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A warning about the use of reduced models of synchronous generators

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***Abstract*—Synchronous generators are an essential component of the electric grid. Recently, the stability of the electric grid has become an area of high interest and intensive research. We discuss the stability of a single generator connected to an infinite bus, and show that certain reduced models fail to predict the behavior of this system.**

I. Introduction

The AC electricity grid was developed at the end of the XIXth century, and has remained very similar until today. The grid is an enormously complex nonlinear and randomly varying system for which rigorous stability analysis is impossible. Many techniques and models that have been developed to assess the stability of a power grid, using rigorous modelling and system theory techniques mixed with practical shortcuts and simplifying assumptions driven by experience, see for instance [1], [2], [3], [4], [5].

In recent years, due to the increasing penetration of renewable energy resources, which connect to the grid via power converters and produce an intermittent power output, it is not clear whether the traditional models and methods for controlling the power grid will succeed to control it. Therefore, there is an increasing interest in the fundamental mathematical models and stability analysis for the grid, see for instance [5], [6], [7], [8], [9], [10].

The *synchronous generator* (SG) is the main power source of the electricity grid. The mathematical model of a SG (see the earlier references and in addition [11], [12], [13], etc.) is complex and difficult to use as a component when we model a large network. Stability analysis is usually done either by simulation, or analytically on simplified models, in which the SGs are connected in a simple network and each SG is represented by reduced order equations, see for instance [5] and [6]. The reduced model of a SG is often obtained by treating the stator currents as fast variables, thus eliminating them from the state variables via the singular perturbation approach (see, for instance, [14]) and keeping only the rotor angle, the rotor angular velocity and the rotor field as relevant state variables, see for instance [1] and [3].

SGs have the important property that once they synchronize, they tend to remain synchronized even without any control. This is important attribute because the electricity grid must maintain nearly constant frequency, and because the ability of a SG to transfer constant power to the grid exists only when the phase difference between it and the grid is constant. Therefore, it is desirable to know if for a given grid which contains SGs and a loads, the SGs tend to synchronize (for initial states in a reasonably large region) and if yes, if the grid frequency and power flows remains stable. To simplify the stability analysis, it is common to use the Park transformation of the voltages and currents, that maps sinusoidal positive sequence signals into a fixed point in the state space.

The most common reduced model, which is known as the classical model, is a second order non linear model. The reference [10] argues that this model is not realistic enough and that a more complicated (but still reduced) model, which they call *improved swing equation* (ISE), should be used. In this paper we show that even the model proposed in [10] is not reliable for stability analysis, because it can't predict an unstable behavior that is predicted by a more faithful model of the same system.

The rest of this paper is organized as follows. In Section II, a fourth order model of a SG connected to infinite bus and having constant field current is presented. The reduction from the above model to the second order ISE is described in Section III. Simulations and local stability analysis that shows different behavior of the models are given in Section IV.

II. Modeling SG connected to infinite bus

In this section we derive the equations for a SG connected to infinite bus and having a constant field (or rotor) current. We start with deriving the equations for single SG, assuming constant field current, following the notation in [15]. Then we derive the equations for a single SG connected to an infinite bus, following [8], [9].

A. Single SG dynamics

The rotor of a SG is a winding (coil) on a magnetic core that spins inside a circular cavity in the stator, having an angle θ with respect to a reference angle, see Figure 1. We denote its self inductance by L_f, its resistance by R_f, the voltage across its terminals by v_f and the current through it (called the field current) by i_f. We assume for simplicity that L_f is independent of θ and i_f. The stator consists of three identical windings that are connected in a star, with phase shifts of 120^0 (see again Figure 1). We consider that there is no neutral connection and no damper windings. The stator windings can be regarded as

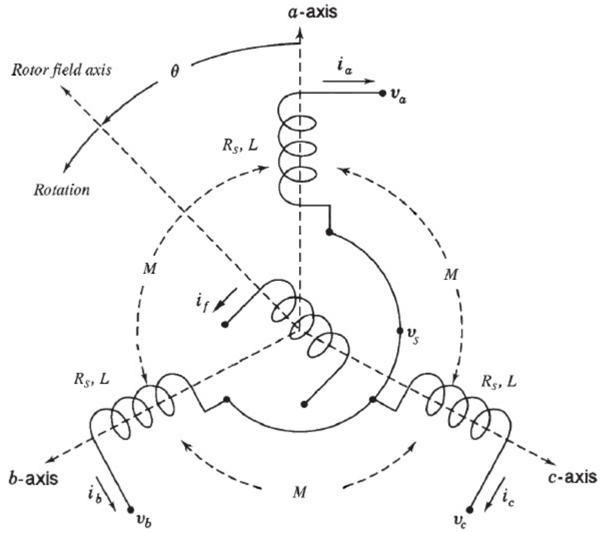

Fig. 1. Structure of an idealized three-phase round rotor synchronous generator, modified from [2, Figure 3.4].

connected coils with self inductance L, mutual inductance $-M$, and resistance R_s (the parameters L_f, R_f, L, M, R_s are positive). We assume no magnetic saturation effects in the iron core and no Eddy currents. The stator terminals are labeled with the letters a,b,c and the vector of voltages on the stator terminals is denoted by $v = [v_a\ v_b\ v_c]^\top$. We denote by v_s the voltage at the unconnected center of the star (see Figure 1) and $v^n = [v_s\ v_s\ v_s]^\top$. We define the vectors

$$\widetilde{\cos}\,\theta = \begin{bmatrix} \cos(\theta) \\ \cos\left(\theta - \frac{2\pi}{3}\right) \\ \cos\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}, \quad \widetilde{\sin}\,\theta = \begin{bmatrix} \sin(\theta) \\ \sin\left(\theta - \frac{2\pi}{3}\right) \\ \sin\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}.$$

We denote the stator flux by $\Phi = [\Phi_a\ \Phi_b\ \Phi_c]^\top$, the stator currents by $i = [i_a\ i_b\ i_c]^\top$ and the rotor current by i_f.

The mutual inductance between the rotor coil and each of the stator coils varies with the rotor angle θ as follows:

$$\begin{bmatrix} M_{a,f} \\ M_{b,f} \\ M_{c,f} \end{bmatrix} = M_f \widetilde{\cos}\,\theta,$$

where $M_f > 0$ is a constant. The vector of flux linkages of the stator windings is

$$\Phi = \begin{bmatrix} L & -M & -M \\ -M & L & -M \\ -M & -M & L \end{bmatrix} i + M_f i_f \widetilde{\cos}\,\theta.$$

Since there is no neutral line, $i_a + i_b + i_c = 0$, so the previous equation can be rewritten as

$$\Phi = L_s i + M_f i_f \widetilde{\cos}\,\theta,$$

where $L_s = L + M$. We will assume that the rotor current is constant (or equivalently, the rotor is composed of a permanent magnet). The stator voltages satisfy

$$v - v^n = -R_s i - \frac{\mathrm{d}\Phi}{\mathrm{d}t} = -R_s i - L_s \frac{\mathrm{d}i}{\mathrm{d}t} + e, \tag{1}$$

where $e = [e_a\ e_b\ e_c]^\top$ is the back electromotive force (EMF) due to the rotor movement (also called synchronous internal voltage), given by

$$e = M_f i_f \dot{\theta} \widetilde{\sin}\,\theta. \tag{2}$$

For a SG with no load, the voltages at each terminal will be sinusoidal functions. In order to represent the voltages and currents in a more convenient way, we apply the Park transformation with respect to the rotor angle:

$$x_{dq0} = \begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = U(\theta) \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = U(\theta) x_{abc},$$

where x_{abc} is a vector in abc coordinates, x_{dq0} is the same vector in the $dq0$ coordinates, and $U(\theta)$ is the following unitary matrix:

$$U(\theta) = \sqrt{\frac{3}{2}} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{4\pi}{3}) \\ \sqrt{1/2} & \sqrt{1/2} & \sqrt{1/2} \end{bmatrix}.$$

Applying the Park transformation to (1) leads to

$$U(\theta)(v - v^n) - U(\theta) e = -R_s U(\theta) i - L_s U(\theta) \frac{\mathrm{d}i}{\mathrm{d}t}. \tag{3}$$

Now we use that, denoting $i_{dq0} = U(\theta) i$,

$$\frac{\mathrm{d}i_{dq0}}{\mathrm{d}\theta} = U(\theta) \frac{\mathrm{d}i}{\mathrm{d}\theta} + \frac{\mathrm{d}U(\theta)}{\mathrm{d}\theta} i = U(\theta) \frac{\mathrm{d}i}{\mathrm{d}\theta} + \begin{bmatrix} i_q \\ -i_d \\ 0 \end{bmatrix}.$$

This implies that

$$\dot{i}_{dq0} = \frac{\mathrm{d}i_{dq0}}{\mathrm{d}\theta} \cdot \frac{\mathrm{d}\theta}{\mathrm{d}t} = U(\theta) \dot{i} + \omega \begin{bmatrix} i_q \\ -i_d \\ 0 \end{bmatrix},$$

where $\omega = \dot{\theta}$. We rewrite (3) as follows:

$$L_s \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} - L_s \omega \begin{bmatrix} i_q \\ -i_d \\ 0 \end{bmatrix} = -R_s \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \begin{bmatrix} e_d - v_d \\ e_q - v_q \\ \tilde{v} \end{bmatrix}, \tag{4}$$

where $\tilde{v} = e_0 - v_0 + v_0^n$. Here we have used that (obviously) $v_d^n = v_q^n = 0$. Since there is no neutral connection, $i_0 = 0$, hence $\tilde{v} = 0$. Applying the Park transformation to (2) gives

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = -\sqrt{\frac{3}{2}} M_f \begin{bmatrix} 0 \\ \omega i_f \end{bmatrix}.$$

We denote $m = \sqrt{\frac{3}{2}} M_f$. If we substitute the last formula into (4), we get

$$L_s \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = -R_s \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega L_s \begin{bmatrix} i_q \\ -i_d \end{bmatrix} - m \begin{bmatrix} 0 \\ \omega i_f \end{bmatrix} - \begin{bmatrix} v_d \\ v_q \end{bmatrix}. \tag{5}$$

The rotational dynamics of the rotor is given by

$$J\dot{\omega} = T_m - T_e - D_p \omega, \tag{6}$$

where J is the moment of inertia of the rotor, $T_m - D_p\omega$ is is the mechanical torque coming from the prime mover, T_e is the electromagnetic torque developed by the generator, and $D_p > 0$ expresses the combination of three effects: the frequency droop control of the prime mover, the damper windings, and the viscous friction. The frequency droop (the largest of the three effects) is used in order to stabilize the frequency of the grid, see [1], [6], [7], [15]. T_e can be found using energy consideration. It is easy to see that the magnetic energy stored in the generator is

$$E_{mag} = \frac{1}{2}\left(\langle i, L_s i\rangle + L_f i_f^2\right) + M_f i_f \langle i, \widetilde{\cos}\theta\rangle .$$

The electromagnetic torque can be calculated as follows:

$$T_e = \frac{\partial E_{mag}}{\partial \theta}|_{\Phi,\Phi_f\ const.} = -\frac{\partial E_{mag}}{\partial \theta}|_{i,i_f\ const.}$$

(see [15]), whence

$$T_e = M_f i_f \left\langle i, \frac{\mathrm{d}\tilde{\cos}\theta}{\mathrm{d}\theta}\right\rangle = M_f i_f \langle i, \widetilde{\sin}\theta\rangle = -m i_f i_q .$$

Using (5), (6) and the last formula, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} L_s i_d \\ L_s i_q \\ J\omega \end{bmatrix} = \begin{bmatrix} -R_s & \omega L_s & 0 \\ -\omega L_s & -R_s & -m i_f \\ 0 & m i_f & -D_p \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} + \begin{bmatrix} -v_d \\ -v_q \\ T_m \end{bmatrix}.$$

This third order nonlinear dynamical system represents the dynamics of a single SG with constant field current, if we ignore the rotor angle θ.

B. Model of a single generator connected to an infinite bus

In this subsection we develop a model for single SG with constant field current and connected to an infinite bus, following [8], [9]. The infinite bus is modeled as a three phase AC voltage source, i.e, the infinite bus is not affected by the synchronous generator that is connected to it. The justification for this model is that the influence of a single SG on a grid is very small. In general, the line that connects the SG to the grid has its impedance that can be modeled (in most cases) as a resistance and an inductance in series, but these values can simply be added to the parameters R_s and L_s of the SG.

The infinite bus voltage (at the SG terminals) is

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} V \begin{bmatrix} \cos(\theta_g) \\ \cos(\theta_g - \frac{2\pi}{3}) \\ \cos(\theta_g - \frac{4\pi}{3}) \end{bmatrix}, \tag{7}$$

where V is the grid line voltage magnitude, and θ_g is the grid angle. Let us define the *power angle* δ which represents the difference between the grid and the synchronous generator angles:

$$\delta = \theta - \theta_g .$$

After applying the Park transformation to (7), we get

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = U(\theta)\sqrt{\frac{2}{3}} V \begin{bmatrix} \cos(\theta_g) \\ \cos(\theta_g - \frac{2\pi}{3}) \\ \cos(\theta_g - \frac{4\pi}{3}) \end{bmatrix} = -V \begin{bmatrix} \sin\delta \\ \cos\delta \\ 0 \end{bmatrix}. \tag{8}$$

It is easy to see that the dynamics of δ is

$$\dot{\delta} = \omega - \omega_g ,$$

where ω_g is the grid frequency. Substituting (8) and the above equation into (4), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} L_s i_d \\ L_s i_q \\ J\omega \\ \delta \end{bmatrix} = \begin{bmatrix} -R_s & \omega L_s & 0 & 0 \\ -\omega L_s & -R_s & -m i_f & 0 \\ 0 & m i_f & -D_p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} V\sin\delta \\ V\cos\delta \\ T_m \\ -\omega_g \end{bmatrix} \tag{9}$$

We refer to this fourth order nonlinear dynamical system as the *fourth order model* (FOM).

III. The improved swing equation

In this section, we show the relation between the ISE and the FOM. We start with the FOM, and by a model reduction process, we get the ISE model. We apply ideas from singular perturbation analysis (see for instance [14]). The FOM (9) has the following structure:

$$\Lambda \dot{z} = F(z) ,$$

where Λ is a diagonal 4×4 matrix with positive coefficients on the diagonal. Because the first two coefficients on the diagonal of Λ are equal to L_s, which in some sense can be regarded as being small, we rewrite this dynamical system in the following form:

$$\begin{cases} \dot{x} = f(x,y,\varepsilon) , \\ \varepsilon\dot{y} = g(x,y,\varepsilon) . \end{cases}$$

By doing this, we have separated the state variables into a vector of fast variables, denoted by y, and another of slow variables, denoted by x. Here, $\varepsilon > 0$ is the small parameter.

We assuming that ε is very small, meaning that for each x, the vector y converges to a temporary equilibrium value (that depends on x) much faster than the rate of change of x. In our specific case, $\varepsilon = L_s$, $x = \begin{bmatrix} \omega \\ \delta \end{bmatrix}$ and $y = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$. By taking the first and second lines from (9), we have

$$g(x,y,\varepsilon) = \begin{bmatrix} -R_s & \omega L_s \\ -\omega L_s & -R_s \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} V\sin\delta \\ V\cos\delta - m i_f \omega \end{bmatrix}.$$

Our assumption that ε is very small means that the subsystem $\varepsilon\dot{y} = g(x,y,\varepsilon)$ is much faster than $\dot{x} = f(x,y,\varepsilon)$ and it is also stable, so that it will reach its temporary equilibrium almost instantly compared to the slow movement of x. The temporary equilibrium point $\hat{y}(x,\varepsilon)$ is the solution of $g(x,\hat{y},\varepsilon) = 0$. The solution of this linear equation (in $\hat{y}$) is

$$\hat{y} = -\begin{bmatrix} -R_s & \omega L_s \\ -\omega L_s & -R_s \end{bmatrix}^{-1}\begin{bmatrix} V\sin\delta \\ V\cos\delta - m i_f \omega \end{bmatrix}$$
$$= \begin{bmatrix} \frac{R_s V\sin\delta - L_s\omega\left(m i_f\omega - V\cos\delta\right)}{L_s^2\omega^2 + R_s^2} \\ \frac{-R_s\left(m i_f\omega - V\cos\delta\right) - L_s V\omega\sin\delta}{L_s^2\omega^2 + R_s^2} \end{bmatrix}.$$

Assuming that R_s is small so that the terms containing R_s are negligible, we obtain the following approximation of $\hat{y}$:

$$\hat{y}_{app} = \begin{bmatrix} \hat{i}_d \\ \hat{i}_q \end{bmatrix} = \begin{bmatrix} \frac{V\cos\delta - m i_f\omega}{L_s\omega} \\ -\frac{V\sin\delta}{L_s\omega} \end{bmatrix}. \tag{10}$$

We substitute $\hat{y}_{app}$ obtained above into $\dot{x} = f(x,y,\varepsilon)$ (in place of y) to get the reduced model

$$\begin{cases} J\dot{\omega}\omega + D_p\omega^2 = -\frac{mi_f V\sin\delta}{L_s} + T_m\omega\,, \\ \dot{\delta} = \omega - \omega_g\,. \end{cases} \tag{11}$$

The power absorbed from the prime mover can be expressed approximately as

$$P_m = (T_m - D_p\omega_g)\omega\,. \tag{12}$$

This crude approximation cannot be justified other than as a means to obtain the so-called improved swing equation, as we shall see below. A more precise expression of P_m is

$$P_m = (T_m - d_p\omega)\omega\,,$$

where $d_p > 0$ is the part of D_p due to the droop control of the prime mover (so that $T_m - d_p\omega$ is the prime mover torque).

If we express $T_m\omega$ from (12) and substitute into the model (11), we obtain

$$\begin{cases} J\dot{\omega}\omega + D_p\omega(\omega-\omega_g) = P_m - \frac{mi_f V\sin\delta}{L_s}\,, \\ \dot{\delta} = \omega - \omega_g\,. \end{cases}$$

This model is known as the *improved swing equation* (ISE) model, see [10], [16]. Note that we did many different approximations to derive it. The references just cited normally assume that P_m is constant, which then causes T_m to be a function of ω. Our perspective is to view T_m as a constant parameter.

Let us see what happens if we do not use the crude approximation (12) and instead return to (11). If we divide the first equation by ω and substitute $\dot{\omega} = \ddot{\delta}$, then we obtain

$$J\ddot{\delta} + D_p\dot{\delta} + \frac{mi_f V\sin\delta}{L_s\left(\omega_g + \dot{\delta}\right)} = T_m - D_p\omega_g. \tag{13}$$

This is an ugly nonlinear ODE. If $R_s = 0$ then its equilibrium points are exactly the same as for the FOM and if we approximate $\omega_g + \dot{\delta} \approx \omega_g$, then we get the classical swing equation. We think that the reduced model (13) (which is equivalent to (11)) is closer to the FOM than the ISE.

IV. Simulations

In this section we present simulations which demonstrate that the ISE model is a good approximate model in many cases. We also show that there are cases in which there is a significant mismatch between the behavior suggested by the ISE model and the FOM. In each simulation result, the behavior of i_d and i_q over time is described for both the FOM and the ISE models. Note that for the ISE model, we use (10) to estimate these currents. We plot the frequency ω and the power angle δ as functions of time for these two models. In all the simulations, variables with a hat correspond to the reduced model. All simulations assume constant P_m. We did the same simulations also under the assumption of constant T_m and the results are very similar.

The first two simulations concern small SGs of 5KW and 1MW, respectively, with parameters taken from [17]. These models were originally meant to represent synchronverters, which are a type of inverters, but from the point of view of the model, this does not matter.

A. 5KW SG

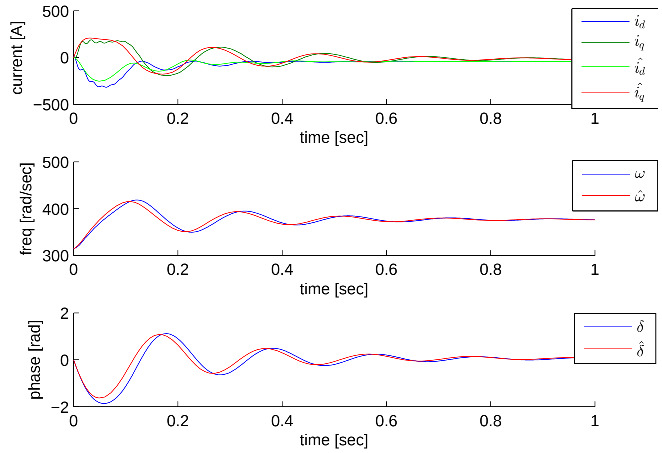

Fig. 2. Simulations for a 5KW SG connected to an infinite bus, using both models. For ω and δ the results are very similar.

As shown in Figure 2, simulations indicates that for the 5KW SG, the behavior of the FOM and the ISE is almost the same. Although the currents at the reduced don't have a ripple that the FOM has, both models converge at the same rate with the same oscillations to the same equilibrium point (The parameters for this simulation are $J = 0.2$ [kgm^2], $D_p = 1.7$ [J/sec], $R_s = 0.152$ [Ω], $L_s = 4.4$ [mH], $mi_f = 1.05$ [$Vsec$], $\omega_g = 60\cdot 2\pi$ [rad/sec], $V = 330$ [V], $Pm = 5$ [kW]).

B. 1MW SG

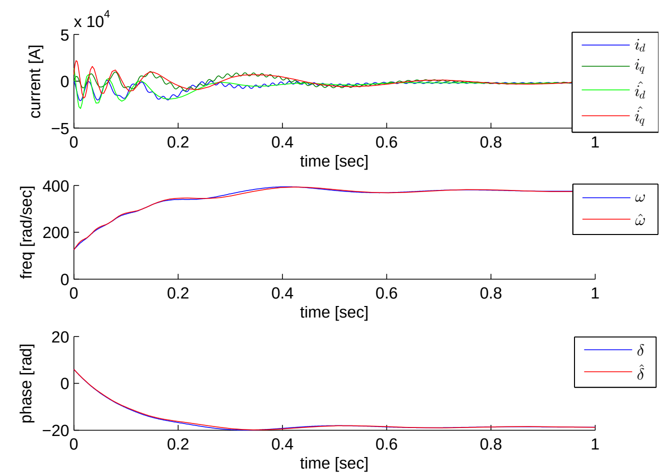

Fig. 3. Simulation for a 1MW SG on an infinite bus, both models

As shown in Figure 3, simulations show that for the 1MW SG, the behavior of the FOM and the ISE are still very similar. Although the FOM currents are much more rippled than the ISE currents, both models converge at the same rate with the same oscillations to the same equilibrium point (The parameters for this simulation are $J = 40.05$ $[kgm^2]$, $D_p = 337$ $[J/sec]$, $R_s = 0.4$ $[\Omega]$, $L_s = 18$ $[mH]$, $mi_f = 1.79$ $[Vsec]$, $\omega_g = 60 \cdot 2\pi$ $[rad/sec]$, $V = 563$ $[V]$, $Pm = 1$ $[MW]$).

C. Non stable behavior of the reduced model

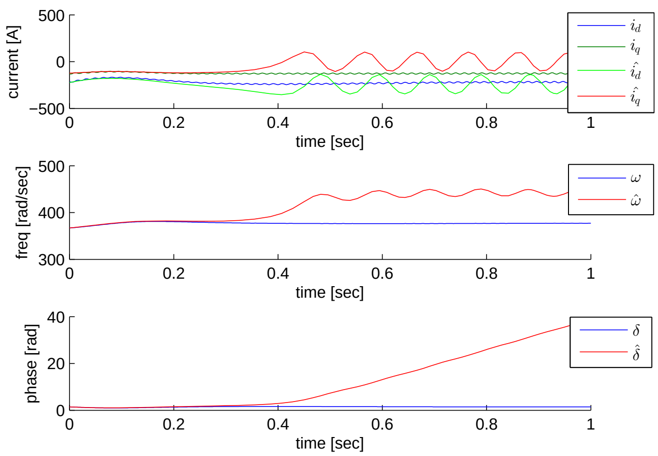

Fig. 4. A simulation example showing different behavior for the FOM and the ISE reduced model for a 50 KW SG

As shown in Figure 4, simulations show that for other parameters set (the parameters for this simulation are $J = 0.2$ $[kgm^2]$, $D_p = 1.7$ $[J/sec]$, $R_s = 0.152$ $[\Omega]$, $L_s = 4.4$ $[mH]$, $mi_f = 1.05$ $[Vsec]$, $\omega_g = 60 \cdot 2\pi$ $[rad/sec]$, $V = 200$ $[V]$, $P_m = 50$ $[KW]$) the behavior of the FOM and the ISE is significantly different. The FOM is stable, since the eigenvalues of the Jacobian around the equilibrium point are $-11.41 \pm 376.9i$, $-508 \pm 837i$. The ISE doesn't have equilibrium point for these parameters. The ISE model have equilibrium point only if $|\sin \delta^e| = \left| \frac{P_m L_s}{mi_f V} \right| \leq 1$ holds.

D. An example for different region of attraction

As shown in Figure 5, simulations show that for some parameters set (The parameters for this simulation are $J = 0.2$ $[kgm^2]$, $D_p = 1.7$ $[J/sec]$, $R_s = 0.152$ $[\Omega]$, $L_s = 1.05$ $[mH]$, $mi_f = 3.5$ $[Vsec]$, $\omega_g = 60 \cdot 2\pi$ $[rad/sec]$, $V = 330$ $[V]$, $P_m = 5$ $[KW]$) the initial condition of this simulation is within the region of attraction of the reduced model, but outside the region of attraction of the FOM. That cause the FOM to diverge while the ISE converges to the equilibrium point.

V. Conclusions

We have investigated the relation between the FOM of a SG with constant field current connected to an infinite bus and the reduced ISE model. Simulations show that sometimes the FOM shows a behavior which does not match the one suggested by the ISE model. We have also seen that for some choices of the parameters the reduced model behaves very similarly to the FOM.

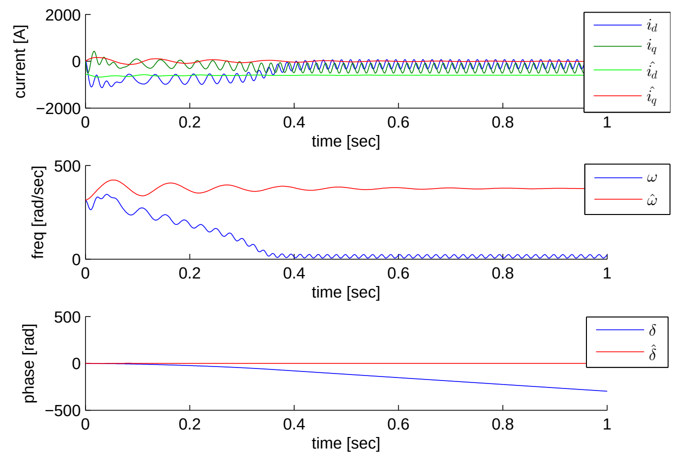

Fig. 5. Simulation example that shows different behavior for the full and the reduced models, for a 5 KW SG.

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