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Assignment 2.5

1) Matrix A has $row_1 + row_2 = row_3$

a. $Ax = (1,0,0)$ Cannot have a Solution

↳ $\begin{bmatrix} row_1 \\ row_2 \\ row_1 + row_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow row_1 . x_1 + row_2 . x_2 = 1 + 0 = 1$

because row 3 Supposed to be 1, $Ax = (1,0,0)$ Can't have a Solution

b. $A = \begin{bmatrix} row_1 \\ row_2 \\ row_1 + row_2 \end{bmatrix}$ $\quad Ax = b \rightarrow \begin{bmatrix} row_1 \\ row_2 \\ row_1 + row_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$b \rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_1 + b_2 \end{bmatrix}$

$row_1 . x_1 = b_1$

$row_2 . x_2 = b_2$

$row_1 . x_1 + row_2 . x_2 = b_3$

c. In elimination

Ex.

↳ $A = \begin{bmatrix} row_1 \\ row_2 \\ row_1 + row_2 \end{bmatrix} \xrightarrow{E_{21}} \begin{bmatrix} row_1 \\ row_2 - row_1 \\ row_1 + row_2 \end{bmatrix} \xrightarrow{E_{31}} \begin{bmatrix} row_1 \\ row_2 - row_1 \\ row_1 + row_2 - row_1 \end{bmatrix} \xrightarrow{E_{32}} \begin{bmatrix} row_1 \\ row_2 - row_1 \\ row_1 + row_2 - row_1 - row_2 \end{bmatrix}$

$row_3 = row_1 - row_1 + row_2 - row_2$

$= 0$

2) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad - bc & -ab + ab \\ cd - cd & -bc + ad \end{bmatrix}$

inverse $\rightarrow \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}^{-1} = \dfrac{1}{ad \cdot bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \dfrac{1}{ad \cdot bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

3) If B is the inverse of A^2 show that AB is the inverse of A

$B = (A.A)^{-1} \rightarrow (A.A)B = I$

$A(AB) = I$

$A . A^{-1} = I \rightarrow AB = A^{-1}$

so AB is the inverse of A

4) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ inverse matrix A by Gauss Jordan method Starting with $[A I]$

↳ $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_2 - r_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_3 - r_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{r_3 - r_2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{r_1 - r_2} \begin{bmatrix} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{r_2 - r_3} \begin{bmatrix} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$

so A^{-1} is $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

5) a. Prove that $A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$ is invertible if $a \neq 0$ and $a \neq b$

elimination

$R_2 \rightarrow r_2 - r_1 = [\,0 \quad a-b \quad 0\,]$

$r_3 \rightarrow r_3 - r_1 = [\,0 \quad a-b \quad a-b\,]$

$r_3 \rightarrow r_3 - r_2 = [\,0 \quad 0 \quad a-b\,]$

Miquel Louis Setiadi, Computer Science 3, 2025330097

b. $C = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}, c = 0$

$= 2\begin{bmatrix} c & c \\ 7 & c \end{bmatrix} - c\begin{bmatrix} c & c \\ 8 & c \end{bmatrix} + c\begin{bmatrix} c & c \\ 8 & 7 \end{bmatrix}$

$= 2(c^2 - 7c) - c(c^2 - 8c) + c(7c - 8c)$

$= 2c^2 - 14c - c^3 + 8c^2 - c^2$

$= -c^3 + 9c^2 - 14c$

$= -c(c^2 - 9c - 14)$

$= -c(c-2)(c-7)$

$c = 0 \quad c = 2 \quad c = 7$

Assignment 2.6

1) $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$

a. $A \to$ upper triangular

$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix} \xrightarrow{r_3 - 3r_1} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}$

b. Find matrix E such $EA = U$

$E_{31} \cdot A = U$

- pivot = 2
- multiplier = $l_{31} = 6/2 = 3$

$\Big\} \; E = E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$

c. $L = E_{31}^{-1} = E^{-1} \to L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

$EA = U$

$E^{-1} \cdot EA = E^{-1} \cdot U$

$I \; A = LU = A$

2) $A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$ Compute L & U such that $A = LU$

$\xrightarrow[R_2 - R_1,\ R_3 - R_1,\ R_4 - R_1]{E_{21}\ E_{31}\ E_{41}} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix} \xrightarrow[R_3 - R_2,\ R_4 - R_2]{E_{32}\ E_{42}} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & (c-a)-(b-a) & (c-a)-(b-a) \\ 0 & 0 & (c-a)-(b-a) & (d-a)-(b-a) \end{bmatrix}$

$\xrightarrow[R_4 - R_3]{E_{43}} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & (c-a)-(b-a) & (c-a)-(b-a) \\ 0 & 0 & 0 & (d-a)-(c-a) \end{bmatrix} = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$

$E_{43} E_{42} E_{32} E_{41} E_{31} E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix} \to L = E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

3) $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

a. Solve $Lc = b$ to find c

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ * $c = L^{-1} \cdot b$

$L^{-1} \to \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$, $c = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$

3) b. Solve $Ux = c$ to find x

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$

- $x_3 = 1$
- $x_2 = 1 - 1 = 0$
- $x_1 = 4 - 0 - 1 = 3$

$x = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

c. $A = LU$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

* $L \cdot c = b$ $\quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$L(Ux) = b$

$(LU)x = b$

$Ax = b$

Miquel Louis Setiadi, Computer Science 3, 2025 30097

Assignment 2.7

1) $A=\begin{bmatrix}1&0\\9&3\end{bmatrix}$ · $A^T=\begin{bmatrix}1&9\\0&3\end{bmatrix}$ · $A^{-1}=\frac{1}{3-0}\begin{bmatrix}3&0\\-9&1\end{bmatrix}=\begin{bmatrix}1&0\\-3&\frac{1}{3}\end{bmatrix}$

$$\left.\begin{array}{l} \cdot\ (A^T)^{-1}=\frac{1}{3-0}\begin{bmatrix}3&-9\\0&1\end{bmatrix}=\begin{bmatrix}1&-3\\0&\frac{1}{3}\end{bmatrix} \\ \cdot\ (A^{-1})^T=\begin{bmatrix}1&-3\\0&1/3\end{bmatrix}\end{array}\right\}\ \text{so } (A^T)^{-1}=(A^{-1})^T$$

2) $A=\begin{bmatrix}1&0\\2&1\end{bmatrix}$ $B=\begin{bmatrix}1&3\\0&1\end{bmatrix}$

· $A^T=\begin{bmatrix}1&2\\0&1\end{bmatrix}$ · $B^T=\begin{bmatrix}1&0\\3&1\end{bmatrix}$ · $AB=\begin{bmatrix}1&3\\2&7\end{bmatrix}$ · $(AB)^T=\begin{bmatrix}1&2\\3&7\end{bmatrix}$

· $B^TA^T=\begin{bmatrix}1&0\\3&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}1&2\\3&7\end{bmatrix}=(AB)^T$

· $A^TB^T=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&0\\3&1\end{bmatrix}=\begin{bmatrix}7&2\\3&1\end{bmatrix}\neq(AB)^T$

3) $S=\begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix}$ factor into $S=LDL^T$ w/ diagonal pivot matrix d

· $S=LU$ $\xrightarrow[l_{21}=-1/2]{r_2+\frac{1}{2}r_1}\begin{bmatrix}2&-1&0\\0&3/2&-1\\0&-1&2\end{bmatrix}\xrightarrow[l_{32}=-2/3]{r_3+2/3\,r_2}\begin{bmatrix}2&-1&0\\0&3/2&-1\\0&0&4/3\end{bmatrix}$ · $E=E_{32}E_{21}=\begin{bmatrix}1&0&0\\1/2&1&0\\0&2/3&1\end{bmatrix}$

· $L=E^{-1}=\begin{bmatrix}1&0&0\\-1/2&1&0\\0&-2/3&1\end{bmatrix}\longrightarrow S=LDL^T\rightarrow\begin{bmatrix}1&0&0\\-1/2&1&0\\0&-2/3&1\end{bmatrix}\begin{bmatrix}2&0&0\\0&3/2&0\\0&0&4/3\end{bmatrix}\begin{bmatrix}1&-1/2&0\\0&1&-2/3\\0&0&1\end{bmatrix}$

$* = U=\begin{bmatrix}1&-1/2&0\\0&1&-2/3\\0&0&1\end{bmatrix}$

4) $A=\begin{bmatrix}0&1&1\\1&0&1\\2&3&4\end{bmatrix}$ find the factorization $PA=LU$

$PA\rightarrow$, $P_{21}\rightarrow\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&1&1\\1&0&1\\2&3&4\end{bmatrix}=\begin{bmatrix}1&0&1\\0&1&1\\2&3&4\end{bmatrix}$

$LU\downarrow$

$\begin{bmatrix}1&0&1\\0&1&1\\2&3&4\end{bmatrix}\xrightarrow[l_{31}=2]{r_3-2r_1}\begin{bmatrix}1&0&1\\0&1&1\\0&3&2\end{bmatrix}\xrightarrow[l_{32}=3]{r_3-3r_2}\begin{bmatrix}1&0&1\\0&1&1\\0&0&-1\end{bmatrix}=U$

$E=E_{32}\cdot E_{31}=\begin{bmatrix}1&0&0\\0&1&0\\-2&-3&1\end{bmatrix}$ $L=E^{-1}=\begin{bmatrix}1&0&0\\0&1&0\\2&3&1\end{bmatrix}$

$PA=LU$

$\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&1&1\\1&0&1\\2&3&4\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\2&3&1\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&1\\0&0&-1\end{bmatrix}$

Assignment 3.1

1) $\det Q=\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$

$\det Q=\frac{1}{\cos^2\theta+\sin^2\theta}=\frac{1}{1}=1$

2) $A=\begin{bmatrix}4&1\\2&3\end{bmatrix}$ | · $A^2=\begin{bmatrix}4&1\\2&3\end{bmatrix}\begin{bmatrix}4&1\\2&3\end{bmatrix}=\begin{bmatrix}18&7\\14&11\end{bmatrix}$ · $\det A^2=\frac{1}{198-98}=\frac{1}{100}$

· $A^{-1}=\frac{1}{12-2}\begin{bmatrix}3&-1\\-2&4\end{bmatrix}=\begin{bmatrix}3/10&-1/10\\-2/10&4/10\end{bmatrix}$ · $\det A^{-1}=\frac{1}{\frac{12}{100}-\frac{2}{100}}=1:\frac{10}{100}=10$

Miquel Louis Setiadi, Computer Science 3, 2025 30097

3) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \longrightarrow \det A = ?$

$\hookrightarrow \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{matrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{matrix} \quad \det A = 0+1+1-0-0-0 = 2$

4) apply row operations to produce an upper triangular U

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \xrightarrow{r_2 - 2r_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \xrightarrow{r_3 + r_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 3 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$

$\xrightarrow[r_4 - r_2]{r_3 - r_2} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

$\det \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix} \rightarrow 1 \cdot 2 \cdot 3 \cdot 6 + 0 + 0 + 0 - 0 - 0 - 0 - 0 = 36$