REHABILITATION BIOMECHANICS

Homework 3

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1 Questions

1.1 Number of joints and links

According to the simplified model, there are **four** rigid bodied segments (pelvis, thigh, shank and foot) and **three** joints (Hip - Spherical, Knee - Revolute and Ankle - Cardan). In addition, there is the reference and end joints that do not include rotations which are used for plotting purposes (*P*) and (*T*).

1.2 Coordinate Systems Definitions

Figure 1.1 defines the *zero-reference position* of the coordinate systems. Each segment is attached a coordinate system. The coordinates (x, y, z) for each segment is parallel to the fixed frame. The Coordinate systems will be placed at the joints and will be denoted heron by h for *Hip*, k for *Knee, a* for *Ankle* and *T* for *Toe*.

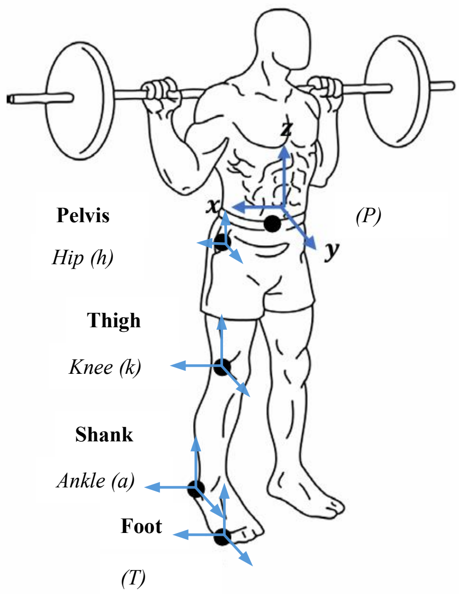

Figure 1.1 - The Zero Reference Position model of the subject, and the coordinate systems definitions. The four segments are in **bold,** and the three joints are in *italic*.

1.3 Body Frame Segment Orientation

The orientation of each segment with respect to its previous frame is defined as follows:

$$v_{pelvis} = [1 \quad 0 \quad 0]_{(P)} \tag{1.1}$$

$$v_{thigh} = [0 \quad 0 \quad -1]_{(h)} \tag{1.2}$$

$$v_{shank} = [0 \quad 0 \quad -1]_{(k)} \tag{1.3}$$

$$v_{foot} = [0 \quad 1 \quad 0]_{(a)} \tag{1.4}$$

1.4 Zero Reference Plot

Given the segment lengths in (1.5), (1.6), (1.7) and (1.8) we plotted the simplified model in MATLAB as shown below.

$$L_{pelvis} = 0.3\ [m] \tag{1.5}$$

$$L_{thigh} = 1\ [m] \tag{1.6}$$

$$L_{shank} = 1\ [m] \tag{1.7}$$

$$L_{foot} = 0.3\ [m] \tag{1.8}$$

We added the coordinate systems to the plot, where each coordinate is in a different color and at an appropriate length (0.1) as shown in the figure below.

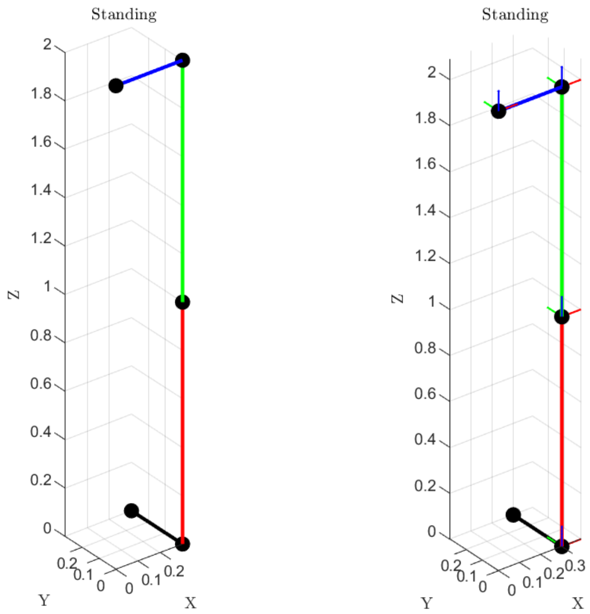

Figure 1.2 - Zero Reference Position of the standing man. The coordinate systems are shown at the joints.

1.5 Rotation Matrix Function

Extrinsic rotations are given by repeatedly rotating a vector about a fixed frame. Intrinsic rotations are given by consecutively rotating a vector about the rotated frame. We created a function named "euler2rotm_extended" that extends the "eul2rotm" built in function by allowing an input with this distinction.

1.6 Joint Rotations Order

The order of biomechanical rotations is flexion (extension) → abduction (adduction) → axial rotation. We will denote $\theta_{i,j}$ as the angle of rotation in the i^{th} joint and the j^{th} biomechanical rotation type.

$$\theta_{h,f} \rightarrow \theta_{h,a} \rightarrow \theta_{h,r} \tag{1.9}$$

$$\theta_{k,f} \tag{1.10}$$

$$\theta_{a,f} \rightarrow \theta_{a,a} \tag{1.11}$$

The rotations are consecutively applied upon the **rotated frames**, so the type of rotation is **intrinsic**. The rotation matrices are given as follows:

$$R_{hip} = R_x(\theta_{h,f})R_y(\theta_{h,a})R_z(\theta_{h,r}) \tag{1.12}$$

$$R_{knee} = R_x(\theta_{k,f}) \tag{1.13}$$

$$R_{ankle} = R_x(\theta_{a,f})R_y(\theta_{a,a}) \tag{1.14}$$

These rotations are given by the following Euler angles:

$$R_{hip}\colon \begin{bmatrix} \theta_{h,f} \\ \theta_{h,a} \\ \theta_{h,r} \end{bmatrix}, \text{"XYZ"}, \text{"intrinsic"} \tag{1.15}$$

$$R_{knee}\colon \begin{bmatrix} \theta_{k,f} \\ 0 \\ 0 \end{bmatrix}, \text{"XYZ"}, \text{"intrinsic"} \tag{1.16}$$

$$R_{ankle}\colon \begin{bmatrix} \theta_{a,f} \\ \theta_{a,a} \\ 0 \end{bmatrix}, \text{"XYZ"}, \text{"intrinsic"} \tag{1.17}$$

1.7 Segment Direction Analytical Derivation

The segment directions with respect to the fixed frame are simply given by the point rotations:

$$\boldsymbol{v}_{pelvis,P} = \boldsymbol{v}_{pelvis} \tag{1.18}$$

$$\boldsymbol{v}_{thigh,P} = R_{hip}\boldsymbol{v}_{thigh} \tag{1.19}$$

$$\boldsymbol{v}_{shank,P} = R_{hip}R_{knee}\boldsymbol{v}_{shank} \tag{1.20}$$

$$\boldsymbol{v}_{foot,P} = R_{hip}R_{knee}R_{ankle}\boldsymbol{v}_{foot} \tag{1.21}$$

1.8 Plot Different States

We used the function from 1.5 and the results in 1.6 and 1.7, to create a new function to plot the body given arbitrary segment lengths and rotations of each joint.

First, we computed using an input vector of rotation angles $[\theta_{h,f}, \theta_{h,a}, \theta_{h,r}, \theta_{k,f}, \theta_{a,f}, \theta_{a,a}]^T$ all rotation matrices given in (1.12) (1.13) and (1.14). Then we found the segment directions using (1.19), (1.20) and (1.21). Finally, we found the positions by adding the previous segment positions to the consecutive vectors multiplied by the corresponding segment length. The directions of the base vectors in each coordinate system are given by the column vector within the orientation matrix.

We then plotted the following tasks – Standing, Sitting, Butterfly and Squat, and compared intrinsic to extrinsic rotation types.

1.8.1 Standing

This is the zero reference position. The chosen task is $[0°, 0°, 0°, 0°, 0°, 0°]^T$. There is no difference between extrinsic and intrinsic rotations simply because there are no rotations at all.

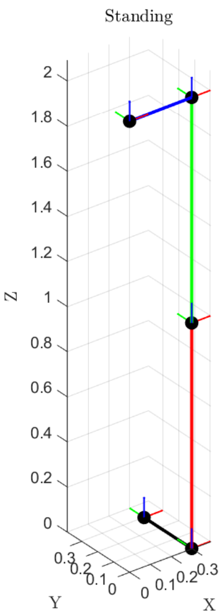

Figure 1.3 - Sitting task position using both intrinsic and extrinsic rotations.

1.8.2 Sitting

This chosen task was $[90°, 0°, 0°, -90°, 0°, 0°]^T$. Also here, there is no difference between extrinsic and intrinsic rotations because there are only single rotations at every joint so there is no significance in the order of multiplications of a single rotation matrix.

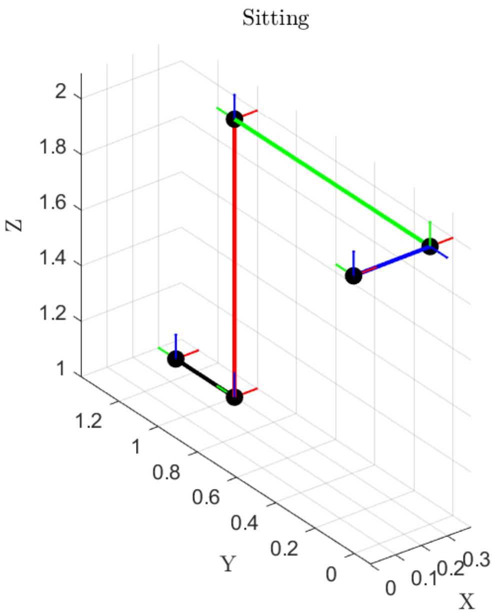

Figure 1.4 - Sitting task position using both intrinsic and extrinsic rotations.

1.8.3 Butterfly

The chosen task was $[90°, -90°, -90°, -162°, 0°, 0°]^T$. There was a difference between extrinsic and intrinsic rotations because the first joint had three consecutive rotations and there is a significance in the order of multiplications of three rotation matrices, i.e., matrix multiplication is not commutative.

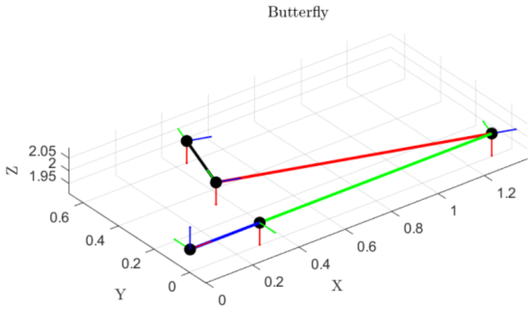

Figure 1.5 - Butterfly task position using intrinsic rotation.

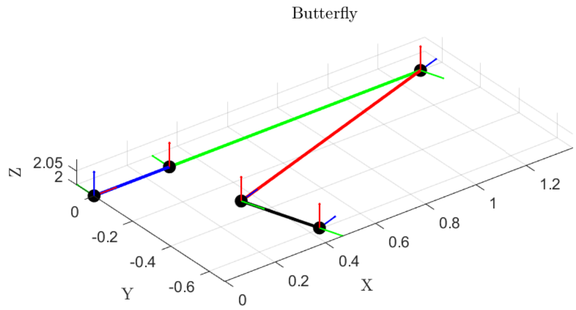

Figure 1.6 - Butterfly task position using extrinsic rotation.

1.8.4 Squat

The chosen task was $[90°, -20°, 0°, -107.2°, 17.2°, 0°]^T$. Also here, the task consists of multiple rotations within the same joint so the order of rotation results in a different configuration.

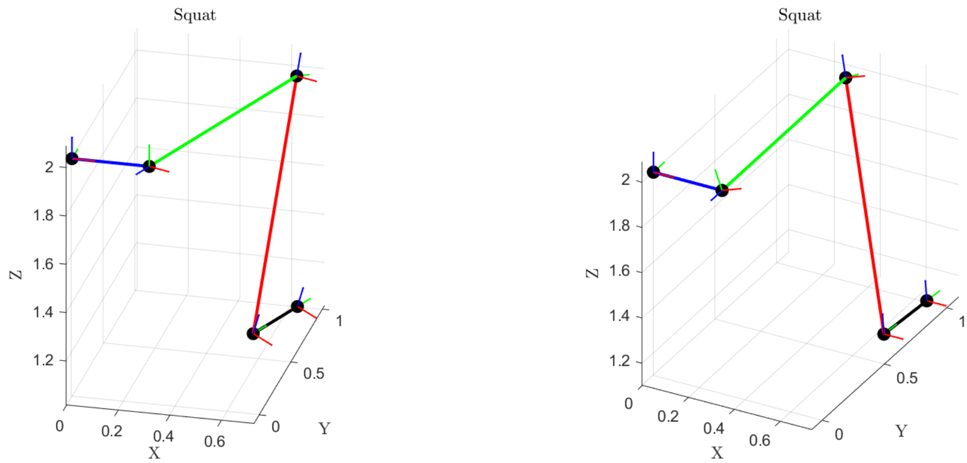

Figure 1.7 - Squat task position using intrinsic (left) and extrinsic (right) rotations.

2 Appendix

The MATLAB files are attached via a zip file, including the “trans_vec.m”, “leg_plotter.m”, “new_plotter.m” and “euler2rotm_extended.m” MATLAB functions. In addition, to get the results run the “main.m” MATLAB script. An additional output folder is included for the resulting figures.