A Hybrid State Estimator For Systems With Limited Number of PMUs

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Abstract—This paper is concerned about effective incorporation of a limited number of phasor measurement unit (PMU) measurements into the state estimation solution in order to enhance the state tracking accuracy and speed. This is particularly important for fast changing system conditions when rapid control action may be necessary based on the onset of voltage instability due to an unexpected event. Under such conditions, SCADA-based state estimators may be too slow in capturing the changes in system state due to the low scan rates of the order of seconds. PMU measurements on the other hand provide refresh rates of milliseconds; however, they may not provide full system observability. In this work, a solution is proposed in order to address the lack of observability by the PMU measurements while maintaining fast tracking capability as well as estimation robustness against pseudo-measurement and PMU errors. Robustness is accomplished via the use of the so-called least absolute value estimator which minimizes L_1 norm of the residuals. Simulation results are provided to illustrate the expected benefits of this implementation on test systems.

Index Terms—Least absolute value (LAV) estimator, phasor measurement units (PMUs), state estimation, synchronized measurements, voltage tracking.

I. INTRODUCTION

BEING one of the most commonly used network applications, static security analysis involves real-time contingency analysis. Thus, an accurate network model and current state of the system need to be continuously tracked by a reliable and accurate state estimator [1]. SCADA measurements are typically received every few seconds and the state estimator is executed every few minutes based on the received measurements and known system topology [2]. Considering the rapidly increasing number of phasor measurement units (PMUs) in power grids, it is inevitable to deploy them in state estimation, which traditionally uses only conventional SCADA measurements, namely voltage magnitude, power injection and flow measurements.

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While the use of phasor measurements is desirable due to their obvious advantages (i.e., increased redundancy, accuracy, and synchronization), their incorporation into existing SCADA-based estimators presents some implementation challenges. One such challenge is due to the significant difference between the refresh rates of SCADA versus PMU measurements. This has long been recognized, and different solutions have been proposed by researchers [3]–[9].

Conventional state estimators use weighted least squares (WLS) estimator, which is computationally efficient, but vulnerable to bad data. Although least absolute value (LAV)-based state estimation is robust and computationally efficient if the measurement set consists of only PMUs [10], it is computationally expensive for today's power grids since the measurement sets consist of both SCADA and PMU measurements. Therefore, there is a need to reconcile these two different categories of measurements and utilize them both fully in order to monitor the system in the best possible way. This paper describes a "hybrid" state estimator, which addresses this need. Here, the word "hybrid" refers to the fact that two different estimation methods are used in handling the PMU and SCADA measurements, which are received at different refresh rates. Hybrid estimators are expected to remain useful during the transition years from an all-SCADA to all-PMU type measurement systems. They can provide benefits over SCADA-only counterparts when monitoring events such as voltage sags, swells or collapses that may be sufficiently slow to monitor by a hybrid estimator but too fast to capture at the current SCADA refresh rates.

PMU measurements are typically updated 30 times a second, while SCADA updates vary from every 2 to 6 s. Fig. 1 shows a magnified view of the period between two consecutive SCADA scans, which are assumed to occur at times "t" and "t+1". Several PMU scans are received in between the two SCADA scans. Most power systems will have only a limited number of PMUs, which is not sufficient to make the entire system observable. Hence, the system will likely be unobservable at instants when only PMU measurements are received. At such instances, an updated version of the most recently scanned SCADA measurements will augment the PMU measurements in order to recover observability. The paper will explain the details of how the SCADA measurements are updated and processed along with the PMU measurements to provide a robust state estimate at each PMU scan.

The proposed "hybrid state estimator" switches between a conventional WLS estimator and a robust weighted LAV (WLAV) estimator depending on the available set of measurements at that instant. This is illustrated in Fig. 1, where at times "t" and "t+1" a conventional WLS-based state estimator,

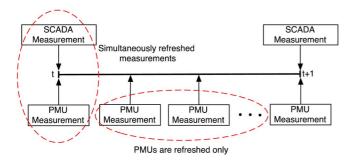


Fig. 1. SCADA and PMU measurements between time instants t and t+1.

which processes both SCADA and PMU measurements, is used to estimate the system state expressed in polar coordinates. Implementation of such an estimator can be easily accomplished by augmenting the SCADA measurement set with the additional PMU measurements [4]. On the other hand, at instances between "t" and t+1", it switches to a WLAV-based state estimator, which is shown to be robust against intentional or unintentional errors in measurements [10]. The WLAV estimator, which uses real and imaginary parts of the bus voltages as system states, is the modified version of LAV estimator, where different weights can be assigned to measurements.

Recently, a WLS method-based estimator, which is designed to incorporate slow-rate SCADA as well as fast-rate PMU measurements, is presented [9]. While the presented setup works well under Gaussian measurement errors, the use of WLS method does not provide any robustness against gross errors, especially in the phasor measurements. Such errors may be unintentional, such as communication or transducer failures, or intentional, such that third parties or hackers may tamper with selective PMUs. While there are post-estimation bad data processing methods to detect, identify, and eliminate bad data when using WLS estimators, these methods come at a high computational cost and cannot be efficiently applied at fast scan rates such as those of PMU measurements.

Preliminary ideas of this hybrid estimator were presented in [11], and a synchro-phasor assisted state estimator (SPASE) was described in [12]. This work investigates the robustness of the estimator under bad-data as well as approximation errors in between SCADA scans and the impact of the number of available PMU measurements on metrics of performance. The proposed method can be used as a complementary diagnosis tool to track the system voltage. It will be possible to monitor voltage collapses, sags, and swells using this hybrid estimator. However, note that this proposed state estimator is not designed to detect faults, instantaneous events, or electrical transients.

This paper is organized as follows. The proposed method is explained in detail in Section II. Simulations are given in Section III, followed by conclusions in Section IV.

II. PROPOSED METHOD

A flowchart of the proposed hybrid estimator is given in Fig. 2. As shown at the top left-hand corner of the flowchart, a conventional WLS estimator is employed at instances when both PMU and SCADA measurements are refreshed simultaneously. Also note that, since SCADA measurements are

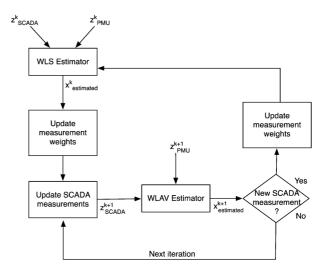


Fig. 2. Flowchart of proposed hybrid estimation method.

not synchronized, they will carry the usual time-skew errors. At subsequent instances between two consecutive SCADA updates, when only the PMU measurements are refreshed, a WLAV estimator is proposed to be utilized. Regretfully, the number of PMUs in most power systems is insufficient to make the entire system observable. Therefore, WLAV estimator will need to utilize a minimum required set of "old" SCADA measurements to maintain observability. Here, WLAV estimator's data interpolation property will be exploited in order to ensure the best possible estimate to be obtained with the refreshed PMU measurements and a minimum number of "old" SCADA measurements.

In compact form, the measurement equations can be written as

$$z = h(x) + e_z \tag{1}$$

where

- n number of buses and m is the number of measurement devices;
- h(.) function that relates the system states and measurements, which includes both linear PMU and nonlinear SCADA measurements;
- z (2 m × 1) measurement vector;
- $x (2n \times 1)$ system state vector;
- e_z (2 m × 1) measurement error vector.

In the presence of SCADA measurements, the state estimation problem is nonlinear and WLAV estimation method is computationally expensive. Therefore, this paper proposes a linearization approach for nonlinear measurements, to increase the performance efficiency of the WLAV estimator. Considering the short duration between two PMU measurement updates, which is 33 ms, and first-order Taylor series expansion of (1) is used as a good approximation as follows:

$$\Delta z = H\Delta x + e \tag{2}$$

where H is the $(2 \text{ m} \times 2n)$ Jacobian matrix

$$\Delta z = z^k - h(x^k)$$
$$\Delta x = x^{k+1} - x^k$$

and e is $(2 \text{ m} \times 1)$ error vector including e_z and errors due to higher order terms in Taylor expansion. Superscript k indicates the time step.

As mentioned previously, SCADA measurements are not updated every time PMU measurements are updated. Since the number of PMUs is not sufficient to guarantee full network observability, at those instances when only PMUs are updated, SCADA measurements should also be employed in state estimation. However, SCADA measurements at those instants are absent. Therefore, either the last updated SCADA measurements should be used or SCADA measurements should be approximately updated. The latter option is used much like in [9], such that SCADA measurements are updated every time the state estimates are updated based on PMU measurements, until a new set of SCADA measurements is received. The update expression is given as follows and illustrated by the "update SCADA measurements" box in Fig. 2:

$$z_{\text{SCADA}}^{k+1} = h_{\text{SCADA}}(x^k) \tag{3}$$

where $z_{\rm SCADA}^{k+1}$ is the vector of updated SCADA measurements at time instant k+1 (note that this update is an approximation since the state vector is updated based on partial information provided by PMUs and old information from SCADA measurements) and $h_{\rm SCADA}(.)$ represents the nonlinear relations between SCADA measurements and system states.

Note that, even though SCADA measurements are updated, their values will still be approximate. In order to minimize the impact of these approximation errors, a WLAV estimator is used in this study. Note that, at this stage, the measurement set will consist of SCADA measurements updated according to (3) and newly received PMU measurements. These are shown as the two inputs to the "WLAV Estimator" box in Fig. 2. WLAV estimation problem can then be set up as follows:

$$\min c^{T} |\bar{r}|$$

$$s.t. \Delta z - H\Delta x = \bar{r}$$
(4)

where c is 2 m \times 1 measurement cost vector and \overline{r} is 2 m \times 1 measurement residual vector.

It is possible to express (4) as an equivalent linear programming (LP) problem by rearranging the equations and defining some new strictly non-negative variables [2], [10]

$$\min c^{T} y$$

$$s.t. My = b$$

$$y \ge \overline{0}$$

$$c^{T} = [Z_{n} W_{m}]$$

$$y = [X_{a} X_{b} U V]^{T}$$

$$M = [H - H I - I]$$

$$b = \Delta z$$
(5)

TABLE I COMPUTATIONAL PERFORMANCES OF WLS- AND WLAV-BASED METHODS

	WLS Based Method	WLAV Based Method
Average Run Time for 14 PMU case	250.5 ms	337.5 ms
Average Run Time for 30 PMU case	262.4 ms	374.8 ms
Average Run Time for 50 PMU case	282.2 ms	363.4 ms
Average Run Time for 70 PMU case	302.7 ms	406.8 ms
Average Run Time for 90 PMU case	309.9 ms	387.7 ms.

where Z_n is the $1 \times 4n$ vector consisting of zeros and W_m is the 1×4 m vector of measurement weights. X_a and X_b are $1 \times 2n$, and U and V are 1×2 m vectors, where

$$\Delta x = X_a^T - X_b^T$$

$$\bar{r} = U^T - V^T.$$
(6)

All entries of the newly defined vectors X_a , X_b , U, and V are constrained to be non-negative in order to create the linear programming formulation without impacting the solution.

Note that W_m is the updated version of measurement weight vector used by WLS estimator. In this study, SCADA measurement weights are divided by 100 to favor the PMU measurements. Note that, instead of 100, any large number can be employed to de-weight the SCADA measurements. The measurement weight update box shown on the left side of Fig. 2 refers to this stage. Weights are changed back to their original values when the SCADA measurements are refreshed, as shown on the right side of Fig. 2.

III. SIMULATIONS AND RESULTS

Here, four case studies will be performed to test the validity of the proposed method. The first one will compare computational performances and accuracies of the proposed hybrid method, which will be called WLAV-based method, and the WLS-based solution [9]. In the second study, performance of the WLAV-based method under bad SCADA measurements will be investigated. Then, a voltage collapse simulation will be performed, and voltage-tracking capability of the proposed method will be presented. Finally, performances of the WLSand WLAV-based methods will be compared under bad PMU measurements. Simulations are carried out using a PC with 4 GB RAM and MAC operating system. LP-based WLAV estimation problem is solved in MATLAB R2010b environment by using GUROBI version 5.0.1. The 140-bus, 233-branch Northeast Power Coordinating Council (NPCC) system [13], [14] is employed for all simulations.

Case 1: This case compares the computational performance and accuracy of the proposed method with those of the WLS based method [9]. Five 10-s-long simulations are performed,

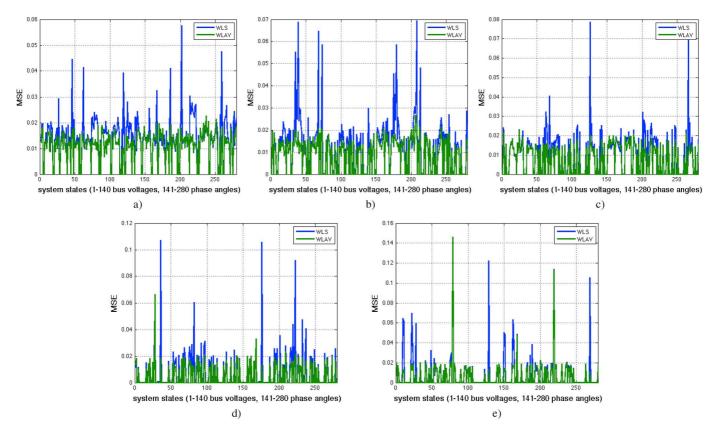


Fig. 3. Comparison of the MSEs for the WLS and WLAV estimators using NPCC system: (a) 14 PMUs, (b) 30 PMUs, (c) 50 PMUs, (d) 70 PMUs, and (e) 90 PMUs.

assuming that SCADA measurements are received every 1 s and PMU measurements are updated 30 times a second. In all simulations, a 140-bus NPCC test system is measured by randomly placed 95 power injection measurements and 205 power flow measurements besides different numbers of branch PMUs (one voltage phasor measurement and one current phasor measurement). Locations of measurements are reassigned in each simulation. Sufficiently high numbers of redundant measurements are placed in the system in order to guarantee network observability and minimize the number of critical measurements. Gaussian errors are added to measurements in all simulations.

In Table I, average durations of each estimation method for different numbers of PMUs are presented. Although the proposed method has a longer solution time, it is still competitive with WLS-based methods. Note that given CPU times are obtained by using an off-the-shelf PC. Large number of measurements used in the study also increases the solution time. Fig. 3 shows the mean square error (MSE) values for each system state for different numbers of PMUs. MSE values are calculated as follows:

$$MSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(x_i^{\text{estimated}} - x_i^{\text{true}}\right)^2}.$$
 (7)

As seen in Fig. 3, MSE values for the WLAV-based method are significantly smaller compared with the WLS-based method, yet their computational performances remain similar. It should be noted that, if the MSE of a state is close to zero, it will imply existence of at least one PMU at the corresponding bus. High

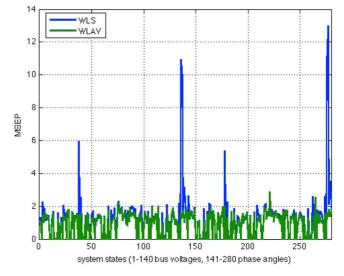


Fig. 4. Performance comparison between WLAV- and WLS-based methods under bad SCADA data.

MSE values for a WLAV-based estimator in Fig. 3(d) and (e), which are the same for WLS estimator, correspond to the buses, which are incident to the critical SCADA measurements. In general, those critical measurements may or may not bias the result of estimation depending on the topology and measurement design of the system.

Case 2: This case considers the performance of the proposed estimator under bad data. Active power injection measurement at bus 126 and the active power flow measurement between

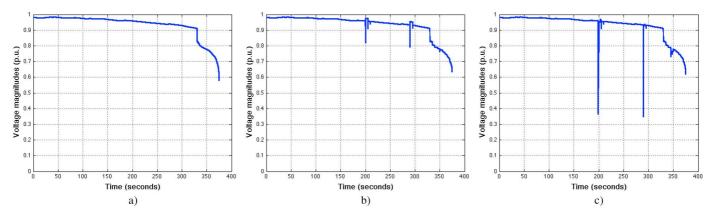


Fig. 5. Comparison of the bus-36 voltage magnitude estimates of the two estimators with the true states. (a) True values. (b) Estimates of WLAV-based estimator. (c) Estimates of WLS-based estimator.

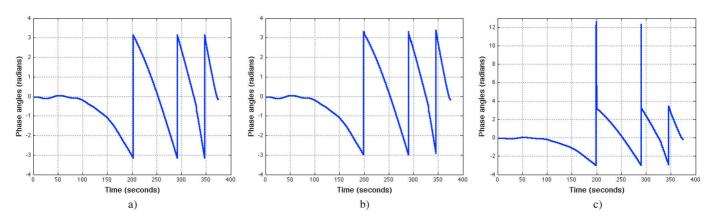


Fig. 6. Comparison of the bus-36 phase angle estimates of the two estimators with the true states. (a) True values. (b) Estimates of WLAV-based estimator. (c) Estimates of WLS-based estimator.

buses 121 and 119 are set to 0 for the entire simulation (10 s). The NPCC system described in Case 1 is employed for the study, and 30 PMUs are placed. Error percentages (MSEP), which are defined as follows, are provided in Fig. 4 for each system state:

$$MSEP = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(100 \times \frac{\left(x_i^{\text{estimated}} - x_i^{\text{true}}\right)}{x_i^{\text{true}}}\right)^2}.$$
 (8)

As seen in Fig. 4, MSEP is very high (10%) for the WLS estimator due to its vulnerability to bad data, while it remains below 2% for the robust WLAV estimator. Fig. 4 presents average performances of the estimators against bad data. When updating states using only PMU measurements, the WLAV-based method effectively rejects gross errors. On the other hand, at time instances when the mixed set of SCADA and PMU measurements are updated, due to the use of the WLS method, highly biased estimates will be obtained. The proposed estimator can disregard the biased estimates artificially, since they will appear once in every second, assuming that SCADA measurements are updated once in every second, while unbiased estimates will be obtained 29 times per second.

Case 3: In this case, a voltage collapse scenario that lasts 375 s is simulated. It is assumed that SCADA measurements are updated once every 5 s. Voltage collapse occurs due to the rapidly increased load at bus 38. Fig. 5(a) shows the time variation of

the voltage magnitude at bus 36, while Fig. 5(b) and (c) presents estimation results for WLAV- and WLS-based methods, respectively. Estimates of bus-36 phase angle are shown in Fig. 6(b) and (c) for WLAV- and WLS-based methods, respectively. Note that bus 36 is not measured by any PMU, while one of its neighbors, bus 33, does have an installed PMU. As evident from Figs. 5 and 6, the WLAV estimator demonstrates superior performance under such rapidly changing operating conditions, due to its robustness.

Sudden changes in bus-36 voltage phase angle measurements, shown in Fig. 6(a), do not occur simultaneously for all buses due to the phase shifts between them. For example, the jump in bus-33 phase angle occurs one time-step ahead of the jumps for bus-36 phase angle. Those instantaneous jumps, which do not occur at the same instant, causes the spikes in estimation results as seen in Figs. 5 and 6. In order to compare the two methods, one can have a closer look at the estimates around time instant 200, as shown in Fig. 7 for voltage magnitudes and Fig. 8 for phase angles. As seen in Fig. 7(a), the WLAV-based method gives biased estimates for a single estimation, which can be filtered easily, while all estimates of WLS-based method are biased between time instants 199 and 200.5, as shown in Fig. 7(b). The same holds true for phase angles, as evident in Fig. 8.

Case 4: Finally, performances of WLS and WLAV based methods are compared once there are bad PMU measurements

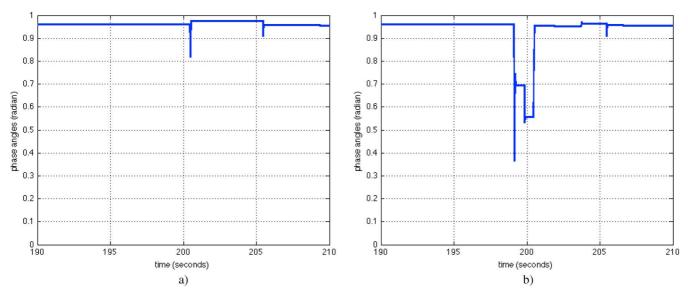


Fig. 7. Comparison of the bus-36 voltage magnitude estimates of the two estimators with the true states between time instants 190 and 210. (a) Estimates of WLAV-based estimator.

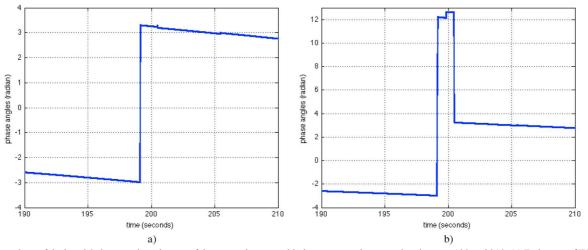


Fig. 8. Comparison of the bus-36 phase angle estimates of the two estimators with the true states between time instants 190 and 210. (a) Estimates of WLAV-based estimator. (b) Estimates of WLS based estimator.

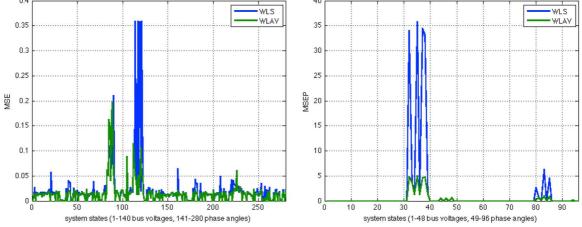


Fig. 9. Performance comparison between WLAV- and WLS-based methods under bad PMU data.

in the measurement set. In addition to the SCADA measurements mentioned in Case 1, 140-bus NPCC system is measured by 30 PMUs. The voltage phasor measurement at bus 121 is set to 0 for the entire simulation period (10 s). Note that bus-121

voltage phasor is measured by another PMU as well. The MSE values of all states and MSEP values of the buses measured by PMUs are presented in Fig. 9(a) and (b), respectively. As seen in Fig. 9, the MSEP values of the proposed method are lower than

those of WLS based method, which indicates higher robustness of the proposed method against bad measurements. However, the estimation results of the proposed WLAV based method are still biased for those buses, which are not directly measured by PMUs. It is noted that as the redundancy of the phasor measurements increases, the proposed state estimation method will become more robust against bad data. Optimal PMU placement for state estimation robustness against bad measurements was discussed earlier in [15].

IV. CONCLUSION

This paper presents a hybrid state estimator that can handle a mixed set of PMU and SCADA measurements. The main contribution of this work is the incorporation of a WLAV-based robust alternative estimator to handle the PMU measurements at their refresh rates.

Since present-day power systems are generally not observable if only PMUs are considered, state estimators have to employ incorrect SCADA measurements to solve the state estimation problem between two SCADA updates. This study shows that WLAV method can be effective in improving the robustness of estimates in between SCADA scans. In the long term, it is expected that power systems will become observable by a redundant set of PMUs, gradually making the bad data analysis after each SCADA scan unnecessary [10].

The proposed method can be used as a complementary diagnosis tool to monitor the system voltage in the case of slow moving voltage problems. It is noted that the proposed method is not fast enough to detect any instantaneous events, such as transients. However, momentary (30 cycles—3 s) and temporary (3 s–1 min) events can be detected, which would be missed by any estimator running at the scan rate of SCADA measurements.

The effectiveness and robustness of this approach is highly related to the number and location of PMUs placed in the system. As more PMUs are placed at strategic locations in the system, more accurate results will be obtained. As a final remark, note that estimated voltages will be less accurate if the stressed bus or area is far away from the majority of the PMUs.

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