Assumptions

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Assumptions: Overview

- Statistical methods for statistical inference (e.g., hypothesis testing and confidence intervals) rely on assumptions.
- A statistical method is said to be **robust** against a failure in an assumption if the method does not depend critically on that assumption.
- That is, if the assumption does not hold exactly, a **robust method** gives results that are still a good approximation to the correct answer.
- Otherwise, we say that the method is **not robust** against a failure in an assumption or is **sensitive** to the assumption.

One-sample T Test

- Assumptions: The observations Y_i are independent and are from a normal population.
- Independence:
  - The one-sample T test is sensitive to the independence assumption.
  - Possible outcomes if $\text{Cov}(Y_i, Y_j) > 0$: higher false positive rates. Why? Under-estimated standard errors $\rightarrow$ inflated statistics.
- Normality:
  - The one-sample T test is robust against non-normality.
  - If the population is not exactly normal but symmetric, the T test still gives a good approximation to the correct result. Why? CLT
- Outlier: The one-sample T test is sensitive to outliers. Why not? Mean

Paired T Test

- The paired T test can be viewed as a one-sample T test based on the differences of the original paired two samples.
- Assumptions of D_i: Same as the one-sample T test (i.e., independence and normality).
- The test is robust against non-normality, but not against dependence.
- Assumptions of (Y_{1i}, Y_{2i}):
  - The pairs are independent of each other pair, but not between Y_{1i} and Y_{2i}.
  - No explicit assumptions about the distributions of Y_{1i} and Y_{2i}.
  - Assumption made on $Y_{1i} - Y_{2i}$.

Unpaired Two Sample T Test

- Assumptions: Normality for each sample, equal variance, independence within each sample, and independence between the two samples.
- The test is robust against non-normality, especially if the two populations being sampled are symmetric.
- The test is not robust against dependence.
- The relative difference in sample sizes plays a role in the robustness of the equal variance assumption.
- If the sample sizes are approximately equal, then the T test is robust against differences in variance.
- Outlier: not resistant against outliers

Assessment of Assumptions

- Assessing independence
- Assessing normality
- Assessing equal variance

Assessing Independence

- Consider how the data were collected; that is, the study or experimental design.
- Paired two sample versus unpaired two sample studies.
- The notion of independent observations is closely related to the idea of an i.i.d. sample.
- Examples of dependence: within a cluster, across space, or over time.
- Not always obvious, as the answer depends on the kinds of scientific questions of interest and the corresponding populations under study.

Assessing Normality

- Histogram
- Difficulties
  - Small sample size: Unreliable representation of the population.
  - How to distinguish between bell-shaped and mound-shaped, but non-normal distribution?

QQ Plot

- A **quantile-quantile (QQ) plot** is more reliable tool to assess normality.
- A QQ plot is also known as a **quantile comparison plot**.
- How to construct a QQ plot for a sample $y_1, y_2, \ldots, y_n$?
  (1) Sort the observations in ascending order:

  $$y_{(1)}, y_{(2)}, \cdots, y_{(n)}.$$

  (2) Compute quantiles of $N(0, 1)$:

  $$z_{[(1-.5)/n]}, z_{[(2-.5)/n]}, \cdots, z_{[(n-.5)/n]}.$$

  (3) Plot pairs of

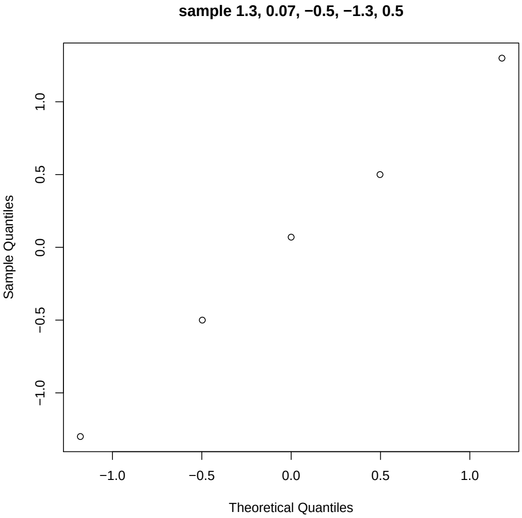
  $$(z_{[(1-.5)/n]}, y_{(1)}), (z_{[(2-.5)/n]}, y_{(2)}), \ldots, (z_{[(n-.5)/n]}, y_{(n)}).$$

Example: One Sample of Size 5

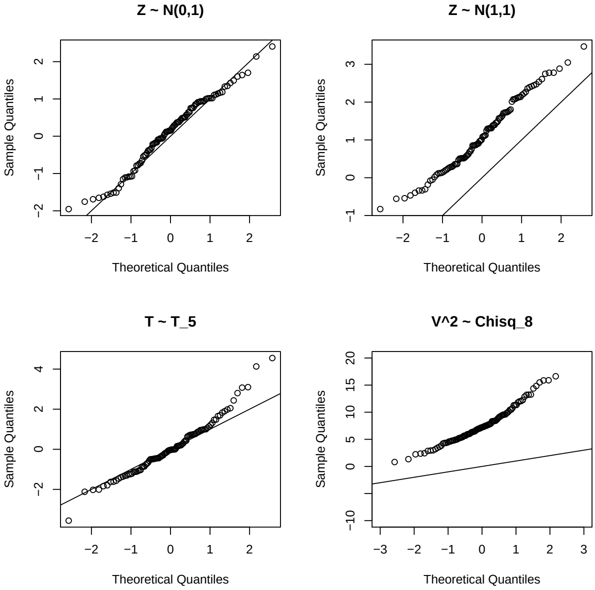
- A small sample of size 5: $1.3, 0.07, -0.5, -1.3, 0.5$.
  (1) Sort the observations in ascending order:
  $$-1.3, -0.5, 0.07, 0.5, 1.3$$
  (2) Compute quantiles of $N(0,1)$:
  $$z_{[0.1]} = -1.28, z_{[0.3]} = -0.52, z_{[0.5]} = 0, z_{[0.7]} = 0.52, z_{[0.9]} = 1.28$$
  - R code: qqnorm( ); compare to 45^o line.

Example: One Sample of Size 5

(3) Plot the quantile pairs:



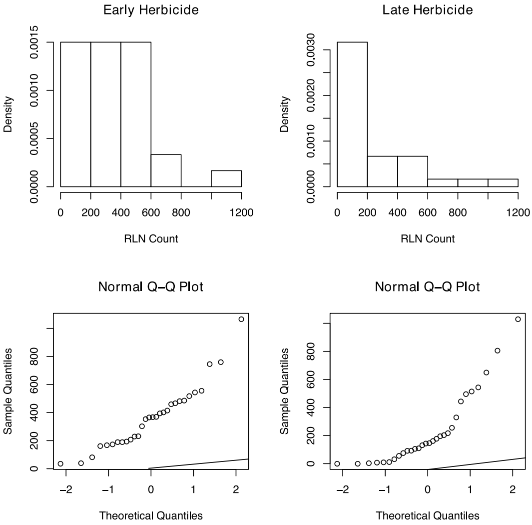
Example: Samples of Size 100



Assessing Equal Variance

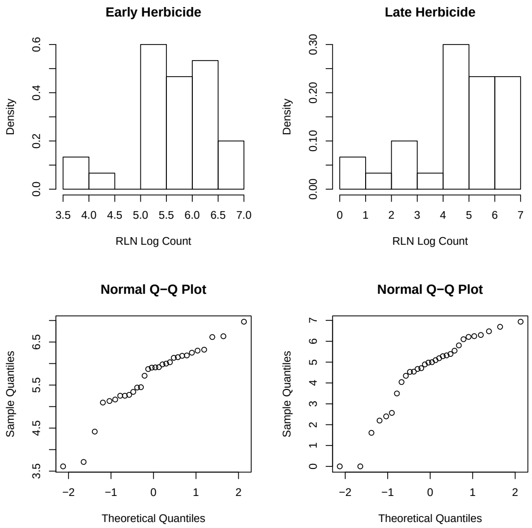
- Equal variance is also called **homoscedasticity** and unequal variance **heteroscedasticity**.
- For an unpaired T test:
  - Robust against unequal variances if the sample sizes are roughly equal.
  - Rule of thumb: If the ratio of s_1^2 and s_2^2 is within two or three, there is probably little need to pursue a formal test.
- Graphical methods: histogram, (side-by-side) box plot.

Example: RLN Count



Normality holds?

Example: RLN ln(Count)



Example: RLN ln(Count)

- Apply a log transformation to the count data: $n_1 = n_2 = 30$ and

$$\bar{y}_1^\dagger = 6.67, \bar{y}_2^\dagger = 4.55, (s_1^\dagger)^2 = 0.60, (s_2^\dagger)^2 = 3.32$$

- Expected log counts: $\mu_1^\dagger$ versus $\mu_2^\dagger$.
- $H_0 : \mu_1^\dagger = \mu_2^\dagger$ vs. $H_\alpha : \mu_1^\dagger \neq \mu_2^\dagger$.
- Two sample test on log-transformed data.

Remarks on Transformation

- Transform the data so that the transformed data might align better with the assumptions than the original data.
- Transformation affects normality and equal variance.
- For count data, log or square root transformations are common.
- Caution: Transform so that the assumptions of the analysis are better met, not that the p-value is the smaller.