model:

$$Y_{ijl} = \mu + \alpha_i + \varepsilon_{ij} + \delta_{ijl}$$

where

$i = 1, \ldots, k$	indexes treatment levels
$j = 1, \ldots, n$	indexes experimental units (plots) for each treatment
$l = 1, \ldots, s$	indexes subsamples (within each plot)
$\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$	corresponds to plot error, e.g. variation from plot to plot
$\delta_{ijl} \sim \mathcal{N}(0, \sigma_\delta^2)$	corresponds to subsample error, e.g. variation between subsamples

ANOVA table

Source	df	SS	MS	$\mathbb{E}$(MS)
Treatment	$k-1$	SSTrt	MSTrt	$\sigma_\delta^2 + s\sigma_\varepsilon^2 + ns\sum_{i=1}^k \alpha_i^2/(k-1)$
Plot Error	$k(n-1)$	SSPE	MSPE	$\sigma_\delta^2 + s\sigma_\varepsilon^2$
Subsampling Error	$kn(s-1)$	SSSSE	MSSSE	σ_δ^2
Total	$kns-1$	SSTot		

where

$$\begin{aligned}
\text{SSTrt} &= sn\sum_{i=1}^{k}(\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot})^2 \\
\text{SSPE} &= s\sum_{i=1}^{k}\sum_{j=1}^{n}(\bar{y}_{ij\cdot} - \bar{y}_{i\cdot\cdot})^2 \\
\text{SSSSE} &= \sum_{i=1}^{k}\sum_{j=1}^{n}\sum_{l=1}^{s}(y_{ijl} - \bar{y}_{ij\cdot})^2 \\
\text{SSTot} &= \sum_{i=1}^{k}\sum_{j=1}^{n}\sum_{l=1}^{s}(y_{ijl} - \bar{y}_{\cdot\cdot\cdot})^2
\end{aligned}$$

Note:

- The LSD for comparing two treatment means is given by:

$$\text{LSD} = T_{k(n-1),\alpha/2}\sqrt{\text{MSPE}}\sqrt{\frac{2}{ns}}$$

- We test $H_0 : \alpha_i = 0$ for all i using

$$F = \frac{\text{MSTrt}}{\text{MSPE}}$$

  and compare with $F_{k-1,k(n-1)}$.