Example which shows that the system with two generators is not always stable.

We will explore the system with the following parameters:

$L_s = 3.1929, \qquad \mathrm{L_g} = 3.1929\,, \qquad J = 19.9051$

$R_s = 1.1786, \qquad \mathrm{D}_p = 0.2433\,, \qquad Mi_f = -4.0035$

$R_l = 3.2130,\ T_m = 1.3796,\ \mathrm{R_g} = 0$

Where the system is:

$$\begin{bmatrix} \Lambda\dot{z}_1 \\ \Lambda\dot{z}_2 \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \mathcal{A}(\omega_1) & -\mathcal{B}(\delta) & 0 \\ -\mathcal{B}(-\delta) & \mathcal{A}(\omega_2) & 0 \\ -e^T & e^T & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \delta \end{bmatrix} + T_m \begin{bmatrix} e \\ e \\ 0 \end{bmatrix}$$

Where:

$$z = \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix}, \Lambda = \begin{bmatrix} L_{tot} & 0 & 0 \\ 0 & L_{tot} & 0 \\ 0 & 0 & J \end{bmatrix},$$

$$\mathcal{A}(\omega) = \begin{bmatrix} -R_{tot} & \omega L_{tot} & 0 \\ -\omega L_{tot} & -R_{tot} & -mi_f \\ 0 & mi_f & -D_p \end{bmatrix}, e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathcal{B}(\delta) = \begin{bmatrix} R_l\cos(\delta) & -R_l\sin(\delta) & 0 \\ R_l\sin(\delta) & R_l\cos(\delta) & 0 \\ 0 & 0 & 0 \end{bmatrix} R_{tot} = R_l + R_s,\ L_{tot} = L_s + L_g$$

First, let's find the system equilibrium points:

As we shoed, the system equilibrium points must satisfy:

$z_1 = z_2, \quad \delta = \pi k$

1. We will start with $\boldsymbol{\delta = 0}$:

We will find ω_0 with the cubic equation from page 12 at the notes:

$$D_p L_{tot}^2 \omega_0^3 - T_m L_{tot}^2 \omega_0^2 + \left(D_p R_T^2 + m^2 i_f^2 R_T\right)\omega_0 - T_m R_T^2 = 0$$

Where $R_T = R_{tot} + R_l$

Solving this equation give the following solution:

$$\omega_0 = \{2.42 \pm 1.9565j, 0.8303\}$$

The only real solution gives $\omega_0 = 0.8303$

We will calculate $i_{q1_0} = i_{q2_0}$ with the dynamics of the third line:

$$m i_f i q_0 - D_p w_0 + T_m = 0$$

$$i q_0 = \frac{D_p w_0 - T_m}{m i_f} = 0.2941$$

We will calculate $i_{d1_0} = i_{d2_0}$ with the dynamics of the second line:

$$-\omega_0 L_{tot} i d_0 - R_{tot} i_{q0} - m i_f \omega_0 + R_l \sin(\delta_0) i_{d0} + R_l \cos(\delta_0)\, i q_0$$

$$i d_0 = \frac{-(R_{tot} + R_l) i_{q0} - m i_f \omega_0}{\omega_0 L_{tot}} = 0.2051$$

In order to validate those results, let's calculate

$$\begin{bmatrix} \mathcal{A}(\omega_0) & -\mathcal{B}(0) & 0 \\ -\mathcal{B}(0) & \mathcal{A}(\omega_0) & 0 \\ -e^T & e^T & 0 \end{bmatrix} \begin{bmatrix} z_0 \\ z_0 \\ \delta_0 \end{bmatrix} + T_m \begin{bmatrix} e \\ e \\ 0 \end{bmatrix} = 10^{-15} \begin{bmatrix} 0.9599 \\ 0.0544 \\ -0.1434 \\ 0.9599 \\ 0.0544 \\ -0.1434 \\ 0 \end{bmatrix}$$

This is MATLAB numerical error.

2. Now, $\boldsymbol{\delta = \pi}$:

We will get the same results as at the previous section, but with

$$R_T = R_{tot} - R_l$$

Solving:

$$D_p L_{tot}^2 \omega_0^3 - T_m L_{tot}^2 \omega_0^2 + \left(D_p R_T^2 + m^2 i_f^2 R_T\right)\omega_0 - T_m R_T^2 = 0$$

Gives:

$\omega_0 = \; 5.3124$

$$iq_0 = \frac{D_p w_0 - T_m}{mi_f} = \; 0.0218$$

Now, $i_{d0} = \frac{-(R_{tot}-R_l)i_{q0}-mi_f\omega_0}{\omega_0 L_{tot}} = 0.6262$

In order to validate those results, let's calculate

$$\begin{bmatrix} \mathcal{A}(\omega_0) & -\mathcal{B}(\pi) & 0 \\ -\mathcal{B}(\pi) & \mathcal{A}(\omega_0) & 0 \\ -e^T & e^T & 0 \end{bmatrix} \begin{bmatrix} z_0 \\ z_0 \\ \delta_0 \end{bmatrix} + T_m \begin{bmatrix} e \\ e \\ 0 \end{bmatrix} = \; 10^{-14} \begin{bmatrix} -0.4624 \\ -0.1078 \\ -0.0003 \\ -0.4624 \\ -0.1078 \\ -0.0003 \\ 0 \end{bmatrix}$$

This is MATLAB numerical error.

Now we will calculate the Jacobian of this system:

$$J = \; \begin{bmatrix} \tilde{\mathcal{A}}(\omega_1) & \mathcal{B}(\delta)/L_{tot} & \mathcal{C}(z_2) \\ \mathcal{B}(-\delta)/L_{tot} & \tilde{\mathcal{A}}(\omega_2) & -\mathcal{C}(z_1) \\ -e^T & e^T & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \delta \end{bmatrix}$$

Where:

$$\tilde{\mathcal{A}}(z_1) = \begin{bmatrix} -R_{tot}/L_{tot} & \omega & i_q \\ -\omega & -R_{tot}/L_{tot} & -(mi_f + L_{tot}i_d) \\ 0 & mi_f/J & -D_p/J \end{bmatrix}$$

$$\mathcal{C}(z) = \frac{R_l}{L_{tot}} \begin{bmatrix} i_q cos(\delta\,) \; + \; i_d sin(\delta) \\ -i_d cos(\delta\,) \; + \; i_q sin(\delta) \\ 0 \end{bmatrix}$$

We will substitute our parameters and our equilibrium points into J and calculate (numerically) its eigenvalues:

For $\boldsymbol{\delta} = \; \mathbf{0}$:

We get: $\lambda = \begin{bmatrix} -1.1788 + 0.8639i \\ -1.1788 - 0.8639\mathrm{i} \\ -0.2000 + 0.8388i \\ -0.2000\ \mathrm{i}\ 0.8388\mathrm{i} \\ 0.0093 + 0.2769\mathrm{i} \\ 0.0093 - 0.2769\mathrm{i} \\ -0.0364 \end{bmatrix}$

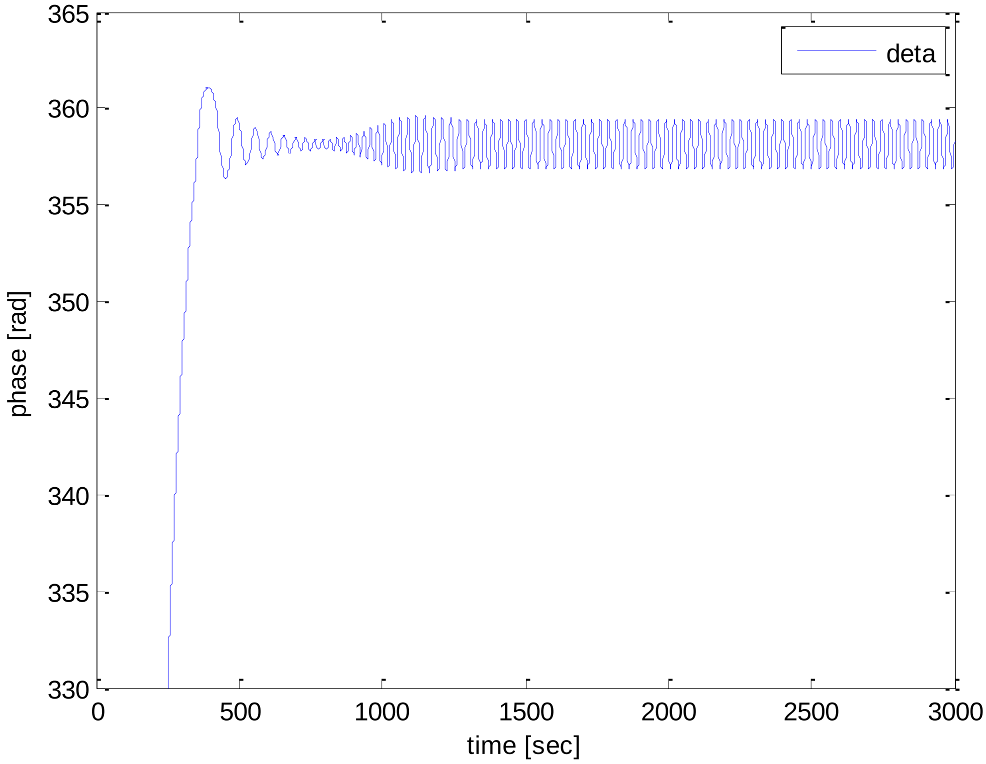
This shows that this equilibrium point is not stable.

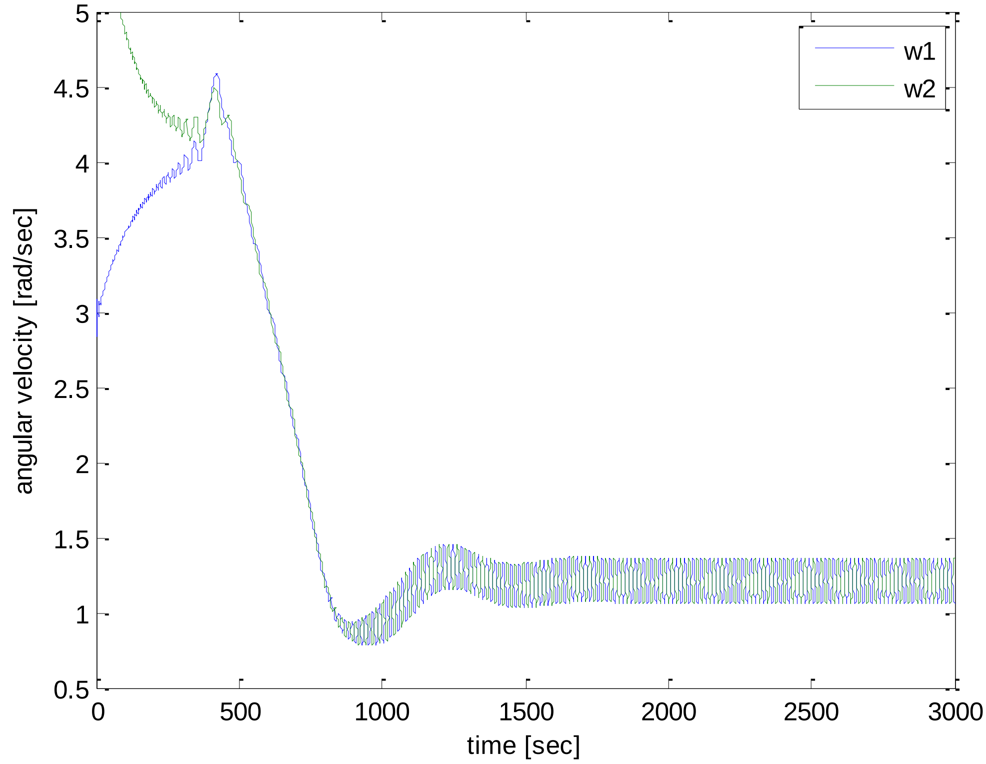
For $\boldsymbol{\delta = \pi}$:

We get: $\lambda = \begin{bmatrix} -1.1932 + 5.3135\mathrm{i} \\ -1.1932 - 5.3135\mathrm{i} \\ -0.1850 + 5.3124\mathrm{i} \\ -0.1850 - 5.3124\mathrm{i} \\ 0.0729 \\ -0.0114 \\ -0.0806 \end{bmatrix}$

This shows that this equilibrium point is not stable.

We will show simulation of system with these parameters set which starts from arbitrary initial point:


365
360
355
350
345
340
335
330
phase [rad]
0
500
1000
1500
2000
2500
3000
time [sec]
deta


w1
w2
angular velocity [rad/sec]
time [sec]
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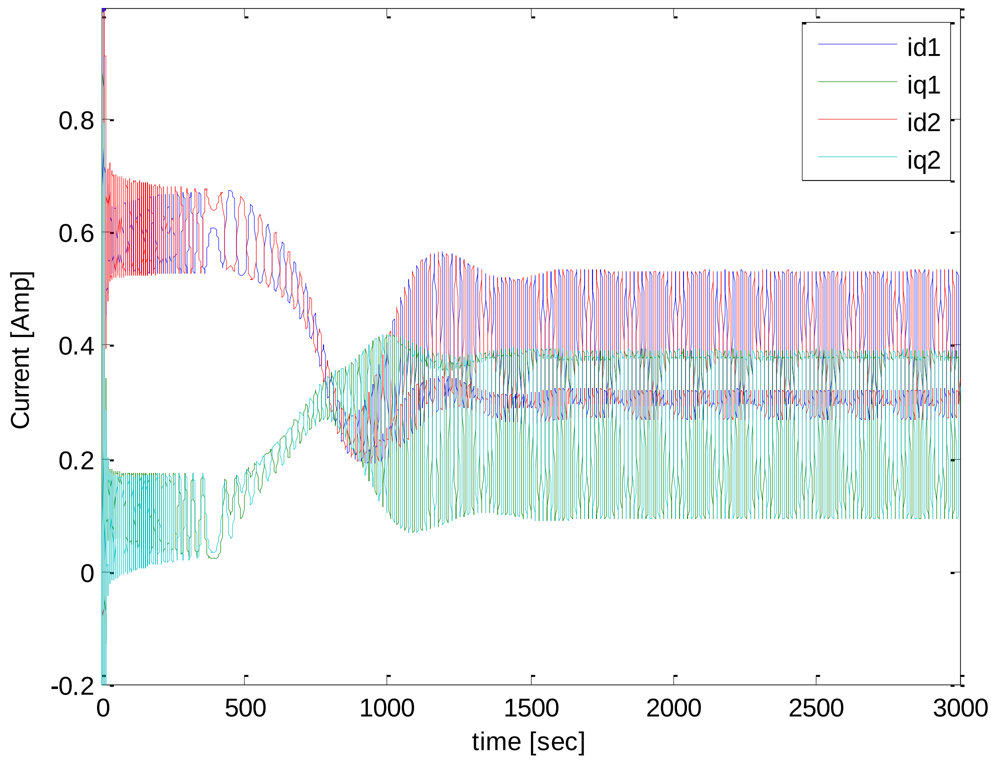
500
1000
1500
2000
2500
3000
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


id1
iq1
id2
iq2
Current [Amp]
time [sec]
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500
1000
1500
2000
2500
3000
-0.2
0
0.2
0.4
0.6
0.8