Kinematics, dynamics and control of robots - Homework project 1

All calculations and programing should be done **parametrically**. Numerical values should only be substituted for creating the figures.

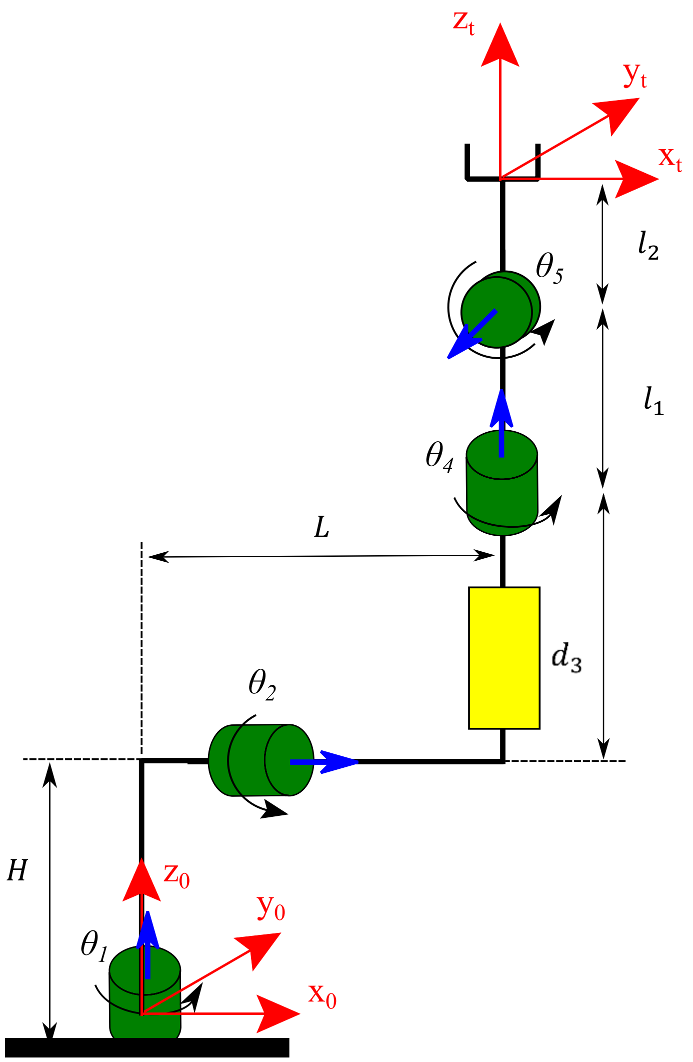

Figure 1 shows a spatial serial manipulator with 5 DOFs - $\theta_1, \theta_2, d_3, \theta_4, \theta_5$

1. Solve the forward kinematics problem for the robot – calculate the homogeneous transformation matrix (4×4) from the tool frame $(\hat{\boldsymbol{x}}_t, \hat{\boldsymbol{y}}_t, \hat{\boldsymbol{z}}_t)$ to the world frame $(\hat{\boldsymbol{x}}_0, \hat{\boldsymbol{y}}_0, \hat{\boldsymbol{z}}_0)$ as a function of the joint variables.
    - The world and tool frames are defined in the figure 1.
    - The revolute joints are drawn in their zero position.
    - The positive direction of the joint angles is as described in the drawing.

2. Solve the inverse kinematics problem – given the full homogeneous transformation matrix representing a possible position and orientation of the robot tool, calculate the values of the joint variables.
   Find all possible solutions and show the multiple solutions in a qualitative drawing.

3. Calculate the full Jacobian matrix (6×5) for the robot in the world frame and the tool frame.

For the following sections, assume $\boldsymbol{\theta_4, \theta_5 = 0}$ and $\boldsymbol{l_1 = l_2 = 0}$.

4. Find all the singular states of the robot with respect to the task of tool position, sketch the states showing the singular direction.

5. Find the forces and torques in the joints, considering pointed mass $\boldsymbol{M}$ held by the gripper, gravitation on $\hat{z}_0$ direction, links and joints mass are negatable.

6. Plan the motion of the tool from point A to point B (coordinates in next page) over 2 seconds, considering the mechanical limitations of the joints (see next page). Plan the trajectory using the following velocity profiles:
    - Constant linear velocity.
    - Trapezoidal velocity profile – constant acceleration to maximal velocity, motion at constant velocity, and constant deceleration to stop. The time of acceleration and deceleration should be equal and the motion at constant velocity will be 2/3 of the total time.
    - Polynomial velocity profile that assures zero velocity and acceleration at the start and end of the motion.

For each of the motion profiles, present the following:

a. Analytic expressions for the position, velocity and acceleration of the end effector as a function of time.
b. Figures of position, velocity and acceleration of the end effector as a function of time.
c. Figures of the joint motion – position (degrees or meters accordingly), velocity and acceleration as functions of time.
   The velocities and accelerations of the joints should be calculated in two methods (and presented on the same figure):
   1. Numeric differentiation of the joint positions $\boldsymbol{q}(t)$.
   2. From the relation $\dot{\boldsymbol{x}} = J_L\dot{\boldsymbol{q}}$ (where J_L is the linear part of the Jacobian matrix) for the velocities and $\ddot{\boldsymbol{x}} = \dot{J}_L\dot{\boldsymbol{q}} + J_L\ddot{\boldsymbol{q}}$ for the accelerations.

   A list of functions to be submitted is in the following page.

   Submission in pairs by 25.06.2024 on the course moodle page.

<u>Numerical values for computer simulation:</u>

Link lengths:

$$H = 0.2[m], \qquad L = 0.1[m], \qquad l_1 = 0, \qquad l_2 = 0$$

Initial and final positions (in meters):

$$x_A = 0.4, \qquad y_A = 0, \qquad z_A = 0.8$$
$$x_B = 0.25, \qquad y_B = -0.5, \qquad z_B = 1$$

Mechanical limitations:

$$-180^0 < \theta_1 < 180^0, \qquad -90° < \theta_2 < 90°, \qquad d_2 > 0$$

Functions that need to be programed and submitted:

1) x=forward_kin(q)

   q – row vector of the joints' parameters

   x – position row vector of the tool
2) q=inverse_kin(x,elbows)

   x - position vector of the tool

   elbows – decision values vector for the different solutions,

   1 for elbow up, -1 for elbow down.
3) J=jacobian_mat(q)

   J - the full Jacobian matrix.
4) J_dot=jacobian_mat_dot(q,q_dot)

   J_dot – the jacobian time derivative.

   q_dot – the joins' parameters time derivative.
5) x=x_plan(prof,t)

   prof – decision value for the different velocity profile.

   x – the position of the tool's origin in time t.
6) v=v_plan(prof,t)

   v – the velocity of the tool's origin in time t.
7) a=a_plan(prof,t)

   a – the acceleration of the tool's origin in time t.
8) q=q_plan(prof,t)

   q – the joints parameters in time t.
9) q_dot=q_dot_plan(prof,t)

   q_dot – the joints velocity in time t
10) q_dot2=q_dot2_plan(prof,t)

    q_dot2 – the second time derivative of the joints' parameters vector.

Remarks

- All physical parameters (lengths) should be define as global parameters:

```
global l1 l2 l3
```

- It can be useful for you if the functions 5-10 could work with vector time t and give a suitable vector/matrix output.
- It can be useful if function 1 and 2 could work with suitable vector/matrix as number of vectors q and x.
- You may add functions and make some changes to the functions without changing the general concept .
- Vectors x,q,q_dot,q_dot2,v,a are row vector (or matrix if the function work with time vector).
- In the start of every function add a short description of the function use.
- Useful Matlab functions: diff/gradient, atan2 ^., /. ,*. ,.

There should be no symbolic calculations or variables in any of these functions!