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Adaptive impulsive cluster synchronization in community network with nonidentical nodes

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In this paper, cluster synchronization in community network with nonidentical nodes is investigated. Through introducing proper adaptive strategy into impulsive control scheme, adaptive impulsive controllers are designed for achieving the cluster synchronization. In this adaptive impulsive control scheme, for any given networks, the impulsive gains can adjust themselves to proper values according to the proposed adaptive strategy when the impulsive intervals are fixed. The impulsive instants can be estimated by solving a sequence of maximum value problems when the impulsive gains are fixed. Both community networks without and with coupling delay are considered. Based on the Lyapunov function method and mathematical analysis technique, two synchronization criteria are derived. Several numerical examples are performed to verify the effectiveness of the derived theoretical results.

Keywords: Cluster synchronization; community network; impulsive control.

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1. Introduction

Complex dynamical networks have been adopted to describe many physical and social systems consisting of interactive individuals, in which the nodes denote the individuals and the edges denote the interactions among the nodes. Further, the dynamical behavior of an isolated individual is governed by the node dynamics and the interaction between a pair of individuals is denoted by the coupling between the nodes. Recently, many large-scale real systems have been found that they have community structures and can be modeled by community networks, such as, biological networks,[1] school friendship networks,[2] congressional cosponsorship networks,[3] protein–protein interaction networks,[4] World Wide Web[5] and so on. In some community networks, the nodes in the same community usually play the same or similar roles and have identical node dynamics, and those in different communities play different roles and have nonidentical node dynamics. For example, in protein–protein interaction networks, communities correspond to functional groups/clusters, i.e. to proteins having the same or similar functions, which are expected to be involved in the same processes.[6] Further, due to the roles of the nodes in the community network, the nodes in the same community usually have the same goals and tend to synchronize with each other, and the nodes in different communities tend to synchronize with different states. That is, the community network achieves cluster synchronization.[7–18]

Synchronization is an interesting and important collective behavior of dynamical networks. In practical applications, synchronization can be beneficial. For example, in computer science, especially in parallel computing, synchronization means the coordination of simultaneous threads or processes to complete a task of obtaining a correct runtime order while avoiding unexpected race conditions.[19] On the other hand, the real networks usually cannot synchronize themselves due to their complexity or the external effects. Thus, how to design simple and effective controllers for achieving synchronization of networks or cluster synchronization of community networks is an important issue and deserves further studies.

Impulsive control, as a typical discontinuous control scheme, has been widely adopted to design effective controllers. In impulsive control scheme, the controllers are applied on the nodes only at a sequence of discrete instants. That is, the impulsive controllers have a relatively simple structure, which means they are easy to implement and low-cost. For any given dynamical network or community network, the key point for designing impulsive controllers is to derive the conditions with respect to the impulsive instants, impulsive gains and system parameters for achieving synchronization or cluster synchronization. That is, how to choose proper impulsive instants and impulsive gains. Deng *et al.* investigated the cluster synchronization in community network with nonidentical nodes via impulsive control.[17] Several synchronization conditions are derived, from which the values of impulsive gains and intervals can be estimated for given community network. As we know, some constants with respect to node dynamics and topology of network have a great influence on the choice of the impulsive gains and intervals. Thus, the controllers with fixed impulsive gains and instants cannot be valid for different dynamical networks. Therefore, how to design unified impulsive controllers is a challenging and interesting problem and deserves further studies.

The main contributions of this paper are twofold. First, proper adaptive strategy is introduced in impulsive controllers, i.e. a parameter with adaptive updating law is introduced to estimate the constant with respect to node dynamics and topology of community network. Second, a method for choosing impulsive gains and an algorithm for determining the impulsive instants are both provided. That is, the constant with respect to node dynamics and topology of network need not to be calculated beforehand. The impulsive gains or instants is chosen or estimated according to the proposed adaptive strategy. In this sense, the designed adaptive impulsive controllers are unified for different community networks.

The rest of this paper is organized as follows. Section 2 introduces the community network models and some preliminaries. Section 3 studies the cluster synchronization in community network with nonidentical nodes through designing proper adaptive impulsive controllers and derives the synchronization criteria. Section 4 performs several numerical simulations to verify the results. Section 5 concludes this paper.

2. Model Description and Preliminaries

Consider a community network consisting of N nonidentical nodes and m $(2 \leq m < N)$ communities, where each node is an n-dimensional system, which is described by

$$\dot{x}_i(t) = f_{\phi_i}(x_i(t)) + c\sum_{l=1}^{m}\sum_{j\in G_l} a_{ij}Hx_j(t), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state variable of node i, $H = \mathrm{diag}(h_1, h_2, \ldots, h_n)$ is the inner coupling matrix, $c > 0$ is the coupling strength, G_l $(l = 1, 2, \ldots, m)$ denotes the set of all nodes belong to the lth community. The matrix $A = (a_{ij}) \in R^{N\times N}$ is the zero-row-sum outer coupling matrix, which denotes the network topology and is defined as, if there is a connection between node j and node i $(i \neq j)$, then $a_{ij} \neq 0$; otherwise, $a_{ij} = 0$. The function ϕ is defined as $\phi : \{1, 2, \ldots, N\} \to \{1, 2, \ldots, m\}$, if node $i \in G_l$, then $\phi(i) = l$, denoted as $\phi_i = l$. The function $f_{\phi_i} : R^n \to R^n$ describes the dynamics of each isolated individual node in the ϕ_ith community.

On the other hand, coupling delay exists in many real networks due to transmission of the exchanged signal and should be considered. Then, the community network with time-varying coupling delay is described by

$$\dot{x}_k(t) = f_{\phi_i}(x_i(t)) + c\sum_{l=1}^{m}\sum_{j\in G_l} a_{ij}Hx_j(t-\tau(t)), \tag{2}$$

where $\tau(t)$ is the time-varying coupling delay.

In this paper, the node dynamics in different communities are assumed to be nonidentical, i.e. if $\phi_i \neq \phi_j$, then $f_{\phi_i} \neq f_{\phi_j}$. Let $s_{\phi_i}(t)$ be a solution of an isolated node in the ϕ_ith community, i.e. $\dot{s}_{\phi_i}(t) = f_{\phi_i}(s_{\phi_i}(t))$, $i = 1, 2, \ldots, N$. The objective here is to apply proper controllers onto the networks (1) and (2) such that

$$\lim_{t\to\infty} \|x_i(t) - s_{\phi_i}(t)\| = 0,$$

where $\|\cdot\|$ denotes Euclidean norm, i.e. the networks (1) and (2) achieve cluster synchronization.

For achieving the cluster synchronization, proper adaptive impulsive controllers are designed and applied onto the networks (1) and (2). Then, the controlled networks are described by

$$\begin{aligned} \dot{x}_i(t) &= f_{\phi_i}(x_i(t)) + c\sum_{l=1}^{m}\sum_{j\in G_l} a_{ij}Hx_j(t), \quad t \neq t_k, \\ x_i(t_k^+) - x_i(t_k^-) &= b_k(t_k)(x_i(t_k^-) - s_{\phi_i}(t_k)), \quad t = t_k \end{aligned} \tag{3}$$

and

$$\begin{aligned} \dot{x}_i(t) &= f_{\phi_i}(x_i(t)) + c\sum_{l=1}^{m}\sum_{j\in G_l} a_{ij}Hx_j(t-\tau(t)), \quad t \neq t_k, \\ x_i(t_k^+) - x_i(t_k^-) &= b_k(t_k)(x_i(t_k^-) - s_{\phi_i}(t_k^-)), \quad t = t_k, \end{aligned} \tag{4}$$

where $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots$, $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$, $x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$. $b_k(t)$ is the impulsive gain, defined as: if $t = t_k$, then $b_k(t) \neq 0$, otherwise, $b_k(t) = 0$. Discrete impulsive instant set $\{t_k\}$ satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, $t_k \to \infty$ as $k \to \infty$. Any solutions of (3) and (4) are assumed to be left continuous at each t_k, i.e. $x_i(t_k^-) = x_i(t_k)$.

For convenience, let $G_l = \{r_{l-1}+1, \ldots, r_l\}(l = 1, 2, \ldots, m)$ with $r_0 = 0$ and $r_m = N$. Suppose that $A \in R^{N\times N}$ can be divided into the following block form

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{2m} & \cdots & A_{mm} \end{pmatrix},$$

where $A_{pq} \in R^{(r_p - r_{p-1})\times(r_q - r_{q-1})}$, $p, q = 1, 2, \ldots, m$. Throughout this paper, the following definitions and assumption are required for deriving main results.

Definition 1. Matrix $A_{pp} \in R^{(r_p - r_{p-1})\times(r_p - r_{p-1})}$ is said to belong to class C_1, denoted as $A_{pp} \in C_1$, $p = 1, 2, \ldots, m$, if A_{pp} is a zero-row-sum, symmetric and irreducible matrix with non-negative off-diagonal elements.

Definition 2. Matrix $A_{pq} \in R^{(r_p - r_{p-1})\times(r_q - r_{q-1})}$ is said to belong to class C_2, denoted as $A_{pq} \in C_2$, $p, q = 1, 2, \ldots, m$, $p \neq q$, if its each row-sum is zero.

Definition 3. If $A_{pp} \in C_1$ and $A_{pq} \in C_2$, $p, q = 1, 2, \ldots, m$ $(p \neq q)$, then matrix A is said to belong to class C_3, denoted as $A \in C_3$.

Assumption 1. Suppose that the functions $f_l(x)$ satisfy Lipschitz condition, i.e. there exists a positive constant $L > 0$ such that

$$\|f_l(y) - f_l(x)\| \leq L\|y - x\|, \quad l = 1, 2, \ldots, m,$$

for any $x, y \in R^n$, where $\|\cdot\|$ denotes Euclidean norm.

It is easy to verify that many typical benchmark chaotic systems such as the Lorenz system, Chen system and Lü system all satisfy Assumption 1.

Assumption 2. Suppose that the time-varying coupling delay $\tau(t)$ is differentiable and there exists a constant $\mu < 1$ such that $\dot{\tau}(t) \leq \mu$.

3. Main Results

Let $e_i(t) = x_i(t) - s_{\phi_i}(t)$ be synchronization errors, $e(t) = (e_1^T(t), \ldots, e_N^T(t))^T$, $A^s = (A^T + A)/2$, $\tau_k = t_k - t_{k-1}$ be impulsive intervals, $\beta_k(t) = (1 + b_k(t))^2$, λ_1 and λ_2 be the largest eigenvalues of $LI + cA^s \otimes H$ and $(L + c(1-\mu)^{-1})I + c(A \otimes H)^T(A \otimes H)$, respectively. From the definition of $b_k(t)$, one has $\beta_k(t) = 1$ for $t \neq t_k$. In what follows, the coupling matrix A considered in this paper is assumed to belong to C_3.

According to Definition 3, it is clear that

$$\sum_{j=1}^{N} a_{ij} H s_{\phi_j}(t) = 0$$

and

$$\sum_{j=1}^{N} a_{ij} H s_{\phi_j}(t - \tau(t)) = 0.$$

Then, one has the following error systems

$$\begin{aligned} \dot{e}_i(t) &= \tilde{f}_{\phi_i}(e_i(t)) + c\sum_{j=1}^{N} a_{ij} H e_j(t), \quad t \neq t_k, \\ e_i(t_k^+) &= (1 + b_k(t_k)) e_i(t_k^-), \quad t = t_k \end{aligned} \tag{5}$$

and

$$\begin{aligned} \dot{e}_i(t) &= \tilde{f}_{\phi_i}(e_i(t)) + c\sum_{j=1}^{N} a_{ij} H e_j(t - \tau(t)), \quad t \neq t_k, \\ e_i(t_k^+) &= (1 + b_k(t_k)) e_i(t_k^-), \quad t = t_k, \end{aligned} \tag{6}$$

where $\tilde{f}_{\phi_i}(e_i(t)) = f_{\phi_i}(x_i(t)) - f_{\phi_i}(s_{\phi_i}(t))$.

Theorem 1. *Suppose that Assumption 1 holds and $A \in C_3$. If there exists a constant $\alpha > 0$ such that*

$$\ln \beta_k(t_k) + \alpha + \hat{L}(t_k)\tau_k < 0, \quad k = 1, 2, \ldots, \tag{7}$$

where $\dot{\hat{L}}(t) = \delta \sum_{i=1}^{N} e_i^T(t) e_i(t)$ and $\delta > 0$ is the adaptive gain, then the cluster synchronization of the network (3) is achieved.

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t) e_i(t) + \frac{\beta_k(t)}{2\delta}(\hat{L}(t) - \lambda_1)^2$$

for $t \in (t_{k-1}, t_k]$, $k = 1, 2, \ldots$.

When $t \in (t_{k-1}, t_k)$, the derivative of $V(t)$ along the solution of (5) gives

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \frac{1}{\delta}(\hat{L}(t) - \lambda_1)\dot{\hat{L}}(t) \\
&= \sum_{i=1}^{N} e_i^T(t)\tilde{f}_{\phi_i}(e_i(t)) + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_i^T(t)He_j(t) + \frac{1}{\delta}(\hat{L}(t) - \lambda_1)\dot{\hat{L}}(t) \\
&\le e^T(t)(LI + cA^s \otimes H)e(t) + (\hat{L}(t) - \lambda_1)e^T(t)e(t) \\
&\le \hat{L}(t)e^T(t)e(t) \\
&\le \hat{L}(t_k)V(t),
\end{aligned}$$

which gives

$$V(t) \le V(t_{k-1}^+)\exp(\hat{L}(t_k)(t - t_k)), \quad t \in (t_{k-1}, t_k). \tag{8}$$

When $t = t_k$, one has

$$\begin{aligned}
V(t_k^+) &= \frac{1}{2}\sum_{i=1}^{N} e_i^T(t_k^+)e_i(t_k^+) + \frac{\beta_k(t_k)}{2\delta}(\hat{L}(t_k^+) - \lambda_1)^2 \\
&= \frac{(1 + b_k(t))^2}{2}\sum_{i=1}^{N} e_i^T(t_k^-)e_i(t_k^-) + \frac{\beta_k(t_k)}{2\delta}(\hat{L}(t_k^-) - \lambda_1)^2 \\
&= \beta_k(t_k)V(t_k^-).
\end{aligned} \tag{9}$$

For $k = 1$, from inequalities (8) and (7), one has

$$\begin{aligned}
&V(t_1^-) \le V(t_0)\exp(\hat{L}(t_1)\tau_1), \\
&V(t_1^+) \le \beta_1(t_1)V(t_1^-) \le \beta_1 V(t_0)\exp(\hat{L}(t_1)\tau_1).
\end{aligned}$$

For $k = 2$, we have

$$\begin{aligned}
V(t_2^-) &\le V(t_1^+)\exp(\hat{L}(t_2)\tau_2) \\
&\le \beta_1(t_1)V(t_0)\exp(\hat{L}(t_2)\tau_2 + \hat{L}(t_1)\tau_1), \\
V(t_2^+) &\le \beta_2(t_2)V(t_2^-) \\
&\le \beta_2(t_2)\beta_1(t_1)V(t_0)\exp(\hat{L}(t_2)\tau_2 + \hat{L}(t_1)\tau_1) \\
&= V(t_0)\prod_{j=1}^{2}(\beta_j(t_j)\exp(\hat{L}(t_j)\tau_j)).
\end{aligned}$$

By mathematical induction, one has

$$V(t_k^+) \le V(t_0)\prod_{j=1}^{k}\beta_j(t_j)\exp(\hat{L}(t_j)\tau_j).$$

If condition (7) holds, one has

$$\beta_j(t_j)\exp(\hat{L}(t_j)\tau_j) \le \exp(-\alpha), \quad j = 1, 2, \dots,$$

and

$$V(t_k^+) \le V(t_0)\exp(-k\alpha),$$

which implies

$$\lim_{k\to\infty} V(t_k^+) = 0.$$

Then, for $t \in (t_k, t_{k+1}]$, one has

$$V(t) \le V(t_k^+)\exp(\hat{L}(t_{k+1})(t - t_k)),$$

which gives $V(t) \to 0$ as $t \to \infty$, i.e. the cluster synchronization is achieved. This completes the proof. □

Theorem 2. *Suppose that Assumptions 1 and 2 hold and $A \in C_3$. If there exists a constant $\alpha > 0$ such that*

$$\ln\beta_k(t_k) + \alpha + \hat{L}(t_k)\tau_k < 0, \quad k = 1, 2, \dots, \tag{10}$$

where $\dot{\hat{L}}(t) = \delta\sum_{i=1}^N e_i^T(t)e_i(t)$ and $\delta > 0$ is the adaptive gain, then the cluster synchronization of the network (4) is achieved.

Proof. Consider the following Lyapunov function

$$V(t) = \frac{1}{2}\sum_{i=1}^N e_i^T(t)e_i(t) + \frac{\beta_k(t)}{2\delta}(\hat{L}(t) - \lambda_2)^2 + \frac{c\beta_k(t)}{1-\mu}\int_{t-\tau(t)}^t \sum_{i=1}^N e_i^T(\theta)e_i(\theta)d\theta$$

for $t \in (t_{k-1}, t_k]$, $k = 1, 2, \dots$.

When $t \in (t_{k-1}, t_k)$, one has

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^N e_i^T(t)\dot{e}_i(t) + \frac{1}{\delta}(\hat{L}(t) - \lambda_2)\dot{\hat{L}}(t) + \frac{c}{1-\mu}\sum_{i=1}^N e_i^T(t)e_i(t)\\
&\quad - \frac{c(1-\dot{\tau}(t))}{1-\mu}\sum e_i^T(t-\tau(t))e_i(t-\tau(t))\\
&\le e^T(t)((L + c(1-\mu)^{-1})I + c(A\otimes H)^T(A\otimes H))e(t) + (\hat{L}(t) - \lambda_2)e^T(t)e(t)\\
&\quad - \frac{c(\mu - \dot{\tau}(t))}{1-\mu}e^T(t-\tau(t))e(t-\tau(t))\\
&\le \hat{L}(t)e^T(t)e(t)\\
&\le \hat{L}(t_k)V(t),
\end{aligned}$$

which gives

$$V(t) \le V(t_{k-1})\exp(\hat{L}(t_k)(t - t_{k-1})), \quad t \in (t_{k-1}, t_k).$$

For $t = t_k$,

$$V(t_k^+) = \sum_{i=1}^{N} e_i^T(t_k^+)e_i(t_k^+) + \frac{\beta_k(t_k)}{2\delta}(\hat{L}(t_k^+) - \lambda_1)^2 + \frac{c\beta_k(t_k)}{1-\mu}\int_{t_k-\tau(t_k)}^{t_k} \sum_{i=1}^{N} e_i^T(\theta)e_i(\theta)d\theta$$
$$= \beta_k(t_k)V(t_k^-).$$

Similar to the proof of Theorem 2, the proof is completed. □

Remark 1. Generally, if the constant α and the impulsive intervals $\tau_k = t_k - t_{k-1}$ are fixed, one can choose the impulsive gain according to the following inequalities

$$-\exp\left(-\frac{\alpha + \hat{L}(t_k)\tau_k}{2}\right) - 1 + \varepsilon \le b_k(t_k) \le \exp\left(-\frac{\alpha + \hat{L}(t_k)\tau_k}{2}\right) - 1 - \varepsilon,$$

such that the conditions (7) and (10) hold, where ε is a small positive constant.

Remark 2. On the other hand, if the impulsive gains $b_k(t_k)$ and the constant α are fixed, one can estimate the control instants t_k through finding the maximum value of t_k subject to $t_k < t_{k-1} - (\ln \beta_k(t_k) + \alpha)\hat{L}^{-1}(t_k)/2$ with $t_0 = 0$, $k = 1, 2, 3, \ldots$.

4. Numerical Illustrations

Consider a community network consisting of 19 nodes and three communities, which is shown in Fig. 1. The outer coupling matrix A in Fig. 1 is defined as: if nodes i and j $(j \neq i)$ is connected by solid line, then $a_{ij} = 1$; if nodes i and j $(j \neq i)$ is connected by dashed line, then $a_{ij} = -1$. Choose the node dynamics of the first community as Chen system[21]:

$$\begin{aligned}\dot{x}_1 &= 35(x_2 - x_1),\\ \dot{x}_2 &= (28 - 35)x_1 - x_1x_3 + 28x_2,\\ \dot{x}_3 &= x_1x_2 - 3x_3,\end{aligned}$$

the second community as Lü system[22]:

$$\begin{aligned}\dot{x}_1 &= 36(x_2 - x_1),\\ \dot{x}_2 &= -x_1x_3 + 20x_2,\\ \dot{x}_3 &= x_1x_2 - 3x_3,\end{aligned}$$

the third community as Lorenz system[23]:

$$\begin{aligned}\dot{x}_1 &= 10(x_2 - x_1),\\ \dot{x}_2 &= 28x_1 - x_1x_3 - x_2,\\ \dot{x}_3 &= x_1x_2 - 8x_3/3\end{aligned}$$

and the inner coupling matrix as an identity matrix, i.e. $\Gamma = \mathrm{diag}\{1, 1, 1\}$.

Example 1. Consider the cluster synchronization of community network (3) without coupling delay.

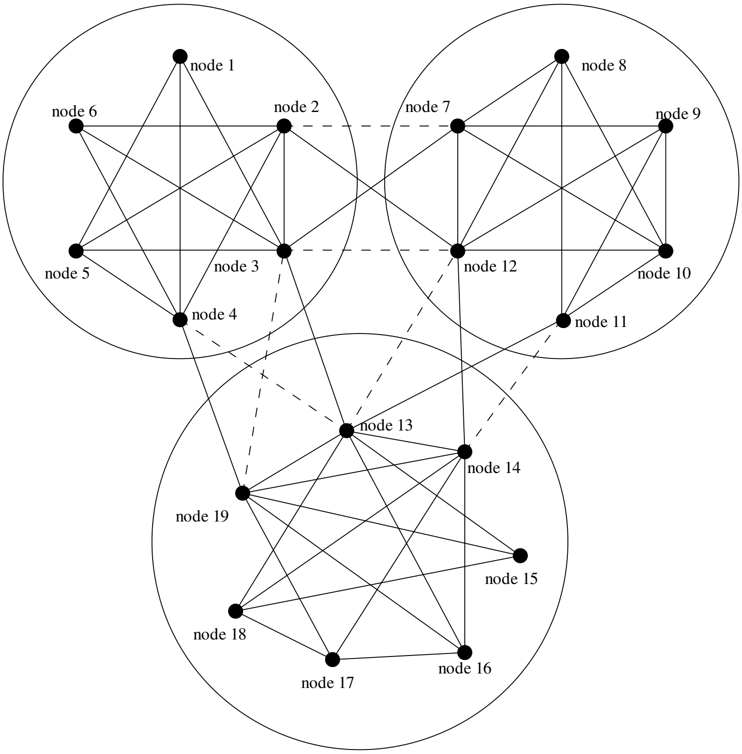

Fig. 1. A community network consists of 19 nodes with three communities. The top-left, top-right and bottom are the first, second and third communities, respectively. The outer coupling matrix A is defined as: if nodes i and j ($j \neq i$) is connected by solid line, then $a_{ij} = 1$; if nodes i and j ($j \neq i$) is connected by dashed line, then $a_{ij} = -1$.

First, choose $\tau_k = 0.2$, $\alpha = 0.001$, $\delta = 0.001$ and $\hat{L}(0) = 0.1$. According to Remark 1, choose $b_k(t_k) = \exp(-\frac{\alpha+\hat{L}(t_k)\tau_k}{2}) - 1 - \varepsilon$ with $\varepsilon = 0.001$. Moreover, choose the initial values of state variables $x_i(t)$ and synchronization orbits $s_{\phi_i}(t)$ randomly. Figure 2 shows the orbits of $x_i(t)$ and $s_{\phi_i}(t)$ in three communities, respectively. Figure 3 shows the impulsive gains $b_k(t_k)$ versus k.

Second, choose $b_k(t_k) = -0.9$, $\alpha = 0.001$, $\delta = 0.001$ and $\hat{L}(0) = 0.1$. Figure 4 shows the orbits of synchronization errors $e_i(t)$, $i = 1, 2, \ldots, 19$. Figure 5 shows the values of impulsive intervals τ_k versus k.

Example 2. Consider the cluster synchronization of community network (4) with time-varying coupling delay. Choose the coupling delay as $\tau(t) = 0.2\sin(t)$.

First, choose $\tau_k = 0.2$, $\alpha = 0.001$, $\delta = 0.0004$ and $\hat{L}(0) = 0.01$. According to Remark 1, choose $b_k(t_k) = \exp(-\frac{\alpha+\hat{L}(t_k)\tau_k}{2}) - 1 - \varepsilon$ with $\varepsilon = 0.001$. Choose the initial values of state variables $x_i(t)$ and synchronization orbits $s_{\phi_i}(t)$ randomly. Figure 6 shows the orbits of $x_i(t)$ and $s_{\phi_i}(t)$ in three communities, respectively. Figure 7 shows the impulsive gains $b_k(t_k)$ versus k.

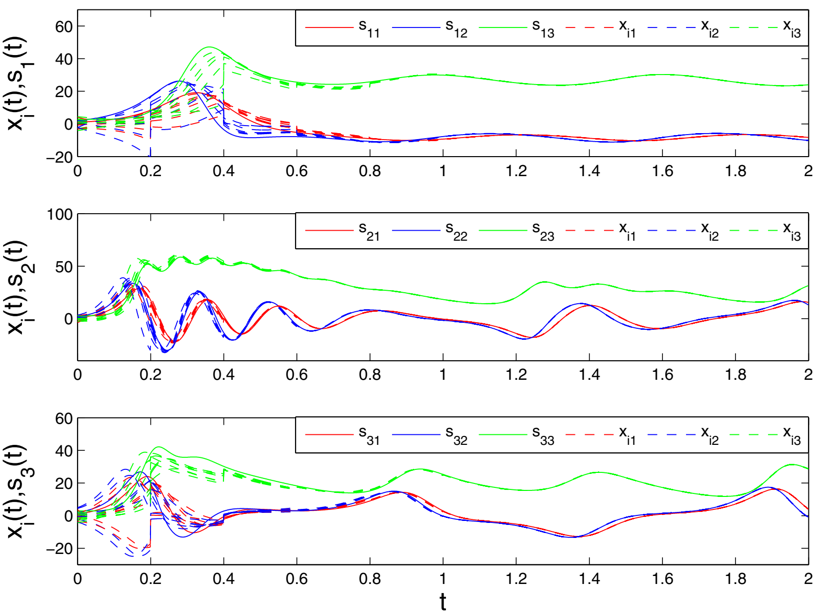

Fig. 2. (Color online) The orbits of $x_i(t)$ and $s_{\phi_i}(t)$ in three communities, $i = 1, 2, \ldots, 19$. The top, middle and bottom denote the first, second and third communities, respectively.

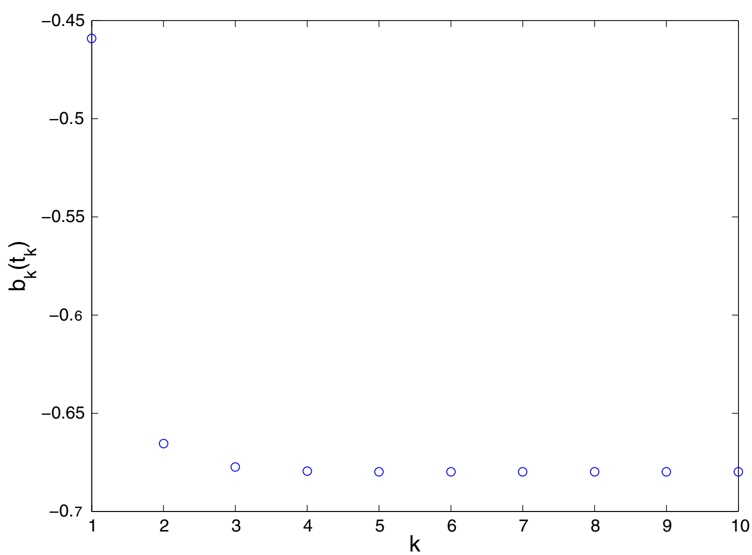

Fig. 3. (Color online) The impulsive gain $b_k(t_k)$ versus k.

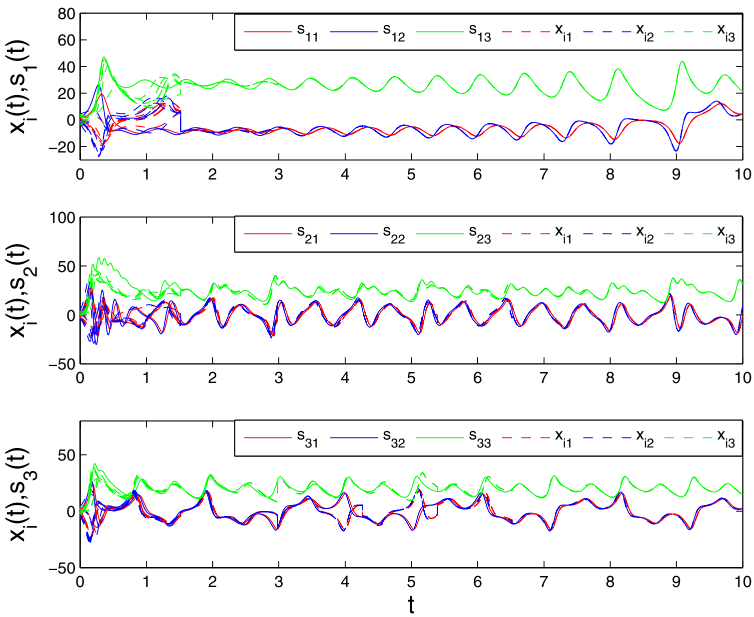

Fig. 4. (Color online) The orbits of $x_i(t)$ and $s_{\phi_i}(t)$ in three communities, $i = 1, 2, \ldots, 19$. The top, middle and bottom denote the first, second and third communities, respectively.

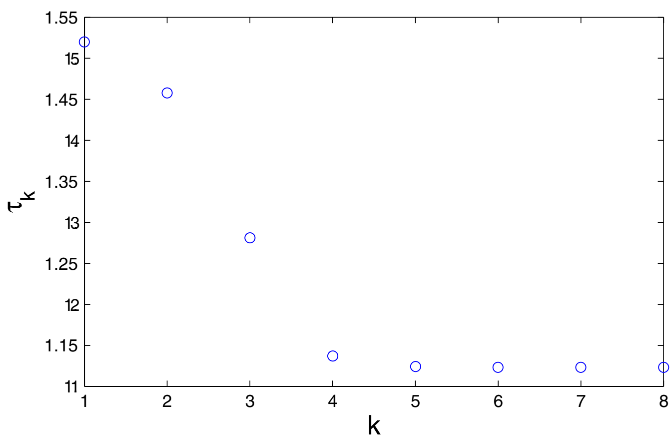

Fig. 5. (Color online) The impulsive interval τ_k versus k.

Second, choose $b_k(t_k) = -0.8$, $\alpha = 0.001$, $\delta = 0.001$ and $\hat{L}(0) = 0.05$. Figure 8 shows the orbits of synchronization errors $e_i(t)$, $i = 1, 2, \ldots, 19$. Figure 9 shows the values of impulsive intervals τ_k versus k.

Remark 3. In the above examples, it is clear that the Lipschitz constant L, the largest eigenvalues λ_1 and λ_2 in the theorems need not be calculated. That is, the

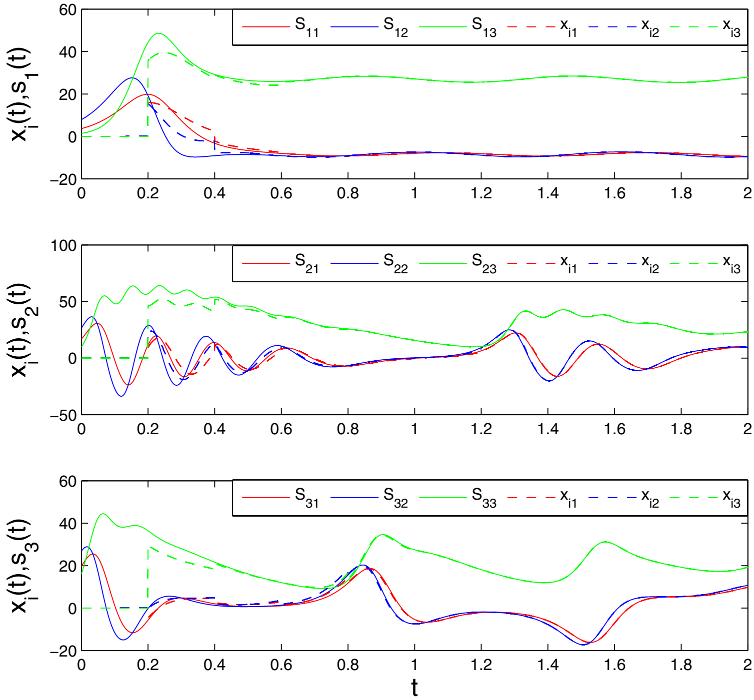

Fig. 6. (Color online) The orbits of $x_i(t)$ and $s_{\phi_i}(t)$ in three communities, $i = 1, 2, \ldots, 19$. The top, middle and bottom denote the first, second and third communities, respectively.

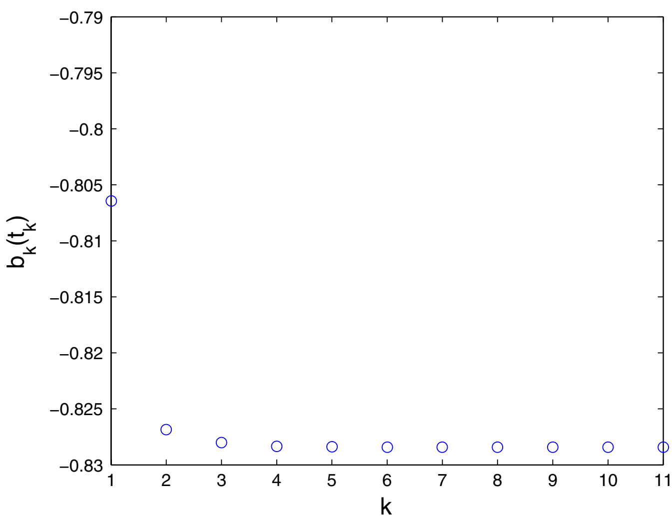

Fig. 7. (Color online) The impulsive gain $b_k(t_k)$ versus k.

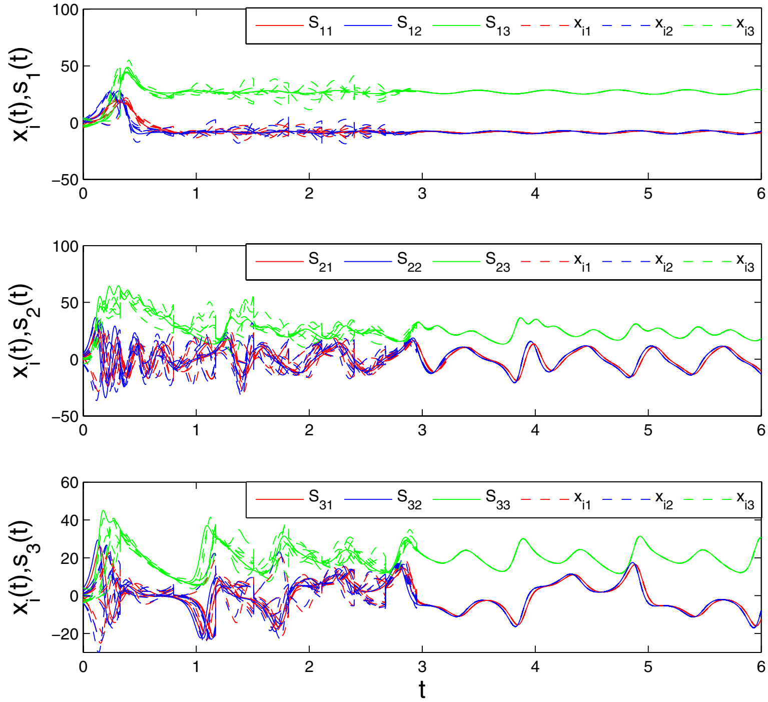

Fig. 8. (Color online) The orbits of $x_i(t)$ and $s_{\phi_i}(t)$ in three communities, $i = 1, 2, \ldots, 19$. The top, middle and bottom denote the first, second and third communities, respectively.

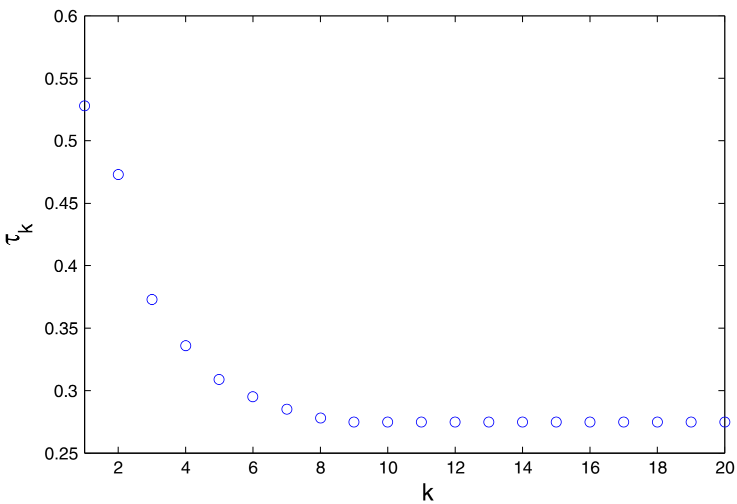

Fig. 9. (Color online) The impulsive interval τ_k versus k.

designed adaptive impulsive controllers are universal for different networks to some extent.

5. Conclusions

In this paper, cluster synchronization in community network with nonidentical nodes has been studied via designing proper adaptive impulsive controllers. In the adaptive impulsive control scheme, the impulsive gains and intervals (or instants) can adaptively change according to the adaptive laws. That is, for any given community networks, the impulsive gains can adjust themselves to proper values and the impulsive instants can be estimated according to Remarks 1 and 2. According to the Lyapunov function method and mathematical analysis technique, both community network without and with coupling delay have been considered and synchronization criteria have been derived. All the derived results have been illustrated to be effective by several numerical simulations.

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