Hi, my name is Elad Venezian, and I am here today to talk about reduced models of synchronies generators.

So let’s get started.

**Introduction**

First of all, short introduction:

The AC electricity grid was developed at the end of the XIXth century, and has remained very similar until today. The grid is an enormously complex nonlinear system, and therefore, rigorous stability analysis is impossible. Many techniques and models that have been developed to assess the stability of the grid, mixed with practical shortcuts instead of pure theoretical models.

In recent years, there is increasing penetration of renewable energy resources. These resources don’t supply a constant AC frequency current, so they connect to the grid via power converters. The convertors have a different behavior than the traditional SG. Therefore, it is not clear whether the traditional methods for controlling the grid will succeed. That causes to an increasing interest in the fundamental mathematical models for the grid.

The synchronous generator is the main power source of the electricity grid. Its mathematical model is complex and difficult to use when we model a large network. Therefore, stability analysis is usually done either by simulation, or analytically on simplified reduced order models.

The ability of a SG to transfer constant power to the grid exists only when the phase difference between it and the grid is constant. In addition, the electricity grid must maintain nearly constant frequency. Therefore, it is desirable to know if for a given grid, the SGs tend to synchronize, and if this is the case, if the grid frequency remains stable.

This work deals with the configuration of a single SG connected to an infinite bus.

The most common reduced model, which is known as the classical model, is a second order model. In a paper by Monshizadeh at el, they argue that this model is not realistic enough and suggest that more complicated model, the ISE model, should be used. In this work we show that even the ISE is not reliable enough, because it fails to predict an unstable behavior that is predicted by a more faithful model.

**Synchronous Generator**

SG composed of two major parts: stator and rotor. The stator consists of three identical windings that are connected in a star, with phase shifts of 1200 . We label each phase with the letters ‘a’ ‘b’ and ‘c’. The stator windings can be regarded as connected coils with self-inductance *L*, mutual inductance −*M*, and resistance *Rs*.

The rotor of a SG is a coil on a magnetic core that spins inside a circular cavity in the stator. It has the angle θ with respect to the ‘a’ phase of the stator. We denote its self-inductance by *L f* , its resistance by *R f* , the voltage across its terminals by *vf* and the current through it (called the field current) by *if*.

We consider that there is no neutral connection and no damper windings. We assume no magnetic saturation effects in the iron core and no Eddy currents.

**PARK**

For a SG with no load, the voltages at each terminal will be sinusoidal functions. In order to represent the voltages and currents in a more convenient way, we apply the Park transformation. Park transformation is a mathematical [transformation](https://en.wikipedia.org/wiki/Transform_(mathematics)) that rotates the reference frame of three-phase systems in an effort to simplify the analysis.  In the case of sinusoidal three-phase signal, the Park transform reduces the three [AC](https://en.wikipedia.org/wiki/Alternating_current) quantities to a fixed point in the dq0 three dimensional state. In case of no neutral, the 0 coordinate is zero, so Park transformation reduce the 3 dimensional vector in the abc frame into a two dimensional point in the dq0 frame.

**SINGLE SG MODEL**

Assuming that the field current is a constant, which means that the field dynamics is very fast, or equivalently, the rotor is composed of a permanent magnet. After calculate the relations between the voltages, currents, fluxes and the movement of the rotor, we get the following model …

Please note, that this model is independent on .

The mechanical torque coming from the prime mover is *Tm* – *Dp*ω. We assume that *Tm* is constant. The feedback term *Dp*ω is used in order to control the frequency of the grid. If any viscous friction is present, it can be absorbed into the term *Dp*ω.

**Improved swing equation**

In order to develop the ISE model, the paper by Monshizadeh at el start with the mechanical dynamics of the rotor (which are similar to the third line of the previous model assuming that T\_e=mif\*iq). Substituting the terms for the mechanical and the electromagnetic torque developed by the generator which are: … yields:

**Classical model**

If we assumes that we are interesting in the model near the nominal value of omega, we can approximate and , in order to simple the ISE model. This yields the following model…

This model is known as the “classical model” at Monshizadeh at el, at Sauer and Pi, and by the book by Machowski Bialek and Bumby

**Infinite bus**

The infinite bus is modeled as a three phase AC voltage source, which means that the infinite bus is not affected by the synchronous generator that is connected to it. The justification for this model is that the influence of a single SG on a grid is very small.

In general, the line that connects the SG to the grid has its impedance that can be modeled (in most cases) as a resistance and an inductance in series, but these values can simply be added to the parameters *Rs* and *Ls* of the SG.

The infinite bus voltage (at the SG terminals) is….

where *V* is the grid line voltage magnitude, and θ*g* is the grid angle. Let’s define the *power angle* δ which represents the difference between the grid and the rotor angle:

Substituting these voltages into the third order model of SG yields:

**Infinite bus ISE**

Using phasor analysis of the electrical power that transformed from the SG to the infinite bus, shows that the electrical power that transformed from the SG to the infinite bus is… (See the book by Machowski Bialek and Bumby) Substituting this term in the ISE model yields:

If we substituting it in the classical model which was presented previously we get:…

The main argument of the paper of Monshizadeh at el, is that the classical model is not realistic enough and that the ISE model should be used instead. Next I will show how to derive the ISE from the FOM without using the phasor analysis, which is problematic when we want to use the model for analysis of the phase stability, and later, show examples that show that even the ISE is not predicate important behaviors that the FOM does.

**Equilibrium point**

**Model reduction1**

we show the relation between the ISE and the FOM. We start with the FOM, and by a model reduction process, we get the ISE model. We apply ideas from singular perturbation analysis. The FOM has the following structure: ….

Because the first two coefficients on the diagonal of Λ are equal to *Ls*, which in some sense can be regarded as being small, we rewrite this dynamical system in the following form ….

By doing this, we have separated the state variables into a vector of fast variables, denoted by *y*, and another of slow variables, denoted by *x*

We assuming that ε is very small, meaning that for each *x*, the vector *y* con- verges to a temporary equilibrium value (that depends on *x*) much faster than the rate of change of *x*. In our case:….

Our assumption that ε is very small means that the subsystem ε*y* ̇ = *g*(*x*,*y*,ε) is much faster than *x* ̇ = *f* (*x*, *y*, ε ) and it is also stable, so that it will reach its temporary equilibrium almost instantly compared to the slow movement of *x*. The temporary equilibrium point *y*ˆ(*x*,ε) is the solution of *g*(*x*,*y*ˆ,ε) = 0. The solution of this linear equation (in *y*ˆ) is ….

assuming that *Rs* is small so that the terms containing *Rs* are negligible, we obtain the following approximation of *y*ˆ:….

**Model reduction2**

Substitute the values of the approximation of iq yelds:…

The power absorbed from the prime mover can be expressed approximately as *Pm* =(*Tm*−*Dp*ω*g*)ω. This crude approximation cannot be justified other than as a means to obtain the so-called improved swing equation, as we shall see below. A more precise expression of *Pm* is the former with omega instead omega\_g.

The references just cited normally assume that *Pm* is constant, which then causes *Tm* to be a function of ω. Our perspective is to view *Tm* as a constant parameter.

**Simulations**

In this section we present simulations which demonstrate that the ISE model is a good approximate model in many cases. We also show that there are cases in which there is a significant mismatch between the behavior suggested by the ISE model and the FOM. In each simulation result, the behavior of *id* and *iq* over time is described for both the FOM and the ISE models. Note that for the ISE model, we use the algebraic equation to estimate these currents. We plot the frequency ω and the power angle δ as functions of time for these two models. In all the simulations, variables with a hat correspond to the reduced model. All simulations assume constant *Pm*. We redid the same simulations also under the assumption of constant *Tm* and the results are very similar.

The first two simulations concern small SGs of 5KW and 1MW, respectively, with parameters taken from [3]. These models were originally meant to represent synchronverters, which are a type of inverters, but from the point of view of the model, this does not matter.

simulations indicates that for the 5KW SG, the be- havior of the FOM and the ISE is almost the same. Although the currents for the ISE have less ripple than for the FOM, both models converge at the same rate with the same oscillations to the same equilibrium point. (The parameters for this simulation can be found in the appendix, table A.1).

*4.2.2 1MW SG*

As shown in Figure 4.2, simulations show that for the 1MW SG, the behavior of the FOM and the ISE are still very similar. Although the FOM currents are much more rippled than the ISE currents, both models converge at the same rate with the same oscillations to the same equilibrium point. (The parameters for this simulation can be found in the appendix, table A.2).

*4.2.3 Non stable behavior of the reduced model*

As shown in … simulations show that for other parameters set the behavior of the FOM and the ISE is significantly different. The FOM is locally

stable, since the eigenvalues of the Jacobian around the equilibrium point are −11.41 ± 376.9*i*, −508 ± 837*i*.



*4.2.4 An example for different regions of attraction*

As shown in …, simulations show that for some parameters set. the initial condition of this simulation is within the region of attraction of the reduced model, but outside the region of attraction of the FOM. This causes the FOM to diverge while the ISE converges to the equilibrium point.