Two Factor Design (Completely Randomized)

For the fixed effects model, we have

$$Y_{ijl} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijl} \quad \text{where}$$

$i = 1, \ldots, a$	indexes levels of first factor (factor A)
$j = 1, \ldots, b$	indexes levels of second factor (factor B)
$l = 1, \ldots, n$	indexes plots (for each factor combination)
$\varepsilon_{ijl} \sim$ i.i.d. $\mathcal{N}(0, \sigma^2_\varepsilon)$	represents plot error

ANOVA table:

Source	df	SS
A	$a-1$	SSA
B	$b-1$	SSB
AB	$(a-1)(b-1)$	SSAB
Error	$ab(n-1)$	SSError
Total	$abn-1$	SSTot

$$\begin{aligned}
\text{SSA} &= bn\sum_{i=1}^{a}(\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot})^2 \\
\text{SSB} &= an\sum_{j=1}^{b}(\bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot\cdot\cdot})^2 \\
\text{SSAB} &= n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij\cdot} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} + \bar{y}_{\cdot\cdot\cdot})^2 \\
\text{SSError} &= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{l=1}^{n}(y_{ijl} - \bar{y}_{ij\cdot})^2 \text{ or by subtraction} \\
\text{SSTotal} &= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{l=1}^{n}(y_{ijl} - \bar{y}_{\cdot\cdot\cdot})^2
\end{aligned}$$

If we assume $\sum_{i=1}^{a}\alpha_i = 0$, $\sum_{j=1}^{b}\beta_j = 0$, $\forall j$: $\sum_{i=1}^{a}(\alpha\beta)_{ij} = 0$, and $\forall i$: $\sum_{j=1}^{b}(\alpha\beta)_{ij} = 0$, then the expected mean squares are:

$$\begin{aligned}
\mathbb{E}(\text{MSA}) &= \sigma^2_\varepsilon + \frac{bn}{a-1}\sum_i \alpha_i^2 \\
\mathbb{E}(\text{MSB}) &= \sigma^2_\varepsilon + \frac{an}{b-1}\sum_j \beta_j^2 \\
\mathbb{E}(\text{MSAB}) &= \sigma^2_\varepsilon + \frac{n}{(a-1)(b-1)}\sum_{ij}(\alpha\beta)^2_{ij} \\
\mathbb{E}(\text{MSError}) &= \sigma^2_\varepsilon
\end{aligned}$$

Thus, we test each of the main effects and the interaction by using the MS for Error.
The model parameters are estimated by

$$\begin{aligned}
\hat{\mu} &= \bar{y}_{\cdot\cdot\cdot}, \\
\hat{\alpha}_i &= \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot}, \\
\hat{\beta}_j &= \bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot\cdot\cdot}, \\
\widehat{(\alpha\beta)}_{ij} &= \hat{\mu}_{ij} - (\hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j) = \bar{y}_{ij\cdot} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} + \bar{y}_{\cdot\cdot\cdot}
\end{aligned}$$