

## Assignment # 13+14

### Modern Physics II & III

#### Chapter 39: Light Waves Behaving as Particles

#### Chapter 40: Quantum Mechanics

#### Important Formulas and Concepts:

Symmetry, Wave Particle Duality,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}}$$

$$L_n = mv_n r_n = n \frac{h}{2\pi}$$

$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$$

$$I = \sigma T^4 \quad (\text{Stefan-Boltzmann law for blackbody})$$

$$\lambda_m T = 2.90 \times 10^{-3} = \text{constant}$$

$$I = \int_0^\infty I(\lambda) d\lambda \quad (\text{Total Intensity})$$

$$I(\lambda) = \frac{2\pi h c^2}{\lambda^5 (\exp \frac{hc}{\lambda kT} - 1)}$$

#### Question 1:

For crystal diffraction experiments, wavelengths on the order of 0.20 nm are often appropriate. Find the energy in electron volts for a particle with this wavelength if the particle is;

- (a) a photon
- (b) an electron
- (c) an alpha particle ( $m = 6.64 \times 10^{-27} \text{ kg}$ )

#### Question 2:

Determine  $\lambda_m$  the wavelength at the peak of the Planck distribution, and the corresponding frequency  $f$ , at these temperatures:

- (a) 3.00 K
- (b) 300
- (c) 3000 K
- (d) Analyse the wavelengths at these three temperatures and arrange them in ascending order.

#### Question 3:

A 20.0-kg satellite circles the earth once every 2.00 h in an orbit having a radius of 8060 km.

Assuming that Bohr's angular-momentum result  $L = n \frac{h}{2\pi}$  applies to satellites just as it does to an electron in the hydrogen atom.

- (a) Find the quantum number  $n$  of the orbit of the satellite.

- (b) Show from Bohr's angular momentum result and Newton's law of gravitation that the radius of an earth-satellite orbit is directly proportional to the square of the quantum number,  $r = kn^2$ , where  $k$  is the constant of proportionality.
- (c) Using the result from part (b), find the distance between the orbit of the satellite in this problem and its next "allowed" orbit. (Calculate a numerical value.)
- (d) Comment on the possibility of observing the separation of the two adjacent orbits.
- (e) Do quantized and classical orbits correspond for this satellite? Which is the "correct" method for calculating the orbits?

**Question 4:**

Write the Planck distribution law in terms of frequency  $f$ , rather than the wavelength  $\lambda$ , to obtain  $I(f)$ .

- (a) Show that

$$\int_0^\infty I(\lambda) d\lambda = \frac{2\pi^5 k^4}{15 c^2 4^3} T^4$$

Where  $I(\lambda)$  is the Planck distribution formula. For the solution, you will be in need of following tabulated integral;

$$\int_0^\infty \frac{x^3}{\exp^{\alpha x} - 1} dx = \frac{1}{240} \left( \frac{2\pi}{\alpha} \right)^4$$

- (b) The result of part (a) is  $I$  and has the form of Stefan-Boltzmann law,  $I = \sigma T^4$ . Evaluate the constants in part (a).

**Question 5:**

The wave function  $\psi(x,t) = A \exp^{i(k_1 x - \omega_1 t)} + A \exp^{i(k_2 x - \omega_2 t)}$  is a superposition of two free-particle wave functions. Both  $k_1$  and  $k_2$  are positive.

- (a) Show that this wave function satisfies the Schrodinger equation for a free particle of mass  $m$ .
- (b) Find the probability distribution function for  $\psi(x,t)$ .

**Question 6:**

Compute  $|\psi|^2$  for  $\psi = \psi \sin \omega t$ , where  $\psi$  is time independent and  $\omega$  is real constant. Is this a wave function for a stationary state? Why or why not