Lots of small offspring or fewer larger ones?

Gatto, Casagrandi and De Leo (2002)

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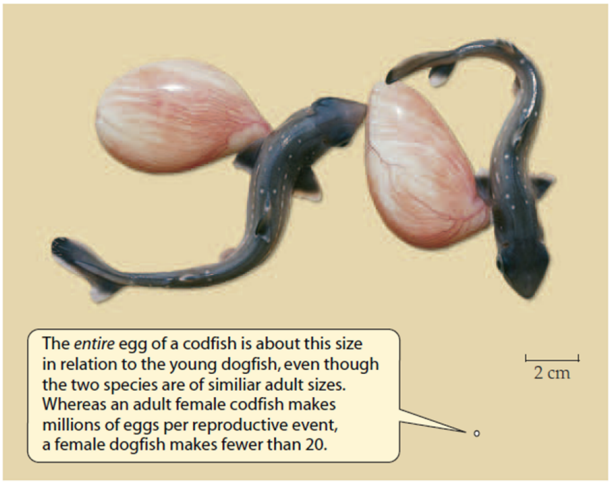
The Problem

A common debate on life history strategies is whether it is better producing many "small" offspring, each with small probability of surviving to the juvenile/adult stage, or fewer "large" offspring, each with more relevant probability of surviving.

Let's try to analyze this problem by using a simple demographic model of a *semelparous*, asexual organism (so we do not have to deal with sex ratio).



Assumptions

We are going to make the following assumptions/definitions:

- δ is the total biomass [grams] invested in reproduction;
- x is the body size (biomass) of each offspring - therefore, the number of offspring produced by an adult individual is: δ/x;
- Offspring with body size x smaller than γ [grams] are too small and die;

- For offspring larger than γ [grams], chance of survival until maturity is an increasing and saturating function of body size x, as described by the following function:

$$\sigma(x) = \alpha \cdot \frac{x - \gamma}{1 + \beta * (x - \gamma)}$$

Let's further assume that natural selection will favor the population with the highest growth rate λ.

Assignment

Assuming that:

- $\delta = 30$ [grams]
- $\alpha = 0.1$ [grams^{-1}]
- $\gamma = 1.00$ [grams]
- $\beta = 0.25$ [grams^{-1}]

compute numerically (and, if you are able, analytically) the optimal body mass of offspring and the corresponding number of offspring

Hint:

- First, draw qualitatively (by hand on paper!) the graph of survival $\sigma(x)$ as a function of offspring size x.
- Then, derive and draw the qualitative graph of the population finite growth rate as a function of body mass x.
- We will then use R to *plot* the graphs of σ and λ as a function of x
- We will find (visually, numerically and analytically) the optimal offspring body weight x_{opt}.
- You will explore which of the 4 parameters (alpha, beta, gamma, delta) has the most influence on x_{opt}.

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