

## Midterm Exam 2013

Preview (1) (20%)	Basic Composite Behaviour
(2) (30%)	Basic Laminate Behaviour
(3) (40%)	Stress and Deformation Under Load
(4) (10%)	Short Answer Question
(5) (0%)	Philosophy

If you are not sure about your answer(s), you should provide comments or reasoning.

### 1. (20%) Basic Composite Behaviour

A  $[0_2/\pm 25/0]_s$  laminate (10 layers) with a core of 1cm ( $2z_c=1\text{cm}$ ) is designed using T300/N5208 carbon epoxy composite material with the following properties:

$$\begin{array}{ll}
 E_x = 181 \text{ GPa} & Q_{xx} = 181.81 \text{ GPa} \\
 E_y = 10.3 \text{ GPa} & Q_{yy} = 10.35 \text{ GPa} \\
 E_s = 7.17 \text{ GPa} & Q_{xy} = 2.90 \text{ GPa} \\
 \nu_x = 0.28 & Q_{ss} = 7.17 \text{ GPa} \quad \text{ply thickness, } h_0 = 0.125 \text{ mm}
 \end{array}$$

The resulting  $[A]$  and the  $[a]$  matrix for this laminate are as follows:

$$[A] = \begin{bmatrix} 200.4 & 15.18 & 0 \\ 15.18 & 16.68 & 0 \\ 0 & 0 & 20.53 \end{bmatrix} \text{ MPa-m} \quad [a] = \begin{bmatrix} 5.36 & -4.88 & 0 \\ -4.88 & 64.40 & 0 \\ 0 & 0 & 48.71 \end{bmatrix} \text{ 1/GPa-m}$$

$$[D] = \begin{bmatrix} 5680.4 & 421.2 & 17.1 \\ 421.2 & 468.7 & 4.4 \\ 17.1 & 4.4 & 572.1 \end{bmatrix} \text{ N-m} \quad [d] = \begin{bmatrix} 0.189 & -0.169 & -0.004 \\ -0.169 & 2.286 & -0.013 \\ -0.004 & -0.013 & 1.748 \end{bmatrix} \text{ 1/kN-m}$$

This laminate is used to design a long beam in the 1-direction. We want to compare this design to a solid bar of aluminum to see which has the higher bending stiffness. Each has a width of  $b=2\text{cm}$  and length  $= 2\text{m}$  (in the 1-direction for the composite beam).

The aluminum bar has a rectangular cross section with thickness  $h=1\text{cm}$  and Young's Modulus  $E=73 \text{ GPa}$ .

**Calculate the 2 bending stiffnesses and determine which beam has the highest bending stiffness in the long direction.**

## 2. (30%) Basic Laminate Behaviour

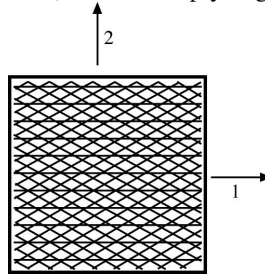
Consider the following six laminates:

$$\begin{array}{ll} \mathbf{A}: [0_5/\pm 30/\pm 45/90]_S & \mathbf{D}: [+30/-30/+30/-30]_S \\ \mathbf{B}: [90/0_2/(\pm 45)_2/0_3]_S & \mathbf{E}: [-30/+30/-30/+30]_S \\ \mathbf{C}: [0_2/90/0_2/90]_S & \mathbf{F}: [+30_2/-30_2]_S \end{array}$$

- Change one ply angle (one number) in laminate **A** such that it will now have an  $A_{22}$  stiffness the same as laminate **B**.
- Reorder the plies of laminate **C** (without changing the angles of those plies) in order to maximize the  $D_{11}$  term.
- Order laminates **D**, **E** and **F**, from highest to lowest, for bending stiffness in the 2-direction.
- Which, of the 6 laminates, are cross-ply laminates.

## 3. (40%) Stress and Deformation Under Load

A composite laminate is provided for you, but the layup is not known, except that it is a symmetric layup. The plate is approximately square, as shown in the diagram (do not infer ply angles from this diagram).



Material: Kevlar/Epoxy

Known material property matrices:

$$\begin{aligned} [\mathbf{A}] &= \begin{bmatrix} 46.54 & 10.00 & 5.57 \\ 10.00 & 40.92 & 11.13 \\ 5.57 & 11.13 & 10.51 \end{bmatrix} \text{ MPa-m} & [\mathbf{a}] &= \begin{bmatrix} 23.25 & -3.27 & -8.85 \\ -3.27 & 34.78 & -35.09 \\ -8.85 & -35.09 & 136.94 \end{bmatrix} \text{ 1/GPa-m} \\ [\mathbf{D}] &= \begin{bmatrix} 8.77 & 1.18 & 0.64 \\ 1.18 & 2.85 & 1.28 \\ 0.64 & 1.28 & 1.25 \end{bmatrix} \text{ N-m} & [\mathbf{d}] &= \begin{bmatrix} 0.121 & -0.041 & -0.020 \\ -0.041 & 0.659 & -0.651 \\ -0.020 & -0.651 & 1.475 \end{bmatrix} \text{ 1/N-m} \end{aligned}$$

3(a) Does this laminate have a balanced layup? State the reason for your answer.

3(b) Given the loading condition,

$$N_1 = 10,000 \quad N_2 = 0 \quad N_6 = 0 \quad \text{N/m}$$

qualitatively describe the resulting deformation of the laminate?  
(similar to the diagrams depicted on page 34, Figure 3.8, of the notes).

3(c) Given the loading condition,

$$N_1 = 0 \quad N_2 = 100 \quad N_6 = 1,000 \quad \text{N/m}$$

which direction will have the highest off-axis axial compressive strain (calculate)?

#### 4. (10%) Short Answer Question

A laminate is subjected to positive bending load in the 1-direction. The stresses are shown here:

Ply Number	$\theta$ (degrees)	On-Axis stresses at ply centroid		
		$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$\sigma_s$ (MPa)
8	?	365.4	5.2	0
7	?	-5.3	20.2	0
6	?	348.6	4.9	0
5	?	-5.1	19.2	0
core				
4	?	5.1	-19.2	0
3	?	-348.6	-4.9	0
2	?	5.3	-20.2	0
1	?	-365.4	-5.2	0

What is the probable laminate layup? Choose between the following 6 choices and state why:

- (a) [0/45/-45/90]<sub>s</sub>
- (b) [0/90/0/90]<sub>s</sub>
- (c) [90/0/90/0]<sub>s</sub>
- (d) [(45/-45)<sub>2</sub>]<sub>s</sub>
- (e) [(-45/45)<sub>2</sub>]<sub>s</sub>
- (f) [0/45/0/-45]<sub>s</sub>

#### (0%) Philosophy

“There's a bit of magic in everything, and some loss to even things out”

“Take a walk on the wild side”

'Life is like Sanskrit read to a pony”

**Lou Reed**, *New York rock singer 1942 to October 27, 2013*

