Optimal Dispatch of Reactive Power for Voltage Regulation and Balancing in Unbalanced Distribution Systems

Daniel B. Arnold[1], Michael Sankur[1], Roel Dobbe[2], Kyle Brady[2], Duncan S. Callaway[3], and Alexandra Von Meier[2]

[1]Department of Mechanical Engineering, [2]Department of Electrical Engineering, [3]Energy Resources Group
University of California, Berkeley

Abstract

Optimization of distributed power assets is a powerful tool that has the potential to assist utility efforts to ensure customer voltages are within pre-defined tolerances and to improve distribution system operations. While convex relaxations of Optimal Power Flow (OPF) problems have been proposed for both balanced and unbalanced networks, these approaches do not provide universal convexity guarantees and scale inefficiently as network size and the number of constraints increase. In balanced networks, a linearized model of power flow, the LinDistFlow model, has been successfully employed to solve approximate OPF problems quickly and with high degrees of accuracy. In this work, an extension of the LinDistFlow model is proposed for unbalanced distribution systems, and is subsequently used to formulate an approximate unbalanced OPF problem that uses VAR assets for voltage balancing and regulation. Simulation results on the IEEE 13 node test feeder demonstrate the ability of the unbalanced LinDist3Flow model to perform voltage regulation and balance system voltages.

Problem Definition

- OPF formulations for balanced systems utilize convex relaxations or approximations
- Many approaches based on *DistFlow* and *LinDistFlow* models in seminal work of [1]
- Second order cone relaxations difficult to extend to 3 phase systems
- Semidefinite relaxations fail to converge in many situations
- Lack of suitable linearizations that relate voltages to complex power flows

Contributions

- Developed a linearized model of unbalanced power flow that relates voltage magnitudes to complex power flows
- This approximation can be viewed as an extension of the *LinDistFlow* model to 3 phase systems (we refer to our approximation as the *LinDist3Flow* model)
- *LinDist3Flow* model allows problems to be addressed which hitherto could not be addressed with OPF techniques
- Here, we incorporate the *LinDist3Flow* model into an OPF that uses reactive power resources to balance voltage magnitude across phases and provide voltage support
- Simulations conducted on the IEEE 13 node feeder

LinDistFlow Model for Balanced Systems

Voltage/Power relationship between adjacent nodes m and n:

$$(\mathbb{V}_m = \mathbb{V}_n + \mathbb{Z}_{mn}\mathbb{I}_n)\,(\mathbb{V}_m = \mathbb{V}_n + \mathbb{Z}_{mn}\mathbb{I}_n)^*$$

let: $y_m = |\mathbb{V}_n|^2$, $\mathbb{V}_n\mathbb{I}_n^* = P_n + jQ_n$, and neglect losses:

$$y_m \approx y_n + 2\Re\{\mathbb{Z}_{mn}(P_n - jQ_n)\}$$

Linear in y_n, y_m, P_m, Q_n

LinDist3Flow Model for Unbalanced Systems

Consider A-phase voltage drop:

$$\mathbb{V}_{a,m} = \mathbb{V}_{a,n} + \mathbb{Z}_{aa,mn}\mathbb{I}_{a,n} + \mathbb{Z}_{ab,mn}\mathbb{I}_{b,n} + \mathbb{Z}_{ac,mn}\mathbb{I}_{c,n}$$

Repeating steps above result in nonlinear and nonconvex system

Solution:

$$\frac{\mathbb{V}_{a,n}}{\mathbb{V}_{b,n}} \approx 1\angle 120^\circ,\ \frac{\mathbb{V}_{a,n}}{\mathbb{V}_{c,n}} \approx 1\angle -120^\circ,\ \frac{\mathbb{V}_{b,n}}{\mathbb{V}_{c,n}} \approx 1\angle 120^\circ$$

Results in:

$$Y_m \approx Y_n + \mathbb{M}_{mn}\mathbb{P}_n + \mathbb{N}_{mn}\mathbb{Q}_n \quad (1)$$

$$\mathbb{M}_{mn} = \begin{bmatrix} 2r^{aa}_{mn} & -r^{ab}_{mn} + \sqrt{3}x^{ab}_{mn} & -r^{ac}_{mn} - \sqrt{3}x^{ac}_{mn} \\ -r^{ba}_{mn} - \sqrt{3}x^{ba}_{mn} & 2r^{bb}_{mn} & -r^{bc}_{mn} + \sqrt{3}x^{bc}_{mn} \\ -r^{ca}_{mn} + \sqrt{3}x^{ca}_{mn} & -r^{cb}_{mn} - \sqrt{3}x^{cb}_{mn} & 2r^{cc}_{mn} \end{bmatrix} \quad (2)$$

$$\mathbb{N}_{mn} = \begin{bmatrix} 2x^{aa}_{mn} & -x^{ab}_{mn} - \sqrt{3}r^{ab}_{mn} & -x^{ac}_{mn} + \sqrt{3}r^{ac}_{mn} \\ -x^{ba}_{mn} + \sqrt{3}r^{ba}_{mn} & 2x^{bb}_{mn} & -x_{bc} - \sqrt{3}r^{bc}_{mn} \\ -x^{ca}_{mn} - \sqrt{3}r^{ca}_{mn} & -x^{cb}_{mn} + \sqrt{3}r^{cb}_{mn} & 2x^{cc}_{mn} \end{bmatrix} \quad (3)$$

Linear in $Y_n = [y_{a,n}, y_{b,n}, y_{c,n}]^T$, $\mathbb{P}_n = [P_{a,n}, P_{b,n}, P_{c,n}]^T$, and $\mathbb{Q}_n = [Q_{a,n}, Q_{b,n}, Q_{c,n}]^T$

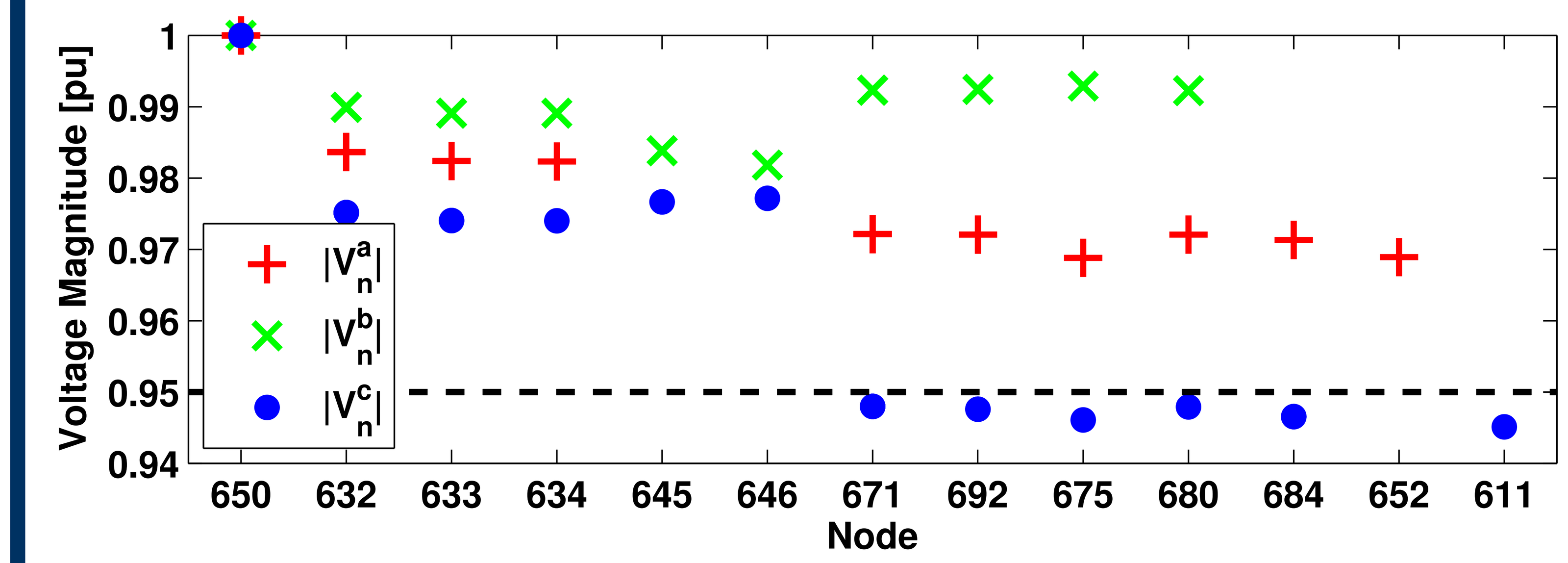
New OPF Formulation

Dispatch reactive power to balance voltage magnitudes across phases:

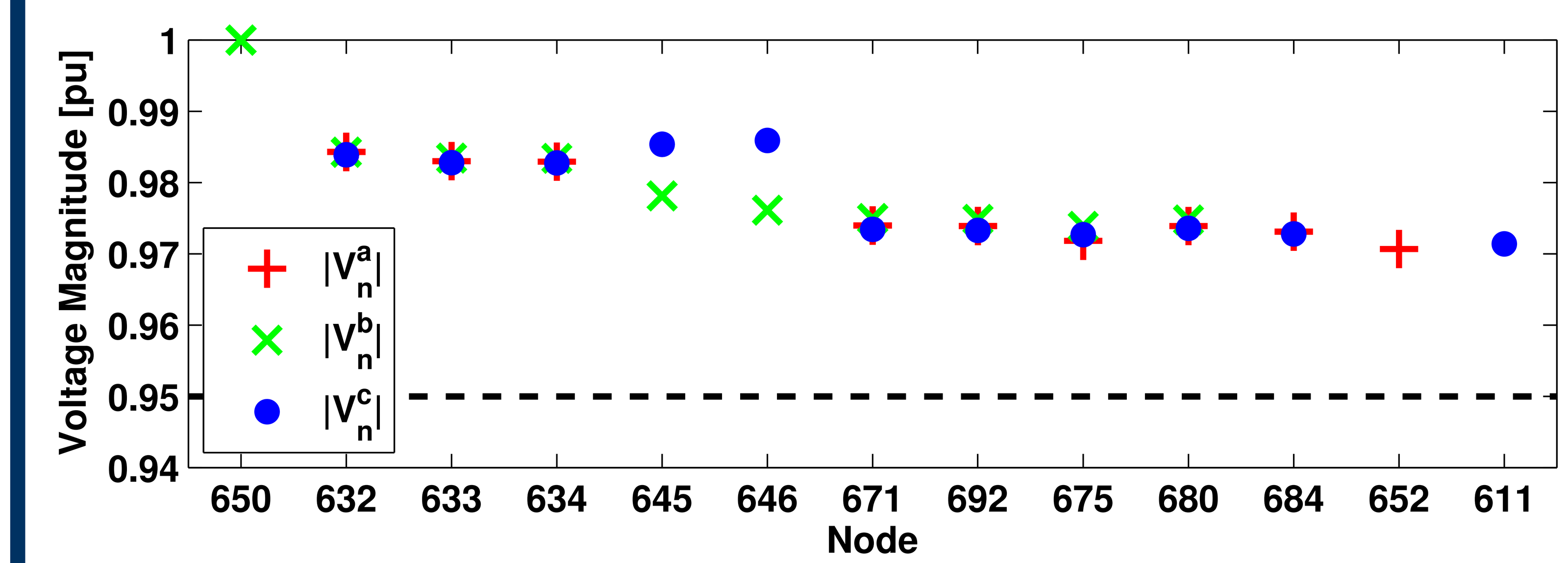
$$\underset{u_n, y_n, Q_n, P_n}{\text{minimize}} \sum_{n\in\mathcal{N}} \left[\sum_{\substack{\phi,\psi\in\{a,b,c\} \\ \phi\neq\psi}} \left(y_n^\phi - y_n^\psi\right)^2 \right] + \rho\left(u_n^\phi\right)^2 \quad (4)$$

$$\text{subject to } (1)-(3),$$
$$\underline{y} \le y_n \le \overline{y}, \quad \forall n \in \mathcal{N},$$

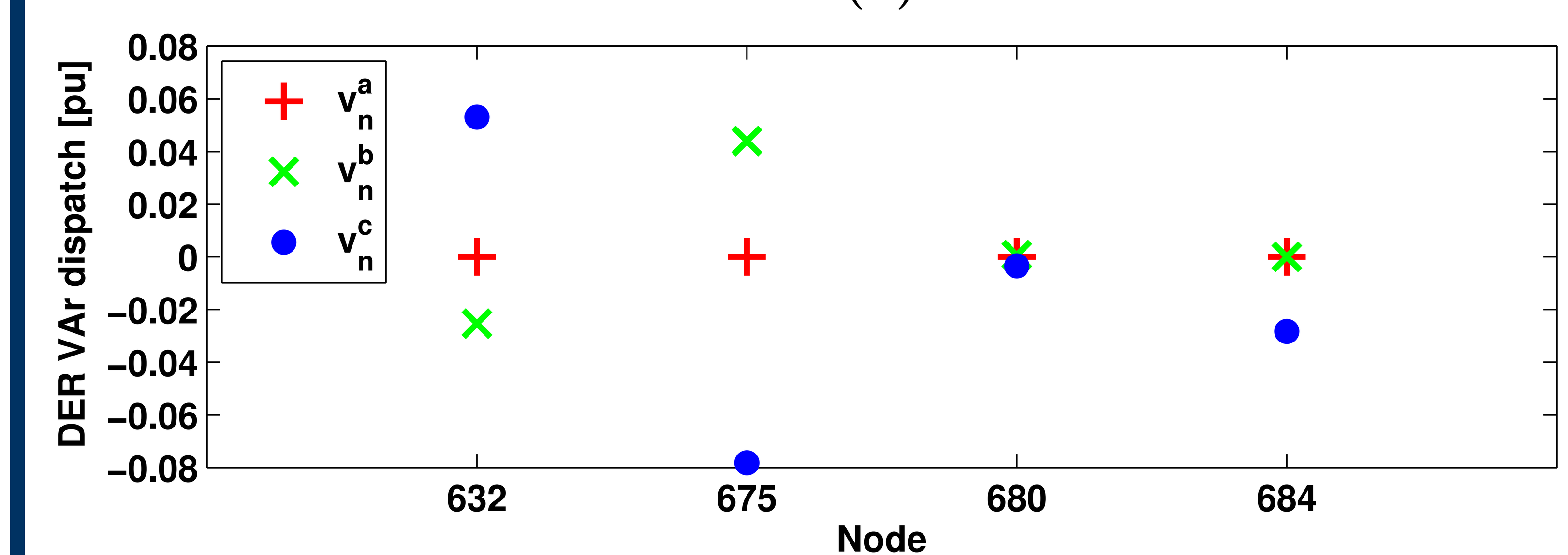
Simulation Results



Voltage magnitudes on 13 node feeder for base (no control) case



Voltage magnitudes on 13 node feeder with VAR dispatch from (4)



Optimal inverter VAR dispatch from (4)

References

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Contact Information:
Daniel Arnold (dbarnold@lbl.gov), Michael Sankur (msankur@lbl.gov)