On the Internal Model Principle in the Coordination of Nonlinear Systems

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***Abstract*—The role of the internal model principle is investigated in this paper for the coordination of relative-degree-one and relative-degree-two nonlinear systems. For relative-degree-one systems that are incrementally (output-feedback) passive, we propose internal-model-based distributed control laws which guarantee output synchronization to an invariant manifold driven by autonomous synchronized internal models. For relative-degree-two systems, we consider a different internal-model-based distributed control framework for solving a formation control problem where the agents have to track a reference signal available only to the leader agent. In both cases, the local controller is also able to reject the disturbance signals generated by a local exosystem.**

***Index Terms*—Cooperative control, disturbance rejection, nonlinear systems, passivity, synchronization.**

I. Introduction

IN THIS paper, we study the coordination control problems for relative-degree-one and for relative-degree-two nonlinear systems based on the internal model principle. To investigate the role of internal model principle in various coordination problems, we consider an output synchronization problem for relative-degree-one systems and we look at a formation control problem for relative-degree-two systems. The passivity-based approach is central to our internal-model-based distributed controls, and passivity and internal models are the unifying theme in our investigation.

In the synchronization problem, one investigates conditions under which the state variables of all the subsystems asymptotically converge to each other, while in the formation control problem, one studies the distributed control laws which ensures that the position or the velocity of all the subsystems converge to the desired position or velocity. Passivity [1], [4], [18], [17] or the weaker notion of semi-passivity [14], [15] has been studied in both the synchronization and formation control problems.

In the context of output synchronization, the notion of passive nonlinear systems has been exploited to show that the relative output measurements in a network suffice to ensure output synchronization, that is, the output of all the subsystems asymptotically converge to each other. This distributed control structure is shown in Fig. 1 where each subsystem Σ_i is passive with output y_i and input u_i, and the arrows schematically denote the communicating agents. This trivial synchronization solution is mainly due to the property of passive systems where the sum of storage functions (which become the Lyapunov function of the closed-loop systems) is nonincreasing under such interconnection and it is constant only if the outputs are equal. If one assumes a stronger notion of strictly incremental passivity for each subsystem, where the strictness corresponds to the incremental stability of the autonomous systems, then the output synchronization implies the state synchronization [18].

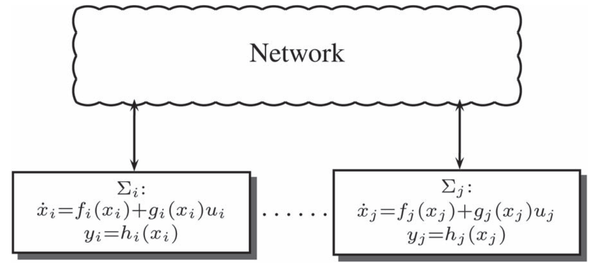

Fig. 1. Trivial solution to the output synchronization of N nonlinear passive systems Σ_i, $i = 1, \ldots, N$. The cloud represents the network where the information exchange takes place. In this configuration, for every agent i, the input u_i is equal to $-\sum_k d_{ik}(y_i - y_k)$, where $d_{ik} = 1$ if agents i and k exchange information and $d_{ik} = 0$ otherwise.

An alternative solution to the output synchronization is explored in this paper which is motivated by the recent results in [21] and [22]. It is shown in [22] that the existence of an internal model is a necessary and sufficient condition for output synchronization of linear networked systems and an internal-model-based distributed controller is proposed for solving the output synchronization problem for linear networked systems. Our first main contribution of this paper is to show that the results of [21] and [22] can be extended to the nonlinear case and, at the same time, be generalized to solve the disturbance rejection problem, akin to that for the single agent case [9], [10]. We refer the interested reader to the recent paper [7] for a different approach to the generalization of [22] to nonlinear systems, and to [3] for another passivity-based approach to nonlinear output agreement problems inspired by [12].

In this paper, we propose a distributed control law that uses the local information of the networked exosystems and the relative output measurements of the subsystems and of the exosystems as shown in Fig. 2. In this figure, the synchronization of the systems Σ_i is achieved via the networked internal model Ξ_i^r which exchanges relative output measurements

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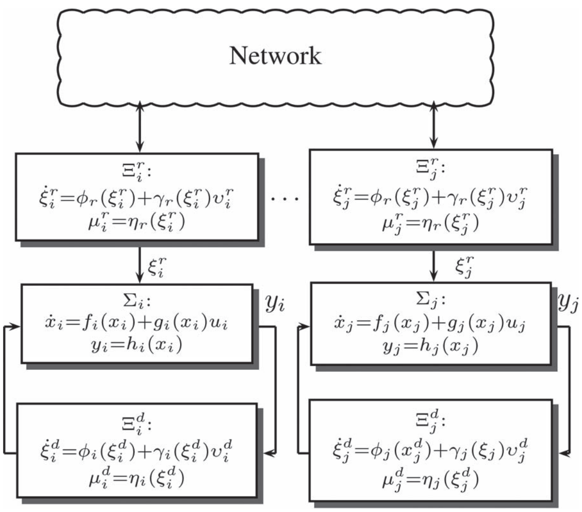

Fig. 2. Distributed control framework for the internal-model-based output synchronization of relative-degree-one systems as discussed in Section III. The cloud represents the network where the information exchange takes place. In this configuration, for every agent i, the internal-model input υ_i^r is equal to $-\sum_k d_{ik}(\mu_i^r - \mu_k^r)$, where $d_{ik} = 1$ if agents i and k exchange information and $d_{ik} = 0$ otherwise. In our main result in Proposition 3 and 4, the proposed local control input u_i is defined based on ξ_i^r, υ_i^r, and the local measurement of y_i, while υ_i^d is defined based on ξ_i^r and y_i (for dealing with the disturbance rejection problem).

with its neighbors, while, at the same time, the lower internal model Ξ_i^d is responsible to reject an input disturbance signal generated by an exosystem (refer to Section III for details). We generalize our preliminary results appeared in [5] on the synchronization of passive systems by extending our previous controllers with additional dynamics that are able to reject disturbances generated by an exosystem, and to accommodate for relaxed co-coercive systems which strictly include the class of incrementally passive systems.

The proposed control approach enforces, in some sense, a forced synchronization to the subsystems via networked exosystems. In particular, the subsystems asymptotically converge to an invariant manifold driven by an autonomous synchronized exosystem. As a result, this particular distributed control structure offers an additional feature that is not inherent in the trivial solution. By assigning a particular internal model, one can shape the behavior of the synchronized systems. We illustrate this in the controlled synchronization of a network of Goodwin oscillators.

Another distinguishing feature of our approach is the heterogeneity of the dynamical systems in the network. We cope with such heterogeneity by proposing internal-model-based controllers that depend on the solution of *coupled* regulator equations.

In the context of formation control, Bai, Arcak, and Wen in [2] have discussed the role of passivity for the coordination of networked systems. Using that framework, we consider a formation control problem where the followers, which are modeled by relative-degree-two systems, asymptotically track the leader's velocity and rendezvous.[1] In this problem setting, we consider an internal-model-based distributed control framework that shares the same distinguishing features, although some differences arise with respect to the output synchronization problem. Instead of using a networked exosystem as before, we explore an internal-model-based distributed control approach that—in the case no disturbance affects the systems—uses only the relative measurement of the system output.

For every system, an internal model will be used to track the reference velocity signal that is only available to the leader and to reject disturbance signals generated by an exosystem. In this way, we generalize our preliminary result on the internal-model-based formation control in [5], where the disturbance rejection problem was neglected. We do not include a collision-avoidance mechanism, because, as pointed out in [2], this would raise additional technical issues that go beyond the scope of this paper.

In Section II, we provide a few preliminaries about the class of systems under consideration and the synchronization problem we are interested in. In Section III, the synchronization problem of systems with relative degree one is studied. The formation control problem in the presence of incrementally passive leaders is examined in Section IV. Conclusions are drawn in Section V.

II. Preliminaries

Consider N systems connected over a undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a set of N nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of M edges connecting the nodes. The standing assumption throughout this paper is that the graph $\mathcal{G}$ is *connected*. Each system i, with $i = 1, 2, \ldots, N$, is associated with the node i of the graph and the edges connect the nodes or systems which communicate. Label one end of each edge in E with a positive sign and the other one by a negative sign. Now, consider the kth edge in E, with $k \in \{1, 2, \ldots, M\}$. Unless otherwise explicitly stated, we let i, j be the two nodes connected by the edge k. Associated with each node in the graph is a dynamical nonlinear system whose model is described below.

A. Nonlinear Plants

Each subsystem is described by

$$\Sigma_i : \begin{cases} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ y_i &= h_i(x_i) \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{m_i}$, $u_i, y_i \in \mathbb{R}^q$. For every i, we assume there exists a regular storage function[2] $H_{\Sigma_i} : \mathbb{R}^{m_i} \times \mathbb{R}^{m_i} \to \mathbb{R}_+$ and a number $\rho_i \in \mathbb{R}$ such that

$$\frac{\partial H_{\Sigma_i}(x_i, x_i')}{\partial x_i}\left(f(x_i) + g(x_i)u_i\right) + \frac{\partial H_{\Sigma_i}(x_i, x_i')}{\partial x_i'}\left(f(x_i') + g(x_i')u_{i'}\right)$$

[1] As in [2], we could, in addition, address the problem of guaranteeing a desired shape, but this is not done for the sake of simplicity. Nevertheless, we continue to refer to the problem as a "formation control" problem.

[2] A storage function $S(x_k, x_k')$, with $S : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}_+$, is said to be regular ([12]) if for any sequence (x_k, x_k'), $k = 1, 2, \ldots$, such that x_k' is bounded and $\|x_k\| \to +\infty$, we have that $S(x_k, x_k') \to +\infty$ as $k \to +\infty$.

$$\begin{aligned}&\le -\rho_i \left\|h_i(x_i) - h_i(x_i')\right\|^2 \\ &\quad + \langle h_i(x_i) - h_i(x_i'), u_i - u_i'\rangle\end{aligned} \tag{2}$$

for all $(x_i, x_i') \in \mathbb{R}^{m_i} \times \mathbb{R}^{m_i}$ and $(u_i, u_i') \in \mathbb{R}^q \times \mathbb{R}^q$.

The system Σ_i is said to be incrementally output-feedback passive (or ρ_i-*relaxed cocoercive* in the terminology of [16]). If (2) holds with $\rho_i > 0$, Σ_i is also said to be *incrementally output-strictly passive* and *incrementally passive* if $\rho_i \ge 0$.

In this paper, we will also consider a slight extension of the system above, namely, the relative degree two system

$$\Sigma_i : \begin{cases} \dot{q}_i = x_i \\ \dot{x}_i = f_i(x_i) + g_i(x_i)u_i \\ y_i = q_i, \end{cases} \tag{3}$$

where the subsystem in the x_i coordinates satisfies the dissipation inequality (2).

The reasons to consider these two classes of systems in a single work are manifold. Incrementally passive systems of the form (1) have been extensively used to model interconnected systems, such as biochemical networks [16], [19] and oscillators [18]. On the other hand, systems of the form (3) arise in a variety of problems, including formation control as well as frequency regulation in power systems[3] [1]–[3]. The passivity property of the x_i-subsystems allows us to consider important control problems, such as distributed output tracking and disturbance rejection (to be formulated below), for both classes of systems in a unified manner. In this way, internal model-based controllers emerge as a ubiquitous technique to control systems (1) and (3) at once, instead of a case-by-case study. In that respect, the results presented in this paper can be similarly applied to the systems considered in, for example, [1], [2], [16], [18], and [19].

B. Disturbance Rejection and Reference Tracking

We are interested in problems of coordination in the presence of disturbances in the input channel. Thus, we will replace, at times, the x_i-system in (1) and (3) by

$$\dot{x}_i = f_i(x_i) + g_i(x_i)(u_i + d_i) \tag{4}$$

where d_i is a disturbance signal which we assume to be generated by an exosystem W_i^d given by

$$W_i^d : \begin{cases} \dot{w}_i^d = s_i^d(w_i^d) \\ d_i = c_i^d(w_i^d) \end{cases} \tag{5}$$

where $w_i^d \in \mathbb{R}^n$, $d_i \in \mathbb{R}^p$. We are also interested in the problem of having the outputs of the systems to track a reference signal r generated by an exosystem W^r described by

$$W^r : \begin{cases} \dot{w}^r = s^r(w^r) \\ r = c^r(w^r), \end{cases} \tag{6}$$

where $w^r \in \mathbb{R}^m$, $r \in \mathbb{R}^p$. The functions s_i^d, s^r, c_i^d, and c^r are assumed to be sufficiently smooth.

The problem we are interested in can be formulated as follows: given the systems (1) (respectively, (3)), possibly in the presence of disturbances (see Eq. (4)), find distributed control laws generating the inputs u_i in such a way that $\lim_{t\to+\infty}(y_i(t) - y_j(t)) = 0$ for all $i \neq j$ (respectively, $\lim_{t\to+\infty}(y_i(t) - r(t)) = 0$, for all i).

C. (Incrementally) Passive Internal Models

It is well known that in problems of trajectory tracking and disturbance rejection, the internal model plays a fundamental role. Throughout this paper, we use internal models for solving coordination problems for systems (1) and (3).

Each internal model i is described by

$$\Xi_i : \begin{cases} \dot{\xi}_i = \phi(\xi_i) + \gamma(\xi_i)\upsilon_i \\ \mu_i = \eta(\xi_i), \end{cases} \tag{7}$$

where the state ξ_i is of appropriate dimension, the input $\upsilon_i \in \mathbb{R}^p$, the output $\mu_i \in \mathbb{R}^p$, and the functions ϕ, γ, η are assumed to be locally Lipschitz, satisfying $\phi(\mathbf{0}) = \mathbf{0}$, $\gamma(\mathbf{0}) \neq 0$, $\eta(\mathbf{0}) = \mathbf{0}$. In several parts of this paper, we may use a superscript d in Ξ_i^d and its state variables ξ_i^d for indicating that the internal model will be used to reject the disturbance signal d_i. Similarly, a superscript r will be used in Ξ_i^r and ξ_i^r when the internal model is used for tracking the reference signal r. For each internal model Ξ_i, we assume the following assumption:

Assumption 1: There exists a storage function $H_{\Xi_i} : \mathbb{R}^n \to \mathbb{R}_+$ which is positive definite and radially unbounded such that

$$\frac{\partial H_{\Xi_i}(\xi_i)}{\partial \xi_i}\left(\phi(\xi_i) + \gamma(\xi_i)\upsilon_i\right) \le \eta(w_i)^T \upsilon_i. \tag{8}$$

Such a system Ξ_i is called a *passive system*. In what follows, we denote the inner product $x^T y$ also by $\langle x, y\rangle$.

The autonomous system Ξ_i (with $\upsilon_i = 0$) is called *incrementally observable* if for every trajectories ξ_i, ξ_j such that $\eta(\xi_i) = \eta(\xi_j)$, then $\xi_i = \xi_j$. The notion of incremental observability is a variation to the standard notion of zero-state observability.

Although not named in this way, incremental observability is a common assumption in problems of synchronization (see, for example, [16, Corollary 1] and [6] for the same property).

D. Relative Measurements

As discussed in the Introduction, we consider internal model-based coordination control for relative-degree-one and -two systems. These systems require different relative measurements. For systems with relative degree one, we use the relative measurements of the internal model output μ in (7). More precisely, for the kth edge of the graph, we can define the relative measurement ζ_k by

$$\zeta_k = \begin{cases} \mu_i - \mu_j & \text{if } i \text{ is the positive end of the edge } k \\ \mu_j - \mu_i & \text{if } i \text{ is the negative end of the edge } k. \end{cases}$$

[3]Note that for a mechanical system in a formation control problem, q represents the generalized displacements and x denotes the generalized velocity, while in a power system regulation problem, q represents the bus voltage angle and ω is the frequency.

For systems with relative degree two, where the relative measurement is done on the plant output y, we define the relative measurement on the kth edge as follows:

$$z_k = \begin{cases} y_i - y_j & \text{if } i \text{ is the positive end of the edge } k \\ y_j - y_i & \text{if } i \text{ is the negative end of the edge } k. \end{cases}$$

Using the incidence matrix D, the variables ζ and z can also be concisely represented as

$$\zeta = (D^T \otimes I_p)\mu \text{ and } z = (D^T \otimes I_q)y \tag{9}$$

where $\mu = [\mu_1^T \ldots \mu_N^T]^T$ and $y = [y_1^T \ldots y_N^T]^T$.

E. Synchronization of Internal Models

Let us recall the synchronization for linear systems as discussed in [17]. In the synchronization problem of [17, Theor. 4], each system Ξ_i in (7) is assumed to be linear, identical, and passive. For such settings, each (passive) internal model Ξ_i is of the form

$$\begin{cases} \dot{\xi}_i = S\xi_i + B\upsilon_i \\ \mu_i = C\xi_i \qquad i = 1, 2, \ldots, N \end{cases} \tag{10}$$

where $\xi_i \in \mathbb{R}^n$, $\upsilon_i, \mu_i \in \mathbb{R}^p$, and the passivity of Ξ_i imply that the following assumption holds:

Assumption 2: There exists an $(n \times n)$ matrix $P = P^T > 0$ such that $S^TP + PS \leq 0$, $B^TP = C$.

The synchronization problem of internal models can then be stated as designing each control law υ_i, $i = 1, 2, \ldots, N$, using only the information available to the agent i such that for every i, $\xi_i - \xi_0 \to 0$, where ξ_0 is the trajectory of the autonomous system $\dot{\xi}_0 = S\xi_0$ which is initialized by the average of the initial states, that is, $\xi_0(0) = (1/N)\sum_i \xi_i(0)$.

In addition to output synchronization, it is well known that the *states* of interconnected passive systems synchronize under the observability assumption [4]. The largest invariant set of the interconnected systems when (C, S) is observable, is the set $\{\xi \in \mathbb{R}^{nN} : \xi_1 = \ldots = \xi_N\}$. The exponential synchronization under static output feedback control laws and time-varying graphs has been investigated in [17].

The generalization to the nonlinear internal model is given in the following proposition:

Proposition 1: Consider the internal models Ξ_i as in (7). Suppose that for every i, Ξ_i satisfies Assumption 1 (i.e., it is passive) and is incrementally observable. Let the communication graph be undirected and connected, and consider the vector of relative measurements $\zeta = (D^T \otimes I_p)\mu$ as in (9) with $\mu = [\mu_1^T \ldots \mu_N^T]^T$. Then, for every initial condition $\xi(0) \in \mathbb{R}^{nN}$, there exists $\omega_0 \in \mathbb{R}^n$ such that the solution of

$$\dot{\xi}_i = \phi(\xi_i) - \gamma(\xi_i)\sum_{k=1}^{M} d_{ik}\zeta_k, i = 1, 2, \ldots, N$$

satisfies $\lim_{t\to+\infty} \|\xi_i(t) - \xi_0(t)\| = 0$, where $\xi = [\xi_1^T\ \xi_2^T \cdots \xi_N^T]^T$, and ξ_0 is the solution of $\dot{\xi}_0 = \phi(\xi_0)$, $\xi_0(0) = \omega_0 \in \mathbb{R}^n$.

Proof: Let $H(\xi) = \sum_i H_{\Xi_i}(\xi_i)$ be the Lyapunov function of the interconnected systems where H_{Ξ_i} satisfies (8). Following the standard passivity argument and the assumptions on the graph, we have

$$\begin{aligned} \dot{H} &= \sum_i \frac{\partial H_{\Xi_i}(\xi_i)}{\partial \xi_i}\left(\phi(\xi_i) - \gamma(\xi_i)\sum_{k=1}^{M} d_{ik}\zeta_k\right) \\ &\leq -\sum_i \eta(\xi_i)\sum_{k=1}^{M} d_{ik}\zeta_k = -\mu^T(D \otimes I_p)\zeta \\ &= -\mu^T(DD^T \otimes I_p)\mu \leq -\lambda_2 \|(\Pi \otimes I_p)\mu\|^2 \end{aligned}$$

where[4] λ_2 is the smallest nonzero eigenvalue of the Laplacian DD^T and $\Pi = I_N - (\mathbf{1}_N\mathbf{1}_N^T/N)$. Since H is proper (by the assumption on each H_i), the above inequality implies that the state trajectory ξ stays in a compact set. With the application of the LaSalle invariance principle, ξ converges to the ω-limit set $\Omega(\xi(0))$ where we have that $(\Pi \otimes I_p)\mu = 0$. In the ω-limit set $\Omega(\xi(0))$, the trajectory ξ is a solution to

$$\begin{bmatrix} \dot{\xi}_1 \\ \vdots \\ \dot{\xi}_N \end{bmatrix} = \begin{bmatrix} \phi(\xi_1) \\ \vdots \\ \phi(\xi_N) \end{bmatrix} \tag{11}$$

from an initial condition $\xi(0) \in R^{nN}$ such that $\eta(\xi_1) = \eta(\xi_2) = \ldots = \eta(\xi_N)$. By the incremental observability of Ξ_i, it implies that $\xi_1 = \xi_2 = \ldots = \xi_N$. Therefore, the trajectory of ξ in the ω-limit set can be described by $\mathbf{1}_N \otimes \xi_0$, where ξ_0 is the solution to $\dot{\xi}_0 = \phi(\xi_0)$, $\xi_0(0) = \omega_0$, where $\omega_0 \in \mathbb{R}^n$. □

Remark 1: The result is related to others appearing in recent literature. In [19, Theor. 2], output synchronization is proven with no such assumption as incremental observability but requires the systems to be incrementally output feedback passive and the network to satisfy a strong coupling assumption (namely, the algebraic connectivity of the graph should be larger than a certain constant).

On the other hand, incremental observability must be assumed to prove state synchronization [6]. Strict incremental passivity for a network of systems has been studied in [13] (see also [12]) and used to prove exponential synchronization under integral coupling in an all-to-all graph.

Remark 2: The assumption on the symmetry of the graph is not needed. In fact, consider the control law $\upsilon_i = \sum_j a_{ij}(\mu_j - \mu_i)$, where a_{ij} are the entries of the adjacency matrix of a weight-balanced and weakly connected directed graph.[5] Then, $\dot{H} = -\mu^T(L \otimes I_p)\mu \leq -\tilde{\lambda}_2\|(\Pi \otimes I_p)\mu\|^2$, where L is the Laplacian of the graph and $\tilde{\lambda}_2$ is the smallest nonzero eigenvalue of $(L + L^T)/2$. Then, the thesis descends as before. The assumption can be relaxed even further—see, for example, [11] and [20].

[4] We are exploiting the following well-known property. Given an $n \times n$ positive semidefinite matrix M, let $\mathcal{E}(M)$ denote the eigenspace corresponding to the zero eigenvalue. Let $\pi_{\mathcal{E}(M)}$ be the orthogonal projection onto $\mathcal{E}(M)$. Then, $x^TMx \geq \lambda_2(M)\|x - \pi_{\mathcal{E}(M)}(x)\|^2$.

[5] We recall that a graph is weight-balanced if the out-degree of each node is equal to its in-degree. It is weakly connected if for each pair of its nodes there is a (not necessarily oriented) path connecting the nodes.

III. Internal-Model-Based Output Synchronization

In this section, we discuss the solvability of the output synchronization problem using the internal model approach. This is motivated by the approach that is proposed in [22] where the existence of an internal model is a necessary and sufficient condition for the output synchronization of linear networked systems. Let us discuss the result of [22] (see also [2, Sec. 3.6]), which is adapted to the passive linear systems.

Given N, heterogeneous passive linear systems

$$\Sigma_i : \begin{cases} \dot{x}_i = F_i x_i + G_i u_i \\ y_i = L_i x_i, i = 1, 2, \ldots, N \end{cases} \tag{12}$$

with the storage function $H_{\Sigma_i} = x_i^T P_i x_i$, $P_i = P_i^T > 0$ such that $F_i^T P_i + P_i F_i \leq 0$, $P_i G_i = L_i^T$, with (L_i, F_i) detectable and with a graph $\mathcal{G}$ (which here, as usual in this paper, we assume static undirected and connected), find a feedback control law u_i for each system i which: i) uses relative measurements concerning only the systems which are connected to the system i via the graph G and ii) such that output synchronization is achieved, that is, $\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = 0$ for all $i, j \in \{1, 2, \ldots, N\}$.

Excluding the trivial case where the closed-loop system has an attractive set of equilibria where the outputs are all zero, the authors of [22] show that the output synchronization problem for N heterogeneous systems is solvable if and only if there exist matrices S, C such that $\lim_{t\to\infty} \|y_i(t) - Ce^{-St}\xi_0\| = 0$ for each $i \in \{1, 2, \ldots, N\}$, for some ξ_0 and (S, C) is observable. Moreover, provided that $\sigma(S) \subset j\mathbb{R}$, the controllers which solve the regulation problem are

$$u_i = -K_i(y_i - L_i \Pi_i \xi_i) = \Gamma_i \xi_i = -K_i(y_i - C\,\xi_i) + \Gamma_i \xi_i \tag{13}$$

where $K_i > 0$, Π_i, Γ_i are matrices which solve the regulator equations

$$\begin{aligned} F_i \Pi_i + G_i \Gamma_i &= \Pi_i S \\ L_i \Pi_i &= C \end{aligned} \tag{14}$$

and $\xi_i \in \mathbb{R}^p$, $i = 1, \ldots N$ are the internal model states that synchronize via communication channels. The latter are described by

$$\dot{\xi}_i = S\xi_i - B \sum_{k=1}^{M} d_{ik} \zeta_k \qquad i = 1, \ldots, N \tag{15}$$

where d_{ik} is the (i, k) entry of the incidence matrix D, $\zeta = (D^T \otimes I_p)(I_N \otimes C)\xi$, and the triple (S, B, C) satisfies Assumption 2. The controllers (13)–(15) are a modified form of the ones in [22, Eq. (10)] where, in the latter, the local controller communicates the entire exosystem state ξ_i to its connecting nodes and the local controller uses state-feedback and state-observer.

Proposition 2: The controllers (13)–(15) solve the output synchronization problem for the N heterogenous passive linear systems as given in (12).

Proof: By denoting $\varepsilon_i = x_i - \Pi_i \xi_i$ and using (12)–(15), we have

$$\dot{\varepsilon}_i = (F_i - G_i K_i L_i)\varepsilon_i - \Pi_i B \sum_k d_{ik} \zeta_k \tag{16}$$

where $F_i - G_i K_i L_i$ is Hurwitz (by the passivity and detectability assumption of Σ_i). On the other hand, following the result in Proposition 1, the internal model state ξ_i, $i = 1, \ldots N$ as in (15) synchronizes asymptotically which implies also that $\zeta \to 0$ as $t \to \infty$. Hence, we can conclude from (16) that for all i, $\varepsilon_i \to 0$ as $t \to \infty$. Using the second equation in (14), it follows then that the output synchronizes asymptotically. □

We note that in the above internal-model-based distributed controller, it uses the distributed control strategy as shown in Fig. 2 where the relative measurement of the internal model output is exchanged between agents. This framework provides an alternative solution to the classical output synchronization of passive systems, such as the one proposed in [15] and [17] as discussed before in the Introduction and shown in Fig. 1.

In the following text, we show that the aforementioned result can be extended to the case of nonlinear incrementally passive systems. Consider N heterogenous nonlinear plants connected over an undirected and connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each system Σ_i, with $i = 1, 2, \ldots, N$, is described as in (1) and is incrementally passive, that is, there exists a regular function $H_{\Sigma_i} : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}_+$ such that (2) holds.

Suppose that for every $i = 1, \ldots N$, there exists an internal model Ξ_i as in (7), with state variable ξ_i and maps ϕ, γ and η such that (8) holds. Suppose also that there exist functions $\pi_i(\xi_i), \alpha_i(\xi_i)$ such that

$$\frac{\partial \pi_i(\xi_i)}{\partial \xi_i} \phi(\xi_i) = f_i\left(\pi_i(\xi_i)\right) + g_i\left(\pi_i(\xi_i)\right) \alpha_i(\xi_i) \tag{17}$$

$$g_i\left(\pi_i(\xi_i)\right) = \frac{\partial \pi_i(\xi_i)}{\partial \xi_i} \gamma(\xi_i) \tag{18}$$

hold for all $\xi_i \in \mathbb{R}^n$. Finally, let the output maps agree in such a way that for $i \neq j$

$$h_i\left(\pi_i(\xi_i)\right) = h_j\left(\pi_j(\xi_i)\right) \tag{19}$$

holds for all $\xi_i \in \mathbb{R}^n$.

The equations above characterize the invariant manifold

$$\begin{aligned} \mathcal{M} = \{(x_i, \xi_i), i = 1, 2, \ldots, N : x_i = \pi_i(\xi_i), \\ \xi_i = \xi_j, \forall i, j = 1, 2, \ldots, N\} \end{aligned}$$

over which $y_i = y_j$ for all i, j. In fact, the variable $\varepsilon_i = x_i - \pi_i(\xi_i)$ satisfies

$$\begin{aligned} \dot{\varepsilon}_i &= f_i\left(\varepsilon_i + \pi_i(\xi_i)\right) + g_i\left(\varepsilon_i + \pi_i(\xi_i)\right) u_i \\ &\quad - f_i\left(\pi_i(\xi_i)\right) - g_i\left(\pi_i(\xi_i)\right)\left(\alpha_i(\xi_i) + v_i\right) \\ y_i &= h_i\left(\varepsilon_i + \pi_i(\xi_i)\right). \end{aligned} \tag{20}$$

Hence, if initially $\varepsilon_i = 0$, the feedforward input $u_i = \alpha_i(\xi_i) + v_i$ renders the manifold above a controlled invariant one provided that on this manifold $v_i = v_j$ (the latter needs to have

$\xi_i = \xi_j$ for all i, j as well). On this manifold, output synchronization is achieved since $y_i = h_i(\pi_i(\xi_i)) = h_j(\pi_j(\xi_j)) = y_j$. Moreover, the feedforward control becomes equal to $\alpha_i(\xi_i)$. In the proposition below, we propose a distributed controller that makes the manifold above globally attractive and guarantees $\upsilon_i = \upsilon_j$ for all i, j on this manifold, thus providing a solution to the nonlinear output synchronization problem.

Proposition 3: Consider the internal model as in (7) satisfying Assumption 1. Assume that for every i, Σ_i in (1) is incrementally output-feedback passive and (17)–(19) hold. Then, system (1) is closed loop with the internal-model-based distributed controllers

$$\begin{aligned}\dot{\xi}_i &= \phi(\xi_i) + \gamma(\xi_i)\upsilon_i\\ \upsilon_i &= \sum_{k=1}^{M} d_{ik}\zeta_k\\ u_i &= \alpha_i(\xi_i) + \upsilon_i - K_i\left(y_i - h_i\left(\pi_i(\xi_i)\right)\right)\end{aligned} \tag{21}$$

where $K_i > -\rho_i$ and $\zeta = (D^T \otimes I_p)\mu$ as in (9), satisfy $\lim_{t\to+\infty} \|y_i(t) - y_j(t)\| = 0$ for each $i \neq j$. More precisely, there exists $\omega_0 \in \mathbb{R}^n$ such that for every $i = 1, 2, \ldots, N$, $\lim_{t\to+\infty} \|y_i(t) - h_0(\xi_0(t))\| = 0$, where $h_0 = h_i \circ \pi_i$ and ξ_0 is the solution of $\dot{\xi}_0 = \phi(\xi_0)$, $\xi_0(0) = \omega_0$.

Remark 3: In (21), the ξ_i subsystem represents the internal model that at steady state generates the feedforward input $\alpha_i(\xi_i)$. The innovation term $y_i - h_i(\pi_i(\xi_i))$ that appears in u_i renders the manifold $\mathcal{M}$ introduced above globally attractive. The condition (18) allows us to neutralize the effect of the diffusive term υ_i in the dissipation inequality by canceling it out via u_i. Without (18) and without having υ_i in u_i, we could not exploit the incremental passivity of the system in the dissipation inequality.

Proof: Denoting $x_i' = \pi_i(\xi_i)$, we can observe that

$$\begin{aligned}\dot{x}_i' &= f_i(x_i') + g_i\left(x_i'\right)\alpha_i(\xi_i) + \frac{\partial \pi_i(\xi_i)}{\partial \xi_i}\gamma(\xi_i)\upsilon_i\\ &= f_i(x_i') + g_i(x_i')\left(\alpha_i(\xi_i) + \upsilon_i\right)\end{aligned} \tag{22}$$

where we have used (18) in the last equation.

Using H_{Σ_i}, (2), and (22), it can be computed that $\dot{H}_{\Sigma_i}(x_i, x_i') \leq -\rho_i\|h_i(x_i) - h_i(x_i')\|^2 + \langle h_i(x_i) - h_i(x_i'), u_i - \alpha_i(\xi_i) - \upsilon_i\rangle$. Thus, by assigning $u_i = \alpha_i(\xi_i) + \upsilon_i - K_i\big(h_i(x_i) - h_i(x_i')\big)$, we arrive at $\dot{H}_{\Sigma_i}(x_i, x_i') \leq -(K_i + \rho_i)\| h_i(x_i) - h_i(x_i')\|^2$.

Finally, by defining $H(\xi, x) = \sum_i H_{\Xi_i}(\xi_i) + \sum_i H_{\Sigma_i}(x_i, \pi_i(\xi_i))$ and using the above inequality and following the same argument as in the proof of Proposition 1 as far as the derivative of $\sum_i H_{\Xi_i}$ is concerned, we obtain that

$$\dot{H} \leq -\sum_{i=1}^{N}(K_i + \rho_i)\left(\|h_i(x_i) - h_i\left(\pi_i(\xi_i)\right)\|^2\right) - \lambda_2\left\|(\Pi \otimes I_p)\mu\right\|^2.$$

Since the trajectories of ξ are bounded according to Proposition 1 and using the regularity of H_{Σ_i}, this inequality implies that the state trajectories x are also bounded and remain in a compact set.

Moreover, by the LaSalle invariance principle, the state trajectory (ξ, x) converges to the ω-limit set $\Omega(\xi(0), x(0))$ where both $\lambda_2\|(\Pi \otimes I_p)\mu\|^2$ and $\sum_i \|h_i(x_i) - h_i(\pi_i(\xi_i))\|^2$ are equal to zero. Similar to the arguments as in the proof of Proposition 1, the trajectory of ξ in the ω-limit set can be described by $\mathbf{1}_N \otimes \xi_0$, where ξ_0 is the solution to $\dot{\xi}_0 = \phi(\xi_0)$, $\xi_0(0) = \omega_0$. Since the output maps agree with each other on the ω-limit set, that is, by (19), $h_i \circ \pi_i = h_j \circ \pi_j$, $i \neq j$, then the synchronization of ξ_i implies that the outputs of the heterogeneous nonlinear system also synchronize. □

We note that (17) and (19) are the nonlinear counterparts of the *regulator equations* in (14). On the other hand, (18) is a new condition on the exosystem that facilitates the control design. In this case, the function γ has to be designed such that (18) holds. Another consequence of (18) is that the dimension of the input υ_i is the same as that of u_i. In the case of linear systems, (18) becomes $G_i\Pi_i = \Pi_i B$, which has to be added to (14). The corresponding control law, which solves the output synchronization for (12), is

$$u_i = -K_i\left(y_i - L_i\Pi_i\xi_i\right) + \Gamma_i\xi_i + \sum_{k=1}^{M} d_{ik}\zeta_k,$$

(cf., the control law in (13)).

The reason for the discrepancy between the control law (13) and the one proposed here as well as the need for the extra condition (19) is owing the nonlinear nature of the system under consideration. In fact, suppose that (19) does not hold. Then, the dynamics (20) would become

$$\begin{aligned}\dot{\varepsilon}_i = f_i\left(\varepsilon_i + \pi_i(\xi_i)\right) + g_i\left(\varepsilon_i + \pi_i(\xi_i)\right)u_i\\ -f_i\left(\pi_i(\xi_i)\right) - g_i\left(\pi_i(\xi_i)\right)\alpha_i(\xi_i) - \frac{\partial \pi_i}{\partial \xi_i}\gamma(\xi_i)\upsilon_i.\end{aligned} \tag{23}$$

If the system was the linear system (12), then the dynamics above would be written as

$$\dot{\varepsilon}_i = (F_i - G_iK_iL_i)\varepsilon_i - \Pi_iB\upsilon_i. \tag{24}$$

With the passivity property of (12) and the detectability of the pair (L_i, F_i), the matrix $(F_i - G_iK_iL_i)$ is Hurwitz. Hence, $\upsilon_i \to 0$ implies $\varepsilon_i \to 0$ by the trivial input-to-state stability of asymptotically stable linear systems. In the nonlinear case, the input-to-state stability is not guaranteed and the argument sketched above does not apply. The nonlinear case requires slightly different arguments, controllers, and assumptions summarized in Proposition 3.

Example 1: The following subsystem Σ_i is a Goodwin oscillator example as used in [16] with a modification on the inputs:[6]

$$\Sigma_i : \begin{bmatrix}\dot{x}_{i,1}\\ \dot{x}_{i,2}\\ \dot{x}_{i,3}\end{bmatrix} = \underbrace{\begin{bmatrix}-0.5x_{i,1} + \frac{1}{x_{i,3}^{19}+1}\\ -x_{i,2} + x_{i,1}\\ -x_{i,3} + x_{i,2}\end{bmatrix}}_{f(x_i)} + u_i$$

$$y_i = x_i,$$

[6]Here, we also use a different Hill coefficient than that reported in [16] namely $p = 19$ instead of $p = 17$.

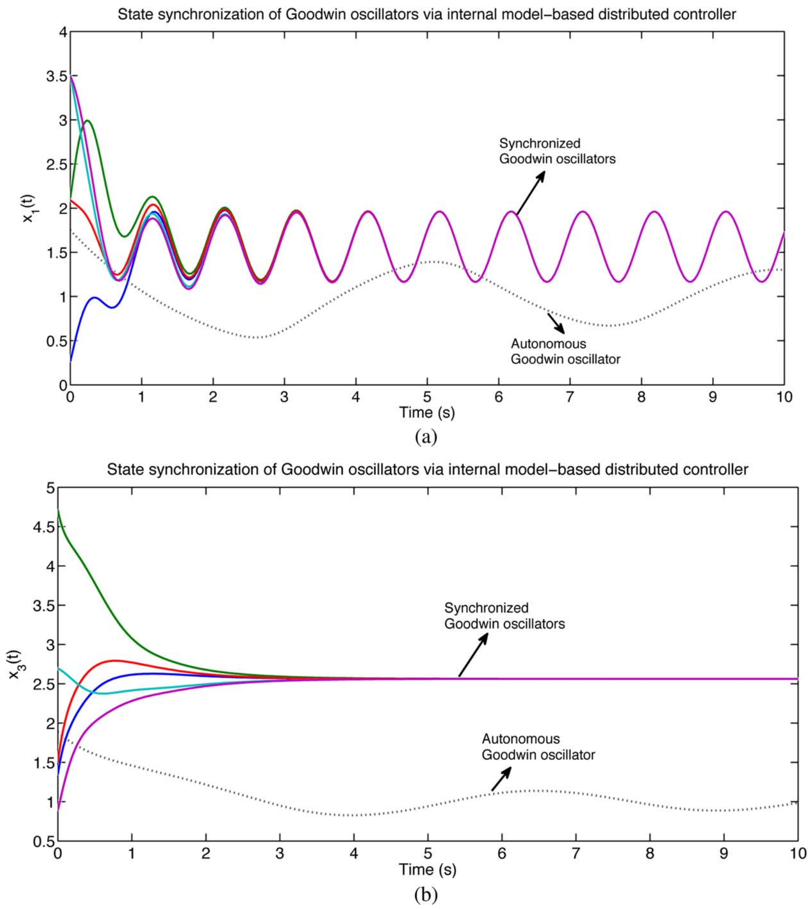

Fig. 3. Numerical simulation results of the interconnected Goodwin oscillators with internal-model-based distributed controllers. The dashed-line shows the trajectory of an autonomous Goodwin oscillator which exhibits a periodic orbit with a different period from the controlled one. In particular, the states $x_{i,1}$ and $x_{i,2}$ (not shown here) converge to a sinusoid of frequency ω while $x_{i,3}$ converges to a constant. (a) The plot of $x_{i,1}$ and (b) the plot of $x_{i,3}$.

where $x_i = (x_{i,1}\ x_{i,2}\ x_{i,3})^T \in \mathbb{R}^3_+$ and $u_i \in \mathbb{R}^3$. When $u_i = 0$ (i.e., it is autonomous), this Goodwin oscillator exhibits a periodic orbit as shown in Fig. 3. It can be checked that using

$$H_{\Sigma_i}(x_i, x_i') = \frac{1}{2}\left((x_{i,1} - x_{i,1}')^2 + (x_{i,2} - x_{i,2}')^2 + (x_{i,3} - x_{i,3}')^2\right),$$

the Goodwin oscillator Σ_i is incrementally output-feedback passive with $\rho = -1.64432$, that is, (2) holds.

It has been shown in [16] that output synchronization of Goodwin oscillators can be realized by directly coupling the input–output pairs via an undirected and balanced graph. Despite the simplicity of this approach, it cannot be used to control the resulting synchronized oscillation behavior. In the following treatment, we will use the result in Proposition 3 such that the steady-state trajectories of $x_{i,1}$, $x_{i,2}$, and $x_{i,3}$ follow a biased sinusoidal signal with a predefined frequency ω. Let the internal model Ξ_i be given by

$$\dot{\xi}_i = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xi_i + \gamma(\xi_i)\upsilon_i$$
$$\mu_i = \eta(\xi_i),$$

where $\omega > 0$ and $\xi_i = \begin{bmatrix} \xi_{i,1} \\ \xi_{i,2} \\ \xi_{i,3} \end{bmatrix}$ and the functions γ, η will be designed to fulfill the regulator equation. By choosing

$$\pi(\xi_i) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xi_i$$

it is immediate to check that (17) holds when we take

$$\alpha(\xi_i) = \begin{bmatrix} 0.5\xi_{i,1} - \frac{1}{\xi_{i,3}^{19}+1} + \omega\xi_{i,2} \\ \xi_{i,2} - (1+\omega)\xi_{i,1} \\ \xi_{i,3} - \xi_{i,2} \end{bmatrix}.$$

For satisfying (18), the matrix $\gamma(\xi_i)$ must be given by

$$\gamma(\xi_i) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Finally, by taking $\eta(\xi_i) = \gamma(\xi_i)^T\xi_i$, the passivity of Ξ_i is immediate by using $H_{\Xi_i}(\xi_i) = (1/2)\xi_i^T\xi_i$. Using this internal model and the distributed controller as in Proposition 3 with gains $K_i > -\rho$, the Goodwin oscillators asymptotically synchronize to a biased sinusoidal signal with the desired frequency ω. In Fig. 3, the numerical simulation of five Goodwin oscillators interconnected over a ring graph and controlled by the proposed internal-model-based distributed controllers with gains $K_i = 2$ and $\omega = 2\pi$ rad/s is shown.

A. Output Synchronization With Disturbance Rejection

Consider now the case when disturbance signals generated by an exosystem (5) are added to the control input of each subsystem Σ_i in (1), i.e.,

$$\Sigma_i : \begin{cases} \dot{x}_i &= f_i(x_i) + g_i(x_i)(u_i + d_i) \\ y_i &= h_i(x_i) \end{cases} \tag{25}$$

where, for simplicity, the disturbance signal d_i is generated by a linear exosystem

$$W_i^d : \begin{cases} \dot{w}_i^d = S_i^d w_i^d \\ d_i = C_i^d w_i^d \end{cases} \tag{26}$$

with the exosystem state $w_i^d \in \mathbb{R}^n$ and the disturbance signal $d_i \in \mathbb{R}^q$. If W_i^d is marginally stable and observable, that is, there exists a symmetric and positive-definite matrix $Q_i \in \mathbb{R}^{n\times n}$ such that $(S_i^d)^T Q_i + Q_i S_i^d \leq 0$ and the pair (C_i^d, S_i^d) is observable, then the output synchronization can still be solved as given in the following proposition:

Proposition 4: Consider the internal model as in (7) satisfying Assumption 1. Assume that for every i, Σ_i in (25) is incrementally output-feedback passive and (17)–(19) hold. Then, system (25) in closed loop with the internal-model-based distributed controllers

$$\begin{aligned} \dot{\xi}_i^d &= S_i^d \xi_i^d + B_i^d \left(y_i - h_i\left(\pi_i(\xi_i^r)\right)\right) \\ \dot{\xi}_i^r &= \phi\left(\xi_i^r\right) + \gamma\left(\xi_i^r\right) \upsilon_i \\ \upsilon_i &= \sum_{k=1}^{M} d_{ik}\zeta_k \\ u_i &= \alpha_i\left(\xi_i^r\right) + \upsilon_i - K_i\left(y_i - h_i\left(\pi_i(\xi_i^r)\right)\right) - C_i^d \xi_i^d \end{aligned} \tag{27}$$

where $K_i > -\rho_i$, $B_i^d = Q_i^{-1}(C_i^d)^T$ and $\zeta = (D^T \otimes I_p)\mu^r$ with $\mu^r = [(\mu_1^r)^T \dots (\mu_N^r)^T]^T$ and $\mu_i^r = \eta(\xi_i^r)$, satisfies $\lim_{t\to+\infty} \|y_i(t) - y_j(t)\| = 0$ for each $i \neq j$. More precisely, there exists $\omega_0 \in \mathbb{R}^n$ such that for every $i = 1, 2, \dots, N$, $\lim_{t\to+\infty} \|y_i(t) - h_0(\xi_0^r(t))\| = 0$, where $h_0 = h_i \circ \pi_i$ and ξ_0^r is the solution of $\dot{\xi}_0^r = \phi(\xi_0^r)$, $\xi_0^r(0) = \omega_0$.

Proof: The proof follows similarly to the proof of Proposition 3. In particular, we note that by denoting $\xi_i^{d'} = w_i^d - \xi_i^d$, where w_i^d satisfies (26), we have

$$\begin{aligned} \dot{\xi}_i^{d'} &= S_i^d \xi_i^{d'} - B_i^d \left(y_i - h_i\left(\pi_i(\xi_i^r)\right)\right) \\ d_i - C_i^d \xi_i^d &= C_i^d \xi_i^{d'}. \end{aligned}$$

Using $H_{\Xi_i^d} = (\xi_i^{d'})^T Q_i \xi_i^{d'}$, it follows immediately that:

$$\dot{H}_{\Xi_i^d} \leq -\left\langle d_i - C_d \xi_i^d, y_i - h_i(\pi_i\left(\xi_i^r\right)).\right\rangle.$$

Consider H as in the proof of Proposition 3. The function $H_{\Sigma_i}(x_i, x_i')$, where now $x_i' = \pi_i(\xi_i^r)$, in the presence of the disturbance d_i satisfies $\dot{H}_{\Sigma_i}(x_i, x_i') \leq -\rho_i \|h_i(x_i) - h_i(x_i')\|^2 + \langle h_i(x_i) - h_i(x_i'), u_i + d_i - \alpha_i(\xi_i) - \upsilon_i\rangle$. The choice of u_i in (27) yields $\dot{H}_{\Sigma_i}(x_i, x_i') \leq -(\rho_i + K_i)\|h_i(x_i) - h_i(x_i')\|^2 + \langle h_i(x_i) - h_i(x_i'), -C_i^d\xi_i^d + d_i\rangle = -(\rho_i + K_i)\|h_i(x_i) - h_i(\pi_i(\xi_i^r))\|^2 + \langle h_i(x_i) - h_i(\pi_i(\xi_i^r)), -C_i^d\xi_i^d + d_i\rangle$. Thus, similar to Proposition 3

$$\sum_i \dot{H}_{\Xi_i^d} + \dot{H} \leq -\lambda_2 \left\|(\Pi \otimes I_p)\mu\right\|^2 - \sum_{i=1}^{N} \left(K_i + \rho_i\right) \left\|h_i(x_i) - h_i(\pi_i(\xi_i^r))\right\|^2.$$

Finally, we prove the claim by using the same arguments as in the last part of the proof of Proposition 3. □

Remark 4: The output agreement problem with disturbance decoupling of incrementally passive networked systems has been also considered in [3]. The approach is different from the one pursued here, in that [3] considers the constrained setting for which the input u generated by the controller must belong to the space $\mathcal{R}(D^T \otimes I)$ and the only measured variables are the relative measurements that lie in $\mathcal{R}(D \otimes I)$. To deal with this setting (that, in particular, prevents using absolute measurements of the state variables), the approach of [12] to output regulation of an incrementally passive system has proved to be resolutive. In the context of networked systems, the approach results in controllers associated with the edges of the graph that use relative measurements achieve the solution of the problem. Furthermore, the approach pursued in [3] allows establishing a connection with optimal flow control problems.

IV. Internal Model-Based Control of a Relative Degree-Two Systems

Let us consider a relative degree-two system as follows:

$$\Sigma_i : \begin{cases} \dot{q}_i = x_i \\ \dot{x}_i = f_i(x_i) + G_i(u_i + d_i) \\ y_i = q_i \end{cases} \tag{28}$$

where $q_i, x_i, y_i, u_i \in \mathbb{R}^m$, d_i is a disturbance signal which is added to the input channel and G_i is a full-rank constant matrix. Compared to (4), we restrict the systems to systems with a constant input vector. This considerably simplifies the analysis.

As in the previous section, the disturbance is assumed to be generated by the exosystem

$$W_i^d : \begin{cases} \dot{w}_i^d = S_i^d w_i^d \\ d_i = C_i^d w_i^d \end{cases} \tag{29}$$

which is (marginally) stable and observable, that is, there exists a symmetric and positive-definite matrix $Q_i \in \mathbb{R}^{n\times n}$ such that $(S_i^d)^T Q_i + Q_i S_i^d \leq 0$, and the pair (C_i^d, S_i^d) is observable.

Using a distributed control strategy, we are interested in the problem of tracking a reference velocity that is remotely generated and is available only to a leader subsystem Σ_1. In particular, we show in our main proposition below that when $d = 0$, the problem can be solved by only using the relative position measurement $q_i - q_j$, and when $d \neq 0$, then, in addition, the controller for subsystem Σ_i requires the absolute measurement of its own velocity x_i in order to reject the disturbances.

We could have as well cast a problem in which the reference signal to track is generated locally by the synchronization of different exosystems, as in the previous sections. We are taking this (minor) detour from the path followed so far, because relative degree-two systems of the form (28) appear frequently in formation control problems (see, for example, [1] and [2]), where tracking a reference signal that is prescribed by one (or more) leader is a more common problem. Nevertheless, a similar tracking problem as formulated in the previous section can also be solved using a slightly modified approach. This is not pursued for the sake of conciseness.

For clarity of presentation, we assume that the reference velocity $r \in \mathbb{R}^p$ is generated by the linear exosystem

$$W^r : \begin{cases} \dot{w}^r = S^r w^r \\ r = C^r w^r \end{cases} \tag{30}$$

which is (marginally) stable, that is, there exists a symmetric and positive-definite matrix P s.t. $(S^r)^T P + P S^r \leq 0$, and the pair (C^r, S^r) is observable. The reference velocity signal r is only available to Σ_1 and we define the velocity tracking error for Σ_1 as $e_1 = x_1 - r$. For the case of reference velocity generated by a nonlinear exosystem, we refer the interested reader to [5].

The other systems generate an estimate μ_i^r of the reference signal via an internal model (cf., (7))

$$\Xi_i^r : \begin{cases} \dot{\xi}_i^r = S^r \xi_i^r + B^r \upsilon_i^r \\ \mu_i^r = C^r \xi_i^r \end{cases} \tag{31}$$

where $B^r = P^{-1}(C^r)^T$. It can be checked that Ξ_i^r is incrementally passive with the storage function $H_{\Xi_i^r} = (1/2)(\xi_i^r)^T P \xi_i^r$. For $i \neq 1$, we denote the velocity tracking error for the subsystem Σ_i by

$$e_i := x_i - \mu_i^r.$$

For each system Σ_i, $i = 1, 2, \ldots, N$, we assume output strict passivity, that is, (2) holds with $\rho_i > 0$. We further assume that the output with respect to which passivity holds is $h_i(x_i) = x_i$. Note that $e_i - e_i' = x_i - x_i'$.

We now show that the regulator equations (analogous to those in (17)–(19)) are solvable for every Σ_i w.r.t. the corresponding autonomous internal model Ξ_i^r (for $i \neq 1$) or w.r.t. the exosystem W^r (for $i = 1$). For every $i \neq 1$, by defining $\pi_i(\xi_i^r) = C^r \xi_i^r$, $\alpha_i(\xi_i^r) = G_i^{-1}(C^r S^r \xi_i^r - f_i(C^r \xi_i^r))$, and $\Gamma_i = G_i^{-1} C^r B^r$, the following regulator equations hold:

$$\left.\begin{array}{c} \frac{\partial \pi_i(\xi_i^r)}{\partial \xi_i^r} S^r \xi_i^r = f_i\left(\pi_i\left(\xi_i^r\right)\right) + G_i \alpha_i\left(\xi_i^r\right) \\ 0 = \pi_i\left(\xi_i^r\right) - C^r \xi_i^r \end{array}\right\} \tag{32}$$

and

$$G_i \Gamma_i = \frac{\partial \pi_i\left(\xi_i^r\right)}{\partial \xi_i^r} B^r. \tag{33}$$

Similarly, for $i = 1$, we can define $\pi_1(w^r) = C^r w^r$ and $\alpha_1(w^r) = G_1^{-1}(C^r S^r w^r - f_1(C^r w^r))$ which fulfill

$$\left.\begin{array}{c} \frac{\partial \pi_1(w^r)}{\partial w^r} S^r w^r = f_1\left(\pi_1(w^r)\right) + G_1 \alpha_1(w^r) \\ 0 = \pi_1(w^r) - C^r w^r \end{array}\right\}. \tag{34}$$

Proposition 5: Consider the systems (28), with G_i full rank for all i, satisfying (2) with $g_i(\xi_i) = G_i$ and $\rho_i > 0$. Assume that the reference velocity r is generated by the exosystem (30). Consider also the internal models (31) for $i \neq 1$. Assume that there exist maps $\pi_i(\xi_i)$, $\alpha_i(\xi_i)$, $i = 2, \ldots, N$, $\pi_1(w^r)$, and $\alpha_1(w^r)$ such that (32)–(34) hold.

Let the distributed control u in (28) be generated by

$$\dot{\hat{x}}_i = f_i(x_i) + G_i u_i + G_i C_i^d \xi_i^d + K_i(x_i - \hat{x}_i) \tag{35}$$
$$\dot{\xi}_i^d = S_i^d \xi_i^d + L_i(x_i - \hat{x}_i) \tag{36}$$
$$\dot{\xi}_i^r = S^r \xi_i^r + B^r \upsilon_i^r, \quad i \neq 1 \tag{37}$$
$$u_i = \begin{cases} \alpha_1(w^r) + \upsilon_1^r - C_1^d \xi_1^d & \text{if } i = 1 \\ \alpha_i\left(\xi_i^r\right) + (\Gamma_i + I)\upsilon_i^r - C_i^d \xi_i^d & \text{if } i \neq 1 \end{cases} \tag{38}$$

where $\upsilon_i^r = -(D_i \otimes I_p) z$, for all i with $z = (D^T \otimes I_p) y$ and with D_i be the ith row of the incidence matrix D, and the matrices K_i, L_i are chosen such that

$$\begin{bmatrix} -K_i & -G_i C_i^d \\ -L_i & S_i^d \end{bmatrix}$$

is Hurwitz. Then, the solution $(z, x, \hat{x}, \xi^r, \xi^d)$ of the closed-loop system is defined for all $t \geq 0$ and converges asymptotically to the set where $z = 0$, $x = \pi(\xi^r)$, $\xi^d = w^d$, and $\xi_i^r = w^r$, $\mu_i^r = r$ for all $i \neq 1$.

Proof: By using (32)–(34) and by defining $\bar{x}_i := \pi_i(\xi_i^r) = \mu_i^r$ (thus implying that $e_i = x_i - \bar{x}_i$), $\bar{u}_i := \alpha_i(\xi_i^r) + \Gamma_i \upsilon_i^r - \bar{d}_i$ for $i \neq 1$, and $\bar{x}_1 := \pi_1(w^r) = r$ (which also implies that $e_1 = x_1 - \bar{x}_1$), $\bar{u}_1 := \alpha_1(w^r) - \bar{d}_1$, (where $\bar{d}_i$ is an additional control signal for rejecting the disturbance signals to be defined later), we have that for every i

$$\dot{\bar{x}}_i = \begin{cases} f_1(\bar{x}_1) + G_1 \bar{u}_1 + G_1 \bar{d}_1 & \text{if } i = 1 \\ f_i(\bar{x}_i) + G_i \bar{u}_i + G_i \bar{d}_i & \text{if } i \neq 1 \end{cases}$$
$$0 = \begin{cases} \bar{x}_1 - C^r w^r & i = 1 \\ \bar{x}_i - C^r \xi_i^r & i \neq 1. \end{cases} \tag{39}$$

By the incremental passivity property in (2), given $u_i, \bar{u}_i, \upsilon_i^r$ and the solutions $x_i, \bar{x}_i$ to (28) and (39), respectively, it follows that for $i \neq 1$:

$$\begin{aligned} &\frac{\partial H_{\Sigma_i}(x_i, \bar{x}_i)}{\partial x_i} \left(f_i(x_i) + G_i u_i + G_i d_i\right) \\ &\quad + \frac{\partial H_{\Sigma_i}(x_i, \bar{x}_i)}{\partial \bar{x}_i} \left(f_i(\bar{x}_i) + G_i \bar{u}_i + G_i \bar{d}_i\right) \\ &\qquad \leq -\rho_i \|e_i\|^2 + e_i^T (u_i - \bar{u}_i) \\ &\qquad\quad + e_i^T (d_i - \bar{d}_i). \end{aligned} \tag{40}$$

By the incremental passivity of Ξ_i^r, using $H_{\Xi_i^r}(\xi_i^r, w^r) = (1/2)(\xi_i^r - w^r)^T P (\xi_i^r - w^r)$, it holds true that for $i \neq 1$

$$\begin{aligned} &\frac{\partial H_{\Xi_i^r}\left(\xi_i^r, w^r\right)}{\partial \xi_i^r} \left(S^r \xi_i^r + B^r \upsilon_i^r\right) + \frac{\partial H_{\Xi_i^r}\left(\xi_i^r, w^r\right)}{\partial w^r} S^r w^r \\ &\qquad \leq \left(\mu_i^r - r\right)^T \upsilon_i^r. \end{aligned} \tag{41}$$

Set $H_r(\xi^r, w^r) = \sum_{i=2}^N H_{\Xi_i^r}(\xi_i^r, w^r)$ and $\tilde{\mu} := \mu - \mathbf{1}_N \otimes r$, where $\mu = \left[r^T \ (\mu_2^r)^T \ldots (\mu_N^r)^T\right]^T$ and $\xi^r = \left[(\xi_2^r)^T \ldots (\xi_N^r)^T\right]^T$.

Observe that $\tilde{\mu}_1 = \mathbf{0}$. Denoting $\upsilon := [(\upsilon_1^r)^T(\upsilon_2^r)^T \dots (\upsilon_N^r)^T]^T$, (41) implies

$$\dot{H}_r(\xi^r, w^r) \le \tilde{\mu}^T \upsilon.$$

Recall that the relative position measurement between connected systems is given by $z = (D^T \otimes I_p)q$. Since $\dot{q} = x = e + \mu$ and $(D^T \otimes I_p)\mathbf{1}_N \otimes r = \mathbf{0}$, it follows that $\dot{z} = (D^T \otimes I_p)(e + \tilde{\mu})$. Hence, the function $V(z) = (1/2)z^T z$ fulfills the equality

$$\dot{V}(z) = z^T (D^T \otimes I_p)(e + \tilde{\mu}).$$

By defining $H_\Sigma(x, \bar{x}) := \sum_{i=1}^N H_{\Sigma_i}(x_i, \bar{x}_i)$ and bearing in mind (40), routine computations show that

$$\dot{H}_\Sigma \le -(x - \bar{x})^T \rho (x - \bar{x}) + e^T(u - \bar{u}) + e^T(d - \bar{d})$$

where $\rho := \text{block.diag}\{\rho_1 I_m, \dots, \rho_N I_m\}$.

Using $\tilde{x}_i := x_i - \hat{x}_i$ and $\tilde{\xi}_i^d := \xi_i^d - w_i^d$, the state equation of $(\tilde{x}_i, \tilde{\xi}_i^d)$ is given by

$$\Phi_i : \begin{cases} \dot{\tilde{x}}_i = -G_i C_i^d \tilde{\xi}_i^d - K_i \tilde{x}_i \\ \dot{\tilde{\xi}}_i^d = S_i^d \tilde{\xi}_i^d - L_i \tilde{x}_i \end{cases}$$

where we have exploited the dynamical equations (28), (29), (35), and (36).

By hypothesis, the matrix $\begin{bmatrix} -K_i & -G_i C_i^d \\ -L_i & S_i^d \end{bmatrix}$ is Hurwitz which implies that there exists a positive definite matrix Q_i such that

$$\begin{bmatrix} -K_i & -G_i C_i^d \\ -L_i & S_i^d \end{bmatrix}^T Q_i + Q_i \begin{bmatrix} -K_i & -G_i C_i^d \\ -L_i & S_i^d \end{bmatrix} \le -2\epsilon_i I$$

where $\epsilon_i > (1/\rho_i)\|C_i^d\|^2$ (this particular choice of ϵ_i will be useful in bounding the total derivative of the Lyapunov function below). Hence, by defining $H_{\Phi_i} := (1/2)[\tilde{x}_i^T \ (\tilde{\xi}_i^d)^T] Q_i \left[\begin{smallmatrix} \tilde{x}_i \\ \tilde{\xi}_i^d \end{smallmatrix}\right]$, we have

$$\dot{H}_{\Phi_i} \le -\epsilon_i \|\tilde{x}_i\|^2 - \epsilon_i \left\|\tilde{\xi}_i^d\right\|^2.$$

Let us define the combined Lyapunov function for all Φ_i, $i = 1, \dots, N$, by $H_\Phi(\tilde{x}, \tilde{\xi}^d) = \sum_i H_{\Phi_i}(\tilde{x}_i, \tilde{\xi}_i^d)$ which satisfies

$$\dot{H}_\Phi \le -\tilde{x}^T \epsilon_x \tilde{x} - (\tilde{\xi}^d)^T \epsilon_\xi \tilde{\xi}^d$$

where $\epsilon_x := \text{diag}\{\epsilon_1 I_m, \dots, \epsilon_N I_m\}$ and $\epsilon_\xi := \text{diag}\{\epsilon_1 I_n, \dots, \epsilon_N I_n\}$.

Now, we let the additional control signals $\bar{d}_i$ introduced at the beginning of the proof be given by

$$\begin{aligned} \bar{d}_1 &= C_1^d \xi_1^d, \\ \bar{d}_i &= C_i^d \xi_i^d \quad i = 2, \dots, N. \end{aligned} \tag{42}$$

Combining all of the storage functions H_r, H_Σ, H_Φ, and V into the function $H = V + H_r + H_\Sigma + H_\Phi$, we have

$$\begin{aligned} \dot{H} \le\ & z^T (D^T \otimes I_p)(e + \tilde{\mu}) + \tilde{\mu}^T \upsilon - e^T \rho e \\ & + e^T(u - \bar{u}) - e^T C^d \tilde{\xi}^d - \tilde{x}^T \epsilon_x \tilde{x} - (\tilde{\xi}^d)^T \epsilon_\xi \tilde{\xi}^d \end{aligned}$$

where $C^d := \text{block.diag}\{C_i^d\}$. It can be checked that, by definition of u in (38), $\bar{u}$ at the beginning of the proof, and $\bar{d}$ in (42), $u - \bar{u} = \upsilon = -(D \otimes I_p)z$. Hence, we get

$$\begin{aligned} \dot{H} &\le -e^T \rho e - \tilde{x}^T \epsilon_x \tilde{x} - (\tilde{\xi}^d)^T \epsilon_\xi \tilde{\xi}^d - e^T C^d \tilde{\xi}^d \\ &\le -\frac{1}{2} e^T \rho e - \frac{1}{2} \tilde{x}^T \epsilon_x \tilde{x} - \frac{1}{2} (\tilde{\xi}^d)^T \epsilon_\xi \tilde{\xi}^d. \end{aligned}$$

With the regularity of H and the boundedness of w^r and w^d, it follows that $x, \bar{x}, \hat{x}, \xi^r, \xi^d$ are bounded[7] and defined for all $t \ge 0$.

With the LaSalle invariance principle, one can conclude that the solutions of the system converge to the largest invariant set $\mathcal{M}$, where $(e, \tilde{x}, \tilde{\xi}^d) = 0$. On such an invariant set, we have $e = x - \bar{x} = 0 = f(x) - f(\bar{x})$ and

$$\begin{aligned} \dot{z} &= (D^T \otimes I_p)\tilde{\mu} \\ 0 &= \text{diag}\{G_i\}(D \otimes I_p)z \\ \dot{\xi}_i^r &= S^r \xi_i^r - B^r (D_i \otimes I_p)z \quad \text{if } i \ne 1. \end{aligned}$$

Since G_i is full-rank for all $i = 1, \dots N$, it follows that $(D \otimes I_p)z = 0$. Taking its time derivative and using the dynamics of z in $\mathcal{M}$, we obtain $(DD^T \otimes I_p)\tilde{\mu} = 0$. Since the kernel of $DD^T \otimes I_p$ is the span of $1 \otimes I_p$ and noting that $\tilde{\mu}_1 = 0$, it follows then that $\tilde{\mu} = 0$, that is, the output of the internal model is equal to the reference signal in the invariant set $\mathcal{M}$. Moreover, it also implies that $z = 0$ and $\xi_i^r = w^r$ in $\mathcal{M}$ which means that the agents are synchronized. □

The structure of the controller (35)–(37) can be explained as a combination of internal models (in the ξ_i^d, ξ_i^r variables) and an observer-based system ($\hat{x}_i$) that helps in reconstructing the disturbance d_i to reject. In this respect, the construction pursued in this section slightly differs from the one in the previous section, although they both rest on the common ground of incremental passivity and internal model properties. The need for a new design is largely due to the relative degree-two nature of the system to be controlled.

Remark 5: The control law (35)–(38) can be shown to be robust to coarse measurements. This feature is investigated in [8].

V. Conclusion

In this paper, we have discussed the role of the internal model principle and of the passivity property in the design of distributed control laws for nonlinear output synchronization and formation control problems for networked nonlinear systems.

We have shown that the proposed framework provides a systematic way to deal with a variety of control problems for relative-degree-one and -two incrementally passive systems. In particular, we have shown how to achieve controlled synchronization to desired steady-state regimes despite the action of

[7]The boundedness of w^r implies the boundedness of ξ^r. With (28), the latter implies the boundedness of μ. Since $r = C^r w^r$ and r is bounded, $\bar{x}$ is bounded by (39). With the regularity of H_Σ, it implies that x is bounded. The properness of H_Φ implies that $\tilde{x}$ and $\tilde{\xi}^d$ are bounded and, thus, $\hat{x}$ andξ_d are bounded. The boundedness of ξ^r follows from the properness of H_r.

disturbances on the input channel. The controlled synchronization of a network of Goodwin oscillators has been considered as a case study.

Incremental passivity and internal models have been used also in other contexts [3], resulting in "flow-regulating" controllers placed at the edges. A unifying point of view on these different yet related approaches is worth investigating. Incremental passivity is related to several notions that have been used to study the emergence of coordination in complex networks (differential passivity, contraction, convergent systems, transverse stability, etc.). The role of internal models in connection with these classes of systems has yet to be fully explored. Passivity has been used extensively in the study of sustained oscillations in networks (see, for example, [18]). Whether disturbances can destroy oscillations and whether internal-model-based controllers can help restore the oscillatory regime is a stimulating research topic.

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