Three Phase State Estimation Using Hybrid Particle Swarm Optimization

Sara Nanchian, *Student Member, IEEE*, Ankur Majumdar, *Student Member, IEEE* and Bikash C. Pal, *Fellow, IEEE*

***Abstract*—This paper proposes a method for three phase state estimation in power distribution network including on-load tap changers (OLTC) for voltage control. The OLTC tap positions are essentially discrete variables from the state estimation (SE) point of view. Estimation of these variables in SE presents a formidable challenge. The proposed methodology combines discrete and continuous state variables (voltage magnitudes, angles and tap positions). A hybrid particle swarm optimization (HPSO) is applied to obtain the solution. The method is tested on standard IEEE 13-bus and 123-bus unbalanced test system models. The proposed algorithm accurately estimates the network bus voltage magnitudes and angles and discrete tap values. The HPSO based tap estimation provides more accurate estimation of losses in the network which helps in fair allocation of cost of losses in arriving at overall cost of the electricity.**

***Index Terms*—Three phase state estimation, Tap estimation, Hybrid Particle Swarm optimization (HPSO).**

I. Introduction

THE state estimation (SE) is described as a process of finding network voltage magnitudes and angles so that all the other network quantities such as transformer and feeder loadings etc. can be obtained from them. SE is a standard computational task in supervisory control and data acquisition (SCADA) for transmission system. At transmission level generalised framework [1] is well adopted for state estimation with emphasis on bad data analysis including handling switch status error. There is always an assumption of balanced system at transmission level so single phase positive sequence network data are adequate. The distribution network is hardly balanced and symmetric so three phase state estimation is necessary. Until recently it was not much important to have mandatory state estimation at distribution system. But with growing controllable devices and components, it is now important to have state estimator for efficient operation of the distribution network.

There have been growing literatures in distribution system state estimation. Chen and others [2] have provided a thorough model of distribution transformer including core loss to obtain more accurate state estimates. Standard weighted least square (WLS) is suggested. The quality and adequacy of measurements being less than required-several papers focused on accurately estimating the loads through additional measurement placements. References [3], [4], [5] have proposed the concept of minimum additional measurements for improving accuracy of the SE in WLS framework. Because of the radial nature of the distribution network it was found computationally much easier to formulate feeder current as state variables [6], [7]. The results are very convincing and with slight modification in the form of inclusion of constraints in the optimisation objective, small meshed network can also be solved. Small meshed network arises from closing normally open point to extend the feed. The authors in [8] later ex0tended the branch current based estimation to obtain set of locations where placement of meters provide maximum impact in terms of accuracy. References [9], [10], [11], [12] have demonstrated the impact of various degrees of unbalanced in the network loads and parameters and topological uncertainties on the accuracies of SE. Zero injection is taken as constraints in the nodes having no measurement. The authors in [13] and [14] have demonstrated further the impact of inaccurate parameters, untransposed lines, ignored transformer vector groups etc. not only on the accuracy of the estimated quantities but on the bad data rejection capability as well. This led to the requirement of synchronised phasor measurements at the distribution level. Recently Haughton and Heydt [15] proposed a linear state estimator for full three phase unbalanced distribution system. It assumes that synchronised PMU based measurement and other smart demand measurements are available. It is indicated smart meters may record and transmit active and reactive power, energy consumption over time intervals, e.g., 5, 15, 60 min, so real time state estimation is not possible. The authors in [16] used graph theory approach to place measurement to guarantee observability and then solve the node current injection equations to obtain node voltage and angle which appears to work as robust approach to obtain the estimates. The radial nature of the distribution network is exploited by the backward forward sweep method proposed in [17] which is somewhat along the lines of load estimation as performed in radial distribution system power flow computation. It refines the pseudo measurement by power summation at node. Reference [18] provides faster algorithm for estimating loads. The approach is to reduce the complexity of the network into several load estimation zones connected through physical measurements. None of the above references considered primary distribution transformer on-load tap changing (OLTC) position as estimated variable. This exclusion of transformer tap position as estimated variable from the above references encouraged us to develop a three phase state estimation algorithm including the tap as discrete variable and obtain the exact tap position which is not possible by conventional weighted least square (WLS) method.

Sara Nanchian, Ankur Majumdar and Bikash C. Pal are with the Department of Electrical and Electronics Engineering, Imperial College London, SW7 2AZ London, U.K. email: (s.nanchian11@ imperial.ac.uk; ankur.majumdar@imperial.ac.uk; b.pal@imperial.ac.uk)

This work was supported by EP/F037686/1, Power Network Research Academy by EPSRC, UK, and Scottish and Southern Energy Networks, U.K.

Distribution network has also other problem of rapid voltage fluctuation because of the infeed from DG. The output from DG being not predictable (solar and wind) the voltage variation in the network is very severe. So the voltage control is as important as feeder flow control. On load tap changing transformer (OLTC) in the primary substation is also subjected to rapid control actions. It is very important to include tap position as an estimated variable so that network control can be performed more effectively. There have not been much literature in tap observability detection. The authors in [19] proposed a robust algorithm based on Givens rotation of the gain matrix to obtain the state and tap position under erroneous zero injection. The method is demonstrated to work well in Brazilian system models of varying complexity. The authors in [20] used node voltages , angles and tap position as state variables under the assumption that the taps are continuous. However, in practice OLTC has discrete tap positions so inaccurate estimate produces inaccurate computed values of other network quantities such as feeder flows, losses etc. So, the incorporation of the discrete tap variables in SE is required for effective network control.

Commonly, SE algorithm is developed based on the assumption of balanced network model which assumes that line parameters and loads are balanced. Based on this assumption the analysis is applied on the single phase model, simply using the positive sequence network for estimation of the system states. However, in distribution networks the presence of single and two-phase laterals, untransposed three-phase circuits and unbalanced loads affect the accuracy of the estimated states when assumption of balanced system is applied. This results in unexpected system performance or undesirable operating situations. So, an appropriate three-phase estimator is required to accurately obtain the states with taps as discrete variables [7], [21]. The motivation of this work came from the need of a comprehensive three phase state estimator including tap as discrete variable to obtain most accurate network nodal voltages and angles under unbalanced situation.

SE is commonly formulated as a weighted least square (WLS) problem [22]. However, transformer tap positions make the problem a mixed integer nonlinear one. So, the solution of the problem through normal equation framework is not possible as the objective function is not differentiable [23].

Authors in references [24] and [25] have assumed continuous taps instead of discrete and have incorporated rounding technique or sensitivity method to address the complexity of discrete values. However, this assumption reduces the accuracy of the SE and the solution does not represent the actual network taps positions. Therefore, the estimation of transformer tap positions has to be obtained from the solution of the mixed integer non-linear optimization problem containing continuous and discrete values [26], [27].

There has been growing interest in the application of heuristic algorithms such as neural networks (NN), genetic algorithms (GA), honey bee mating optimization (HBMO), and particle swarm optimization (PSO) in recent years. To overcome the computational difficulty of such complex optimization problem, these algorithms have been successfully applied to wide range of optimization problem where they can handle mixed integer variables of the objective functions as they do not need the function to be continuous and differentiable [28].

PSO appears to be a very effective technique compared to GA and other evolutionary algorithms as it is simple in concept and implementation. It has limited number of parameters in comparison to other heuristic optimization methods. It can be easily applied to diverse issues where it can produce satisfactory solutions and stable convergence characteristics [29]. However, PSO has weak form of selection that increases the amount of time to get to the effective area in the solution space.

A hybrid form of PSO (HPSO) is used to overcome this situation [30] combining the feature from PSO and GA. HPSO has been applied in balanced state estimation problem [31]. HPSO uses a selection method which is based on the evolution from generation to generation. Reference [31] considered a balanced three phase distribution network model without considering unbalanced nature of the system and discrete tap as state variables.

Our research proposes a full three phase state estimator including unbalanced load, network model and the discrete taps as additional estimated variables based on HPSO optimization technique in order to estimate the discrete value of the transformer taps. The contribution of our research lies in handling the complexity of the unbalanced system and correctly computing the discrete values of the tap. The performance of the proposed method has been tested on IEEE 13-bus and 123-bus test system models and the results are presented.

Like all the methods based on heuristic search, the HPSO method takes longer time (tens of minutes) to converge in a reasonably sized network model. This apparently weakens the case for HPSO for SE in real time. It is important to note that state estimation in distribution SCADA is not done in real time. Since the life expectancy of the tap changing mechanism is influenced by the number of operations of the taps, it is also not allowed to have frequent tap operation when rapid changes and loads and generation take place. HPSO can be applied for network computation used for operational planning purposes such as hour ahead contingency analysis and reactive power control scheduling, loss estimation for dynamic pricing purpose. The HPSO based solution in this way is still attractive to operate the network with higher efficiency even though the calculation is not done in real time. Our paper demonstrates the value of this concept through comprehensive modelling, computation and analysis in this context. We have also made several modifications of traditional PSO and HPSO techniques to obtain solutions faster.

This paper is organized as follows: Section II provides an overview of SE in distribution system. In Section III the PSO solution method is described. Section IV provides an overview of HPSO. Section V presents the HPSO approach in the context of three phase unbalanced state estimation for distribution network model. Section VI demonstrates the results and discussions of the state estimation for IEEE 13-bus and IEEE 123-bus model networks. Section VII concludes the paper.

II. Distribution system state estimation (DSSE)

The solution to DSSE problem is formulated as a minimization of the following objective function subject to satisfying several equality and inequality constraints. The goal is to obtain the bus voltage magnitudes, angles and tap positions that minimizes weighted square of the difference between the measured quantity and the estimated quantity which are functions of estimated states. It is expressed as:

$$min\ J(x) = \sum_{i=1}^{m} w_{ii} r_i^2 * \tag{1}$$

Subject to:

$$z_i = h_i(x) + r_i * \tag{2a}$$
$$c(x) = 0 * \tag{2b}$$
$$g_{min} \leq g(x) \leq g_{max} \tag{2c}$$

Where,

- x State variables such as voltage magnitudes, angles and tap positions.
- m Number of measurements.
- w_{ii} Weighting factor of measurement variable i, $z = \left[z_1^a\ z_1^b\ z_1^c\ \ldots\ z_i^a\ z_i^b\ z_i^c\ \ldots\ z_m^a\ z_m^b\ z_m^c\right]$.
- z_i Measured value of i^{th} measurement.
- h_i i^{th} measurement as a function of state x.
- r_i i^{th} measurement error.

In three phase system $x = \left[V_i^k\ \delta_i^k\ t_i^k\right]$, where $V_i^k = \left[V_i^a\ V_i^b\ V_i^c\right]^\top$ is the vector of three-phase voltage magnitude at bus i, $\delta_i^k = \left[\delta_i^a\ \delta_i^b\ \delta_i^c\right]^\top$ denotes the phase angles of bus i except the reference bus and $t_i^k = \left[t_i^a\ t_i^b\ t_i^c\right]^\top$ is the transfomer's tap vector if present at i^{th} bus.

In three-phase model, the power injected at bus i for phase k can be written as:

$$P_i^k = V_i^k \sum_{l=1}^{3} \sum_{j=1}^{n} V_j^l \left[G_{ij}^{k,l} \cos\left(\delta_i^k - \delta_j^l\right) * + B_{ij}^{k,l} \sin\left(\delta_i^k - \delta_j^l\right)\right] * \tag{3}$$

$$Q_i^k = V_i^k \sum_{l=1}^{3} \sum_{j=1}^{n} V_j^l \left[G_{ij}^{k,l} \sin\left(\delta_i^k - \delta_j^l\right) * - B_{ij}^{k,l} \cos\left(\delta_i^k - \delta_j^l\right)\right] * \tag{4}$$

Where $G+jB$ is the system admittance matrix, n is number of buses and l is the number of phases that can be 1, 2 or 3 phase. The power flow from bus i to bus j for phase k can be written as follows:

$$P_{ij}^k = V_i^k \sum_{l=1}^{3} V_i^l \left[G_{ij}^{k,l} \cos\left(\delta_i^k - \delta_i^l\right) + B_{ij}^{k,l} \sin\left(\delta_i^k - \delta_i^l\right)\right] - V_i^k \sum_{l=1}^{3} V_j^l \left[G_{ij}^{k,l} \cos\left(\delta_i^k - \delta_j^l\right) + B_{ij}^{k,l} \sin\left(\delta_i^k - \delta_j^l\right)\right] * \tag{5}$$

$$Q_{ij}^k = -V_i^k \sum_{l=1}^{3} V_i^l \left[G_{ij}^{k,l} \sin\left(\delta_i^k - \delta_i^l\right) - B_{ij}^{k,l} \cos\left(\delta_i^k - \delta_i^l\right)\right] - V_i^k \sum_{l=1}^{3} V_j^l \left[G_{ij}^{k,l} \sin\left(\delta_i^k - \delta_j^l\right) - B_{ij}^{k,l} \cos\left(\delta_i^k - \delta_j^l\right)\right] * \tag{6}$$

Where,

- P_i^k , Q_i^k Active and reactive power injection of phase k in bus i.
- P_{ij}^k , Q_{ij}^k Active and reactive power flow of phase k from bus i to bus j.
- V_i^l Voltage magnitude of phase l at bus i.
- δ_i^l Angle of phase l in bus i.

A. Equality constraints $c(x)$

The equality constraints are the set of equation corresponding to virtual measurements.

$$0 = P_{Gi}^k - P_{Di}^k - \sum_{l=1}^{3} \sum_{j=1}^{n} V_i^k V_j^l \left[G_{ij}^{k,l} \cos\left(\delta_i^k - \delta_j^l\right) * + B_{ij}^{k,l} \sin\left(\delta_i^k - \delta_j^l\right)\right] * \tag{7}$$

$$0 = Q_{Gi}^k - Q_{Di}^k - \sum_{l=1}^{3} \sum_{j=1}^{n} V_i^k V_j^l \left[G_{ij}^{k,l} \sin\left(\delta_i^k - \delta_j^l\right) * - B_{ij}^{k,l} \cos\left(\delta_i^k - \delta_j^l\right)\right] * \tag{8}$$

Where P_{Gi}^k and Q_{Gi}^k are the real and reactive power injected at bus i respectively, the load demand at the same bus is represented by P_{Di}^k and Q_{Di}^k [32]. Indices n is number of buses and l is the number of phases which can be 1, 2 or 3 phase.

B. Inequality constraints

These are the set of constraints of continuous and discrete variables that represent the system operational and security limits, such as setting upper and lower limits for control variables. These are as follows:

- **Bus voltage** - Voltage magnitudes at each bus in the network:
$$V_{min,i}^k \leq V_i^k \leq V_{max,i}^k$$
- **Bus angle** - The bus angle at each bus in the network:
$$-\delta_{min,i}^k \leq \delta_i^k \leq \delta_{max,i}^k$$
- **Transformer taps** - Transformer taps settings:
$$t_{min,i}^k \leq t_i^k \leq t_{max,i}^k$$

III. PARTICLE SWARM OPTIMIZATION (PSO)

The PSO is a population based optimization method first proposed by Kennedy and Eberhart [33], which is used to search the solution space of a given problem to find the candidate solution which can maximize or minimize a particular objective function. Some of the attractive features of the PSO include the ease of application and the fact that no gradient information is required which allows the PSO to be used on functions where the gradient is either unavailable or computationally expensive to obtain. It generates high quality solutions and has stable convergence characteristic than other stochastic methods; so can be used to solve a wide array of optimization problems. It uses a number of particles that constitute a swarm. Each particle traverses the search space looking for the global minimum (or maximum). During flight, each particle adjusts its position according to its own experience and the experience of neighbouring particles, making use of the best position encountered by itself (*pbest*) and its neighbours (*gbest*). Let X and V represent a particle position and its corresponding velocity in a search space, respectively. The best previous position of a particle is recorded and represented as *pbest*. The index of the best particle amongst all the particles in the group is represented as *gbest*. The modified velocity and position of each particle are calculated as follows [34]:

$$\begin{aligned} V_{ij} = WV_{ij}^{0} + C_1 R_1 \times \left(X_{pbest} - X_{ij}^{0}\right) * \\ + C_2 R_2 \times \left(X_{gbest} - X_{ij}^{0}\right) * \end{aligned} \tag{9}$$

$$X_{ij} = X_{ij}^{0} + V_{ij} * \tag{10}$$

i $\quad i^{th}$ particle
j $\quad$ Dimention of the velocity associated with particle i

Where V_{ij} and X_{ij} are respectively the velocity and position of the particle at iteration j, W is the weighting function, C_1 and C_2 are the weighting factors and R_1 and R_2 are random numbers between 0 and 1. In equation (9), X_{pbest} is the $pbest$ of particle and X_{gbest} is the $gbest$ of the group.

A. Selection of parameters for PSO and simulation condition

The selection of key parameters to set up PSO such as W, C_1, C_2 and V_{max} is an important task as they govern the rate of convergence.

1) Weighting function (W): W is a weighting factor which is related to the velocity of the particle during the previous iteration. It controls the amount of the previous velocity that particle takes in the next iteration. This value is equal to 1.0 for the original PSO. The concept of velocity will be lost if this value is set to zero. Shi and Eberhart [35] investigated the effect of W in the range $(0.0, 1.4)$. The faster convergence is obtained by setting this value between $(0.8, 1.2)$. However, setting of inertia weight that decreased from 0.9 to 0.4 generated satisfactory result [34].

The weighting function is obtained as:

$$W = W_{ini} - \frac{(W_{ini} - W_{fin}) \times iter}{iter_{max}} * \tag{11}$$

Where W_{ini} and W_{fin} are the initial and final weight respectively, $iter_{max}$ is the maximum iteration number and $iter$ is the current iteration number.

2) Acceleration coefficients (C_1 and C_2): C_1 and C_2 affect the maximum step size of a particle in a single iteration. C_1 regulates the maximum step size of a particle in the direction of the $pbest$ while C_2 regulates the maximum step size in the direction of the $gbest$ [34] as shown in (9).

The particle velocity is limited by V_{max} to minimize the possibility of the particle escaping the search space. If the search space is defined by the bounds $[X_{min}, X_{max}]$, the value of V_{max} is typically set to:

$$V_{max} = K \times (X_{max} - X_{min}) * \tag{12}$$

Where,

$$0.1 \leq K \leq 1.0$$

Initially, PSO has been performed several times with different values of the key parameters of W, C_1, C_2 and V_{max} to achieve the satisfactory results.

B. Improving the speed of convergence

We have made appropriate modification to update the position of the particle by changing the direction when reaching the search space boundary in order to improve the speed of convergence. The direction of the particle should be modified in such a way that it keeps the particle inside its range when the velocity takes the particle out of its boundary $[X_{min}, X_{max}]$. So the new position of the particle will be updated based on the following equation instead of (10).

$$X_{ij} = X_{ij}^{0} - V_{ij} * \tag{13}$$

the optimal solution be close to the boundary the α factor helps to reach the solution faster.

$$X_{ij} = X_{ij}^{0} - \alpha V_{ij} * \tag{14}$$

where, α is an optimally chosen number between 0 and 1. We have discussed about a suitable value of alpha later in the discussion section.

C. General algorithm for PSO

1) Generate initial population of particles with random velocities and positions. In this study, these particles are bus voltage magnitudes and angles and transformer tap positions.
2) These initial particles must be feasible candidate solutions that satisfy the practical operation constraints. Set upper and lower limits for these particles (v, δ, t).
3) For each particle of the population, calculate the error based on (2).
4) Obtain value of each particle in the population using the evaluation function (fitness function) and the penalty function. The value of fitness function is obtained as follows:

$$FF(x) = \frac{1}{1 + J(x) + Penalty(x)} * \tag{15}$$

Where $J(x)$ comes from Eq (1) and $Penalty(x)$ as follows:

$$Penalty(x) = \rho \sum_{i}^{N} (x_i - x_0)^2 * \tag{16}$$

- N Number of penalised control variables.
- ρ Scalar quadratic penalty weight.
- x_0 Control variable current value (pu).
- x_i Control variable penalty offset (pu).

5) Compare each particles value with its $pbest$. The best value among the $pbest$ is denoted as $gbest$.
6) Update the time counter $t = t + 1$.
7) Update the inertia weight W given by (11).
8) Modify the velocity V of each particle according to (9).
9) Modify the position of each particle according to (10) and (14). If a particle violates its position limits in any dimension, set its position at the proper limit according to (14).
10) Each particle is evaluated according to its updated position. If the value of each particle is better than the previous $pbest$, the current value is set to be $pbest$. If the best $pbest$ is better than $gbest$, this value is set to be $gbest$.
11) If the maximum of a variation of the state variable $\triangle x$ is smaller than 0.001 and the iteration has reached maximum number of iteration specified go to Step #12. Otherwise go to Step #6 until the end criteria are satisfied.
12) The particle that generates the latest $gbest$ is the optimal value.

IV. Hybrid Particle Swarm Optimization

The PSO algorithm has a weak selection process which very much depends on $pbest$ and $gbest$, therefore, the searching area is limited by $pbest$ and $gbest$. This leads to increase in the amount of time that it takes to get to the effective area in the solution space. The hybrid particle swarm optimization (HPSO) method has been introduced based on tournament selection method of genetic algorithm (GA), in order to improve the weakness of PSO method in this area. The introduction of the tournament selection mechanism in PSO algorithm, the effect of $pbest$ and $gbest$ is gradually eliminated by the selection which can result in the wider search area. The purpose of the selection in evolutionary algorithm is to influence the execution of the algorithm on a specific region of search space; usually one that delivered promising solutions in the recent past. Particle positions with low values are replaced by those with high values using the tournament selection method [30]. Therefore, the number of highly evaluated particles is increased while the number of lowly evaluated particles is reduced at each iteration. It should be mentioned that although, the particle position is substituted by another particle position, the information related to each particle is reserved. Therefore, the intensive search in a current effective area and dependence on the past high evaluation position are realized [30].

V. Distribution system state estimation by HPSO

This section presents the application of the HPSO algorithm for solving the DSSE problem. The set of tasks involve:

A. Network configuration and network data

The presence of unsymmetrical-network components and unbalanced-load on unbalanced distribution system makes it essential to consider the exact model of the system components (three-phase model). It includes the information about line resistance, reactance, tap setting connectivity information etc.

Therefore, the following provides information regarding three phase model of various components of the network such as line, transformers and switches [36].

1) Line model: The distribution overhead lines and underground cables are three-phase, two-phase or single-phase and are most often untransposed serving unbalanced loads. In addition, since the voltage drops due to the mutual coupling of the lines is playing important part in the analysis of the distribution network, it is important to compute the impedance of the overhead and underground line segment as accurate as possible. Therefore, it is necessary to retain the self and mutual impedance terms of the conductors and take into account the ground return path for the unbalanced currents [37].

A modified Carson's equation has been applied to model the overhead lines and underground cables.

$$Z_{ii} = r_i + 0.095 + j0.121 \times \left(Ln \frac{1}{GMR_i} + 7.934 \right) \Omega/\text{mile} \; * \tag{17}$$

$$Z_{ij} = 0.095 + j0.121 \times \left(Ln \frac{1}{D_{ij}} + 7.934 \right) \Omega/\text{mile} * \tag{18}$$

Where,

- Z_{ii} Self-impedance of conductor i in Ω/mile.
- Z_{ij} Mutual impedance between conductors i and j in Ω/mile.
- r_i Resistance of conductor i in Ω/mile.
- GMR_i Geometric mean radius of conductor i in feet.
- D_{ij} Distance between conductors i and j in feet.

While using the modified Carson's equations there is no need to make any assumptions, such as transposition of the lines. By assuming an untransposed line and including the actual phasing of the line and correct spacing between conductors, the most accurate values of the phase impedances, self and mutual, are determined.

While applying modified Carson's equations (17) and (18) to a three phase overhead or underground circuit which consists of n phases and neutral conductors the resulting impedance matrix will be $n \times n$. For most applications, it is necessary to have the 3×3 phase impedance matrix. Therefore, the following Kron's reduction is applied in order to breakdown the impedance matrix into the 3×3 phase frame matrix. In this approach, all the lines will be modelled by 3×3 phase impedance matrix and for two phase and single phase lines the missing phases are modelled by setting the impedance element to zero [37].

$$Z_{ij} = Z_{ij} - \frac{Z_{in} \times Z_{nj}}{Z_{nn}} * \tag{19}$$

2) Transformer model: The three phase transformer banks are commonly used in the distribution network and provide the final voltage transformation to the customers load. The conventional transformer models based on a balanced three-phase assumption can no longer be used when system is unbalanced. Three phase transformers are modeled by an admittance matrix which depends on the connection type. The admittance matrix of a transformer is sub-divided into sub-matrices for both self and mutual admittances between the primary and secondary. In the analysis of the distribution feeder, it is required to model the various three phase transformer connections correctly. The comprehensive calculations of three phase transformers and their various connections can be found in reference [37].

3) Switch model: Switches are modelled as branches with zero impedance when the switch is closed or as branches with zero admittance when it is open. The operational constraints imposed by the open or closed status of the switching branches will be as follows:

- When the switch between bus i and bus j is closed for branch i-j, the voltages and angles for bus i and bus j for all the three phases are equal.

$$V_i - V_j = 0*$$
$$\delta_i - \delta_j = 0*$$

- When the switch is open between bus i and bus j, the active and reactive power flow to the switch will be zero.

$$P_{ij} = 0*$$
$$Q_{ij} = 0*$$

These have been included as equality constraints in the state estimation formulation and a weighted quadratic penalty function defined in (16) is applied to solve the above equality constraint problem in which a penalty factor is added to the objective function moment any constraint violation occurs.

4) Load model: The loads on distribution network are generally unbalanced and can be connected in a grounded wye configuration or an ungrounded delta configuration. It is also possible to have three-phase, two-phase, or single-phase loads with varying degree of unbalance. The loads are commonly indicated by complex power consumed per phase and supposed to be line-to- neutral for wye load and line-to-line for delta load [37].

In this paper, loads are modelled as constant impedance, constant current and constant complex power or any combination of the three. Typically, the load values are given as nominal power delivered to the load and must be converted into the appropriate constant model parameters. The general form of ZIP model is as follows [38]:

$$P_L = P_n \left[c_1^P + c_2^P \left(\frac{V}{V_n} \right) + c_3^P \left(\frac{V}{V_n} \right)^2 \right] * \qquad (20)$$

$$Q_L = Q_n \left[c_1^Q + c_2^Q \left(\frac{V}{V_n} \right) + c_3^Q \left(\frac{V}{V_n} \right)^2 \right] * \qquad (21)$$

The two-phase and single-phase loads are modelled by setting the values of the complex power to zero for the non-existent phases for both the wye and delta connected loads [37].

B. Power flow calculation

Three-phase power flow program suitable for distribution system has been set up with network information and load data to generate measurement data as an input to the state estimator. The normally distributed noise component has been added to these measurements to produce input for state estimator. Usually 1% to 3% error is associated with true measurements while the error for pseudo measurements (load data) is considered to vary between 20% and 50%.

C. State estimation based on HPSO algorithm

The following steps have been followed to estimate the state by HPSO estimator.

1) Set up a set of HPSOs parameters such as number of particles N, weighting function W, acceleration coefficients C_1 and C_2 and maximum number of iterations.
2) Generate an initial population of the state variables (V, δ and t) with random velocities and positions in the solution space.
3) Set upper and lower limits for state variables (V, δ and t).
4) For each state variable if the variable is within the set limits go to the following step. Otherwise, that variable is infeasible.
5) The state has been calculated based on the minimization between measurements and calculated value based on HPSO algorithm.

Fig. 1 illustrates the distribution system state estimation using HPSO algorithm.

VI. Results and discussions

A. Case studies

The proposed modified HPSO approach was implemented in MATLAB by Intel(R)Xeon(R) processor @2.40 GHz with 12.0 GB of RAM and has been tested on IEEE 13 bus and 123 bus test systems to validate the proposed algorithms. The network parameters and load data are obtained from [39], [40]. The topologies of the test systems are shown in Fig. 2 and Fig. 3. The systems consist of both overhead lines and underground cables. The overhead lines and underground cables have been modelled with modified Carson's equations. There are both three-phase and single-phase loads. The three-phase loads are either star or delta connected. The loads in the system have been modelled in ZIP-model. A three phase transformer is modelled as three individual single-phase transformers.

Measurements were generated using three-phase power flow program with addition of normally distributed noise component to generate noisy measurements. The error of true measurement was assumed 3% while the error in pseudo measurement (load data) was considered 20%.

Several trials are performed for appropriate parameter value selection. Appropriate combination of W_{ini}, W_{fin}, C_1, C_2 have been applied. The following values have been chosen for HPSO estimator; $W_{ini} = 0.9$, $W_{fin} = 0.4$, $C_1 = C_2 = 2.05$. The values are very close to those recommended by other authors in [31] and [34]. In order to choose the best value

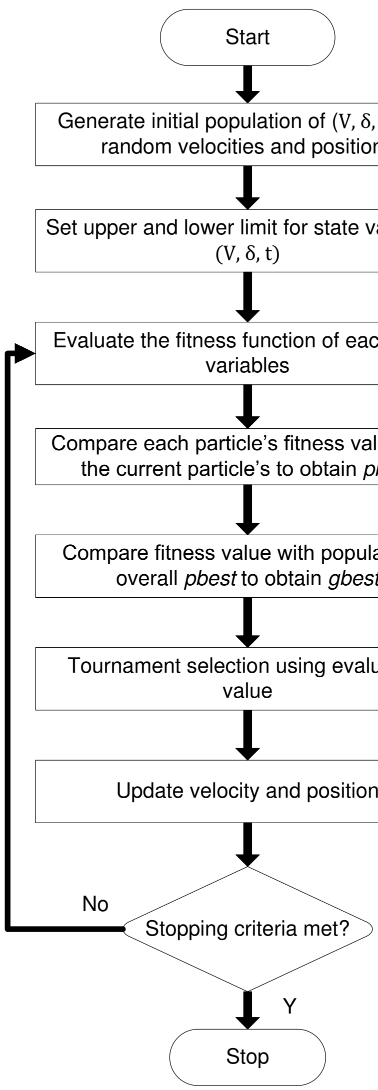

Fig. 1: Setting up of HPSO

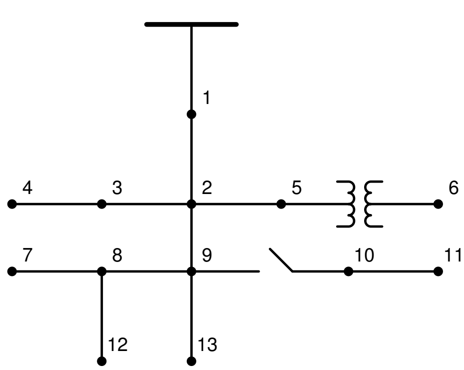

Fig. 2: IEEE-13 bus distribution system

for alpha, the algorithm was run for different values of alpha between 0 and 1 in step of 0.1 in order to obtain the value

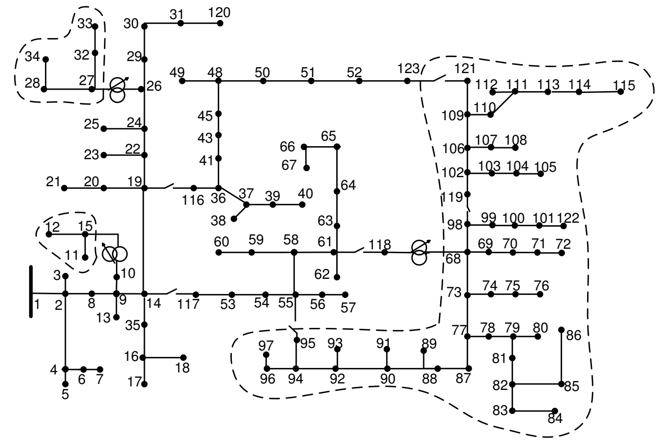

Fig. 3: IEEE-123 bus distribution system

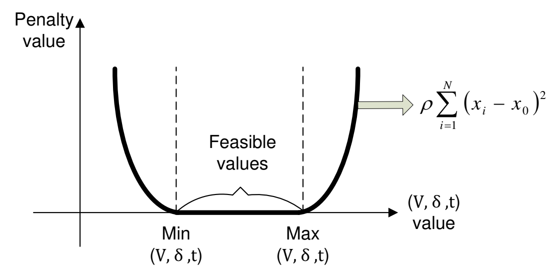

Fig. 4: Proposed penalty function

with the least convergence time. It is found that $\alpha = 0.2$ produces the best convergence speed for this example system. The number of particles and the number of iterations are 200 and 1000 respectively. The lower and upper limits of control variables corresponding to the coding on the HPSO is set in such a way that the inequality constraints of the control variables are satisfied as listed in (22). The shape of the proposed penalty function is displayed in Fig. 4.

$$0.95 \text{ pu} \leq V_i^a,\ V_i^b,\ V_i^c \leq 1.05 \text{ pu}* \tag{22a}$$
$$-30^\circ \leq \delta_i^a,\ \delta_i^b,\ \delta_i^c \leq +30^\circ* \tag{22b}$$
$$0.9 \leq t_i^a,\ t_i^b,\ t_i^c \leq 1.1* \tag{22c}$$

The tap ratio of transformer is specified in the range of 0.9 to 1.1 in steps of 0.00625.

The zero injections, voltages and angles across the closed switch and voltage at the regulator bus have been taken as equality constraints in the state estimation formulation. A weighted quadratic penalty function is added to the objective function to take care of each of these equality constraints.

The switch between bus 9 and bus 10 is assumed to be closed for 13 bus test system. Fig. 5, 6 and 7 show the real and estimated vector of voltages and Table I shows the real and estimated vector of angles at different phases of the IEEE 13 bus model distribution network. There are 5 switches in the 123 bus system. The switch between buses 121 and 123 is assumed to be open while other switches are assumed to

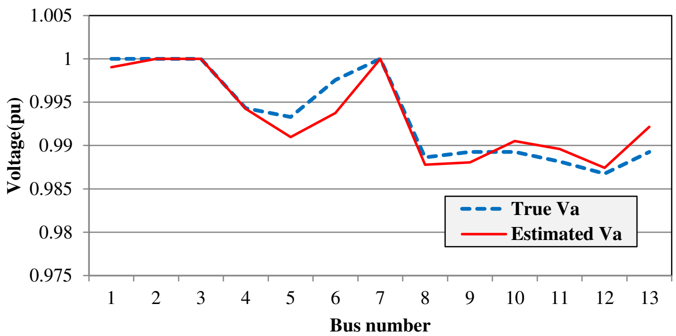

Fig. 5: True and estimated voltage for phase a

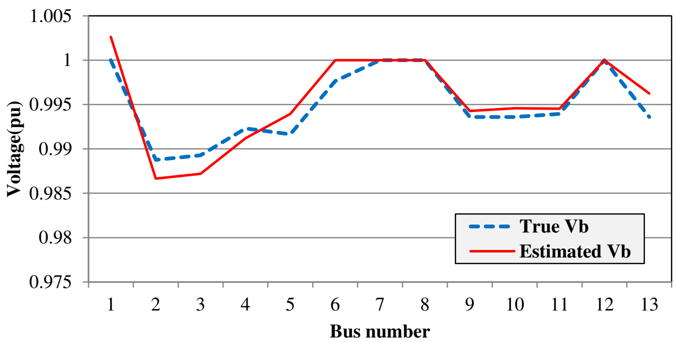

Fig. 6: True and estimated voltage for phase b

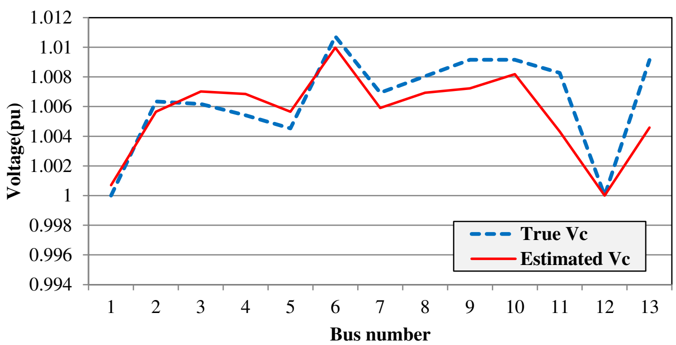

Fig. 7: True and estimated voltage for phase c

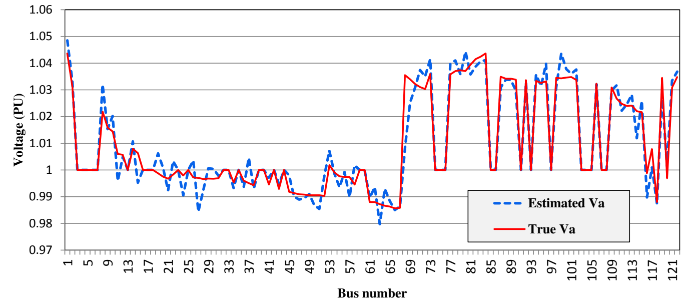

Fig. 8: True and estimated voltage for phase a

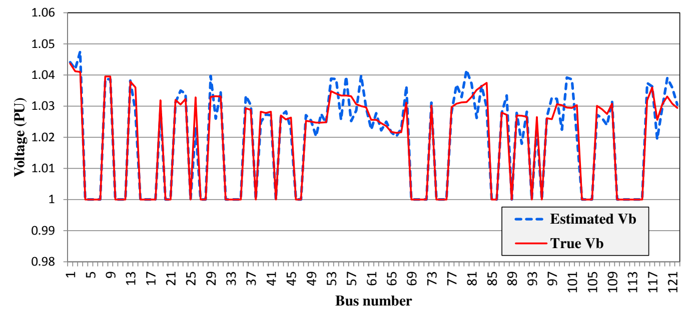

Fig. 9: True and estimated voltage for phase b

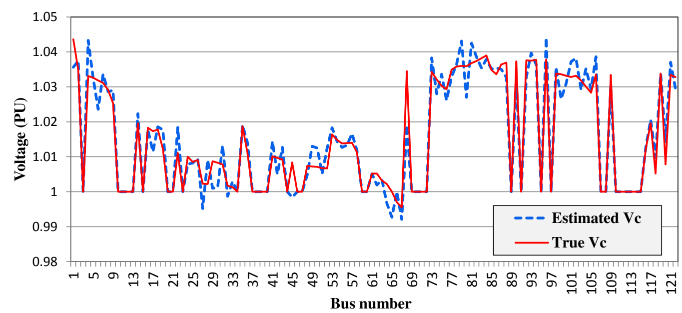

Fig. 10: True and estimated voltage for phase c

be closed for this network. Fig. 8, 9 and 10 show the real and estimated vector of voltages of the IEEE 123 bus model distribution network. In Figs. 5 to 10 the value of voltage has been taken as 1 where there is no phase available for the given bus on the graphs. Also, in Table I, the dashes show that there is no phase available for the given bus. The obtained results are found to be very satisfactory within the allowable error range. The main advantage of the HPSO based method is in correct estimation of the transformer taps as it is shown in Table II for 13 bus test system and Table III for 123 bus system, which is the most important findings from this research reported here.

B. Discussions

We have applied both PSO and HPSO to obtain tap position estimates. Both methods have been tested on IEEE 13 and 123 bus test systems. PSO takes on an average of 6-7 hours to obtain the solution on 13-bus test system. We have modified the particle position update equation for improving the speed of convergence as given in (14). Therefore, the execution time reduced to $2-3$ hours. HPSO on the other hand takes $18-20$ minutes to converge for 13-bus test system. The convergence characteristic of HPSO and improved PSO with correction are shown in Fig 12 for 13-bus test system. As can be seen from convergence characteristic, HPSO method is much faster than

TABLE I: True and estimated angle

Bus number	True δ phase a	Estimated δ phase a	True δ phase b	Estimated δ phase b	True δ phase c	Estimated δ phase c
1	0	0	-120	-120	120	120
2	-	-	-119.57	-118.01	119.36	117.44
3	-	-	-119.62	-118.3	119.42	117.88
4	-0.2412	-0.2507	-119.68	-118.41	119.55	118.15
5	-0.264	-0.2541	-119.7	-118.21	119.55	117.86
6	-0.4974	-0.4869	-119.87	-118.64	119.39	117.11
7	-	-	-	-120	119.39	117.51
8	-0.6488	-0.6297	-	-120	119.41	117.62
9	-0.631	-0.6513	-119.25	-118.27	119.42	117.45
10	-0.631	-0.6494	-119.25	-117.8	119.42	117.65
11	-0.6725	-0.6845	-119.26	-117.78	119.45	117.49
12	-0.6242	-0.6332	-	-	-	-
13	-0.6317	-0.6193	-119.25	-118.19	119.42	117.26

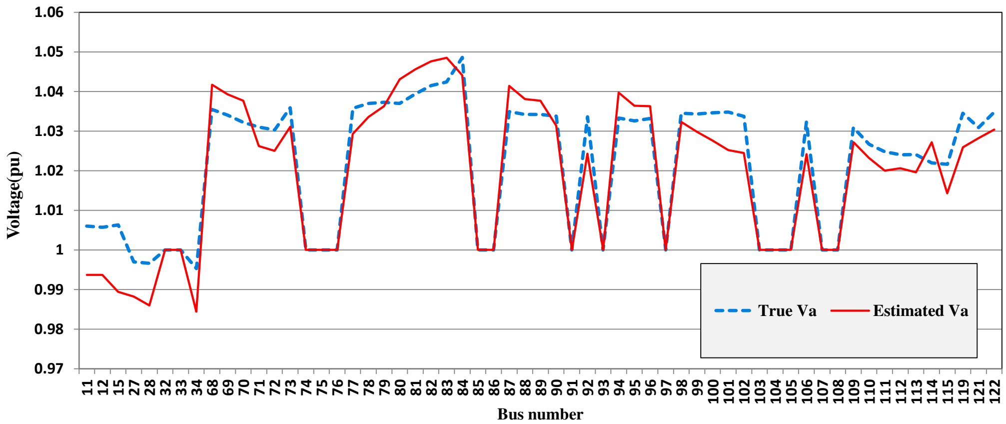

Fig. 11: True and estimated voltage for phase a

TABLE II: True and estimated tap position for 13 Bus system at bus 6

Phase	a	b	c
True Tap position	2	2	1
Estimated tap position HPSO	2	2	1

TABLE III: True and estimated tap position for 123 Bus system

State variable	Real value	WLS	HPSO
T15-Phase a	-1	-1	-1
T27-Phase a	0	0	0
T27- Phase c	-1	-1	-1
T68-Phase a	8	7	8
T68-Phase b	1	1	1
T68-Phase c	5	4	5

PSO method.

In the ideal smart grid environment in principle the state estimator in distribution system should be running in real time. There are several practical limitations to realize this in practice. Even the data from modern smart meters are communicated to distribution SCADA every few hours. The most automated distribution network in the UK does not transmit data more than twice a day. The telecommunication infrastructure in most automated distribution network is such that it is not possible to transmit half hourly measured and stored smart meter data more than twice a day to the control centre. The authors in [41] have documented in details the standard and practice of data transmission from smart meters in various countries across the world. Given this situation, one obvious question arises : is there any possibility for real time state estimation in power distribution system in the forcible future? The frequency of tap operation is also another practical consideration. Considering the operating life of transformer, after 3000 operations, it requires maintenance and after 30000 operations the transformer tap changing mechanism should be replaced with a new one. In that case changing the tap position every few seconds will damagingly result in shortening the transformer operating life. So it is convincing that unlike transmission system the state estimation in distribution system will not run every 2-5 seconds.

Also, the HPSO method with correction for improving the speed has been applied to IEEE 123 bus test system model. It takes on an average of 4 hours to obtain the solution. However, by identifying and considering only those buses where voltage and angles are affected significantly by changing the tap changer position the execution time has reduced to 90 minutes as a result of the reduction in the number of state variables. Those buses have been enclosed through a dashed line in Fig 3 and the results are shown in Fig 11, 13 and 14. One way to improve the speed of convergence of HPSO is to parallelize the algorithm. In the context of computation in power transmission, it has been explored to obtain improvement in speed of computation. The authors in [42] have achieved 4.8 times faster than normal PSO for transmission system state estimation. Because of the similarity of the structure of the equations and between the inter relationship between variables it is possible to achieve similar results based on the application of parallel PSO for the given network in this study. This is not pursued further because of the lack of access to parallel computing cluster. The comparison of the results of tap estimation from WLS and HPSO reveals that estimation of the continuous values of tap changer may result in an inaccurate tap position since it is based on the rounding technique while HPSO provides the exact position of transformer taps by estimating the discrete values of transformer tap positions. So HPSO output provides more accurate voltage and angle estimates thus helps in obtaining better estimate of line and transformer loading and losses. This is very useful in dynamic price setting for efficient electricity market operation and also for optimum scheduling of voltage and var control resources. The power losses in the lines for 123 node system have been computed and the results are shown in Fig 15 and 16. It is clearly seen that HPSO based technique provides more accurate loss figures. Given the customer has to bear the cost of the losses HPSO based estimated voltage and angles and tap positions when used to calculate operational losses for pricing, they will result in more fairer value. This clearly justifies the novelty and benefit of this research contribution. One network operator (Scottish and Southern Energy) in the UK has already

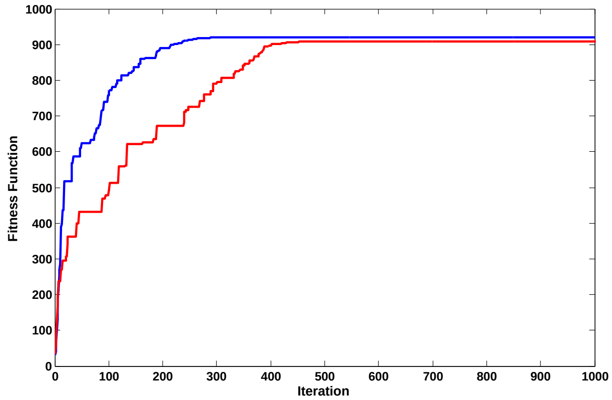

Fig. 12: Convergence characteristic of HPSO and PSO

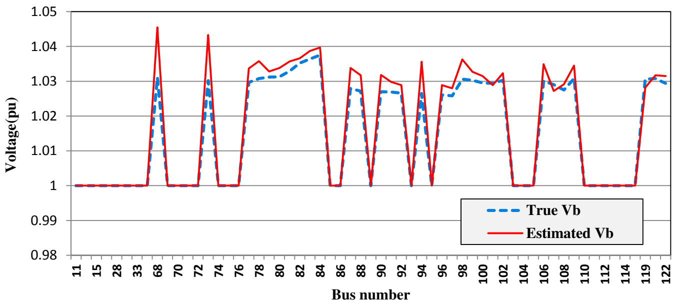

Fig. 13: True and estimated voltage for phase b

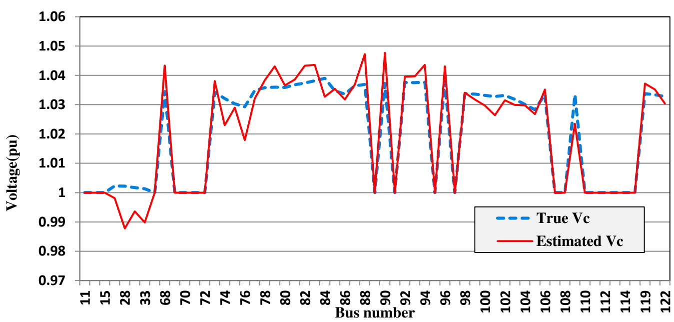

Fig. 14: True and estimated voltage for phase c

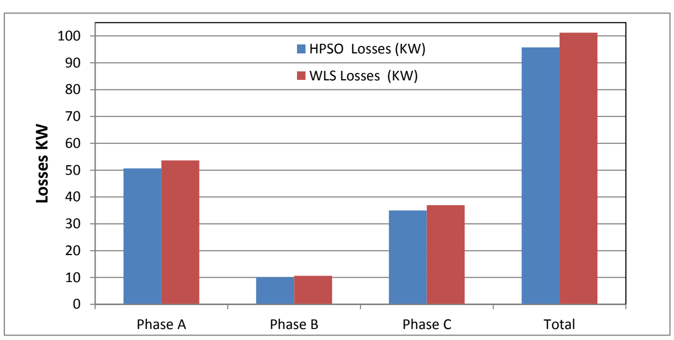

Fig. 15: KW Losses for HPSO and WLS

found this approach useful for network loss calculation in their 33/11 kV network. Compared to WLS based approach the computed network losses are noticeably different which is very useful outcome of this proposed methodology.

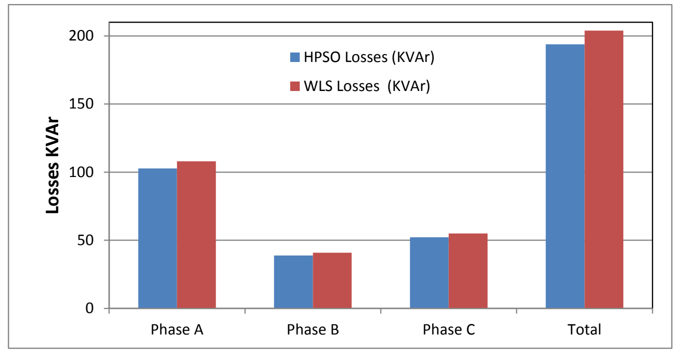

Fig. 16: KVAr Losses for HPSO and WLS

VII. Conclusions

The technique proposed in this paper has addressed the transformer taps estimation in the context of distribution state estimation (SE). It has, for the first time, applied HPSO method to estimate transformer tap positions without any assumption and also in unbalanced three phase distribution system. The simulation results on IEEE 13-bus and IEEE 123-bus standard system models showed that the HPSO method can generate reliable estimate for transformer taps with discrete variables in distribution network while minimizing the objective function. It is also demonstrated that it performs better when compared to PSO. The paper also contributes to novel strategies to expedite solution from PSO and HPSO. The solution from HPSO is accurate and is very useful for computation of various quantities accurately used in the operational planning time scale i.e. voltage and var control (VVC) and pricing in short term market.

Moreover, operational losses from HPSO is much accurate in this case lower than WLS method. In that sense this method is better than WLS method as it provides higher accuracy. Such lower losses will help lowering the overall cost of electricity. In a distribution network where about 6% of electricity generated is lost, accurate estimation of that has huge technical and commercial benefits. The technique proposed in this paper definitely will help realize those benefits.

No gross error is assumed in tap measurement, the tap position measured can be corrupted while being telemetered. A further work continues for bad tap error detection in a large practical power distribution network model. Also, our immediate future plan is to explore mixed integer non-linear (MINLP) optimization solver for distribution SE problem.

Acknowledgment

We like to thank EPSRC,UK and Scottish and Southern Energy Networks (SSE) for sponsoring this research activity.

References

[1] O. Alsac, N. Vempati, B. Stott, and A. Monticelli, "Generalized state estimation [power systems]," in *Power Industry Computer Applications., 1997. 20th International Conference on*, pp. 90–96, May 1997.

[2] T.-H. Chen, M.-S. Chen, T. Inoue, P. Kotas, and E. Chebli, "Three-phase cogenerator and transformer models for distribution system analysis," *Power Delivery, IEEE Transactions on*, vol. 6, pp. 1671–1681, Oct 1991.

[3] I. Roytelman and S. Shahidehpour, "State estimation for electric power distribution systems in quasi real-time conditions," *Power Delivery, IEEE Transactions on*, vol. 8, pp. 2009–2015, Oct 1993.

[4] A. Sakis Meliopoulos and F. Zhang, "Multiphase power flow and state estimation for power distribution systems," *Power Systems, IEEE Transactions on*, vol. 11, pp. 939–946, May 1996.

[5] K. Li, "State estimation for power distribution system and measurement impacts," *Power Systems, IEEE Transactions on*, vol. 11, pp. 911–916, May 1996.

[6] C. Lu, J. Teng, and W.-H. Liu, "Distribution system state estimation," *Power Systems, IEEE Transactions on*, vol. 10, pp. 229–240, Feb 1995.

[7] M. Baran and A. Kelley, "State estimation for real-time monitoring of distribution systems," *IEEE Transactions on Power Systems*, vol. 9, pp. 1601–1609, Aug 1994.

[8] H. Wang and N. Schulz, "A revised branch current-based distribution system state estimation algorithm and meter placement impact," *Power Systems, IEEE Transactions on*, vol. 19, pp. 207–213, Feb 2004.

[9] C. Hansen and A. Debs, "Power system state estimation using three-phase models," *Power Systems, IEEE Transactions on*, vol. 10, pp. 818–824, May 1995.

[10] A. Rankovi, B. M. Maksimovi, and A. T. Sari, "A three-phase state estimation in active distribution networks," *International Journal of Electrical Power Energy Systems*, vol. 54, no. 0, pp. 154 – 162, 2014.

[11] C. Muscas, S. Sulis, A. Angioni, F. Ponci, and A. Monti, "Impact of different uncertainty sources on a three-phase state estimator for distribution networks," *Instrumentation and Measurement, IEEE Transactions on*, vol. 63, pp. 2200–2209, Sept 2014.

[12] P. P. Barbeiro, H. Teixeira, J. Krstulovic, J. Pereira, and F. Soares, "Exploiting autoencoders for three-phase state estimation in unbalanced distributions grids," *Electric Power Systems Research*, vol. 123, no. 0, pp. 108 – 118, 2015.

[13] A. Meliopoulos, B. Fardanesh, and S. Zelingher, "Power system state estimation: modeling error effects and impact on system operation," in *System Sciences, 2001. Proceedings of the 34th Annual Hawaii International Conference on*, pp. 682–690, Jan 2001.

[14] S. Zhong and A. Abur, "Effects of nontransposed lines and unbalanced loads on state estimation," in *Power Engineering Society Winter Meeting, 2002. IEEE*, vol. 2, pp. 975–979 vol.2, 2002.

[15] D. Haughton and G. Heydt, "A linear state estimation formulation for smart distribution systems," *Power Systems, IEEE Transactions on*, vol. 28, pp. 1187–1195, May 2013.

[16] D. Thukaram, J. Jerome, and C. Surapong, "A robust three-phase state estimation algorithm for distribution networks," *Electric Power Systems Research*, vol. 55, no. 3, pp. 191 – 200, 2000.

[17] M. F. M. Jnior, M. A. Almeida, M. C. Cruz, R. V. Monteiro, and A. B. Oliveira, "A three-phase algorithm for state estimation in power distribution feeders based on the powers summation load flow method," *Electric Power Systems Research*, vol. 123, no. 0, pp. 76 – 84, 2015.

[18] I. Dzafic, D. Ablakovic, and S. Henselmeyer, "Real-time three-phase state estimation for radial distribution networks," in *Power and Energy Society General Meeting, 2012 IEEE*, pp. 1–6, July 2012.

[19] R. Pires, L. Mili, and F. Lemos, "Constrained robust estimation of power system state variables and transformer tap positions under erroneous zero-injections," *Power Systems, IEEE Transactions on*, vol. 29, pp. 1144–1152, May 2014.

[20] M. Shiroie and S. Hosseini, "Observability and estimation of transformer tap setting with minimal pmu placement," in *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE*, pp. 1–4, July 2008.

[21] S. Wang *et al.*, "An improved branch current-based three-phase state estimation algorithm for distribution systems with DGs," in *2012 IEEE Innovative Smart Grid Technologies - Asia (ISGT Asia)*, pp. 1–6, May 2012.

[22] A. Abur and A. Gomez-Exposito, *Power System State Estimation: Theory and Implementation*. CRC press, 2004.

[23] Y. Fukuyama, "State estimation and optimal setting of voltage regulator in distribution systems," in *2001 IEEE Power Engineering Society Winter Meeting*, vol. 2, pp. 930–935, 2001.

[24] P. Teixeira *et al.*, "State estimation of voltage and phase-shift transformer tap settings," in *1991 Conference Proceedings Power Industry Computer Application Conference*, pp. 416–422, May 1991.

[25] G. Korres *et al.*, "Transformer tap setting observability in state estimation," *IEEE Transactions on Power Systems*, vol. 19, pp. 699–706, May 2004.

[26] H. Maalouf *et al.*, "Mixed-integer quadratic programming based rounding technique for power system state estimation with discrete and continuous variables," *Electric Power Components and Systems*, vol. 41, no. 5, pp. 555–567, 2013.

[27] M. Liu *et al.*, "An extended nonlinear primal-dual interior-point algorithm for reactive-power optimization of large-scale power systems with discrete control variables," *IEEE Transactions on Power Systems*, vol. 17, pp. 982–991, Nov 2002.

[28] A. Arefi *et al.*, "Distribution harmonic state estimation based on a modified pso considering parameters uncertainty," in *2011 IEEE Trondheim PowerTech*, pp. 1–7, June 2011.

[29] K. Lee and J. Park, "Application of particle swarm optimization to economic dispatch problem: Advantages and disadvantages," in *2006 IEEE PES Power Systems Conference and Exposition (PSCE '06)*, pp. 188–192, Oct 2006.

[30] P. Angeline, "Using selection to improve particle swarm optimization," in *1998 IEEE International Conference on Evolutionary Computation Proceedings. IEEE World Congress on Computational Intelligence.*, pp. 84–89, May 1998.

[31] S. Naka *et al.*, "A hybrid particle swarm optimization for distribution state estimation," *IEEE Transactions on Power Systems*, vol. 18, pp. 60–68, Feb 2003.

[32] M. R. AlRashidi *et al.*, "Enhanced Particle Swarm Optimization Approach for Solving the Non-Convex Optimal Power Flow," *World Academy of Science, Engineering and Technology*, vol. 4, no. 2, pp. 580 – 584, 2010.

[33] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *1995 IEEE International Conference on Neural Networks Proceedings*, vol. 4, pp. 1942–1948, Nov 1995.

[34] F. van den Bergh, *An analysis of particle swarm optimizers*. PhD thesis, University of Pretoria, November 2001.

[35] Y. Shi and R. Eberhart, "A modified particle swarm optimizer," in *1998 IEEE International Conference on Evolutionary Computation Proceedings. IEEE World Congress on Computational Intelligence.*, pp. 69–73, May 1998.

[36] A.-R. Ghasemi *et al.*, "A Modified Newton-Raphson Algorithm of Three-Phase Power Flow Analysis in Unsymmetrical Distribution Networks with Distributed Generation," *Journal of Basic and Applied Scientific Research*, vol. 9, pp. 36–46, September 2013.

[37] W. H.Kersting, *Distribution system modeling and analysis*. CRC press, 2002.

[38] K. Wang *et al.*, "Research on time-sharing zip load modeling based on linear bp network," in *2013 5th International Conference on Intelligent Human-Machine Systems and Cybernetics (IHMSC)*, vol. 1, pp. 37–41, Aug 2013.

[39] W. Kersting, "Radial distribution test feeders," in *Power Engineering Society Winter Meeting, 2001. IEEE*, vol. 2, pp. 908–912 vol.2, 2001.

[40] W. H. Kersting, "Radial distribution test feeders," tech. rep., IEEE PES Distribution System Analysis Subcommittee, 2000.

[41] K. Samarakoon, J. Wu, J. Ekanayake, and N. Jenkins, "Use of delayed smart meter measurements for distribution state estimation," in *Power and Energy Society General Meeting, 2011 IEEE*, pp. 1–6, July 2011.

[42] H.-M. Jeong, H.-S. Lee, and J.-H. Park, "Application of parallel particle swarm optimization on power system state estimation," in *Transmission Distribution Conference Exposition: Asia and Pacific, 2009*, pp. 1–4, Oct 2009.

Sara Nanchian (S'12) received the M.Sc. degree in Electrical Power Engineering from University of Manchester, in 2008. Currently, she is working towards her Ph.D. in the Department of Electrical and Electronic Engineering, Imperial College London, London, U.K. Her current research interests include state estimation for active distribution network.

Ankur Majumdar (S'12) received the B.E.E.(with honours) degree from Jadavpur University, Calcutta, India, in Electrical Engineering in 2009 and the M.Tech. degree from the Indian Institute of Technology, Delhi, India in Electric Power Systems in 2011. Currently, he is a Research Assistant and working towards his Ph.D. in the Department of Electrical and Electronic Engineering, Imperial College London, London, U.K. His research interests include state estimation, smart grid security and power system analysis.

Bikash. C. Pal (M'00-SM'02-F'13) received the B.E.E.(with honours) degree from Jadavpur University, Calcutta, India, the M.E. degree from the Indian Institute of Science, Bangalore, India, and the Ph.D. degree from the Imperial College London, London, U.K, in 1990, 1992, and 1999, respectively, all in electrical engineering. Currently, he is a Professor in the Department of Electrical and Electronic Engineering, Imperial College London, London, U.K. His current research interests include state estimation, power system dynamics, and Flexible ac transmission system controllers. He is the Editor-in-Chief of IEEE Transactions on Sustainable Energy and a Fellow of the IEEE for his contribution to power system stability and control.