v}^{(K)}$ , we have

$$\mathbf{h}_{v}^{(K)} = \sum_{u \in \mathbf{\Lambda}_{v}^{(K)}} \frac{\partial \mathbf{h}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}, \tag{15}$$

where  $\mathbf{\Lambda}_v^{(K)} = \left\{ \Gamma_v^{(1)}, \Gamma_v^{(2)}, \cdots, \Gamma_v^{(K)} \right\}$  is the node set containing all nodes within the K-hop neighbor of v.  $\Gamma_v^{(k)}$  denotes the set of k-hop neighbors of node v, and  $\Gamma_v^{(1)} = \Gamma_v$ .

Based on Equation (14), the feature-label influence in Equation (6) can be expressed as follows:

$$I_{fl}(v, u; K) = \mathbf{1}_{y_u}^T \left[ \frac{\partial \mathbf{h}_v^{(K)}}{\partial \mathbf{h}_u^{(0)}} \right] \mathbf{h}_u^{(0)}$$

$$= \mathbf{1}_{y_u}^T \left[ \prod_{l=K}^1 \mathbf{W}^{(l)} \cdot \rho \cdot \sum_{p=1}^{\Psi_{v \to u}} \prod_{l=K}^1 a_{v_p^l, v_p^{l-1}} \right] \mathbf{h}_u^{(0)}$$

$$= \rho \cdot \mathbf{1}_{y_u}^T \left[ \prod_{l=K}^1 \mathbf{W}^{(l)} \right] \mathbf{h}_u^{(0)} \cdot \sum_{p=1}^{\Psi_{v \to u}} \prod_{l=K}^1 a_{v_p^l, v_p^{l-1}}$$

$$= C \cdot \sum_{p=1}^{\Psi_{v \to u}} \prod_{l=K}^1 a_{v_p^l, v_p^{l-1}},$$
(16)

where  $C = \rho \mathbf{1}_{y_u}^T [\prod_{l=1}^k \mathbf{W}^{(l)}] \mathbf{h}_u^{(0)}$  is a constant for a given GCN model. Comparing GCN with LP, we can find their iteration processes are similar, except that LP has no model parameters (which is constant for a trained GCN model). Specifically, we can calculate the label influence  $I_l$  as follows:

$$I_l(v, u; K) = \frac{\partial y_v^{(K)}}{\partial y_u^{(0)}} = \sum_{p=1}^{\Psi_{v \to u}} \prod_{l=K}^1 a_{v_p^l, v_p^{l-1}}.$$
 (17)

Hence, we can build the following relationship between the feature-label influence in GNN and label influence in LP:

$$I_{fl}(v, u; K) = C \cdot I_l(v, u; K). \tag{18}$$

## Proof of Theorem 2

Let  $\tilde{\mathbf{h}}_{v}^{(K)}$  be u's final node representation after the attack. Substituting Equation 15, we have that the attack's objective function in Equation (5) is equivalent to the following form:

$$\max_{\tilde{\mathbf{A}}_{v}} \left( \left[ \sum_{u \in \tilde{\mathbf{A}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)} \right]_{c} - \left[ \sum_{u \in \tilde{\mathbf{A}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)} \right]_{y_{v}} \right)$$

$$s.t., \quad \sum_{s} |\tilde{A}_{v,s} - A_{v,s}| \leq \Delta.$$

Based on Assumption 1, we further deduce attack's objective function as follows:

$$\begin{split} & [\sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}]_{c} - [\sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}]_{y_{v}} \\ & \approx [\sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}]_{c} - [\sum_{z \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{z}^{(0)}} \cdot \mathbf{h}_{z}^{(0)}]_{y_{v}} \\ & = \sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} [\frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}]_{c} - \sum_{z \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} [\frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{z}^{(0)}} \cdot \mathbf{h}_{z}^{(0)}]_{y_{v}} \\ & = \sum_{u \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} [\mathbf{1}_{y_{u}}^{T} \cdot \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{u}^{(0)}} \cdot \mathbf{h}_{u}^{(0)}] - \sum_{z \in \tilde{\mathbf{\Lambda}}_{v}^{(K)}} [\mathbf{1}_{y_{z}}^{T} \frac{\partial \tilde{\mathbf{h}}_{v}^{(K)}}{\partial \mathbf{h}_{z}^{(0)}} \cdot \mathbf{h}_{z}^{(0)}], \end{split}$$

We note that the two terms  $[\mathbf{1}_{y_u}^T \cdot \frac{\partial \tilde{\mathbf{h}}_v^{(K)}}{\partial \mathbf{h}_u^{(0)}} \cdot \mathbf{h}_u^{(0)}]$  and  $[\mathbf{1}_{y_z}^T \cdot \frac{\partial \tilde{\mathbf{h}}_v^{(K)}}{\partial \mathbf{h}_z^{(0)}} \cdot \mathbf{h}_z^{(0)}]$  are exactly the feature-label influence after the attack, and we denote them as  $\tilde{I}_{fl}(v, u; K)$  and  $\tilde{I}_{fl}(v, z; K)$ , respectively. Based on the relationship between the feature-label influence and label influence in Equation (18), we thus have the following attack's objective function in terms of label influence:

$$\max_{\tilde{\mathbf{A}}_{v}} \left( \sum_{\substack{u \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{u} = c}} \tilde{I}_{fl}(v, u; K) - \sum_{\substack{z \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{z} = y_{v}}} \tilde{I}_{fl}(v, z; K) \right) \\
\Leftrightarrow \max_{\tilde{\mathbf{A}}_{v}} \left( \sum_{\substack{u \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{u} = c}} \tilde{I}_{l}(v, u; K) - \sum_{\substack{z \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{z} = y_{v}}} \tilde{I}_{l}(v, z; K) \right).$$

$$= \max_{\tilde{\mathbf{A}}_{v}} \left( \sum_{\substack{u \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{u} = c}} \tilde{Y}_{v \to u} \prod_{l = K}^{1} \tilde{a}_{v_{p}^{l}, v_{p}^{l-1}} - \sum_{\substack{z \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{z} = y_{v}}} \tilde{Y}_{v \to z} \prod_{l = K}^{1} \tilde{a}_{v_{p}^{l}, v_{p}^{l-1}} \right)$$

$$= \sum_{\substack{u \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{u} = c}} \tilde{Y}_{v \to u} \prod_{l = K}^{1} \tilde{a}_{v_{p}^{l}, v_{p}^{l-1}} - \sum_{\substack{z \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{z} = y_{v}}} \tilde{Y}_{v \to z} \prod_{l = K}^{1} \tilde{a}_{v_{p}^{l}, v_{p}^{l-1}} \right)$$

$$= \sum_{\substack{v \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{u} = c}} \tilde{Y}_{v \to u} \prod_{l = K}^{1} \tilde{a}_{v_{p}^{l}, v_{p}^{l-1}} - \sum_{\substack{z \in \tilde{\mathbf{A}}_{v}^{(K)} \\ y_{z} = y_{v}}} \tilde{Y}_{v \to z} \prod_{l = K}^{1} \tilde{a}_{v_{p}^{l}, v_{p}^{l-1}} \right)$$

where  $\tilde{\Psi}_{v\to u}$  is the total number of paths of length K+1 from v to u, and  $\tilde{a}_{v_p^l,v_p^{l-1}}$  is the normalized weight after perturbing the graph structure.

# Example for calculating label influence $I_f$ when K=2

We take Figure 8 as an example to illustrate how the label influence terms are calculated when adding an new edge or deleting an existing edge when K = 2.

The original graph is shown in Figure 8(a). There are two classes of nodes in the graph, which are green color and yellow color, respectively. v is the target node with a label- $y_v$ . The candidate nodes for v to add edges are  $\mathcal{N}_A = \{u6, u7, u8\}$ , and the candidate nodes for v to delete edges are  $\mathcal{N}_B = \{u2, u3\}$ .

In the clean graph,  $\mathbf{\Lambda}_{v}^{(2)} = [v, u1, u2, u3, u4, u5, u6, u7]; \{u \in \mathbf{\Lambda}_{v}^{(2)}, y_{u} = c\} = \{u5, u6, u7, u8\};$ 

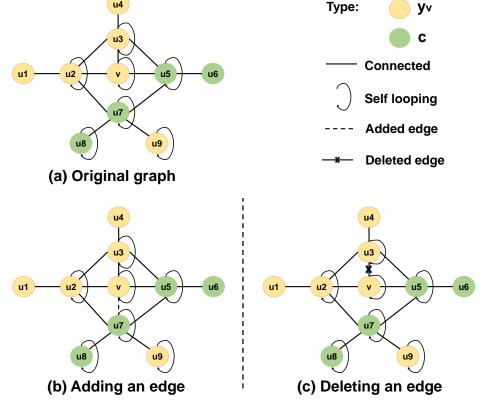


Figure 8: An example for calculating label influence

$$\{z\in \mathbf{\Lambda}_{v}^{(2)}, y_{z}=y_{v}\}=\{u1,u2,u3,u4\}. \text{ We have}$$
 
$$\varPsi_{v\to u}=\{v-u5-u5,v-v-u5,v-u3-u5,\\v-u5-u6,\\v-u2-u7,v-u5-u7\},$$
 
$$\varPsi_{v\to z}=\{v-v-v,v-u5-v,v-u2-v,v-u3-v,\\v-u2-u1,\\v-u2-u2,v-v-u2,v-u3-u2,\\v-u3-u3,v-v-u3,v-u2-u3,v-u5-u3,\\v-u3-u4\}$$

Adding an edge. We first compute the exact label influence and then calculate the approximate label influence. Figure 8(b) is the graph structure after adding an edge between u7 and v.

In the perturbed graph,  $\tilde{\mathbf{\Lambda}}_{v}^{(2)} = [v, u1, u2, u3, u4, u5, u6, u7, u8, u9]$ , and  $\Delta \mathbf{\Lambda}_{v}^{(2)} = \{v, u2, u5, u7, u8, u9\}$ .  $\{u \in \Delta \mathbf{\Lambda}_{v}^{(2)}, y_{u} = c\} = \{u5, u7, u8\}$ ; and  $\{z \in \Delta \mathbf{\Lambda}_{v}^{(2)}, y_{z} = y_{v}\} = \{v, u2, u9\}$ .

The new added paths are passing through u7. Specifically, all the new added paths  $\Psi'_{v\to u} = \{v - u7 - u5, v - u7 - u7, v - v - u7, v - u7 - u8\}.$ 

All the new added paths  $\Psi'_{v\to z} = \{v - u7 - v, v - u7 - u2, v - u7 - u9\}.$ 

Exact label influence calculation. We first precisely calculate the label influence as follows:

$$\begin{split} &\sum_{u \in \tilde{\Lambda}_{v}^{(2)}} \tilde{I}_{l}(v, u; 2) - \sum_{z \in \tilde{\Lambda}_{v}^{(2)}} \tilde{I}_{l}(v, z; 2) \\ &= [\sum_{u \in \Lambda_{v}^{(2)}} \sum_{p=1}^{\Psi_{v \to u}} \prod_{l=2}^{1} \tilde{d}_{v_{p}^{-1}}^{-1} \tilde{d}_{v_{p}^{-1}}^{-1} - \sum_{z \in \Lambda_{v}^{(2)}} \sum_{p=1}^{\Psi_{v \to z}} \prod_{l=2}^{1} \tilde{d}_{v_{p}^{-1}}^{-1} \tilde{d}_$$

Note that  $C_A(exact) = -0.3102$  is the exact constant label influence in the original paths, and  $I_A(u7) = 0.0336$  is the label influence in the new added paths.

Approximate constant label influence calculation. When calculating the label influence in the original paths in Equation (20) after adding the edge between v and u7, we notice that only paths including u7 are affected, as u7's degree increases by 1. Here, for efficient calculation, we ignore the effect caused by u7's degree, which means that we do not need to normalize the weights of edges associated with u7. For simplicity, we only calculate the first label influence term and the approximate constant label influence is calculated as follow:

$$\begin{split} &\sum_{\substack{u \in \Lambda_v^{(2)} \\ y_u = c}} \tilde{I}_l(v, u; 2) - \sum_{\substack{z \in \Lambda_v^{(2)} \\ y_z = y_v}} \tilde{I}_l(v, z; 2) \\ &\approx \left( d_{u5}^{-\frac{3}{2}} d_v'^{-\frac{1}{2}} + d_{u5}^{-\frac{1}{2}} d_v'^{-\frac{3}{2}} + d_{u5}^{-\frac{1}{2}} d_{u3}^{-1} d_v'^{-\frac{1}{2}} + d_{u6}^{-\frac{1}{2}} d_{u5}^{-1} d_v'^{-\frac{1}{2}} + d_{u7}^{-\frac{1}{2}} d_{u5}^{-1} d_v'^{-\frac{1}{2}} \right) \\ &- \left( d_{u2}^{-\frac{3}{2}} d_v'^{-\frac{1}{2}} + d_{u2}^{-\frac{1}{2}} d_v'^{-\frac{3}{2}} + d_{u2}^{-\frac{1}{2}} d_{u3}^{-1} d_v'^{-\frac{1}{2}} + d_{u3}^{-\frac{3}{2}} d_v'^{-\frac{1}{2}} + d_{u3}^{-\frac{1}{2}} d_v'^{-\frac{3}{2}} + d_{u3}^{-\frac{1}{2}} d_v'^{-\frac{1}{2}} \right) \\ &+ d_{u3}^{-\frac{1}{2}} d_{u5}^{-1} d_v'^{-\frac{1}{2}} + d_{u1}^{-\frac{1}{2}} d_u^{-1} d_v'^{-\frac{1}{2}} + d_{u4}^{-\frac{1}{2}} d_u^{-1} d_v'^{-\frac{1}{2}} + d_v'^{-1} d_u^{-1} + d_v'^{-1} d_{u5}^{-1} + d_v'^{-1} d_{u5}^{-1} \right) \\ &= C_A(approx) \\ &= (0.2632 - 0.5665) \\ &= -0.3032, \end{split}$$

where  $d_{u7}$  is used instead of the value of  $d'_{u7} = d_{u7} + 1$ , and  $d'_v = d_v + 1$ .

We notice that the approximate value -0.3032 is close to the precise value -0.3102, which thus demonstrates that our approximate label influence for adding an edge is effective.

**Deleting an edge.** Figure 8(c) is the graph after deleting an edge between u3 and v.

In the perturbed graph,  $\tilde{\mathbf{\Lambda}}_{v}^{(2)} = [v, u1, u2, u3, u4, u5, u6, u7]$ , and  $\Delta \mathbf{\Lambda}_{v}^{(2)} = \{v, u2, u3, u4, u5\}$ .  $\{u \in \Delta \mathbf{\Lambda}_{v}^{(2)}, y_{u} = c\} = \{u5\}$ ; and  $\{z \in \Delta \mathbf{\Lambda}_{v}^{(2)}, y_{z} = y_{v}\} = \{v, u2, u3, u4\}$ .

The new deleted paths are passing through u3. Specifically, all the new deleted paths  $\Psi'_{v\to u}=\{v-u3-u5\}$ 

All the new deleted paths  $\Psi'_{v\to z} = \{v-u3-v, v-u3-u2, v-v-u3, v-u3-u3, v-u3-u4\}$ . Exact label influence calculation. We first precisely calculate the label influence as follows:

$$\begin{split} &\sum_{\substack{u \in \Lambda_{v}^{(2)} \\ y_{u} = c}} \tilde{I}_{l}(v, u; 2) - \sum_{\substack{z \in \Lambda_{v}^{(2)} \\ y_{z} = y_{v}}} \tilde{I}_{l}(v, z; 2) \\ = &[\sum_{\substack{u \in \Lambda_{v}^{(2)} \\ y_{u} = c}} \tilde{I}_{l}(v, u; 2) - \sum_{\substack{z \in \Lambda_{v}^{(2)} \\ y_{z} = y_{v}}} \tilde{I}_{l}(v, z; 2)] - [\sum_{\substack{u \in \Delta\Lambda_{v}^{(2)} \\ y_{u} = c}} \tilde{I}_{l}(v, z; 2) - \sum_{\substack{z \in \Delta\Lambda_{v}^{(2)} \\ y_{z} = y_{v}}} \tilde{I}_{l}(v, z; 2)] \\ = &[\sum_{\substack{u \in \Lambda_{v}^{(2)} \\ y_{u} = c}} \sum_{p=1}^{\Psi_{v \to u}} \prod_{l=2}^{1} \tilde{d}_{v_{p}^{l}}^{-\frac{1}{2}} \tilde{d}_{v_{p}^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Lambda_{v}^{(2)} \\ y_{z} = y_{v}}} \sum_{p=1}^{\Psi_{v \to z}} \prod_{l=2}^{1} \tilde{d}_{v_{p}^{l}}^{-\frac{1}{2}} \tilde{d}_{v_{p}^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Lambda_{v}^{(2)} \\ y_{z} = y_{v}}} \sum_{p=1}^{\Psi_{v \to z}} \prod_{l=2}^{1} \tilde{d}_{v_{p}^{l}}^{-\frac{1}{2}} \tilde{d}_{v_{p}^{l-1}}^{-\frac{1}{2}} - \sum_{\substack{z \in \Lambda_{v}^{(2)} \\ y_{z} = y_{v}}} \sum_{p=1}^{\Psi_{v \to z}} \prod_{l=2}^{1} \tilde{d}_{v_{p}^{l}}^{-\frac{1}{2}} \tilde{d}_{v_{p}^{l-1}}^{-\frac{1}{2}} \\ = &[\left(d_{u_{5}}^{-\frac{3}{2}} d_{v}^{\prime}^{-\frac{1}{2}} + d_{u_{5}}^{-\frac{1}{2}} d_{v}^{-\frac{3}{2}} + d_{u_{5}}^{-\frac{1}{2}} d_{u_{3}}^{-1} d_{v}^{\prime}^{-\frac{1}{2}} + d_{u_{6}}^{-\frac{1}{2}} d_{v_{b}^{l}}^{-\frac{1}{2}} + d_{u_{7}}^{-\frac{1}{2}} d_{v_{p}^{l-1}}^{-\frac{1}{2}} + d_{u_{7}}^{-\frac{1}{2}$$

Note that  $C_B(exact) = -0.5426$  is the exact constant label influence in the original paths, and  $I_B(u3) = -0.2860$  is the label influence in the new added paths.

Approximate constant label influence calculation. When calculating the label influence in the original paths in Equation (21) after deleting the edge between v and u3, we notice that only paths including u3 are affected, as u3's degree decreases by 1. Here, for efficient calculation, we ignore the effect caused by u3's degree, which means that we do not need to normalize the weights of edges associated with u3. For simplicity, we only calculate the first label influence term and the approximate constant label influence is calculated as follow:

$$\begin{split} &\sum_{\substack{u \in \Lambda_v^{(2)} \\ y_u = c}} \tilde{I}_l(v, u; 2) - \sum_{\substack{z \in \Lambda_v^{(2)} \\ y_z = y_v}} \tilde{I}_l(v, z; 2) \\ &\approx \left( d_{u5}^{-\frac{3}{2}} d_v'^{-\frac{1}{2}} + d_{u5}^{-\frac{1}{2}} d_v'^{-\frac{3}{2}} + d_{u5}^{-\frac{1}{2}} d_{u3}^{-1} d_v'^{-\frac{1}{2}} + d_{u6}^{-\frac{1}{2}} d_{u5}^{-1} d_v'^{-\frac{1}{2}} + d_{u7}^{-\frac{1}{2}} d_{u2}^{-1} d_v'^{-\frac{1}{2}} + d_{u7}^{-\frac{1}{2}} d_{u5}^{-1} d_v'^{-\frac{1}{2}} \right) \\ &- \left( d_{u2}^{-\frac{3}{2}} d_v'^{-\frac{1}{2}} + d_{u2}^{-\frac{1}{2}} d_v'^{-\frac{3}{2}} + d_{u2}^{-\frac{1}{2}} d_{u3}^{-1} d_v'^{-\frac{1}{2}} + d_{u3}^{-\frac{3}{2}} d_v'^{-\frac{1}{2}} + d_{u3}^{-\frac{1}{2}} d_v'^{-\frac{3}{2}} + d_{u3}^{-\frac{1}{2}} d_v'^{-\frac{1}{2}} \right. \\ &+ d_{u3}^{-\frac{1}{2}} d_{u5}^{-1} d_v'^{-\frac{1}{2}} + d_{u1}^{-\frac{1}{2}} d_{u2}^{-1} d_v'^{-\frac{1}{2}} + d_{u4}^{-\frac{1}{2}} d_{u3}^{-1} d_v'^{-\frac{1}{2}} + d_v'^{-1} d_{u5}^{-1} + d_v'^{-1} d_{u5}^{-1} + d_v'^{-1} d_{u3}^{-1} \right) \\ &= C_B(approx) = -0.5304 \end{split}$$

where  $d_{u3}$  is used instead of the value of  $d'_{u3} = d_{u3} - 1$ , and  $d'_v = d_v - 1$ .

We can notice that the approximate value -0.5304 is close to the precise value -0.5426, which thus demonstrates that our approximate label influence for deleting an edge is effective.