Some Additional Multiple Comparisons Procedures

Suppose there are k treatments, let $t_{ii'} = \dfrac{\bar{y}_{i\cdot} - \bar{y}_{i'\cdot}}{s_\varepsilon\sqrt{\frac{1}{n_i} + \frac{1}{n_{i'}}}}$, and let $n = n_i$ if the design is balanced.

The Holm-Bonferroni method

Put the unadjusted p-values in order from smallest to largest: $p_{(1)} < p_{(2)} < \cdots < p_{(r)}$.

- If $p_{(1)} > \alpha/r$ then accept all null hypotheses. Otherwise, reject the corresponding H_0 and look at $p_{(2)}$.
- If $p_{(2)} > \alpha/(r-1)$ then accept the remaining null hypotheses. Otherwise reject the hypothesis corresponding to $p_{(2)}$ and look at $p_{(3)}$: compare $p_{(3)}$ to $\alpha/(r-2)$, etc.
- In general, at the ith stage, compare $p_{(i)}$ to $\alpha/(r-i+1)$.

Least Significant Difference

- Compare $t_{ii'}$ to $T_{\mathrm{dfErr},\,\alpha/2}$.
- This is equivalent to comparing $|\bar{y}_{i\cdot} - \bar{y}_{i'\cdot}|$ to $T_{\mathrm{dfErr},\,\alpha/2}\, s_\varepsilon\sqrt{2/n}$ if the design is balanced. If the experiment is unbalanced, the harmonic mean of the sample sizes is sometimes used instead: $|\bar{y}_{i\cdot} - \bar{y}_{i'\cdot}|$ is compared to $T_{\mathrm{dfErr},\,\alpha/2}\, s_\varepsilon\sqrt{2/n^*}$ with $(n^*)^{-1} = \frac{1}{k}\sum_{i=1}^k n_i^{-1}$
- Fisher's LSD, also called a Protected LSD, first performs the F test in ANOVA, and proceeds with means comparisons only if the F is significant.

Tukey

- For a balanced experiment, compare $|\bar{y}_{i\cdot} - \bar{y}_{i'\cdot}|$ to $Q_{k,\mathrm{dfErr},\alpha}\, s_\varepsilon\sqrt{1/n}$ where Q is chosen such that

$$\mathbb{P}\left\{\max_{i,i'} \frac{|(\bar{Y}_{i\cdot} - \mu_i) - (\bar{Y}_{i'\cdot} - \mu_{i'})|}{s_\varepsilon/\sqrt{n}} \le Q\right\} = 1 - \alpha\ .$$

- For an unbalanced experiment use the "Tukey-Kramer" method: compare $t_{ii'}$ to $Q_{k,\mathrm{dfErr},\alpha}/\sqrt{2}$ (but note that the quantile Q was derived under a balanced design).

Hayter: Like Fisher's LSD, use an overall F test, but then use $Q_{k-1,\mathrm{dfErr},\alpha}/\sqrt{2}$ instead of $T_{\mathrm{dfErr},\alpha/2}$.

Scheffé: Compare $|t_{ii'}|$ to $\sqrt{(k-1)F_{k-1,\mathrm{dfErr},\alpha}}$.

Contrasts

Scheffé's method and Tukey's method can be extended to constrasts.

- Tukey: $\mathbb{P}\left\{\sum_{i=1}^k c_i\mu_i \in \sum_1^k c_i\bar{Y}_{i\cdot} \pm Q\, s_\varepsilon/\sqrt{n}\sum_1^k |c_i|/2 \quad \forall \mathbf{c} \text{ s.t. } \mathbf{c}'\mathbf{1} = 0\right\} = 1 - \alpha$
- Scheffé: $\mathbb{P}\left\{\sum_{i=1}^k c_i\mu_i \in \sum_1^k c_i\bar{Y}_{i\cdot} \pm \sqrt{(k-1)F_{k-1,\mathrm{dfErr},\alpha}}\, s_\varepsilon\sqrt{\sum c_i^2/n_i} \quad \forall \mathbf{c} \text{ s.t. } \mathbf{c}'\mathbf{1} = 0\right\} = 1 - \alpha$