Modern Physics Revision

Question 1: A laser produces light of wavelength 520 nm in an ultrashort pulse.

(a) What is the energy of the photon in this laser pulse?

(b) If the uncertainty is 1.0% of the photon energy, what is the value of ΔE?

(c) What is the minimum duration of the pulse Δt?

Solution:

1(a)

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^{8})}{520 \times 10^{-9}} = 3.82 \times 10^{-19}\ \mathrm{J}$$

1(b)

Given uncertainty is 1.0%, so

$$\Delta E = 0.01E = 0.01(3.82 \times 10^{-19}) = 3.82 \times 10^{-21}\ \mathrm{J}$$

1(c)

Energy-time uncertainty principle:

$$\Delta E \Delta t = \frac{\hbar}{2}$$

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{1.055 \times 10^{-34}}{2(3.82 \times 10^{-21})} = 1.38 \times 10^{-14}\ \mathrm{s}$$

Question 2: A horizontal beam of laser light of wavelength 650 nm passes through a narrow slit that has width 0.0620 mm. The intensity of the light is measured on a vertical screen that is 2.00 m from the slit.

(a) What is the minimum uncertainty in the vertical component of the momentum of each photon in the beam after the photon has passed through the slit?

(b) Use the result of part (a) to estimate the width of the central diffraction maximum that is observed on the screen.

Solution:

2(a)

Heisenberg uncertainty principle:

$$\Delta y \Delta p_y = \frac{\hbar}{2}$$

$$\Delta p_y = \frac{\hbar}{2\Delta y} = \frac{1.055 \times 10^{-34}}{2(6.20 \times 10^{-5})} = 8.51 \times 10^{-31}\ \mathrm{kgm/s}$$

2(b)

Photon momentum along beam:

$$p_x = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{6.50 \times 10^{-7}} = 1.02 \times 10^{-27}\ \mathrm{kgm/s}$$

Angle estimate:

$$\theta = \frac{\Delta p_y}{p_x} = \frac{8.51 \times 10^{-31}}{1.02 \times 10^{-27}} = 8.35 \times 10^{-4}$$

So the distance is:

$$y = L\theta = (2.00)(8.35 \times 10^{-4}) = 1.67\ \mathrm{mm}$$

Question 3: A hydrogen atom initially in the ground level absorbs a photon, which excites it to the level n = 4. Determine the wavelength and frequency of the photon.

Solution:

Hydrogen energy levels:

$$E_n = -\frac{13.6\ \text{eV}}{n^2}$$

Ground state (n=1)

$$E_1 = -\frac{13.6\ \text{eV}}{(1)^2} = -13.6\ \text{eV}$$

Excited state (n=4)

$$E_4 = -\frac{13.6\ \text{eV}}{(4)^2} = -0.85\ \text{eV}$$

$$\Delta E = E_4 - E_1$$

$$\Delta E = (-0.85) - (-13.6) = 12.75\ \text{eV}$$

$$\lambda = \frac{hc}{\Delta E}$$

$$\lambda = \frac{(4.136\ \times 10^{-15}\ \text{eV})(3\ \times 10^{8})}{12.75}$$

$$\lambda = 9.73\ \times 10^{-8} = 97.3\ \text{nm}$$

For frequency,

$$f = \frac{c}{\lambda} = \frac{3\ \times 10^{8}}{9.73\ \times 10^{8}} = 3.08\ \times 10^{15}\ \text{Hz}$$

Question 4: The wave function $\psi(x, t) = Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)}$ is a superposition of two free-particle wave functions. Both k_1 and k_2 are positive. Find the probability distribution function for $\psi(x, t)$.

Solution:

Wave function is given as:

$$\psi(x, t) = Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)}$$

Complex conjugate of the wave function is:

$$\psi^*(x, t) = A^*e^{-i(k_1x - \omega_1t)} + A^*e^{-i(k_2x - \omega_2t)}$$

Probability distribution is written as:

$$|\psi|^2 = \psi^*\psi = (A^*e^{-i(k_1x - \omega_1t)} + A^*e^{-i(k_2x - \omega_2t)})(Ae^{i(k_1x - \omega_1t)} + Ae^{i(k_2x - \omega_2t)})$$

$$|\psi|^2 = A^*A\ (e^{-i(k_1x - \omega_1t)}e^{i(k_1x - \omega_1t)} + e^{-i(k_2x - \omega_2t)}e^{i(k_2x - \omega_2t)}) + e^{-i(k_1x - \omega_1t)}e^{i(k_2x - \omega_2t)}) + e^{-i(k_2x - \omega_2t)})e^{i(k_1x - \omega_1t)}$$

$$|\psi|^2 = |A|^2\ [e^0 + e^0 + e^{i[(k_2-k_1)x-(\omega_2-\omega_1)t]} + e^{-i[(k_2-k_1)x-(\omega_2-\omega_1)t]}]$$

From Euler's formula: $e^{i\theta} + e^{-i\theta} = 2\cos\theta$, Hence

$$|\psi|^2 = |A|^2\ [1 + 1 + 2\cos((k_2 - k_1)x - (\omega_2 - \omega_1)t)]$$

$$|\psi|^2 = 2|A|^2\ [1 + \cos((k_2 - k_1)x - (\omega_2 - \omega_1)t)]$$

So the probability distribution is:

$$P(x,t) = |\psi(x,t)|^2 = 2|A|^2\ [1 + \cos((k_2 - k_1)x - (\omega_2 - \omega_1)t)]$$

Question 5: A spacecraft of the Trade Federation flies past the planet Coruscant at a speed of 0.700c. A scientist on Coruscant measures the length of the moving spacecraft to be l_0 =75.0 m. The spacecraft later lands on Coruscant, and the same scientist measures the length of the now stationary spacecraft.

(a) What is the Lorentz factor γ at that speed?
(b) What length l does she get?

Solution:

5(a)

The formula for Lorentz factor is:

$$\gamma = \frac{1}{\sqrt{1 - (0.700c/c)^2}} = \frac{1}{\sqrt{1 - 0.49}} \approx 1.40$$

5(b)

$$L = \frac{L_0}{\gamma}$$

$$L_0 = \gamma L = (1.40)(75.0) = 105\text{m}$$

Question 6: Define

(a) Doppler effect
(b) De Broglie wavelength
(c) Heisenberg uncertainty principle

Solution:

6(a)

Doppler effect:
The Doppler effect is the apparent change in observed frequency (or wavelength) of a wave due to relative motion between the source and the observer.

6(b)

de Broglie wavelength:
The de Broglie wavelength is the wavelength associated with a moving particle:

$$\lambda = \frac{h}{p}$$

Where h is Planck's constant and p is momentum.

6(c)

Heisenberg uncertainty principle:
It is impossible to know exactly both the position and the momentum of a particle at the same time. The more precisely one is known, the less precisely the other can be known. Mathematically,

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

And same for energy and time:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$