If each of the factors A, B, and C has two levels, then there are $2^3 = 8$ possible factor-level combinations, listed below.

abc	ab	ac	bc	a	b	c	1

We now define the respective main effects:

$$\begin{aligned}
\mathrm{A}_{\mathrm{main}} &= \frac{1}{4}\Big[(\overline{abc} - \overline{bc}) + (\overline{ab} - \overline{b}) + (\overline{ac} - \overline{c}) + (\overline{a} - \overline{1})\Big] \\
\mathrm{B}_{\mathrm{main}} &= \frac{1}{4}\Big[(\overline{abc} - \overline{ac}) + (\overline{ab} - \overline{a}) + (\overline{bc} - \overline{c}) + (\overline{b} - \overline{1})\Big] \\
\mathrm{C}_{\mathrm{main}} &= \frac{1}{4}\Big[(\overline{abc} - \overline{ab}) + (\overline{ac} - \overline{a}) + (\overline{bc} - \overline{b}) + (\overline{c} - \overline{1})\Big]
\end{aligned}$$

The two-way interactions are given by

$$\begin{aligned}
\mathrm{AB} &= \frac{1}{4}\Big[\Big\{(\overline{abc} - \overline{bc}) - (\overline{ac} - \overline{c})\Big\} + \Big\{(\overline{ab} - \overline{b}) - (\overline{a} - \overline{1})\Big\}\Big] \\
\mathrm{BC} &= \frac{1}{4}\Big[\Big\{(\overline{abc} - \overline{ac}) - (\overline{ab} - \overline{a})\Big\} + \Big\{(\overline{bc} - \overline{c}) - (\overline{b} - \overline{1})\Big\}\Big] \\
\mathrm{AC} &= \frac{1}{4}\Big[\Big\{(\overline{abc} - \overline{bc}) - (\overline{ab} - \overline{b})\Big\} + \Big\{(\overline{ac} - \overline{c}) - (\overline{a} - \overline{1})\Big\}\Big]
\end{aligned}$$

Finally, we define the three-way interaction:

$$\mathrm{ABC} = \frac{1}{4}\Big[\Big\{(\overline{abc} - \overline{bc}) - (\overline{ac} - \overline{c})\Big\} - \Big\{(\overline{ab} - \overline{b}) - (\overline{a} - \overline{1})\Big\}\Big]$$

General model for a three-factor design:

$$Y_{ijkl} = \mu + \alpha_i + \beta_j + \tau_k + (\alpha\beta)_{ij} + (\alpha\tau)_{ik} + (\beta\tau)_{jk} + (\alpha\beta\tau)_{ijk} + \varepsilon_{ijkl}$$

where

$i = 1, \ldots, a$	indexes the levels of first factor (factor A)
$j = 1, \ldots, b$	indexes the levels of second factor (factor B)
$k = 1, \ldots, c$	indexes the levels of third factor (factor C)
$l = 1, \ldots, n_{ijk}$	indexes plots / observations (within each factor combination)
$\varepsilon_{ijkl} \sim$ i.i.d. $\mathcal{N}(0, \sigma^2_\varepsilon)$	corresponds to plot error / within group variation

ANOVA table

Source	df	SS
A	$a-1$	SSA
B	$b-1$	SSB
C	$c-1$	SSC
AB	$(a-1)(b-1)$	SSAB
AC	$(a-1)(c-1)$	SSAC
BC	$(b-1)(c-1)$	SSBC
ABC	$(a-1)(b-1)(c-1)$	SSABC
Error	$abc(n-1)$	SSError
Total	$abcn-1$	SSTotal

where:

$$\begin{aligned}
\text{SSTotal} &= \sum_{ijkl}(\bar{y}_{ijk\cdot} - \bar{y}_{\cdots\cdot})^2 \\
\text{SSError} &= \sum_{ijkl}(y_{ijkl} - \bar{y}_{ijk\cdot})^2 \\
\text{SSA} &= bcn\sum_{i=1}^{a}(\bar{y}_{i\cdots} - \bar{y}_{\cdots\cdot})^2 \\
\text{SSB} &= acn\sum_{j=1}^{b}(\bar{y}_{\cdot j\cdot\cdot} - \bar{y}_{\cdots\cdot})^2 \\
\text{SSC} &= abn\sum_{k=1}^{c}(\bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\cdots\cdot})^2 \\
\text{SSAB} &= cn\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij\cdot\cdot} - \bar{y}_{i\cdots} - \bar{y}_{\cdot j\cdot\cdot} + \bar{y}_{\cdots\cdot})^2 \\
\text{SSAC} &= bn\sum_{i=1}^{a}\sum_{k=1}^{c}(\bar{y}_{i\cdot k\cdot} - \bar{y}_{i\cdots} - \bar{y}_{\cdot\cdot k\cdot} + \bar{y}_{\cdots\cdot})^2 \\
\text{SSBC} &= an\sum_{j=1}^{b}\sum_{k=1}^{c}(\bar{y}_{\cdot jk\cdot} - \bar{y}_{\cdot j\cdot\cdot} - \bar{y}_{\cdot\cdot k\cdot} + \bar{y}_{\cdots\cdot})^2 \\
\text{SSABC} &= n\sum_{ijkl}(\bar{y}_{ijk\cdot} - \bar{y}_{ij\cdot\cdot} - \bar{y}_{i\cdot k\cdot} - \bar{y}_{\cdot jk\cdot} + \bar{y}_{i\cdots} + \bar{y}_{\cdot j\cdot\cdot} + \bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\cdots\cdot})^2
\end{aligned}$$

and

$$\begin{aligned}
\bar{y}_{i\cdots} &= \frac{1}{bcn}\sum_{jkl} y_{ijkl}, \quad \bar{y}_{\cdot j\cdot\cdot} = \frac{1}{acn}\sum_{ikl} y_{ijkl}, \quad \bar{y}_{\cdot\cdot k\cdot} = \frac{1}{abn}\sum_{ijl} y_{ijkl}, \\
\bar{y}_{ij\cdot\cdot} &= \frac{1}{cn}\sum_{kl} y_{ijkl}, \quad \bar{y}_{i\cdot k\cdot} = \frac{1}{bn}\sum_{jl} y_{ijkl}, \quad \bar{y}_{\cdot\cdot jk} = \frac{1}{an}\sum_{il} y_{ijkl}, \\
\bar{y}_{ijk\cdot} &= \frac{1}{n}\sum_{l} y_{ijkl}, \quad \bar{y}_{\cdots\cdot} = \frac{1}{abcn}\sum_{ijkl} y_{ijkl},
\end{aligned}$$

Note that if we assume $\sum_i \alpha_i = 0$, $\sum_j \beta_j = 0$, $\sum_k \tau_k = 0$, $\sum_i (\alpha\beta)_{ij} = 0$ for each j, $\sum_j (\alpha\beta)_{ij} = 0$ for each i, and similar for $(\alpha\tau)_{ik}$, $(\beta\tau)_{jk}$ and $(\alpha\beta\tau)_{ijk}$, then

$$\begin{aligned}
\hat{\alpha}_i &= \bar{y}_{i\cdots} - \bar{y}_{\cdots\cdot} \\
\hat{\beta}_j &= \bar{y}_{\cdot j\cdot\cdot} - \bar{y}_{\cdots\cdot} \\
\hat{\tau}_k &= \bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\cdots\cdot} \\
\widehat{(\alpha\beta)}_{ij} &= \bar{y}_{ij\cdot\cdot} - \bar{y}_{i\cdots} - \bar{y}_{\cdot j\cdot\cdot} + \bar{y}_{\cdots\cdot} \\
\widehat{(\alpha\tau)}_{ik} &= \bar{y}_{i\cdot k\cdot} - \bar{y}_{i\cdots} - \bar{y}_{\cdot\cdot k\cdot} + \bar{y}_{\cdots\cdot} \\
\widehat{(\beta\tau)}_{jk} &= \bar{y}_{\cdot jk\cdot} - \bar{y}_{\cdot j\cdot\cdot} - \bar{y}_{\cdot\cdot k\cdot} + \bar{y}_{\cdots\cdot} \\
\widehat{(\alpha\beta\tau)}_{ijk} &= \bar{y}_{ijk\cdot} - \bar{y}_{ij\cdot\cdot} - \bar{y}_{i\cdot k\cdot} - \bar{y}_{\cdot jk\cdot} + \bar{y}_{i\cdots} + \bar{y}_{\cdot j\cdot\cdot} + \bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\cdots\cdot}
\end{aligned}$$