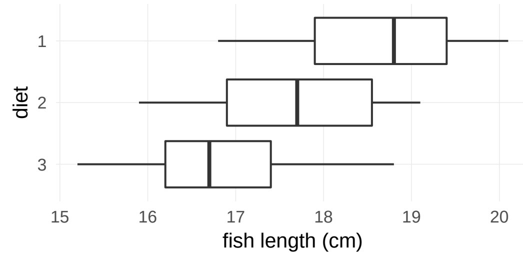
Linear Contrasts in One-Way ANOVA

Data Example

Three diets with seven observations per diet. The data are fish lengths, in cm.



raw data:

Y_1	18.2	20.1	17.6	16.8	18.8	19.7	19.1
Y_2	17.4	18.7	19.1	16.4	15.9	18.4	17.7
Y_3	15.2	18.8	17.7	16.5	15.9	17.1	16.7

summary statistics:

$\bar{y}_{1\cdot} = 18.61 \quad s_1^2 = 1.358$
$\bar{y}_{2\cdot} = 17.66 \quad s_2^2 = 1.410$
$\bar{y}_{3\cdot} = 16.84 \quad s_3^2 = 1.393$
$\bar{y}_{\cdot\cdot} = 17.70 \quad N = 21$

corresponding ANOVA table:

Source	df	SS	MS
Trt	2	11.01	5.505
Error	18	24.96	1.387
Total	20	35.97	

and $F = \dfrac{\text{MSTrt}}{\text{MSErr}} = \dfrac{5.505}{1.387} = 3.97$. Comparing this with $F_{2,18}$ yields a p-value of 0.037. Thus, there is moderate evidence against H_0: $\alpha_i = 0$ for all i.

Examples of Contrasts

- Comparison of Diet 1 and Diet 2:

$$\bar{y}_{1\cdot} - \bar{y}_{2\cdot}$$

- Comparison of the average of Diets 1 and 2 with Diet 3:

$$\frac{1}{2}\left(\bar{y}_{1\cdot} + \bar{y}_{2\cdot}\right) - \bar{y}_{3\cdot}$$

Orthogonal Polynomial Contrasts

The table lists contrast coefficients (c_i), for k treatments, for polynomial degrees $1, \ldots, k-1$, and assuming a balanced design and equal spacing in the treatment means μ_i.

k	degree	$\bar{y}_{1\cdot}$	$\bar{y}_{2\cdot}$	$\bar{y}_{3\cdot}$	$\bar{y}_{4\cdot}$	$\bar{y}_{5\cdot}$	$\bar{y}_{6\cdot}$
2	1	−1	+1				
3	1	−1	0	+1			
	2	+1	−2	+1			
4	1	−3	−1	+1	+3		
	2	+1	−1	−1	+1		
	3	−1	+3	−3	+1		
5	1	−2	−1	0	+1	+2	
	2	+2	−1	−2	−1	+2	
	3	−1	+2	0	−2	+1	
	4	+1	−4	+6	−4	+1	
6	1	−5	−3	−1	+1	+3	+5
	2	+5	−1	−4	−4	−1	+5
	3	−5	+7	+4	−4	−7	+5
	4	+1	−3	+2	+2	−3	+1
	5	−1	+5	−10	+10	−5	+1

Another Set of Orthogonal Contrasts

These are not polynomial contrasts, but can be used to form sets that are occasionally of use.

k	number	$\bar{y}_{1\cdot}$	$\bar{y}_{2\cdot}$	$\bar{y}_{3\cdot}$	$\bar{y}_{4\cdot}$	$\bar{y}_{5\cdot}$
2	1	+1	−1			
3	1	+1	−1	0		
	2	+1	+1	−2		
4	1	+1	−1	0	0	
	2	+1	+1	−2	0	
	3	+1	+1	+1	−3	
5	1	+1	−1	0	0	0
	2	+1	+1	−2	0	0
	3	+1	+1	+1	−3	0
	4	+1	+1	+1	+1	−4

etc.