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# TERM PROJECT

## MECH-530

### Progress Report 5

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November 3, 2014

**OVERVIEW:** This progress report features the addition of a module to analyze various failure criterion, notably Maximum Stress, Quadratic Polynomial and Hashin Criterion. The purpose of this analysis is to determine how close the laminate is to failure as well as the mode of the eventual failure, where applicable.

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# 1 Preliminary Calculations and Design Criterion

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Based on the following given parameters, the applied load vectors were obtained. Thus,  $M_1 = -1159.09$  N while  $M_2 = M_6 = 0$  N, and  $N_1 = 4545.45$  N/m (tensile load) while  $N_2 = N_6 = 0$  N/m.

- $L = 51$  cm (length of skateboard)
- $b = 11$  cm (width of skateboard)
- $P_1 = 1000$  N downwards (arbitrary load)

$$\begin{aligned} M_1 &= \frac{M}{b} \\ &= \frac{P_1 \cdot L}{4 \cdot b} \\ &= \frac{-1000 \cdot 0.51}{4 \cdot 0.11} \\ &= -1159.0909 \text{ N} \end{aligned}$$

$$\begin{aligned} N_1 &= \frac{0.5 \cdot |P_1|}{b} \\ &= \frac{1000}{2 \cdot 0.11} \\ &= 4545.4545 \text{ N/m} \end{aligned}$$

Refer to the tables in Sections A and B for the stresses, strains and failure  $R$  values per top and bottom of each ply.

## 2 Program Output

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Program output to follow...

```
In [1]: run -i MECH530_main.py
```

PLYBOOK (READ FROM EXCEL FILE AND SUMMARIZED HERE)

The laminate is given by the following plybook where the highest ply number '13' indicates the top layer, while the first ply number '1' indicates the bottom layer.

Unique Ply #,	Fiber/Matrix,	Orientation, (degrees)	Thickness (mm)
13	AS4/PEEK	0	0.125
12	AS4/PEEK	0	0.125
11	AS4/PEEK	25	0.125
10	AS4/PEEK	-25	0.125
9	AS4/PEEK	0	0.125
8	AS4/PEEK	0	0.125
7	CORE	0	10.000
6	AS4/PEEK	0	0.125
5	AS4/PEEK	0	0.125
4	AS4/PEEK	-25	0.125
3	AS4/PEEK	25	0.125
2	AS4/PEEK	0	0.125
1	AS4/PEEK	0	0.125

#### PLIES AND THICKNESSES

- Total number of plies in the laminate: 13
- Total thickness of laminate is: 11.500 mm
- The core thickness is 2 Zc = 10.000 mm
- Note that the CORE will be omitted in the following stress and safety factor tables
- Laminate contains 1 Fiber/Matrix combination. The material properties for this combination shall be listed below.

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#### MATERIALS AND MATERIAL PROPERTIES

RESIN/MATRIX 1 of 1: For AS4/PEEK, the given material properties are:

-Stiffness and Strength:

Ex = 134.0000 GPa, Ey = 8.9000 GPa, Es = 5.1000 GPa and nu\_x = 0.2800

Xt = 2130.0000 MPa, Xc = 1100.0000 MPa, Yt = 80.0000 MPa, Yc = 200.0000 MPa and Sc = 160.0000 MPa.

-The 'on-axis' matrices are given by the following:

S\_on =  
[[ 0.0 -0.0021 0.0]  
[ -0.0021 0.1124 0.0]  
[ 0.0 0.0 0.1961]] [1/GPa]

Q\_on =  
[[ 134.7014 2.5050 0.0]  
[ 2.5050 8.9466 0.0]

```
[      0.0      0.0    5.1000]] [GPa]
```

-The linear combinations of the modulus, independent of ply angle are the following:

```
Us_1 =    0.0661 [1/GPa]
Us_2 =   -0.0562 [1/GPa]
Us_3 =   -0.0099 [1/GPa]
Us_4 =   -0.0120 [1/GPa]
Us_5 =    0.1563 [1/GPa]
```

-The linear combinations of the modulus, dependent on ply angle are the following:

```
Uq_1 =    57.0443 GPa
Uq_2 =    62.8774 GPa
Uq_3 =    14.7797 GPa
Uq_4 =    17.2848 GPa
Uq_5 =    19.8797 GPa
```

-The 'Stiffness' [A] and 'Compliance' [a] matrices are given by the following:

```
A =
[[    0.1821    0.0124    0.0]
 [    0.0124    0.0160    0.0]
 [         0.0         0.0    0.0163]] [GN/m]
```

```
a =
[[    5.7979   -4.5109   -0.0]
 [   -4.5109   66.0994   -0.0]
 [         0.0         0.0  61.2628]] [m/GN]
```

-The 'In-Plane Flexural Modulus' [D] and 'In-Plane Flexural Compliance' [d] matrices are given by the following:

```
D =
[[    5.2718    0.3594    0.0130]
 [    0.3594    0.4622    0.0032]
 [    0.0130    0.0032    0.4720]] [kNm]
```

```
d =
[[ 200.3143 -155.6995  -4.4529]
 [-155.6995 2284.5488 -11.2113]
 [ -4.4529 -11.2113 2118.8941]] [1/MNm]
```

-----  
INPUTS:

Would you like to input a resultant applied stress? ON/OFF/NO

OFF

Enter the applied stress resultant vector [N1, N2, N6] [N/m].

N1 = 4545.4545

N2 = 0

N6 = 0

Would you like to input a resultant applied moment? ON/OFF/NO

OFF

Enter the applied moment resultant vector [M1, M2, M6] [N].

M1 = -1159.0909

M2 = 0

M6 = 0

-----  
Curvature K =  
[[ -0.2322      0.1805      0.0052]] [1/m]

OFF-AXIS APPLIED RESULTANTS:

(INPUT) Off-axis Applied stress resultant N =  
[[4545.4545      0.0      0.0]] [N/m]

(INPUT) Off-axis Applied moment resultant M =  
[[-1159.0909      0.0      0.0]] [N]

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FAILURE CRITERION AND ANALYSIS

MAXIMUM STRESS:

-Minimum R = 6.331 and laminate fails in Fiber Compression at TOP of ply 13  
-The load vectors which cause the failure are:

R\*N =  
[[ 28.7795      0.0      0.0]] [kN/m]

R\*M =  
[[ -7.3388      0.0      0.0]] [kN]

QUADRATIC POLYNOMIAL:

-Minimum R = 4.664 and laminate fails at TOP of ply 13  
-The load vectors which cause the failure are:

R\*N =  
[[ 21.2022      0.0      0.0]] [kN/m]

R\*M =  
[[ -5.4066      0.0      0.0]] [kN]

HASHIN CRITERION:

-Minimum R = 6.331 and laminate fails in Fiber Compression at TOP of ply 13  
-The load vectors which cause the failure are:

R\*N =  
[[ 28.7795      0.0      0.0]] [kN/m]

R\*M =  
[[ -7.3388      0.0      0.0]] [kN]  
-----

In [2]: `# stress_df`

In [3]: `# failure_df`

In [11]: `# print stress_df.to_latex()`

In [12]: `# print failure_df.to_latex(float_format=lambda x:"%.3f" % x)`



# A Stress and Strain State throughout Laminate

Position	Ply	Angle (°)	$\epsilon_1$	$\epsilon_2$	$\epsilon_6$	$\epsilon_x$	$\epsilon_y$	$\epsilon_s$	$\sigma_x$ (GPa)	$\sigma_y$ (GPa)	$\sigma_s$ (GPa)
TOP	13	0	-0.001309	0.001017	0.000030	-0.001309	0.001017	0.000030	-0.173735	0.005822	0.000151
BOT	13	0	-0.001280	0.000995	0.000029	-0.001280	0.000995	0.000029	-0.169882	0.005693	0.000148
TOP	12	0	-0.001280	0.000995	0.000029	-0.001280	0.000995	0.000029	-0.169882	0.005693	0.000148
BOT	12	0	-0.001251	0.000972	0.000028	-0.001251	0.000972	0.000028	-0.166029	0.005564	0.000145
TOP	11	25	-0.001251	0.000972	0.000028	-0.000843	0.000564	0.001721	-0.112111	0.002937	0.008777
BOT	11	25	-0.001222	0.000950	0.000028	-0.000823	0.000551	0.001681	-0.109508	0.002868	0.008573
TOP	10	-25	-0.001222	0.000950	0.000028	-0.000844	0.000572	-0.001645	-0.112318	0.003005	-0.008391
BOT	10	-25	-0.001193	0.000927	0.000027	-0.000824	0.000559	-0.001606	-0.109650	0.002934	-0.008192
TOP	9	0	-0.001193	0.000927	0.000027	-0.001193	0.000927	0.000027	-0.158323	0.005306	0.000138
BOT	9	0	-0.001164	0.000904	0.000026	-0.001164	0.000904	0.000026	-0.154470	0.005177	0.000135
TOP	8	0	-0.001164	0.000904	0.000026	-0.001164	0.000904	0.000026	-0.154470	0.005177	0.000135
BOT	8	0	-0.001135	0.000882	0.000026	-0.001135	0.000882	0.000026	-0.150618	0.005047	0.000132
TOP	6	0	0.001187	-0.000923	-0.000026	0.001187	-0.000923	-0.000026	0.157615	-0.005282	-0.000132
BOT	6	0	0.001216	-0.000945	-0.000026	0.001216	-0.000945	-0.000026	0.161468	-0.005411	-0.000135
TOP	5	0	0.001216	-0.000945	-0.000026	0.001216	-0.000945	-0.000026	0.161468	-0.005411	-0.000135
BOT	5	0	0.001245	-0.000968	-0.000027	0.001245	-0.000968	-0.000027	0.165320	-0.005540	-0.000138
TOP	4	-25	0.001245	-0.000968	-0.000027	0.000860	-0.000583	0.001678	0.114435	-0.003061	0.008558
BOT	4	-25	0.001274	-0.000991	-0.000028	0.000880	-0.000597	0.001717	0.117102	-0.003132	0.008757
TOP	3	25	0.001274	-0.000991	-0.000028	0.000859	-0.000575	-0.001753	0.114293	-0.002995	-0.008939
BOT	3	25	0.001303	-0.001013	-0.000028	0.000879	-0.000588	-0.001793	0.116895	-0.003064	-0.009143
TOP	2	0	0.001303	-0.001013	-0.000028	0.001303	-0.001013	-0.000028	0.173026	-0.005799	-0.000145
BOT	2	0	0.001332	-0.001036	-0.000029	0.001332	-0.001036	-0.000029	0.176879	-0.005928	-0.000148
TOP	1	0	0.001332	-0.001036	-0.000029	0.001332	-0.001036	-0.000029	0.176879	-0.005928	-0.000148
BOT	1	0	0.001361	-0.001058	-0.000030	0.001361	-0.001058	-0.000030	0.180732	-0.006057	-0.000151

## B Failure Criterion R Values

			Maximum Stress				Quad Poly		Hashin Criterion				
Position	Ply	Angle	FT	FC	MT	MC	S	(+)	(-)	FT	FC	MT	MC
(°)													
TOP	13	0	0.000	6.331	13.741	0.000	1057.104	4.664	-10.599	0.000	6.331	13.740	0.000
BOT	13	0	0.000	6.475	14.052	0.000	1080.595	4.770	-10.840	0.000	6.475	14.051	0.000
TOP	12	0	0.000	6.475	14.052	0.000	1080.595	4.770	-10.840	0.000	6.475	14.051	0.000
BOT	12	0	0.000	6.625	14.378	0.000	1105.154	4.881	-11.091	0.000	6.625	14.377	0.000
TOP	11	25	0.000	9.812	27.243	0.000	18.230	6.912	-13.631	0.000	9.812	15.151	0.000
BOT	11	25	0.000	10.045	27.890	0.000	18.663	7.077	-13.955	0.000	10.045	15.511	0.000
TOP	10	-25	0.000	9.794	26.620	0.000	19.067	6.931	-13.817	0.000	9.794	15.501	0.000
BOT	10	-25	0.000	10.032	27.267	0.000	19.531	7.099	-14.153	0.000	10.032	15.878	0.000
TOP	9	0	0.000	6.948	15.078	0.000	1157.780	5.119	-11.631	0.000	6.948	15.077	0.000
BOT	9	0	0.000	7.121	15.454	0.000	1186.019	5.246	-11.921	0.000	7.121	15.453	0.000
TOP	8	0	0.000	7.121	15.454	0.000	1186.019	5.246	-11.921	0.000	7.121	15.453	0.000
BOT	8	0	0.000	7.303	15.850	0.000	1215.669	5.380	-12.226	0.000	7.303	15.848	0.000
TOP	6	0	13.514	0.000	0.000	37.863	1215.669	11.683	-5.141	13.513	0.000	0.000	37.836
BOT	6	0	13.192	0.000	0.000	36.959	1186.019	11.405	-5.019	13.191	0.000	0.000	36.934
TOP	5	0	13.192	0.000	0.000	36.959	1186.019	11.405	-5.019	13.191	0.000	0.000	36.934
BOT	5	0	12.884	0.000	0.000	36.098	1157.780	11.139	-4.902	12.883	0.000	0.000	36.073
TOP	4	-25	18.613	0.000	0.000	65.339	18.696	13.557	-6.802	13.191	0.000	0.000	16.892
BOT	4	-25	18.189	0.000	0.000	63.851	18.270	13.248	-6.647	12.890	0.000	0.000	16.507
TOP	3	25	18.636	0.000	0.000	66.769	17.898	13.375	-6.781	12.909	0.000	0.000	16.288
BOT	3	25	18.221	0.000	0.000	65.283	17.500	13.078	-6.630	12.622	0.000	0.000	15.925
TOP	2	0	12.310	0.000	0.000	34.490	1105.154	10.643	-4.684	12.310	0.000	0.000	34.466
BOT	2	0	12.042	0.000	0.000	33.739	1080.595	10.411	-4.581	12.041	0.000	0.000	33.716
TOP	1	0	12.042	0.000	0.000	33.739	1080.595	10.411	-4.581	12.041	0.000	0.000	33.716
BOT	1	0	11.785	0.000	0.000	33.020	1057.104	10.189	-4.484	11.785	0.000	0.000	32.997