Solution Assignment 1

Waves I

Chapter 15: Mechanical Waves

Question 1 Solution:

(a) By comparing the equation (A) with standard equation of wave i.e;
y(x,t) = 0.00327 Sin(72.1x – 2.72t) ------------------------------(A)
y(x,t) = Asin(kx – ωt) --(Standard Equation)
y_m = 0.00327 m = 3.27 mm

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{72.1} = 0.0871$ m = 8.71 cm

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.72} = 2.31$ s

$f = \frac{1}{T} = \frac{1}{2.31} = 0.433$ Hz

(c) $v = \frac{\omega}{k} = \frac{2.72}{72.1} = 0.0377$ m/s = 3.77 cm/s

(d) y = 0.00327 Sin(72.1 × 0.225 – 2.72 × 18.9)
y = (0.00327 m) Sin(-35.1855 rad)
y = 0.00192 m = 1.92 mm

(e) $u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$
u = (-2.72).(3.27) Cos(-35.1855)

u = 7.20 mm/s

(f)

Question 2 Solution:

v = 8.00 m/s, A = 0.0700 m, λ = 0.320 m

(a) v = f.λ
$f = \frac{v}{\lambda} = \frac{8}{0.32} = 25.0$ Hz

(b) For a wave travelling in the -x direction
$y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)$

at x = 0, $y(0, t) = A\cos 2\pi\left(\frac{t}{T}\right)$
So y = A and t = 0. This equation describes the wave specified in the problem.
Substitute values in equation we will get following equation.

$y(x,t) = (0.700\text{ m})\cos 2\pi\left(\frac{x}{0.320}m + \frac{t}{0.0400}s\right)$

$y(x,t) = (0.700\text{ m})\cos\left(19.6\,m^{-1}x + 157\frac{rad}{s}t\right)$

(c) From part (b)
$y(x,t) = (0.700\text{ m})\cos 2\pi\left(\frac{x}{0.320}m + \frac{t}{0.0400}s\right)$
plug in x = 0.360 m
and t = 0.150s
$y(x,t) = (0.700\text{ m})\cos 2\pi\left(\frac{0.360}{0.320}m + \frac{0.15}{0.04}s\right)$

$y(x,t) = (0.700\text{ m})\ \mathrm{Cos}2\pi(4.875\,rad) = +0.0495\text{ m} = +4.95\text{ cm}.$

(d) In part (c) t = 0.150 s

$y = A$ which means that $\cos 2\pi\,(\frac{x}{\lambda} + \frac{t}{T}) = 1$

$\cos\theta = 1$ for $\boldsymbol{\theta} = 0, 2\pi, 4\pi, \ldots\ldots = n(2\pi)$ for n = 0,1,2,3,……….

So, $y = A$ when $2\pi\,(\frac{x}{\lambda} + \frac{t}{T}) = n(2\pi) = (\frac{x}{\lambda} + \frac{t}{T}) = n$

$t = T\,(n - \frac{x}{\lambda}) = (0.04\text{ s})(n - \frac{0.360\,m}{0.320\text{ m}})$

$t = = (0.04\text{ s})(n - 1.125)$

For n = 4, t = 0.1150 s (before the instant in part(c)

For n = 5, t = 0.15550 s (the first occurrence of y = A after the instant in part (c). This the elapsed time is 0.1550 s – 0.1500 = 0.0050 s

So, for part (d) y = A at 0.115 s and next at 0.155 s; the difference between these two times is 0.04 s, which is the period. At t = 0.150 s the particle at x = 0.360 m is at y = 4.95 cm and travelling upward. It takes $\frac{T}{4}$ = 0.01 s for it to travel from y = 0 to y = A, so our answer of 0.005 s is reasonable.

Question 3 Solution:

(a) The tension F in the rope is the weight of the hanging mass:

$F = mg = 1.50\text{ kg} \times 9.80\,\frac{m}{s^2} = 14.7\text{ N}$

$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{\mu}}$

Here we have $\mu = \frac{m}{L}$ and $v = f.\lambda$

$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{14.7}{0.0550}} = 16.3\,\frac{m}{s}$

(b) $v = f.\lambda$

$\lambda = \frac{v}{f} = \frac{16.3}{120} = 0.136\text{ m}.$

(c) $v = \sqrt{\frac{F}{\mu}}$ where F = mg. Doubling m increase v by a factor of $\sqrt{2}$. $\lambda = \frac{v}{f}$, f remains 120 Hz and v increase by a factor of $\sqrt{2}$, so λ increase by a factor of $\sqrt{2}$.

Question 4 Solution:

$v = \sqrt{\frac{F}{\mu}}$ Here we have $\mu = \frac{m}{L}$ and $v = f.\lambda$

$\mathbf{F} = \mu v^2 = \frac{m}{L}\,(f\,\lambda)^2$

$F = \frac{0.120}{2.50}\,(40 \times 0.750)^2 = 43.2\text{ N}$

Question 5 Solution:

$\omega = 2\pi f$ and $\mu = \frac{m}{L} = \frac{3.00\times 10^{-3}}{0.8}$

(a) $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

$P_{av} = \frac{1}{2}\sqrt{\mu F}\,(2\pi f)^2 A^2 = 0.223$ W

(b) P_{av} = is proportional to A^2, so halving the amplitude quarters the average power to 0.056 W.