

## 1: Hybrid Gibbs Sampler to estimate Poisson Distribution $\lambda$ (30%)

### Derivation:

For these Poisson distributed random variables (r.v.s) ( $n = 500$ ) with mean parameter  $\lambda$ , unobserved variables are  $\lambda, y_1, y_2, \dots, y_{78}$ , in which  $y_i$ 's denote the r.v.s which are more than or equal to five.

$$\begin{aligned} \text{Prior : } \pi(\lambda) &\propto \frac{1}{\lambda}; \\ P(X, Y|\lambda) &= \prod_{i=1}^{422} \frac{e^{-\lambda} \lambda^{x_i}}{x_i!} \prod_{j=1}^{78} \frac{e^{-\lambda} \lambda^{y_j}}{y_j!} I(y_j \geq 5); \\ P(\lambda|X, Y) &\propto P(X, Y|\lambda)P(\lambda) \propto e^{-n\lambda} \lambda^{\sum_i x_i + \sum_j y_j - 1} \\ P(y_j|X, \lambda) &= \frac{e^{-\lambda} \lambda^{y_j}}{y_j!} I(y_j \geq 5) \propto \frac{\lambda^{y_j}}{y_j!} I(y_j \geq 5) \end{aligned}$$

Then we can know that  $\lambda|X, Y \propto \text{Gamma}(\sum_i x_i + \sum_j y_j, n)$ .

The MH-Step to sample 78 unobserved  $y_j$ 's is

$$\begin{aligned} y_j^* &= \begin{cases} y_j^{(t)} - 1, & \text{with prob. } \frac{1}{3} \\ y_j^{(t)}, & \text{with prob. } \frac{1}{3} \\ y_j^{(t)} + 1, & \text{with prob. } \frac{1}{3} \end{cases} \\ r &= \min \left\{ \frac{[\lambda^{(t+1)}]^{y_j^*} / y_j^*}{[\lambda^{(t+1)}]^{y_j^{(t)}} / y_j^{(t)}} I(y_j^* \geq 5), 1 \right\} \end{aligned}$$

where  $r$  is the accept-reject ratio.

**Result:**  $\hat{\lambda} = 1.674$ .

## 2: Bayesian Inference (30%)

## 3: Hybrid Gibbs Sampler (40%)