2016 ICSEE International Conference on the Science of Electrical Engineering

A warning about the use of reduced models of synchronous generators

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***Abstract*—Synchronous generators are an essential component of the electric grid. Recently, the stability of the electric grid has become an area of high interest and intensive research. We discuss the stability of a single generator connected to an infinite bus, and show that reduced models fail to predict the behavior of this system.**

I. Introduction

The AC electricity grid was developed at the end of the XIXth century, and has remained very similar until today. The grid is an enormously complex nonlinear and randonly varying system for which rigorous stability analysis is impossible. Many techniques and models that have been developed to assess the stability of a power grid, using rigorous modelling and system theory techniques mixed with practical shortcuts and simplifying assumptions driven by experience, see for instance [1], [2], [3], [4], [5].

In recent years, due to the increasing penetration of renewable energy resources, which connect to the grid via power converters and produce an intermittent power output, it is not clear whether the traditional models and methods for controlling the power grid will succeed to control it. Therefore, there is an increasing interest in the fundamental mathematical models and stability analysis for the grid, see for instance [5], [6], [7], [8], [9], [10].

The *synchronous generator* (SG) is the main power source of the electricity grid. The mathematical model of a SG (see the earlier references and in addition [11], [12], [13], etc.) is complex and difficult to use as a component when we model a large network. Stability analysis is usually done either by simulation, or analytically on simplified models, in which the SGs are connected in a simple network and each SG is represented by reduced order equations, see for instance [5] and [6]. The reduced model of a SG is often obtained by treating the stator currents as fast variables, thus eliminating them from the state variables via the singular perturbation approach (see, for instance, [14]) and keeping only the rotor angle, and the rotor angular velocity and the rotor field as relevant state variables, see for instance [1] and [3].

SGs have the important property that once they synchronize, they tend to remain synchronized even without any control. This is important attribute because the electricity grid must maintain nearly constant frequency, and because the ability of a SG to transfer constant power to the grid exists only when the phase difference between it and the grid is constant. Therefore, it is desirable to know if for a given grid which contains SGs and a loads, the SGs tend to synchronize (for initial states in a reasonably large region) and if yes, if the grid frequency and power flows remains stable. To simplify the stability analysis, it is common to use the Park transformation of the voltages and currents, that maps sinusoidal positive sequence signals into a fixed point in the state space.

The most common reduced model, which is known as the classical model, is a second order non linear model. The reference [10] argues that this model is not realistic enough and that a more complicated (but still reduced) model, which they call *improved swing equation*, should be used. In this paper we show that even the model proposed in [10] is not reliable for stability analysis, because it can't predict an unstable behavior that is predicted by more faithful model of the same system.

The rest of this paper is organized as follows. In Section II, a fourth order model of a SG connected to infinite bus is presented. The reduction from the fourth order model to the second order *improved swing equation* is described in Section III. Simulations and local stability analysis that shows different behavior of the models are given in Section IV.

II. Modeling SG connected to infinite bus

In this section we derive the equations for a SG connected to infinite bus. We start with deriving the equations for single SG, assuming constant field current, following the notation in [15]. Then we derive the equations for a single SG connected to an infinite bus, following [8], [9].

A. Single SG dynamics

The rotor of a SG is a winding (coil) on a magnetic core that spins inside a circular cavity in the stator, having an angle θ with respect to a reference angle, see Figure 1. We denote its self inductance by L_f, its resistance by R_f, the voltage across its terminals by v_f and the current through it (called the field current) by i_f. We assume for simplicity that L_f is independent of θ and i_f. The stator consists of three identical windings that are connected in a star, with phase shifts of 120^0 (see again Figure 1). We consider that there is no neutral connection and no damper windings. The stator windings can be regarded as connected coils with self inductance L, mutual inductance $-M$,

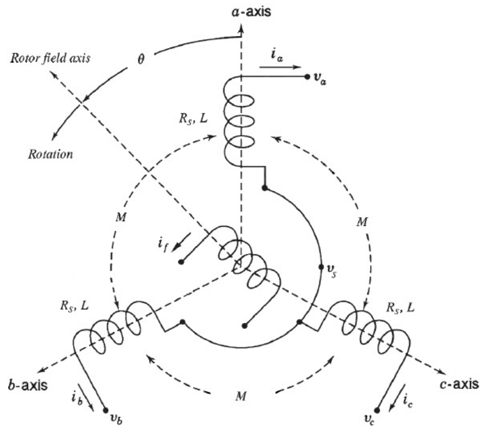

Fig. 1. Structure of an idealized three-phase round rotor synchronous generator, modified from [2, Figure 3.4].

and resistance R_s (the parameters L_f, R_f, L, M, R_s are positive). We assume no magnetic saturation effects in the iron core and no Eddy currents. The stator terminals are labeled with the letters a, b, c and the vector of voltages on the stator terminals is denoted by $v = [v_a \ v_b \ v_c]^\top$. We denote by v_s the voltage at the unconnected center of the star (see Figure 1) and $v^n = [v_s \ v_s \ v_s]^\top$. We define the vectors

$$\widetilde{\cos}\theta = \begin{bmatrix} \cos(\theta) \\ \cos\left(\theta - \frac{2\pi}{3}\right) \\ \cos\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}, \quad \widetilde{\sin}\theta = \begin{bmatrix} \sin(\theta) \\ \sin\left(\theta - \frac{2\pi}{3}\right) \\ \sin\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}.$$

We denote the stator flux by $\Phi = [\Phi_a \ \Phi_b \ \Phi_c]^\top$, the stator currents by $i = [i_a \ i_b \ i_c]^\top$ and the rotor current by i_f.

The mutual inductance between the rotor coil and each of the stator coils varies with the rotor angle θ as follows:

$$\begin{bmatrix} M_{a,f} \\ M_{b,f} \\ M_{c,f} \end{bmatrix} = M_f \widetilde{\cos}\theta,$$

where $M_f > 0$ is a constant. The vector of flux linkages of the stator windings is

$$\Phi = \begin{bmatrix} L & -M & -M \\ -M & L & -M \\ -M & -M & L \end{bmatrix} i + M_f i_f \widetilde{\cos}\theta.$$

Since there is no neutral line, $i_a + i_b + i_c = 0$, so the previous equation can be rewritten as

$$\Phi = L_s i + M_f i_f \widetilde{\cos}\theta,$$

where $L_s = L + M$. We will assume that the rotor current is constant (or equivalently, the rotor is composed of a permanent magnet). The stator voltages satisfy

$$v - v^n = -R_s i - \frac{\mathrm{d}\Phi}{\mathrm{d}t} = -R_s i - L_s \frac{\mathrm{d}i}{\mathrm{d}t} + e, \tag{1}$$

where $e = [e_a \ e_b \ e_c]^\top$ is the back electromotive force (EMF) due to the rotor movement (also called synchronous internal voltage), given by

$$e = M_f i_f \dot{\theta} \widetilde{\sin}\theta. \tag{2}$$

For a SG with no load, the voltages at each terminal will be sinusoidal functions. In order to represent the voltages and currents in a more convenient way, we apply the Park transformation with respect to the rotor angle:

$$x_{dq0} = \begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = U(\theta) \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = U(\theta) x_{abc},$$

where x_{abc} is a vector in abc coordinates, x_{dq0} is the same vector in the $dq0$ coordinates, and $U(\theta)$ is the following unitary matrix:

$$U(\theta) = \sqrt{\frac{3}{2}} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{4\pi}{3}) \\ \sqrt{1/2} & \sqrt{1/2} & \sqrt{1/2} \end{bmatrix}.$$

Applying the Park transformation to (1) leads to

$$U(\theta)(v - v^n) - U(\theta)e = -R_s U(\theta) i - L_s U(\theta) \frac{\mathrm{d}i}{\mathrm{d}t}. \tag{3}$$

Now we use that, denoting $i_{dq0} = U(\theta) i$,

$$\frac{\mathrm{d}i_{dq0}}{\mathrm{d}\theta} = U(\theta) \frac{\mathrm{d}i}{\mathrm{d}\theta} + \frac{\mathrm{d}U(\theta)}{\mathrm{d}\theta} i = U(\theta) \frac{\mathrm{d}i}{\mathrm{d}\theta} + \begin{bmatrix} i_q \\ -i_d \\ 0 \end{bmatrix}.$$

This implies that

$$\dot{i}_{dq0} = \frac{\mathrm{d}i_{dq0}}{\mathrm{d}\theta} \cdot \frac{\mathrm{d}\theta}{\mathrm{d}t} = U(\theta) \dot{i} + \omega \begin{bmatrix} i_q \\ -i_d \\ 0 \end{bmatrix},$$

where $\omega = \dot{\theta}$. We rewrite (3) as follows:

$$L_s \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} - L_s \omega \begin{bmatrix} i_q \\ -i_d \\ 0 \end{bmatrix} = -R_s \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \begin{bmatrix} e_d - v_d \\ e_q - v_q \\ \tilde{v} \end{bmatrix}, \tag{4}$$

where $\tilde{v} = e_0 - v_0 + v_0^n$. Here we have used that (obviously) $v_d^n = v_q^n = 0$. Since there is no neutral connection, $i_0 = 0$, hence $\tilde{v} = 0$. Applying the Park transformation to (2) gives

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = -\sqrt{\frac{3}{2}} M_f \begin{bmatrix} 0 \\ \omega i_f \end{bmatrix}.$$

We denote $m = \sqrt{\frac{3}{2}} M_f$. If we substitute the last formula into (4), we get

$$L_s \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = -R_s \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega L_s \begin{bmatrix} i_q \\ -i_d \end{bmatrix} - m \begin{bmatrix} 0 \\ \omega i_f \end{bmatrix} - \begin{bmatrix} v_d \\ v_q \end{bmatrix}. \tag{5}$$

The rotational dynamics of the rotor is given by

$$J\dot{\omega} = T_m - T_e - D_p \omega, \tag{6}$$

where J is the moment of inertia of the rotor, $T_m - D_p\omega$ is is the mechanical torque coming from the prime mover, T_e is the electromagnetic torque developed by the generator, and D_p is a the frequency droop coefficient. The feedback term $D_p\omega$ is used in order to control the frequency of the grid, see [1], [6], [7], [15]. T_e can be found using energy consideration. It is easy to see that the magnetic energy stored in the generator is

$$E_{mag} = \frac{1}{2}\left(\langle i, L_s i\rangle + L_f i_f^2\right) + M_f i_f \langle i, \widetilde{\cos}\theta\rangle \,.$$

The electromagnetic torque can be calculated as follows:

$$T_e = \frac{\partial E_{mag}}{\partial \theta}|_{\Phi,\Phi_f\ const.} = -\frac{\partial E_{mag}}{\partial \theta}|_{i,i_f\ const.}$$

(see [15]), whence

$$T_e = M_f i_f \left\langle i, \frac{\mathrm{d}\tilde{\cos}\theta}{\mathrm{d}\theta}\right\rangle = M_f i_f \langle i, \widetilde{\sin}\theta\rangle = -m i_f i_q \,.$$

Using (5), (6) and the last formula, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} L_s i_d \\ L_s i_q \\ J\omega \end{bmatrix} = \begin{bmatrix} -R_s & \omega L_s & 0 \\ -\omega L_s & -R_s & -m i_f \\ 0 & m i_f & -D_p \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} + \begin{bmatrix} -v_d \\ -v_q \\ T_m \end{bmatrix}.$$

This third order nonlinear dynamical system represents the dynamics of a single SG with constant field current.

B. Model of a single generator connected to an infinite bus

In this subsection we develop a model for single SG connected to an infinite bus, following [8], [9]. The infinite bus is modeled as a three phase AC voltage source, i.e, the infinite bus is not affected by the synchronous generator that is connected to it. The justification for this model is that the influence of a single SG on a grid is very small. In general, the line that connects the SG to the grid has its impedance that can be modeled (in most cases) as a resistance and an inductance in series, but these values can simply be added to the parameters R_s and L_s of the SG.

The infinite bus voltage (at the SG terminals) is

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}}V\begin{bmatrix} \cos(\theta_g) \\ \cos(\theta_g - \frac{2\pi}{3}) \\ \cos(\theta_g - \frac{4\pi}{3}) \end{bmatrix}, \tag{7}$$

where V is the grid line voltage magnitude, and θ_g is the grid angle. Let us define the *power angle* δ which represents the difference between the grid and the synchronous generator angles:

$$\delta = \theta - \theta_g \,.$$

After applying the Park transformation to (7), we get

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = U(\theta)\sqrt{\frac{2}{3}}V\begin{bmatrix} \cos(\theta_g) \\ \cos(\theta_g - \frac{2\pi}{3}) \\ \cos(\theta_g - \frac{4\pi}{3}) \end{bmatrix} = -V\begin{bmatrix} \sin\delta \\ \cos\delta \\ 0 \end{bmatrix}. \tag{8}$$

It is easy to see that the dynamics of δ is

$$\dot{\delta} = \omega - \omega_g \,,$$

where ω_g is the grid frequency. Substituting (8) and the above equation into (4), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} L_s i_d \\ L_s i_q \\ J\omega \\ \delta \end{bmatrix} = \begin{bmatrix} -R_s & \omega L_s & 0 & 0 \\ -\omega L_s & -R_s & -m i_f & 0 \\ 0 & m i_f & -D_p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} V\sin\delta \\ V\cos\delta \\ T_m \\ \omega_g \end{bmatrix} \tag{9}$$

This fourth order nonlinear dynamical system is the model for one SG connected to an infinite bus.

III. THE IMPROVED SWING EQUATION

We separate the state variables into a fast variables and a slow variables, in order to use singular perturbation analysis (see for instance [14]). Now the structure of the system is:

$$\begin{cases} \dot{x} = f(x,y,\varepsilon) \\ \varepsilon\dot{y} = g(x,y,\varepsilon) \end{cases}$$

where x is the vector of slow state variables, y is the vector of fast state variables and $\varepsilon > 0$ is a small parameter.

In our case, assuming that L_s is very small in a suitable sense, define $\varepsilon = L_s$, $x = \begin{bmatrix} \omega \\ \delta \end{bmatrix}$ and $y = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$ so that

$$g(x,y,\varepsilon) = \begin{bmatrix} -R_s & \omega L_s \\ -\omega L_s & -R_s \end{bmatrix}\begin{bmatrix} \omega \\ \delta \end{bmatrix} + \begin{bmatrix} V\sin\delta \\ V\cos\delta - m i_f \omega \end{bmatrix}.$$

We assume that ε is very small, meaning that $\varepsilon\dot{y} = g(x,y,\varepsilon)$ is much faster than $\dot{x} = f(x,y,\varepsilon)$, and it will reach its equilibrium (assuming that the equilibrium point exist and attractive) almost instantly with regard to x. The equilibrium point $y_0 = \hat{y}(x, \varepsilon = 0)$ is the solution of $g(x,y,\varepsilon) = 0$. The solution of this linear equation (in $\hat{y}$) is

$$\begin{aligned} \hat{y} &= -\begin{bmatrix} -R_s & \omega L_s \\ -\omega L_s & -R_s \end{bmatrix}^{-1}\begin{bmatrix} V\sin(\delta) \\ V\cos(\delta) - m i_f \omega \end{bmatrix} \\ &= \begin{bmatrix} \frac{R_s V\sin(\delta) - L_s\omega\left(m i_f \omega - V\cos(\delta)\right)}{L_s^2\omega^2 + R_s^2} \\ \frac{-R_s\left(m i_f \omega - V\cos(\delta)\right) - L_s V\omega\sin(\delta)}{L_s^2\omega^2 + R_s^2} \end{bmatrix} \end{aligned}$$

Assuming that $R_s \ll L_s\omega_g$ and $R_s m i_f \ll L_s V$, and assuming that our dynamics are close to the equilibrium, namely $\omega \simeq \omega_g$.

$$\hat{y} = \begin{bmatrix} \hat{i}_d = \\ \hat{i}_q \end{bmatrix} = \begin{bmatrix} \frac{V\cos(\delta) - m i_f \omega}{L_s\omega} = \\ -\frac{V\sin(\delta)}{L_s\omega} \end{bmatrix} \tag{10}$$

Now, substitute $\hat{y}$ in $\dot{x} = f(x,y,\varepsilon)$ to have a reduced model:

$$\begin{cases} J\dot{\omega}\omega + D_p\omega\omega = -\frac{m i_f V\sin(\delta)}{L_s} + T_m\omega \\ \dot{\delta} = \omega - \omega_g \end{cases}$$

We assume that the active torque is due to fix mechanical power and a correction factor for the viscous friction and droop:

$$T_m = \frac{P_m}{\omega} + D_p\omega_g$$

and then the model is:

$$\begin{cases} J\dot{\omega}\omega + D_p\omega(\omega - \omega_g) = P_m - \frac{m i_f V\sin(\delta)}{L_s} + \\ \dot{\delta} = \omega - \omega_g \end{cases}$$

This model is known as the *improved swing equation* model, see [10], [16].

IV. SIMULATIONS

Here we present a simulation that demonstrates that the improved swing equation model is a good reduction for many cases, but there are cases in which there is mismatch between the behavior suggested by the improved swing equations model and the fourth order model (9). For each simulation, we will show the currents i_d and i_q over time for both the fourth order model and the improved swing equation model, where for the improved swing equation model, we estimate these currents by (10). In addition we will show the frequency over time for the two models, and the phase over time of the fourth order model and the improved swing equation model.

A. 5KW SG

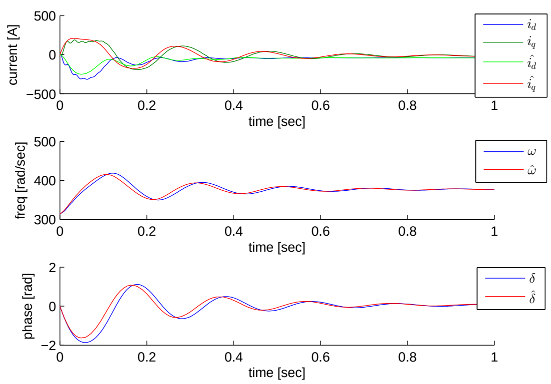

Fig. 2. Simulation for infinite bus with single 5KW SG

As shown in Figure 2, simulations show that for the 5KW SG, the behavior of the fourth order model and the reduced model is almost the same. Although the currents at the reduced don't have a ripple that the fourth order model has, both models converge at the same rate with the same oscillations to the same equilibrium point (The parameters for this simulation are $J = 0.2$ $[kgm^2]$, $D_p = 1.7$ $[J/sec]$, $R_s = 0.152$ $[\Omega]$, $L_s = 4.4$ $[mH]$, $mi_f = 1.05$ $[Vsec]$, $\omega_g = 60 \cdot 2\pi$ $[rad/sec]$, $V = 330$ $[V]$, $Pm = 5$ $[kW]$).

B. 1MW SG

As shown in Figure 3, simulations show that for the 1MW SG, the behavior of the fourth order model and the reduced model are still very similar. Although the fourth order model currents are much rippled than the reduced model currents, both models converge at the same rate with the same oscillations to the same equilibrium point (The parameters for this simulation are $J = 40.05$ $[kgm^2]$, $D_p = 337$ $[J/sec]$, $R_s = 0.4$ $[\Omega]$, $L_s = 18$ $[mH]$, $mi_f = 1.79$ $[Vsec]$, $\omega_g = 60 \cdot 2\pi$ $[rad/sec]$, $V = 563$ $[V]$, $Pm = 1$ $[MW]$).

C. Non stable behavior of the reduced model

As shown in Figure 4, simulations show that for other parameters set (The parameters for this simulation are $J = 0.2$

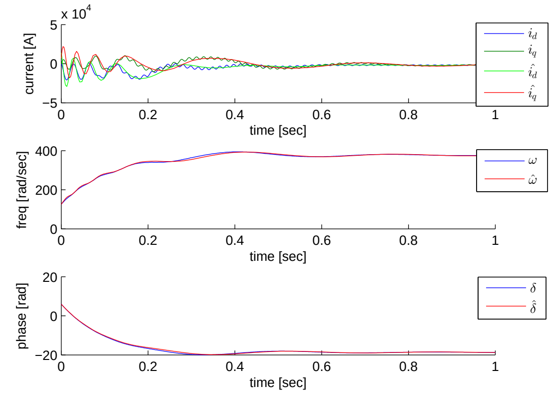

Fig. 3. Simulation for infinite bus with single 1MW SG

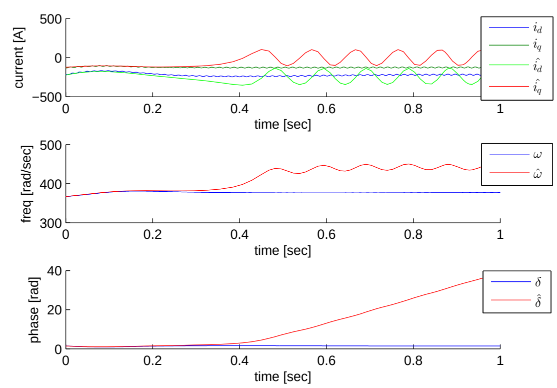

Fig. 4. Simulation example that shows different behavior for the reduced model

$[kgm^2]$, $D_p = 1.7$ $[J/sec]$, $R_s = 0.152$ $[\Omega]$, $L_s = 4.4$ $[mH]$, $mi_f = 1.05$ $[Vsec]$, $\omega_g = 60 \cdot 2\pi$ $[rad/sec]$, $V = 200$ $[V]$, $Pm = 50$ $[kW]$) the behavior of the fourth order model and the reduced model is significantly different. while the four order model is stable (the eigenvalues of the Jacobian around the equilibrium point are:

$$[-11.41 + 376.9i, -11.41 - 376.9i, -508 + 837i, -508 - 837i]$$

the reduced model is not stable (the eigenvalues of the Jacobian around the equilibrium point are: $[-14.6 + 9.4979i,\ 6.1 - 9.4i]$).

D. Different region of attraction example

As shown in Figure 5, simulations show that for some parameters set (The parameters for this simulation are $J = 0.2$ $[kgm^2]$, $D_p = 1.7$ $[J/sec]$, $R_s = 0.152$ $[\Omega]$, $L_s = 1.05$ $[mH]$, $mi_f = 3.5$ $[Vsec]$, $\omega_g = 60 \cdot 2\pi$ $[rad/sec]$, $V = 330$ $[V]$, $Pm = 5$ $[kW]$) the initial condition of this simulation is within the region of attraction of the reduced model, but outside the region of attraction of the fourth model. That cause the fourth order model to diverge while the reduce model converges to the equilibrium point.

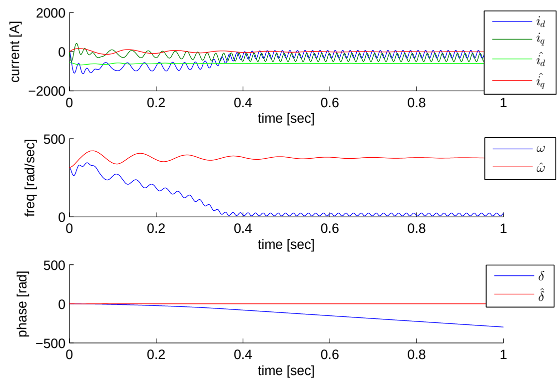

Fig. 5. Simulation example that shows different behavior for the reduced model

V. Conclusions

We have showed the relation between the fourth order model of SG connected to an infinite bus and the improved swing equation model. Simulations are carried out to show that the fourth order model gives rise to a behavior that does not match what is suggested by the improved swing equation.

Acknowledgment

The authors would like to thank...

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