Whole Plots randomized as a CRD

model:

$$Y_{ijk} = \mu + \alpha_i + \varepsilon_{ij} + \tau_k + (\alpha\tau)_{ik} + \delta_{ijk}$$

where $i = 1, \ldots, a$ indexes the levels of A (say whole-plot treatment)
$k = 1, \ldots, c$ indexes the levels of C (say subplot treatment)
$j = 1, \ldots, b$ indexes whole plots for each whole-plot treatment level
$\varepsilon_{ij} \sim \mathcal{N}(0, \sigma^2_\varepsilon)$ represents whole plot error; and $\delta_{ijk} \sim \mathcal{N}(0, \sigma^2_\delta)$ represents subplot error.

ANOVA

Source	df	SS	MS	$\mathbb{E}$MS
A	$a-1$	SSA	MSA	$\sigma^2_\delta + c\sigma^2_\varepsilon + \frac{bc\sum_i \alpha_i^2}{a-1}$
Whole Plot Error	$a(b-1)$	SSWPErr	MSWPErr	$\sigma^2_\delta + c\sigma^2_\varepsilon$
C	$c-1$	SSC	MSC	$\sigma^2_\delta + \frac{ab\sum_k \tau_k^2}{c-1}$
AC	$(a-1)(c-1)$	SSAC	MSAC	$\sigma^2_\delta + \frac{b\sum_i\sum_k(\alpha\tau)^2_{ik}}{(a-1)(c-1)}$
Sub Plot Error	$a(b-1)(c-1)$	SSSPErr	MSSPErr	σ^2_δ
Total	$abc-1$	SSTot		

The Sub Plot Error is most easily calculated by subtraction. The WPError is equivalent to the SS for whole plots nested in A. In that sense, the WPError reminds us of the sample or plot error in a design with subsampling. All other SS are calculated the usual way.

Tests

To test H_0: $(\alpha\tau)_{ik} = 0$ for all i and k, we use: $F = \frac{\text{MSAC}}{\text{MSSPE}}$

To test H_0: $\tau_k = 0$ for all k, we use: $F = \frac{\text{MSC}}{\text{MSSPE}}$

To test H_0: $\alpha_i = 0$ for all i, we use: $F = \frac{\text{MSA}}{\text{MSWPE}}$

Almost always, tests involving whole plot treatments are less powerful than tests involving subplot treatments.

Means Comparisons

within the same whole-plot treatment:	$\text{var}(\bar{Y}_{1\cdot1} - \bar{Y}_{1\cdot2}) = \frac{2\sigma^2_\delta}{b}$
across whole-plot treatments:	$\text{var}(\bar{Y}_{1\cdot1} - \bar{Y}_{2\cdot1}) = \frac{2(\sigma^2_\delta + \sigma^2_\varepsilon)}{b}$
whole-plot treatments means:	$\text{var}(\bar{Y}_{1\cdot\cdot} - \bar{Y}_{2\cdot\cdot}) = \frac{2(\sigma^2_\delta + c\sigma^2_\varepsilon)}{bc}$
subplot treatments means:	$\text{var}(\bar{Y}_{\cdot\cdot1} - \bar{Y}_{\cdot\cdot2}) = \frac{2\sigma^2_\delta}{ab}$

Whole Plots randomized as an RCBD

model:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} + \tau_k + (\alpha\tau)_{ik} + \delta_{ijk}$$

where $i = 1, \ldots, a$ indexes the levels of A (whole-plot treatment);
$j = 1, \ldots, b$ indexes blocks (1 whole plot per block per whole-plot treatment level);
$k = 1, \ldots, c$ indexes the levels of C (subplot treatment);
$\varepsilon_{ij} \sim \mathcal{N}(0, \sigma^2_\varepsilon)$ represents whole plot error; and $\delta_{ijk} \sim \mathcal{N}(0, \sigma^2_\delta)$ represents subplot error.

ANOVA

Source	df	SS	MS	EMS
Block	$b-1$	SSBlk	MSBlk	$\sigma^2_\delta + c\sigma^2_\varepsilon + \frac{ab\sum_j \beta_j^2}{b-1}$
A	$a-1$	SSA	MSA	$\sigma^2_\delta + c\sigma^2_\varepsilon + \frac{bc\sum_i \alpha_i^2}{a-1}$
WP Error	$(a-1)(b-1)$	SSWPErr	MSWPErr	$\sigma^2_\delta + c\sigma^2_\varepsilon$
C	$c-1$	SSC	MSC	$\sigma^2_\delta + \frac{ab\sum_k \tau_k^2}{c-1}$
AC	$(a-1)(c-1)$	SSAC	MSAC	$\sigma^2_\delta + \frac{b\sum_i\sum_k(\alpha\tau)^2_{ik}}{(a-1)(c-1)}$
SP Error	$a(b-1)(c-1)$	SSSPErr	MSSPErr	σ^2_δ
Total	$abc-1$	SSTot		

- The SSWPError is identical to the SS for the A by Block interaction (which is always the case in a block design).
- The df for SPError can also be written as:

  $$a(b-1)(c-1) = (b-1)(c-1) + (a-1)(b-1)(c-1)$$

  This gives a hint of an alternative way to find the SSSPError.
- Tests and means comparisons are the same regardless of whether the whole plots are randomized as a CRD or as an RCBD.