Advanced Regression Methods for Independent Data

STAT/BIOST 570, 2020

Practical Aspects of Generalized Linear Models

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Generalized Linear Models in R

We'll present some examples to illustrate how to fit and check GLMs using R.

In R, GLMs are implemented in the glm function

```
glm(formula, family = gaussian, data, weights, subset,
    na.action, start = NULL, etastart, mustart, offset,
    control = list(...), model = TRUE, method = "glm.fit",
    x = FALSE, y = TRUE, singular.ok = TRUE, contrasts = NULL, ...)
```

where the family argument is specified as one of

```
binomial(link = "logit") # accepts the links logit, probit, cauchit, log, cloglog
gaussian(link = "identity") # accepts identity, log, inverse
Gamma(link = "inverse") # accepts inverse, identity, log
inverse.gaussian(link = "1/mu^2") # accepts 1/mu^2, inverse, identity, log
poisson(link = "log") # accepts log, identity, sqrt
```

Pseudo-examples:

```
glm(y^x1+x2), family = Gamma) # Gamma GLM with inverse link (default) glm(y^x1+x2), family = Gamma(link="log")) # Gamma GLM with log link glm(y^x1+x2), family = poisson) # Poisson GLM with log link (default)
```

- FEV: forced expiratory volume. FEV1: amount of air you can force from your lungs in one second.
- Data from 654 children and youths ages 3–19 in East Boston, 1980. (Childhood Respiratory Disease Study).
- For more information visit: http://www.statsci.org/data/general/fev.html

```
> url <- "http://www.statsci.org/data/general/fev.txt"
> data <- read.table(file = url, header = T, sep="\t", stringsAsFactors = F)
> data$Sex <- factor(data$Sex, levels=c("Male", "Female"), labels=c(0,1))
> data$Smoker <- factor(data$Smoker, levels=c("Non", "Current"), labels=c(0,1))
> data$HeightC <- data$Height - 60
> data$AgeC <- data$Age - 10
> data$HeightCSq <- data$HeightC^2</pre>
```

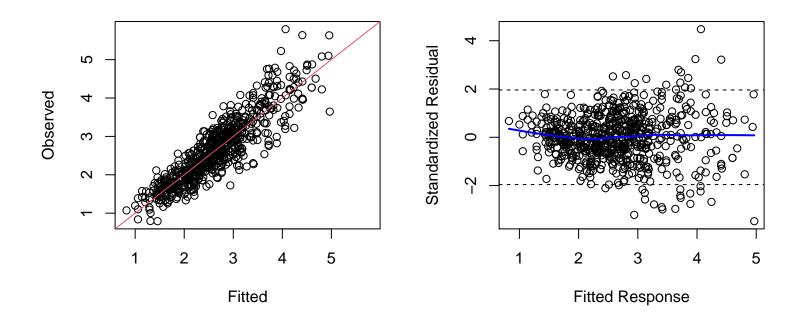
Based on an exploratory analysis, we last looked at a normal linear model with regression function:

$$\begin{split} \mathsf{E}(\mathsf{FEV1} \mid \mathbf{X}) = & \beta_0 + \beta_1 \mathsf{Height} + \beta_2 \mathsf{Height}^2 + \beta_3 \mathsf{Age} + \\ & (\beta_4 + \beta_5 \mathsf{Height} + \beta_6 \mathsf{Height}^2 + \beta_7 \mathsf{Age}) I(\mathsf{Sex=Female}) + \\ & (\beta_8 + \beta_9 \mathsf{Height} + \beta_{10} \mathsf{Height}^2 + \beta_{11} \mathsf{Age})) I(\mathsf{Smoker=Yes}) \end{split}$$

```
> model3 <- lm(
+ FEV ~ (HeightC+HeightCSq+AgeC)*(Sex+Smoker),
+ data=data)</pre>
```

We were mainly concerned about the assumption of homoskedasticity

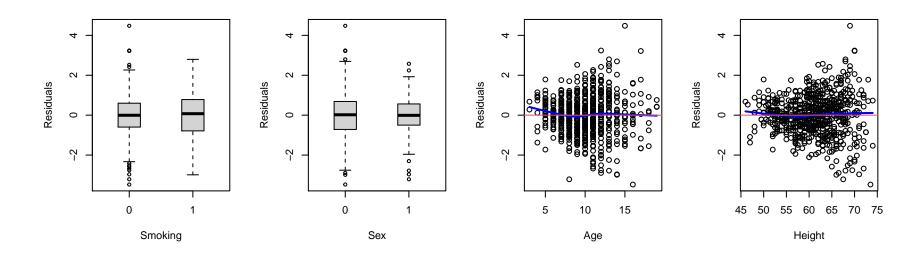
```
> limits3 <- range(data$FEV, fitted(model3))
> par(mfrow = c(1,2))
> plot(fitted(model3), data$FEV, xlab="Fitted", ylab="Observed", xlim=limits3, ylim=limits3)
> abline(a=0, b=1, col=2)
> library(MASS)
> resid_mod3 <- stdres(model3)
> plot(fitted(model3), stdres(model3),
+ ylab = "Standardized Residual", xlab = "Fitted Response")
> lines(lowess(fitted(model3), resid_mod3), lwd = 2, col = "blue")
> abline(h = 1.96, lty = 2)
> abline(h = -1.96, lty = 2)
```



Lack of homoscedasticity seems to be the main issue here, so the normal linear model doesn't hold

Residuals vs covariates: variance of response is not constant

```
> par(mfrow = c(1,4))
> plot(resid_mod3 ~ data$Smoker, xlab = "Smoking", ylab = "Residuals",
+ col = 'lightgray', boxwex = .35)
> plot(resid_mod3 ~ data$Sex, xlab = "Sex", ylab = "Residuals",
+ col = 'lightgray', boxwex = .35)
> plot(data$Age, resid_mod3, xlab = "Age", ylab = "Residuals",)
> lines(lowess(data$Age, resid_mod3), lwd = 2, col = "blue")
> abline(h=0, col=2)
> plot(data$Height, resid_mod3, xlab = "Height", ylab = "Residuals",)
> lines(lowess(data$Height, resid_mod3), lwd = 2, col = "blue")
> abline(h=0, col=2)
```



Remember that under the gamma GLM

$$var(Y_i \mid \mathbf{x}_i) = \alpha E(Y_i \mid \mathbf{x}_i)^2,$$

which might be appropriate for our data

It is OK to use the identity link here since the outcome is away from zero

Let's consider a gamma GLM with identity link:

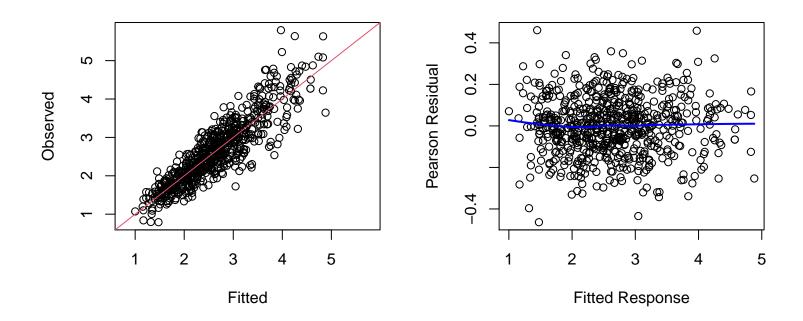
```
> FEV_Gamma <- glm(  + FEV \sim (HeightC+HeightCSq+AgeC)*(Sex+Smoker), \\ + family = Gamma(link="identity"), \\ + data=data)  In this case, \widehat{\alpha} is
```

```
> summary(FEV_Gamma)$dispersion
```

[1] 0.02015759

For the gamma GLM to be appropriate, we need to *not* find patterns in the Pearson residuals

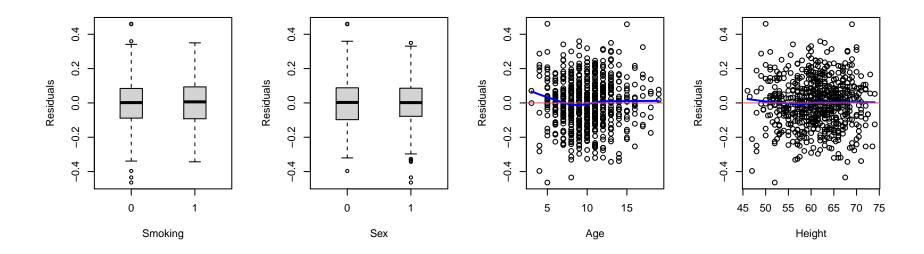
```
> resid_Gamma <- residuals(FEV_Gamma, type = "pearson")
> fitted_Gamma <- fitted(FEV_Gamma)
> limits <- range(data$FEV, fitted_Gamma)
> par(mfrow = c(1,2))
> plot(fitted_Gamma, data$FEV, xlab="Fitted", ylab="Observed", xlim=limits, ylim=limits)
> abline(a=0, b=1, col=2)
> library(MASS)
> plot(fitted_Gamma, resid_Gamma,
+ ylab = "Pearson Residual", xlab = "Fitted Response")
> lines(lowess(fitted_Gamma, resid_Gamma), lwd = 2, col = "blue")
> abline(h = 1.96, lty = 2)
> abline(h = -1.96, lty = 2)
```



Looks pretty good!

Residuals vs covariates look good too:

```
> par(mfrow = c(1,4))
> plot(resid_Gamma ~ data$Smoker, xlab = "Smoking", ylab = "Residuals",
+ col = 'lightgray', boxwex = .35)
> plot(resid_Gamma ~ data$Sex, xlab = "Sex", ylab = "Residuals",
+ col = 'lightgray', boxwex = .35)
> plot(data$Age, resid_Gamma, xlab = "Age", ylab = "Residuals",)
> lines(lowess(data$Age, resid_Gamma), lwd = 2, col = "blue")
> abline(h=0, col=2)
> plot(data$Height, resid_Gamma, xlab = "Height", ylab = "Residuals",)
> lines(lowess(data$Height, resid_Gamma), lwd = 2, col = "blue")
> abline(h=0, col=2)
```



We had the following question: is there any association between FEV and smoking after controlling for other variables in this model?

We can test the hull hypothesis

$$H_0: \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = 0$$

We can do this by comparing FEV_Gamma with a model that does not include the Smoker variable, say FEV_Gamma_red:

```
> FEV_Gamma_red <- glm(
+ FEV ~ (HeightC+HeightCSq+AgeC)*Sex,
+ family = Gamma(link="identity"),
+ data=data)</pre>
```

```
> library(lmtest)
> waldtest(FEV_Gamma_red, FEV_Gamma, test = "Chisq") # Wald test
Wald test
Model 1: FEV ~ (HeightC + HeightCSq + AgeC) * Sex
Model 2: FEV ~ (HeightC + HeightCSq + AgeC) * (Sex + Smoker)
 Res.Df Df Chisq Pr(>Chisq)
    646
1
    642 4 3.0416
                      0.5509
> anova(FEV_Gamma_red, FEV_Gamma, test = "Rao") # Score test
Analysis of Deviance Table
Model 1: FEV ~ (HeightC + HeightCSq + AgeC) * Sex
Model 2: FEV ~ (HeightC + HeightCSq + AgeC) * (Sex + Smoker)
 Resid. Df Resid. Dev Df Deviance Rao Pr(>Chi)
       646
               13.462
       642 13.404 4 0.058237 0.056386 0.5923
```

```
> anova(FEV_Gamma_red, FEV_Gamma, test = "LR") # LR test
Analysis of Deviance Table
Model 1: FEV ~ (HeightC + HeightCSq + AgeC) * Sex
Model 2: FEV ~ (HeightC + HeightCSq + AgeC) * (Sex + Smoker)
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)
        646
                13.462
        642
                13.404 4 0.058237 0.5766
> # LR test 'by hand', from the individual model deviances
> (dev_MOvsM1 <- FEV_Gamma_red$deviance - FEV_Gamma$deviance)</pre>
[1] 0.05823693
> (scaled_dev_MOvsM1 <- dev_MOvsM1/summary(FEV_Gamma)$dispersion)</pre>
[1] 2.889083
> 1-pchisq(scaled_dev_MOvsM1, 4)
[1] 0.5765554
```

Therefore the larger model is *not* a better fit, and so we lack evidence that FEV is associated with smoking status after controlling for other variables in this model.

A Count Data Example

We will use the data from Deb and Trivedi (1997), Demand for Medical Care by the Elderly: A Finite Mixture Approach, Journal of Applied Econometrics.

Data on 4406 individuals, aged 66 and over, who are covered by Medicare (a public insurance program in the US for people over 65). Taken from the National Medical Expenditure Survey (NMES, 1987 and 1988)

These data were also used by Zeileis (2006) Object-oriented Computation of Sandwich Estimators, Journal of Statistical Software, which describes some functionality of the sandwich R package

You can download the data from https://www.jstatsoft.org/article/view/v016i09/0

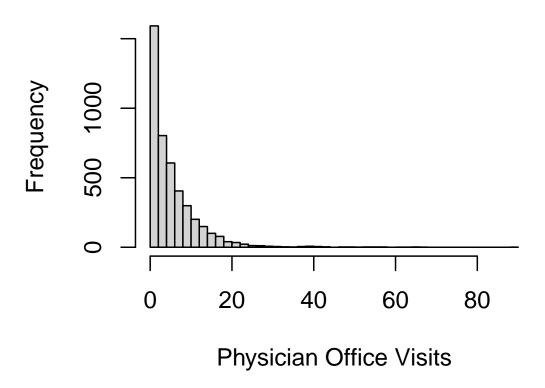
A Count Data Example

From Deb and Trivedi (1997):

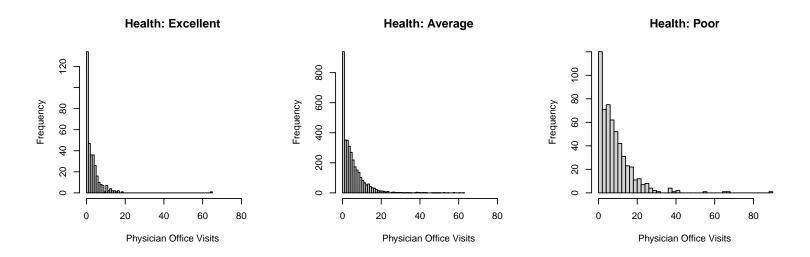
```
OFP
                # of physician office visits
                # of non-physician office visits
OFNP
OPP
                # of physician hospital outpatient visits
OPNP
                # of non-physician hospital outpatient visits
EMR
                # of emergency room visits
HOSP
                # of hospital stays
EXCLHLTH
                = 1 if self-perceived health is excellent
POORHLTH
                = 1 if self-perceived health is poor
NUMCHRON
                # of chronic conditions (cancer, heart attack, gall bladder problems,
                emphysema, arthritis, diabetes, other heart disease)
ADLDIFF
                = 1 if the person has a condition that limits activities of daily living
NOREAST
                = 1 if the person lives in northeastern US
MIDWEST
                = 1 if the person lives in the midwestern US
WEST
                = 1 if the person lives in the western US
AGF
                age in years (divided by 10)
BLACK
                = 1 if the person is African American
MALE
                = 1 if the person is male
MARRIED
                = 1 if the person is married
SCHOOL
                # of years of education
FAMINC
                family income in $10000
EMPLOYED
                = 1 if the person is employed
PRIVINS
                = 1 if the person is covered by private health insurance
MEDICAID
                = 1 if the person is covered by Medicaid
```

```
> load("DebTrivedi.rda")
> str(DebTrivedi)
'data.frame': 4406 obs. of 19 variables:
$ ofp : int 5 1 13 16 3 17 9 3 1 0 ...
$ ofnp : int 0 0 0 0 0 0 0 0 0 ...
$ opp : int 0 2 0 5 0 0 0 0 0 ...
$ opnp : int 0 0 0 0 0 0 0 0 0 ...
$ emer : int 0 2 3 1 0 0 0 0 0 ...
$ hosp : int 1 0 3 1 0 0 0 0 0 0 ...
$ health : Factor w/ 3 levels "average", "excellent", ..: 1 1 3 3 1 3 1 1 1 1 ...
 $ numchron: int 2 2 4 2 2 5 0 0 0 0 ...
 $ adldiff : Factor w/ 2 levels "no", "yes": 1 1 2 2 2 2 1 1 1 1 ...
 $ region : Factor w/ 4 levels "midwest", "noreast", ...: 3 3 3 3 3 1 1 1 1 ...
 $ age
          : num 6.9 7.4 6.6 7.6 7.9 6.6 7.5 8.7 7.3 7.8 ...
$ black : Factor w/ 2 levels "no", "yes": 2 1 2 1 1 1 1 1 1 1 ...
 $ gender : Factor w/ 2 levels "female", "male": 2 1 1 2 1 1 1 1 1 1 ...
 $ married : Factor w/ 2 levels "no", "yes": 2 2 1 2 2 1 1 1 1 1 ...
 $ school : int 6 10 10 3 6 7 8 8 8 8 ...
 $ faminc : num 2.881 2.748 0.653 0.659 0.659 ...
 $ employed: Factor w/ 2 levels "no", "yes": 2 1 1 1 1 1 1 1 1 1 ...
 $ privins : Factor w/ 2 levels "no", "yes": 2 2 1 2 2 1 2 2 2 ...
 $ medicaid: Factor w/ 2 levels "no","yes": 1 1 2 1 1 2 1 1 1 1 ...
Zeileis (2006) used the model
ofp ~ health + age + gender + married + faminc + privins
and so will focus on these variables for illustrative purposes
```

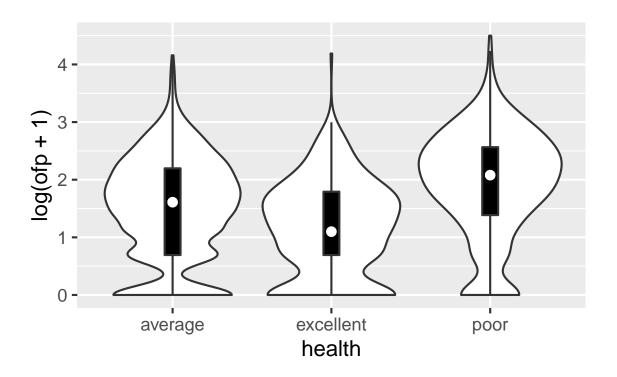
> hist(DebTrivedi\$ofp, 50, xlab="Physician Office Visits", main="")



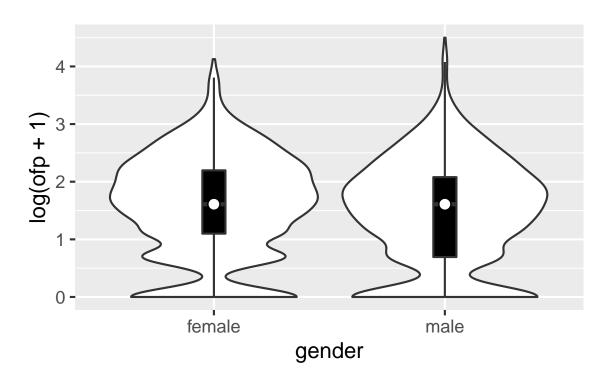
```
> par(mfrow=c(1,3))
> ofp_range <- range(DebTrivedi$ofp); xlab <- "Physician Office Visits"
> with(DebTrivedi,
+ {hist(ofp[health=="excellent"], 50, xlab=xlab, xlim=ofp_range, main="Health: Excellent")
+ hist(ofp[health=="average"], 50, xlab=xlab, xlim=ofp_range, main="Health: Average")
+ hist(ofp[health=="poor"], 50, xlab=xlab, xlim=ofp_range, main="Health: Poor")
+ })
```



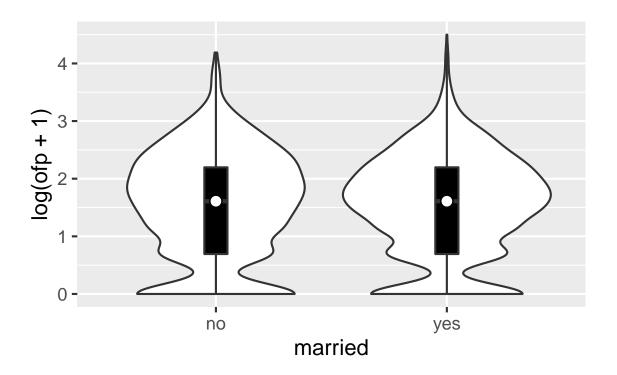
```
> ggplot(DebTrivedi, aes(x = health, y = log(ofp+1))) +
+ geom_violin() +
+ geom_boxplot(width = .1, fill = "black", outlier.colour = NA) +
+ stat_summary(fun.y = median, geom = "point", fill = "white", shape = 21, size = 2.5)
```



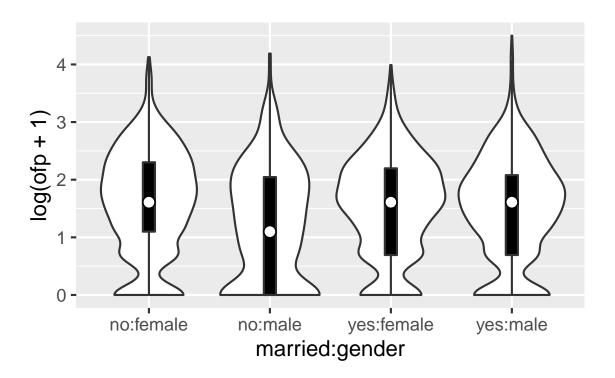
```
> ggplot(DebTrivedi, aes(x = gender, y = log(ofp+1))) +
+ geom_violin() +
+ geom_boxplot(width = .1, fill = "black", outlier.colour = NA) +
+ stat_summary(fun.y = median, geom = "point", fill = "white", shape = 21, size = 2.5)
```

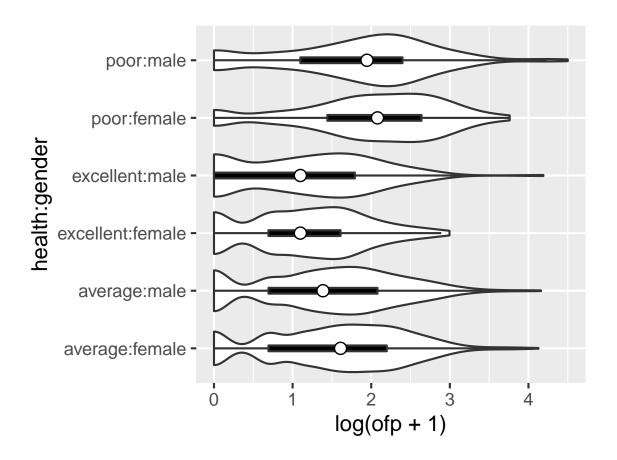


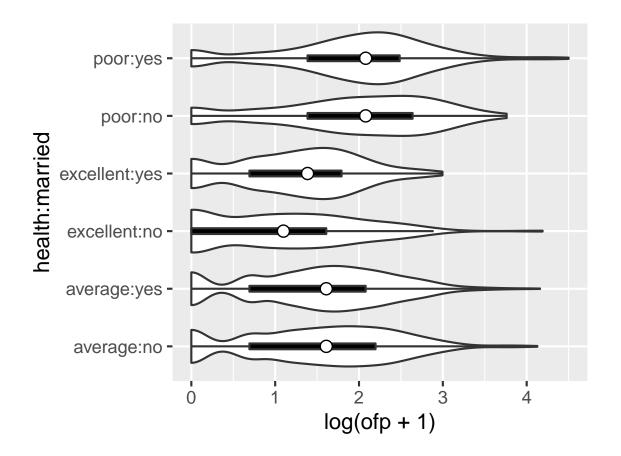
```
> ggplot(DebTrivedi, aes(x = married, y = log(ofp+1))) +
+ geom_violin() +
+ geom_boxplot(width = .1, fill = "black", outlier.colour = NA) +
+ stat_summary(fun.y = median, geom = "point", fill = "white", shape = 21, size = 2.5)
```



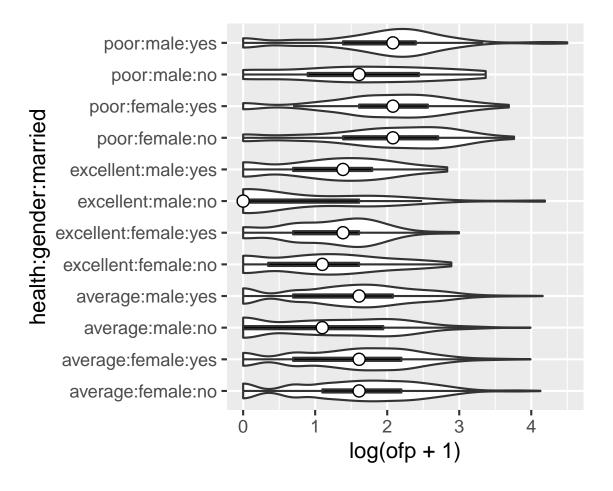
```
> ggplot(DebTrivedi, aes(x = married:gender, y = log(ofp+1))) +
+ geom_violin() +
+ geom_boxplot(width = .1, fill = "black", outlier.colour = NA) +
+ stat_summary(fun.y = median, geom = "point", fill = "white", shape = 21, size = 2.5)
```







```
> ggplot(DebTrivedi, aes(x = health:gender:married, y = log(ofp+1))) + geom_violin() +
+ geom_boxplot(width = .1, fill = "black", outlier.colour = NA) +
+ stat_summary(fun.y = median, geom = "point", fill = "white", shape = 21, size = 2.5) +
+ coord_flip()
```

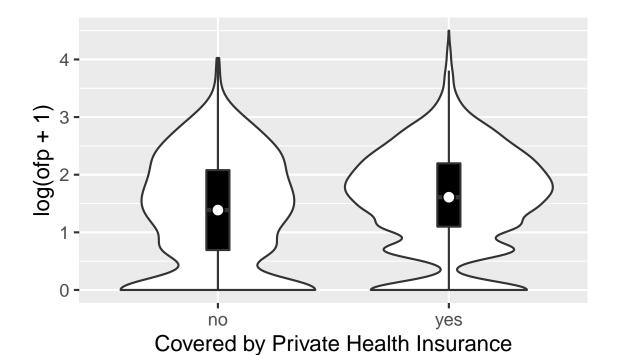


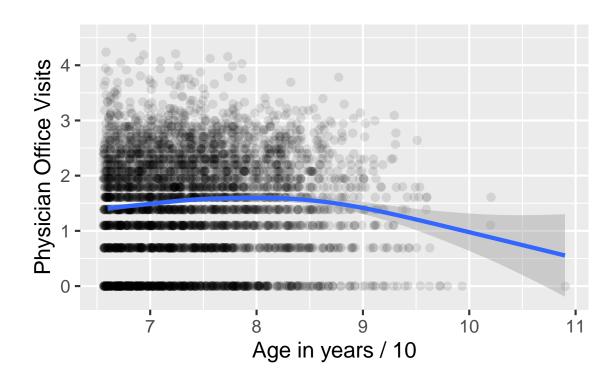
```
> ggplot(DebTrivedi, aes(x = privins, y = log(ofp+1))) +

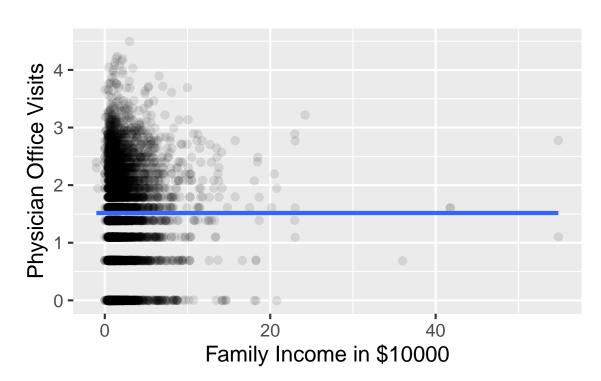
+ geom_violin() + geom_boxplot(width = .1, fill = "black", outlier.colour = NA) +

+ stat_summary(fun.y = median, geom = "point", fill = "white", shape = 21, size = 2.5) +

+ xlab("Covered by Private Health Insurance")
```







Fitting Generalized Linear Models

We will start with a Poisson GLM.

The following mean model seems reasonable given our previous data exploration

```
> DebTrivedi <- within(DebTrivedi, married <- relevel(married, ref = 2))
> DebTrivedi$age2 <- DebTrivedi$age^2
> glm_pois <- glm(ofp ~ faminc + age + age2 + privins + health*gender*married,
+ data = DebTrivedi, family = poisson(link = "log"))</pre>
```

Fitting Generalized Linear Models

```
> library(lmtest)
> coeftest(glm_pois)
z test of coefficients:
                                       Estimate Std. Error z value Pr(>|z|)
(Intercept)
                                    -5.56485456 0.77334663 -7.1958 6.209e-13 ***
                                     0.00495037 0.00217587 2.2751 0.022899 *
faminc
                                     1.87532273 0.20309799 9.2336 < 2.2e-16 ***
age
                                    -0.12526063 0.01326983 -9.4395 < 2.2e-16 ***
age2
privinsyes
                                     0.28442945
                                                0.01653981 17.1967 < 2.2e-16 ***
healthexcellent
                                    -0.54992640
                                                0.06569215
                                                            -8.3713 < 2.2e-16 ***
healthpoor
                                                0.03482476 15.0660 < 2.2e-16 ***
                                     0.52466828
                                     0.01735944 0.01975549 0.8787 0.379556
gendermale
                                                             8.5655 < 2.2e-16 ***
                                     0.16834455 0.01965372
marriedno
healthexcellent:gendermale
                                     0.16488771 0.08306020
                                                             1.9852 0.047127 *
healthpoor:gendermale
                                     0.00710958 0.04593409
                                                             0.1548 0.876996
healthexcellent:marriedno
                                    -0.07182619 0.08418571 -0.8532 0.393555
healthpoor:marriedno
                                     0.00014626 0.04263093
                                                             0.0034 0.997263
gendermale:marriedno
                                    -0.38182992  0.03675448  -10.3887  < 2.2e-16 ***
healthexcellent:gendermale:marriedno 0.34843884 0.13282644
                                                             2.6233 0.008709 **
healthpoor:gendermale:marriedno
                                     0.07532543 0.07678206
                                                             0.9810 0.326578
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

As expected, one of the three-way interactions turned out to be significant

Comparing Models

We now present likelihood-based tests to test a reduced model.

Say we want to test whether marital status modifies the association of health/gender and the number of visits. In this reduced model the effect of marital status is additive:

```
> glm_pois_red <- glm(ofp ~ faminc + age + age2 + privins + married + health*gender,
         data = DebTrivedi, family = poisson)
> library(lmtest)
> coeftest(glm_pois_red)
z test of coefficients:
                         Estimate Std. Error z value Pr(>|z|)
(Intercept)
                       -5.6176099 0.7719123 -7.2775 3.400e-13 ***
                        0.0046488 0.0021769 2.1355
faminc
                                                    0.03272 *
                        1.8981875 0.2026714 9.3658 < 2.2e-16 ***
age
                       -0.1261455 0.0132412 -9.5267 < 2.2e-16 ***
age2
privinsyes
                        0.2839862 0.0165118 17.1990 < 2.2e-16 ***
marriedno
                        -0.5937832  0.0410910 -14.4505 < 2.2e-16 ***
healthexcellent
healthpoor
                        0.5280521 0.0202545 26.0709 < 2.2e-16 ***
                       gendermale
healthexcellent:gendermale 0.2613004 0.0603016 4.3332 1.469e-05 ***
healthpoor:gendermale
                        0.0015054 0.0331895 0.0454 0.96382
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

Comparing Models

```
> waldtest(glm_pois_red, glm_pois, test = "Chisq") # Wald test
Wald test
Model 1: ofp ~ faminc + age + age2 + privins + married + health * gender
Model 2: ofp ~ faminc + age + age2 + privins + health * gender * married
 Res.Df Df Chisq Pr(>Chisq)
1 4395
2 4390 5 128.41 < 2.2e-16 ***
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
> anova(glm_pois_red, glm_pois, test = "LR") # LR test
Analysis of Deviance Table
Model 1: ofp ~ faminc + age + age2 + privins + married + health * gender
Model 2: ofp ~ faminc + age + age2 + privins + health * gender * married
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)
      4395
                25200
1
2
      4390
                25066 5 134.13 < 2.2e-16 ***
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

Comparing Models

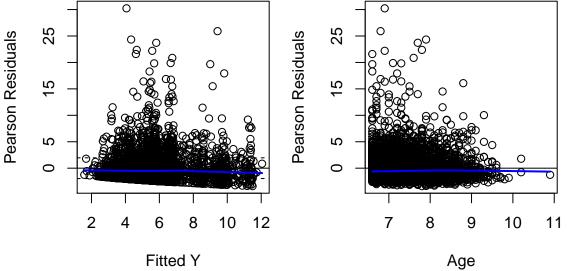
Therefore the larger model is a better fit, which if the model was correct would provide evidence that marital status modifies the association of the some combinations of health/gender and the number of visits.

Note that these tests rely on the Poisson model being correctly specified, in particular its mean/variance relationship

Diagnostics

Something we will want to check are diagnostics, in particular residuals

```
> pearson <- residuals(glm_pois, type = "pearson")
> yhat <- fitted(glm_pois)
> par(mar=c(4,4,1,1))
> par(mfrow = c(1,2))
> plot(yhat, pearson, ylab = "Pearson Residuals", xlab = "Fitted Y")
> lines(lowess(yhat, pearson), lwd = 2, col = "blue")
> abline(h=0); abline(h = 1.96, lty = 2); abline(h = -1.96, lty = 2)
> plot(DebTrivedi$age, pearson, ylab = "Pearson Residuals", xlab = "Age")
> abline(h=0); lines(lowess(DebTrivedi$age, pearson), lwd = 2, col = "blue")
```

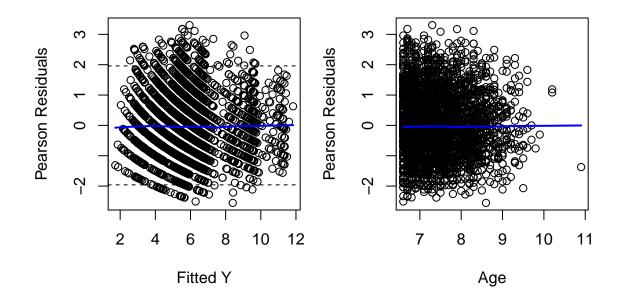


If the Poisson model was correct, how should the residuals look like?

Diagnostics

One way to find out: simulate from a Poisson model as if your estimated model was the true one, fit model with new response and check residuals

Diagnostics



Don't look anything like what we obtained with the actual response variable, so Poisson model doesn't seem OK here