Randomized Complete Block Design

model:

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

where

$i = 1, \ldots, k$	indexes treatment levels
$j = 1, \ldots, b$	indexes blocks
$\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$	corresponds to plot error, e.g. residual variation

ANOVA table

Source	df	SS	MS	$\mathbb{E}$(MS)
Blocks	$b-1$	$k\sum_j(\bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot})^2$	MSBlk	$\sigma_\varepsilon^2 + k\sum_{j=1}^{b}\beta_j^2/(b-1)$
Treatment	$k-1$	$b\sum_i(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$	MSTrt	$\sigma_\varepsilon^2 + b\sum_{i=1}^{k}\alpha_i^2/(k-1)$
Error	$(k-1)(b-1)$	$\sum_i\sum_j(y_{ij} - \bar{y}_{i\cdot} - \bar{y}_{\cdot j} + \bar{y}_{\cdot\cdot})^2$	MSErr	σ_ε^2
Total	$kb-1$	$\sum_i\sum_j(y_{ij} - \bar{y}_{\cdot\cdot})^2$		

Tests

- We test H_0: $\alpha_i = 0$ for all i using
$$F = \frac{\text{MSTrt}}{\text{MSError}}$$
and compare to an $F_{k-1,(b-1)(k-1)}$ distribution.

- We test H_0: $\beta_j = 0$ for all j using
$$F = \frac{\text{MSBlk}}{\text{MSError}}$$
and compare to an $F_{b-1,(b-1)(k-1)}$ distribution.
Intepretation of the test for H_0: all $\beta_j = 0$ requires care.

Notes:

- Under the usual assumptions $\sum_i \alpha_i = 0$ and $\sum_j \beta_j = 0$ we have $\hat{\mu} = \bar{y}_{\cdot\cdot}$, $\hat{\alpha}_i = \bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot}$ and $\hat{\beta}_j = \bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot}$.

- The word "complete" in the name "randomized complete block design" means that every treatment appears at least once in each block. This is different from the meaning in "completely randomized design" where the word "completely" refers to the fact that the design is randomized as much as possible, subject to the constraint that each treatment must show up a prescribed number of times.

- The "plot error" represents the variability from experimental unit (EU) to EU within the same block and the same treatment, that is, residual variation between EUs not explained by block differences and treatment differences.