MATH3320 Project Report
Topic: Image Compression

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1. Introduction

In recent years, with the rapid developement in technology, multimedia product of digital information grows increasingly fast, which requires a large memory space and sufficient bandwidth in the storage and transmission process. Therefore, data compression becomes extremely vital for reducing the data redundancy to save more hardware space and transmission bandwidth.

Image compression is the process of removing redundant and irrelevant information, and efficiently encoding or reverting what remains without affecting or degrading its quality. The objective of image compression is to store or transmit data in an efficient form and to reduce the storage quantity as much as possible.

One useful techniques in image compression is to decompose an image into linear combination of elementary images with specific properties. By truncating some less important components in the image decomposition, we can compress an image to reduce the image size and achieve transform coding.

In this paper, we will discuss some useful image decomposition methods, demonstrate the applications of these decomsotion methods for image compression and analyze their advantages, disadvantages and applicability.

2. Image Decomposition methods

2.1 Singular Value Decomposition (SVD)

2.1.1 Definition

Every $m \times n$ image g has a singular value decomposition.
For any $g \in M_{m\times n}$, the singular value decomposition (SVD) of g is a matrix factorization give by

$$g = U\Sigma V^T$$

with $U \in M_{m\times m}$ and $V \in M_{n\times n}$ both unitary, and $\Sigma \in \mathbb{R}^{m\times n}$ is a diagonal matrix with diagonal elements $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$ with r $\leq$ min (m,n).
The diagonal elements are called singular values.
Denote the columns of U by $\{\vec{u_1}, \vec{u_2}, \ldots, \vec{u_m}\}$
Denote the columns of V by $\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\}$
We have $g = U\Sigma V^T = \sum_{i=1}^{r} \sigma_i \vec{u_i}\vec{v_i}^T$, where $\vec{u_i}\vec{v_i}^T$ is called the eigenimages of g under SVD.

2.1.2 Error Computation

For any k with $0 \leq k \leq r$, we define

$$g_k = \sum_{j=1}^{k} \sigma_j \vec{u_j}\vec{v_j}^T$$

where g_k is called a rank-k approximation of g.
This low rank matrix approximation can be applied to image compression.

Here we apply Frobenius norm to compute the error of approximation: Let $f = \sum_{j=1}^{r} \sigma_j \vec{u_j}\vec{v_j}^T$ be the SVD of an $M \times N$ image f. For any k with $k < r$, and $f_k = \sum_{j=1}^{k} \sigma_j \vec{u_j}\vec{v_j}^T$, we have

$$\| f - f_k \|_F^2 = \sum_{j=k+1}^{r} \sigma_j^2$$

2.2 Haar Transform

2.3 Walsh Transform

2.4 Discrete Fourier Transform (DFT)

2.5 Even Discrete Cosine Transform (EDCT)

3. Results