Some MCMC tests

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Agenda

1. stationarity
2. Conditions for stationarity
3. Practical tests

basic definitions

- A Markov process is when $P(\theta_{t+1}|\theta_t, \theta_{t-1}, .., \theta_1) = P(\theta_{t+1}|\theta_t)$

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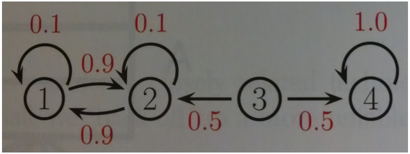
- In general, most of the following conditions for stationarity cannot be a priori constructed and all the practical test are necessary but not sufficient.

irreducibility



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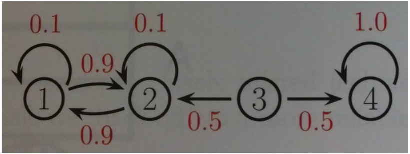

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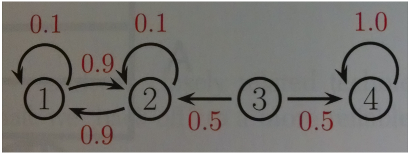

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- keep in mid that you can have a stationary distribution but that is not unimodal - consider the oscillation between states 1 and 2, within each chain - stationarity does not mean unimodality of the distribution.

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- According to Jackman, ergodic also means that the finite time is proportional to the probability distribution.

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- AKA Conductance.

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- $$\hat{R} = \sqrt{\frac{V}{W}} \approx 1$$

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- **Heidelberger and Wlech** checks two things. Tries to reject the null of stationarity with a one sided Cramer-Von Mises. And if the sample size is large enough to measure the mean.