Discussion week 8

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10/31/2021

The basic Logic of Hypothesis Testing - Contradiction

- Story 1:
  Sherlock Holmes: "When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth.".

- Story 2:
  You read in a history textbook about someone who was born in 1856. Prove that this person is not still alive.
  You make the observation that most people don't live past 90 years, and the oldest person ever (verified) lived to be 122. If he were, he would be either $2017-1856=161$ or 160 years old, which contradicts our observation.

- Story 3:
  How do I know if it is raining outside without getting to see the weather, since I am now sitting in the classroom?
  I can set up two hypotheses; the first one, it is not raining outside, the second one is it is raining outside. There are so many students in the class, I think if it is raining outside, I could at least see one student with umbrella. Since I am not seeing it now, it is not raining outside.

The logic of hypothsis testing:

I set up two hypotheses, and think about the situation if these two are true. If what I see (from the data) is unlikely to happen under a hypothesis, I would not prefer this hypothesis. In other words, we are looking for the evidence, if the data is contradicting the hypothesis or not.

Practice 1:

A potato chip producer has just received a truckload of potatoes from its main supplier. If the producer finds convincing evidence that more than 8% of the potatoes in the shipment have blemishes, the truck will be sent away to get another load from the supplier. A supervisor selects a simple random sample of $500.$ potatoes from the truck. An inspection reveals that

47 of the potatoes have blemishes.

(a) State null and alternative hypotheses relevant to determining whether the shipment should be returned.
Answer:
The null is that the true percentage of blemished potatoes in the truckload is equal to 8%.
The alternative is that the true percentage of blemished potatoes in the truckload is greater than 8%.

(b) Perform a test of the stated hypotheses at the 5% significance level. Make sure to compute the test statistic and P-value and make a conclusion in the context of the problem.
Answer:
The observed percentage is $\frac{47}{500} = 9.4\%$. Under the null, the EV of the percentage is 8%. Under the null, the SD of the box is $\sqrt{0.08 * 0.92} = 0.27$, so the SE of the percentage is $0.27 \times \sqrt{500} = 1.2\%$. Since the sample size is large, the approximate distribution of the sample percentage is normal, by the CLT. So we can do a z − test. The test statistic is z = $\frac{9.4\% - 8.0\%}{1.2\%} = 1.17$. Since the alternative is greater than, the P-value is the chance of falling to the right of this value under a standard normal curve. From the table, the chance of falling to the left of 1.17 is 0.8790, so the chance of falling to the right is $1 - 0.8790 = 0.121$, or 12.1%. Because the P-value is greater than 5%, we fail to reject the null. There is not convincing evidence that the shipment contains more than 8% blemished potatoes.

(c) What type of error could we have made given our conclusions in (b)? Describe the implications of that error in the context of the question.
Answer:
We decided to not reject the null. The null is either true or false. If the null is true, we made the correct conclusion by not rejecting it. If the null is false, we should have rejected, so we made the wrong conclusion, and that would be a Type II error. If we decided the shipment was ok, but in reality, more than 8% of the potatoes were blemished, we'd end up with some low-quality potato chips.

Practice 2:

Indicate whether the questions below are True or False, and give a brief explanation.

(a) The p-value is the probability that the null hypothesis is true.

Ansewr:
False. The null is either true or it isn't. The p-value is the probability of a test statistic as or more extreme than what was observed, assuming the null is true.

(b) Smaller p-values are more evidence for the null hypothesis.
Answer:
False. Smaller p-values are more evidence against the null, since a small p-value means the data we observed was unlikely if the null was true.

(c) If we do not find sufficient evidence against the null, we say that we accept the null.
Answer:
False. We assume the null from the start, so if we do not have evidence against the null, this is not the same as finding sufficient evidence for the null.

(d) The observed significance level will change depending on the particular sample taken.
Answer:
True. The observed significance level is the same as the p-value, and the p-value is computed based on the data from the sample, so different samples will give different observed significance level.

Practice 3

Anne reads that the average price of regular gas in her state is $4.06 per gallon. She thinks it's lower in her city. To test this, she selects an SRS of 8 gas stations and records the price per gallon for regular gas at each station. The data are below (in dollars):

$$4.13, 4.01, 4.09, 4.05, 3.97, 3.99, 4.05, 3.98$$

(a) State null and alternative hypotheses appropriate to Anne's question.
Answer:
The null is that the average gas price in her city is equal to $4.06. The alternative is that the average gas price in her city is less than $4.06.

(b) Perform a test of the hypotheses you stated in part (a) at a significance level of 10%. You may assume that the distribution of gas prices in the city is approximately normal. Make sure to compute the test statistic and P-value, and make a conclusion in context.
Answer:
The sample average price is $\$4.03$, and the sample SD is $\$0.06$. The EV of the average is the average of the box under the null, which is $\$4.06$. The SD of the box can be estimated by the sample SD, or $\$0.06$, so the SE of the average is $\frac{0.06}{\sqrt{8}} = 0.02$. Since the sample size is small and the population is normal, we should use a T-test. The test statistic is $t = \frac{4.03-4.06}{0.02} = -1.5$,The alternative is less than, so we need the chance of falling to the left of that value under a T distribution with $8-1=7$ df. 1.5 falls between 10% and 5%, so that is the P-value. At a significance level of 10%, we would reject the null, and conclude that the gas does seem cheaper in her city.

(c) Anne sees an article in the paper that a census of all of the gas stations in Anne’s city found an average price of $\$4.02$ per gallon. What error, if any, occured in Anne’s hypothesis test?
Answer:
Even though the estimate of $\$4.03$ missed the truth by a little bit, Anne still correctly rejected the null. So no mistake was made.