# 12

# Power System Stability

## 12.1 INTRODUCTION

The stability of an interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance. Conversely, instability means a condition denoting loss of synchronism or falling out of step. Stability considerations have been recognized as an essential part of power system planning for a long time. With interconnected systems continually growing in size and extending over vast geographical regions, it is becoming increasingly more difficult to maintain synchronism between various parts of a power system.

The dynamics of a power system are characterised by its basic features given below:

1. Synchronous tie exhibits the typical behaviour that as power transfer is gradually increased a maximum limit is reached beyond which the system cannot stay in synchronism, i.e., it falls out of step.

2. The system is basically a spring-inertia oscillatory system with inertia on the mechanical side and spring action provided by the synchronous tie wherein power transfer is proportional to sin $\delta$ or $\delta$ (for small $\delta$; $\delta$ being the relative internal angle of machines).

3. Because of power transfer being proportional to sin $\delta$, the equation determining system dynamics is nonlinear for disturbances causing large variations in angle $\delta$. Stability phenomenon peculiar to non-linear systems as distinguished from linear systems is therefore exhibited by power systems (stable up to a certain magnitude of disturbance and unstable for larger disturbances).

Accordingly power system stability problems are classified into three basic types*—steady state, dynamic and transient.

*There are no universally accepted precise definitions of this terminology. For a definition of some important terms related to power system stability, refer to IEEE Standard Dictionary of Electrical and Electronic Terms, IEEE, New York, 1972.

The study of steady state stability is basically concerned with the determination of the upper limit of machine loadings before losing synchronism, provided the loading is increased gradually.

Dynamic instability is more probable than steady state instability. Small disturbances are continually occurring in a power system (variations in loadings, changes in turbine speeds, etc.) which are small enough not to cause the system to lose synchronism but do excite the system into the state of natural oscillations. The system is said to be dynamically stable if the oscillations do not acquire more than certain amplitude and die out quickly (i.e., the system is well-damped). In a dynamically unstable system, the oscillation amplitude is large and these persist for a long time (i.e., the system is underdamped). This kind of instability behaviour constitutes a serious threat to system security and creates very difficult operating conditions. Dynamic stability can be significantly improved through the use of power system stabilizers. Dynamic system study has to be carried out for 5–10 s and sometimes up to 30 s. Computer simulation is the only effective means of studying dynamic stability problems. The same simulation programmes are, of course, applicable to transient stability studies.

Following a sudden disturbance on a power system rotor speeds, rotor angular differences and power transfer undergo fast changes whose magnitudes are dependent upon the severity of disturbance. For a large disturbance, changes in angular differences may be so large as to cause the machines to fall out of step. This type of instability is known as transient instability and is a fast phenomenon usually occurring within 1 s for a generator close to the cause of disturbance. There is a large range of disturbances which may occur on a power system, but a fault on a heavily loaded line which requires opening the line to clear the fault is usually of greatest concern. The tripping of a loaded generator or the abrupt dropping of a large load may also cause instability.

The effect of short circuits (faults), the most severe type of disturbance to which a power system is subjected, must be determined in nearly all stability studies. During a fault, electrical power from nearby generators is reduced drastically, while power from remote generators is scarcely affected. In some cases, the system may be stable even with a sustained fault, whereas other systems will be stable only if the fault is cleared with sufficient rapidity. Whether the system is stable on occurrence of a fault depends not only on the system itself, but also on the type of fault, location of fault, rapidity of clearing and method of clearing, i.e., whether cleared by the sequential opening of two or more breakers or by simultaneous opening and whether or not the faulted line is reclosed. The transient stability limit is almost always lower than the steady state limit, but unlike the latter, it may exhibit different values depending on the nature, location and magnitude of disturbance.

Modern power systems have many interconnected generating stations, each with several generators and many loads. The machines located at any one point in a system normally act in unison. It is, therefore, common practice in stability studies to consider all the machines at one point as one large machine. Also machines which are not separated by lines of high reactance are lumped together and considered as one equivalent machine. Thus a multimachine system can often be reduced to an equivalent few machine system. If synchronism is lost, the machines of each group stay together although they go out of step with other groups. Qualitative behaviour of machines in an actual system is usually that of a two machine system. Because of its simplicity, the two machine system is extremely useful in describing the general concepts of power system stability and the influence of various factors on stability. It will be seen in this chapter that a two machine system can be regarded as a single machine system connected to infinite system.

Stability study of a multimachine system must necessarily be carried out on a digital computer.

## 12.2 DYNAMICS OF A SYNCHRONOUS MACHINE

The kinetic energy of the rotor at synchronous machine is

$$\text{KE} = \frac{1}{2} J\omega^2_{sm} \times 10^{-6} \text{ MJ}$$

where $J$ = rotor moment of inertia in kg-m$^2$

$\omega_{sm}$ = synchronous speed in rad (mech)/s

But $\omega_s = \left(\frac{P}{2}\right) \omega_{sm}$ = rotor speed in rad (elect)/s

where $P$ = number of machine poles

$$\therefore \quad \text{KE} = \frac{1}{2}\left(J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right)\omega_s$$

$$= \frac{1}{2} M \omega_s$$

where $M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$

= moment of inertia in MJ-s/elect rad

We shall define the inertia constant H such that

$$G\text{H} = \text{KE} = \frac{1}{2} M \omega_s \text{ MJ}$$

where $G$ = machine rating (base) in MVA (3-phase)

H = *inertia constant* in MJ/MVA or MW-s/MVA

It immediately follows that

$$M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \text{ MJ-s/elect rad} \tag{12.1}$$

$$= \frac{GH}{180 f} \text{MJ-s/elect degree}$$

M is also called the inertia constant.

Taking $G$ as base, the inertia constant in pu is

$$M(\text{pu}) = \frac{H}{\pi f} \text{ s}^2\text{/elect rad} \tag{12.2}$$

$$= \frac{H}{180f} \text{ s}^2\text{/elect degree}$$

The inertia constant H has a characteristic value or a range of values for each class of machines. Table 12.1 lists some typical inertia constants.

**Table 12.1** Typical inertia constants of synchronous machines*

| *Type of Machine* | | *Intertia Constant H Stored Energy in MW Sec per MVA*** |
|---|---|---|
| Turbine Generator | | |
| Condensing | 1,800 rpm | 9-6 |
| | 3,000 rpm | 7-4 |
| Non-Condensing | 3,000 rpm | 4-3 |
| Water wheel Generator | | |
| Slow-speed (< 200 rpm) | | 2-3 |
| High-speed (> 200 rpm) | | 2-4 |
| Synchronous Condenser*** | | |
| Large | | 1.25 |
| Small | | 1.00 |
| Synchronous Motor with load varying from 1.0 to 5.0 and higher for heavy flywheels | | 2.00 |

It is observed from Table 12.1 that the value of H is considerably higher for steam turbogenerator than for water wheel generator. Thirty to sixty per cent of the total inertia of a steam turbogenerator unit is that of the prime mover, whereas only 4 –15% of the inertia of a hydroelectric generating unit is that of the waterwheel, including water.

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** Where range is given, the first figure applies to the smaller MVA sizes.

*** Hydrogen-Cooled, 25 per cent less.

### *The Swing Equation*

Figure 12.1 shows the torque, speed and flow of mechanical and electrical powers in a synchronous machine. It is assumed that the windage, friction and iron-loss torque is negligible. The differential equation governing the rotor dynamics can then be written as

$$J \frac{d^2\theta_m}{dt^2} = T_m - T_e \text{ Nm} \tag{12.3}$$

where

$\theta_m$ = angle in rad (mech)

$T_m$ = turbine torque in Nm; it acquires a negative value for a motoring machine

$T_e$ = electromagnetic torque developed in Nm; it acquires negative value for a motoring machine

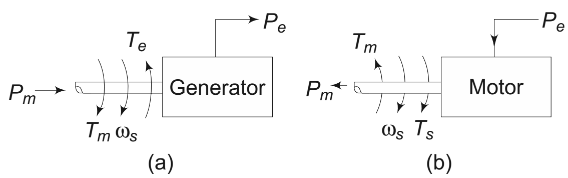



**Fig. 12.1** Flow of mechanical and electrical powers in a synchronous machine

While the rotor undergoes dynamics as per Eq. (12.3), the rotor speed changes by insignificant magnitude for the time period of interest (1s) [Sec. 12.1]. Equation (12.3) can therefore be converted into its more convenient power form by assuming the rotor speed to remain constant at the synchronous speed ($\omega_{sm}$). Multiplying both sides of Eq. (12.3) by $\omega_{sm}{}'$ we can write

$$J\omega_{sm} \frac{d^2\theta_m}{dt^2} \times 10^{-6} = P_m - P_e \text{ MW} \tag{12.4}$$

where

$P_m$ = mechanical power input in MW

$P_e$ = electrical power output in MW; stator copper loss is assumed negligible.

Rewriting Eq. (12.4)

$$\left(J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right) \frac{d^2\theta_e}{dt^2} = P_m - P_e \text{ MW}$$

where $\theta_e$ = angle in rad (elect)

or
$$M\frac{d^2\theta_e}{dt^2} = P_m - P_e \tag{12.5}$$

It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference. Let

$\delta = \theta_e - \omega_s t$; rotor angular displacement from synchronously rotating reference frame (called *torque angle/power angle*) (12.6)

From Eq. (12.6)

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \tag{12.7}$$

Hence Eq. (12.5) can be written in terms of $\delta$ as

$$M\,\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW} \tag{12.8}$$

With M as defined in Eq. (12.1), we can write

$$\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW} \tag{12.9}$$

Dividing throughout by $G$, the MVA rating of the machine,

$$\text{M(pu)}\,\frac{d^2\delta}{dt^2} = P_m - P_e;$$

in pu of machine rating as base (12.10)

where

$$\text{M(pu)} = \frac{H}{\pi f}$$

or $$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu} \tag{12.11}$$

This equation (Eq. (12.10)/Eq. (12.11)), is called the *swing equation* and it describes the rotor dynamics for a synchronous machine (generating/motoring). It is a second-order differential equation where the damping term (proportional to $d\delta/dt$) is absent because of the assumption of a lossless machine and the fact that the torque of *damper winding* has been ignored. This assumption leads to pessimistic results in transient stability analysis—damping helps to stabilize the system. Damping must of course be considered in a dynamic stability study. Since the electrical power $P_e$ depends upon the sine of angle $\delta$ (see Eq. (12.29)), the swing equation is a non-linear second-order differential equation.

**Multimachine System**

In a multimachine system a common system base must be chosen.
Let

$G_{\text{mach}}$ = machine rating (base)

$G_{\text{system}}$ = system base

Equation (12.11) can then be written as

$$\frac{G_{\text{mach}}}{G_{\text{system}}}\left(\frac{H_{\text{mach}}}{f}\frac{d^2\delta}{dt^2}\right) = (P_m - P_e)\,\frac{G_{\text{mach}}}{G_{\text{system}}}$$

or

$$\frac{H_{\text{system}}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu in system base} \tag{12.12}$$

where

$$H_{\text{system}} = H_{\text{mach}}\left(\frac{G_{\text{mach}}}{G_{\text{system}}}\right) \tag{12.13}$$

= machine inertia constant in system base

### Machines Swinging Coherently

Consider the swing equations of two machines on a *common system base*.

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \text{ pu} \tag{12.14}$$

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \text{ pu} \tag{12.15}$$

Since the machine rotors swing together (coherently or in unison)

$$\delta_1 = \delta_2 = \delta$$

Adding Eqs (12.14) and (12.15)

$$\frac{H_{\text{eq}}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{12.16}$$

where

$$\begin{aligned} P_m &= P_{m1} + P_{m2} \\ P_e &= P_{e1} + P_{e2} \\ H_{\text{eq}} &= H_1 + H_2 \end{aligned} \tag{12.17}$$

The two machines swinging coherently are thus reduced to a single machine as in Eq. (12.16). The equivalent inertia in Eq. (12.17) can be written as

$$H_{\text{eq}} = H_{1\text{ mach}}\, G_{1\text{ mach}}/G_{\text{system}} + H_{2\text{ mach}}\, G_{2\text{ mach}}/G_{\text{system}} \tag{12.18}$$

The above results are easily extendable to any number of machines swinging coherently.

**Example 12.1**

A 50 Hz, four pole turbogenerator rated 100 MVA, 11 kV has an inertia constant of 8.0 MJ/MVA.

(a) Find the stored energy in the rotor at synchronous speed.

(b) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, find rotor acceleration, neglecting mechanical and electrical losses.

(c) If the acceleration calculated in part (b) is maintained for 10 cycles, find the change in torque angle and rotor speed in revolutions per minute at the end of this period.

*Solution*

(a) Stored energy = $G$H = 100 × 8 = 800 MJ

(b) $P_a = 80 - 50 = 30 \text{ MW} = \text{M} \dfrac{d^2\delta}{dt^2}$

$$\text{M} = \frac{G\text{H}}{180 f} = \frac{800}{180 \times 50} = \frac{4}{45} \text{ MJ-s/elect deg}$$

$$\therefore \quad \frac{4}{45}\frac{d^2\delta}{dt^2} = 30$$

or

$$\alpha = \frac{d^2\delta}{dt^2} = 337.5 \text{ elect deg/s}^2$$

(c) 10 cycles = 0.2 s

Change in $\delta = \frac{1}{2}(337.5) \times (0.2)^2 = 6.75$ elect degrees

$$= 60 \times \frac{337.5}{2 \times 360^\circ} = 28.125 \text{ rpm/s}$$

$\therefore$ Rotor speed at the end of 10 cycles

$$= \frac{120 \times 50}{4} + 28.125 \times 0.2$$

$$= 1505.625 \text{ rpm}$$

## 12.3 POWER ANGLE EQUATION

In solving the swing equation (Eq. (12.10)), certain simplifying assumptions are usually made. These are:

1. Mechanical power input to the machine ($P_m$) remains constant during the period of electromechanical transient of interest. In other words, it means that the effect of the turbine governing loop is ignored being much slower than the speed of the transient. This assumption leads to pessimistic result—governing loop helps to stabilize the system.

2. Rotor speed changes are insignificant—these have already been ignored in formulating the swing equation.

3. Effect of voltage regulating loop during the transient is ignored, as a consequence the generated machine emf remains constant. This assumption also leads to pessimistic results—voltage regulator helps to stabilize the system.

Before the swing equation can be solved, it is necessary to determine the dependence of the electrical power output ($P_e$) upon the rotor angle.

### Simplified Machine Model

For a nonsalient pole machine, the per phase induced emf-terminal voltage equation under steady conditions is

$$E = V + jX_d I_d + jX_q I_q;\ X_d > X_q \tag{12.19}$$

where $$I = I_d + I_g \tag{12.20}$$

and usual symbols are used.

Under transient condition

$$X_d \rightarrow X'_d < X_d$$

but

$X'_q = X_q$ since the main field is on the d-axis

$X'_d < X_q$ ; but the difference is less than in Eq. (12.19)

Equation (12.19) during the transient modifies to

$$E' = V + jX'_d I_d + jX_q I_q \tag{12.21}$$

$$= V + jX_q(I - I_d) + jX_d{}' I_d$$

$$= (V + jX_q I) + j(X'_d - X_q)I_d \tag{12.22}$$

The phasor diagram corresponding to Eqs. (12.21) and (12.22) is drawn in Fig. 12.2.

Since under transient condition, $X'_d < X_d$ but $X_q$ remains almost unaffected, it is fairly valid to assume that

$$X'_d \approx X_q \tag{12.23}$$

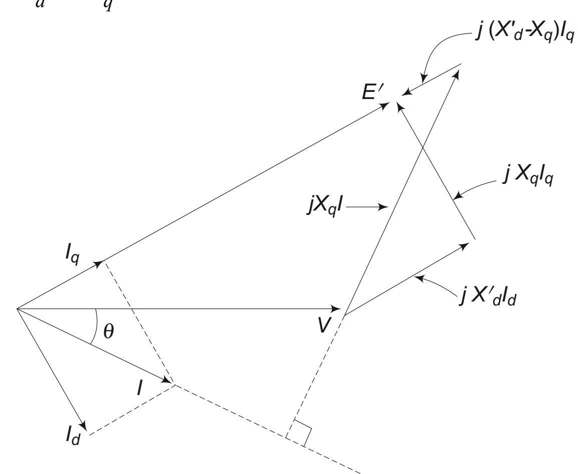



**Fig. 12.2** Phasor diagram—salient pole machine

Equation (12.22) now becomes

$$E' = V + jX_q I$$
$$= V + jX_d' I \quad (12.24)$$

The machine model corresponding to Eq. (12.24) is drawn in Fig. 12.3 which also applies to a cylindrical rotor machine where $X_d' = X_q' = X_s'$(transient synchronous reactance)

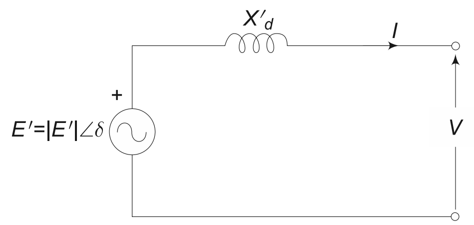



**Fig. 12.3** Simplified machine model

The simplified machine of Fig. 12.3 will be used in all stability studies.

## Power Angle Curve

For the purposes of stability studies $|E'|$, transient emf of generator motor, remains constant or is the independent variable determined by the voltage regulating loop but $V$, the generator determined terminal voltage is a dependent variable. Therefore, the nodes (buses) of the stability study network pertain to the emf terminal in the machine model as shown in Fig. 12.4, while the machine reactance ($X_d'$) is absorbed in the system network as different from a load flow study. Further, the loads (other than large synchronous motors) will be replaced by equivalent static admittances (connected in shunt between transmission network buses and the reference bus). This is so because load voltages vary during a stability study (in a load flow study, these remain constant within a narrow band).

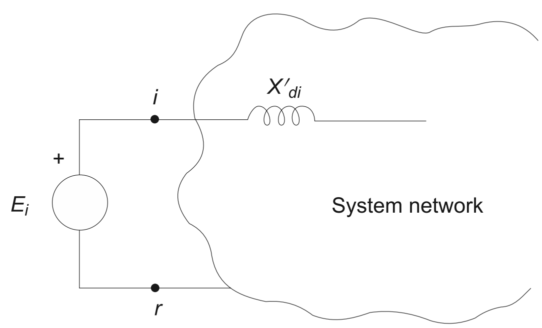



**Fig. 12.4**

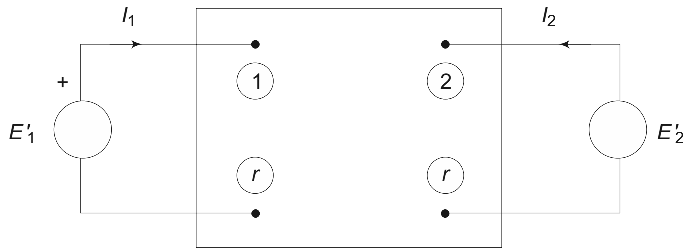



**Fig. 12.5** Two-bus stability study network

For the 2-bus system of Fig. 12.5

$$\mathbf{Y_{BUS}} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}; \; Y_{12} = Y_{21} \tag{12.25}$$

Complex power into bus is given by

$$P_i + jQ_i = E_i I_i^*$$

At bus 1

$$P_1 + jQ_1 = E_1' \, (Y_{11}E_1')^* + E_1 \, (Y_{12}E_2')^* \tag{12.26}$$

But

$$E_1' = |E_1'| \angle \delta_1; \; E_2' = |E_2'| \angle \delta_2$$

$$Y_{11} = G_{11} + jB_{11}; \; Y_{12} = |Y_{12}| \angle \theta_{12}$$

Since in solution of the swing equation only real power is involved, we have from Eq. (12.26)

$$P_1 = |E_1'|^2 \, G_{11} + |E_1'| \, |E_2'| \, |Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12}) \tag{12.27}$$

A similar equation will hold at bus 2.

Let

$$|E_1'|^2 G_{11} = P_c$$

$$|E_1'| \, |E_2'| \, |Y_{12}| = P_{\max}$$

$$\delta_1 - \delta_2 = \delta$$

and $\qquad \phi_{12} = \pi/2 - \gamma$

Then Eq. (12.27) can be written as

$$P_1 = P_c + P_{\max} \sin(\delta - \gamma); \; \textit{Power Angle Equation} \tag{12.28}$$

For a purely reactive network

$$G_{11} = 0 \; (\therefore P_c = 0); \text{ lossless network}$$

$$\theta_{12} = \pi/2, \; \therefore \gamma = 0$$

Hence

$$P_e = P_{\max} \sin \delta \tag{12.29a}$$

where $$P_{\max} = \frac{|E_1'|\;|E_2'|}{X};$$

simplified *power angle equation* (12.29b)

where $X$ = transfer reactance between nodes (i.e., between $E_1'$ and $E_2'$)

The graphical plot of power angle equation (Eq.(12.29)) is shown in Fig. 12.6.

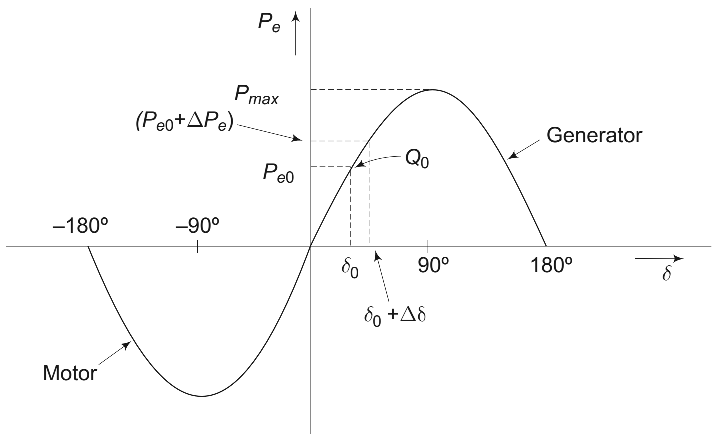



**Fig. 12.6** Power angle curve

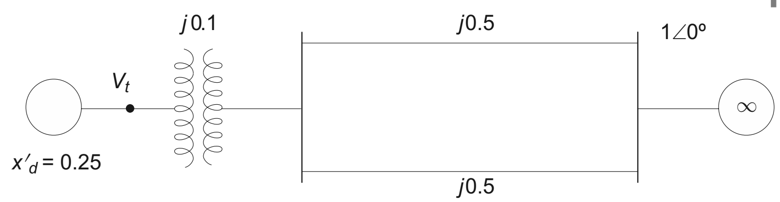
The swing equation (Eq. (12.10)) can now be written as

$$\frac{\mathrm{H}}{\pi f}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_{\max}\sin\delta\,\mathrm{pu} \tag{12.30}$$

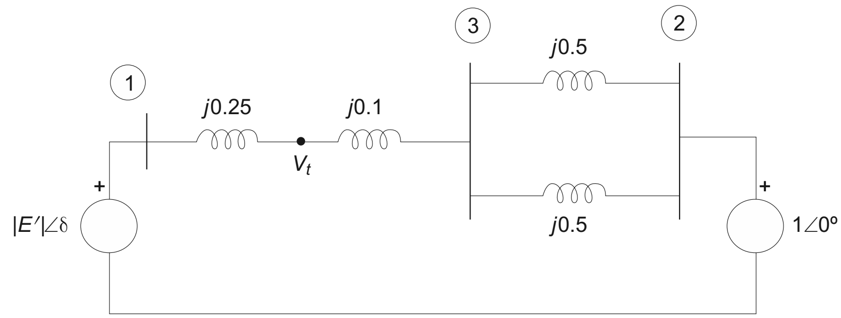
which, as already stated, is a non-linear second-order differential equation with no damping.

## 12.4 NODE ELIMINATION TECHNIQUE

In stability studies, it has been indicated that the buses to be considered are those which are excited by the internal machine voltages (transient emf's) and not the load buses which are excited by the terminal voltages of the generators. Therefore, in $Y_{\mathrm{BUS}}$ formulation for the stability study, the load buses must be eliminated. Three methods are available for bus elimination. These are illustrated by the simple system of Fig. 12.7(a) whose reactance diagram is drawn in Fig. 12.7(b). In this simple situation, bus 3 gets easily eliminated by parallel combination of the lines. Thus




(a)




(b)

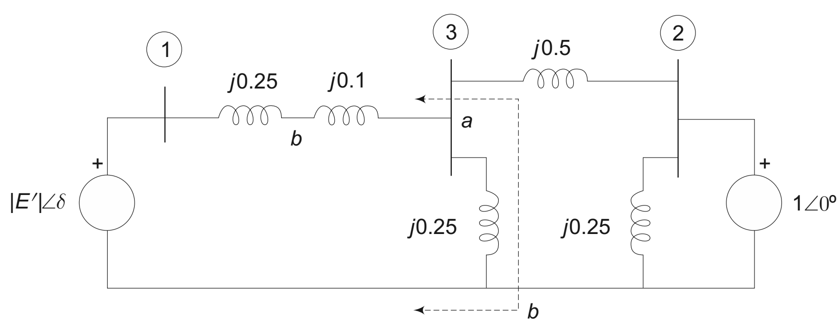
**Fig. 12.7** A simple system with its reactance diagram

$$X_{12} = 0.25 + 0.1 + \frac{0.5}{2}$$
$$= 0.6$$

Consider now a more complicated case wherein a 3-phase fault occurs at the midpoint of one of the lines in which case the reactance diagram becomes that of Fig. 12.8 (a).

## Star-Delta Conversion

Converting the star at the bus 3 to delta, the network transforms to that of Fig. 12.8(b) wherein




(a)

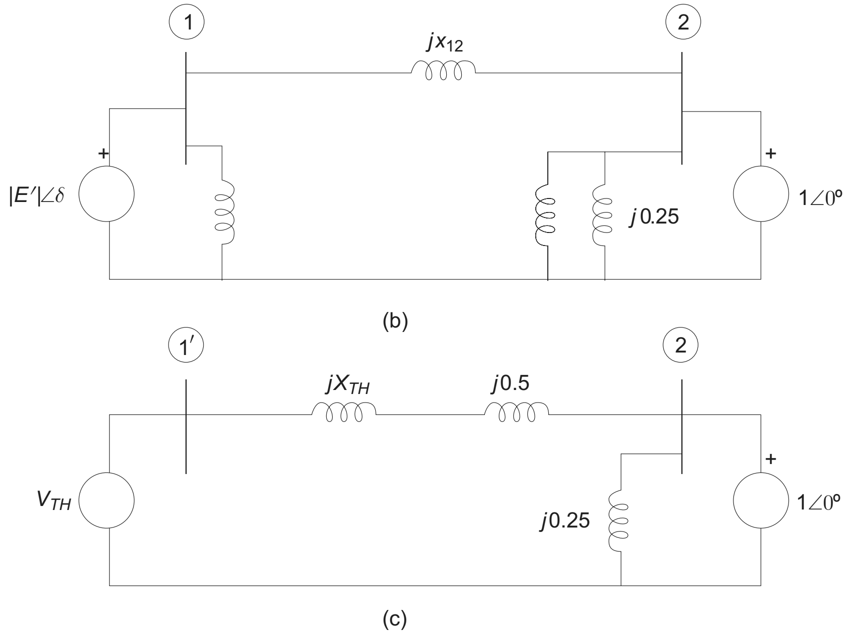



**Fig. 12.8**

$$X_{12} = \frac{0.25 \times 0.35 + 0.35 \times 0.5 + 0.5 \times 0.25}{0.25}$$

$$= 1.55$$

This method for a complex network, however, cannot be mechanized for preparing a computer programme.

### Thevenin's Equivalent

With reference to Fig. 12.8(a), the Thevenin's equivalent for the network portion to the left of terminals *a b* as drawn in Fig. 12.8(c) wherein bus 1 has been modified to 1′.

$$V_{\mathrm{Th}} = \frac{0.25}{0.25 + 0.35} \; |E'| \angle\delta$$

$$= 0.417 \; |E'| \angle\delta$$

$$X_{\mathrm{Th}} = \frac{0.35 \times 0.25}{0.35 + 0.25} = 0.146$$

Now

$$X_{12} = 0.146 + 0.5 = 0.646^*$$

*This value is different from that obtained by star delta transformation as $V_{\mathrm{Th}}$ is no longer $|E'| \angle\delta$; in fact it is 0.417 $|E'| \angle\delta$.

This method obviously is cumbersome to apply for a network of even small complexity and cannot be computerized.

### Node Elimination Technique

Formulate the bus admittances for the 3-bus system of Fig. 12.8(a). This network is redrawn in Fig. 12.9 wherein instead of reactance branch, admittances are shown. For this network,

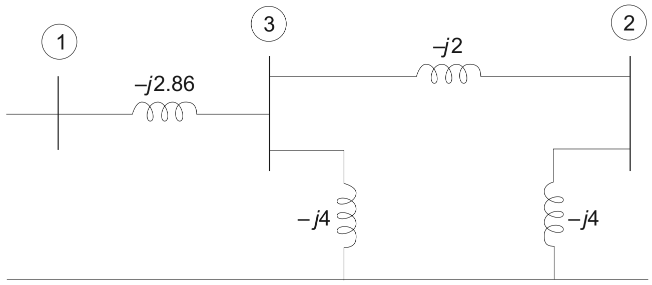



**Fig. 12.9**

$$Y_{\text{BUS}} = j \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} -2.86 & 0 & 2.86 \\ 0 & -6 & 2 \\ 2.86 & 2 & -8.86 \end{bmatrix}$$

The bus 3 is to be eliminated.

In general for a 3-bus system

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{12.31}$$

Since no source is connected at the bus 3

$$I_3 = 0$$

or $$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 = 0$$

or $$V_3 = -\frac{Y_{31}}{Y_{33}}V_1 - \frac{Y_{32}}{Y_{33}}V_2 \tag{12.32}$$

Substituting this value of $V_3$ in the remaining two equations of Eq. (12.31), thereby eliminating $V_3$,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3$$

$$= \left(Y_{11} - \frac{Y_{13}Y_{31}}{Y_{33}}\right)V_1 + \left(Y_{12} - \frac{Y_{13}Y_{32}}{Y_{33}}\right)V_2$$

In compact form

$$\mathbf{Y}_{\text{BUS}}\ (\text{reduced}) = \begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{bmatrix} \tag{12.33}$$

where

$$Y'_{11} = Y_{11} - \frac{Y_{13}Y_{31}}{Y_{33}} \tag{12.34a}$$

$$Y'_{12} = Y'_{21} = Y_{12} - \frac{Y_{13}Y_{32}}{Y_{33}} \tag{12.34b}$$

$$Y'_{22} = Y_{22} - \frac{Y_{23}Y_{32}}{Y_{33}} \tag{12.34c}$$

In general, in eliminating node $n$

$$Y_{kj}\ (\text{new}) = Y_{kj}\ (\text{old}) - \frac{Y_{kn}(\text{old})Y_{nj}(\text{old})}{Y_{nn}(\text{old})} \tag{12.35}$$

Applying Eq. (12.34) to the example in hand

$$Y_{\text{BUS}}\ (\text{reduced}) = j\begin{bmatrix} -1.937 & 0.646 \\ 0.646 & -5.549 \end{bmatrix}$$

It then follows that

$$X_{12} = \frac{1}{0.646} = 1.548\ (\approx 1.55)$$

## Example 12.2

In the system shown in Fig. 12.10, a three-phase static capacitive reactor of reactance 1 pu per phase is connected through a switch at motor bus bar. Calculate the limit of steady state power with and without reactor switch closed. Recalculate the power limit with capacitive reactor replaced by an inductive reactor of the same value.

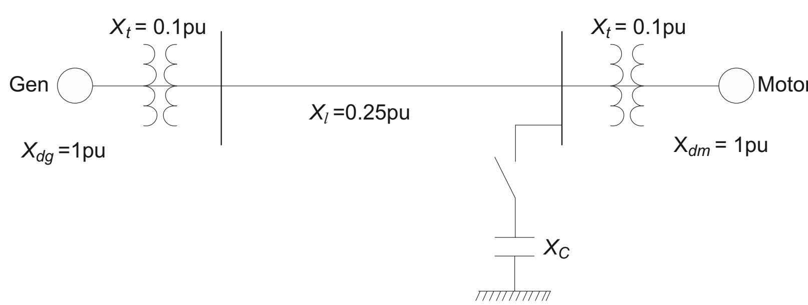



Fig. 12.10

Assume the internal voltage of the generator to be 1.2 pu and that of the motor to be 1.0 pu.

*Solution*

(1) Steady state power limit without reactor

$$= \frac{|E_g||E_m|}{X(\text{total})} = \frac{1.2 \times 1}{1 + 0.1 + 0.25 + 0.1 + 1} = 0.49 \text{ pu}$$

(2) Equivalent circuit with capacitive reactor is shown in Fig. 12.11 (a).

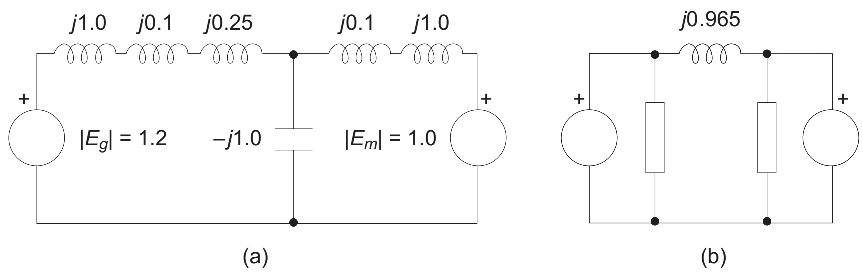



**Fig. 12.11**

Converting star to delta, the network of Fig. 12.11(a) is reduced to that of Fig. 12.11(b) where

$$jX(\text{transfer}) = \frac{j1.35 \times j1.1 + j1.1 \times (-j1.0) + (-j1.0) \times j1.35}{-j1.0}$$

$$= j0.965$$

$$\text{Steady state power limit} = \frac{1.2 \times 1}{0.965} = 1.244 \text{ pu}$$

(3) With capacitive reactance replaced by inductive reactance, we get the equivalent circuit of Fig. 12.12. Converting star to delta, we have the transfer reactance of

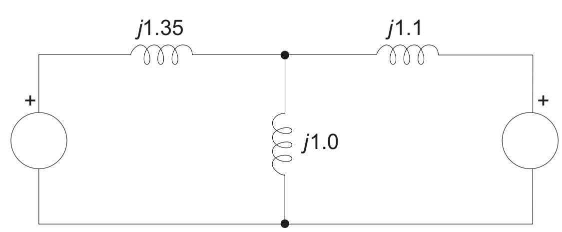



**Fig. 12.12**

$$jX(\text{transfer}) = \frac{j1.35 \times j1.1 + j1.1 \times j1.0 + j1.0 \times j1.35}{j1.0}$$

$$= j3.935$$

Steady state power limit $= \dfrac{1.2 \times 1}{3.935} = 0.304$ pu

**Example 12.3**

The generator of Fig. 12.7(a) is delivering 1.0 pu power to the infinite bus ($|V|$ = 1.0 pu), with the generator terminal voltage of $|V_t|$ = 1.0 pu. Calculate the generator emf behind transient reactance. Find the maximum power that can be transferred under the following conditions:

(a) System healthy
(b) One line shorted (3-phase) in the middle
(c) One line open.

Plot all the three power angle curves.

*Solution*

Let $$V_t = |V_t| \angle\alpha = 1\angle\alpha$$

From power angle equation

$$\frac{|V_t||V|}{X} \sin\alpha = P_e$$

or $$\left(\frac{1\times 1}{0.25+0.1}\right) \sin\alpha = 1$$

or $$\alpha = 20.5^\circ$$

Current into infinite bus,

$$I = \frac{|V_t|\angle\alpha - |V|\angle 0^\circ}{jX}$$

$$= \frac{1\angle 20.5^\circ - 1\angle 0^\circ}{j0.35}$$

$$= 1 + j0.18 = 1.016 \angle 10.3^\circ$$

Voltage behind transient reactance,

$$E' = 1\angle 0^\circ + j0.6 \times (1 + j0.18)$$

$$= 0.892 + j0.6 = 1.075 \angle 33.9^\circ$$

(a) System healthy

$$P_{\max} = \frac{|V||E'|}{X_{12}} = \frac{1\times 1.075}{0.6} = 1.79 \text{ pu}$$

$\therefore$ $$P_e = 1.79 \sin\delta \qquad \text{(i)}$$

(b) One line shorted in the middle:

As already calculated in this section,

$$X_{12} = 1.55$$

$$\therefore \quad P_{max} = \frac{1 \times 1.075}{1.55} = 0.694 \text{ pu}$$

or $$P_e = 0.694 \sin \delta \qquad \text{(ii)}$$

(c) One line open:

It easily follows from Fig. 12.7(b) that

$$X_{12} = 0.25 + 0.1 + 0.5 = 0.85$$

$$\therefore \quad P_{max} = \frac{1 \times 1.075}{0.85} = 1.265$$

or $$P_e = 1.265 \sin \delta \qquad \text{(iii)}$$

The plot of the three power angle curves (Eqs. (i), (ii) and (iii)) is drawn in Fig. 12.13. Under healthy condition, the system is operated with $P_m = P_e = 1.0$ pu and $\delta_0 = 33.9°$, i.e., at the point P on the power angle curve 1.79 sin $\delta$. As one line is shorted in the middle, $P_m$ remains fixed at 1.0 pu (governing system act instantaneously) and is further assumed to remain fixed throughout the transient (governing action is slow), while the operating point instantly shifts to Q on the curve 0.694 sin $\delta$ at $\delta = 33.9°$. Notice that because of machine inertia, the rotor angle can not change suddenly.

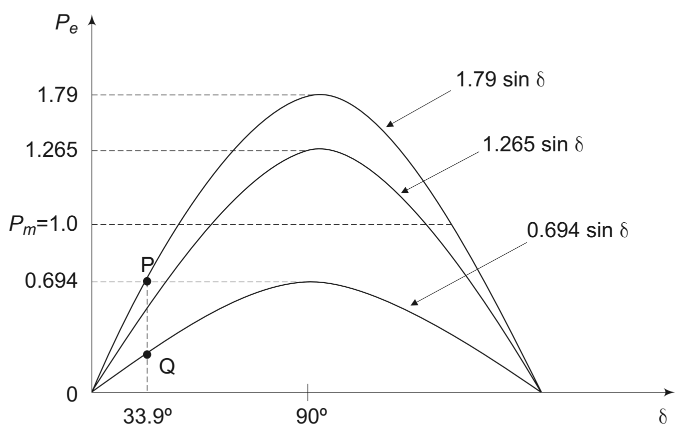



**Fig. 12.13** Power angle curves

## 12.5 SIMPLE SYSTEMS

### Machine Connected to Infinite Bus

Figure 12.14 is the circuit model of a single machine connected to infinite bus through a line of reactance $X_e$. In this simple case

$$X_{\text{transfer}} = X'_d + X_e$$

From Eq. (12.29b)

$$P_e = \frac{|E'||V|}{X_{\text{transfer}}} \sin \delta = P_{\max} \sin \delta \tag{12.36}$$

The dynamics of this system are described in Eq. (12.11) as

$$\frac{\mathrm{H}}{\pi f} \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_e \text{ pu} \tag{12.37}$$

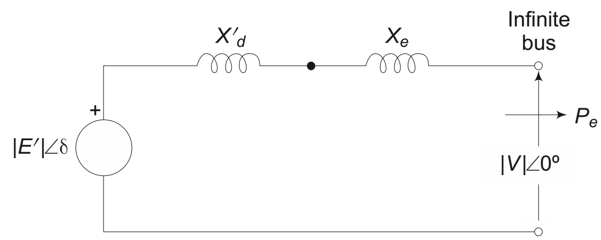



**Fig. 12.14** Machine connected to infinite bus

### Two Machine System

The case of two finite machines connected through a line ($X_e$) is illustrated in Fig. 12.15 where one of the machines must be generating and the other must be motoring. Under steady condition, before the system goes into dynamics and

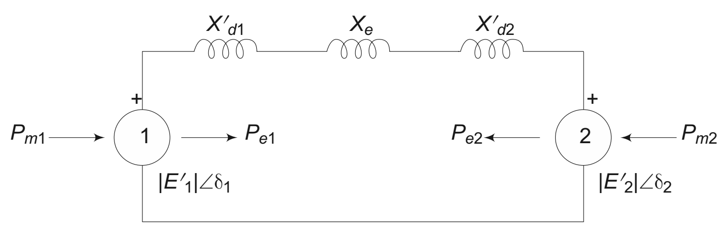



**Fig. 12.15** Two-machine system

$$P_{m1} = -P_{m2} = P_m \tag{12.38a}$$

the mechanical input/output of the two machines is assumed to remain constant at these values throughout the dynamics (governor action assumed slow). During steady state or in dynamic condition, the electrical power output of the generator must be absorbed by the motor (network being lossless). Thus at all time

$$P_{e1} = -P_{e2} = P_e \tag{12.38b}$$

The swing equations for the two machines can now be written as

$$\frac{\mathrm{d}^2\delta_1}{\mathrm{d}t^2} = \pi f \left(\frac{P_{m1} - P_{e1}}{H_1}\right) = \pi f \left(\frac{P_m - P_e}{H_1}\right) \tag{12.39a}$$

and $$\frac{d^2\delta_2}{dt^2} = \pi f\left(\frac{P_{m2} - P_{e2}}{H_2}\right) = \pi f\left(\frac{P_e - P_m}{H_2}\right) \tag{12.39b}$$

Subtracting Eq. (12.39b) from Eq. (12.39a)

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \pi f\left(\frac{H_1 + H_2}{H_1 H_2}\right)(P_m - P_e) \tag{12.40}$$

or $$\frac{H_{eq}}{\pi \mathrm{f}}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{12.41}$$

where $$\delta = \delta_1 - \delta_2 \tag{12.42}$$

$$H_{eq} = \frac{H_1 H_2}{H_1 + H_2} \tag{12.43}$$

The electrical power interchange is given by expression

$$P_e = \frac{|E_1'||E_2'|}{X_{d1}' + X_e + X_{d2}'}\sin\delta \tag{12.44}$$

The swing equation Eq. (12.41) and the power angle equation Eq. (12.44) have the same form as for a single machine connected to infinite bus. Thus a two-machine system is equivalent to a single machine connected to infinite bus. Because of this, the single-machine (connected to infinite bus) system would be studied extensively in this chapter.

### Example 12.4

In the system of Example 12.3, the generator has an inertia constant of 4 MJ/MVA, write the swing equation upon occurrence of the fault. What is the initial angular acceleration? If this acceleration can be assumed to remain constant for $\Delta t$ = 0.05s, find the rotor angle at the end of this time interval and the new acceleration.

*Solution*

Swing equation upon occurrence of fault

$$\frac{\mathrm{H}}{180f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{4}{180\times 50}\frac{d^2\delta}{dt^2} = 1 - 0.694 \sin\delta$$

or $$\frac{d^2\delta}{dt^2} = 2250\,(1 - 0.694 \sin\delta).$$

Initial rotor angle $\delta_0$ = 33.9º (calculated in Example 12.3)

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0^+} = 2250\ (1 - 0.694\ \sin 33.9º)$$

$$= 1379 \text{ elect deg/s}^2$$

$$\left.\frac{d\delta}{dt}\right|_{t=0^+} = 0;\ \text{rotor speed cannot change suddenly}$$

$$\Delta\delta\ (\text{in } \Delta t = 0.05\text{s}) = \frac{1}{2} \times 1379 \times (0.05)^2$$

$$= 1.7º$$

$$\delta_1 = \delta_0 + \Delta\delta = 33.9 + 1.7º = 35.6º$$

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0.05s} = 2250\ (1 - 0.694\ \sin 35.6º)$$

$$= 1341 \text{ elect deg/s}^2$$

Observe that as the rotor angle increases, the electrical power output of the generator increases and so the acceleration of the rotor reduces.

## 12.6 STEADY STATE STABILITY

The steady state stability limit of a particular circuit of a power system is defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Consider the simple system of Fig. 12.14 whose dynamics is described by equations

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW; Eq. (12.8)}$$

$$M = \frac{H}{\pi f} \text{ in pu system} \tag{12.45}$$

and

$$P_e = \frac{|E||V|}{X_d} \sin\delta = P_{max} \sin\delta \tag{12.46}$$

For determination of steady state stability, the direct axis reactance ($X_d$) and voltage behind $X_d$ are used in the above equations.

The plot of Eq. (12.46) is given in Fig. 12.6. Let the system be operating with steady power transfer of $P_{e0} = P_m$ with torque angle $\delta_0$ as indicated in the figure. Assume a small increment $\Delta P$ in the electric power with the input from the prime mover remaining fixed at $P_m$ (governor response is slow compared to the speed of energy dynamics), causing the torque angle to change to ($\delta_0 + \Delta\delta$). Linearizing about the operating point $Q_0$ ($P_{e0}$, $\delta_0$) we can write

$$\Delta P_e = \left(\frac{\partial \mathbf{P}_e}{\partial \delta}\right)_0 \Delta\delta$$

The excursions of $\Delta\delta$ are then described by

$$\mathrm{M}\ \frac{\mathrm{d}^2\Delta\delta}{\mathrm{d}t^2} = P_m - (P_{e0} + \Delta P_e) = -\ \Delta P_e$$

or

$$\mathrm{M}\ \frac{\mathrm{d}^2\Delta\delta}{\mathrm{d}t^2} + \left[\frac{\partial \mathbf{P}_e}{\partial \delta}\right]_0 \Delta\delta = 0 \tag{12.47}$$

or

$$\left[Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_0\right]\Delta\delta = 0$$

where

$$P = \frac{\mathrm{d}}{\mathrm{d}t}$$

The system stability to small changes is determined from the characteristic equation

$$Mp^2 + \left[\frac{\partial P_e}{\partial \delta}\right]_0 = 0$$

whose two roots are

$$P = \pm\left[\frac{-(\partial P_e/\partial\delta)_0}{M}\right]^{\frac{1}{2}}$$

As long as $(\partial \mathbf{P}_e/\partial\delta)_0$ is positive, the roots are purely imaginary and conjugate and the system behaviour is oscillatory about $\delta_0$. Line resistance and damper windings of machine, which have been ignored in the above modelling, cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as

$$(\partial P_e/\partial\delta)_0 > 0 \tag{12.48}$$

When $(\partial P_e/\partial\delta)_0$ is negative, the roots are real, one positive and the other negative but of equal magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment (disturbance) and the synchronism is soon lost. The system is therefore unstable for

$$(\partial P_e/\partial\delta)_0 < 0$$

$(\partial P_e/\partial\delta)_0$ is known as *synchronizing coefficient*. This is also called *stiffness* (electrical) of synchronous machine.

Assuming $|E|$ and $|V|$ to remain constant, the system is unstable, if