# Model Families for Correlated Data

# LMMs: Conditional Model

Conditional/Hierarchical specification of LMM

$$Y_{ij} = \mathbf{X}_{ij}^T \boldsymbol{\beta} + \mathbf{Z}_{ij}^T \mathbf{b}_i + \epsilon_{ij}$$

- $Y_{ij}$ : the *jth* outcome of the *ith* subject.
- $\beta$  : regression coefficient vector  $(p \times 1)$ .
- $\mathbf{b}_i$ : random effects for the *ith* subject,  $\mathbf{b}_i \sim N\{0, \mathbf{D}(\boldsymbol{\theta})\}$
- $\theta$  is a  $q \times 1$  vector of variance components.
- $\epsilon_{ij}$ : residual, and  $\epsilon_i = (\epsilon_{i1}, \cdots, \epsilon_{in_i})^T \sim N\{0, \mathbf{R}(\boldsymbol{\theta})\}.$
- $(X_{ij}, Z_{ij})$ : covariate design matrices.

## Equivalently:

$$\mathbf{Y}_i | \mathbf{b}_i \sim N(\mathbf{X}_i \boldsymbol{\beta} + \mathbf{Z}_i \mathbf{b}, \mathbf{R}_i)$$
  
 $\mathbf{b}_i \sim N(\mathbf{0}, \mathbf{D})$ 

# LMMs: Marginal Model

$$f_i(\mathbf{y}) = \int f_i(\mathbf{y}_i|\mathbf{b}_i)f(\mathbf{b}_i)d\mathbf{b}_i$$

Then the marginal model is

$$\mathbf{Y}_i \sim N(\mathbf{Z}_i \boldsymbol{eta}, \mathbf{Z}_i \mathbf{D} \mathbf{Z}_i' + \mathbf{R})$$

- Estimation and Inference are derived from the marginal model
- Nearly seamless/interchangeable with conditional model
  - Some constraints on the variance components

# Model Families: Gaussian Case

### Marginal Model:

$$E[Y_{ij}|\mathbf{X}_{ij}] = \mathbf{X}_{ij}\boldsymbol{\beta} \tag{1}$$

#### Conditional Model:

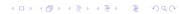
$$E[Y_{ij}|\mathbf{b}_i,\mathbf{X}_{ij}] = \mathbf{X}_{ij}\boldsymbol{\beta} + \mathbf{Z}_{ij}\mathbf{b}_i$$
 (2)

#### Transition Model:

$$E[Y_{ij}|Y_{i,j-1},\ldots,Y_{i,1},\mathbf{X}_{ij}] = \mathbf{X}_{ij}\boldsymbol{\beta} + \alpha Y_{i,j-1}$$
(3)

- (2) follows directly from (1)  $\Rightarrow \beta$  has marginal AND conditional interpretation, simultaneously
  - Marginalize over  $\mathbf{b}_i$  or condition on  $\mathbf{b}_i = \mathbf{0}$

Non-normal data: Connection between marginal/conditional models is no longer straightforward!



## Model Families: General Case

### Marginal Model:

- Responses modeled marginalized over all other responses
- (usually) GEEs
- (possibly) likelihood based models

### Conditionally Specified Models:

- Responses in sequence are conditioned upon other outcomes
- (e.g.) Transition models

### Subject-Specific (Conditional) Model:

- Responses independent conditionally on subject-specific parameters
- (usually) Mixed models
- (possibly) fixed subject specific effects; conditional logistic model