Fixed effect model

Consider two (fixed) experimental factors A and B randomized in a balanced CRD. The model is:

$$Y_{ijl} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijl}$$

The α_i, β_j and $(\alpha\beta)_{ij}$ represent fixed effects for A, B, and the A $\times$ B interaction. The only random part of the model is the error term: $\varepsilon_{ijl} \sim \mathcal{N}(0, \sigma_e^2)$. We also assume: $\sum_i \alpha_i = 0$, $\sum_j \beta_j = 0$, $\sum_i (\alpha\beta)_{ij} = 0$, and $\sum_j (\alpha\beta)_{ij} = 0$. For this model, the ANOVA table looks like:

Source	df	SS	$\mathbb{E}$(MS)
A	$a-1$	SSA	$\sigma_\varepsilon^2 + bn \dfrac{\sum \alpha_i^2}{a-1}$
B	$b-1$	SSB	$\sigma_\varepsilon^2 + an \dfrac{\sum \beta_j^2}{b-1}$
AB	$(a-1)(b-1)$	SSAB	$\sigma_\varepsilon^2 + n \dfrac{\sum (\alpha\beta)_{ij}^2}{(a-1)(b-1)}$
Error	$ab(n-1)$	SSErr	σ_ε^2
Total	$abn-1$		

Thus, we test each of the main effects and the interaction by using the MS for Error.

Random effect model

The two-factor random effects model looks like: $Y_{ijl} = \mu + A_i + B_j + (AB)_{ij} + \varepsilon_{ijl}$ where all terms other than μ are random: $A_i \sim \mathcal{N}(0, \sigma_A^2)$, $B_j \sim \mathcal{N}(0, \sigma_B^2)$, $(AB)_{ij} \sim \mathcal{N}(0, \sigma_{AB}^2)$ and $\varepsilon_{ijl} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, all independent. For this model, we have

$$\begin{aligned}
\mathbb{E}(\text{MSA}) &= \sigma_\varepsilon^2 + n\sigma_{AB}^2 + bn\sigma_A^2 \\
\mathbb{E}(\text{MSB}) &= \sigma_\varepsilon^2 + n\sigma_{AB}^2 + an\sigma_B^2 \\
\mathbb{E}(\text{MSAB}) &= \sigma_\varepsilon^2 + n\sigma_{AB}^2 \\
\mathbb{E}(\text{MSErr}) &= \sigma_\varepsilon^2
\end{aligned}$$

It may be surprising to see that the expected mean squares for A and for B include terms involving σ_{AB}^2.

Mixed effect model

If A is a fixed effect, and B is a random effect then the model is: $Y_{ijl} = \mu + \alpha_i + B_j + (AB)_{ij} + \varepsilon_{ijl}$ where $\sum \alpha_i = 0$, $B_j \sim \mathcal{N}(0, \sigma_B^2)$, $(AB)_{ij} \sim \mathcal{N}(0, \sigma_{AB}^2)$ and $\varepsilon_{ijl} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, all independent. With this model we have:

$$\begin{aligned}
\mathbb{E}(\text{MSA}) &= \sigma_\varepsilon^2 + n\sigma_{AB}^2 + bn \frac{\sum \alpha_i^2}{a-1} \\
\mathbb{E}(\text{MSB}) &= \sigma_\varepsilon^2 + n\sigma_{AB}^2 + an\sigma_B^2 \\
\mathbb{E}(\text{MSAB}) &= \sigma_\varepsilon^2 + n\sigma_{AB}^2 \\
\mathbb{E}(\text{MSErr}) &= \sigma_\varepsilon^2
\end{aligned}$$