One-Way Random Effects Model

A one-way fixed effects model looks like: $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ where the α_i represent fixed effects, e.g. specific treatments or groups.

One-way random effects model

$$Y_{ij} = \mu + A_i + \varepsilon_{ij}$$

where

$i = 1, \ldots, k$	indexes treatments (or groups)
$j = 1, \ldots, n$	indexes experimental units (plots) for each treatment
$A_i \sim \mathcal{N}(0, \sigma_A^2)$	corresponds to the random effect (varying from group to group)
$\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$	corresponds to variation within each group

ANOVA table

Source	df	SS	MS	$\mathbb{E}$(MS)
Trts (Groups)	$k-1$	$n\sum_i (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$	MSTrt	$\sigma_\varepsilon^2 + n\sigma_A^2$
Error	$k(n-1)$	$\sum_{i,j} (y_{ij} - \bar{y}_{i\cdot})^2$	MSErr	σ_ε^2
Total	$kn-1$	$\sum_{i,j} (y_{ij} - \bar{y}_{\cdot\cdot})^2$		

Notes

- The sums of squares are exactly the same as for the one-way fixed effects design.
- In the random effects model, the $\{A_i\}$ represent a sample from some population $\mathcal{N}(0, \sigma_A^2)$ which we are interested in.
- We test H_0: $\sigma_A^2 = 0$ vs H_A: $\sigma_A^2 > 0$ using $F = \text{MSTrt}/\text{MSError}$, analogous to the fixed effects case.
- When the focus is on subsampling, the model is sometimes written

$$Y_{ij} = \mu + \varepsilon_i + \delta_{ij} \quad \text{where } \varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2) \quad \text{and } \delta_{ij} \sim \mathcal{N}(0, \sigma_\delta^2).$$