1: Hybrid Gibbs Sampler to estimate Poisson Distribution λ (30%)

Derivation:

For these Poisson distributed random variables (r.v.s) ($n = 500$) with mean parameter λ, unobserved variables are $\lambda, y_1, y_2, \ldots, y_{78}$, in which $y's$ denote the r.v.s which are more than or equal to five.

$$
\begin{aligned}
&\text{Prior} : \pi(\lambda) \propto \frac{1}{\lambda};\\
&P(X,Y|\lambda) = \prod_{i=1}^{422} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} \prod_{j=1}^{78} \frac{e^{-\lambda}\lambda^{y_i}}{y_i!} I(y_i \geq 5);\\
&P(\lambda|X,Y) \propto P(X,Y|\lambda)P(\lambda) \propto e^{-n\lambda}\lambda^{\sum_i x_i + \sum_j y_j - 1}\\
&P(y_j|X,\lambda) = \frac{e^{-\lambda}\lambda^{y_i}}{y_i!} I(y_i \geq 5) \propto \frac{\lambda^{y_i}}{y_i!} I(y_i \geq 5)
\end{aligned}
$$

Then we can know that $\lambda|X,Y \propto Gamma(\sum_i x_i + \sum_j y_j, n)$.

The MH-Step to sample 78 unobserved $y's$ is

$$
y_j^* = \begin{cases} y_j^{(t)} - 1, & \text{with prob.} \frac{1}{3} \\ y_j^{(t)}, & \text{with prob.} \frac{1}{3} \\ y_j^{(t)} + 1, & \text{with prob.} \frac{1}{3} \end{cases}
$$

$$
r = min\Big\{ \frac{[\lambda^{(t+1)}]^{y_j^*}/y_j^*}{[\lambda^{(t+1)}]^{y_j^{(t)}}/y_j^{(t)}} I(y_j^* \geq 5), 1 \Big\}
$$

where r is the accept-reject ratio.

Result: $\hat{\lambda} = 1.674$.

2: Gibbs Sampler for Clustering (30%)

3: Hybrid Gibbs Sampler (40%)