-High humidity caused corrosion around the connecting bolts.
-Fatigue
- crack ripped airplane appart

Chapter 9

Fatigue Failure

9.1 Fatigue Failure (General Considerations)

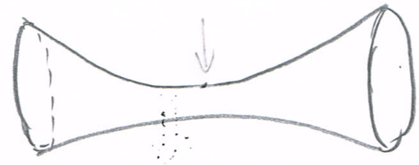
Engineers have long recognized that subjecting a metallic member to a large number of cycles of stress will produce fracture of the member by much smaller stresses than those associated with static failure of the member. Poncelet in 1839 was probably the first to introduce the term *fatigue* and discuss the property by which materials resist repeated cycles of stress. Modern investigators would probably use a term such as *progressive fracture* to replace the term *fatigue*.

Between 1852 and 1869, A. Wohler, a German engineer, designed the first repeated-load testing machines. Wohler discovered that:

1. The number of cycles of stress rather than elapsed time of testing is primary.
   (iron and steel alloys).
2. Ferrous materials stressed below a certain limiting value can withstand an indefinite number of cycles of stress without fracture.

mostly experimental or empirical

The literature of fatigue studies is voluminous. These studies have proceeded along two lines, fundamental research seeking to explain the phenomenon and empirical investigations to provide information for practical design and analysis. Recent progress in studies of crack propagation and use of the electron microscope have enhanced fundamental research efforts, but empirical methods still continue to be widely employed for engineering design. Fatigue fractures originate at points where stress concentrations occur, such as fillets, keyways, holes, and screw threads, or internal inclusions or defects in the material. Cracks begin at these points of stress concentrations and then propagate through the cross section until the remaining uncracked regions are insufficient to resist the applied forces and fracture occurs suddenly.



4 point bending

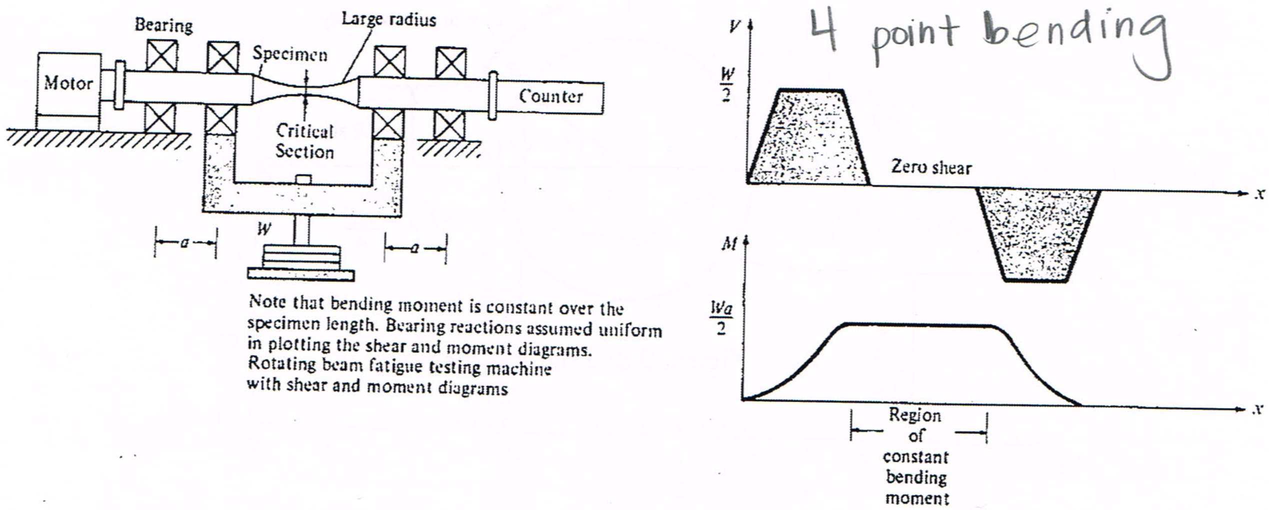


Figure 9.2: Rotating beam fatigue testing machine.

For a given test, the choice of the weight W and the set spacing a of the machine bearings determines the bending moment applied to the specimen as it rotates at a constant angular velocity ω. As shown in Fig. 9.3, the loads are applied in a vertical plane, and for a given point P on the specimen circumference of distance v measured perpendicular to the neutral axis (N.A., which coincides with the u axis) varies with time. This is the one quantity that is time-variant in the flexure formula.[1]

$$S = \frac{M_u v}{I_u} = \frac{Wa}{2}\,\frac{R \sin \omega t}{\pi R^4/4}$$

$$S = \frac{2Wa}{\pi R^3} \sin \omega t \qquad (9.1)$$

$$S = S_r \sin \omega t \qquad (9.2)$$

where S_r is the amplitude given by $\theta = \omega t$

$$S_r = \frac{2Wa}{\pi R^3}$$

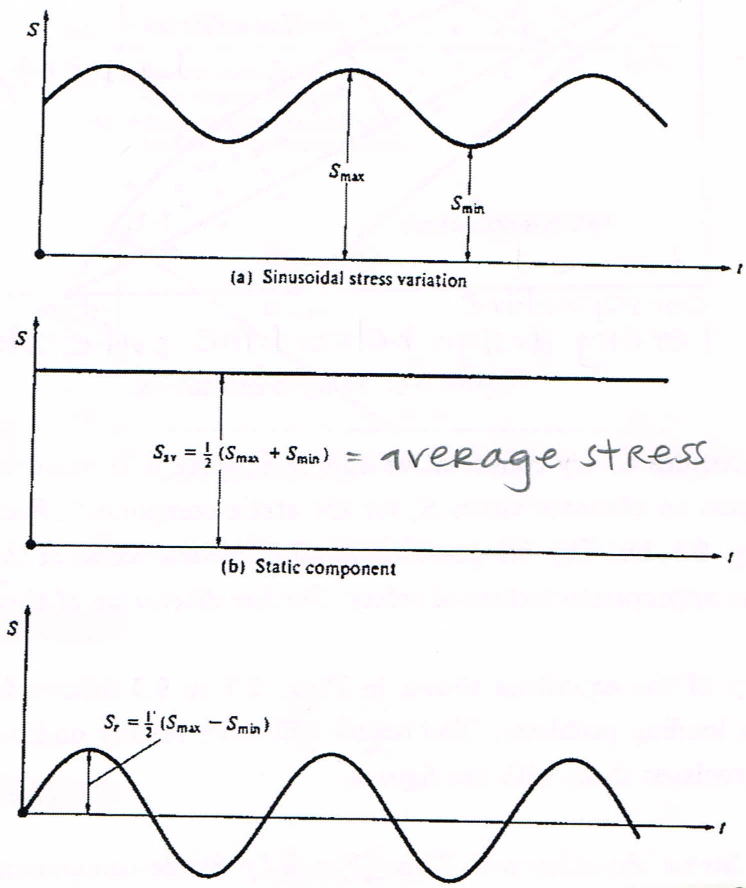
R = −1 means fully reversed cycle (worst case scenario) → apply F in tension undo, then F in compression

Equation 9.2 expresses the fact that the bending stress S at any point P on the specimen circumference will vary sinusoidally with time as shown in Fig. 9.4. This variation between

[1] S will be used as a generic symbol for stress in this section, since repeated referenced is made to the $S-N$ diagram in the huge volume of information available on fatigue.

reversed but not fully. Apply F tension then go back to

Q: know the Relation between these 3 curves.



(c) Complete reversal → means average stress = 0

Figure 9.7: Stress vs time.

av. stress > 0 → tension
av. stress < 0 → compression

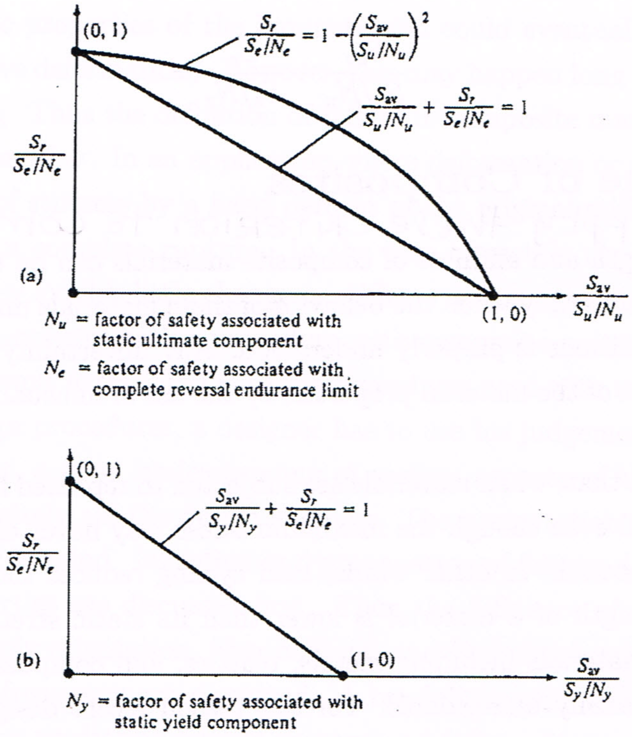


Figure 9.9: Normalized fatigue life curves.

only important eq's in this Ch. *

Goodman straight line:

$$\frac{S_{av}}{S_u} + \frac{S_r}{S_e} = 1 \tag{9.6}$$

Soderberg straight line:

$$\frac{S_{av}}{S_y} + \frac{S_r}{S_e} = 1 \tag{9.7}$$

GERBER, GOODMAN, AND SODERBERG EQUATIONS WITH SAFETY FACTORS (Fig. 9.9). 2 safety factors: ① for average N_u ② for reversal N_e.

Gerber parabola:

$$\frac{S_r}{S_e/N_e} = 1 - \left[\frac{S_{av}}{S_u/N_u}\right]^2 \tag{9.8}$$

Goodman straight line:

$$\frac{S_{av}}{S_u/N_u} + \frac{S_r}{S_e/N_e} = 1 \tag{9.9}$$

Soderberg straight line:

a lowering of elastic properties of the laminate and could eventually lead to its structural failure (e.g., excessive deformation). However, this may happen long before the laminate is in danger of fracturing. Thus the definition of failure in composite materials may change from one application to another. In an application where deformation or a change in stiffness has to be limited, loss of stiffness by a fixed percent of the original stiffness may be the failure criterion rather than complete rupture. In the case of metals the two criteria practically coincide because they exhibit little change in stiffness unless cracking is extensive. For these obvious reasons a successful design procedure with composite materials for fatigue applications cannot be a simple extrapolation of the procedures used with metals. In the absence of well-developed design procedures, a designer has to use his judgement with a proper degree of caution. However, a good understanding of various aspects of the fatigue behaviour of composites will definitely aid the design engineer. The presentation in this section has been made with this view in mind. Initiation and propagation of fatigue damage and its influence on composite properties are discussed first. Then the influence of material variables such as matrix material, ply orientation, fiber content, and fiber finish and testing variables such as mean stress and frequency are discussed. A brief discussion on the trend in developing empirical relations for predicting fatigue damage and fatigue life is also presented.

→ cracks start at bonding area between fiber/matrix.

9.2.1 Fatigue Damage and Its Influence on Composite Properties

There have been several studies on mechanism of damage initiation and propagation during fatigue of composite laminates. It has been established that the damage first initiates by the separation of fibers from the matrix (called *debonding*) in the fiber-rich regions of the plies in which the fibers lie perpendicular or at a large angle to the loading direction. Large stress and strain concentrations at the fiber-matrix interface are responsible for the initiation of these cracks. After initiation the crack usually propagates between fibers primarily along the fiber-matrix interface. A typical cross-ply crack is shown in Fig. 9.10(a). The crack is generally perpendicular to the direction of load and extends over the entire width of the ply. The cross-ply cracks can appear during the first cycle of loading provided that the applied stress exceeds the local ply strength that might happen at applied stresses as low as 20% of the ultimate stress dependent on the laminate construction. The number of cross-ply cracks increases with either the number of cycles or an increase in the stress level. Multiple crack formulations in the cross-plies are shown in Fig. 9.10b.

cross ply cracks occur ⊥ to the fiber and crack ⊥ to loading direction. ↓ can appear in first cycle of loading

The initial damage in randomly oriented fibrous composites commences in a similar manner. In a tensile test on a thin laminate made from chopped-strand mat, the first signs of damage have been observed at about 30% of the expected ultimate tensile strength.

↳ when you apply 30-40% force of your final failure load.

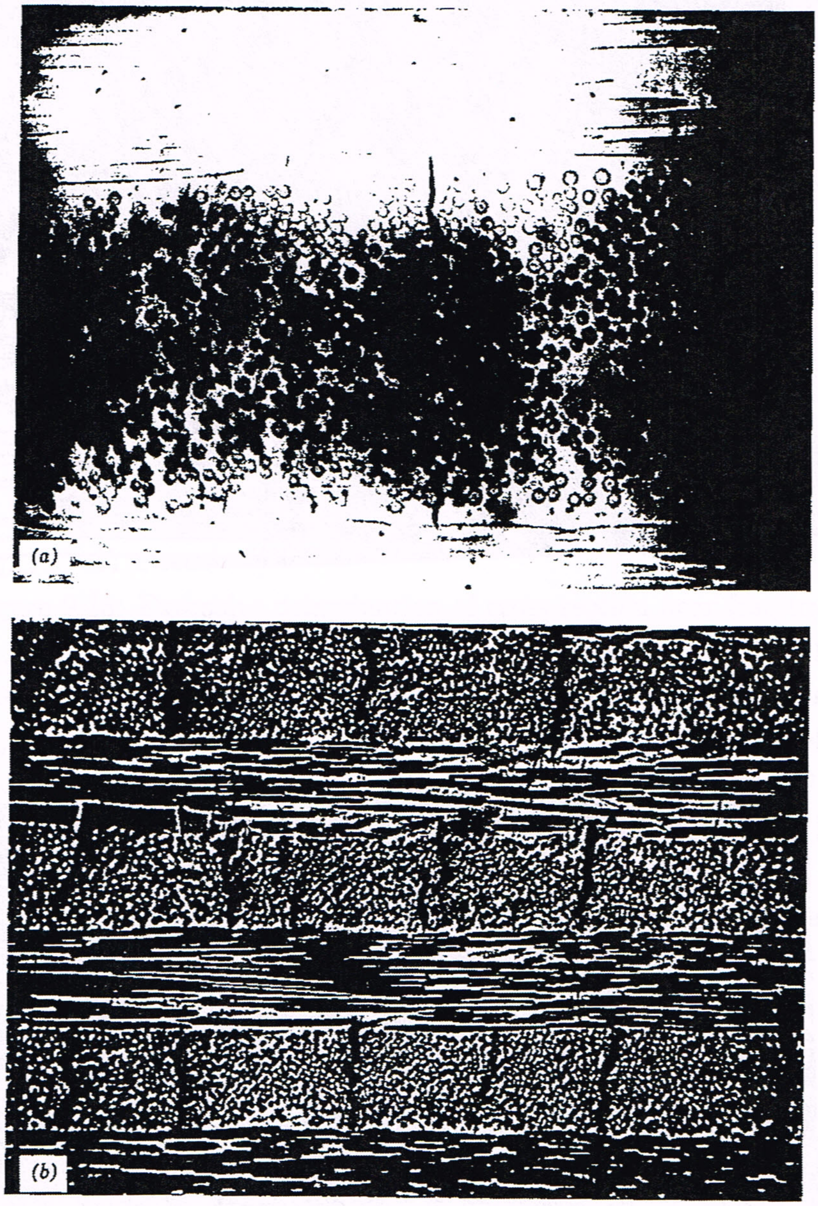


Figure 9.10: Fatigue failure initiation: (a) single cross-ply crack and (b) multiple crack formation in cross plies.

cracks then propagates along fiber face.

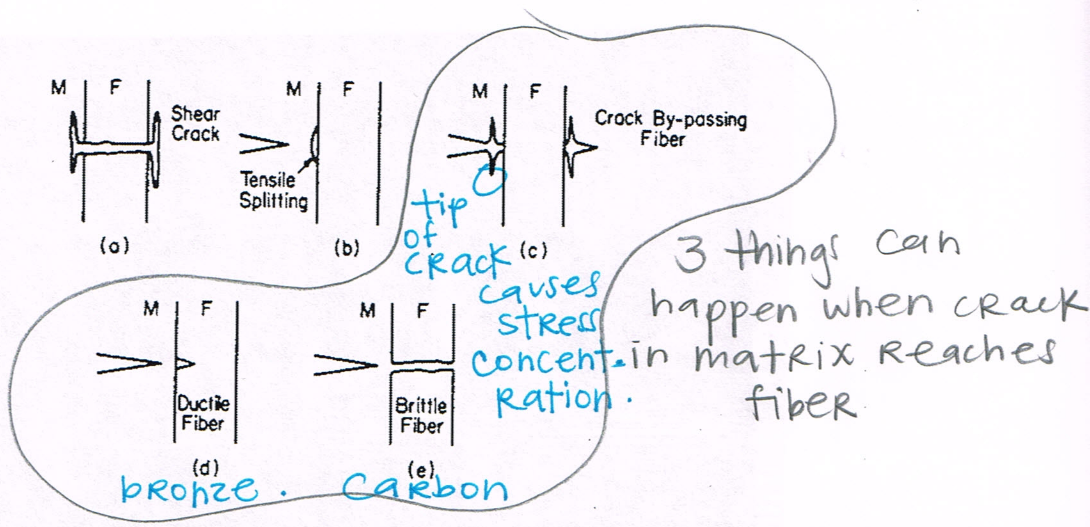


Figure 9.14: Modes of fatigue crack growth in fiber-reinforced materials: (a) shear crack initiation at fiber break, (b) tensile splitting of interface ahead of matrix crack, (c) matrix crack bypassing strong fiber, (d) crack initiation in ductile fiber ahead of matrix crack, and (e) fracture of brittle fiber ahead of matrix crack.

cracks in a specified area. Weight gain from water immersion was employed as a measure of internal damage by early investigators, such as McGarry, but is not always found suitable. Nondestructive inspection techniques are now being developed for detection of fatigue damage. These techniques include ultrasonics, holographic interferometry, and X-ray radiography. Changes in structural properties such as static or dynamic modulus and temperature rise during fatigue loading are also considered indicative of internal damage. However, no clear quantitative correlation between structural properties and internal damage measurements has as yet been established.

Internal cracking results in the lowering of the stiffness and strength of composite materials. Broutman and Sahu had related the changes in residual strength and modulus to the development of cracks in a glass-epoxy cross-ply material (Fig. 9.15). The residual strength and stiffness decrease with the increasing crack density. It has also been pointed out that the stress-strain curve of a virgin cross-ply material can be approximated by two straight lines giving two elastic moduli for the material. The two moduli are referred to as *primary* and *secondary moduli*. The material exhibits a higher (primary) modulus at the beginning of the test because there are no cracks present, and both the longitudinal plies as well as the cross-plies fully contribute to the stiffness of the composite. As the load increases, the cross-ply cracks appear and thus the contribution of the cross-plies to the composite stiffness

② measure change in stiffness

cross ply

unidirectional 0° plies

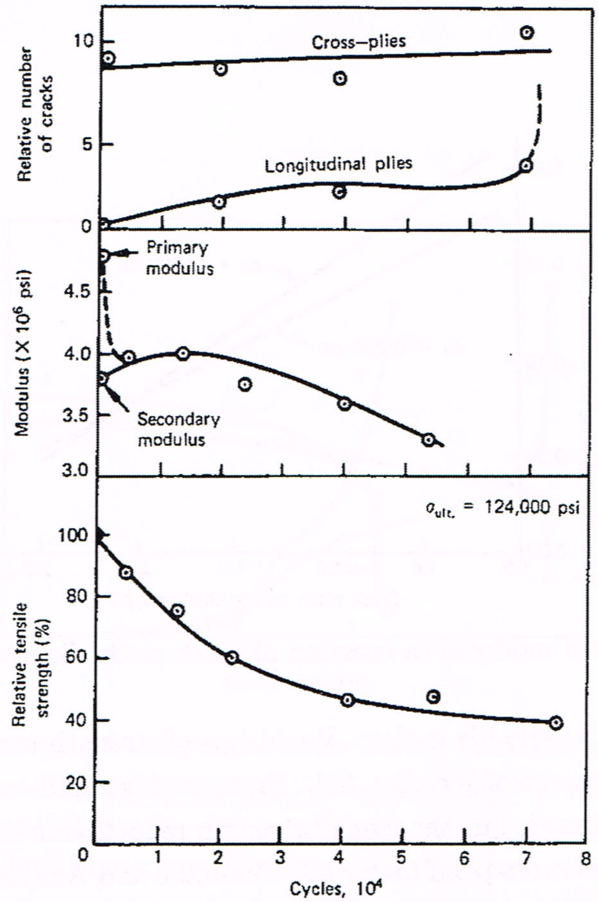


Figure 9.15: Number of cracks and loss of strength and modulus during fatigue. Source: Broutman and Sahu.

decreases, causing a reduction in modulus. In fatigue tests the modulus decreases first in the presence of cross-ply cracks and then longitudinal-ply cracks and delamination cracks. Therefore, with fatigue exposure, the stress-strain curve of the material becomes linear with a modulus close to the secondary modulus of the virgin material. The modulus may become less than the secondary modulus of the material when the longitudinal-ply cracks and delamination cracks develop due to fatigue loading. Dally and Agarwal developed a quantitative relationship between modulus change and crack density for an E-glass-epoxy cross-ply laminate. This relationship is shown in Fig. 9.16, in which the *crack pitch* is defined as the average distance between two consecutive cracks in the cross-plies.

There is a gradual decrease in static strength of the material as it is subjected to an increasing number of cycles at a given stress level. It is obvious from Fig. 9.15 that much of the strength reduction occurs in the first 25% of the fatigue life beyond which the rate of decrease in static strength is reduced until the fatigue life is reached and failure occurs. Once again, the reason for the initial loss of strength is the failure of the cross-plies. Development of longitudinal-ply cracks and delamination cracks is slow, and hence the loss of strength in heat degrades resin properties and matrix stiffness. Cycling at higher ω accelerates this process.

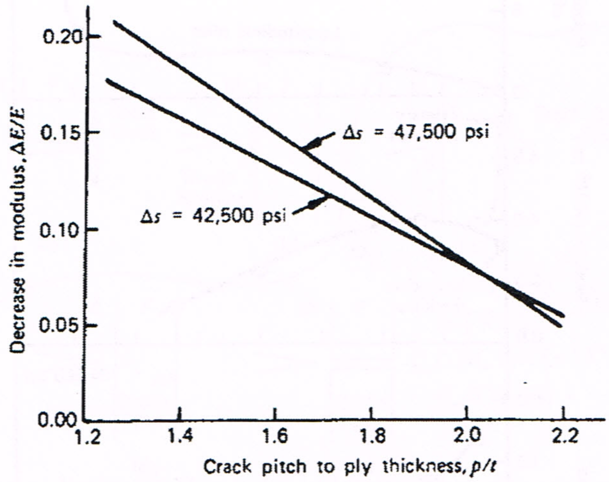


Figure 9.16: Loss of modulus as function of crack pitch. Source: Dally and Agarwal.

the later part of the fatigue life is slow. Rapid loss of strength occurs in the last few cycles of fatigue life when the stronger plies fail. Prior to this rapid loss of strength, individual plies do become weakened, but the overall strength reduction is slow. The curves shown in Fig. 9.15 are typical of cross-plied materials. Tanimoto and Amijima have studied fatigue of glass-cloth-reinforced polyester resins. Their results are very similar to those of Broutman and Sahu, as shown in Fig. 9.15. In addition, they have reported that the residual strength in interlaminar shear follows the same trend as the residual tensile strength. Hahn and Kim, while studying fatigue of glass-epoxy angle-ply laminates, observed that the secant modulus of the material decreases with exposure to fatigue loading and indicated that the decrease in secant modulus is related to internal damage.

ie: prevent temp. rise and extend fatigue life.

Besides internal cracking damage, a rise in temperature also causes a decrease in properties of the material. Dally and Broutman observed that a significant rise in temperature takes place during the fatigue of a cross-ply material, particularly when the frequency is high. Cessna et al. performed constant-deflection flexural tests on glass-reinforced polypropylene and monitored the load decay (proportional to modulus decay) with cycles (Fig. 9.17). They also monitored the temperature rise caused by viscoelastic energy dissipation, which is common for polymer-matrix composites. In addition to indicating progressive fatigue damage, the temperature rise also helps to weaken the material and shorten its fatigue life. By cooling their specimens to maintain isothermal conditions. Cessna et al. were able to extend both the cycles to onset of stiffness change and the fracture life by one order of magnitude.

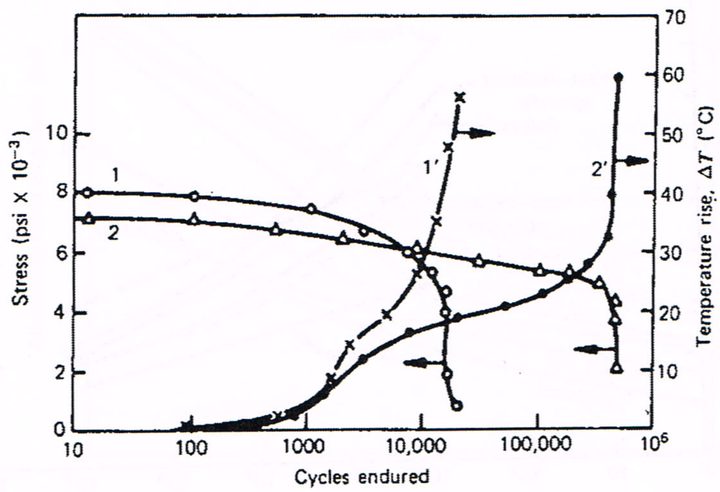


Figure 9.17: Load decay and temperature rise during constant-deflection flexural fatigue. Source: Cessna et al. → Aerospace grade materials are more resistant to temperature changes and fatigue.

9.2.2 Factors Influencing Fatigue Behavior of Composites

Results of fatigue tests are typically presented as a plot of applied stress (S) against number (N) of cycles to failure. This graph is called an $S - N$ *curve.* The ordinate is generally the stress or strain amplitude or the maximum stress or strain in a cycle and is plotted on a linear scale. The abscissa is the number of cycles to failure for a fixed stress cycle and is plotted on a logarithmic scale. Complete separation of the specimen has been taken as the criterion for failure by most investigators. However, another approach is to record fatigue data as loss of stiffness against number of cycles and to present curves of stress versus number of cycles for fixed percent changes in stiffness. The $S - N$ curves for all materials including metals, polymers, and composites have a negative slope. That is, the number of cycles to failure (or the fatigue life) increases as the stress decreases. The exact shape of the curve differs from material to material. For composites, the curve is influenced by various material and testing variables, as follows: (1) matrix material (type of resin), (2) ply orientation, (3) volume fraction of reinforcement, (4) interface properties, (5) type of loading, (6) mean stress, (7) frequency, and (8) environment. The first four factors are material variables, whereas the remaining are test variables.

Boller has investigated the effect of matrix materials on the fatigue strengths of glass-reinforced plastic laminates. The $S - N$ curves are shown in Fig. 9.18, and all the resins were reinforced with style 181 E-glass fabric. This fabric produced a balanced lamina such

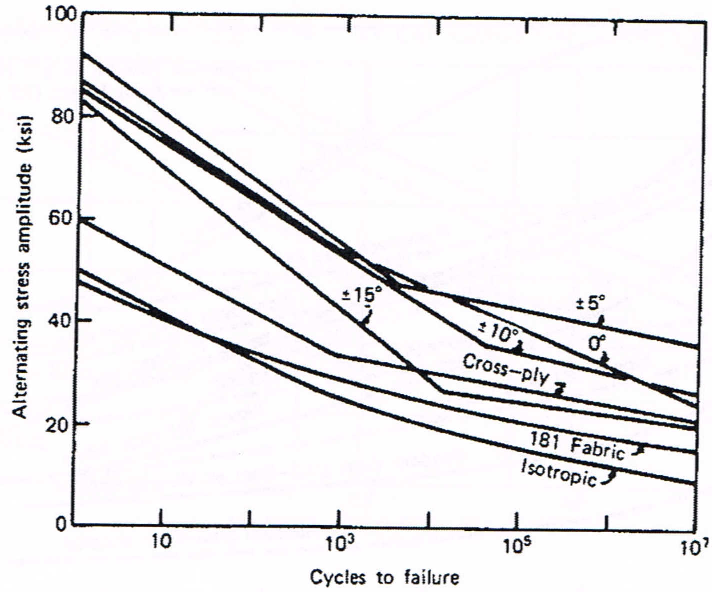
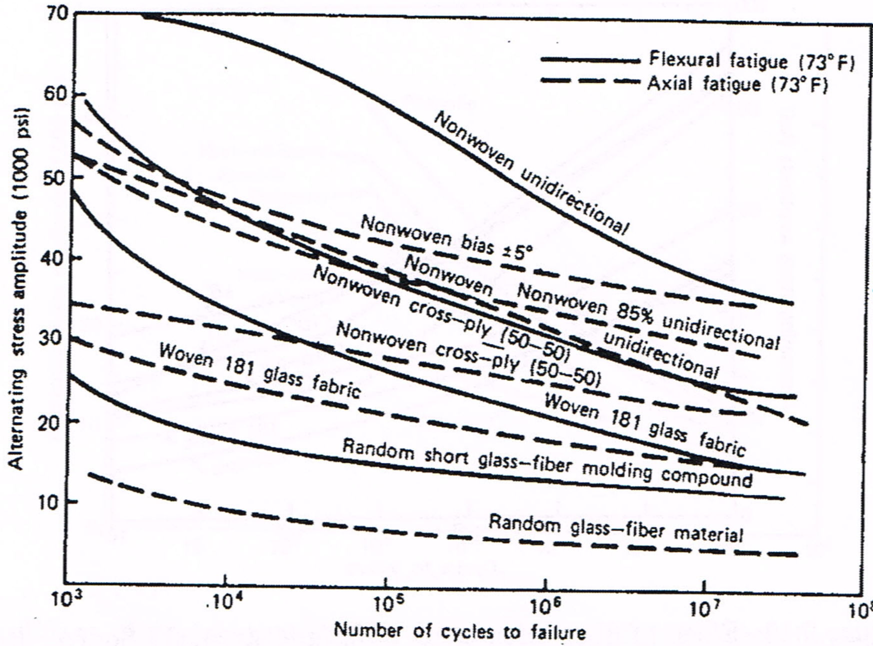


Figure 9.19: Effect of fiber orientation of fatigue strength. Source: Boller.

to woven materials in fatigue because fibers in nonwoven materials are straight and parallel and do not get crimped as in the fabric construction. Thus nonwoven materials possess optimum static and fatigue properties.

Besides the orientation of plies, the stacking sequence also influences the fatigue life. Foye and Baker observed that when positions of the plies in a $[\pm 15/\pm 45]_s$ laminate were changed, a difference in fatigue strength of about 25,000 psi occurred. This was explained by Pagano and Pipes through analysis of interlaminar stresses. They showed that the interlaminar stress normal to the laminate changes from tension to compression by changing and stacking sequence, and this accounts for the difference in fatigue load capability. Delamination was observed to occur in the specimens that developed tensile interlaminar stress. Whitney has made similar observations on the influence of stacking sequence on the fatigue strength and failure mode of composite laminates.

Amijima and Tanimoto have studied the influence of glass content on fatigue properties of laminated glass-fiber composite materials. Their results (Figs. 9.21 and 9.22) clearly show that the fatigue strength of glass-cloth reinforced polyester resin increases with increasing glass content in both axial fatigue (V_f range 29.3 - 54.2%) and rotating bending fatigue (V_f range 11.8 - 30.4%). This increase in fatigue strength occurs with the increase in static strength of the composite as a result of increased fiber volume fraction. Earlier studies by Boller and Davis et al. indicated that the fatigue strength is not related to fiber contents as it varies from 63 to 80% in a glass-cloth-reinforced epoxy. It appears that an optimum



woven fabrics has shorter fatigue life (because woven has a slightly induced curvature

Figure 9.20: $S-N$ curves for different laminate construction. Source: Davis et al.

fatigue strength may be achieved with 70% by weight of glass fibers in the case of fabric laminates.

The effect of interfacial bond strength between the matrix and the reinforcement on the fatigue strength of composites has been studied by Hofer et al. They studied the fatigue behavior of glass-fabric composites having four different finishes, including an untreated surface and surfaces treated with Volan A, A-1100, and S-550 finishes (organosilane coupling agents). The untreated glass exhibited the highest fatigue strength in a dry environment, but it was also the most severely affected in a humid environment. As a result, all fabrics tested showed a similar resistance to fatigue when tested in a humid environment. Thus when laminates are fatigued in real environments, it is difficult to demonstrate the effectiveness of various surface treatments. This is partially a result of the stress system, which is usually such that the composite properties are fiber dominant and not greatly dependent on the interface strength.

Like static strengths, fatigue strengths of composites in longitudinal tension and shear are quite independent. Shear fatigue has been recently studied by many investigators. Pipes's results on a unidirectional glass-epoxy composite are shown in Fig. 9.23. These results indicate that the fatigue strength of glass-epoxy interlaminar shear is superior to that in longitudinal tension when compared to the ultimate static strength. This trend of the increasing V_0 increases fatigue life

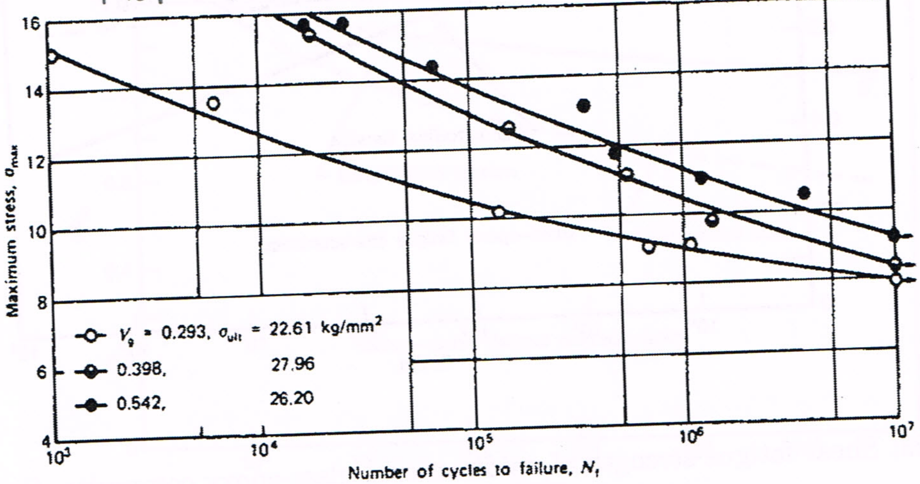


Figure 9.21: Influence of glass content on axial fatigue strength. Source: Tanimoto and Amijima.

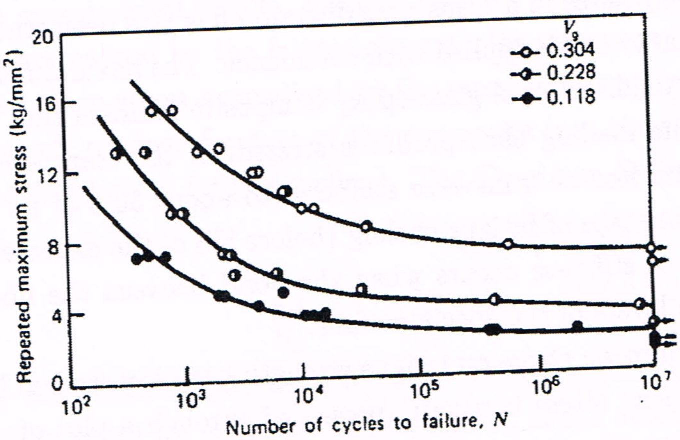


Figure 9.22:. Influence of glass content on rotating bending fatigue strength. Source: Amijima and Tanimoto.

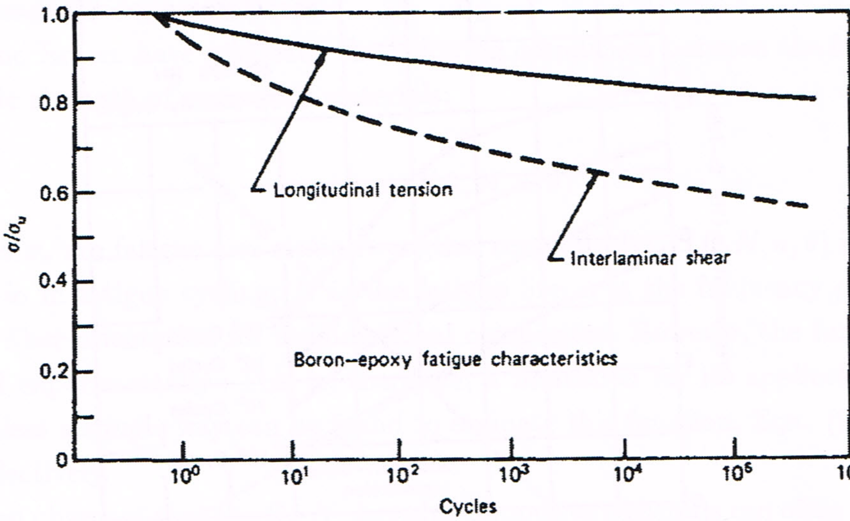


Figure 9.24: Shear fatigue strength of unidirectional boron-epoxy composite. Source: Pipes.

stress, cyclic life decreases as the stress amplitude increases. The influence of mean stress on the fatigue behavior of composites is similar to that of metallic materials. It is observed that like metals, the stress amplitude in composites is related to the mean stress through a linear relationship. The Goodman-Boller relationship, which is usually found to be in good agreement with the experimental results, assumes that for a fixed fatigue life, the decrease in stress amplitude normalized by the fatigue strength (or stress amplitude) at zero mean stress is equal to the mean stress normalized by the stress-rupture strength, defined as the constant stress that will produce fracture of the composite in a time corresponding to the fatigue life (i.e., the duration of fatigue cycling). The Goodman-Boller relationship can be written as

$$\frac{S_A}{S_E} = 1 - \frac{S_M}{S_c} \tag{9.11}$$

not widely used for fatigue

where S_A and S_M are stress amplitude and mean stress, respectively, S_E is the fatigue strength at zero mean stress for equal cyclic life, and S_c is the stress-rupture strength for the time corresponding to the cyclic. Limited experimental results indicate that at elevated temperatures the influence of mean stress may be different from that predicted by the Goodman-Boller relationship.

Dally and Broutman have shown that the frequency of stress cycling significantly influences the temperature rise of specimens during fatigue testing. However, the fatigue life of both cross-ply and isotropic materials is only modestly influenced by frequency effects.

intensity factor. In this case also the fatigue strength has not been related to the fatigue life that can be used for design purposes.

Hashin and Rotem have suggested the following correlation between the fatigue strength and the static strength of composite materials:

$$\sigma_f = \sigma_s\, f(R, N, n, \theta) \tag{9.12}$$

where σ_f and σ_s are fatigue and static strengths, respectively: $f(R, N, n, \theta)$ is a function of R, stress ratio in fatigue cycling; N is the fatigue life; n is the frequency of load cycling; and θ is the fiber orientation for unidirectional composites. However, the function f has to be evaluated experimentally. This is, therefore, a limitation for its applicability to design analysis. Unless a simple way can be found to evaluate this function, Eqn. (9.12) cannot be used very effectively.

It has been observed that the $S-N$ curves of composite materials can often be represented by straight lines with the equation

need to be characterized for diff. materials and layups.

$$\frac{\Delta S}{\sigma_u} = m \log N + b \tag{9.13}$$

where ΔS is the stress range, σ_u is the ultimate tensile strength, and m and b are material constants. Some investigations show that the values of m and b may be close to 0.1 and 1.0, respectively. However, there are not sufficient experimental data to suggest that these values may be used with confidence for design applications.

Another useful relationship to represent fatigue data is a power law:

$$N^k\, \Delta\epsilon = c \tag{9.14}$$

also highly dependent on material type and manufacturing

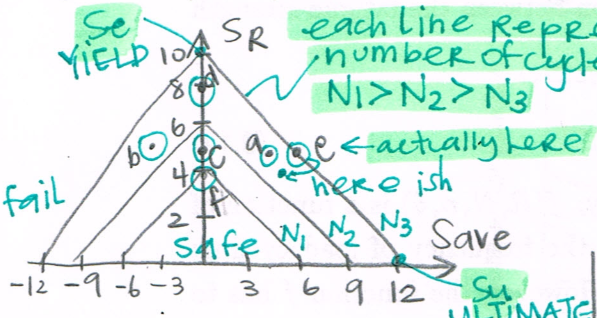
where $\Delta\epsilon$ is the strain range and k and c are material constants. This equation has been found to be very useful for predicting fatigue life of metallic materials. For most metals, k is known to vary from 0.5 to 0.6, and the value of c is related to the ductility of the material. In this manner it is possible to predict the fatigue behavior of metals from their static properties. Results of Agarwal and Dally and Hahn and Kim do follow Eqn. (9.14), but the constant k and c are not universal constants for composite materials. Further, it has not yet been possible to relate these constants to the static properties of materials. Such a correlation may be evolved in the future so as to make Eqn. (9.14) and also Eqn. (9.13) very useful in design procedures.

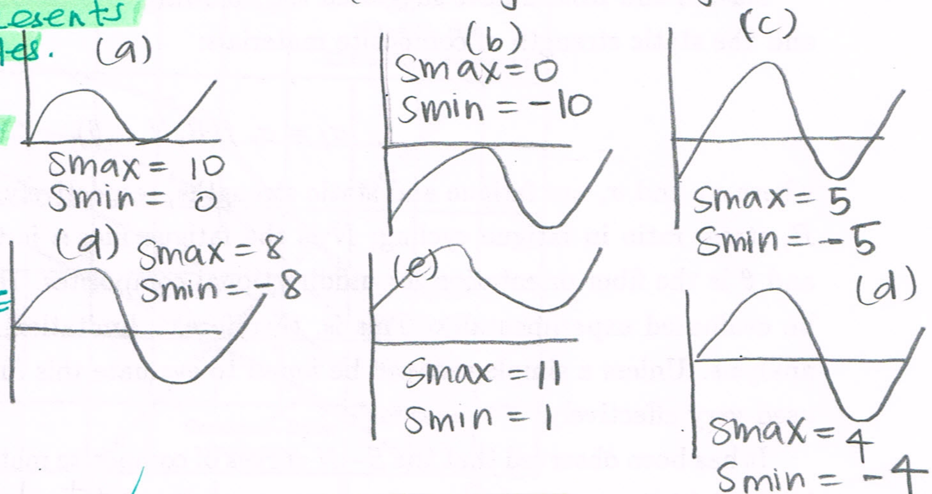
November 11 L

$\boxed{Q}$ A laminate is subjected to different types of loading. The fatigue properties of this Laminate are defined by a Goodman diagram, (figure below). Six different levels of fatigue are proposed as load cases.

Pg 224

- which load level will be the worst case (ie: shortest fatigue life)?
- which load level will be the best case (ie: longest fatigue life)?





SoL: Pg 224

$$S_{av} = \frac{1}{2}(S_{max} + S_{min})$$

$$S_R = \frac{1}{2}(S_{max} - S_{min})$$

a) $S_{av} = 5$ ✓
$S_R = 5$

b) $S_{av} = -5$ ✓
$S_R = 5$

c) $S_{av} = 0$ ✓
$S_R = 5$

d) $S_{av} = 0$ ✓ → average stress
$S_R = 8$ → cyclic stress

e) $S_{av} = 6$ ✓
$S_R = 5$

f) $S_{av} = 0$ ✓
$S_R = 4$

worst: e (shortest Life) ✓
best: f (longest Life) ✓

because diagram is symmetric: you have same strength propрties in tension and compression
↳ so a, b are exactly same result

critical points are d, e
↳ look @ pg 225:

$$\frac{S_{av}}{S_u} + \frac{S_R}{S_e} = 1$$

$S_u = 12$
$S_e = 10$

e): $\frac{6}{12} + \frac{5}{10} = 1$ → means its ON the Line
< 1 → means safe (under Line)
> 1 → means has failed (above Line).

d): $\frac{0}{12} + \frac{8}{10} = 0.8$ → safe

∴ worst case is e and best = f

$\boxed{Q}$ Say $N_1 = 10^5$, $N_2 = 10^4$, $N_3 = 10^3$ cycles, Safety factor for fatigue loading assuming same safety factor for average and cyclic stress.

SoL:

$\left(\frac{S_{av}}{S_u} + \frac{S_R}{S_e}\right)R = 1$ goodman eq^n

so safety factor: $a_{ave} \rightarrow R a_{ave}$

$a_R \rightarrow R a_R$

more generally:

$\frac{R_{av} S_{av}}{S_u} + \frac{R_R S_R}{S_e} = 1$

if $R_{av} = R_R = R$ then: $R = \left(\frac{S_{av}}{S_u} + \frac{S_R}{S_e}\right)^{-1}$

consider design a)

consider S_u, S_e for N_3, very safe. if you used N_2, a would have failed.

this is up to the limit of the true line below.

*this method is for a single ply.

Chapter 10

Hygrothermal Behavior of Laminated Composites

- creates residual stress in laminate
- tends to expand more in matrix direction than fibers

Organic matrix composites are sensitive to change of temperature and to absorption of moisture. A matrix, in most cases, is much more susceptible to hygrothermal deformation than the fibers. Therefore, deformations resulting from hygrothermal effect are much higher in the transverse direction than in the longitudinal direction in a unidirectional layer. Such anisotropy in deformation results in the presence of residual stresses in laminated composites because the multidirectionality of fiber orientation prohibits free deformation. Moreover, the temperature change and moisture absorption can also change mechanical properties of the composites.

*phenolic kevlar needs to be kept in the freezer

10.1 Stress-Strain Relation of Unidirectional Layer Including Hygrothermal Effect

Assuming the thermal expansion is linearly proportional to the change of temperature, we have the following relation:

const. along fibers

" along matrix

$$\begin{Bmatrix} e_x^T \\ e_y^T \\ e_s^T = \gamma_{xy}^T \end{Bmatrix} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T, \quad \Delta T = T - T_0 \tag{10.1}$$

where α_x, α_y, α_{xy} are coefficients of thermal expansion, which are constants when thermal strain $\{e^T\}$ is linearly related to ΔT.

Due to orthotrophy of the unidirectional layer, the shear thermal strain γ_{xy}^T is zero ($\alpha_{xy} = 0$). Equation (10.1) reduces to