Application of Graph Theory in Stability Analysis of Meshed Microgrids

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An isolated microgrid consisting of loads being fed by inverters connected in parallel provides the foundation for reliable and expandable ac power systems. A truly reliable microgrid is one in which the inverters are controlled in a decentralized manner using variables that are measurable locally at the inverters. An effective decentralized control technique for multi inverter microgrids are the active power/voltage frequency and the reactive power/voltage magnitude droop control laws implemented at every inverter in the microgrid. These laws implement a strategy of locally reducing the frequency of the inverter by an amount proportional to the active power drawn from that inverter, and simultaneously reducing the AC voltage amplitude locally by an amount proportional to the reactive power drawn from the inverter; such strategies are termed the droop strategy. These control laws enable the inverters to emulate synchronous generators in conventional power systems and therefore share the load demand of the microgrid in a desired ratio [1].

An important issue that arises when implementing such local laws is the stability of the microgrid for arbitrary meshing of the inverters. In this paper we address the following question: does the droop strategy work successfully for various models of the interconnection of inverters and for arbitrary meshing of such microgrids, or is there a condition that the graph of the network of inverters has to satisfy for stabilization of the frequency of the interconnected network?

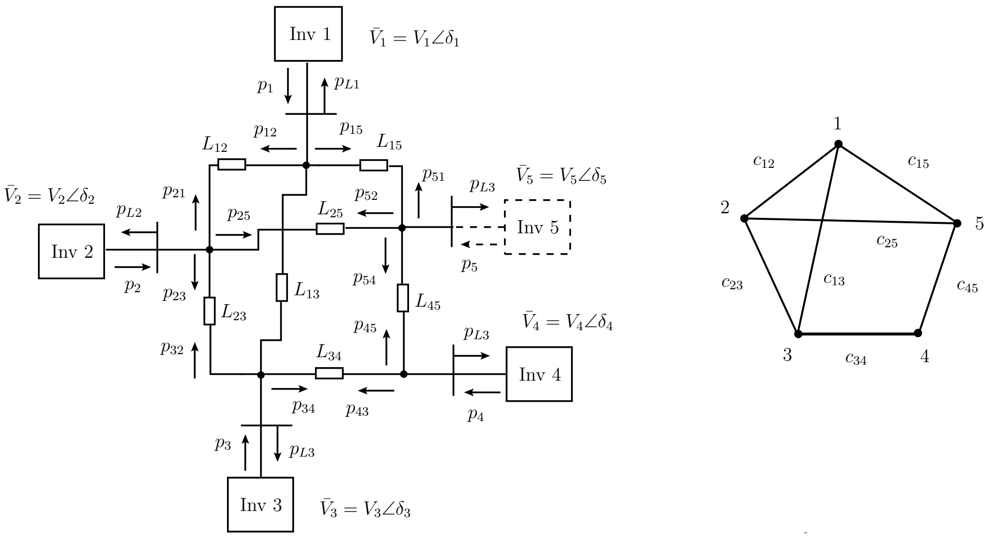
We show that using the steady state power sharing model for inductive connection of two voltage sources, an arbitrary connected network of ideal voltage source inverters stabilize to the same frequency. This is first shown for the case when all loads in the network are associated to a voltage source locally. We consider the case when one of the loads is connected at a node not supported by a source locally. We introduce a fictitious inverter that has a very high droop thus causing negligible power supplied by this source; stability is guaranteed by the above steady state power sharing model for arbitrary connected network for this case also. We state the main result below after introducing some preliminaries of graph theory.

For the purpose of this paper, we need the notion of the Laplacian matrix associated to a graph. Consider Fig. 1(b) that shows a graph G with N nodes with entries c_{mn} along the edge between nodes m and n. The adjacency matrix $\mathbf{A}$ has N rows and N columns with c_{mn} as its (m,n)-th element. Define the diagonal matrix $\mathbf{D}$ of size $N \times N$ whose (m,m)-th element is $\sum_{\{n\}} c_{mn}$ with $\{n\}$ being the set of all nodes connected to node m. The Laplacian matrix $\mathbf{L}$ is defined as $\mathbf{L} = \mathbf{D} - \mathbf{A}$. The Laplacian matrix $\mathbf{L}$ has some key properties that form the crux of the proof in this paper - $\mathbf{L}$ is symmetric and its rows and columns add up to zero. Further, $\mathbf{L}$ is positive semidefinite and the number of eigenvalues at zero coincide with the number of connected components of the graph, see [2].

The following theorem states that for the steady state power sharing law between two nodes, the decentralized active power/frequency droop law results in the stability of an arbitrary meshed network. The frequency of inverter n is denoted by ω_n.

Theorem 1 *Consider an arbitrary meshed connected microgrid of Fig. 1(a) with $\bar{V}_m = V_m \angle \delta_m$ denoting the voltage-phase of the m-th inverter. For a connected pair of inverters m and n, suppose the power flow p_{mn} from inverter m to inverter n satisfies the law*

$$p_{mn} = c_{mn}(\delta_m - \delta_n).$$



(a) Microgrid topology (b) Connected graph

Figure 1: The microgrid topology and the equivalent connected graph

Consider the decentralized control law: $\omega_m = \omega_0 - k_{pm}p_m$ *applied to each inverter. Then all frequencies converge to a steady state frequency* ω_0*. Moreover, as* $k_{pm} \to \infty$*, the rate of convergence to* ω_0 *is increasingly faster. In other words, assuming the above model for power sharing between inverters, the* $p - \omega$ *droop law stabilizes the microgrid.*

Theorem 2 *Consider a connected microgrid with local loads at each inverter and one intermediate load (not supported by an inverter locally). Suppose the power sharing laws are as explained in the above theorem. Then, in this intermediate loading situation also, the decentralized frequency control law* $\omega = \omega_0 - k_p p$ *ensures all inverters' frequencies converge to* ω_0*.*

The above theorems crucially use the following matrix eigenvalue result.

Lemma 1 *Let* $\mathbf{A}_P$ *be the Laplacian matrix associated to the graph* G *in Fig. 1(b) and suppose* c_{mn} *are positive. Let* $\mathbf{A}_k$ *be a diagonal matrix with positive entries along the diagonal. Consider the eigenvalues of* $\mathbf{A}_k\mathbf{A}_P$*. Then, all eigenvalues are non-negative. Moreover, the number eigenvalues at zero is equal to the number of connected components in the graph* G*. Further, as the diagonal entries in* $\mathbf{A_k}$ *approach* $+\infty$*, the nonzero eigenvalues of* $\mathbf{A}_k\mathbf{A}_P$ *approach* $+\infty$*.*

The proof of the lemma uses the positive semidefinite property of the Laplacian matrix (see [2]).

The above theorems indicate that the steady state power sharing model for interconnection of two or more sources with the droop strategy results in stability for any large droop values. An in-depth analysis will be undertaken to examine how the modelling technique affects the stability results for the microgrid.

References

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[2] C. Godsil and G. Royle, *Algebraic Graph Theory.* Springer, 2001.