PHYS1001B College Physics IB

Optics III Interference (Ch. 34)

Introduction

Optical effects that depend on the wave nature of light are grouped under the heading **physical optics.**

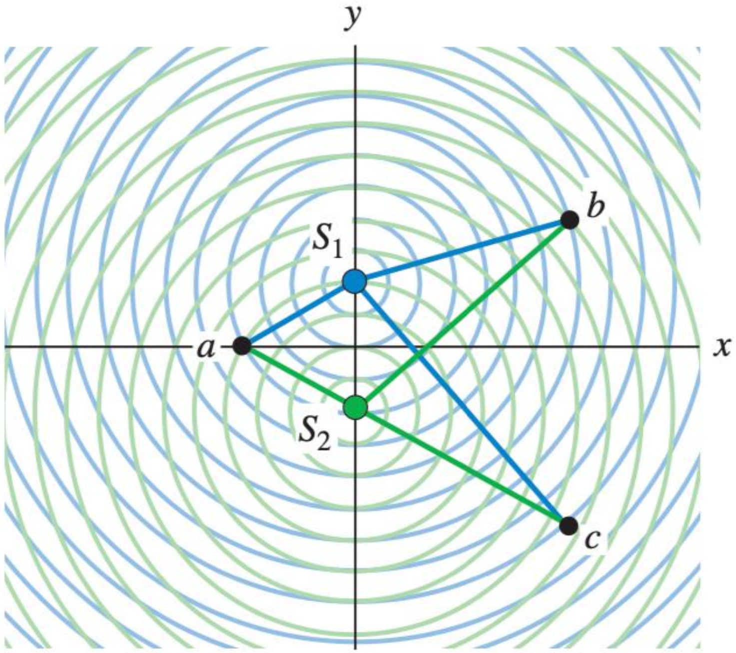
Interference/ diffraction

Outline

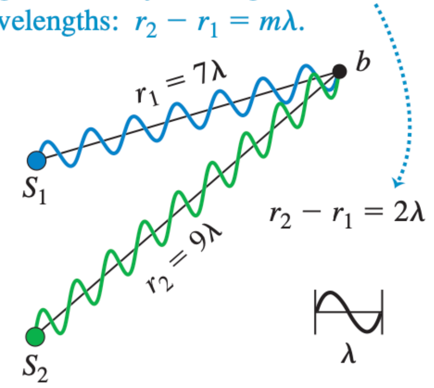
- 35-1 Interference and Coherent Sources
- 35-2 Two-Source Interference of Light
- 35-3 Intensity in Interference Patterns
- 35-4 Interference in Thin Films
- 35-5 The Michelson Interferometer

35-1 Interference and Coherent Sources

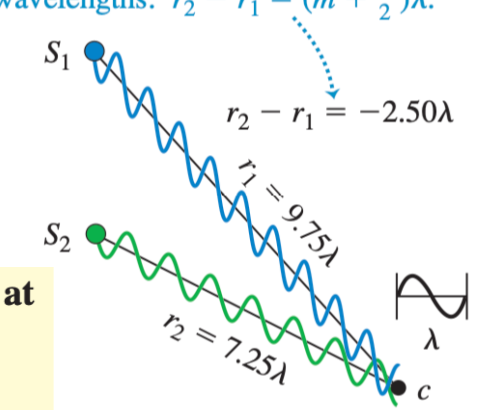
(a) Two coherent wave sources separated by a distance 4λ



(b) Conditions for constructive interference: Waves interfere constructively if their path lengths differ by an integral number of wavelengths: $r_2 - r_1 = m\lambda$.



(c) Conditions for destructive interference: Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.

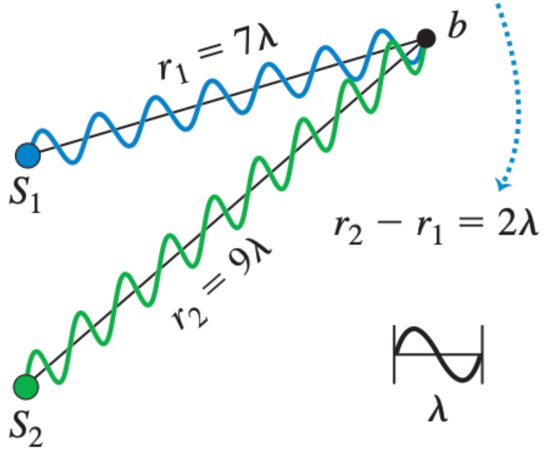


principle of superposition

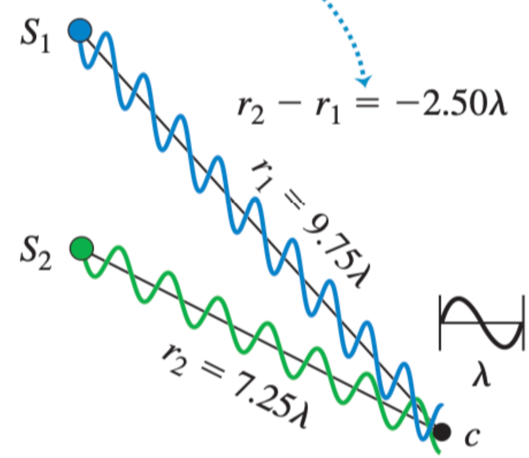
When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

35-1 Interference and Coherent Sources

(b) Conditions for constructive interference:
Waves interfere constructively if their path lengths differ by an integral number of wavelengths: $r_2 - r_1 = m\lambda$.



(c) Conditions for destructive interference:
Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.

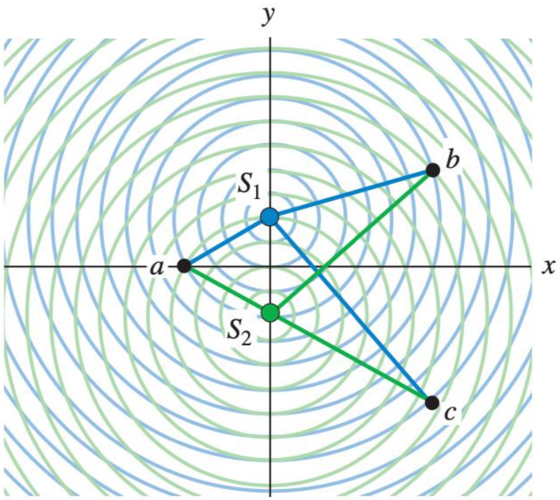


$$r_2 - r_1 = m\lambda \qquad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$ (constructive interference, sources in phase)

$$r_2 - r_1 = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$ (destructive interference, sources in phase)

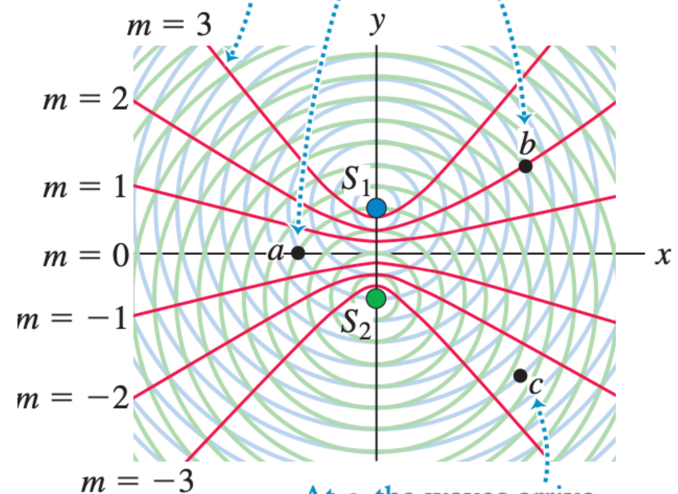
35-1 Interference and Coherent Sources

(a) Two coherent wave sources separated by a distance 4λ



Antinodal curves (red) mark positions where the waves from S_1 and S_2 interfere constructively.

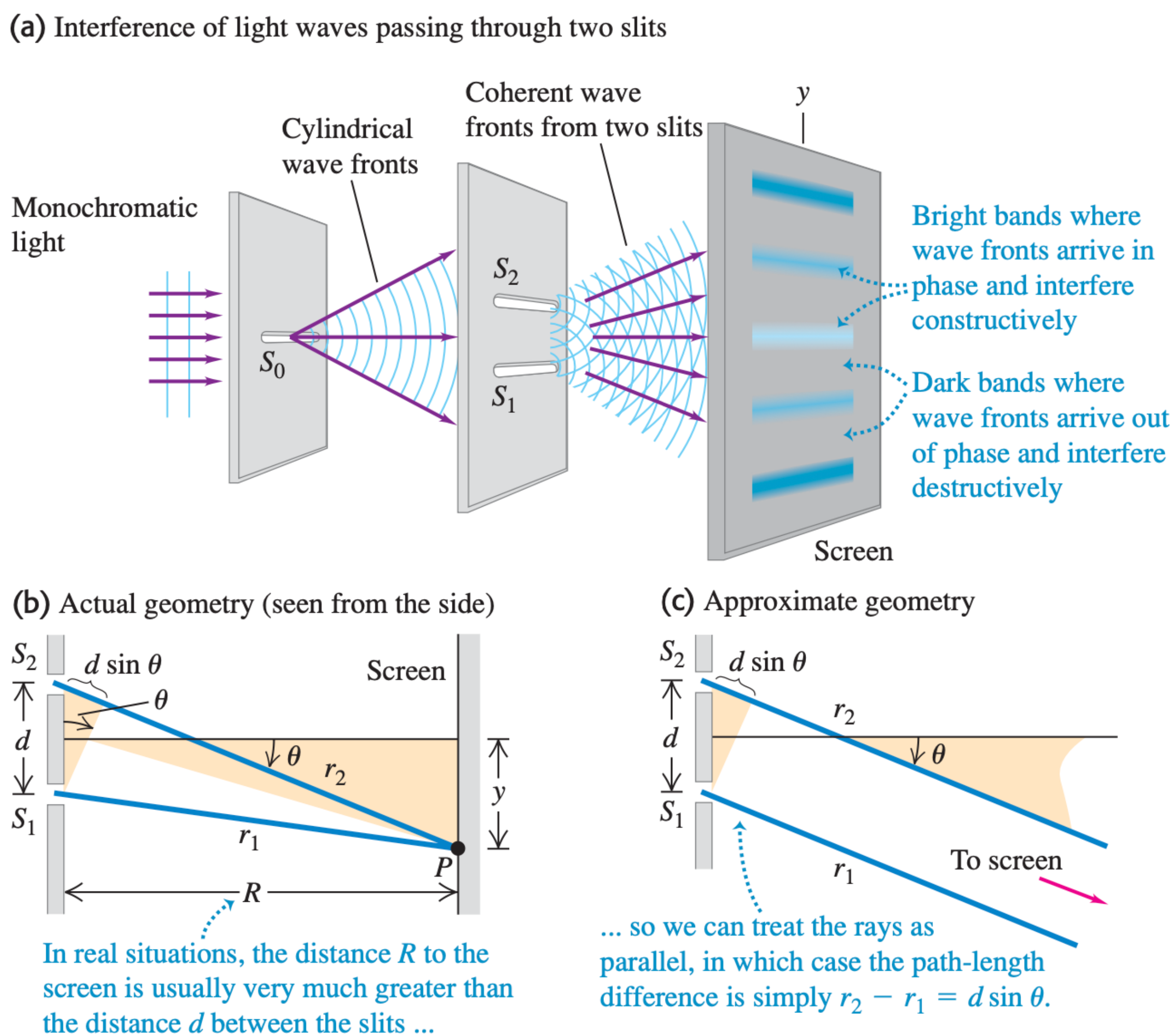
At a and b, the waves arrive in phase and interfere constructively.



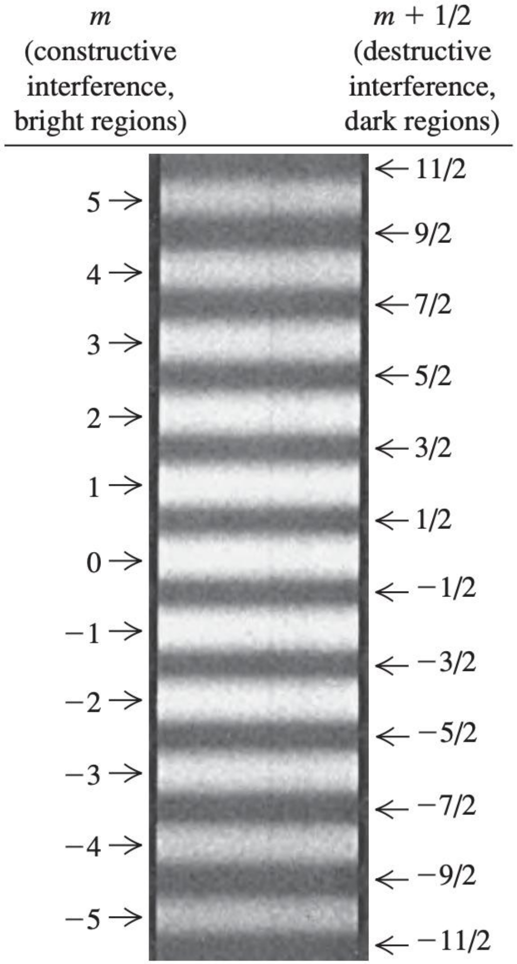
At c, the waves arrive one-half cycle out of phase and interfere destructively.

m = the number of wavelengths λ by which the path lengths from S_1 and S_2 differ.

35-2 Two-Source Interference of Light



35-2 Two-Source Interference of Light



Young's double- slit

Constructive interference

$$d \sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \dots)$$

Small angle

$$y_m = R\frac{m\lambda}{d}$$

Destructive interference

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, \pm 1, \pm 2, \dots)$$

Sample Problem

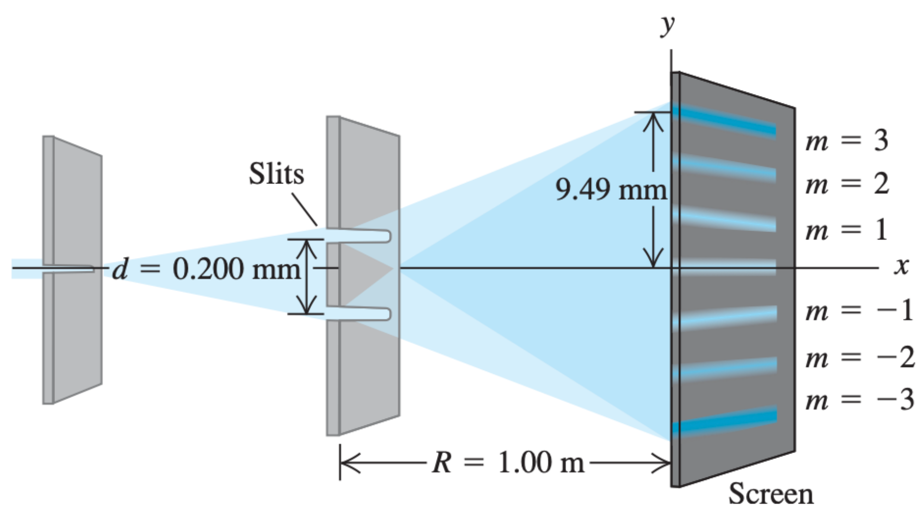
Example 35.1 **Two-slit interference**

Figure 35.7 shows a two-slit interference experiment in which the slits are 0.200 mm apart and the screen is 1.00 m from the slits. The $m = 3$ bright fringe in the figure is 9.49 mm from the central fringe. Find the wavelength of the light.

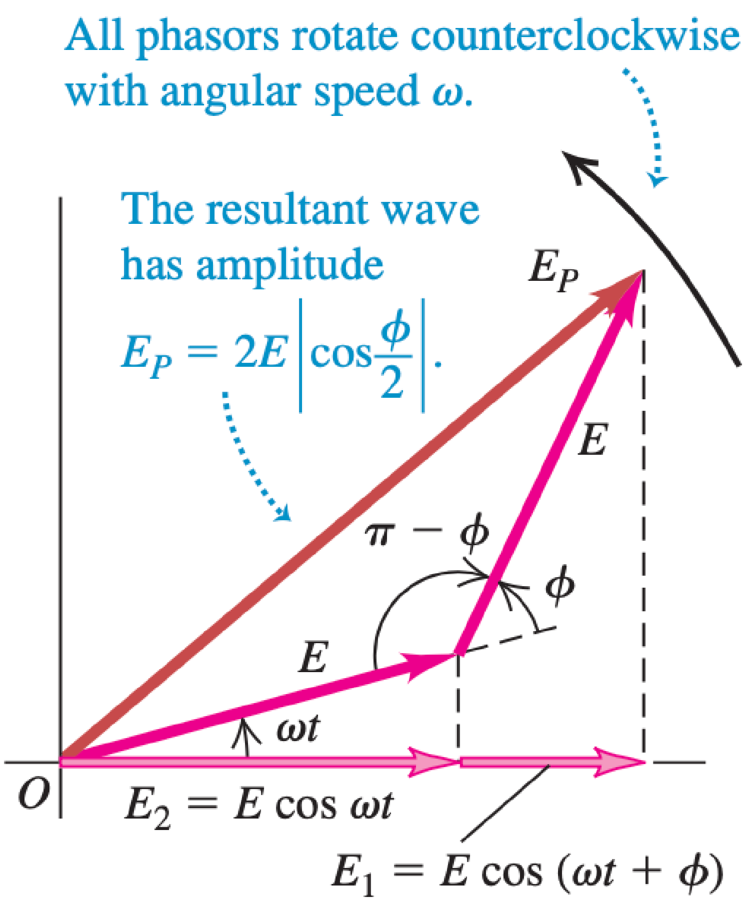
EXECUTE: We solve Eq. (35.6) for λ for the case $m = 3$:

$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3}\ \text{m})(0.200 \times 10^{-3}\ \text{m})}{(3)(1.00\ \text{m})}$$

$$= 633 \times 10^{-9}\ \text{m} = 633\ \text{nm}$$



35-3 Intensity in Interference Patterns



$$E_P{}^2 = E^2 + E^2 - 2E^2\cos(\pi - \phi)$$
$$= E^2 + E^2 + 2E^2\cos\phi$$

$$E_P{}^2 = 2E^2(1 + \cos\phi) = 4E^2\cos^2\left(\frac{\phi}{2}\right)$$

$$E_P = 2E\left|\cos\frac{\phi}{2}\right|$$

$$I = S_{av} = \frac{E_P{}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_P{}^2 = \frac{1}{2}\epsilon_0 c E_P{}^2$$

$$I = I_0\cos^2\frac{\phi}{2}$$

Sample Problem

Example 35.3 A directional transmitting antenna array

Suppose the two identical radio antennas of Fig. 35.8 are moved to be only 10.0 m apart and the broadcast frequency is increased to $f = 60.0$ MHz. At a distance of 700 m from the point midway between the antennas and in the direction $\theta = 0$ (see Fig. 35.8), the intensity is $I_0 = 0.020\ \mathrm{W/m^2}$. At this same distance, find (a) the intensity in the direction $\theta = 4.0°$; (b) the direction near $\theta = 0$ for which the intensity is $I_0/2$; and (c) the directions in which the intensity is zero.

EXECUTE: The wavelength is $\lambda = c/f = 5.00$ m. The spacing $d = 10.0$ m between the antennas is just twice the wavelength (as was the case in Example 35.2), so $d/\lambda = 2.00$ and Eq. (35.14) becomes

$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = I_0 \cos^2[(2.00\pi \text{ rad})\sin\theta]$$

(a) When $\theta = 4.0°$,

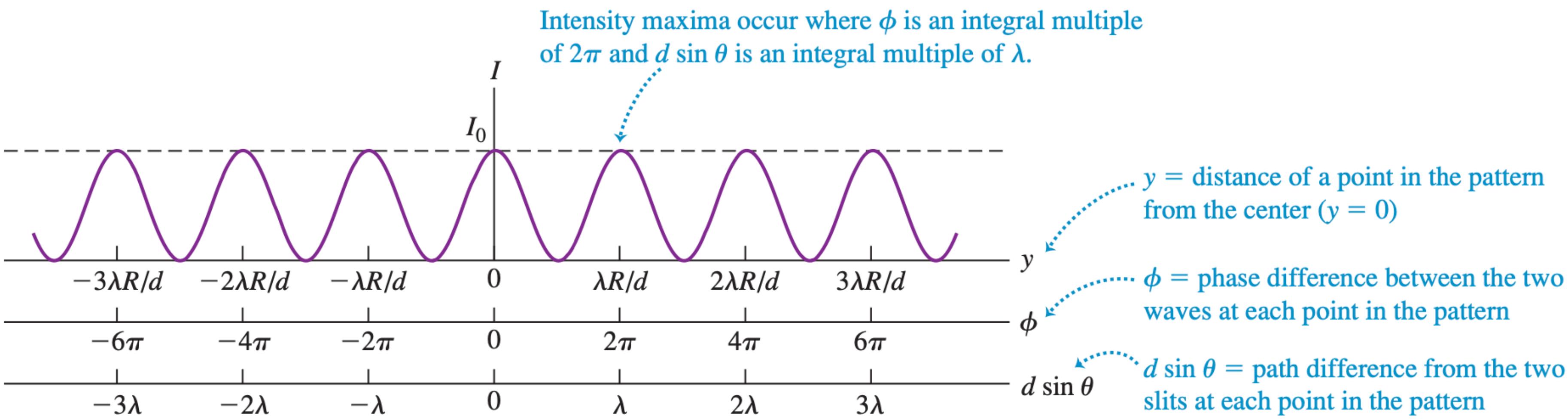
$$I = I_0 \cos^2[(2.00\pi \text{ rad})\sin 4.0°] = 0.82 I_0$$
$$= (0.82)(0.020\ \mathrm{W/m^2}) = 0.016\ \mathrm{W/m^2}$$

(b) The intensity I equals $I_0/2$ when the cosine in Eq. (35.14) has the value $\pm 1/\sqrt{2}$. The smallest angles at which this occurs correspond to $2.00\pi \sin\theta = \pm\pi/4$ rad, so that $\sin\theta = \pm(1/8.00) = \pm 0.125$ and $\theta = \pm 7.2°$.

(c) The intensity is zero when $\cos[(2.00\pi \text{ rad})\sin\theta] = 0$. This occurs for $2.00\pi \sin\theta = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$, or $\sin\theta = \pm 0.250, \pm 0.750, \pm 1.25, \ldots$. Values of $\sin\theta$ greater than 1 have no meaning, so the answers are

$$\theta = \pm 14.5°, \pm 48.6°$$

35-3 Intensity in Interference Patterns



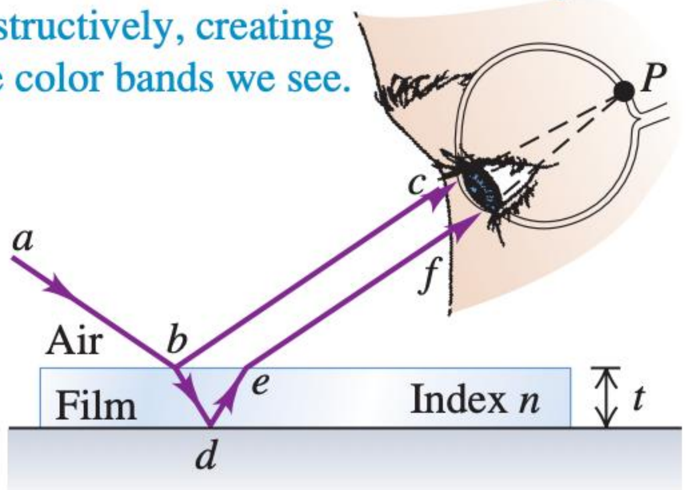
$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)$$ (phase difference related to path difference)

35-4 Interference in Thin Films

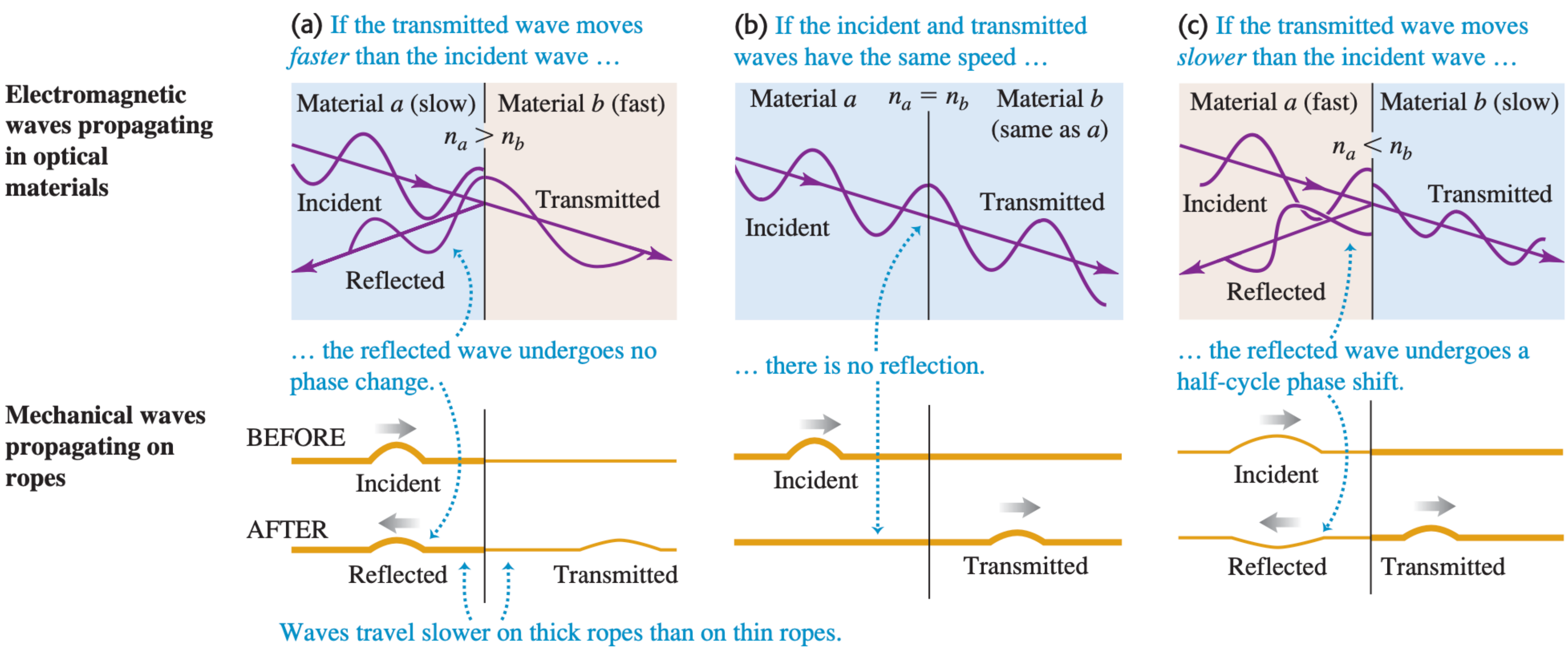
(a) Interference between rays reflected from the two surfaces of a thin film

Light reflected from the upper and lower surfaces of the film comes together in the eye at *P* and undergoes interference.

Some colors interfere constructively and others destructively, creating the color bands we see.



35-4 Interference in Thin Films



35-4 Interference in Thin Films

We can summarize this discussion mathematically. If the film has thickness t, the light is at normal incidence and has wavelength λ in the film; if neither or both of the reflected waves from the two surfaces have a half-cycle reflection phase shift, the conditions for constructive and destructive interference are

$$2t = m\lambda \qquad (m = 0, 1, 2, \dots) \qquad \text{(constructive reflection from thin film, no relative phase shift)} \tag{35.17a}$$

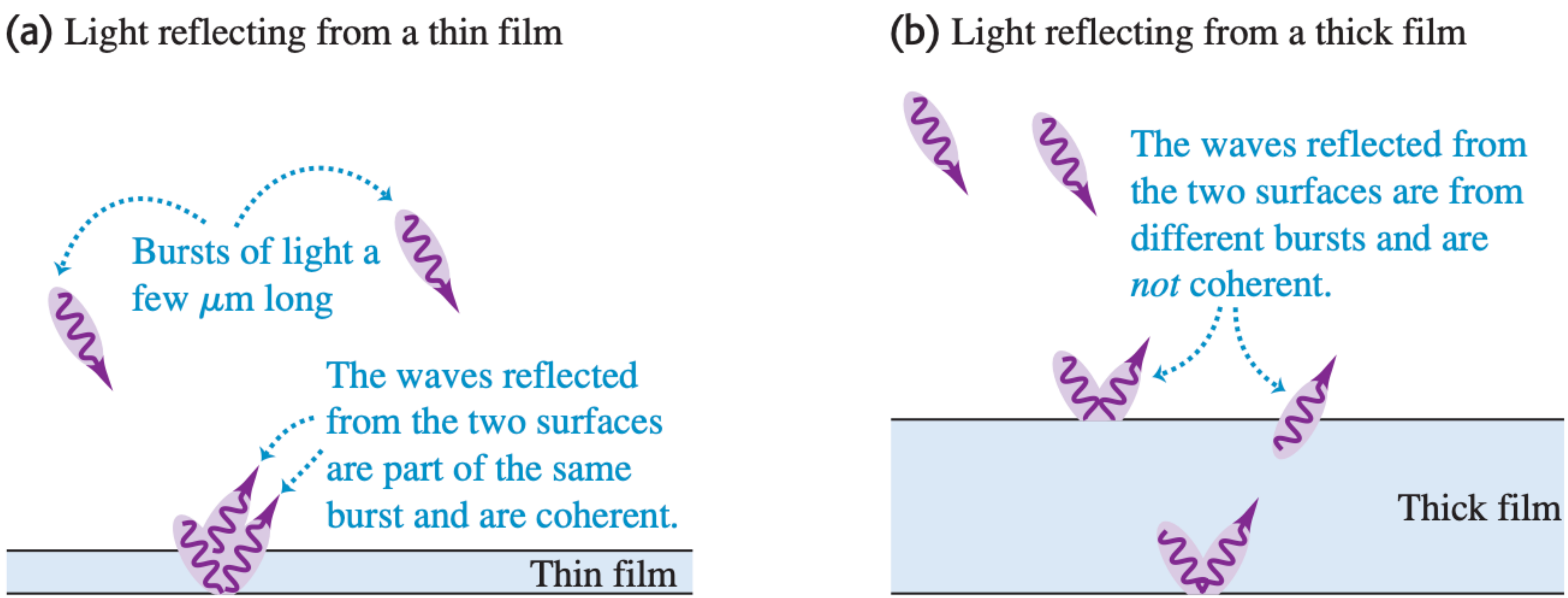
$$2t = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \dots) \qquad \text{(destructive reflection from thin film, no relative phase shift)} \tag{35.17b}$$

If *one* of the two waves has a half-cycle reflection phase shift, the conditions for constructive and destructive interference are reversed:

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \dots) \qquad \text{(constructive reflection from thin film, half-cycle relative phase shift)} \tag{35.18a}$$

$$2t = m\lambda \qquad (m = 0, 1, 2, \dots) \qquad \text{(destructive reflection from thin film, half-cycle relative phase shift)} \tag{35.18b}$$

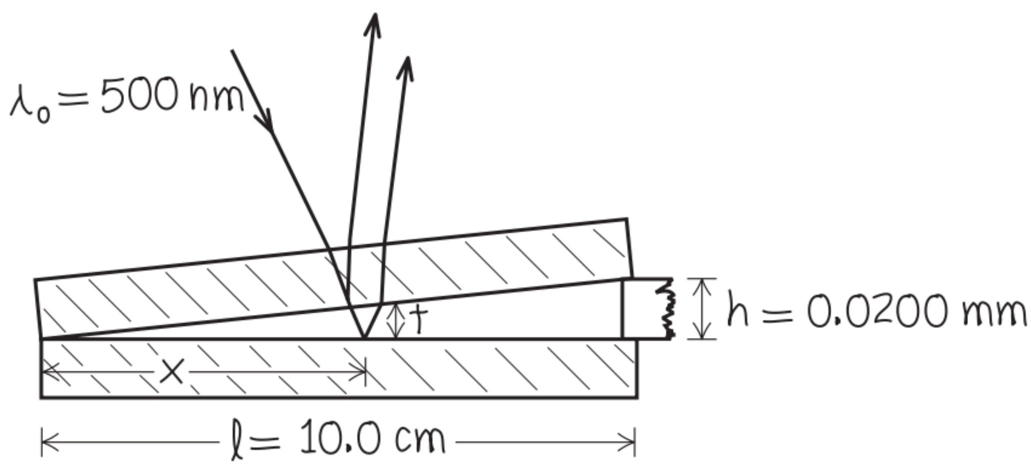
35-4 Interference in Thin Films



Sample Problem

Example 35.4 Thin-film interference I

Suppose the two glass plates in Fig. 35.12 are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of $\lambda = \lambda_0 = 500$ nm.



Sample Problem

EXECUTE: Since only one of the reflected waves undergoes a phase shift, the condition for *destructive* interference (a dark fringe) is Eq. (35.18b):

$$2t = m\lambda_0 \qquad (m = 0, 1, 2, \ldots)$$

From similar triangles in Fig. 35.15 the thickness t of the air wedge at each point is proportional to the distance x from the line of contact:

$$\frac{t}{x} = \frac{h}{l}$$

Combining this with Eq. (35.18b), we find

$$\frac{2xh}{l} = m\lambda_0$$

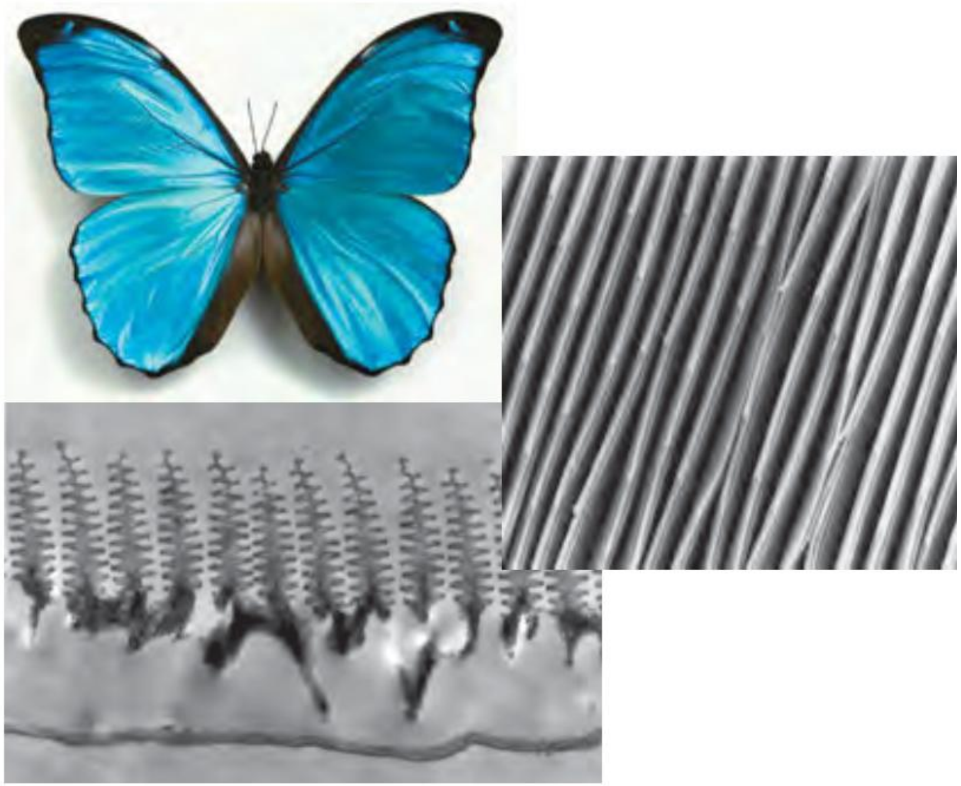
$$x = m\frac{l\lambda_0}{2h} = m\frac{(0.100\ \text{m})(500 \times 10^{-9}\ \text{m})}{(2)(0.0200 \times 10^{-3}\ \text{m})} = m(1.25\ \text{mm})$$

Successive dark fringes, corresponding to $m = 1, 2, 3, \ldots,$ are spaced 1.25 mm apart. Substituting $m = 0$ into this equation gives $x = 0$, which is where the two slides touch (at the left-hand side of Fig. 35.15). Hence there is a dark fringe at the line of contact.

35-4 Interference in Thin Films

Application Interference and Butterfly Wings

Many of the most brilliant colors in the animal world are created by *interference* rather than by pigments. These photos show the butterfly *Morpho rhetenor* and the microscopic scales that cover the upper surfaces of its wings. The scales have a profusion of tiny ridges (middle photo); these carry regularly spaced flanges (bottom photo) that function as reflectors. These are spaced so that the reflections interfere constructively for blue light. The multilayered structure reflects 70% of the blue light that strikes it, giving the wings a mirror-like brilliance. (The undersides of the wings do not have these structures and are a dull brown.)



35-5 The Michelson Interferometer

Michelson interferometers are used to make precise measurements of wavelengths and of very small distances, such as the minute changes in thickness of an axon when a nerve impulse propagates along its length.