PHYS1001B College Physics IB

Optics II Geometric Optics (Ch. 34)

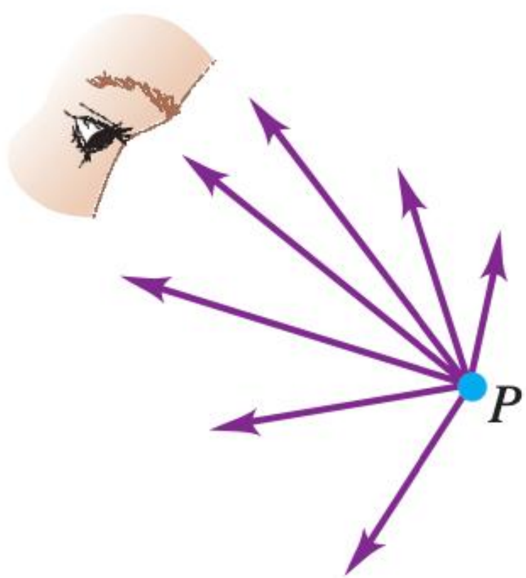
Introduction

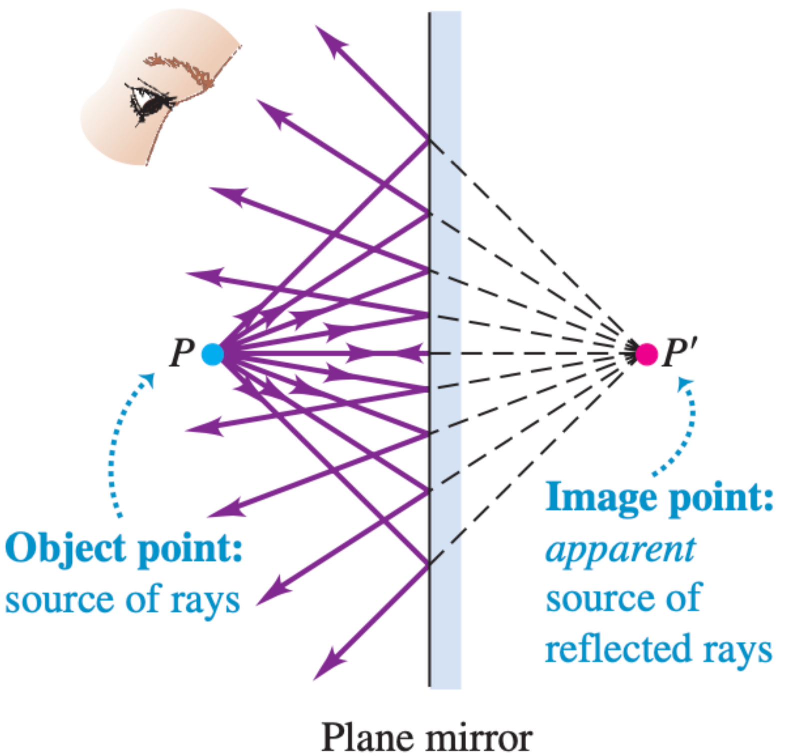
? How do magnifying lenses work? At what distance from the object being examined do they provide the sharpest view?

Outline

- 34-1 Reflection and Refraction at a Plane Surface
- 34-2 Reflection at a Spherical Surface
- 34-3 Refraction at a Spherical Surface
- 34-4 Thin Lenses
- 34-5 Cameras
- 34-6 The Eye
- 34-7 The Magnifier
- 34-8 Microscopes and Telescopes

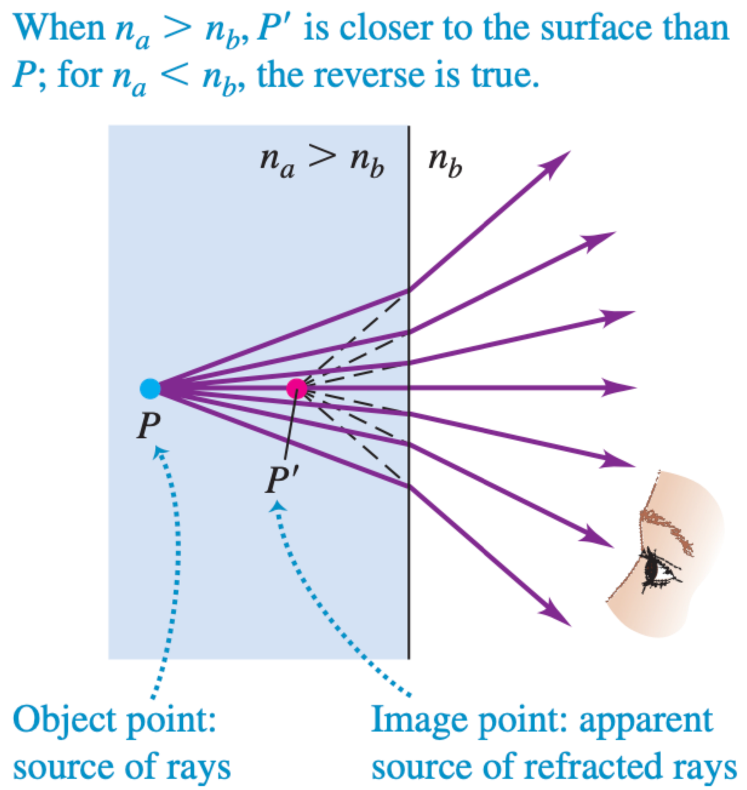
34-1 Reflection and Refraction at a Plane Surface





34-1 Reflection and Refraction at a Plane Surface

34.3 Light rays from the object at point P are refracted at the plane interface. The refracted rays entering the eye look as though they had come from image point P'.



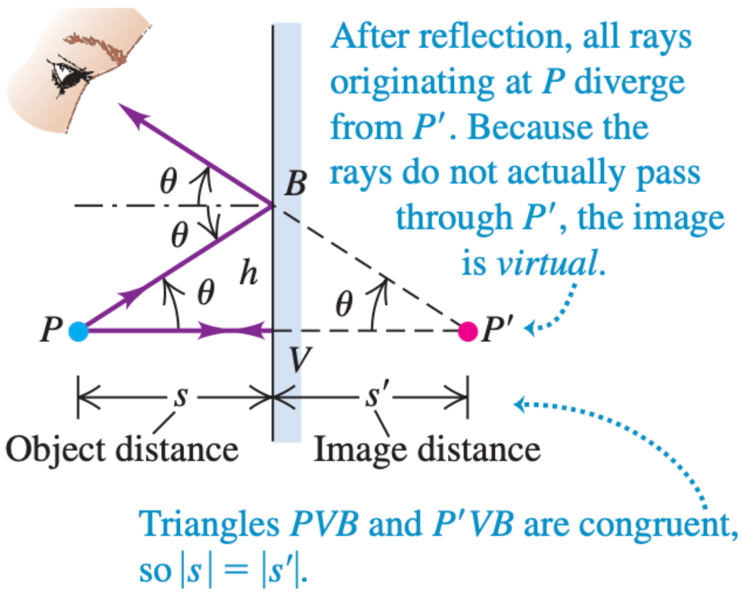
34-1 Reflection and Refraction at a Plane Surface

Sign rules

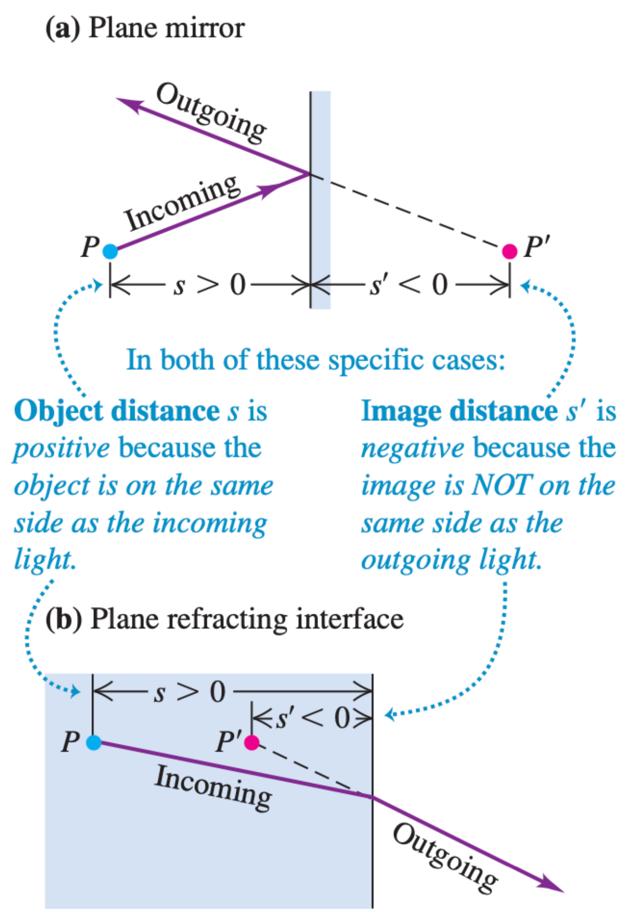
1. **Sign rule for the object distance:** When the object is on the same side of the reflecting or refracting surface as the incoming light, the object distance s is positive; otherwise, it is negative.
2. **Sign rule for the image distance:** When the image is on the same side of the reflecting or refracting surface as the outgoing light, the image distance s' is positive; otherwise, it is negative.
3. **Sign rule for the radius of curvature of a spherical surface:** When the center of curvature C is on the same side as the outgoing light, the radius of curvature is positive; otherwise, it is negative.

34-1 Reflection and Refraction at a Plane Surface

34.4 Construction for determining the location of the image formed by a plane mirror. The image point P' is as far behind the mirror as the object point P is in front of it.

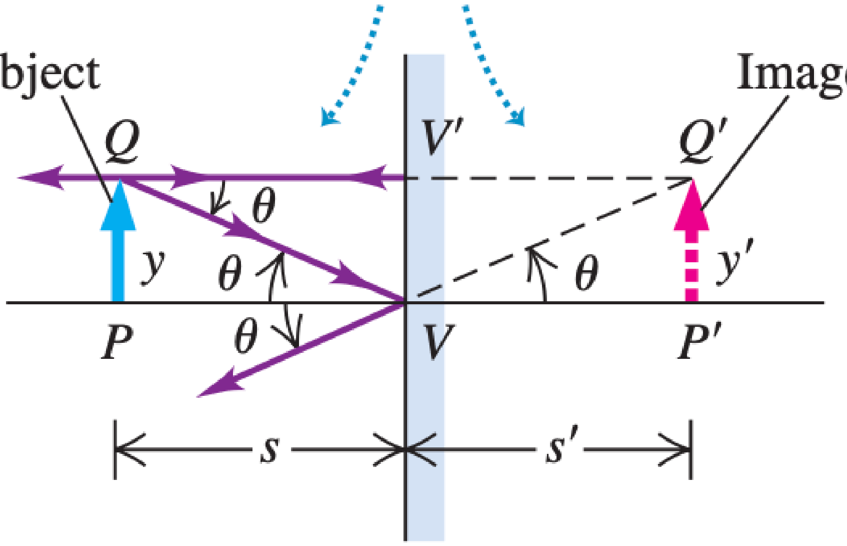


34.5 For both of these situations, the object distance s is positive (rule 1) and the image distance s' is negative (rule 2).



34-1 Reflection and Refraction at a Plane Surface

For a plane mirror, PQV and $P'Q'V$ are congruent, so $y = y'$ and the object and image are the same size (the lateral magnification is 1).

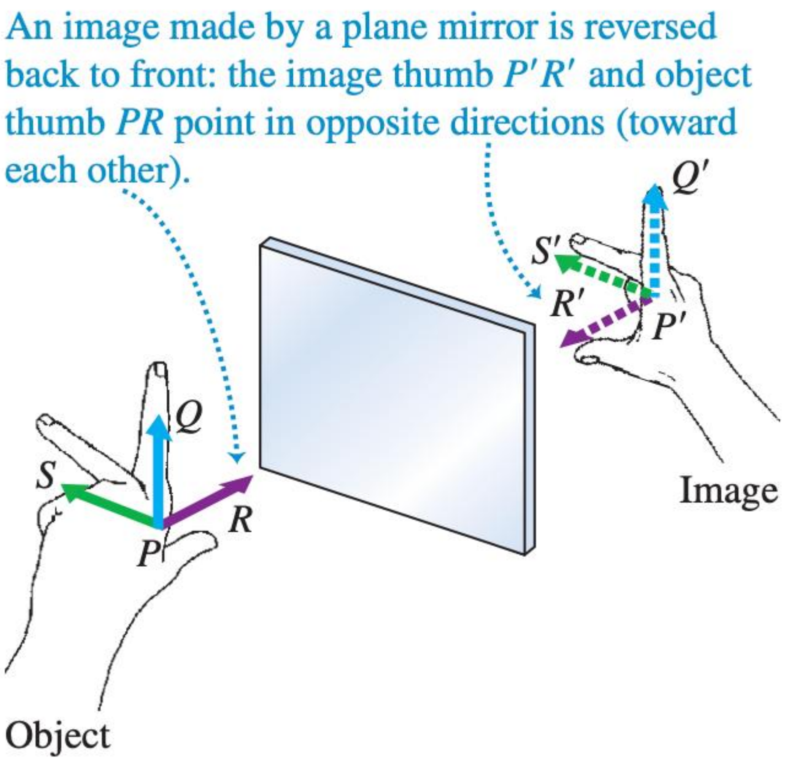


$$m = \frac{y'}{y} \qquad \text{(lateral magnification)}$$

m>0 erect
m<0 inverted

34-1 Reflection and Refraction at a Plane Surface

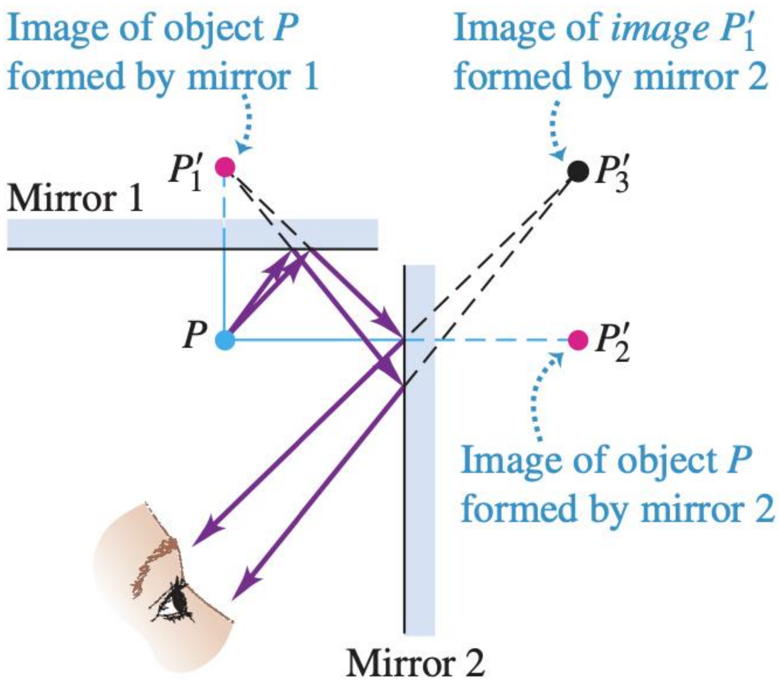
The image formed by a plane mirror is virtual, erect, and reversed



34.8 The image formed by a plane mirror is reversed; the image of a right hand is a left hand, and so on. (The hand is resting on a horizontal mirror.) Are images of the letters H and A reversed?

34-1 Reflection and Refraction at a Plane Surface

Image served as object for a second surface



34.9 Images P_1' and P_2' are formed by a single reflection of each ray from the object at P. Image P_3', located by treating either of the other images as an object, is formed by a double reflection of each ray.

34-1 Reflection and Refraction at a Plane Surface

Test Your Understanding of Section 34.1 If you walk directly toward a plane mirror at a speed v, at what speed does your image approach you? (i) slower than v; (ii) v; (iii) faster than v but slower than $2v$; (iv) $2v$; (v) faster than $2v$.

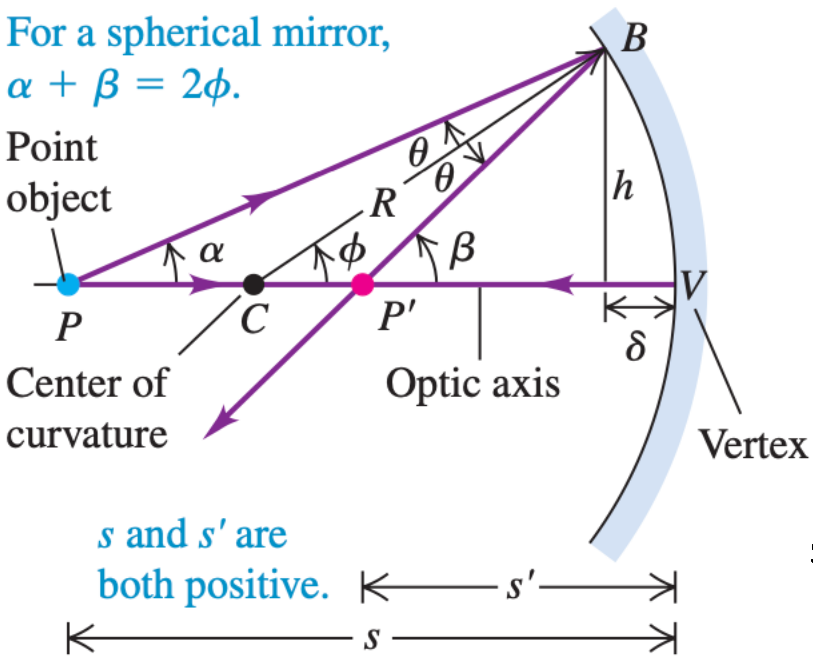
34-2 Reflection at a Spherical Surface

Magnifying mirror: Larger image
Surveillance mirrors: Smaller image

(a)

34-2 Reflection at a Spherical Surface

Concave spherical mirror: Finding position s'



$$\phi = \alpha + \theta \qquad \beta = \phi + \theta$$

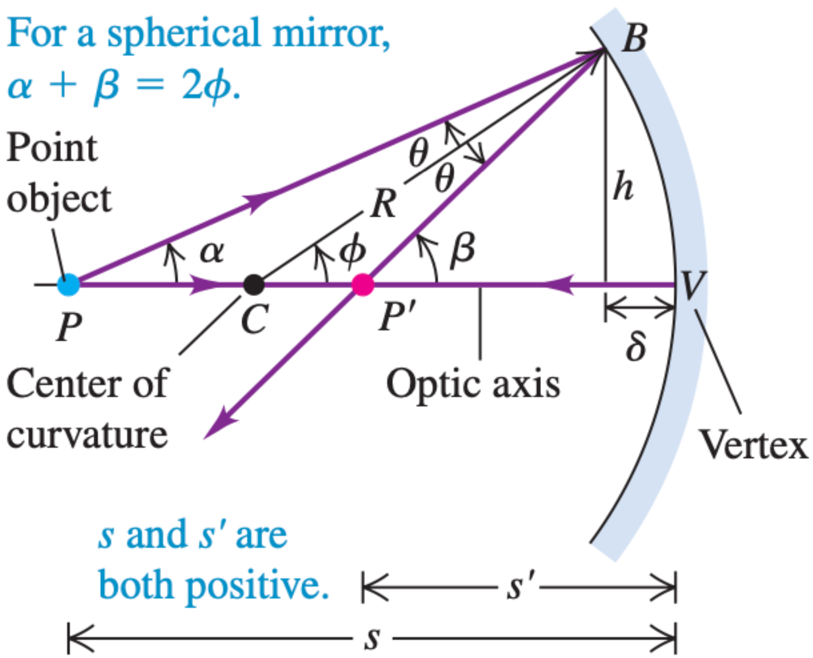
$$\alpha + \beta = 2\phi$$

s: distance to object s': distance to image

$$\tan\alpha = \frac{h}{s - \delta} \qquad \tan\beta = \frac{h}{s' - \delta} \qquad \tan\phi = \frac{h}{R - \delta}$$

34-2 Reflection at a Spherical Surface

Concave spherical mirror: Finding position s’



Small angle and distance δ

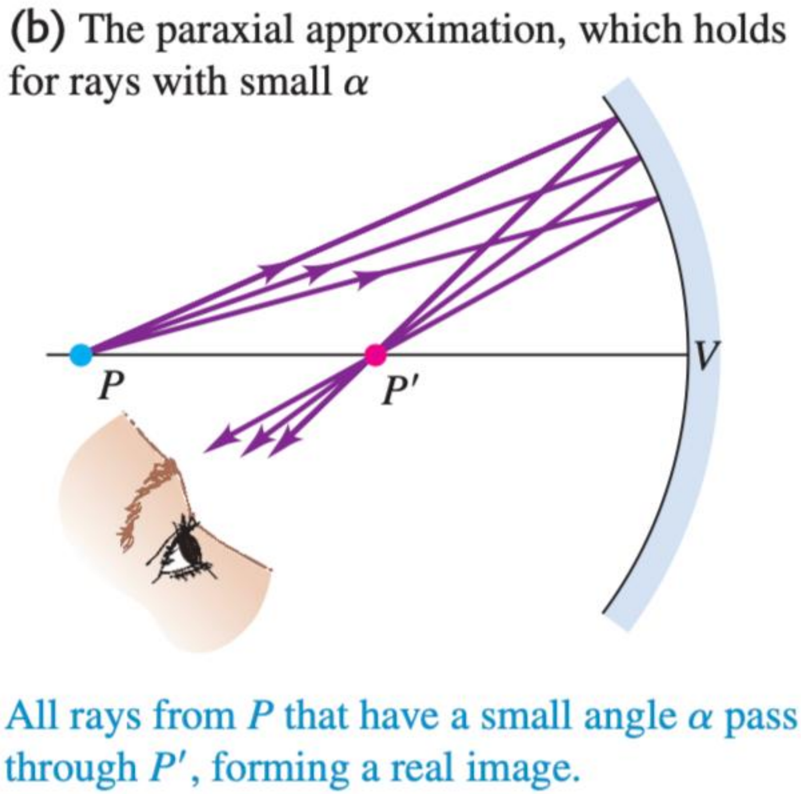
Paraxial approximation
Rays nearly parallel to the axis are paraxial rays.

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

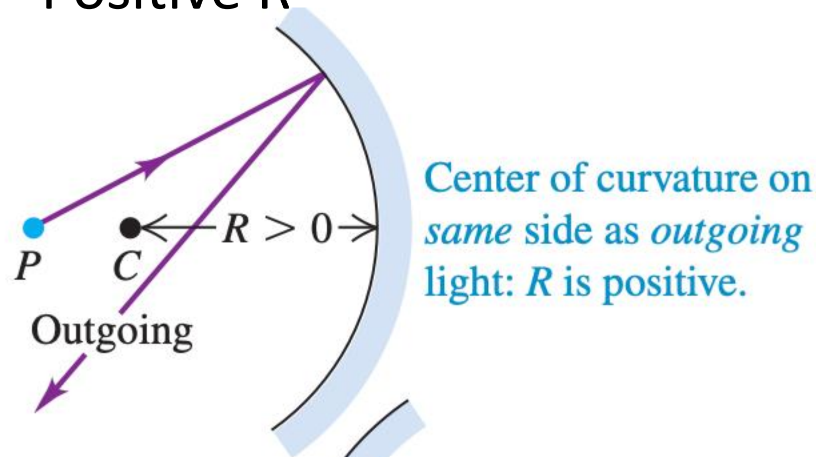
$$\alpha + \beta = 2\phi$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \qquad \text{(object–image relationship, spherical mirror)}$$

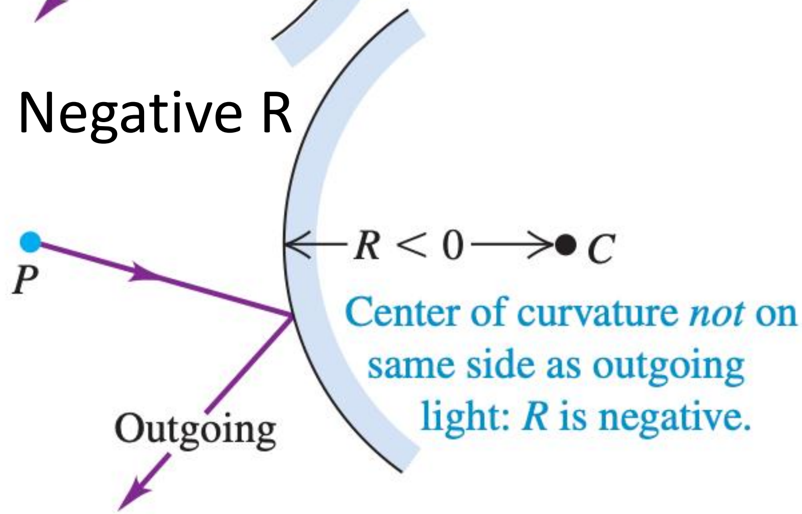
34-2 Reflection at a Spherical Surface



Positive R



Negative R

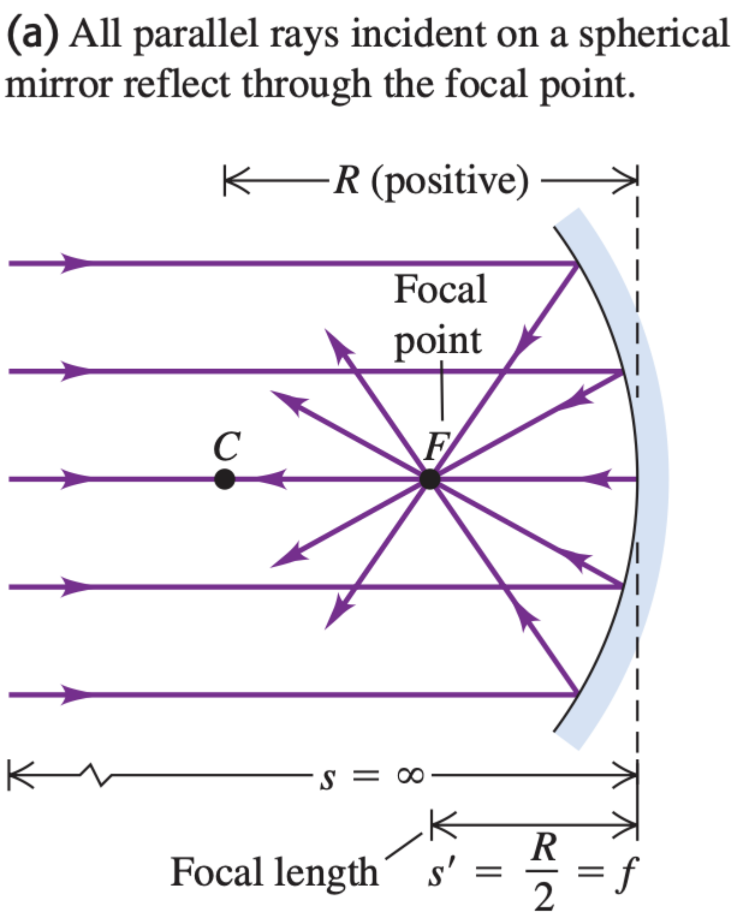


34-2 Reflection at a Spherical Surface

Radius of curvature becomes infinite?
What the mirror look like?

34-2 Reflection at a Spherical Surface

Focal point and Focal Length



When the object point P is very far from the spherical mirror $s = \infty$

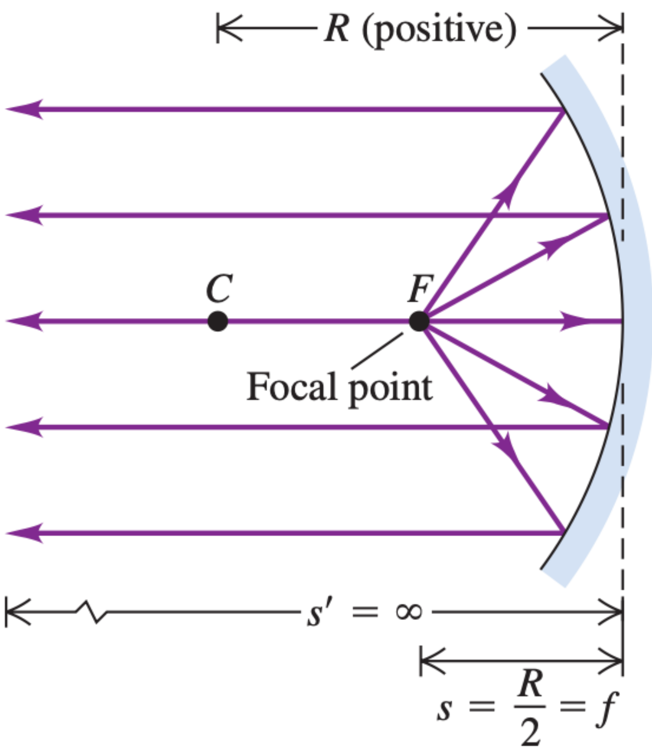
$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \qquad s' = \frac{R}{2}$$

Focal length $$f = \frac{R}{2}$$

34-2 Reflection at a Spherical Surface

Focal point and Focal Length

(b) Rays diverging from the focal point reflect to form parallel outgoing rays.

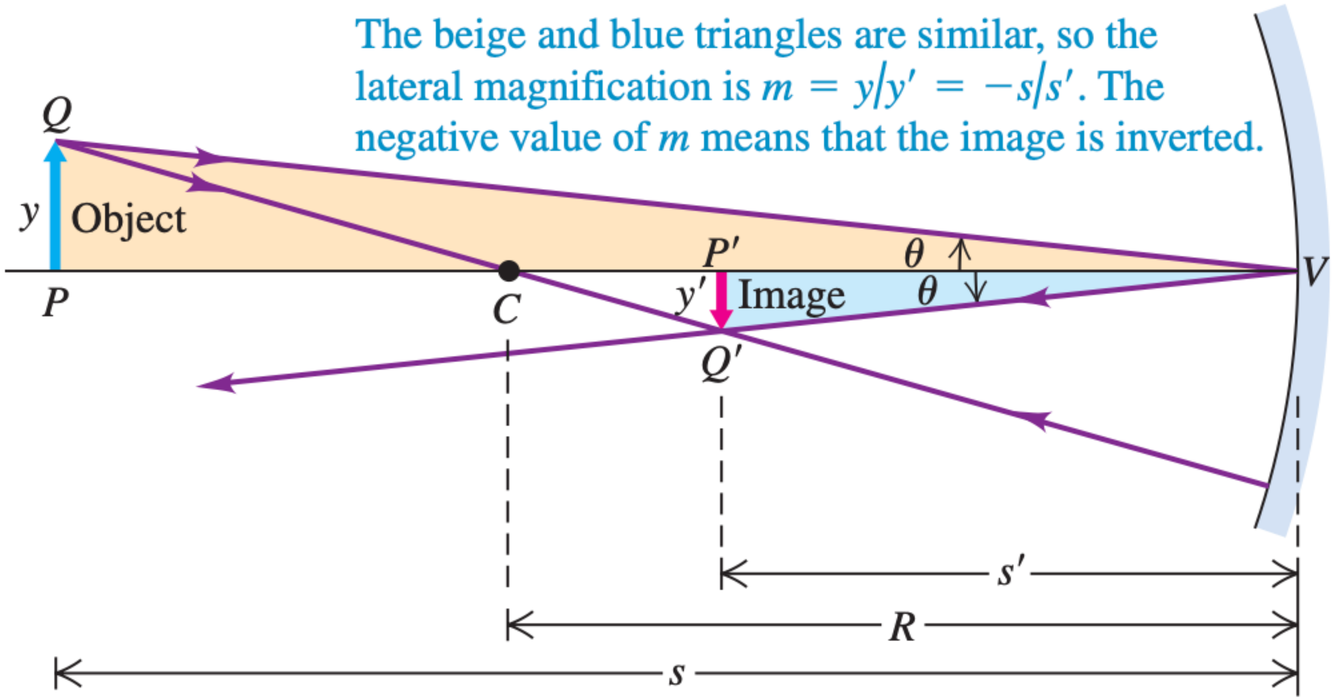


The *object* is placed at the focal point F,

$$\frac{2}{R} + \frac{1}{s'} = \frac{2}{R} \qquad \frac{1}{s'} = 0 \qquad s' = \infty$$

34-2 Reflection at a Spherical Surface

an object with *finite* size

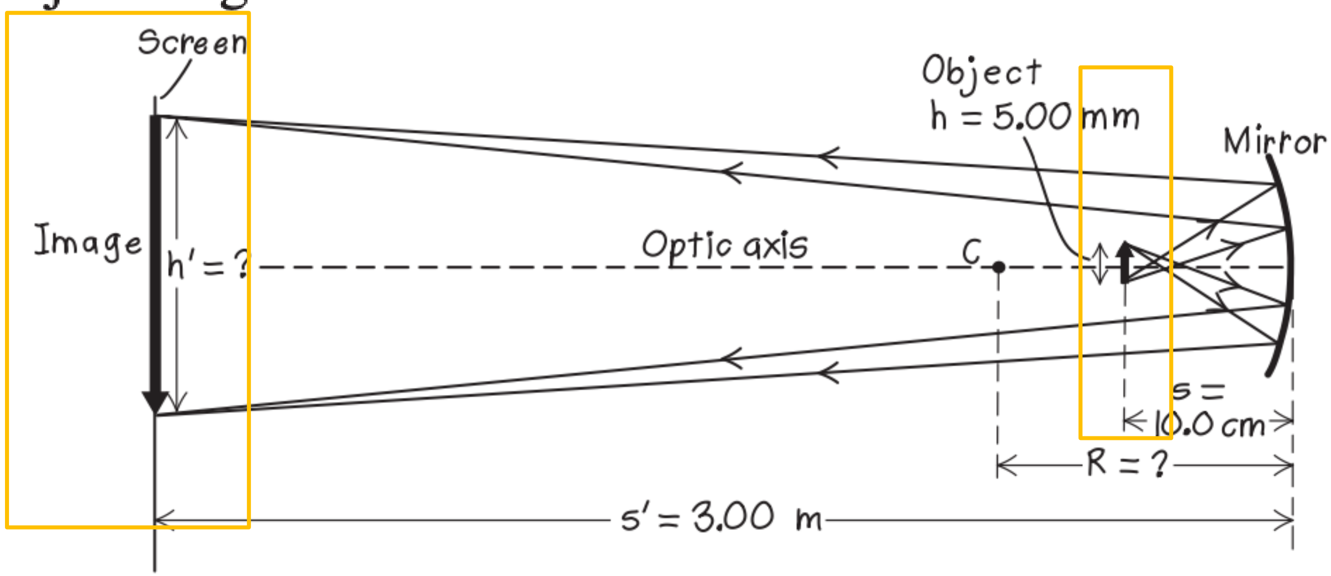


$$m = \frac{y'}{y} = -\frac{s'}{s} \qquad \text{(lateral magnification, spherical mirror)}$$

Sample Problem

Example 34.1 **Image formation by a concave mirror I**

A concave mirror forms an image, on a wall 3.00 m in front of the mirror, of a headlamp filament 10.0 cm in front of the mirror. (a) What are the radius of curvature and focal length of the mirror? (b) What is the lateral magnification? What is the image height if the object height is 5.00 mm?



Sample Problem

Example 34.1 **Image formation by a concave mirror I**

EXECUTE: (a) Both the object and the image are on the concave (reflective) side of the mirror, so both s and s' are positive; we have $s = 10.0$ cm and $s' = 300$ cm. We solve Eq. (34.4) for R:

$$\frac{1}{10.0\text{ cm}} + \frac{1}{300\text{ cm}} = \frac{2}{R}$$

$$R = 2\left(\frac{1}{10.0\text{ cm}} + \frac{1}{300\text{ cm}}\right)^{-1} = 19.4\text{ cm}$$

The focal length of the mirror is $f = R/2 = 9.7$ cm.

(b) From Eq. (34.7) the lateral magnification is

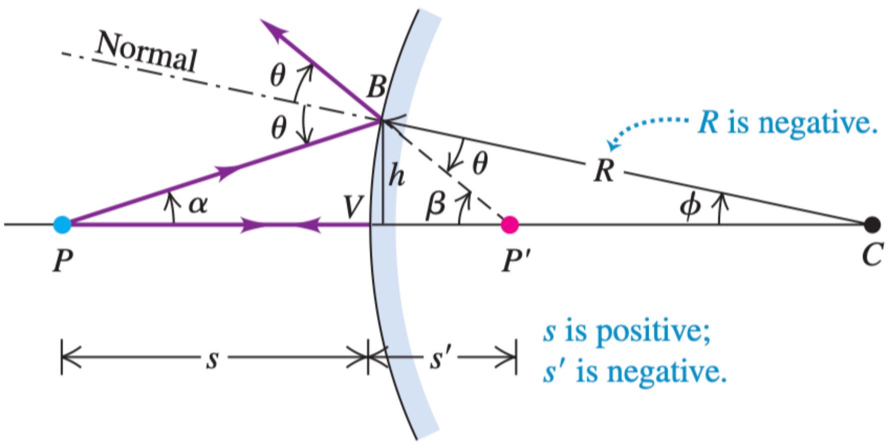
$$m = -\frac{s'}{s} = -\frac{300\text{ cm}}{10.0\text{ cm}} = -30.0$$

Because m is negative, the image is inverted. The height of the image is 30.0 times the height of the object, or $(30.0)(5.00\text{ mm}) = 150$ mm.

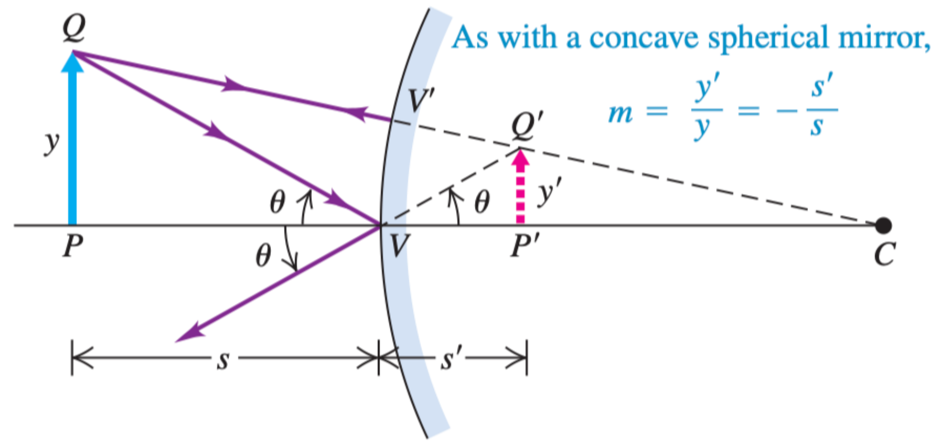
34-2 Reflection at a Spherical Surface

Image formation by a convex mirror

(a) Construction for finding the position of an image formed by a convex mirror



(b) Construction for finding the magnification of an image formed by a convex mirror



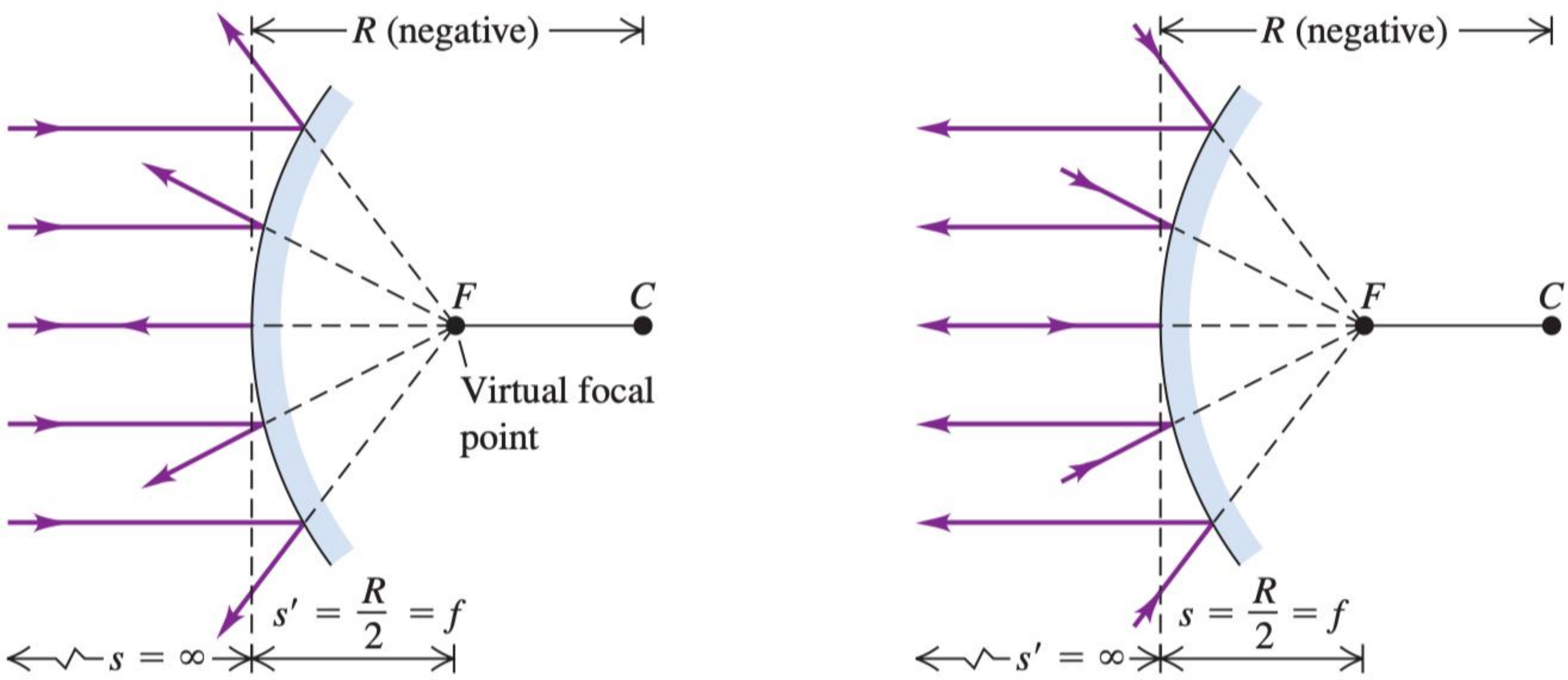
34-2 Reflection at a Spherical Surface

Focal point and focal length

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

(a) Paraxial rays incident on a convex spherical mirror diverge from a virtual focal point.

(b) Rays aimed at the virtual focal point are parallel to the axis after reflection.



Sample Problem

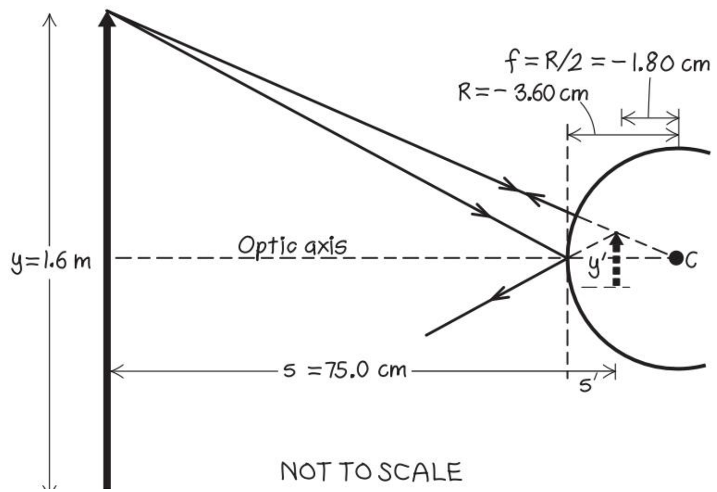
Example 34.3 Santa's image problem

Santa checks himself for soot, using his reflection in a silvered Christmas tree ornament 0.750 m away (Fig. 34.18a). The diameter of the ornament is 7.20 cm. Standard reference texts state that he is a "right jolly old elf," so we estimate his height to be 1.6 m. Where and how tall is the image of Santa formed by the ornament? Is it erect or inverted?

(a) The ornament forms a virtual, reduced, erect image of Santa. **(b)** Our sketch of two of the rays forming the image.

(a)

(b)



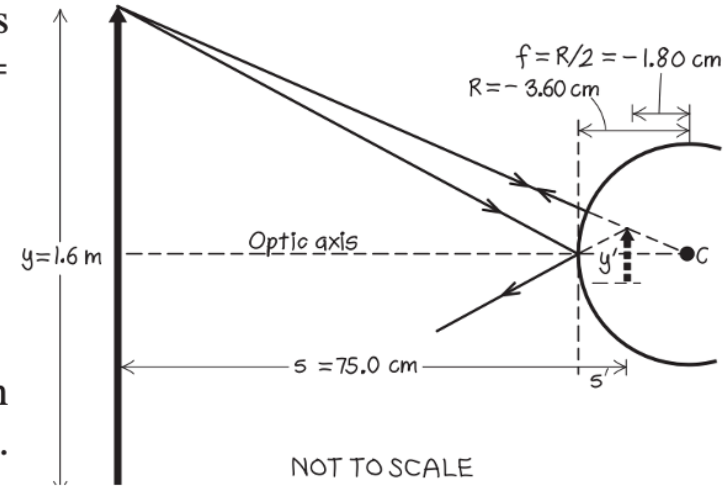
Sample Problem

EXECUTE: The radius of the mirror (half the diameter) is $R = -(7.20 \text{ cm})/2 = -3.60 \text{ cm}$, and the focal length is $f = R/2 = -1.80 \text{ cm}$. From Eq. (34.6),

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-1.80 \text{ cm}} - \frac{1}{75.0 \text{ cm}}$$

$$s' = -1.76 \text{ cm}$$

Because s' is negative, the image is behind the mirror—that is, on the side opposite to the outgoing light (Fig. 34.18b)—and it is virtual.



The image is about halfway between the front surface of the ornament and its center.

From Eq. (34.7), the lateral magnification and the image height are

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{-1.76 \text{ cm}}{75.0 \text{ cm}} = 0.0234$$

$$y' = my = (0.0234)(1.6 \text{ m}) = 3.8 \times 10^{-2} \text{m} = 3.8 \text{ cm}$$

Sample Problem

Example 34.4 **Concave mirror with various object distances**

A concave mirror has a radius of curvature with absolute value 20 cm. Find graphically the image of an object in the form of an arrow perpendicular to the axis of the mirror at object distances of (a) 30 cm, (b) 20 cm, (c) 10 cm, and (d) 5 cm. Check the construction by *computing* the size and lateral magnification of each image.

Sample Problem

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Measurements of the figures, with appropriate scaling, give the following approximate image distances: (a) 15 cm; (b) 20 cm; (c) ∞ or $-\infty$ (because the outgoing rays are parallel and do not converge at any finite distance); (d) -10 cm. To *compute* these distances, we solve Eq. (34.6) for s' and insert $f = 10\text{ cm}$:

$$\text{(a)}\ \frac{1}{30\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = 15\text{ cm}$$

$$\text{(b)}\ \frac{1}{20\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = 20\text{ cm}$$

$$\text{(c)}\ \frac{1}{10\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = \infty\ (\text{or } -\infty)$$

$$\text{(d)}\ \frac{1}{5\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = -10\text{ cm}$$

The signs of s' tell us that the image is real in cases (a) and (b) and virtual in case (d).

Sample Problem

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

The lateral magnifications measured from the figures are approximately (a) $-\frac{1}{2}$; (b) -1; (c) ∞ or $-\infty$; (d) $+2$. From Eq. (34.7),

$$\text{(a) } m = -\frac{15 \text{ cm}}{30 \text{ cm}} = -\frac{1}{2}$$

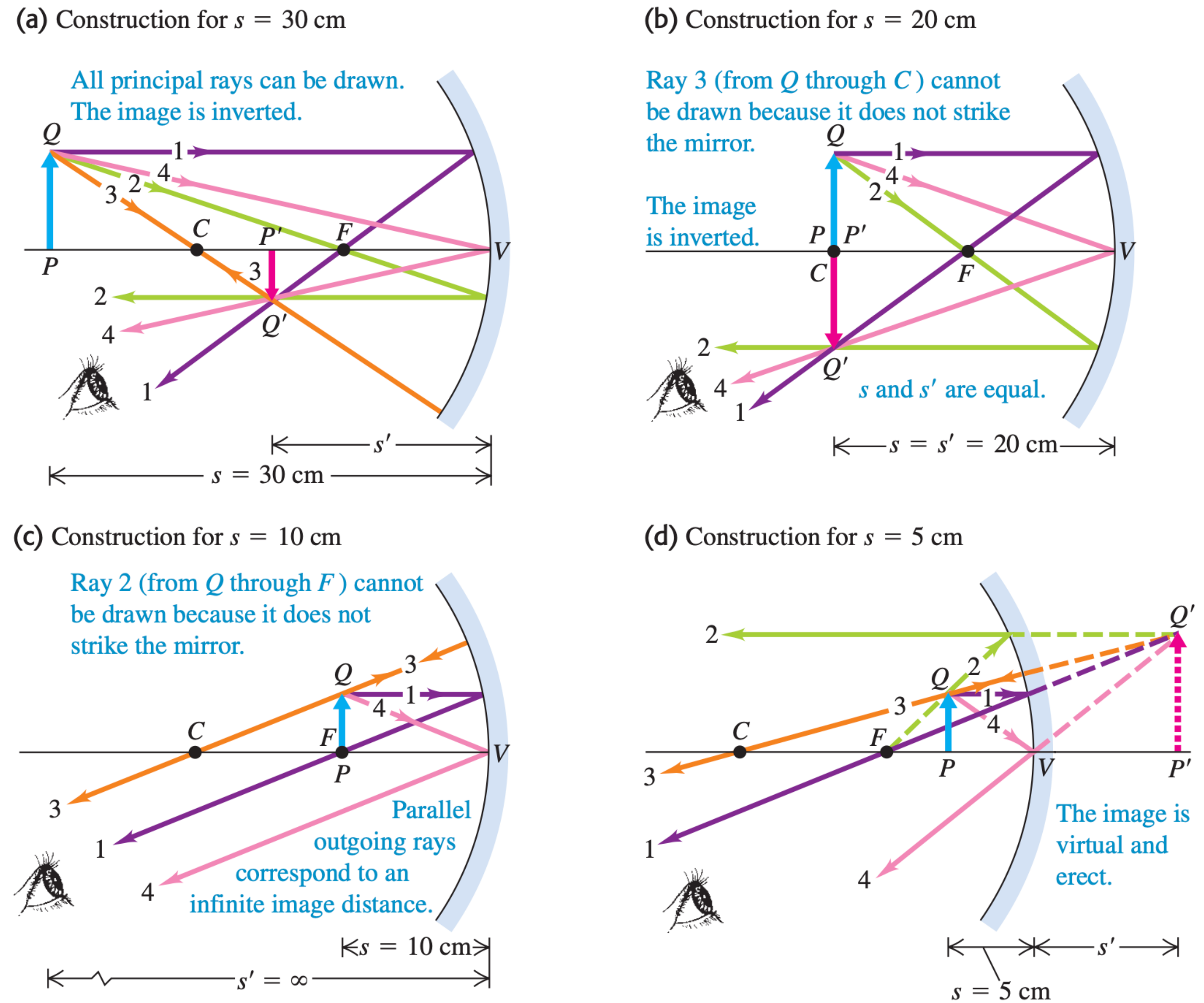
$$\text{(b) } m = -\frac{20 \text{ cm}}{20 \text{ cm}} = -1$$

$$\text{(c) } m = -\frac{\infty \text{ cm}}{10 \text{ cm}} = -\infty \ (\text{or } +\infty)$$

$$\text{(d) } m = -\frac{-10 \text{ cm}}{5 \text{ cm}} = +2$$

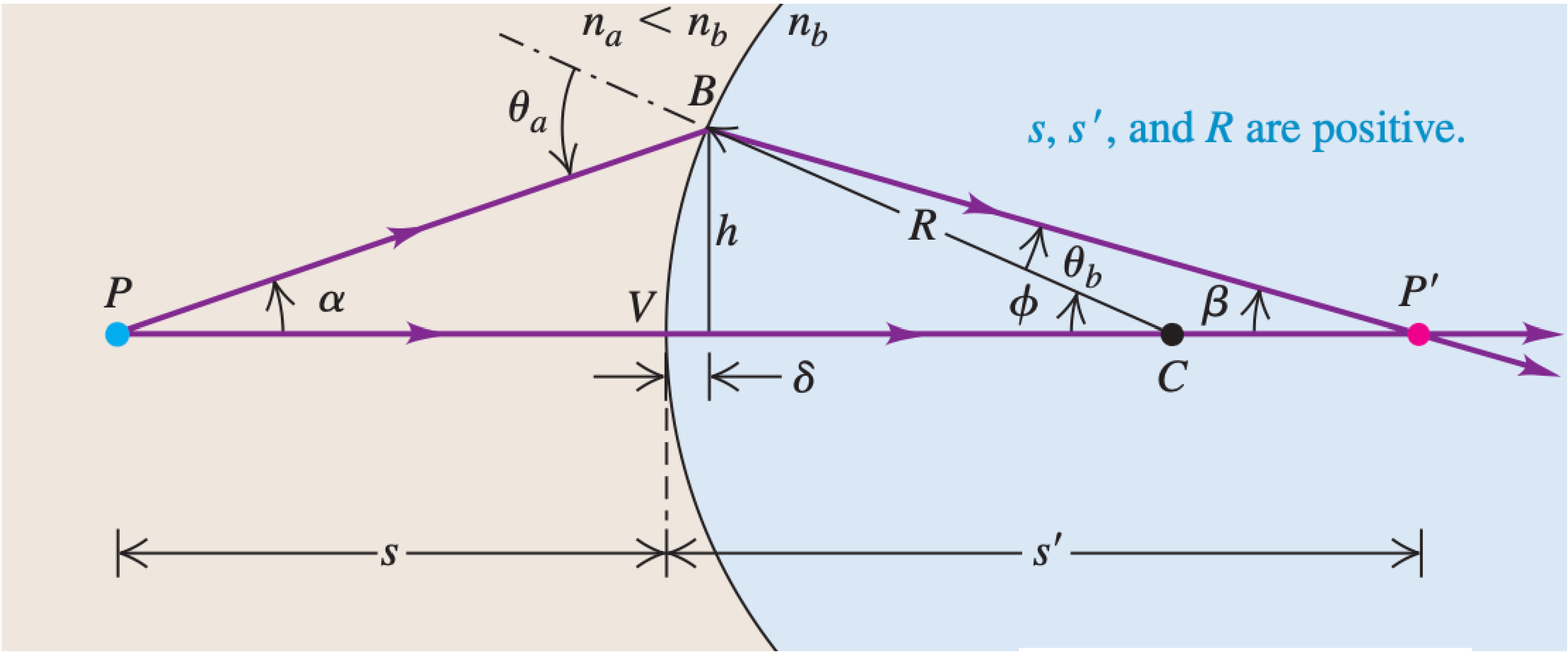
The signs of m tell us that the image is inverted in cases (a) and (b) and erect in case (d).

Sample Problem



34-3 Refraction at a Spherical Surface

Point object

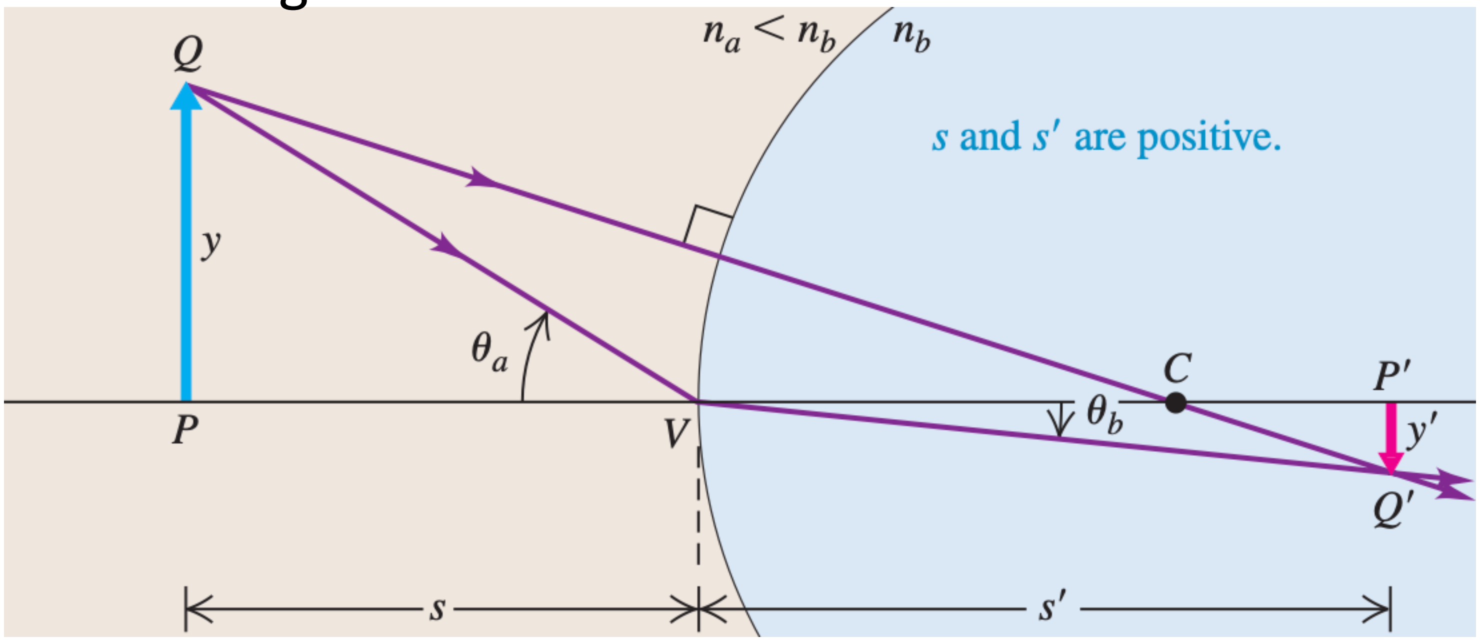


$$\theta_a = \alpha + \phi \qquad \phi = \beta + \theta_b \qquad n_a \sin\theta_a = n_b \sin\theta_b$$

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta} \quad \Rightarrow \quad \alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

34-3 Refraction at a Spherical Surface

Object with height



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$ (object–image relationship, spherical refracting surface)

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$ (lateral magnification, spherical refracting surface)

34-3 Refraction at a Spherical Surface

34.23 Light rays refract as they pass through the curved surfaces of these water droplets.

34-3 Refraction at a Spherical Surface

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \qquad \text{(object–image relationship, spherical refracting surface)}$$

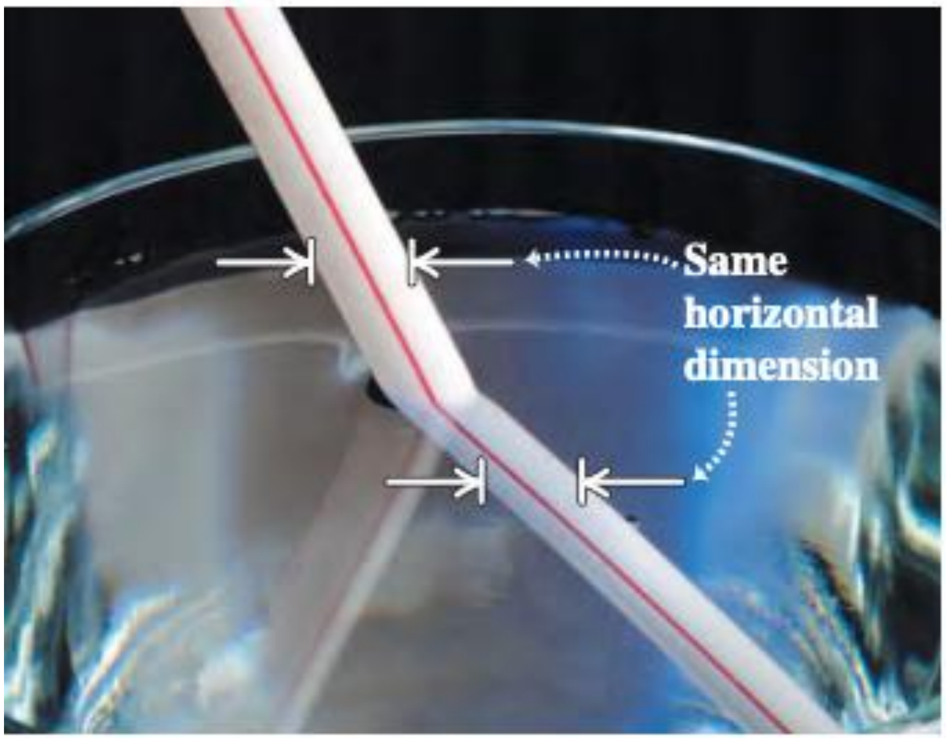
$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \qquad \text{(lateral magnification, spherical refracting surface)}$$

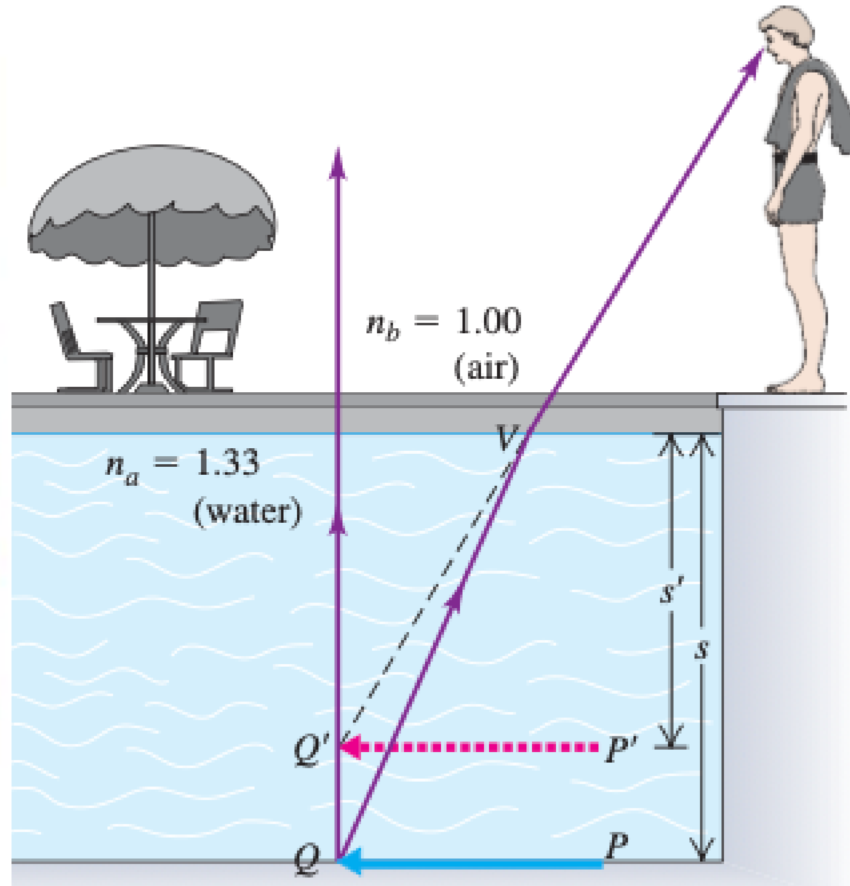
Plane surface $R = \infty$

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \qquad \text{(plane refracting surface)}$$

$$m = 1$$

34-3 Refraction at a Spherical Surface

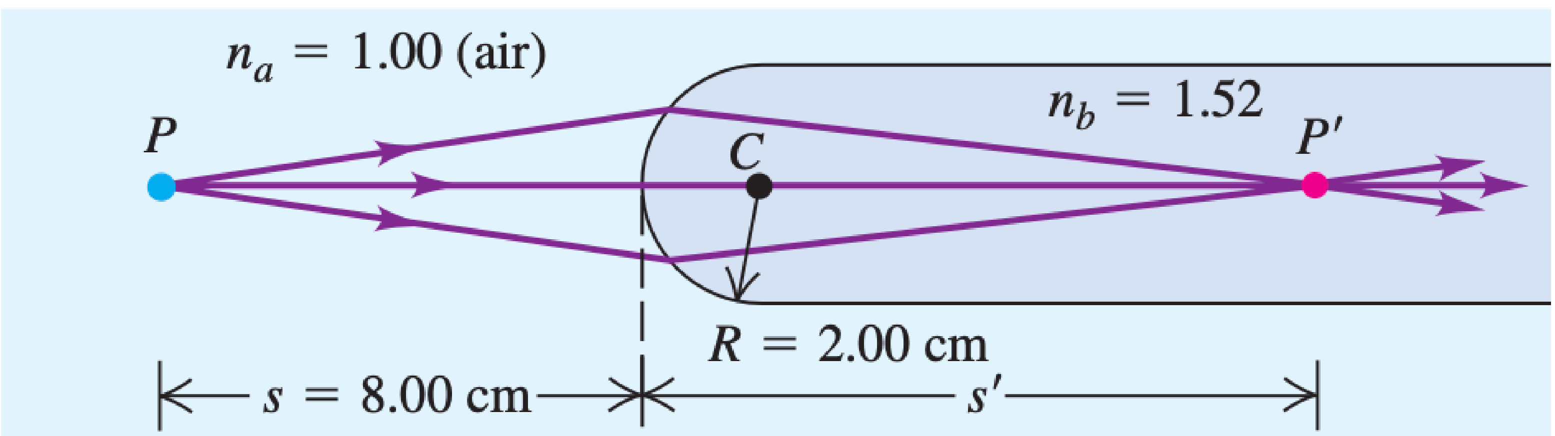




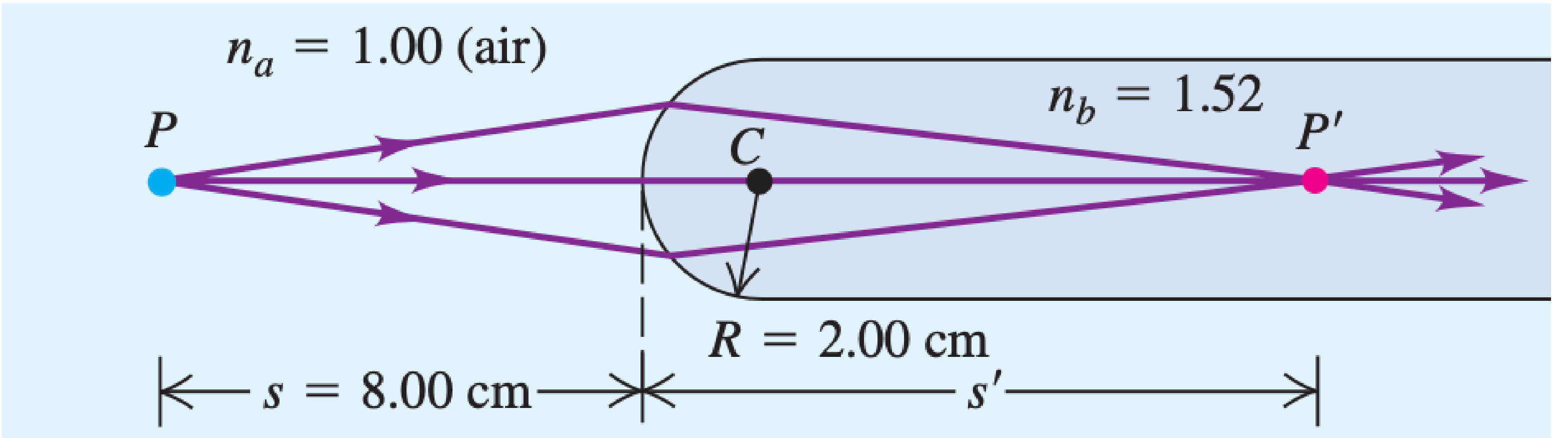
Sample Problem

Example 34.5 **Image formation by refraction I**

A cylindrical glass rod (Fig. 34.24) has index of refraction 1.52. It is surrounded by air. One end is ground to a hemispherical surface with radius $R = 2.00$ cm. A small object is placed on the axis of the rod, 8.00 cm to the left of the vertex. Find (a) the image distance and (b) the lateral magnification.



Sample Problem



EXECUTE: (a) From Eq. (34.11),

$$\frac{1.00}{8.00\ \text{cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.00}{+2.00\ \text{cm}}$$

$$s' = +11.3\ \text{cm}$$

(b) From Eq. (34.12),

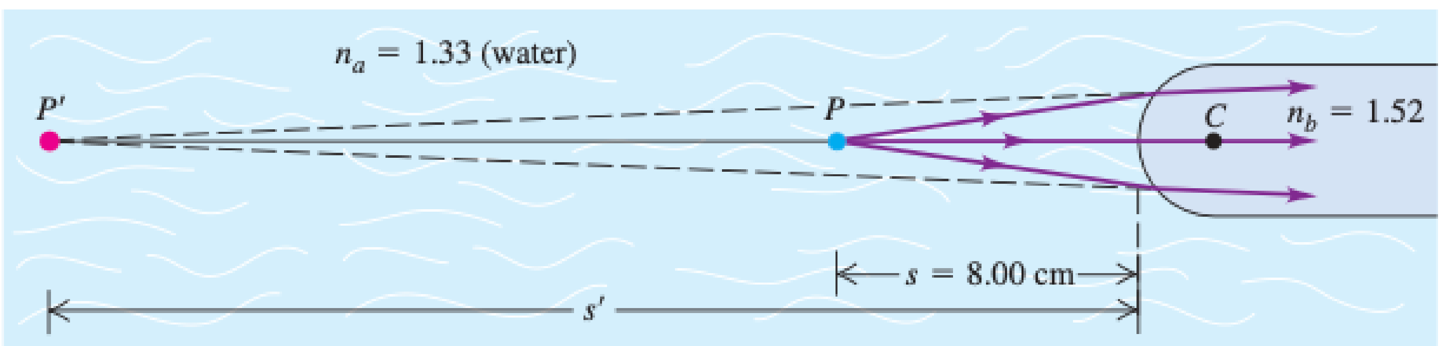
$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(11.3\ \text{cm})}{(1.52)(8.00\ \text{cm})} = -0.929$$

Sample Problem

Example 34.6 **Image formation by refraction II**

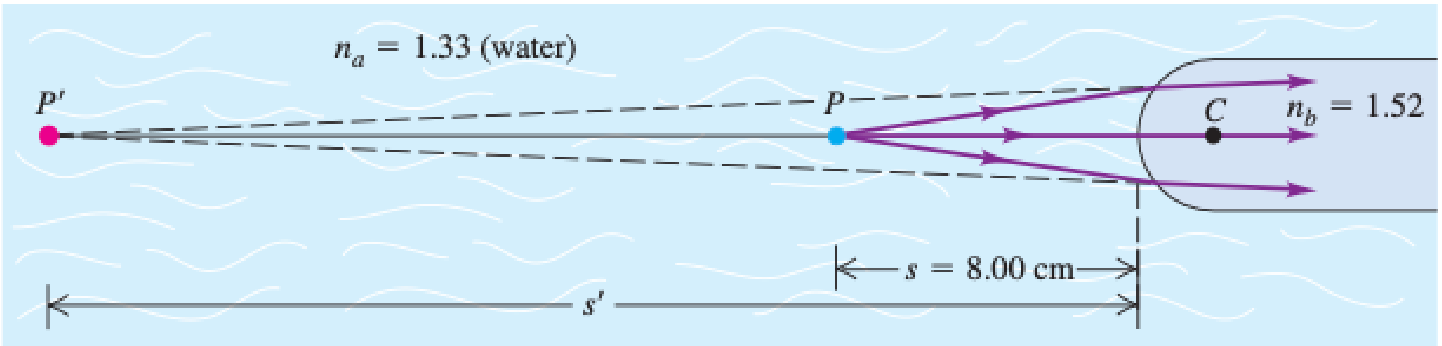
The glass rod of Example 34.5 is immersed in water, which has index of refraction $n = 1.33$ (Fig. 34.25). The object distance is again 8.00 cm. Find the image distance and lateral magnification.

When immersed in water, the glass rod forms a virtual image.



Sample Problem

When immersed in water, the glass rod forms a virtual image.



EXECUTE: Our solution of Eq. (34.11) in Example 34.5 yields

$$\frac{1.33}{8.00\ \text{cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.33}{+2.00\ \text{cm}}$$

$$s' = -21.3\ \text{cm}$$

The lateral magnification in this case is

$$m = -\frac{(1.33)(-21.3\ \text{cm})}{(1.52)(8.00\ \text{cm})} = +2.33$$

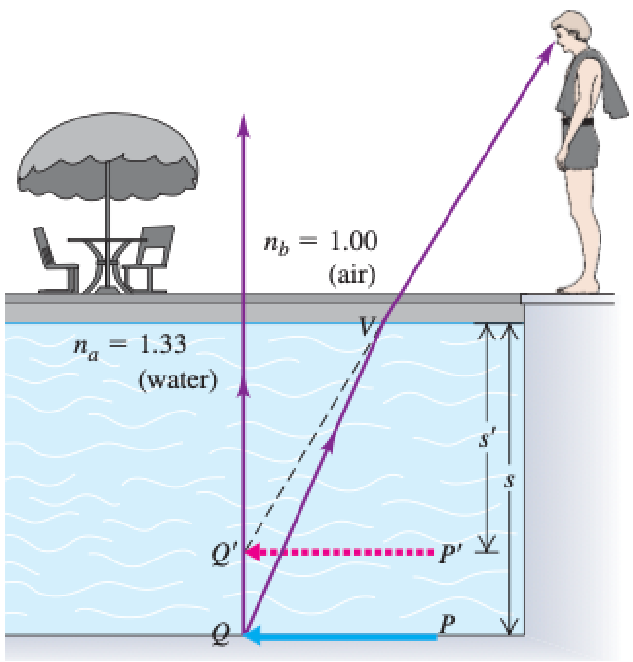
Sample Problem

Example 34.7 Apparent depth of a swimming pool

If you look straight down into a swimming pool where it is 2.00 m deep, how deep does it appear to be?

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{1.33}{2.00\text{ m}} + \frac{1.00}{s'} = 0$$

$$s' = -1.50\text{ m}$$



Summary

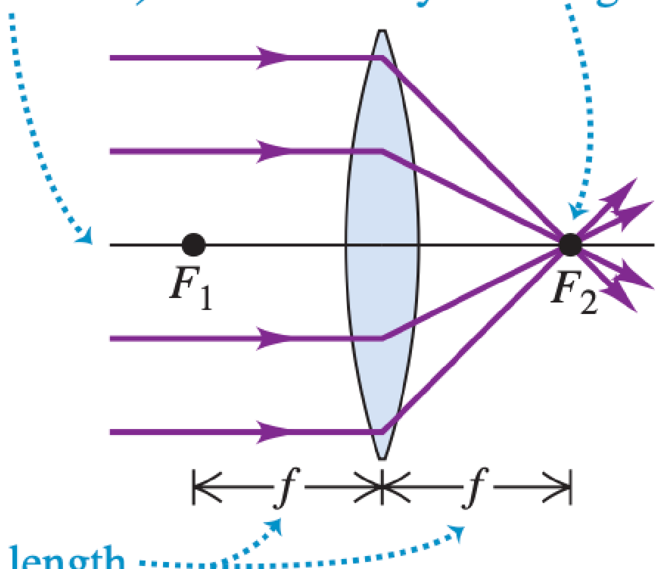
	Plane Mirror	Spherical Mirror
Object and image distances	$\frac{1}{s} + \frac{1}{s'} = 0$	$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$
Lateral magnification	$m = -\frac{s'}{s} = 1$	$m = -\frac{s'}{s}$

	Plane Refracting Surface	Spherical Refracting Surface
Object and image distances	$\frac{n_a}{s} + \frac{n_b}{s'} = 0$	$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$
Lateral magnification	$m = -\frac{n_a s'}{n_b s} = 1$	$m = -\frac{n_a s'}{n_b s}$

34-4 Thin Lenses

Optic axis (passes through centers of curvature of both lens surfaces)

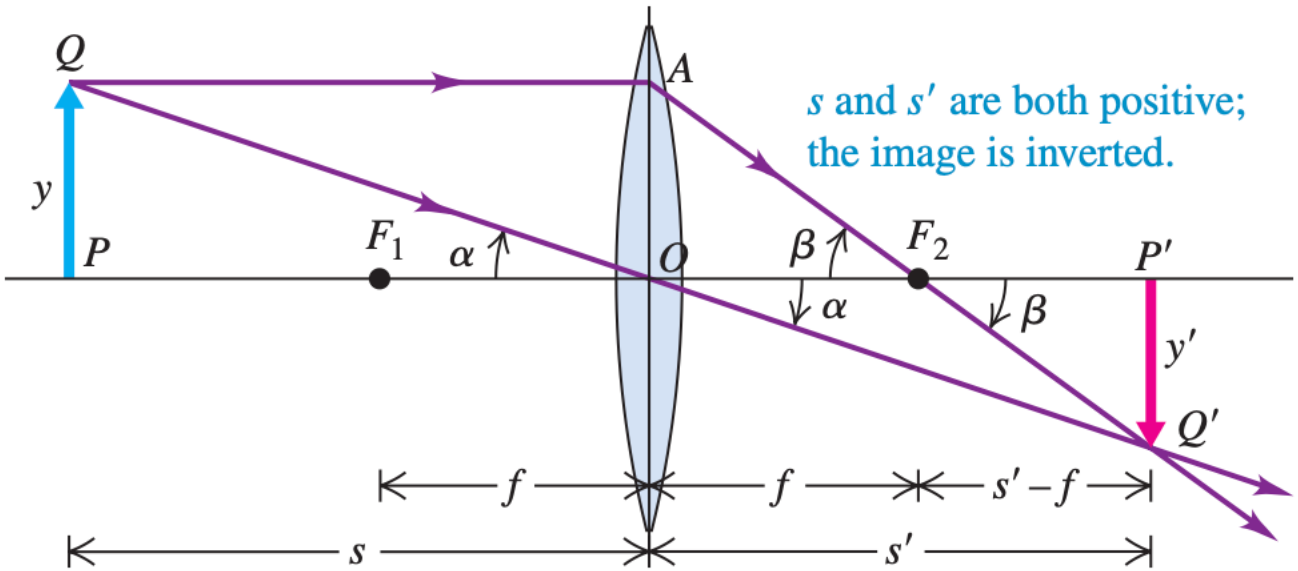
Second focal point: the point to which incoming parallel rays converge



Focal length
- Measured from lens center
- Always the same on both sides of the lens
- Positive for a converging thin lens

34-4 Thin Lenses

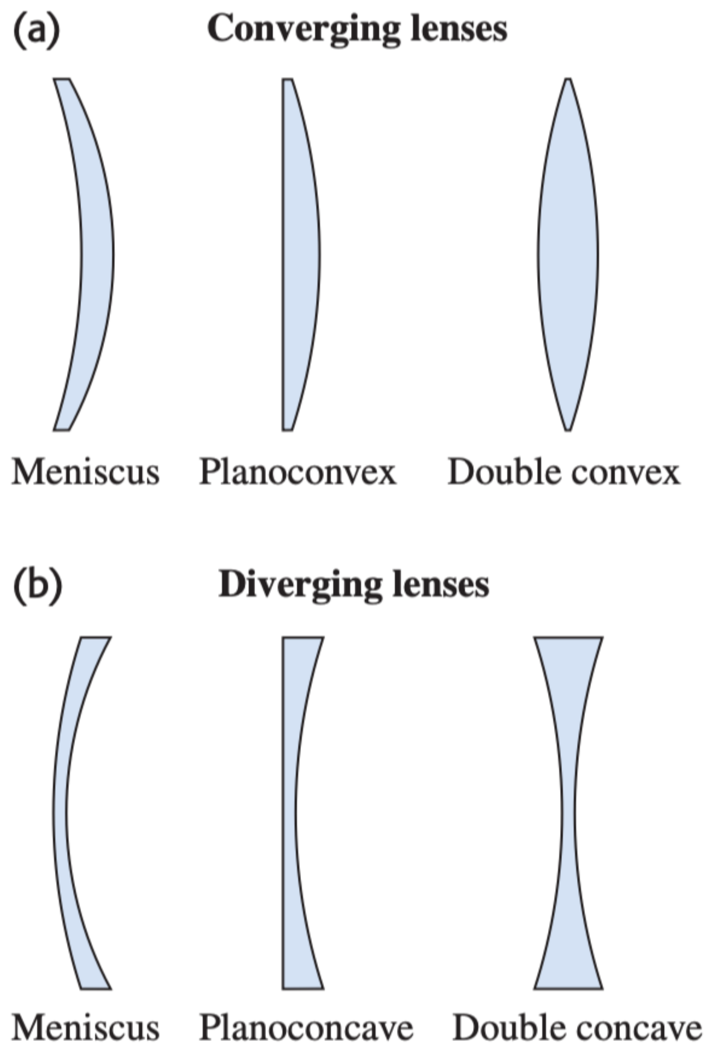
Image of an Extended Object: Converging Lens



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(object–image relationship, thin lens)}$$

$$m = -\frac{s'}{s} \qquad \text{(lateral magnification, thin lens)}$$

34-4 Thin Lenses



Far-sighted people using

Short-sighted people using

34-4 Thin Lenses

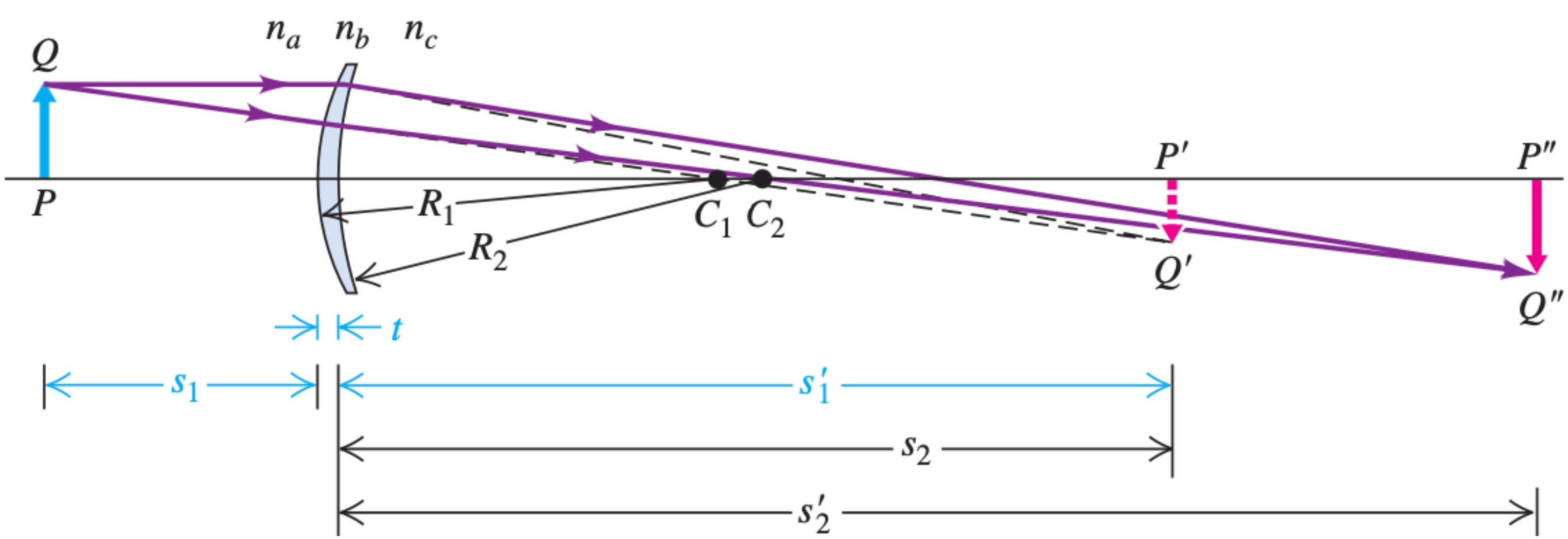
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \quad \text{(object–image relationship, spherical refracting surface)} \tag{34.11}$$

$$\frac{n_a}{s_1} + \frac{n_b}{s'_1} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s'_2} = \frac{n_c - n_b}{R_2}$$

$$\frac{1}{s_1} + \frac{n}{s'_1} = \frac{n - 1}{R_1}$$

$$-\frac{n}{s'_1} + \frac{1}{s'_2} = \frac{1 - n}{R_2}$$



34-4 Thin Lenses

$$\frac{1}{s_1} + \frac{1}{s_2'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \begin{cases} \dfrac{1}{s} + \dfrac{1}{s'} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) \\[2ex] \boxed{\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}} \end{cases}$$

Lensmaker's equation for a thin lens

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Sign of R_1/R_2 is the key

Sample Problem

Example 34.8 Determining the focal length of a lens

(a) Suppose the absolute values of the radii of curvature of the lens surfaces in Fig. 34.35 are both equal to 10 cm and the index of refraction of the glass is $n = 1.52$. What is the focal length f of the lens? (b) Suppose the lens in Fig. 34.31 also has $n = 1.52$ and the absolute values of the radii of curvature of its lens surfaces are also both equal to 10 cm. What is the focal length of this lens?

Fig 34.35

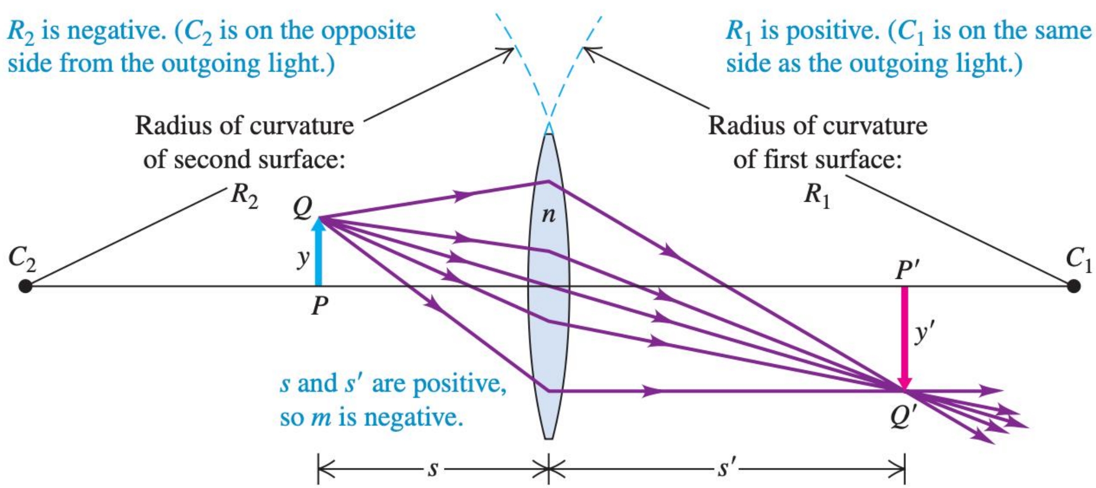
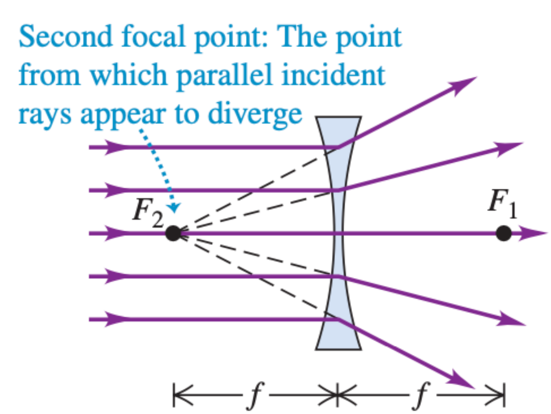
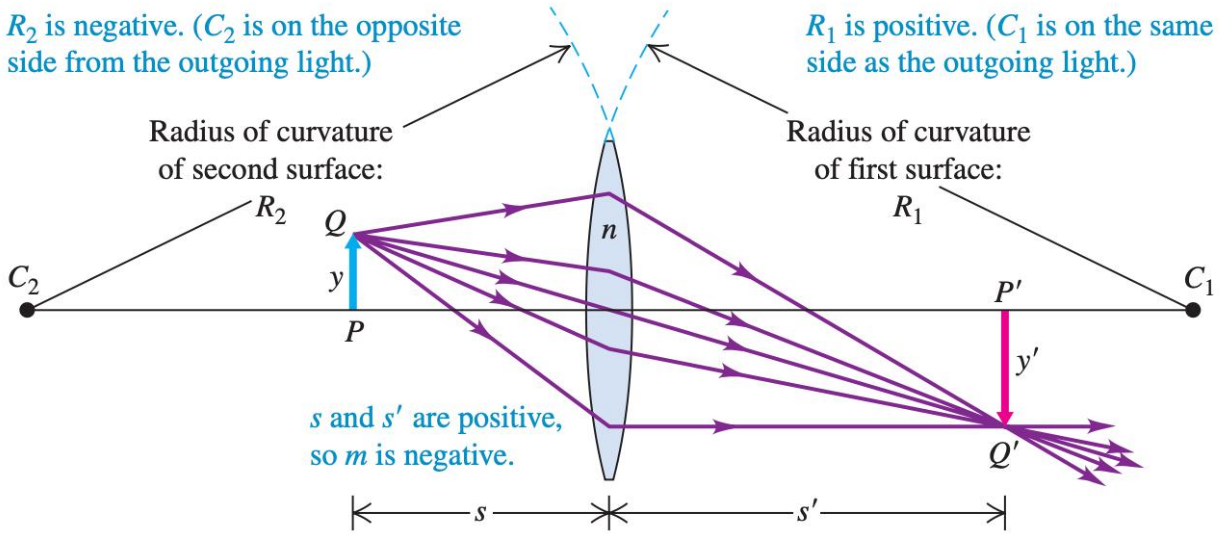


Fig 34.31



Sample Problem *Sign depends on sides of outgoing light*

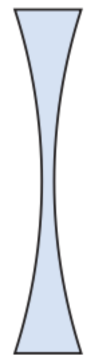


EXECUTE: (a) The lens in Fig. 34.35 is *double convex:* The center of curvature of the first surface (C_1) is on the outgoing side of the lens, so R_1 is positive, and the center of curvature of the second surface (C_2) is on the *incoming* side, so R_2 is negative. Hence $R_1 = +10$ cm and $R_2 = -10$ cm. Then from Eq. (34.19),

$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{+10\text{ cm}} - \frac{1}{-10\text{ cm}}\right)$$

$$f = 9.6\text{ cm}$$

Sample Problem



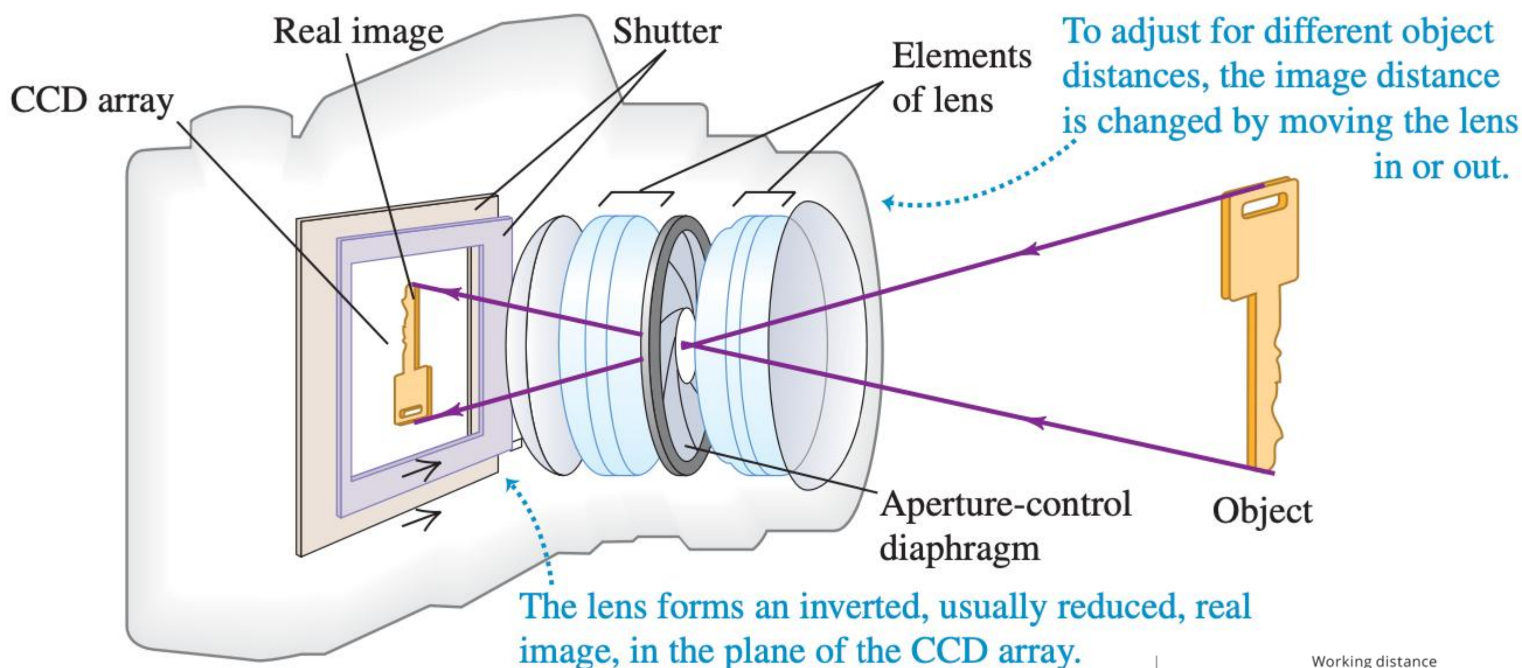
Double concave

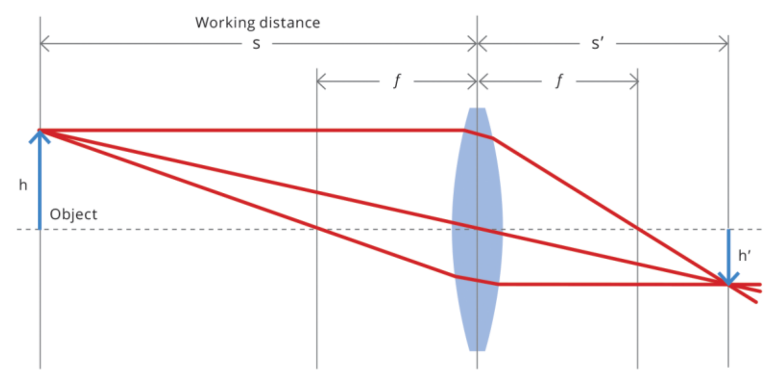
(b) The lens in Fig. 34.31 is *double concave:* The center of curvature of the first surface is on the *incoming* side, so R_1 is negative, and the center of curvature of the second surface is on the outgoing side, so R_2 is positive. Hence in this case $R_1 = -10$ cm and $R_2 = +10$ cm. Again using Eq. (34.19),

$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{-10\text{ cm}} - \frac{1}{+10\text{ cm}}\right)$$

$$f = -9.6\text{ cm}$$

34-5 Cameras


Real image
Shutter
CCD array
Elements of lens
To adjust for different object distances, the image distance is changed by moving the lens in or out.
Aperture-control diaphragm
Object
The lens forms an inverted, usually reduced, real image, in the plane of the CCD array.

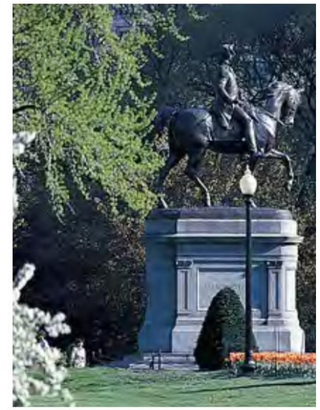

Working distance
s
s′
f
f
h
Object
h′

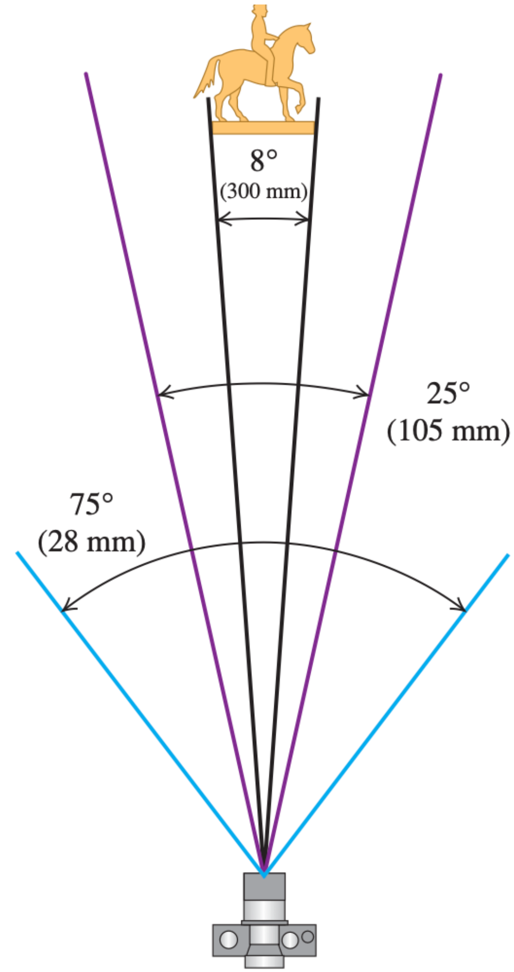
34-5 Cameras

(a) $f = 28$ mm

(b) $f = 105$ mm

(c) $f = 300$ mm





- long focal length, called a *telephoto* lens, gives a small angle of view and a large image of a distant object
- a lens of short focal length gives a small image and a wide angle of view and is called a *wide-angle* lens.

34-5 Cameras

- For the film to record the image properly, the total light energy per unit area reaching the film (the “exposure”) must fall within certain limits.

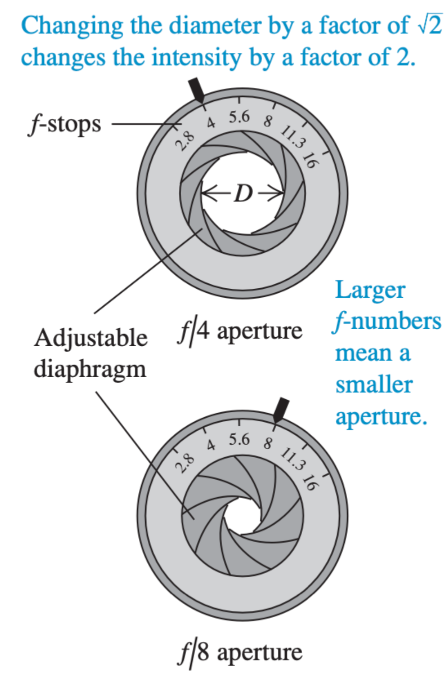
The size of the area that the lens "sees" is proportional to the square of the angle of view of the lens, proportional to $1/f^2$.

Effective area is proportional to aperture diameter D^2.

Intensity of light reaching portional to D^2/f^2

Photographers define

$$f\text{-number} = \frac{\text{Focal length}}{\text{Aperture diameter}} = \frac{f}{D}$$



Sample Problem

Example 34.12 **Photographic exposures**

A common telephoto lens for a 35-mm camera has a focal length of 200 mm; its f-stops range from $f/2.8$ to $f/22$. (a) What is the corresponding range of aperture diameters? (b) What is the corresponding range of image intensities on the film?

EXECUTE: (a) From Eq. (34.20), the diameter ranges from

$$D = \frac{f}{f\text{-number}} = \frac{200\text{ mm}}{2.8} = 71\text{ mm}$$

to

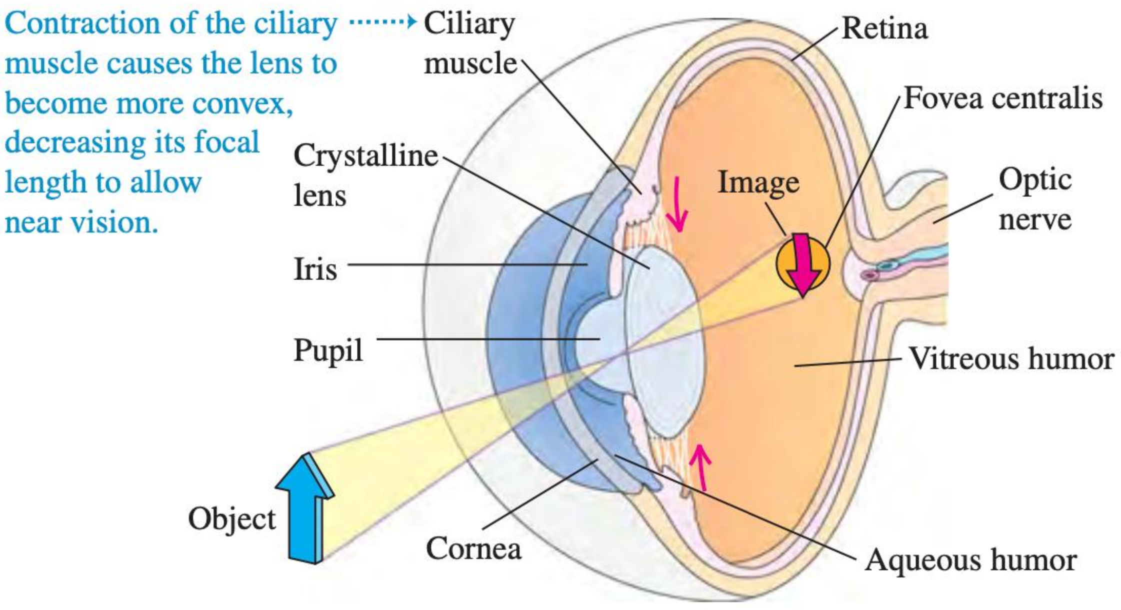
$$D = \frac{200\text{ mm}}{22} = 9.1\text{ mm}$$

(b) Because the intensity is proportional to D^2, the ratio of the intensity at $f/2.8$ to the intensity at $f/22$ is

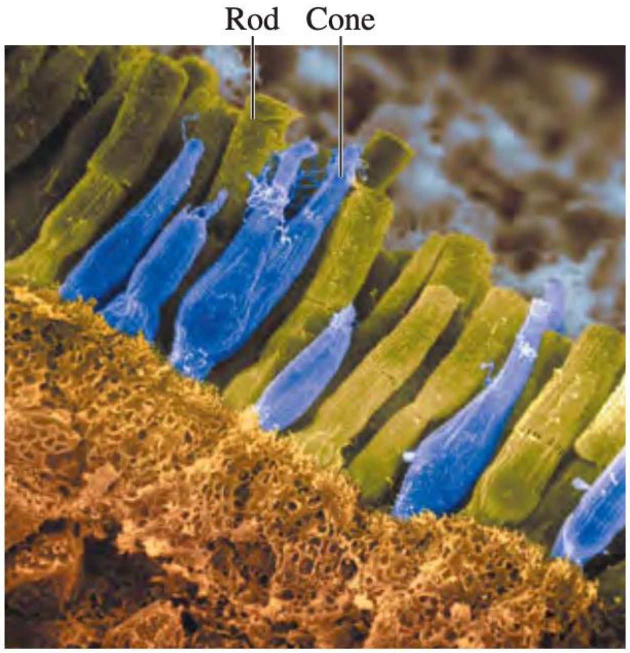
$$\left(\frac{71\text{ mm}}{9.1\text{ mm}}\right)^2 = \left(\frac{22}{2.8}\right)^2 = 62 \qquad (\text{about } 2^6)$$

34-6 The Eye

(a) Diagram of the eye

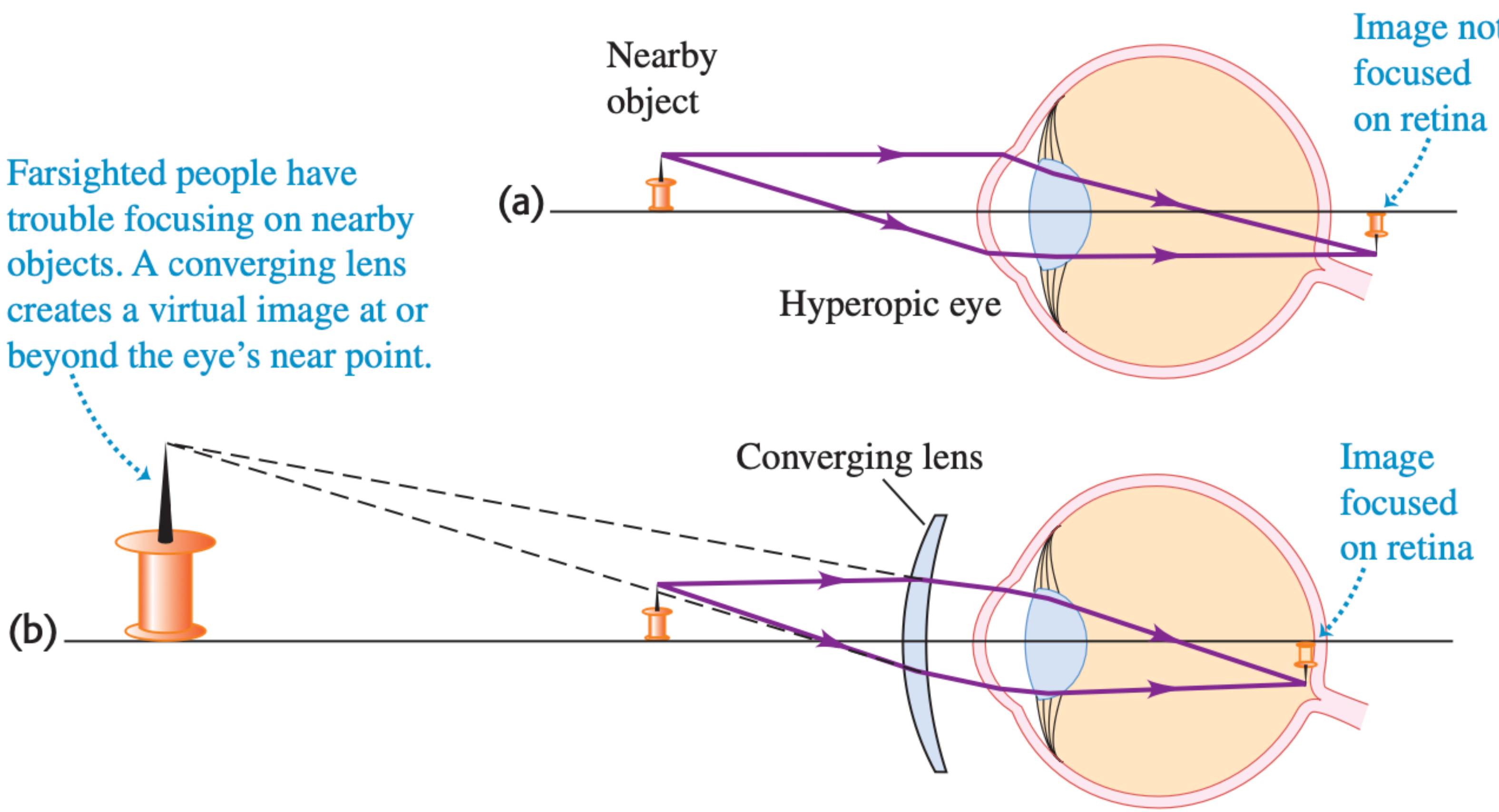


(b) Scanning electron micrograph showing retinal rods and cones in different colors



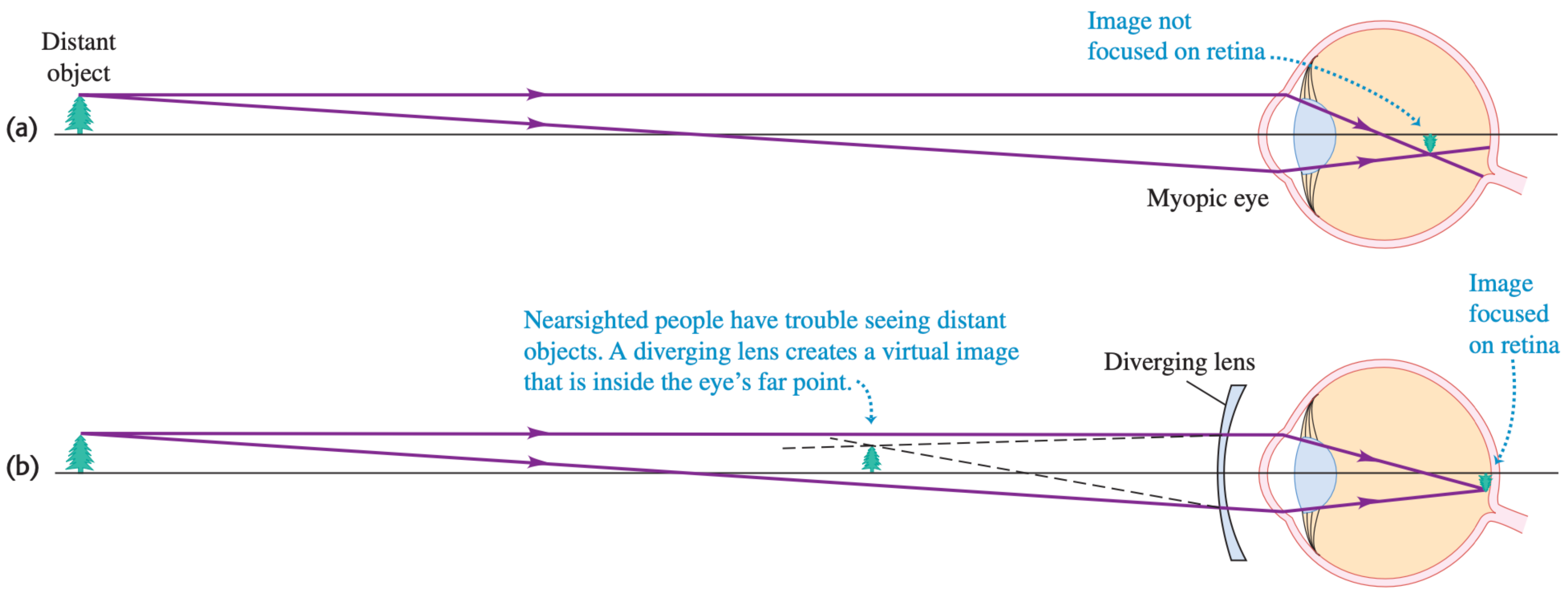
34-6 The Eye

Hyperopic (farsighted) eye



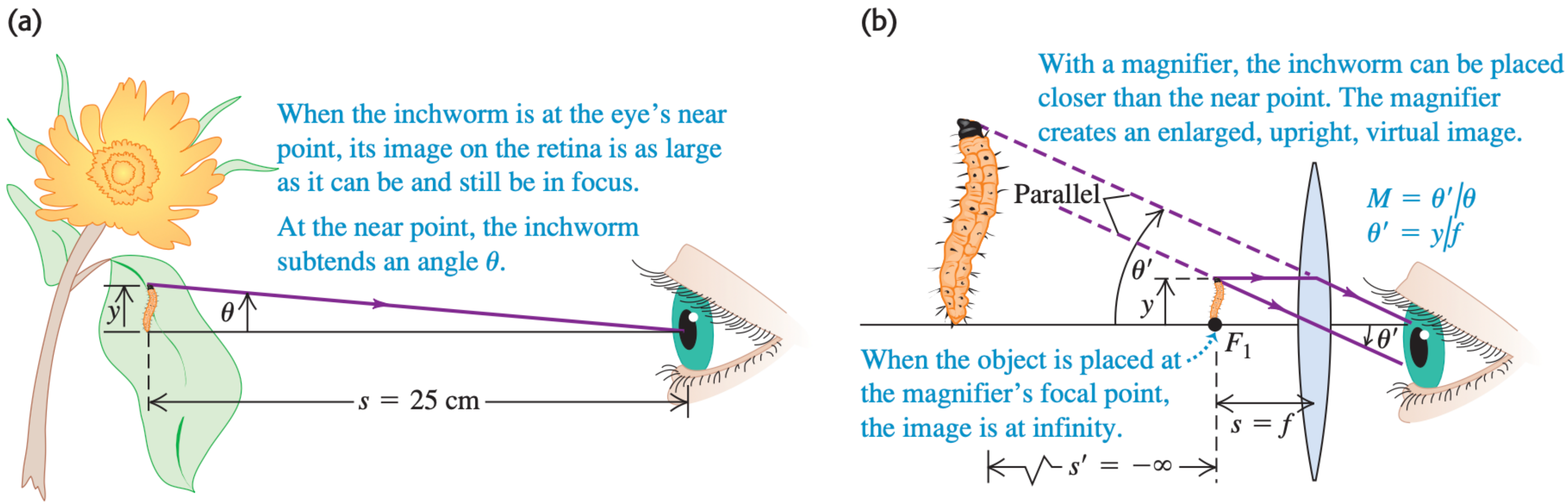
34-6 The Eye

Myopic (nearsighted) eye



34-7 The Magnifier

34.51 **(a)** The angular size θ is largest when the object is at the near point. **(b)** The magnifier gives a virtual image at infinity. This virtual image appears to the eye to be a real object subtending a larger angle θ' at the eye.



34-7 The Magnifier

To find the value of M, we first assume that the angles are small enough that each angle (in radians) is equal to its sine and its tangent. Using Fig. 34.451a and drawing the ray in Fig. 34.51b that passes undeviated through the center of the lens, we find that θ and θ' (in radians) are

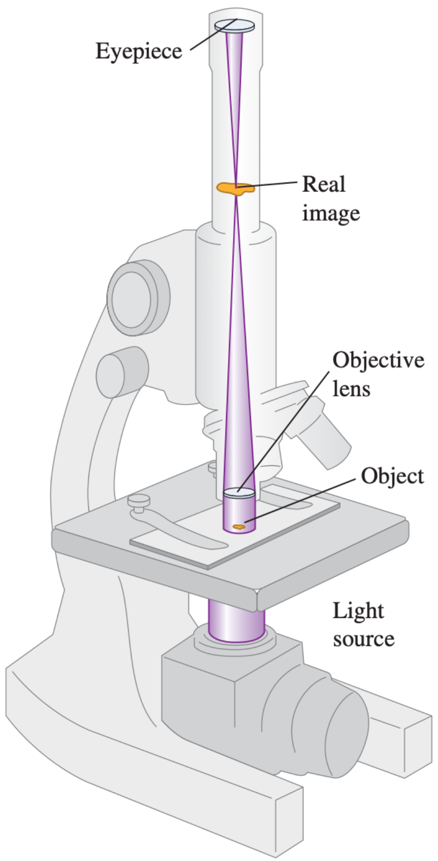
$$\theta = \frac{y}{25\text{ cm}} \qquad \theta' = \frac{y}{f}$$

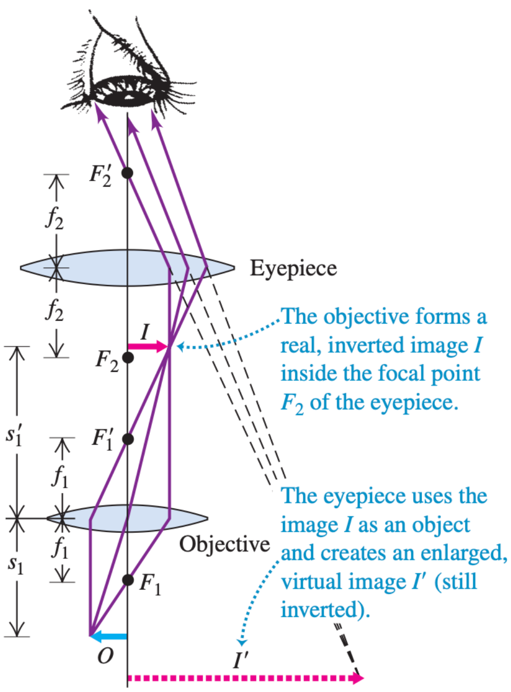
Combining these expressions with Eq. (34.21), we find

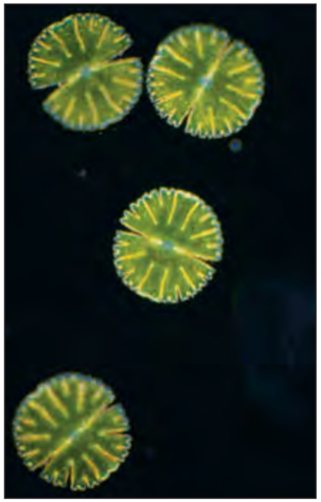
$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25\text{ cm}} = \frac{25\text{ cm}}{f} \qquad \text{(angular magnification for a simple magnifier)} \qquad (34.22)$$

34-8 Microscopes and Telescopes

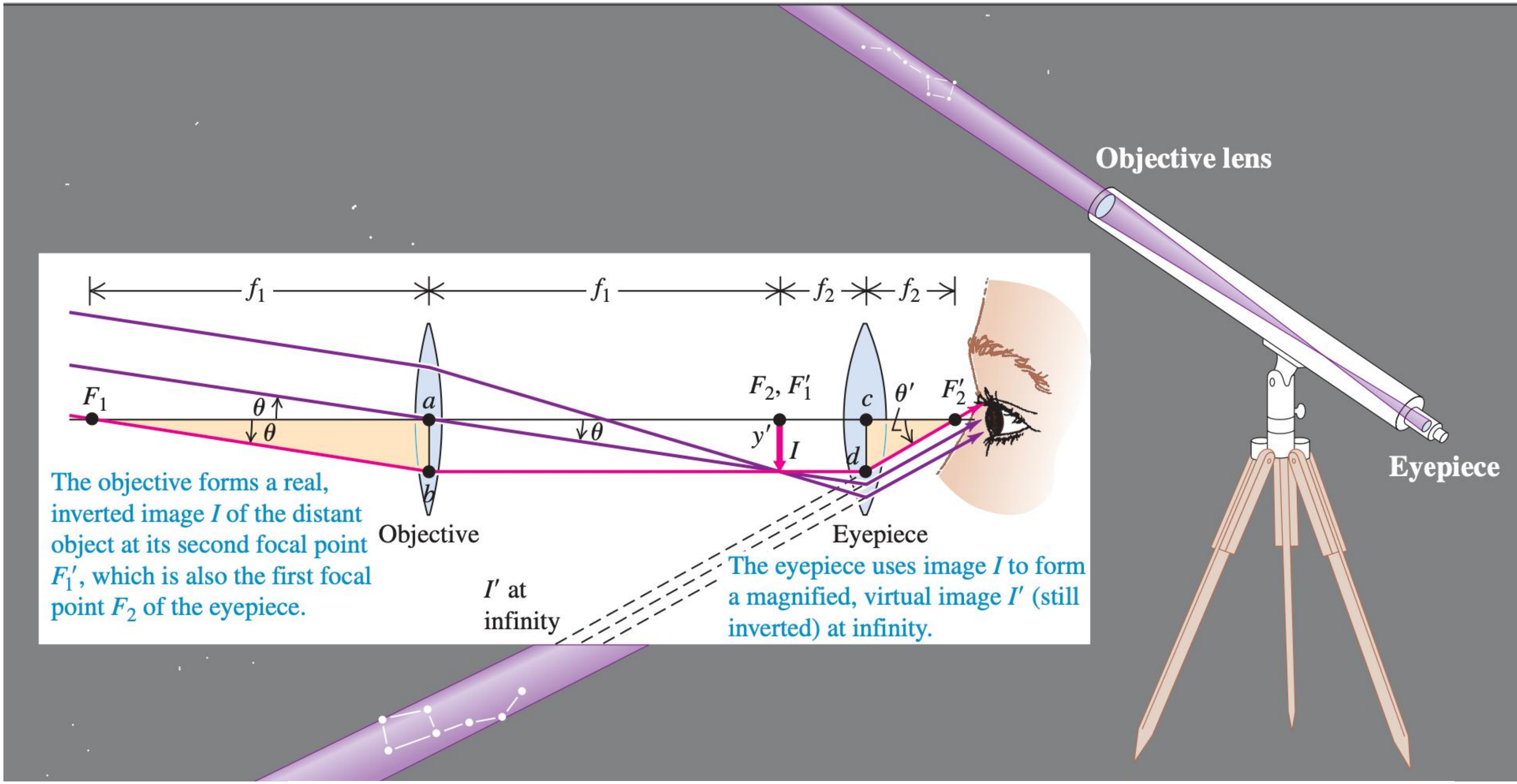
34.52 (a) Elements of a microscope. (b) The object O is placed just outside the first focal point of the objective (the distance s_1 has been exaggerated for clarity). (c) This microscope image shows single-celled organisms about 2×10^{-4} m (0.2 mm) across. Typical light microscopes can resolve features as small as 2×10^{-7} m, comparable to the wavelength of light.







34-8 Microscopes and Telescopes



$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2} \quad \text{(angular magnification for a telescope)}$$