Robust Droop Controller for Accurate Proportional Load Sharing Among Inverters Operated in Parallel

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***Abstract*—In this paper, the inherent limitations of the conventional droop control scheme are revealed. It has been proven that parallel-operated inverters should have the same per-unit impedance in order for them to share the load accurately in proportion to their power ratings when the conventional droop control scheme is adopted. The droop controllers should also generate the same voltage set-point for the inverters. Both conditions are difficult to meet in practice, which results in errors in proportional load sharing. An improved droop controller is then proposed to achieve accurate proportional load sharing without meeting these two requirements and to reduce the load voltage drop due to the load effect and the droop effect. The load voltage can be maintained within the desired range around the rated value. The strategy is robust against numerical errors, disturbances, noises, feeder impedance, parameter drifts and component mismatches. The only sharing error, which is quantified in this paper, comes from the error in measuring the load voltage. When there are errors in the voltage measured, a fundamental tradeoff between the voltage drop and the sharing accuracy appears. It has also been explained that, in order to avoid errors in power sharing, the global settings of the rated voltage and frequency should be accurate. Experimental results are provided to verify the analysis and design.**

***Index Terms*—Droop control, microgrids, parallel operation of inverters, proportional load sharing.**

I. Introduction

NOWADAYS, more and more distributed generation and renewable energy sources, e.g., wind, solar and tidal power, are connected to the public grid via power inverters. They often form microgrids before being connected to the public grid [1]–[4]. Due to the availability of high current power electronic devices, it is inevitable that several inverters are needed to be operated in parallel for high-power and/or low-cost applications. Another reason is that parallel-operated inverters provide system redundancy and high reliability needed by critical customers. A natural problem for parallel-operated inverters is how to share the load among them. A key technique is to use the droop control [5]–[13], which is widely used in conventional power generation systems [14]. The advantage is that no external communication mechanism is needed among the inverters [10], [15]. This enables good sharing for linear and/or nonlinear loads [10], [16]–[20]. In some cases, external communication means are still adopted for load sharing [21] and restoring the microgrid voltage and frequency [3], [9].

The equal sharing of linear and nonlinear loads has been intensively investigated [5], [6], [16], [18], [19] and high accuracy of equal sharing can be achieved. A voltage bandwidth droop control was used to share nonlinear loads in [10] and a small signal injection method was proposed to improve the reactive power sharing accuracy in [20], which can also be extended to harmonic current sharing. Injecting a harmonic voltage according to the output harmonic current can be used to improve the total harmonic distortion (THD) of the voltage [16]. An important contribution was also made in [5], [18], where it was pointed out that the output impedance of the inverters plays a critical role in power sharing and a droop controller for inverters with resistive output impedances was proposed for sharing linear and nonlinear loads [6], [19].

Although significant progress has been made for the equal sharing of linear and nonlinear loads, it is still a problem to share loads accurately in proportion to the power ratings of the inverters. In particular, the accuracy of reactive power sharing (for the $Q-E$ and $P-\omega$ droop) is not high [22]–[24]. Moreover, some approaches developed for equal sharing, e.g., the one proposed in [25], cannot be directly applied to proportional sharing. Another issue is that the output voltage drops due to the increase of the load and also due to the droop control. Hence, the proportional sharing problem needs to be investigated in a systematic way.

It has been recognized that adding an integral action to the droop controller is able to improve the accuracy of load sharing for grid-connected inverters; see [25]–[27]. However, it is still a problem for inverters operated in the standalone mode and also there is an issue associated with the change of the operation mode. A strategy, which involves adding a virtual inductor and estimating the effect of the line impedance, was proposed in [22] to improve the situation via changing the droop coefficients but the strategy is quite complicated and there is still room for improvements. A Q-V dot droop control method was proposed in [24] to improve the accuracy of reactive power sharing following the idea of changing the droop coefficients in [22] but a mechanism to avoid the output voltage variation was necessary, which reduces the accuracy of the reactive power sharing as can be seen from the results given there. All these strategies are sensitive to numerical computational errors, parameter drifts and component mismatches, to the author's best knowledge.

In this paper, it is proven that, in order for parallel-connected inverters to share the load in proportion to their power ratings,

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the inverters should have the same per-unit impedance when the conventional droop controller is used. It also requires that the RMS voltage set-points for the inverters to be the same. Both are very strong conditions and this is the main reason why many different approaches have been proposed to improve the accuracy of power sharing. After thorough analysis of the error in power sharing, a robust droop controller is proposed to achieve accurate proportional load sharing among inverters that are operated in parallel in microgrids. The proposed strategy works for the standalone mode and, naturally, for the grid-connected mode. The accuracy of sharing does not depend on the output impedance of the inverters nor on the RMS voltage set-point and, hence, it is robust to numerical computational errors, disturbances, noises, parameter drifts and component mismatches. Moreover, the controller is able to regulate the load voltage so that the voltage drop due to the load effect and the droop effect is reduced. The sensitivity to the errors in the global settings of the rated frequency and rated voltage is also analyzed, which shows these settings should be accurate. Although the robust droop controller is proposed based on the analysis of inverters with resistive output impedances, it can be applied to inverters with inductive output impedances as well, by using the $Q-E$ and $P-\omega$ droop. Coincidentally, as pointed out by a Reviewer, some aspects of the strategy are similar to the strategy proposed in [28], which was developed from a different starting point and with different reasonings. The strategy proposed in [28] is to force the load voltage to track the reference voltage generated from the conventional droop controller but the strategy proposed here is to improve the strategy that works for grid-connected applications via adding a unit to regulate the load voltage, after thorough analysis of the error in power sharing. The strategy from [28] can compensate the voltage drop due to the load effect but cannot compensate the voltage drop due to the droop effect. The proposed strategy is able to compensate both and, hence, offers much better capability of voltage regulation when the same droop coefficients are used (which should be). Moreover, further insightful understanding about parallel operation of inverters is provided and verified with various experiments in this paper. Probably, this is the first paper in which the error in power sharing has been quantitatively analyzed, following the definition of the relative error in power sharing.

It is worth noting that another important trend to solve the problems associated with the parallel operation of inverters (e.g., sharing accuracy, voltage/frequency deviation, harmonics etc.) is to make the output impedance as accurate as possible over a wide range of frequencies (see [29], [30] and the references therein) and to introduce the secondary control to bring the deviated voltage and frequency back to the rated values [9]. This is an important progress toward the standardization of the operation of microgrids, following what is adopted by the large power systems. However, this inevitably needs the communication among the inverters (even of low bandwidth) and the advantage of droop control is lessened. The secondary control leads to slow responses and/or instability because of the delay introduced in the measurement and communication channel. The complexity of the system is also increased, as evidenced by the number of control loops/levels involved in such systems. This, again, increases the chance of instability. The strategy proposed in this paper is an attempt to embed the secondary control function into the droop control so that the problems can be solved with an integrated neat controller locally and quickly. It allows a high-level secondary control to be hooked up if necessary, e.g., when the voltage needs to be brought back to the rated value.

The rest of the paper is organized as follows. In Section II, the conditions to achieve proportional power sharing are derived and the inherent limitations of the conventional droop control scheme are revealed. An improved droop controller that is robust to computational errors, disturbances, noises, parameter drifts and component mismatches, together with the analysis of power sharing error, are then proposed in Section III. Experimental results are given in Section IV, followed by conclusions made in Section V.

II. Inherent Limitations of the Conventional Droop Control Scheme

Usually, the inverter output impedance is inductive because of the output inductor and/or the highly inductive line impedance. However, in low-voltage applications, the line impedance is predominantly resistive. It is worth noting that control strategies can be used to change the output impedance as well. It can be easily forced to be either resistive or inductive [5], [22]. Arguably, it is better to force the output impedance to be resistive [5], [18], [29] because its impedance does not change with the frequency and the effect of nonlinear loads (harmonic current components) on the voltage THD is reduced. The droop control strategy has different forms according to the type of the output impedance [5], [12], [29]. The $Q-E$ and $P-\omega$ droop is used when the output impedance is inductive; the $Q-\omega$ and $P-E$ droop is used when the output impedance is resistive. Whether to use the $Q-E$ and $P-\omega$ droop or to use the $Q-\omega$ and $P-E$ droop really depends on the application. In this paper, the analysis will be done for the $Q-\omega$ and $P-E$ droop but it can be easily extended to the $Q-E$ and $P-\omega$ droop.

The sketch of two inverters with resistive output impedances R_{o1} and R_{o2} operated in parallel is shown in Fig. 1. The line impedances are omitted because the output impedances of the inverters can be designed to dominate the impedance from the inverter to the ac-bus. The reference voltages of the two inverters are, respectively

$$v_{r1} = \sqrt{2}E_1 \sin(\omega_1 t + \delta_1)$$
$$v_{r2} = \sqrt{2}E_2 \sin(\omega_2 t + \delta_2).$$

Here, E_1 and E_2 are the RMS voltage set-points for the inverters. The power ratings of the inverters are $S_1^* = E^* I_1^*$ and $S_2^* = E^* I_2^*$. They share the same load voltage

$$v_o = v_{r1} - R_{o1} i_1 = v_{r2} - R_{o2} i_2. \tag{1}$$

Note that the load voltage drops when the load increases. This is called the load effect.

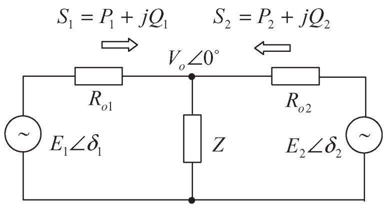

Fig. 1. Two inverters with resistive output impedances operated in parallel.

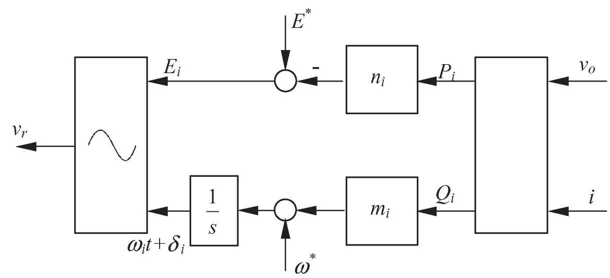

Fig. 2. Conventional droop control scheme.

The analysis in the sequel will be done for the case with two inverters connected in parallel shown in Fig. 1. It can be applied to multiple inverters connected in parallel. As will be explained later, in order to achieve exact power sharing, all the inverters need to share and measure the same load voltage $v_o = V_o\angle 0^\circ$ accurately. In order to measure the same load voltage, each inverter can provide a terminal, which is not internally connected to the output terminal, for voltage measurement. This voltage measurement terminal can be connected to the load terminal with a separate wire to measure the load voltage. In this way, the accuracy of the voltage measured will not be affected by the current flowing through the feeder line. Alternatively, the voltage measurement terminal can be connected to the output terminal externally to measure the local voltage but this would lead to errors in the voltage measured and power sharing, which will be quantified later in details. However, it does open up opportunities for applications with more complicated connections of inverters and/or loads if the sharing accuracy is acceptable. Whether a separate wire is needed for voltage measurement can be determined according to the requirement on the sharing accuracy.

The active and reactive power of each inverter injected into the bus [5], [18] are

$$P_i = \frac{E_i V_o \cos\delta_i - V_o^2}{R_{oi}} \tag{2}$$

$$Q_i = -\frac{E_i V_o}{R_{oi}} \sin\delta_i. \tag{3}$$

In order for the inverters to share the load, the conventional droop controller

$$E_i = E^* - n_i P_i \tag{4}$$

$$\omega_i = \omega^* + m_i Q_i \tag{5}$$

as shown in Fig. 2, is widely used to generate the amplitude and frequency of the voltage reference v_{ri} for Inverter i [6], [19], where ω^* is the rated frequency. Note that, from (3), the reactive power Q_i is proportional to $-\delta_i$ for a small power angle δ_i. In order to make sure that the $Q-\omega$ loop is a negative feedback loop so that it is able to regulate the frequency, the sign before $m_i Q_i$ in (5) is positive, which makes it a boost term. The droop coefficients n_i and m_i are normally determined by the desired voltage drop ratio $n_i P_i^*/E^*$ and frequency boost ratio $m_i Q_i^*/\omega^*$, respectively, at the rated real power P^* and reactive power Q^*. The frequency ω_i is integrated to form the phase of the voltage reference v_{ri}.

In order for the inverters to share the load in proportion to their power ratings, the droop coefficients of the inverters should be in inverse proportion to their power ratings [10], [29], i.e., n_i and m_i should be chosen to satisfy

$$n_1 S_1^* = n_2 S_2^* \tag{6}$$

$$m_1 S_1^* = m_2 S_2^*. \tag{7}$$

It is easy to see that n_i and m_i also satisfy

$$\frac{n_1}{m_1} = \frac{n_2}{m_2}.$$

A. Real Power Sharing

Substituting (4) into (2), the real power of the two inverters can be obtained as

$$P_i = \frac{E^* \cos\delta_i - V_o}{n_i \cos\delta_i + R_{oi}/V_o}. \tag{8}$$

Substituting (8) into (4), the voltage amplitude deviation of the two inverters is

$$\Delta E = E_2 - E_1 = \frac{E^*\cos\delta_1 - V_o}{\cos\delta_1 + \frac{R_{o1}}{n_1 V_o}} - \frac{E^*\cos\delta_2 - V_o}{\cos\delta_2 + \frac{R_{o2}}{n_2 V_o}}. \tag{9}$$

It is known from [10] that the voltage deviation of the two units leads to considerable errors in load sharing. Indeed, in order for

$$n_1 P_1 = n_2 P_2 \qquad \text{or} \qquad \frac{P_1}{S_1^*} = \frac{P_2}{S_2^*}$$

to hold, the voltage deviation ΔE should be zero according to (4). This is a very strict condition because there are always numerical computational errors, disturbances, parameter drifts and component mismatches. This condition is satisfied[1] if

$$\frac{n_1}{R_{o1}} = \frac{n_2}{R_{o2}} \tag{10}$$

$$\delta_1 = \delta_2. \tag{11}$$

In other words, n_i should be chosen to be proportional to its output impedance R_{oi}.

Taking (6) into account, in order to achieve accurate sharing of real power, the (resistive) output impedance should be designed to satisfy

$$R_{o1} S_1^* = R_{o2} S_2^*. \tag{12}$$

[1]Note that this set of conditions is sufficient but not necessary.

Since the per-unit output impedance of Inverter i is

$$\gamma_i = \frac{R_{oi}}{E^*/I_i^*} = \frac{R_{oi}S_i^*}{(E^*)^2}$$

the condition (12) is equivalent to

$$\gamma_1 = \gamma_2.$$

This simply means that the per-unit output impedances of all inverters operated in parallel should be the same in order to achieve accurate proportional real power sharing for the conventional droop control scheme. This is the basis for the virtual output impedance approach [23] to work properly. If this is not met, then the voltage set-points E_i are not the same and errors appear in the real power sharing.

According to (4), the real power deviation ΔP_i due to the voltage set-point deviation ΔE_i is

$$\Delta P_i = -\frac{1}{n_i}\Delta E_i.$$

For two inverters operated in parallel with $P_1 + P_2 = P_1^* + P_2^*$, the relative real power sharing error[2] due to the voltage set-point deviation $\Delta E = E_2 - E_1 = \Delta E_2 - \Delta E_1$ is

$$e_P\% = \frac{P_1}{P_1^*} - \frac{P_2}{P_2^*} = \frac{\Delta P_1}{P_1^*} - \frac{\Delta P_2}{P_2^*} = \frac{E^*}{n_i P_i^*}\frac{\Delta E}{E^*}$$

where $E^*/(n_i P_i^*)$ is the inverse of the voltage drop ratio at the rated power for Inverter i. The smaller the droop coefficient (or the voltage drop ratio), the bigger the sharing error; the bigger the voltage set-point deviation ΔE, the bigger the sharing error. For example, for a voltage drop ratio of $n_i P_i^*/E^* = 10\%$ and a voltage set-point deviation of $\Delta E/E^* = 10\%$, which is very possible for the reasons mentioned before, the error in real power sharing is 100%. The accuracy is very low.

B. Reactive Power Sharing

When the system is in the steady state, the two inverters work under the same frequency, i.e., $\omega_1 = \omega_2$. It is well known that this guarantees the accuracy of reactive power sharing for inverters with resistive output impedances (or the accuracy of real power sharing for inverters with inductive output impedances); see, e.g., [22]. Indeed, from (5), there is

$$m_1 Q_1 = m_2 Q_2.$$

Since the coefficients m_i are chosen to satisfy (7), reactive power sharing proportional to their power ratings is (always) achieved, i.e.,

$$\frac{Q_1}{S_1^*} = \frac{Q_2}{S_2^*}.$$

[2]For generic P_1 and P_2, the relative sharing error should be defined as

$$e_P\% = \left(\frac{P_1}{n_2} - \frac{P_2}{n_1}\right)\frac{n_1+n_2}{P_1+P_2}.$$

Alternatively, according to (3), there is

$$m_1 \frac{E_1 V_o}{R_{o1}} \sin\delta_1 = m_2 \frac{E_2 V_o}{R_{o2}} \sin\delta_2. \tag{13}$$

If $\delta_1 = \delta_2$ and $E_1 = E_2$ then

$$\frac{m_1}{R_{o1}} = \frac{m_2}{R_{o2}}. \tag{14}$$

Theorem: For inverters designed to have resistive output impedances, if the system is stable, then the following two sets of conditions are equivalent:

$$\mathbf{C}_1 : \begin{cases} E_1 = E_2 \\ \frac{n_1}{R_{o1}} = \frac{n_2}{R_{o2}} \end{cases} \iff \mathbf{C}_2 : \begin{cases} \delta_1 = \delta_2 \\ \frac{m_1}{R_{o1}} = \frac{m_2}{R_{o2}}. \end{cases}$$

Proof: If $\mathbf{C}_1$ holds, then proportional real power sharing is achieved according to (4). As a result, (11) holds, according to (9) and (13). Furthermore, reactive power sharing proportional to their ratings is achieved and (14) holds. Conversely, if $\mathbf{C}_2$ holds, then $E_1 = E_2$ according to (13). Furthermore, (10) holds according to (9). This completes the proof. ■

A by-product of this theorem is that n_i and m_i should be proportional. In other words, it is questionable for the conventional droop strategy to achieve different sharing ratios for real power and reactive power. This theorem also indicates that if inverters with resistive output impedances achieve accurate proportional real power sharing under condition $\mathbf{C}_1$, then they also achieve proportional reactive power sharing. If they achieve proportional reactive power sharing under condition $\mathbf{C}_2$, then they also achieve proportional real power sharing. However, this is almost impossible in reality. It is difficult to maintain $E_1 = E_2$ or $\delta_1 = \delta_2$ because there are always numerical computational errors, disturbances and noises. It is also difficult to maintain $\gamma_1 = \gamma_2$ because of different feeder impedances, parameter drifts and component mismatches. The reality is that none of these conditions would be met although the reactive power sharing is accurate (note that conditions $\mathbf{C}_1$ and $\mathbf{C}_2$ are only sufficient but not necessary). A mechanism is needed to guarantee that accurate proportional load sharing can be achieved when these uncertain factors exist.

III. Robust Droop Controller to Achieve Accurate Proportional Load Sharing

A. Proposed Control Strategy

As a matter of fact, the voltage droop (4) can be re-written as

$$\Delta E_i = E_i - E^* = -n_i P_i$$

and the voltage E_i can be implemented via integrating ΔE_i, that is

$$E_i = \int_0^t \Delta E_i dt.$$

This works for the grid-connected mode where ΔE_i is eventually zero (so that the desired power is sent to the grid without error), as proposed in [25]–[27]. However, it does not

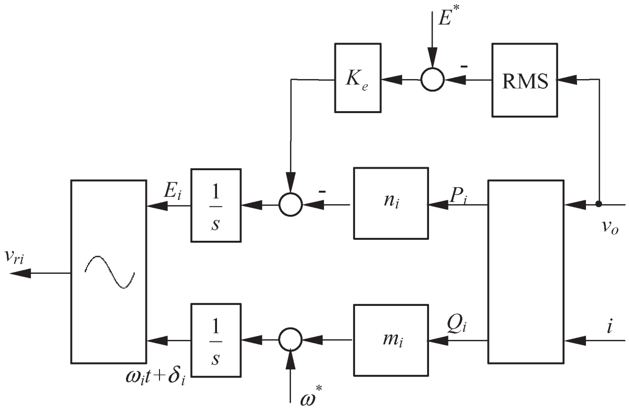

Fig. 3. Proposed robust droop controller to achieve accurate proportional load sharing.

work for the standalone mode because the actual power P_i is determined by the load and ΔE_i cannot be zero. This is why different controllers had to be used for the standalone mode and the grid-connected mode, respectively. When the operation mode changes, the controller needs to be changed as well. It would be advantageous if the change of controller could be avoided when the operation mode changes.

Another issue is that, according to (1), the load voltage v_o drops when the load increases. The voltage also drops due to the droop control, according to (4). The smaller the coefficient n_i, the smaller the voltage drop. However, the coefficient n_i needs to be big to obtain a fast response. In order to make sure that the voltage remains within a certain required range, the load voltage drop $E^* - V_o$ needs to be fed back in a certain way, according to the basic principles of control theory. It can be added to ΔE_i via an amplifier K_e. This actually results in an improved droop controller shown in Fig. 3. This strategy is able to eliminate (at least considerably reduce) the impact of computational errors, noises and disturbances. As to be explained below, it is also able to maintain accurate proportional load sharing and hence robust with respect to parameter drifts, component mismatches and disturbances.

In the steady state, the input to the integrator should be 0. Hence

$$n_i P_i = K_e(E^* - V_o). \tag{15}$$

The right-hand side of the above equation is always the same for all inverters operated in parallel as long as K_e is chosen the same, which can be easily met. Hence

$$n_i P_i = constant$$

which guarantees accurate real power sharing without having the same E_i. This is more natural than the case with the conventional droop controller. The accuracy of real power sharing no longer depends on the inverter output impedances (including the feeder impedance) and is also immune to numerical computational errors and disturbances.

The only possible error in the real power sharing comes from the error in measuring the RMS value of the load voltage. From (15), the real power deviation ΔP_i due to the error ΔV_{oi} in the measurement of the RMS voltage is

$$\Delta P_i = -\frac{K_e}{n_i}\Delta V_{oi}.$$

For two inverters operated in parallel with $P_1 + P_2 = P_1^* + P_2^*$, the relative real power sharing error due to the error in the measurement of the RMS voltage $\Delta V_o = \Delta V_{o2} - \Delta V_{o1}$ is

$$e_P\% = \frac{P_1}{P_1^*} - \frac{P_2}{P_2^*} = \frac{\Delta P_1}{P_1^*} - \frac{\Delta P_2}{P_2^*} = \frac{K_e E^*}{n_i P_i^*}\frac{\Delta V_o}{E^*}.$$

This characterizes the percentage error $e_P\%$ of the real power sharing with respect to the percentage error $\Delta V_o/E^*$ of the RMS voltage measurement. The term $K_e E^*/(n_i P_i^*)$ is the inverse of the voltage drop ratio with respect to the rated voltage at the rated power. If all inverters measure the voltage at the same point accurately, then the error ΔV_o can be made zero and exact proportional sharing can be achieved.

The strategy also reduces the load voltage drop. From (15), the load voltage is

$$V_o = E^* - \frac{n_i}{K_e}P_i = E^* - \frac{n_i P_i}{K_e E^*}E^*$$

where $n_i P_i/(K_e E^*)$ is the voltage drop ratio. Note that the voltage drop ratio here is the overall effective voltage drop ratio, which is much smaller than the drop ratio due to the droop effect and/or the load effect, but the one in both the conventional droop controller and the controller in [28] is just the voltage drop ratio due to the droop effect and does not include the voltage drop ratio due to the load effect. Although the controller in [28] can compensate the voltage drop due to the load effect, it cannot compensate the voltage drop due to the droop effect. The proposed strategy can compensate the voltage drop due to both effects and, hence, offers much better capability of voltage regulation. The voltage drop here is no longer determined by the output impedance originally designed as characterized in (1) but by the parameters n_i, K_e and the actual power P_i. It can be considerably reduced by using a large K_e. If there are errors in the RMS voltage measurement, then the trade-off between the voltage drop and the accuracy of power sharing has to be made because the voltage drop is proportional to n_i/K_e but the sharing error is inverse proportional to n_i/K_e. Here, is a calculation example. Assume that the voltage drop ratio at the rated power is $n_i P_i^*/(K_e E^*) = 10\%$ and the error in the RMS voltage measurement is $\Delta V_o/E^* = 0.5\%$, whether because the local voltages of inverters are measured or because the sensors are not accurate. Then, the error in the real power sharing is $K_e E^*/(n_i S_i^*)(\Delta V_o/E^*) = 0.5\%/10\% = 5\%$, which is reasonable.

B. Importance of Accurate Global Settings for E^* and ω^*

This sub-section is devoted to the sensitivity analysis of the error in the global settings E^* and ω^* for the proposed robust droop controller.

Any small error $\Delta\omega_i$ in ω_i^* would lead to the reactive power deviation (if still stable[3]) of

$$\Delta Q_i = -\frac{1}{m_i}\Delta\omega_i$$

according to (5). For two inverters operated in parallel with $Q_1 + Q_2 = Q_1^* + Q_2^*$, the relative reactive power sharing error due to the error $\Delta\omega = \omega_2^* - \omega_1^* = \Delta\omega_2 - \Delta\omega_1$ is

$$e_Q\% = \frac{Q_1}{Q_1^*} - \frac{Q_2}{Q_2^*} = \frac{\Delta Q_1}{Q_1^*} - \frac{\Delta Q_2}{Q_2^*} = \frac{\omega^*}{m_i Q_i^*}\frac{\Delta\omega}{\omega^*}$$

where $\omega^*/(m_i Q_i^*)$ is the inverse of the frequency boost ratio at the rated reactive power. The smaller the frequency boost ratio, the bigger the reactive power sharing error; the bigger the error $\Delta\omega$, the bigger the sharing error. For example, for a typical frequency boost ratio of $m_i Q_i^*/\omega^* = 1\%$, the error of $\Delta\omega/\omega^* = 1\%$ in the frequency setting would lead to a 100% of error in the reactive power sharing. Hence, the accuracy of reactive power sharing is very sensitive to the accuracy of the global setting for ω^*, which should be made very accurate.

Similarly, according to (15), the real power deviation ΔP_i due to the error ΔE_i^* in E_i^* is

$$\Delta P_i = \frac{K_e}{n_i}\Delta E_i^*.$$

For two inverters operated in parallel with $P_1 + P_2 = P_1^* + P_2^*$, the relative real power sharing error due to the error $\Delta E^* = E_2^* - E_1^* = \Delta E_2^* - \Delta E_1^*$ in the global settings of E^* is

$$e_P\% = \frac{P_1}{P_1^*} - \frac{P_2}{P_2^*} = \frac{\Delta P_1}{P_1^*} - \frac{\Delta P_2}{P_2^*} = -\frac{K_e E^*}{n_i P_i^*}\frac{\Delta E^*}{E^*}.$$

For a typical voltage drop ratio at the rated power of $n_i P_i^*/(K_e E^*) = 10\%$, a 10% error in $\Delta E^*/E^*$ would lead to a -100% error in the real power sharing. Although the error in E^* is less sensitive than the error in ω^*, it is still quite significant. Hence, in practice, it is very important to make sure that the global settings are accurate. Anyway, this is not a problem at all.

IV. Experimental Results

The above strategy has been verified in a laboratory setup, which consists of two single-phase inverters controlled by dSPACE kits and powered by separate 42 V dc voltage supplies. The circuit diagram of each inverter is shown in Fig. 4(a). The inverters are connected to the ac bus via a circuit breaker CB and the load is assumed to be connected to the ac bus. The values of the inductors and capacitors are 2.35 mH and 22 μF, respectively. The switching frequency is 7.5 kHz and the frequency of the system is 50 Hz. The rated voltage is 12 V RMS and $K_e = 10$. The droop coefficients are: $n_1 = 0.4$ and $n_2 = 0.8$; $m_1 = 0.1$ and $m_2 = 0.2$. Hence, it is expected that $P_1 = 2P_2$ and $Q_1 = 2Q_2$. Due to the configuration of the hardware setup, the voltage for Inverter 2 was measured by the controller of Inverter 1 and then sent out via a DAC channel, which was then sampled by the controller of Inverter 2. This brought some latency into the system but the effect was not noticeable.

[3]The maximum $\Delta\omega$ that does not destabilize the system is not known, to the best knowledge of the author.

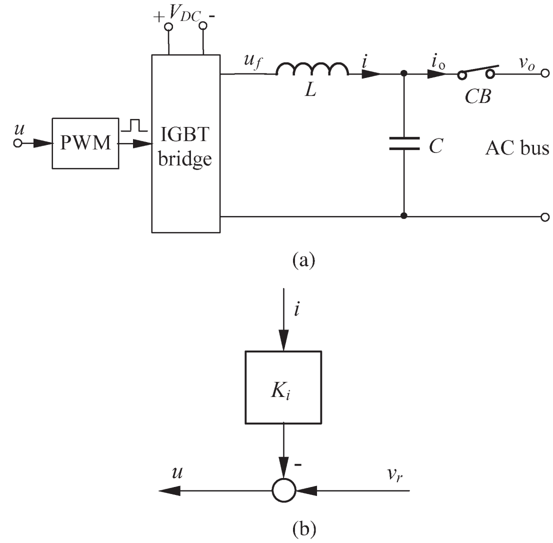

Fig. 4. Singe-phase inverter. (a) Used for physical implementation. (b) Simple controller to achieve a resistive output impedance.

A. Design of a Simple Controller to Achieve Resistive Output Impedance

As the main contribution of the paper is to propose a robust droop controller and to analyze the error in power sharing, the attention is not paid to the design of the inner-loop controller for the inverters. A very simple controller as described below is designed, following the idea of the virtual impedance concept [23], to force the output impedance to be resistive.

As shown in Fig. 4(a), each inverter consists of a single-phase H-bridge inverter powered by a DC source, and an LC filter. The control signal u is converted to a PWM signal to drive the H-bridge so that the average of u_f over a switching period is the same as u, i.e., $u \approx u_f$. Hence, the PWM block and the H-bridge can, and will, be ignored in the controller design. The inductor current i is measured to construct a controller so that the output impedance of the inverter is forced to be resistive and to dominate the impedance between the inverter and the ac bus.

The following two equations hold for the closed-loop system consisting of Fig. 4(a) and (b):

$$u = v_r - K_i i, \quad \text{and} \quad u_f = sLi + v_o.$$

Since the average of u_f over a switching period is the same as u, there is (approximately)

$$v_r - K_i i = sLi + v_o$$

which gives

$$v_o = v_r - Z_o(s) \cdot i$$

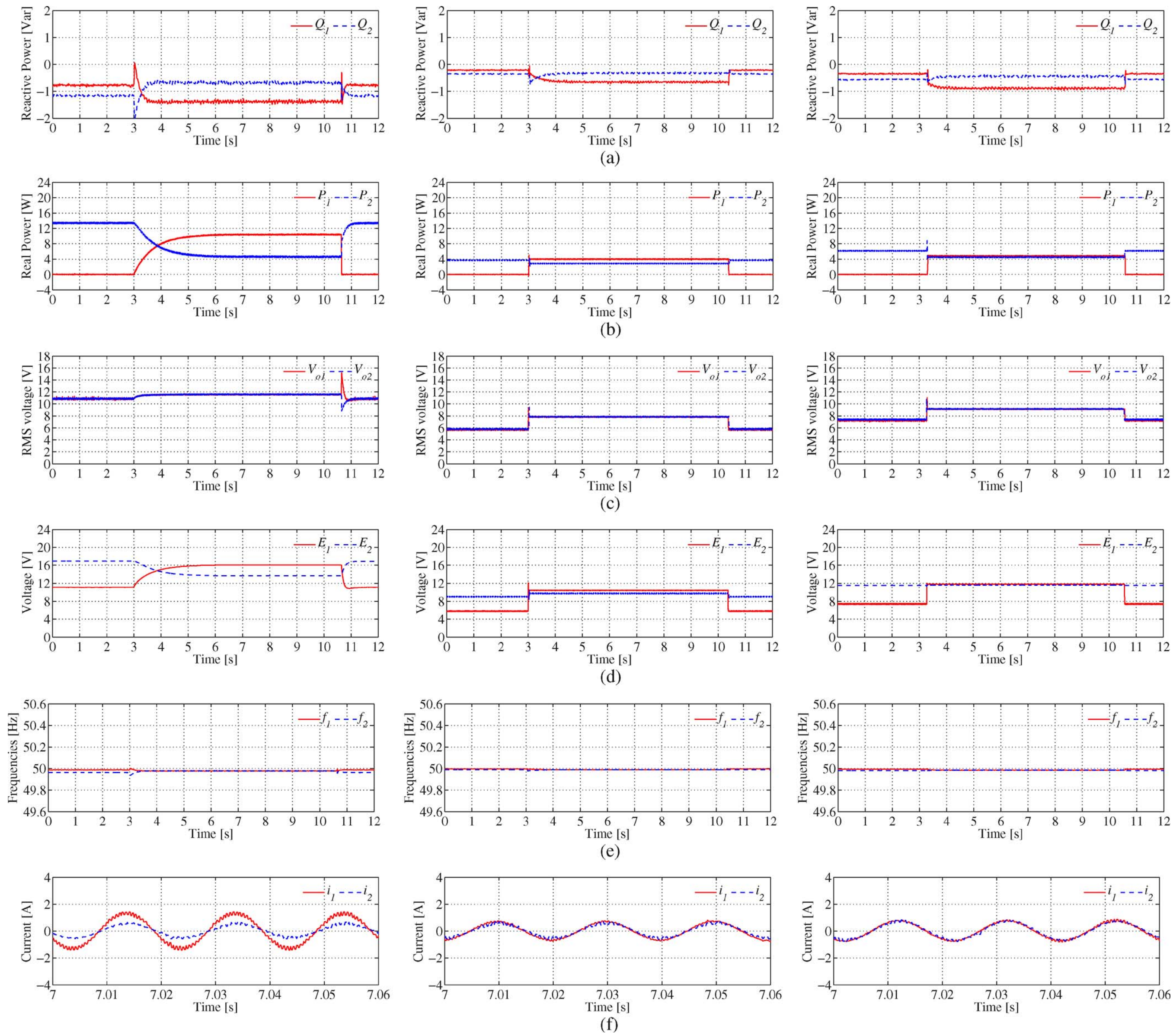

Fig. 5. Experimental results for the case with intentionally different per-unit output impedances to achieve 2 : 1 power sharing: using the proposed robust droop controller (left column), using the conventional droop controller with the same n_i (middle column) and using the conventional droop controller with n_i divided by K_e (right column). (a) Reactive power; (b) real power; (c) RMS value of the load voltage; (d) voltage set-point; (e) frequency; and (f) current in the steady.

with

$$Z_o(s) = sL + K_i.$$

If the gain K_i is chosen big enough, the effect of the inductance is not significant and the output impedance can be made nearly purely resistive over a wide range of frequencies. Then, the output impedance is roughly

$$Z_o(s) \approx R_o = K_i$$

which is independent of the inductance. With this simple current controller, the inverter can be approximated as a controlled ideal voltage supply v_r cascaded with a resistive output impedance R_o as described in (1). This design also indicates that the capacitor can be regarded as a part of the load, instead of a part of the inverter. Note that $v_o \approx u = v_r$ when no load is connected.

B. Experimental Results for the Case With Different Per-Unit Output Impedances to Achieve 2 : 1 Power Sharing

Both K_i were chosen as 4 for the two inverters to intentionally make the per-unit output impedances of the two inverters significantly different. In reality, this could be due to different feeder impedances or component mismatches.

A linear load of about 9 Ω was connected to Inverter 2 initially. Inverter 1 was connected to the system at around $t = 3$ s and was then disconnected at around $t = 10.5$ s. The relevant curves from the experiment with the proposed robust droop controller are shown in the left column of Fig. 5. The same experiment was carried out again with the conventional

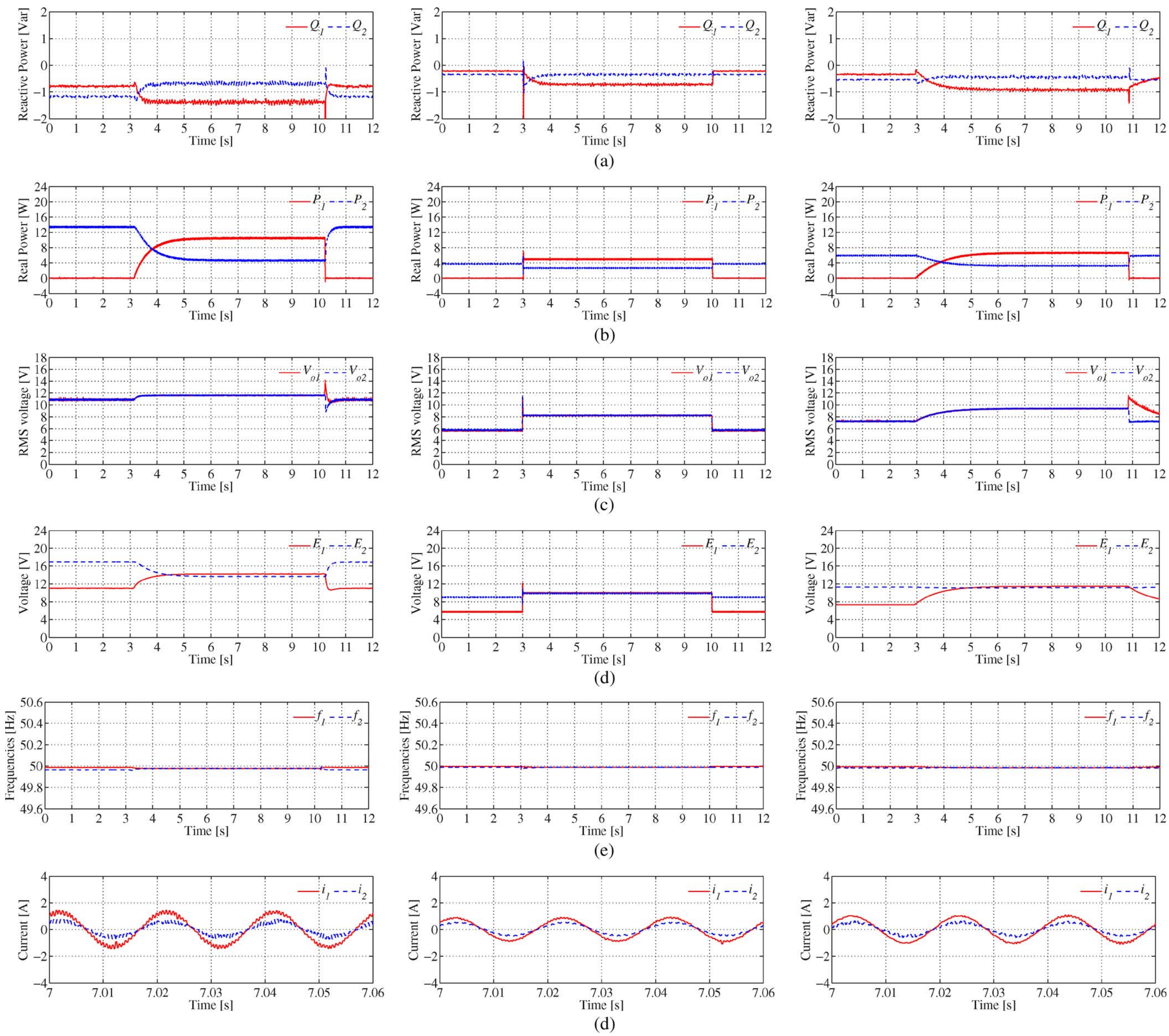

Fig. 6. Experimental results for the case with the same per-unit output impedances to achieve 2 : 1 power sharing: using the proposed robust droop controller with $K_e = 10$ (left column), using the conventional droop controller (middle column) and using the proposed robust droop controller with $K_e = 1$ (right column). (a) Reactive power; (b) real power; (c) RMS value of the load voltage; (d) voltage set-point; (e) frequency; (f) current in the steady.

droop controller using the same droop coefficients n_i and the relevant curves are shown in the middle column of Fig. 5. A third experiment was carried out with the conventional droop controller using the droop coefficients n_i divided by $K_e = 10$ so that the equivalent voltage drop coefficient is the same as the one used in the robust droop controller, expecting that the voltage drop could be comparable to the one obtained with the robust droop controller. The curves are shown in the right column of Fig. 5. In all three cases, the reactive power was shared accurately (in the ratio of 2 : 1), although the actual values are different (because the voltages are different). Inverter 1 was able to pick up the load, gradually in the case of the proposed robust droop controller and very quickly in the case of the conventional droop controller. The proposed robust droop controller was able to share the real power accurately but the conventional one could not. From the two cases using the conventional droop controller, the trade-off between the sharing accuracy and the voltage drop can be clearly seen. When the coefficients n_i were not divided by K_e, i.e., when the voltage drop ratio is bigger, the sharing accuracy is better than the case when it is divided by $K_e = 10$. The proposed robust droop controller has considerably relaxed this trade-off and is able to maintain very good sharing accuracy while maintaining small voltage drop. The voltage from the inverters equipped with the robust droop controller is very close to the rated voltage but the voltage from the inverters equipped with the conventional controller is only 2/3 or 3/4 of the rated voltage. Dividing n_i by $K_e = 10$ reduced the voltage drop but not at a level comparable to the case with the proposed robust droop controller, which could increase the voltage set-points significantly.

The conventional droop controller could not do this because the voltage set-point has to be lower than the rated voltage due to the droop effect. The bigger the voltage drop ratio, the lower the voltage set-point. It can also be clearly seen that $E_1 \neq E_2$ because the per-unit output impedances are different and also there are numerical errors and component mismatches etc. Because of the reduced deviation in the voltage, the reactive power becomes bigger. This leads to a slightly bigger deviation in the frequency but it is expected because of the $Q - \omega$ droop. The current sharing reflects the power sharing well. It is worth noting that there was no need to change the operation mode of Inverter 2 when connecting or disconnecting Inverter 1.

C. Experimental Results for the Case With the Same Per-Unit Output Impedances to Achieve 2 : 1 Power Sharing

The current feedback gains were chosen as $K_{i1} = 2$ and $K_{i2} = 4$ so that the output impedances are consistent with the power sharing ratio 2 : 1. The results from the proposed robust droop controller with $K_e = 10$ are shown in the left column of Fig. 6 and the results from the conventional droop controller are shown in the middle column of Fig. 6. The proposed droop controller was able to share the load according to the sharing ratio and considerably outperformed the conventional droop controller in terms of sharing accuracy and voltage drop. The difference between the voltage set-points can be clearly seen. This indicates the effect of numerical errors, parameter drifts and component mismatches because the voltage set-points were supposed to be identical without these uncertain factors. Comparing the left columns of Figs. 5 and 6, there were no noticeable changes in the performance for the proposed droop controller but the difference in the voltage set-points was decreased. Comparing the middle columns of Figs. 5 and 6, the sharing accuracy and the voltage drop were improved slightly and the voltage set-points became closer to each other for the conventional droop controller when the per-unit output impedances were the same. The results from the proposed robust droop controller with $K_e = 1$ are shown in the right column of Fig. 6 to demonstrate the role of K_e. It can be seen that a large K_e helps speed up the response and reduce the voltage drop.

V. CONCLUSION

In this paper, the inherent limitations of the conventional droop control scheme has been exposed. In order to achieve accurate proportional load sharing among parallel-operated inverters, the inverters should have the same per-unit resistive output impedances and the voltage set-points (E_i) should be the same. These are almost impossible to meet in reality. An improved droop control strategy is then proposed to obtain accurate proportional load sharing for microgrids working in the standalone mode (and naturally also for microgrids working in the grid-connected mode). This strategy does not require the above two conditions to be met in order to achieve accurate proportional sharing. The strategy is also able to compensate the voltage drop due to the load effect and the droop effect and the load voltage can be maintained within the desired range around the rated value. Quantitative analysis of the error in power sharing has been carried out thoroughly. Various experimental results have demonstrated that the strategy proposed here is very effective. The strategy proposed here is demonstrated for inverters with resistive output impedances but it can be applied to inverters with inductive output impedances by using the $Q - E$ and $P - \omega$ droop as well.

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