Recap of Central Results

- **Linear Mixed Models:** Explicitly model the mean as well as sources of correlation.
  - Combines random effects and serial structures
  - Clustered and nested data
  - Mean model and a covariance model
- Estimation via ML and REML:
  - Assume multivariate normality
  - $\widehat{\beta}$ and $\widehat{\theta}$
  - REML: Correction for degrees of freedom in $\widehat{\beta}$

Recap of Central Results (2)

- Estimation (Prediction) of Random effects
  - $\mathbf{b}_i$ for understanding individual trajectories
  - Mixed Model Equations/Empirical Bayes/Penalized Likelihood
  - Shrinkage
- Inference in Mixed Models
  - Wald and LRT
  - Testing on boundary of parameter space
- Model fitting and diagnostics
  - Mean, covariance (random effects and serial correlation)

Key References

- Mixed Models: Laird & Ware (1982, Biometrics)

- REML Estimation: Harville (1974, Biometrika)

- Random Effect Estimation: Harville (1977, JASA) and Robinson (1991, Stat Science)

- Testing VCs: Self & Liang (1987, JASA); Stram and Lee (1994, Biometrics)

- Serial Correlation: Diggle (1988, Biometrics)

- Additional Topics:
    - Influence Diagnostics: Lesaffre & Verbeke (1998, Biometrics)
    - Alternative Random Eff Dist: Butler & Louis (1992, Stat in Med)
    - Non/Semi-parametric Regression: Lin & Zhang (1999, JRSSB)

Where we are in 571...

1. Introduction to Correlated Data
2. **Linear Mixed Models** (HW1)
3. Review of GLMs and Quasi-likelihood
4. **Generalized Estimating Equations** (HW2)
5. **Generalized Linear Mixed Models** (HW3)
6. Missing Data
7. Classical and Modern Multivariate Analysis
8. Group Papers Due