1.f

$$||z_1| - |z_2|| \leq |z_1 - z_2| \Leftrightarrow -|z_1 - z_2| \leq |z_1| - |z_2| \leq |z_1 - z_2|$$

$$\begin{aligned} |z_1| - |z_2| &\leq |z_1 - z_2| \\ |z_1| &\leq |z_1 - z_2| + |z_2| \\ |z_1 - z_2 + z_2| &\leq |z_1 - z_2| + |z_2| \end{aligned}$$

Co zostało udowodnione w **1.e**. Analogicznie:

$$\begin{aligned} &|z_2| - |z_1| \leq |z_2 - z_1| = |z_1 - z_2| \\ &-|z_1 - z_2| \leq |z_1| - |z_2| \end{aligned}$$

3.e

$z = 1 + \cos\alpha + i\sin\alpha$

Założenia: $0 < \alpha < \frac{\pi}{2}$

$$|z| = \sqrt{(1+\cos\alpha)^2 + \sin^2\alpha} = \sqrt{(1+\cos\alpha)^2 + 1 - \cos^2\alpha} =$$
$$\sqrt{(1+\cos\alpha)^2 + (1+\cos\alpha)(1-\cos\alpha)} = \sqrt{(1+\cos\alpha)(1+\cos\alpha+1-\cos\alpha)} =$$
$$\sqrt{2(1+\cos\alpha)} = \sqrt{2\left(1+2\cos^2\frac{\alpha}{2}-1\right)} = \sqrt{4\cos^2\frac{\alpha}{2}} = 2\cos\frac{\alpha}{2}$$

$\cos\frac{\alpha}{2}$ jest zawsze dodatni co wynika z założeń.

Niech $\varphi = \arg(z)$. Wtedy:

$$\cos\varphi = \frac{1+\cos\alpha}{2\cos\frac{\alpha}{2}} = \frac{1+2\cos^2\frac{\alpha}{2}-1}{2\cos\frac{\alpha}{2}} = \cos\frac{\alpha}{2}$$

$$\sin\varphi = \frac{\sin\alpha}{2\cos\frac{\alpha}{2}} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}} = \sin\frac{\alpha}{2}$$

Więc:

$$z = 2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)$$