Consider a hypothetical experiment where we are planning to apply fertilizer to several plots in a field and measure the resulting yield on each plot. We are considering two fertilizers, F1 and F2. There are $2n$ plots in the field, the field layout looks like the following, and each treatment will be used n times:

			...	
			...	

Let μ_1 and μ_2 represent the population mean responses for F1 and F2. To test $H_0 : \mu_1 = \mu_2$ versus the two-sided alternative we will base our tests on the respective sample means, i.e. $\bar{Y}_1$ and $\bar{Y}_2$

Unpaired – Completely Randomized Design

- Randomly assign the treatments such that any plot could receive either of the two treatments. The only restriction: the number of plots for each treatment is n. (How could we do this?)
- For the CRD, we can use:

$$T = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\widehat{\mathrm{Var}}(\bar{Y}_1 - \bar{Y}_2)}} = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 - 1 + n_2 - 1}$$

and s_1^2 and s_2^2 are the sample variances corresponding to F1 and F2, respectively.

Paired – Randomized Complete Block Design

Take advantage of the layout of the field. Label the two plots in each column according to their location:

Loc. 1	Loc. 2	Loc. 3	...	Loc. n
			...	
			...	

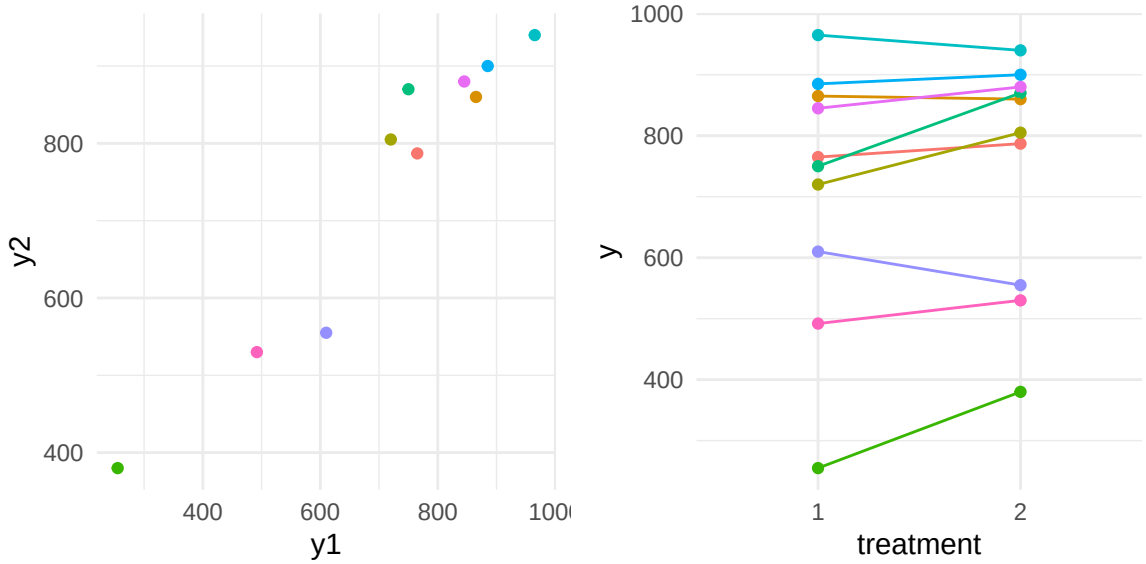
- Apply the treatments in pairs: for each location, randomly assign F1 to the top or bottom. (How could we do this?)
- This is a more restricted randomization than for the unpaired experiment.
- For this paired design, let Y_{1j} and Y_{2j} represent the observations on F1 and F2 respectively within location j, and let $D_j = Y_{1j} - Y_{2j}$. Use:

$$T = \frac{\bar{D}}{\sqrt{\widehat{\mathrm{Var}}(\bar{D})}} = \frac{\bar{D}}{\sqrt{s_D^2/n}}$$

$s_D^2 = \sum(D_i - \bar{D})^2/(n-1)$ is the sample variance of the differences D_i. Note that $\bar{D} = \bar{Y}_1 - \bar{Y}_2$.

Example

Location:	1	2	3	4	5	6	7	8	9	10
Y_1	765	865	720	255	750	965	885	610	845	492
Y_2	787	860	805	380	870	940	900	555	880	530
D	−22	5	−85	−125	−120	25	−15	55	−35	−38


800
600
400
y2
400
600
800
y1
1000
800
600
400
y
1
2
treatment