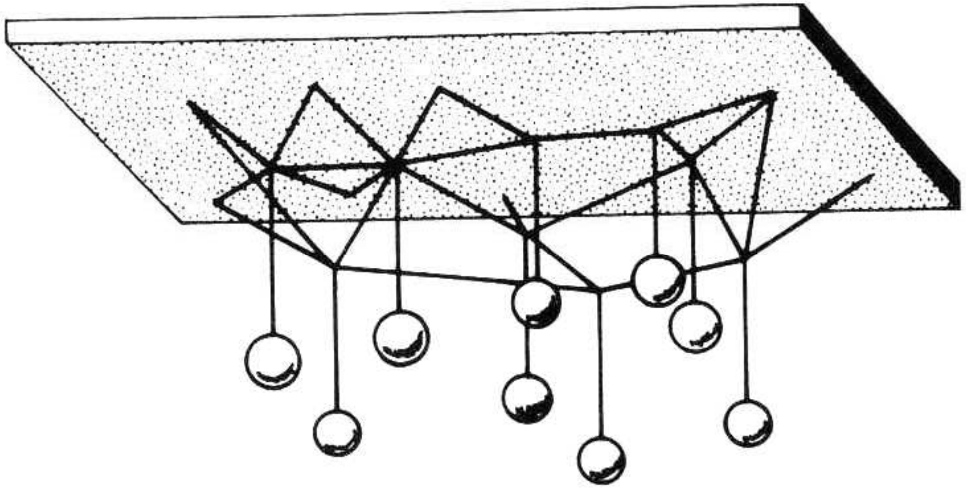
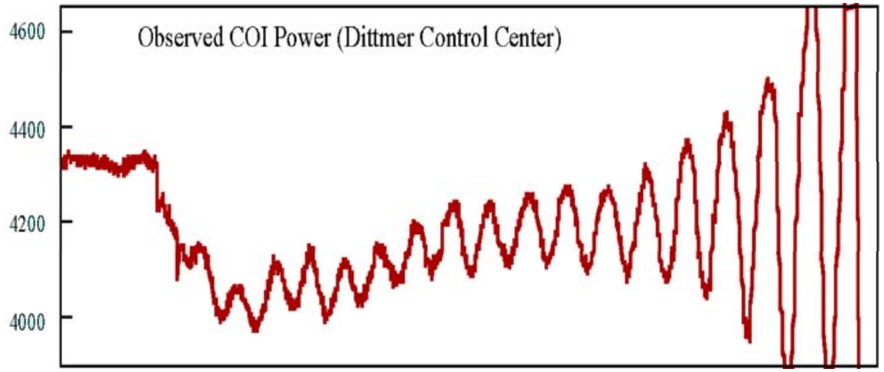
Synchronous generator dynamics



Source: O. Elgerd



Source: BPA

Olof Samuelsson

Outline

– Synchronous generator at steady state

– Synchronization

– Swing equation

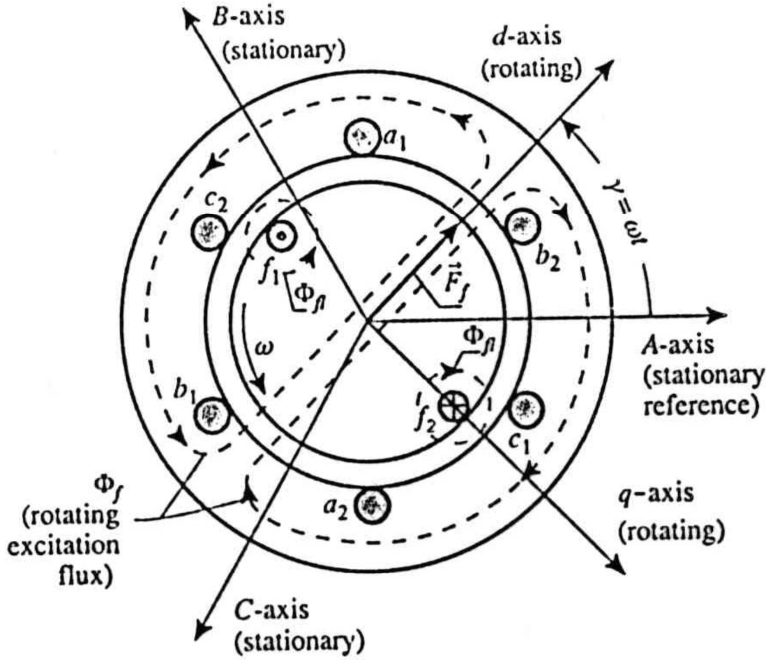
– Transient angle stability

– The Equal Area Criterion

– Small-signal stability

– Frequency control

Synchronous machine



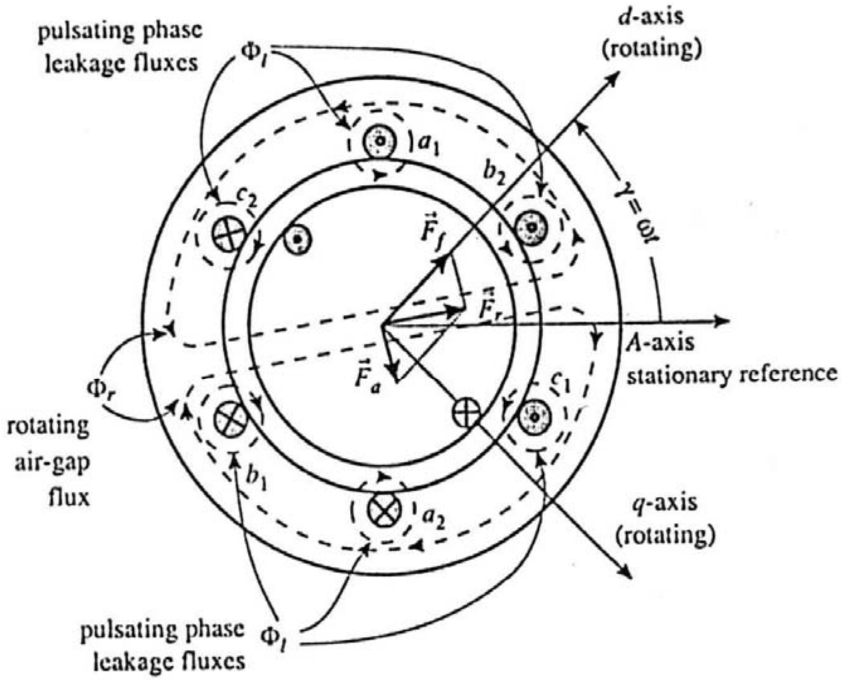
– Rotor

- One field winding fed with DC current

– Stator

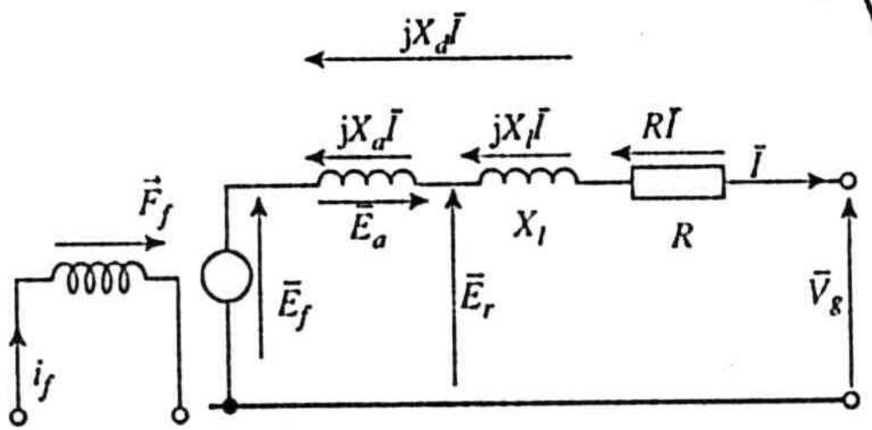
- Three windings 120° apart in space

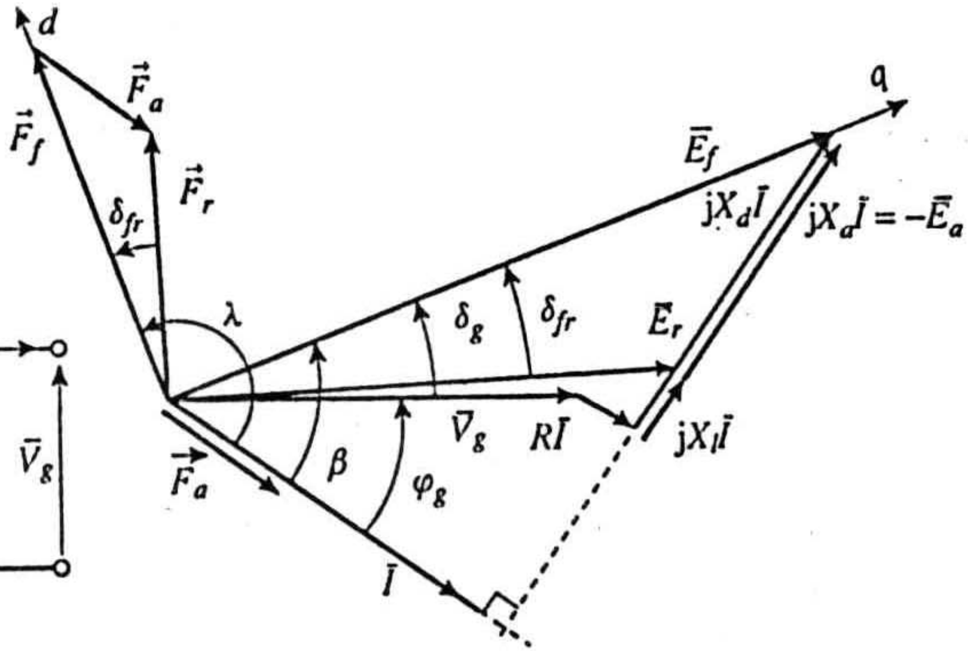
Loaded synchronous generator



- “Armature reaction” flux from load current in stator
- Adds to field flux to form air gap flux
- Stator flux also includes leakage flux

Equivalent circuit 1





Indices:		g	generator
a	armature	l	leakage
d	d-axis	q	q-axis
f	field	r	resulting

Equivalent circuit 2

$E_q(I_f)$	internal voltage (also E_f)
I	stator current
V	terminal voltage

$X_d = X_a + X_l \approx X_a$

$E_q = V + (R_a + jX_d)I$

I lags V by angle $\varphi_{(g)}$

E_q leads V by angle $\delta_{(g)}$

Indices:

a	armature
d	d-axis
f	field
l	leakage
q	q-axis

Load angle δ

δ is a spatial angle between field and air gap flux

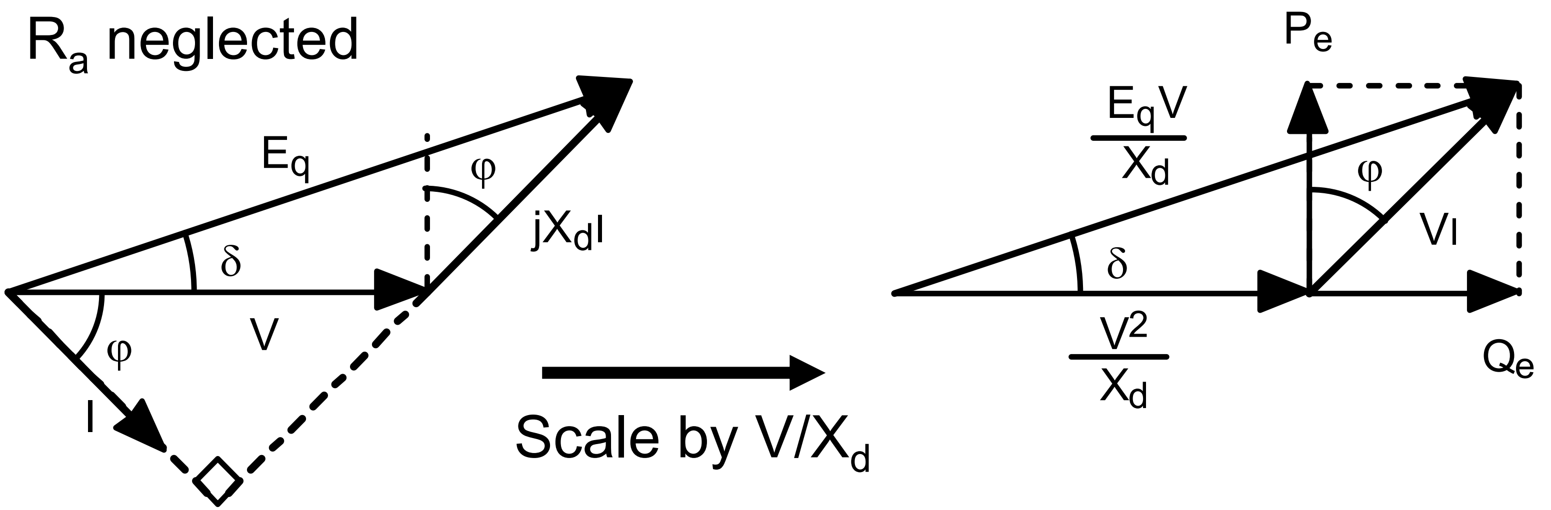
and

a phase angle between E_q and $V+(R_a+jX_l)I$

Note

1. δ is given relative to rotating reference (rotor)
2. δ is a spatial coordinate for a mass – the rotor

Steady state operation

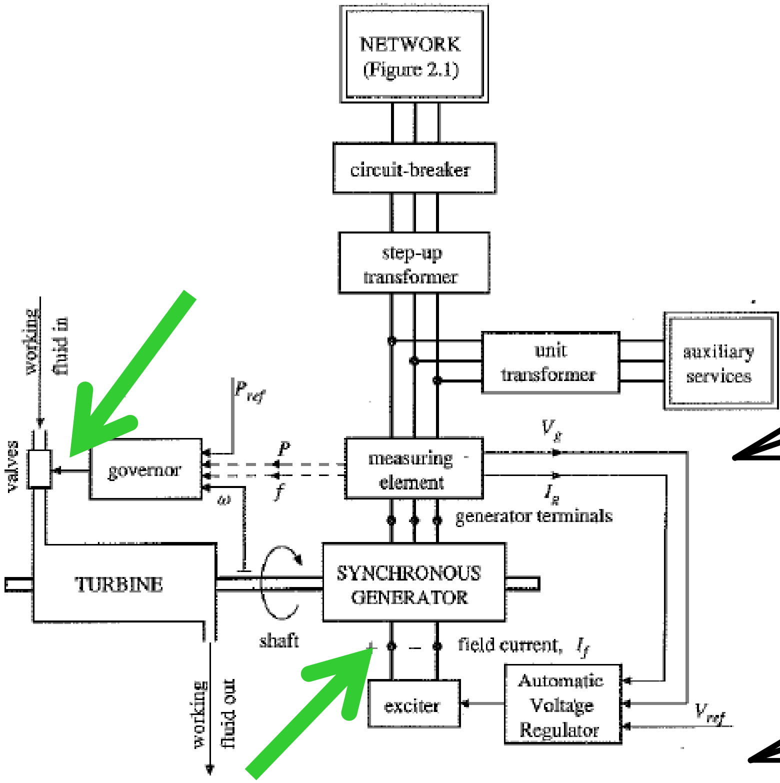


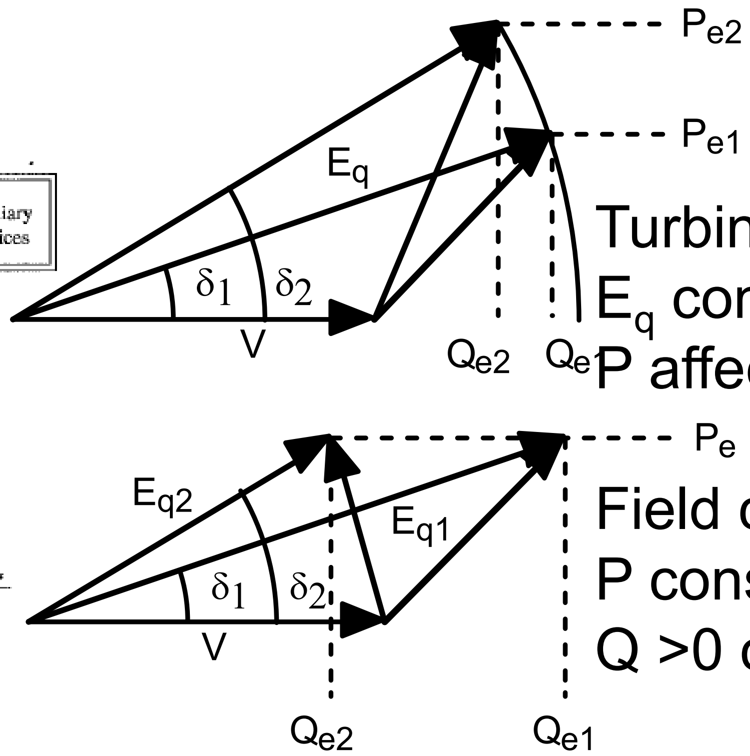
Components of VI:

Vertical $E_qV/X_d\sin\delta = VI\cos\varphi = P_e$

Horizontal $E_qV/X_d\cos\delta - V^2/X_d = VI\sin\varphi = Q_e$

Two control inputs





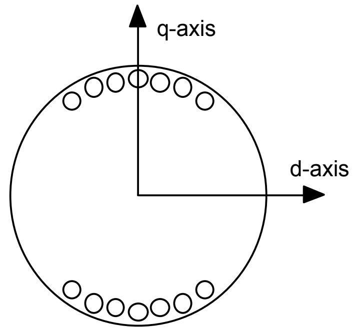
Turbine P → Pe
E_q constant
P affects Q

Field current I_f → E_q
P constant
Q >0 or <0

Synchronous generator rotor types

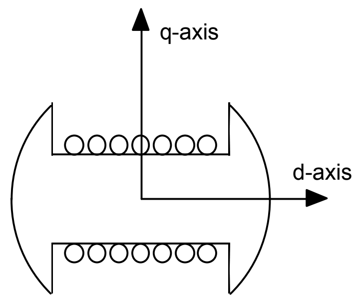
– Round rotor = ”Turbo” rotor

- Two poles
- High speed - 3000 rpm @ 50Hz
- Used with steam turbines (e.g. nuclear)

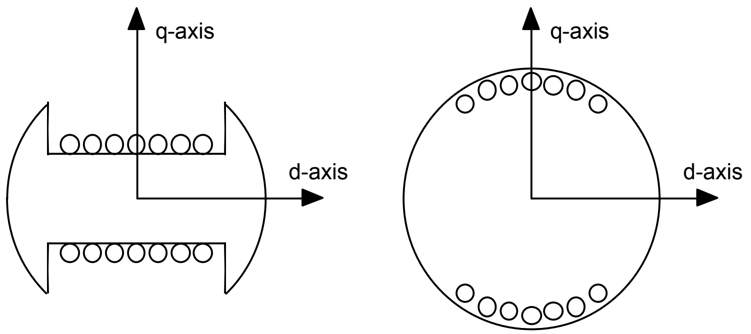


– Salient pole rotor (Swe: utpräglade poler)

- Many poles
- Lower speed - e.g. 150 rpm @ 50 Hz
- Used with hydro turbines
- Gear ratio with more poles: $\omega_{mechanical}=\omega_{electrical}\cdot(2/p)$
- IEA lab generators have four-pole salient pole rotors

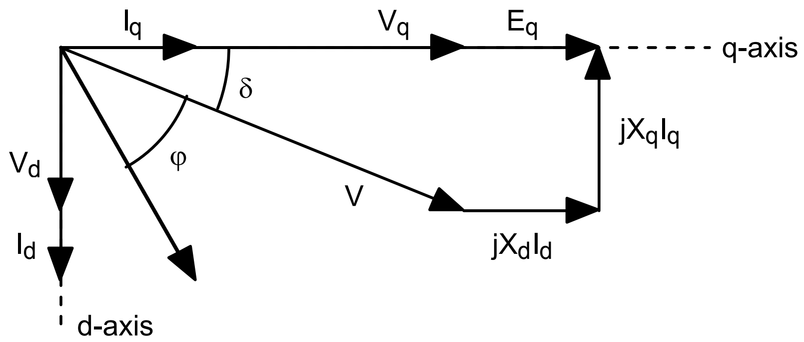


Salient pole rotor



d- and q-axis different

- Geometry
- Flux
- Inductance
- Currents and voltages



$$E_q = V + jX_dI_d + jX_qI_q$$

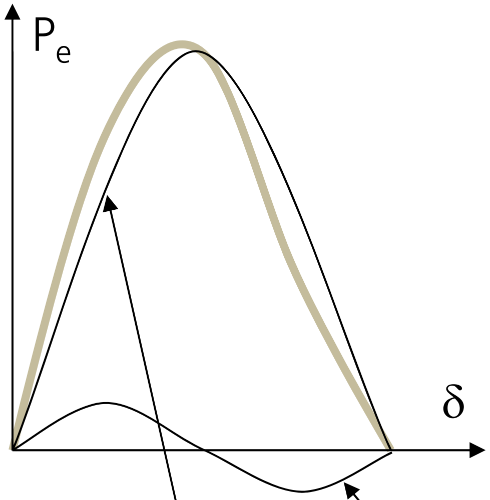
P and Q for salient pole rotor

$P_e + j\,Q_e = (V_d + jV_q)(I_d + jI_q)^*$

$V_d + jV_q = V(\sin\delta + j\cos\delta)$

$I_d = (E_q - V_q)/X_d$

$I_q = V_d/X_q$



$$P_e = \frac{E_q V}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta = P_{\text{field}} + P_{\text{reluctance}}$$

Try $X_d = X_q$!

$$Q_e = \frac{E_q V}{X_d}\cos\delta - V^2\left(\frac{\sin^2\delta}{X_q} + \frac{\cos^2\delta}{X_d}\right)$$

Synchronization

– Connect to an energized network (think Thévenin equiv.)

1. Control prime mover to reach correct speed → right ω_{el}
2. Magnetize field (and armature) winding
3. Make V close to V_{system} (magnitude and angle!)
4. Connect!

– Aim

- Steady-state no-load situation

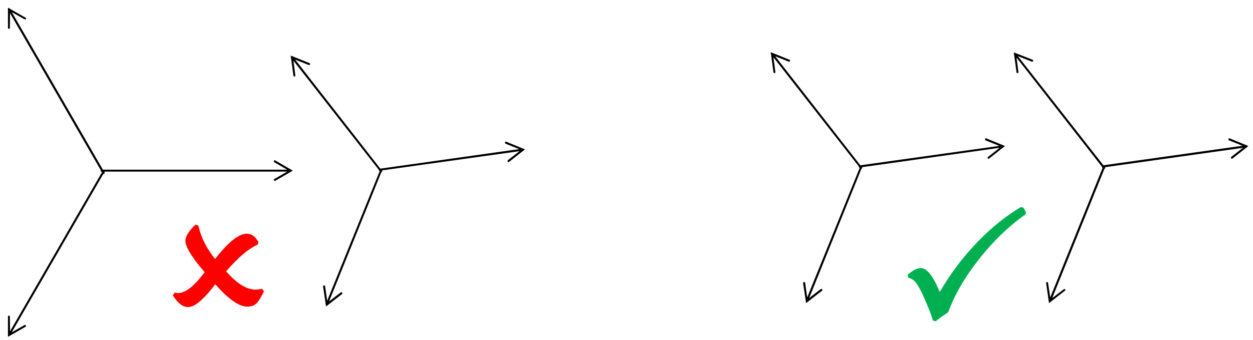
– Careless synchronization

- High currents and high mechanical stress

Synchronization conditions

– V close to V_{system} if the voltages have
  - Same phase order (= wiring correct)
  - Same frequency (= speed correct)
  - Same magnitude (= right magnetization)
  - Same phase (= right timing of connection)

– Think generator Eq and network V as rotating three-phase phasors:



The swing equation

Torque balance for rotor

$$J\frac{d\omega_m}{dt} = T_m - T_e$$

p magnetic rotor poles

$$\omega_m(\text{mech. rad/s}) = \frac{2}{p}\omega_e(\text{elec. rad/s})$$

Multiply torque balance by ω_m

Use ω_e as state and $\omega_e \approx \omega_{s,e}$

Divide by S_{base} to get p.u.

$$\frac{2H}{\omega_{s,e}}\frac{d\omega_e}{dt} = P_m(p.u.) - P_e(p.u.)$$

The inertia constant H

$$H = \frac{\text{Kinetic energy of rotating masses}}{\text{Generator MVA rating}} = \frac{\frac{1}{2} J \omega_m^2}{S_{base}}$$

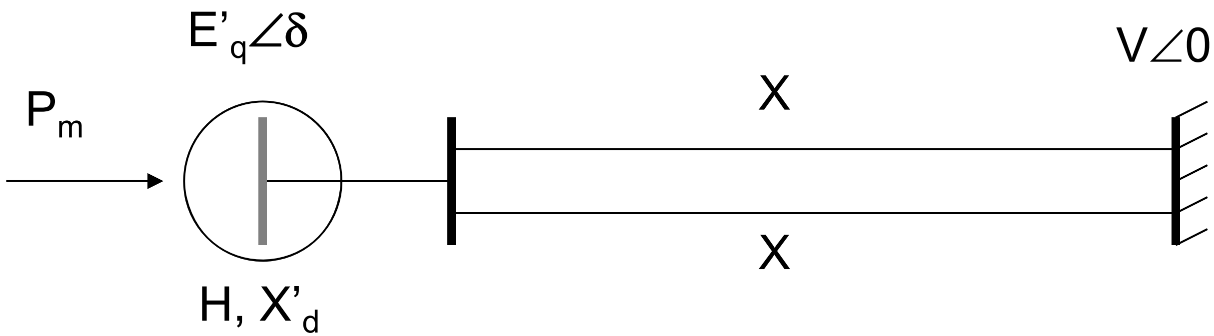
Unit: Ws/VA=s

H on different MVA bases

- Machine base
  - Steam turbines 4-9 s
  - Gas turbines 3-4 s
  - Hydro turbines 2-4 s
  - Synchronous compensator 1-1.5 s
- Common base
  - H ~ generator size (kW-GW)
  - Infinite bus has infinite H

Narrow range!

Single Machine Infinite Bus



<u>"Classical model":</u>

- Fixed E'_q behind X'_d
- Constant P_m
- No damping
- No saliency

<u>"Infinite bus" generator:</u>

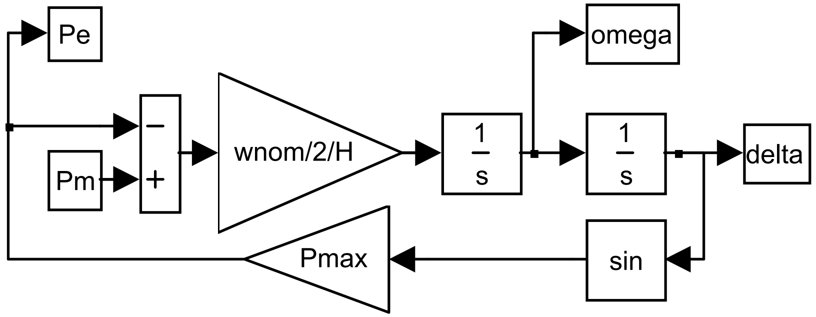
- Infinite H
- Zero impedance
- Fixed voltage $V\angle 0$

$$X_{eq}=X_{line}+X'_d$$

"Classical" dynamic generator model

Synchronous generator connected to infinite bus:

$$\begin{cases} \dfrac{2H}{\omega_{s,e}}\dfrac{d\omega_e}{dt} = P_m - P_e(\delta) \\[2ex] \dfrac{d\delta}{dt} = \omega_e - \omega_{s,e} \end{cases}$$



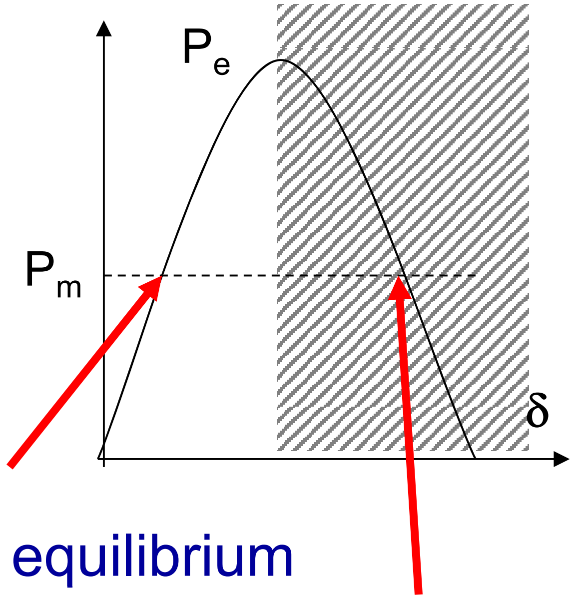
- δ in rad, ω_e in rad/s, $\omega_{s,e}$typically 100π rad/s
- E'_q and X'_d in P_e (δ) for slow transients
- Second order system with poor damping
- Electro-mechanical or "swing" dynamics

Two equilibrium points

$$P_m = P_e(\delta) = \frac{E'_q V}{X_{eq}} \sin \delta$$

Two solutions for δ:

$$\delta = \begin{cases} \delta_0 = \arcsin\left(\dfrac{P_m X_{eq}}{E'_q V}\right) \\ 180° - \delta_0 \end{cases}$$



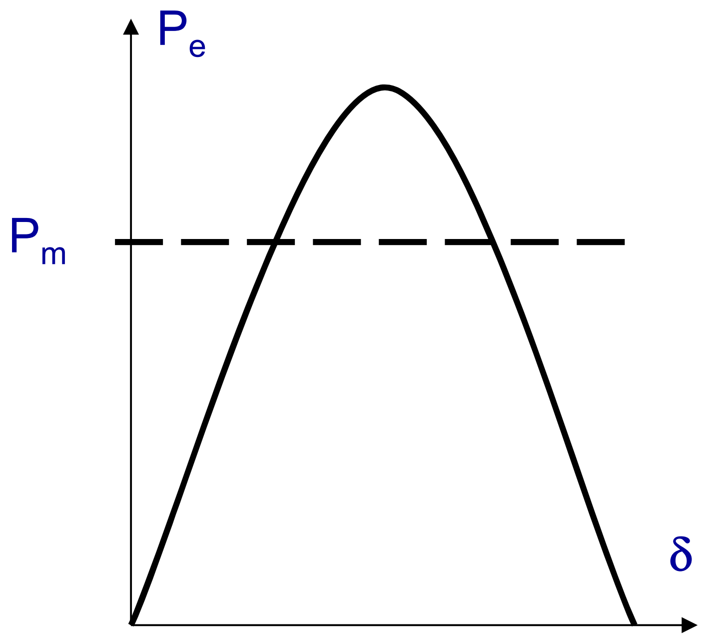
- Synchronizing torque $dP_e/d\delta$
  - $dP_e/d\delta > 0$ for $\delta < 90°$ - stable equilibrium
  - $dP_e/d\delta < 0$ for $\delta > 90°$ - unstable equilibrium

Dynamic response

*	Temporary short-circuit near generator, P_e zero during fault

Response?

1. Second order system
2. No damping
3. Oscillator! δ and ω oscillate (roughly sinusoidally)
4. $\delta(t)$ will lag $\omega(t)$



Demo
sm.mdl
tcl=0.05

Second order response

P_e zero at short-circuit near gen

Step in P_m-P_e

Mechanical states slow

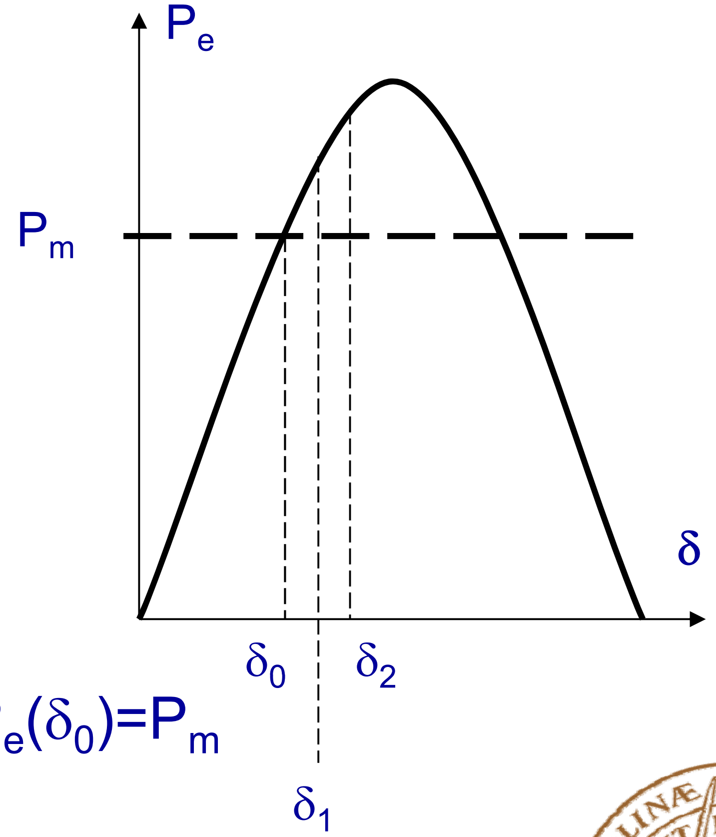
Start at δ_0 and $P_e(\delta_0)$

Acceleration during fault

Fault removed at $\delta=\delta_1$

Overshoot to δ_2 and $P_e(\delta_2)$

Oscillate around equilibrium δ_0 so $P_e(\delta_0)=P_m$



Simulation tcl=0.05, 0.1
PW Example 11.5

Angle stability

δ_0 must be less than steady state limit 90º

δ_2 also has limit – transient angle stability limit

Questions:

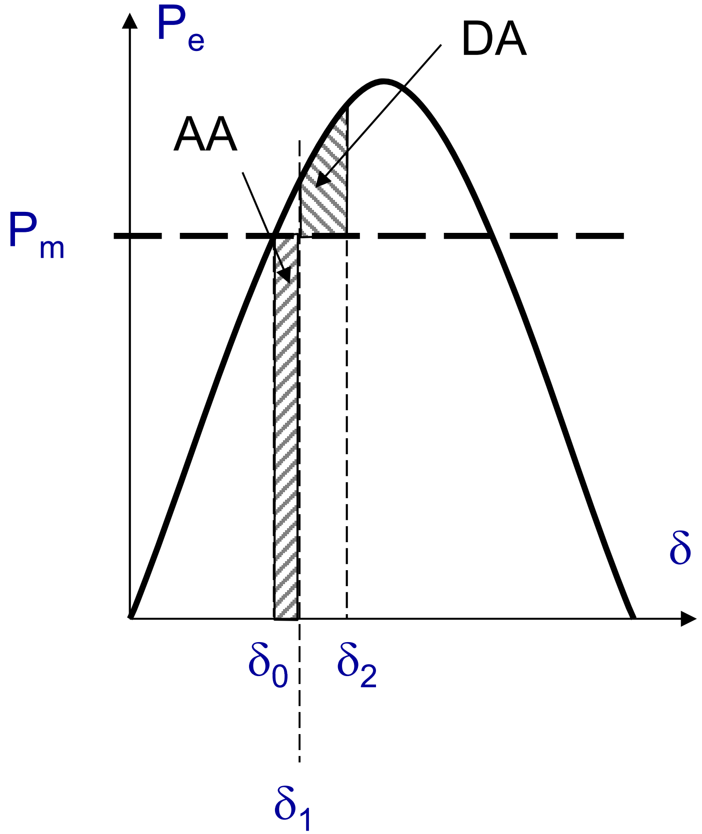
How large can δ_2 be?

What happens when it becomes too large?

What is the largest disturbance that is OK?

Simulation tcl=0.15, 0.1505, 0.151

The Equal Area Criterion



Short-circuit: Pe=zero

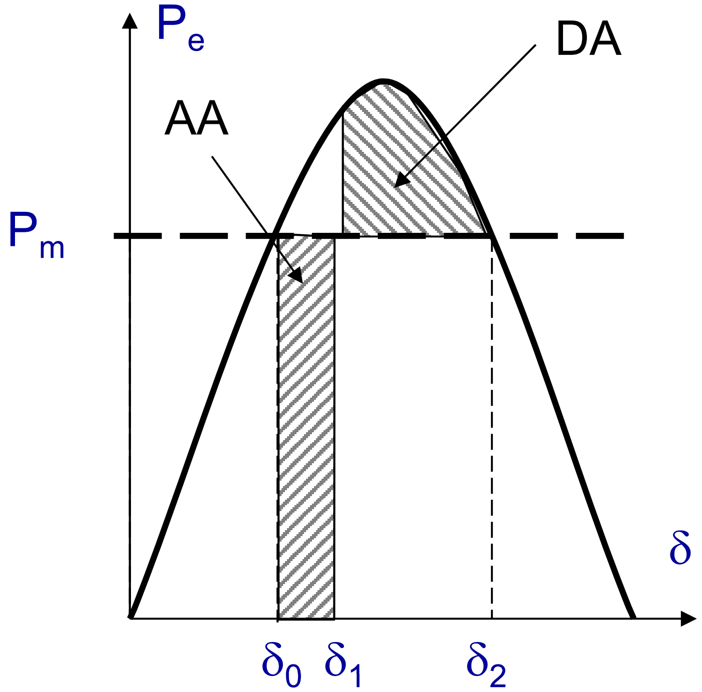
Mark areas between $P_e(\delta)$ and P_m in interval δ_0 to δ_2

Accelerating Area: Below P_m

Decelerating Area : Above P_m

For stable system AA=DA

Transient stability limit



More severe disturbance:

AA larger

Greater δ_2 makes DA larger

Maximum DA at δ_2=180º-δ_0

For larger δ_2 only AA grows…

Simulation

Beyond stability limit

- dω/dt never becomes zero
- Rotor accelerates even more
- Machine <u>transiently unstable</u> = <u>loses synchronism</u>
- Must disconnect and resynchronise

Demo

Equal Area Criterion

- Stability check for **known disturbance**
  Use EAC for δ_2 and check $\delta_2 < \delta_{UEP}$
- **Max disturbance** from stability limit
  Determine disturbance for $\delta_2 = \delta_{UEP}$
- Typical disturbances
  Loss of line, generator or load
  Short-circuit

Stability analysis tools

Analytical – the Equal Area Criterion

- Simple, can be done by hand, but approximate
- Formulated before 1930 by Ivar Herlitz, KTH (First Swedish PhD in engineering)

Time simulation

- Computer application since the beginning
- Voltages and currents as phasors or waveforms
- Multi-machine model with Differential Algebraic Equations
    - Set of **Differential** equations for each generator
    - Power flow for **Algebraic** network equations

Small-signal angle stability

- Linearize at steady state (δ_0, ω_0, P_{m0})
- State space: dx/dt=Ax+Bu
- Compute eigenvalues λ_i of A
- Compute right eigenvectors Φ_i of A
- Applies also to multi-machine models
- Popular application of control theory

Eigenvalues and eigenvectors

- Eigenvalue λ_i:

  Im(λ_i)=resonance oscillation frequency (e.g. 0.35 Hz)

  Re(λ_i)=resonance oscillation damping
  - $\leq$0 for all λ_i system is <u>small-signal stable</u>
  - >0 for any λ_i system is <u>small-signal unstable</u>

- Right eigenvector Φ_i:

  Which generators participate in mode (resonance) i

  E.g. Generators in FI against those in NO and DK

Small-signal damping

- Low >0 for uncontrolled system
- Negative damping from controllers
  - Automatic Voltage Regulators
  - HVDC controllers
- Damping added by dedicated controls
  - Power System Stabilizers (PSS) on generator
  - Power Oscillation Damper (POD) on HVDC or FACTS

FACTS=MW size power electronic devices

System frequency

One eigenvector shows all generator speeds vary together

- The *rigid body* mode – the dynamics of system frequency

All generators synchronize to same ω, but which one?

$$\omega_{system} = \frac{\sum_i H_i \omega_i}{\sum_i H_i}$$

Large generators dominate

Infinite bus is extreme case

- $H=\infty$ so that $\omega_{system}=\omega_{infbus}$

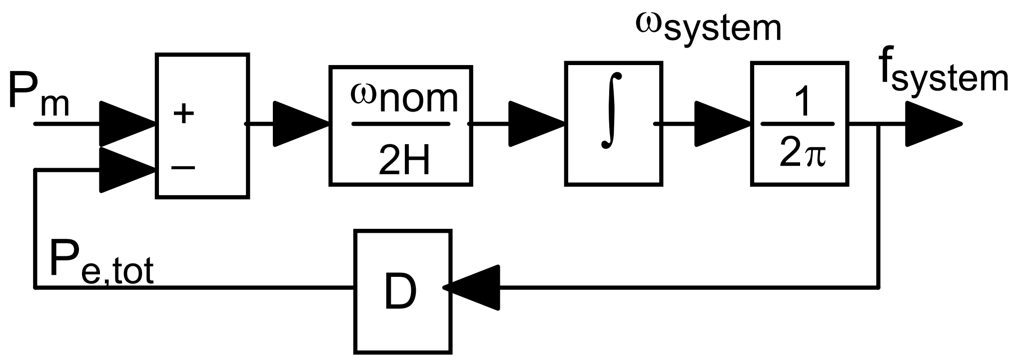
Also *center of inertia* frequency, like center or gravity!

System frequency dynamics

All generators modeled as one with:

$\omega = \omega_{system}$

$$H = \sum_i H_i$$



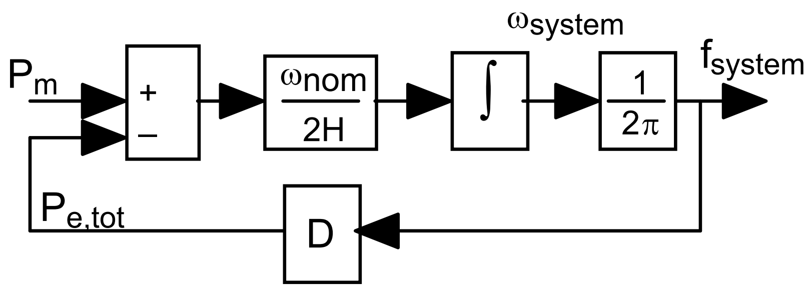
This is single machine, but no infinite bus (to relate δ to)

Electrical load is frequency dependent

Frequency event without control

Generator is suddenly disconnected...

- Step reduction of P_m
- Unbalance: $P_m < P_e$
- ω decreases
- Decrease stops when P_e is reduced to P_m
- Error in ω



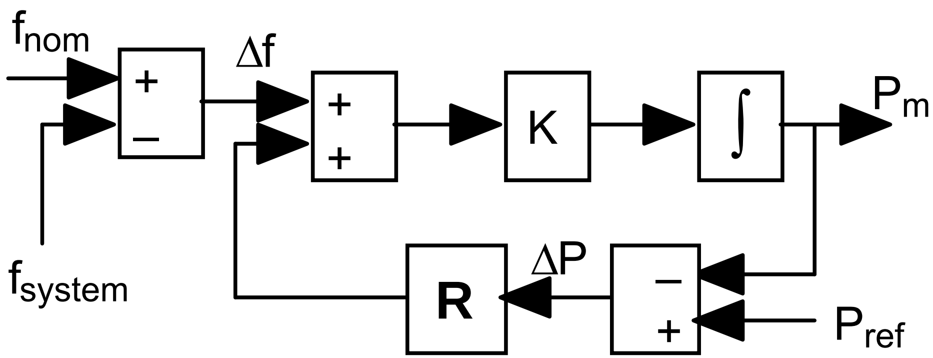
Turbine governor

Proportional frequency control law:

$P_m = P_{ref} + \Delta f / R$

$\Delta f = f_{nom} - f_{system}$

R is <u>speed droop</u>, Hz/MW or p.u./p.u.



R on machine base

All generators usually have the same R given in p.u. on machine base

A disturbance gives same Δf everywhere

All generators do same p.u. contribution

Typical R value is 5%

Δf=0.05 p.u. gives ΔP_m=1 p.u.

PW Example 12.4

R on common base

R for entire system on common base:

$$\frac{1}{R_{total}} = \sum_i \frac{1}{R_i}$$

More generators give greater $1/R_{total}$

In Nordel $1/R_{total} \approx 6000$ MW/Hz

Frequency error tolerance

Instantaneous value of Δf:

± 0.01 Hz in US

± 0.1 Hz in Nordel

± 0.2 Hz in Ireland

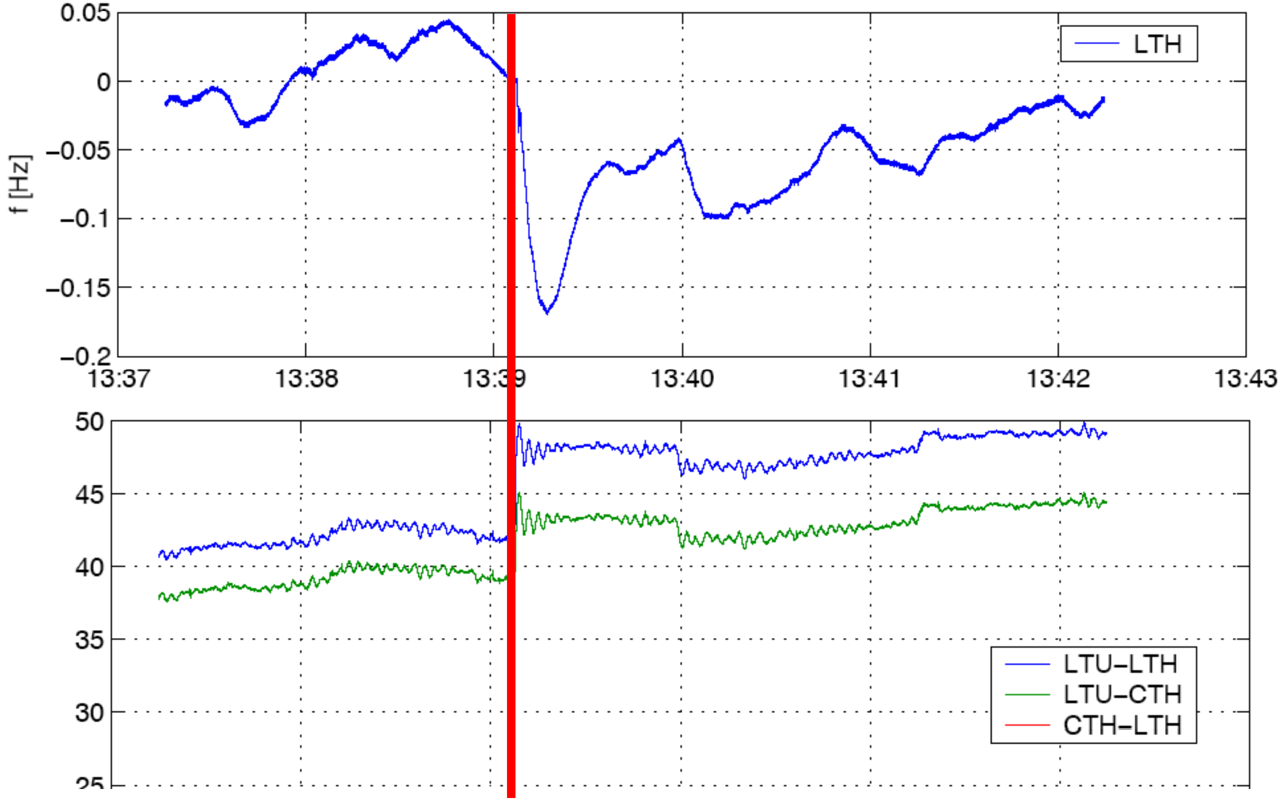
Time integral of :

Time error on clocks <10s in Nordel

600 MW step excites f and angle dynamics

Deviation from 50 Hz at LTH

Phase angle differences (degrees) LTU=Luleå



13:39 13/11: 600 MW generator in Denmark disconnected

Frequency dip and North-South angle oscillations

Conclusions

- Steady state
  - P and Q for round and salient pole rotor
- Transient angle stability
  - Equal Area Criterion and simulations
- Small-signal stability
  - Eigenvalues and eigenvectors
- Frequency dynamics
  - All generators like one
  - Fair sharing: All generators respond equally in p.u. on machine base if same p.u. R