Nov 18 - example from ch. 10:

For $[90_4/0_4]_s$ and $[+45_4/-45_4]_s$ laminates subject to temperature load only (cooling from cure to room temp), find the residual stresses in the fiber and matrix directions for each layer.

Material = T300/5208 graphite epoxy.

Hint: the result should make intuitive sense.

a)

90
0
0
90

b)

45
-45
-45
45

$Q_{xx} = 181.8$ GPa $\qquad h_0 = 0.125$ mm

$Q_{xy} = 2.897 \qquad H = 0.002$ m

$Q_{yy} = 10.35 \qquad \alpha_x = 0.2 \times 10^{-6}$ m/mK

$Q_s = 7.17 \qquad \alpha_y = 22.5 \times 10^{-6}$ m/mK

The laminate cools from 177°C to 22°C $\Rightarrow \Delta T = -155$°C

a) $[90_4/0_4]_s$ eq 10.70 $\rightarrow$

$$\begin{Bmatrix} N_1^N \\ N_2^N \\ N_6^N \end{Bmatrix} = \begin{Bmatrix} P_0 H + q_0 V_1(A) \\ P_0 H - q_0 V_1(A) \\ q_0 V_3(A) \end{Bmatrix} \Delta T$$

$$V_1(A) = \sum^{16} \int_{z_{i-1}}^{z_i} \cos 2\theta_i \, dz = 0$$

$$V_3(A) = \sum^{16} \int_{z_{i-1}}^{z_i} \sin 2\theta_i \, dz = 0$$

10.69: $P_0 = \frac{1}{2}(Q_{xx} + Q_{xy})\alpha_x + \frac{1}{2}(Q_{xy} + Q_{yy})\alpha_y$

$$\begin{Bmatrix} N_1^N \\ N_2^N \\ N_6^N \end{Bmatrix} = \begin{Bmatrix} -51924.5193 \\ -51924.5193 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1^N \\ M_2^N \\ M_6^N \end{Bmatrix} = \begin{Bmatrix} q_0 V_1(B) \\ -q_0 V_1(B) \\ q_0 V_3(B) \end{Bmatrix} \Delta T$$

Symmetric so $V_1(B) = 0$ & $V_3(B) = 0$

so

$$\begin{Bmatrix} M_1^N \\ M_2^N \\ M_6^N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

10.8 $$\begin{Bmatrix} e_1^{oN} \\ e_2^{oN} \\ e_6^{oN} \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{66} \end{bmatrix} \begin{Bmatrix} N_1^N \\ N_2^N \\ N_6^N \end{Bmatrix}$$

Symmetry: $\alpha = a = A^{-1}$

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & \cancel{B} \\ \cancel{B} & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k \end{Bmatrix}$$ $\rightarrow N = A\varepsilon \rightarrow \varepsilon = aN$

$\rightarrow M = Dk \rightarrow k = dM$

get U values from appendix C

$A_{11} = 192.16$ Mpa m $\quad a_{11} = 5.209$

$A_{12} = 5.78$ Mpa m $\quad a_{12} = -0.1567$

$A_{22} = 192.16$ Mpa m $\quad a_{22} = 5.209$

$A_{66} = 14.34$ Mpa m $\quad a_{66} =$ don't need.

$$\begin{Bmatrix} e_1^{oN} \\ e_2^{oN} \\ e_6^{oN} \end{Bmatrix} = \begin{Bmatrix} (a_{11}+a_{12})N_1 \\ (a_{22}+a_{12})N_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -2.6324\times10^{-4} \\ -2.6324\times10^{-4} \\ 0 \end{Bmatrix}$$

Strain due to free expansion:

$$\begin{Bmatrix} e_1^N \\ e_2^N \\ e_6^N \end{Bmatrix} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ 0 \end{Bmatrix} \Delta T = \begin{Bmatrix} -3.1\times10^{-5} \\ -3.487\times10^{-5} \\ 0 \end{Bmatrix} \Rightarrow \text{for } 0^\circ$$

$$\begin{Bmatrix} e_1^N \\ e_2^N \\ e_6^N \end{Bmatrix} = \begin{Bmatrix} -3.487\times10^{-5} \\ -3.1\times10^{-5} \\ 0 \end{Bmatrix} \Rightarrow \text{for } 90^\circ$$

Residual strain:

$$\begin{Bmatrix} e_1^R \\ e_2^R \\ e_6^R \end{Bmatrix} = \begin{Bmatrix} e_1^{oN} \\ e_2^{oN} \\ e_6^{oN} \end{Bmatrix} - \begin{Bmatrix} e_1^N \\ e_2^N \\ e_6^N \end{Bmatrix}$$

for 0°

$$\begin{Bmatrix} e_x^R \\ e_y^R \\ e_s^R \end{Bmatrix} = \begin{Bmatrix} -23.134\times10^{-8} \\ 32.2516\times10^{-4} \\ 0 \end{Bmatrix}$$

for 90°

$$\begin{Bmatrix} e_x^R \\ e_y^R \\ e_s^R \end{Bmatrix} = \begin{Bmatrix} -23.134\times10^{-8} \\ 32.2516\times10^{-4} \\ 0 \end{Bmatrix}$$

residual stress: $\sigma = Q\,\varepsilon$

$0° \Rightarrow$

$$\begin{Bmatrix} \sigma_x^R \\ \sigma_y^R \\ \sigma_s^R \end{Bmatrix} = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} \begin{Bmatrix} -23.134\times10^{-5} \\ 32.2516\times10^{-4} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -32.714 \\ 32.7 \\ 0 \end{Bmatrix}$$

for 90° $\begin{Bmatrix} \sigma_x^R \\ \sigma_y^R \\ \sigma_s^R \end{Bmatrix} = \begin{Bmatrix} 32.7 \\ -32.7 \\ 0 \end{Bmatrix}$

for B $[45_4/-45_4]$

$\Rightarrow$ same procedue:

$V_1(A) = \Sigma V \cos 2\theta dz = 0$

$V_3(A) = \Sigma V \sin 2\theta dz = 0$

N_1^N

$N_2^N \Rightarrow$ same as part A

N_6^N

$V_1(B) = V_3(B) = 0 \Rightarrow$ symmetry

M_1^N

$M_2^N = 0$

M_6^N

Hilroy

$a_{11} = 19.955 \ \frac{1}{Gpam}$ $\quad e_1^{ON} = e_2^{ON} = -2.6221 \times 10^{-4}$

$a_{12} = 14.905$

$a_{22} = 19.955$

* Transform off axis to on axis *

Table 3.4 →
$$\begin{Bmatrix} e_x^{ON} \\ e_y^{ON} \\ e_s^{ON} \end{Bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ -1 & 1 & 0 \end{bmatrix} \begin{Bmatrix} -2.6221 \times 10^{-4} \\ -2.6221 \times 10^{-4} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -2.6221 \times 10^{-4} \\ -2.6221 \times 10^{-4} \\ 0 \end{Bmatrix}$$ ⇒ same result for ±45.

$$\begin{Bmatrix} e_x^R \\ e_y^R \\ e_s^R \end{Bmatrix} = \begin{Bmatrix} -2.6221 \times 10^{-4} + 3.1 \times 10^{-5} \\ -2.6221 \times 10^{-4} + 3.4875 \times 10^{-5} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -23.12 \times 10^{-5} \\ 23.25 \times 10^{-4} \\ 0 \end{Bmatrix}$$

residual stress for 45°:

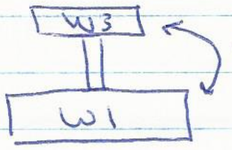
$$\begin{Bmatrix} \sigma_x^R \\ \sigma_y^R \\ \sigma_s^R \end{Bmatrix} = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.34 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} \begin{Bmatrix} e_x^R \\ e_y^R \\ e_s^R \end{Bmatrix} = \begin{Bmatrix} -32.67 \\ 32.7 \\ 0 \end{Bmatrix}$$

for −45° ⇒ get same answer

⇒ same result as for 90°.

Ch 11

possible question (from page 271, eq 11.31)



not symmetric.

⇒ cannot use eq. 11.31,
need to take out factor of 2
and evaluate W_1 & W_3 seperately.

~~flange~~ eq 11.31 ⇒ $\left[2\left(WD_{11}^{(1)}+\left(\frac{h}{2}\right)^2 WA_{11}^{(1)}\right)+\frac{L^3}{12}A_{11}^{(2)}\right]$

$\Downarrow$ comparible to rigidity

11.37: D_x ⇒ comparible to rigidity

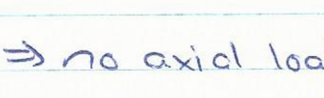
B_x ⇒ <u>ALWAYS</u> zero, no need to evaluate

section 11.3:

11.75 ⇒ $\frac{1}{GJ}$ comparable to $\frac{t\,a_{66}}{J}$

11.84 ⇒ no axial load ⇒ $P = 0$

possible questions



Symmetric: $a_{11} = a_{11}$

$a_{16} = a_{16}$

⇒ 11.84 gives off axis strain

⇒ 11.39 ⇒ <u>$M = D_x k$</u>

Hilroy