For K-dimension

$$f_x(x_1, x_2, \ldots x_n) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp\left[\frac{-1}{2} (\bar{x}-\bar{\mu})^T \Sigma^{-1} (\bar{x}-\bar{\mu}) \right] \quad -(a)$$

Standard Deviation $\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix}$ $-(I)$

$$\rho = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

$$\sigma_{xy} = \rho(\sigma_x \sigma_y) \quad -(II)$$

Substituting (II) in (I)

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho(\sigma_x\sigma_y) \\ \rho(\sigma_y\sigma_x) & \sigma_y^2 \end{bmatrix}$$

Now calculating Σ^{-1} as

$$\Sigma^{-1} = \frac{1}{\Sigma} = \frac{1}{\sigma_x^2\sigma_y^2 - \{\rho(\sigma_x\sigma_y)\rho(\sigma_y\sigma_x)\}} \begin{bmatrix} \sigma_y^2 & -\rho(\sigma_x\sigma_y) \\ -\rho(\sigma_y\sigma_x) & \sigma_x^2 \end{bmatrix} \quad -(III)$$

$$|\Sigma| = (\sigma_x^2\sigma_y^2) - \{(\sigma_x\sigma_y)\rho\}^2$$

$$= \sigma_x^2\sigma_y^2 - \sigma_x^2\sigma_y^2\rho^2 \quad - (IV)$$

$$= \sigma_x^2\sigma_y^2(1-\rho^2)$$

$$\sqrt{|\Sigma|} = \sqrt{\sigma_x^2\sigma_y^2(1-\rho^2)} = \sigma_x\sigma_y\sqrt{1-\rho^2} \quad - (V)$$

Substituting (III) and (V) in (a) we get

$$= \frac{1}{2\pi(\sigma_x\sigma_y\sqrt{1-\rho^2})} \exp\left[\frac{-1}{2}\begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix} \frac{1}{\sigma_x^2\sigma_y^2(1-\rho^2)} \begin{bmatrix} \sigma_y^2 & -\rho\sigma_x\sigma_y \\ -\rho\sigma_x\sigma_y & \sigma_x^2 \end{bmatrix} \begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}\right]$$

$$= \frac{1}{2\pi\sigma_1\sigma_2(1-\rho^2)^{\frac{1}{2}}} \exp\left[\frac{-1}{2}\left[(x-\mu_x)(y-\mu_y)\right] \cdot \frac{1}{\sigma_x^2\sigma_y^2(1-\rho^2)} \begin{bmatrix} \sigma_x^2(x-\mu_x) - \rho\sigma_x\sigma_y(y-\mu_y) \\ -\rho\sigma_x\sigma_y(x-\mu_x) + \sigma_x^2(y-\mu_x) \end{bmatrix}\right]$$

$$= \frac{1}{2\pi\sigma_x\sigma_y(1-\rho)^{\frac{1}{2}}} \exp\left(\frac{-1}{2\sigma_x^2\sigma_y^2(1-\rho^2)}\begin{bmatrix} (x-\mu_x)[\sigma_y^2(x-\mu_x) - \rho\sigma_x\sigma_y(y-\mu_y)]. \\ \sigma_x^2(y-\mu_y)[\rho\sigma_x\sigma_y(x-\mu_x)] \end{bmatrix}\right)$$

$$= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(\frac{-1}{2\sigma_x^2\sigma_y^2(1-\rho^2)} \left(\sigma_y^2(x-\mu_y)^2 - 2\rho\sigma_x\sigma_y(x-\mu_x)(y-\mu_y) + \sigma_x^2(y-\mu_y)^2\right]\right.$$

$$= \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \frac{-1}{2(1-\rho^2)} \left[\frac{\sigma_y^2(x-\mu_x)}{\sigma_x^2\sigma_y^2} - \frac{2\rho\sigma_x\sigma_y(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{\sigma_x^2(y-\mu_y)^2}{\sigma_x^2\sigma_y^2}\right]$$

$$= \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\cdot Z\right)$$

where $Z = \frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}$