Fundamentals of Electric Circuits

CHAPTER 4 Circuit Theorem

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CHAPTER 4 Circuit Theorems

4.2 Linearity Property

4.3 Superposition

4.4 Source Transformation

4.5 Thevenin's theorem

4.6 Norton's Theorem

4.7 Derivations of Thevenin's and Norton's Theorems

4.8 Maximum Power Transfer

4.2 Linearity Property

- Linearity Property: *Homogeneity* & *additivity*
- *Homogeneity*: In a circuit, if the current is increased by constant *k*, the voltage increases correspondingly by *k*

$$\boxed{kiR = kv}$$

- Additivity: It requires that the response to a sum of inputs is the sum of the responses to each input applied separately

$$\boxed{v = (i_1 + i_2)R = i_1R + i_2R = v_1 + v_2}$$

- A resistor satisfies both of these properties, and is a linear element
- A linear circuit is one whose output is linearly related (or directly proportional) to its input, which consists of only linear elements, linear dependent sources, and independent source.

4.2 Linearity Property

$$p = i^2R = v^2/R$$

- The relationship between power and voltage (or current) is nonlinear
- The theorems in this chapter is not applicable to power

Example The linear circuit has no independent sources inside it.

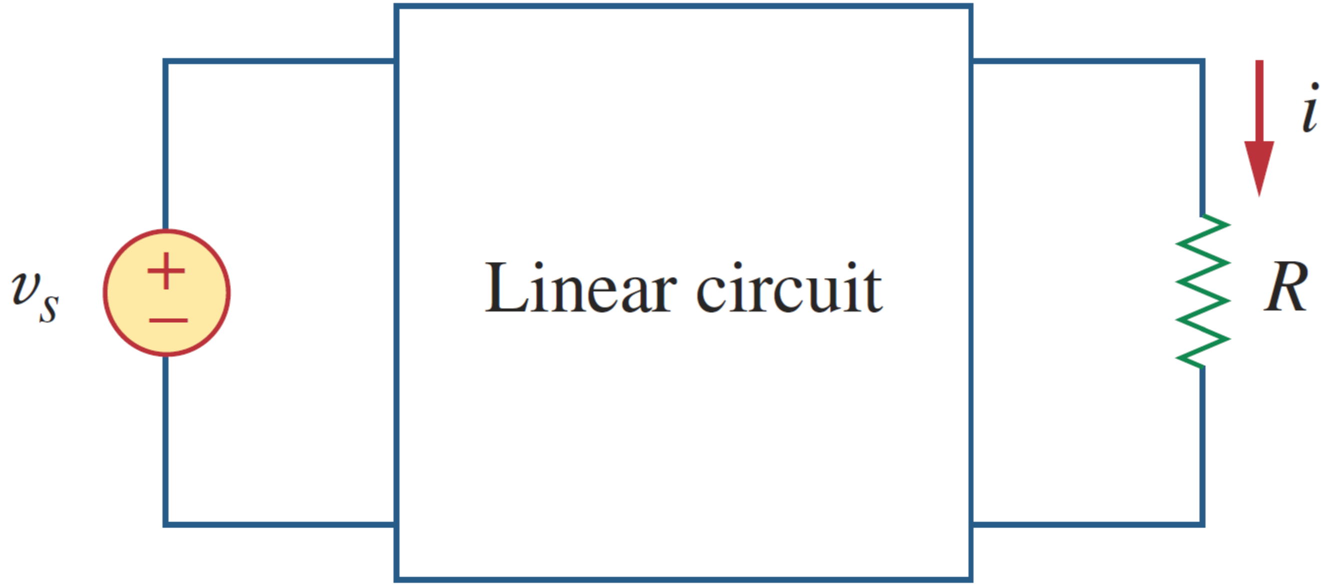


Figure 4.1
A linear circuit with input v_s and output i.

Suppose v_s=10V, i=2A

When v_s = 1V, i=?

When i = 1mA, v_s =?

Example

For the circuit in Fig. 4.2, find I_o when $v_s = 12$ V and $v_s = 24$ V.

Example 4.1

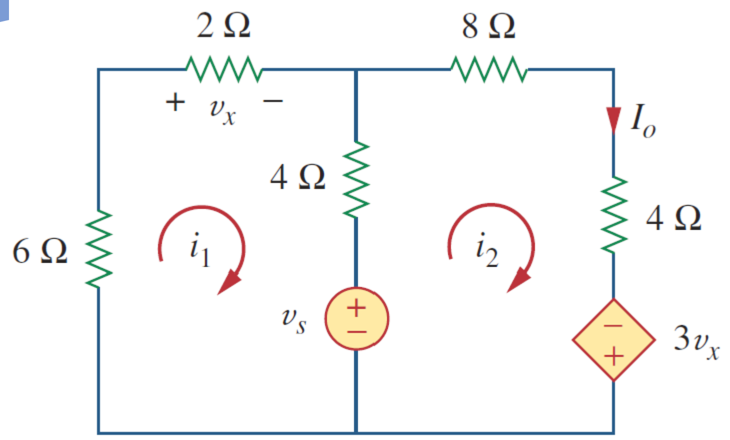


Figure 4.2
For Example 4.1.

Solution:

Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0 \quad \textbf{(4.1.1)}$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \quad \textbf{(4.1.2)}$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \quad \textbf{(4.1.3)}$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \quad \Rightarrow \quad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

$$I_o = i_2 = \frac{12}{76}\text{A}$$

When $v_s = 24$ V,

$$I_o = i_2 = \frac{24}{76}\text{A}$$

showing that when the source value is doubled, I_o doubles.

Example

Practice Problem 4.1

For the circuit in Fig. 4.3, find v_o when $i_s = 30$ and $i_s = 45$ A.

Answer: 40 V, 60 V.

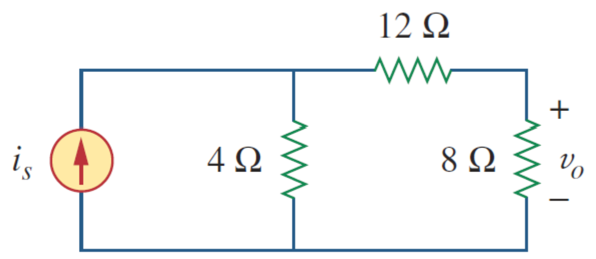
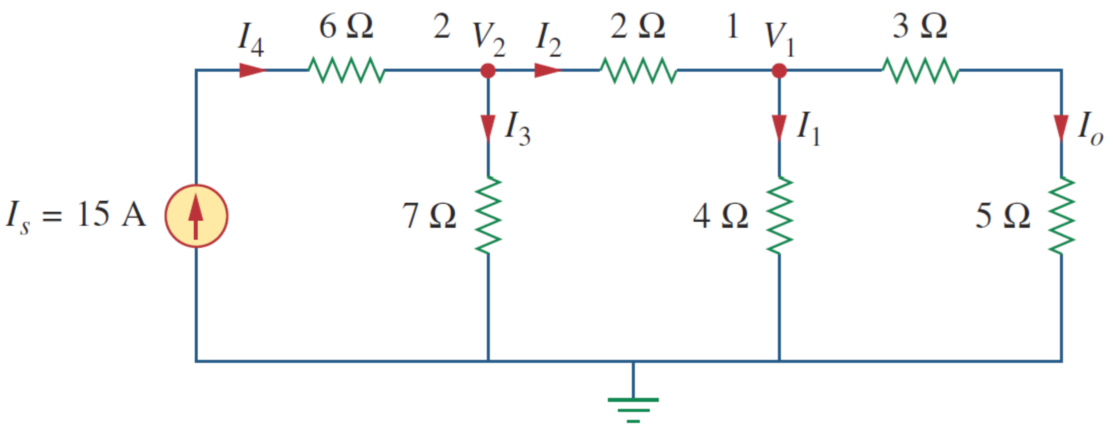


Figure 4.3
For Practice Prob. 4.1.

Example

Example 4.2

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.



Solution:

If $I_o = 1$ A, then $V_1 = (3 + 5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \qquad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A, the actual source current of 15 A will give $I_o = 3$ A as the actual value.

Example

Practice Problem 4.2

Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig. 4.5.

Answer: 16 V.

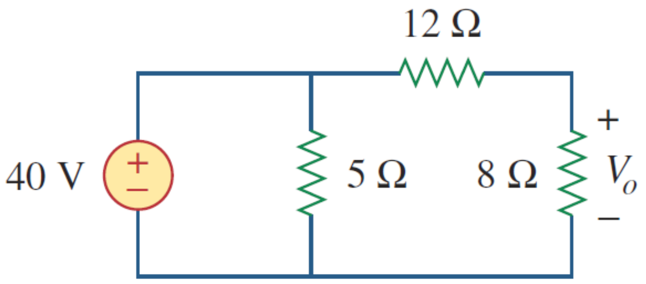


Figure 4.5
For Practice Prob. 4.2.

4.3 Superposition

- If there are two or more independent sources, there are two ways to solve the circuit:
  - Nodal or mesh analysis
  - **Use superposition** (based on the linearity property)
- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) the element due to each independent source acting alone.
- That is, calculate the contribution of each independent source separately, and then add them up.
- Superposition helps reduce a complex circuit to simpler circuits.

Applying Superposition Principle

Keep two things in mind:

1. Apply one independent source at a time while all other independent sources are ***turned off***

   Turn off ***: Replace every voltage source by 0V (short circuit),***

   Replace every current source by 0A (open circuit)

2. Dependent sources are left intact because they are controlled by circuit variables.

The steps are:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to the active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Use the superposition theorem to find v in the circuit of Fig. 4.6.

Example 4.3

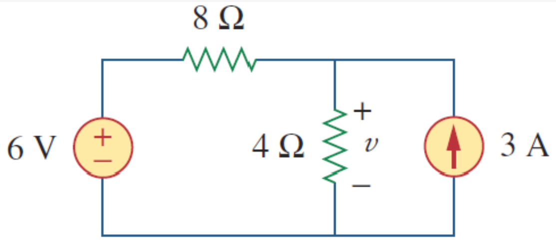
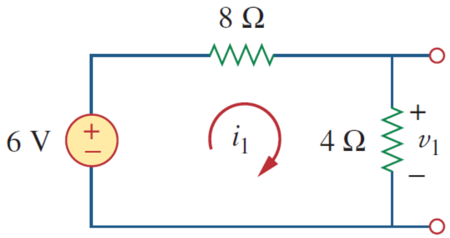
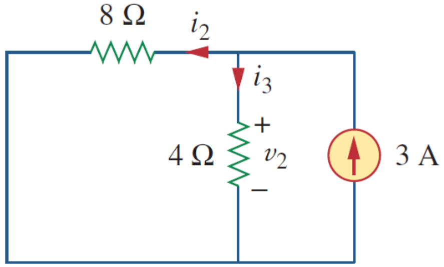


Figure 4.6
For Example 4.3.



(a)



(b)

Figure 4.7
For Example 4.3: (a) calculating v_1, (b) calculating v_2.

Solution:

Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Example

Practice Problem 4.3

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.

Answer: 7.4 V.

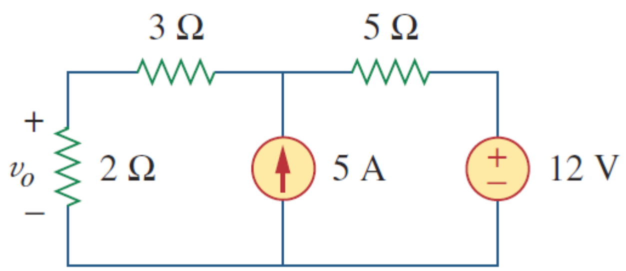
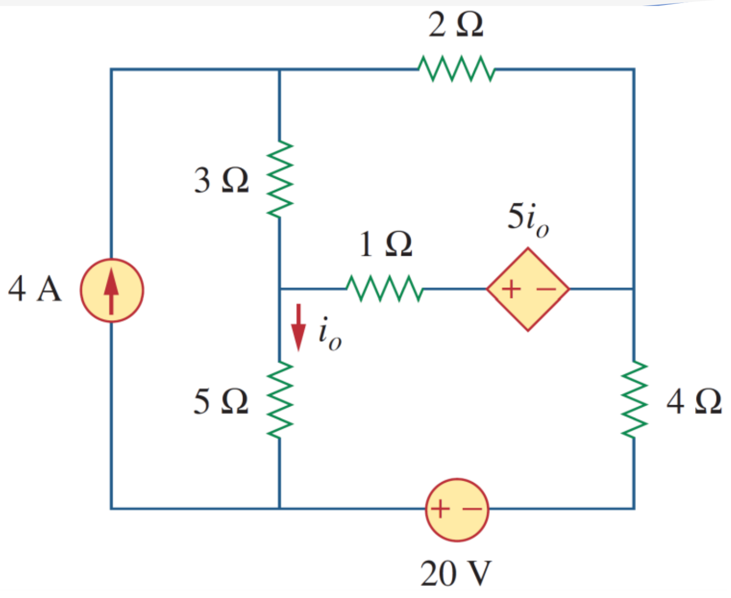


Figure 4.8
For Practice Prob. 4.3.

Example

Example 4.4 Find i_o in the circuit of Fig. 4.9 using superposition.



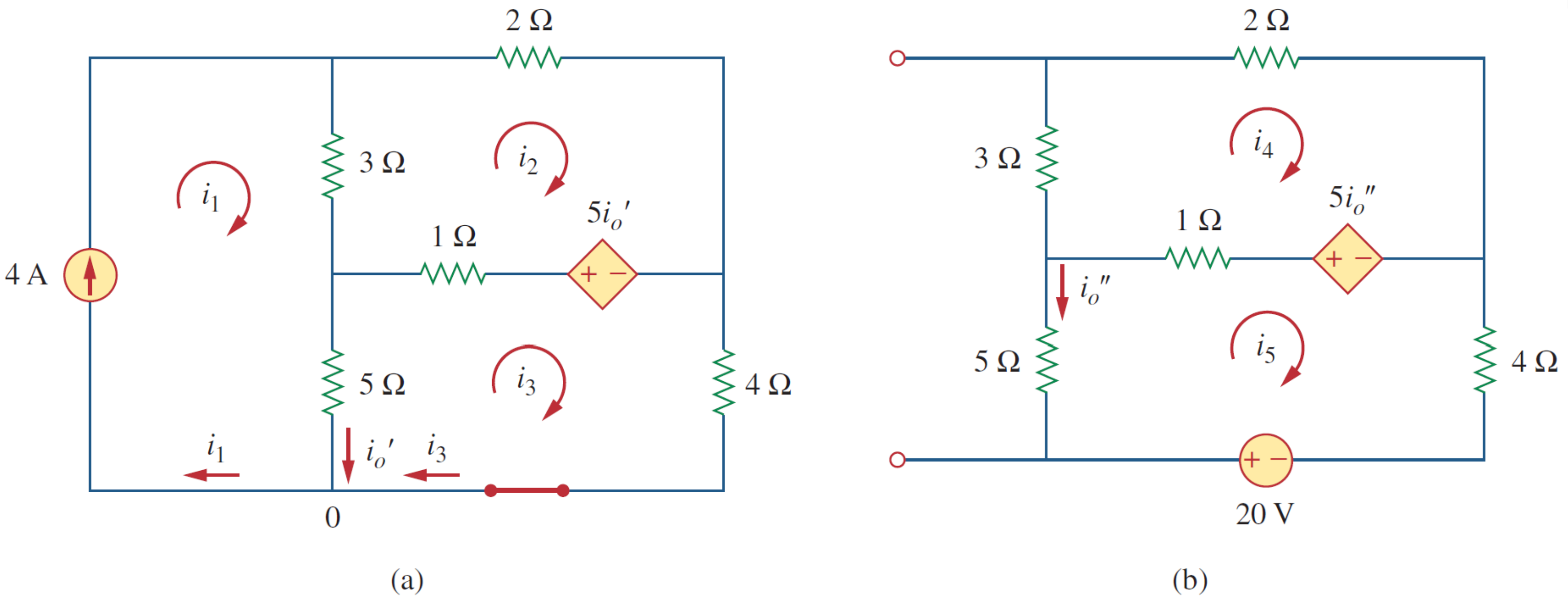
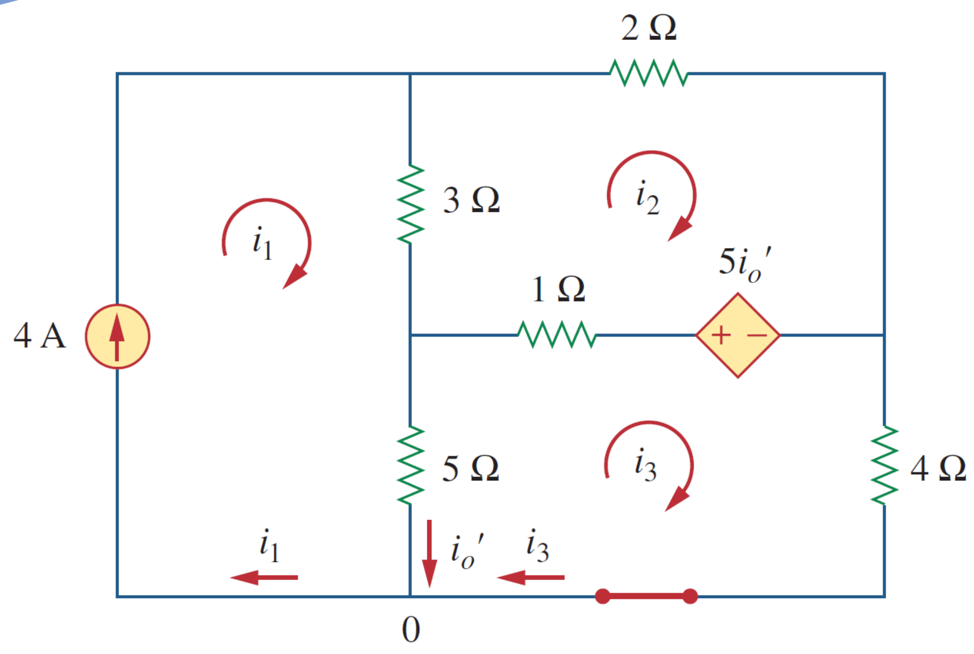


Figure 4.10
For Example 4.4: Applying superposition to (a) obtain i'_o, (b) obtain i''_o.

Example

Example 4.4 Find i_o in the circuit of Fig. 4.9 using superposition.



For loop 1

$$i_1 = 4 \text{ A} \tag{4.4.2}$$

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \tag{4.4.3}$$

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \tag{4.4.4}$$

But at node 0,

$$i_3 = i_1 - i'_o = 4 - i'_o \tag{4.4.5}$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i'_o = 8 \tag{4.4.6}$$

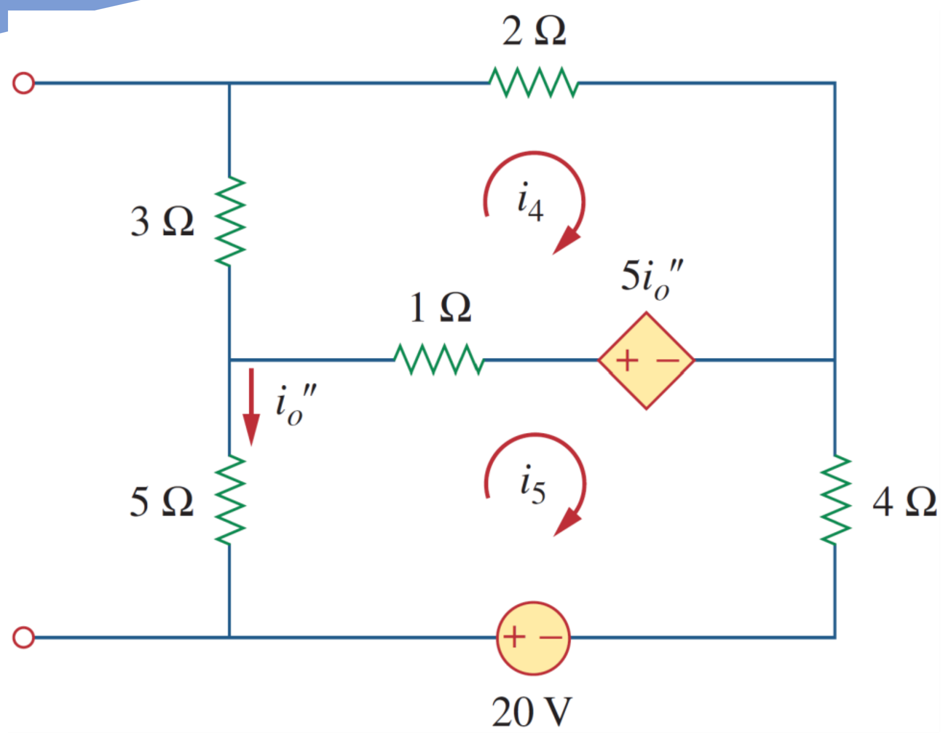
$$i_2 + 5i'_o = 20 \tag{4.4.7}$$

which can be solved to get

$$i'_o = \frac{52}{17} \text{ A} \tag{4.4.8}$$

Example

Example 4.4 Find i_o in the circuit of Fig. 4.9 using superposition.



To obtain i_o'', we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i_o'' = 0 \tag{4.4.9}$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i_o'' = 0 \tag{4.4.10}$$

But $i_5 = -i_o''$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i_o'' = 0 \tag{4.4.11}$$

$$i_4 + 5i_o'' = -20 \tag{4.4.12}$$

which we solve to get

$$i_o'' = -\frac{60}{17}\text{ A} \tag{4.4.13}$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706\text{ A}$$

Example

Practice Problem 4.4

Use superposition to find v_x in the circuit of Fig. 4.11.

Answer: $v_x = 31.25$ V.

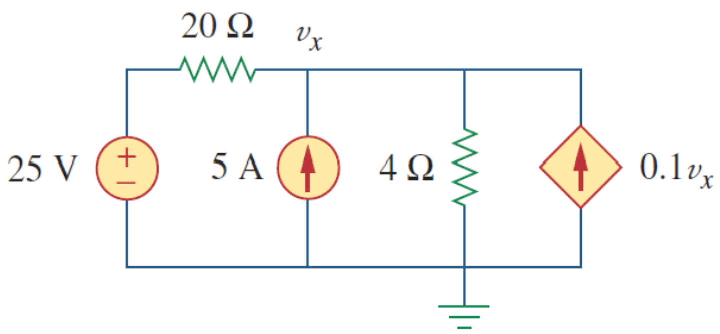


Figure 4.11
For Practice Prob. 4.4.

Example

Example 4.5 For the circuit in Fig. 4.12, use the superposition theorem to find i.

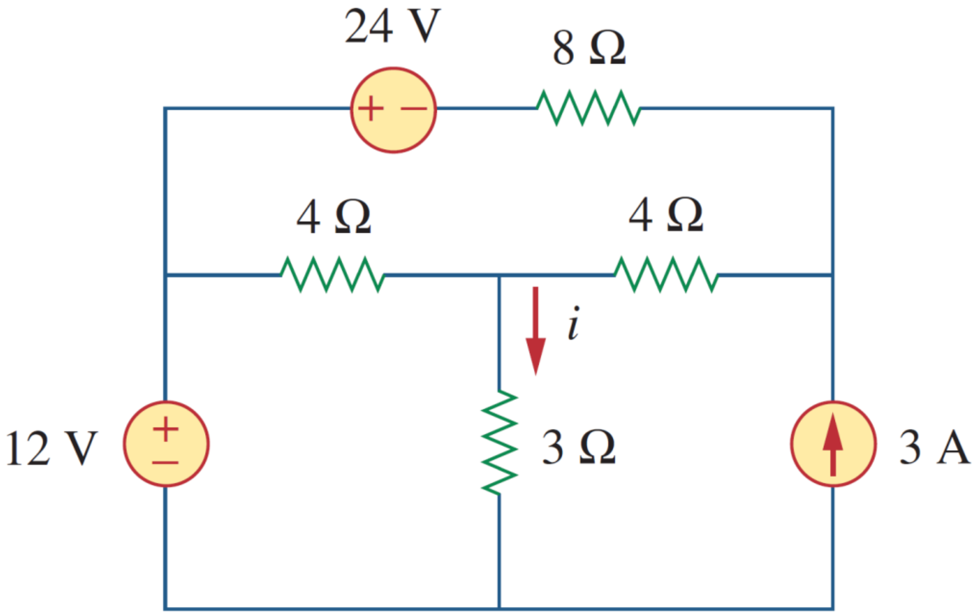
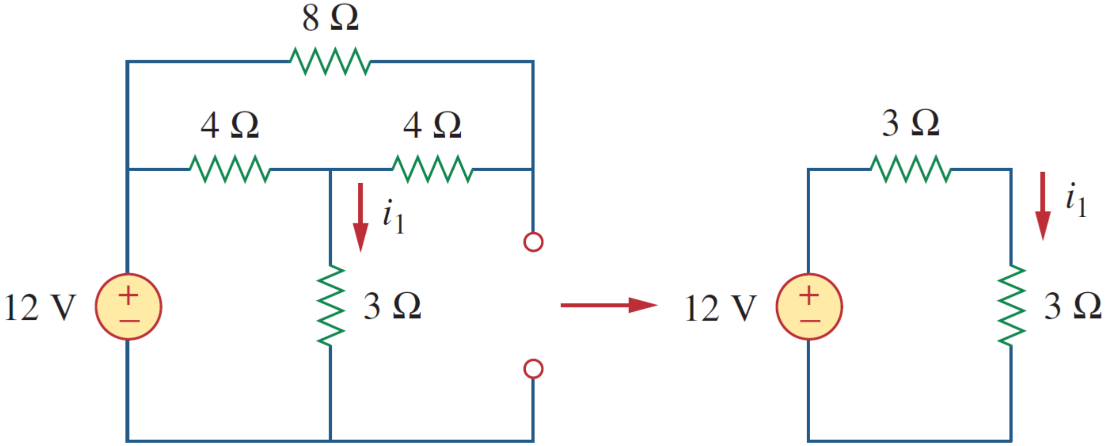
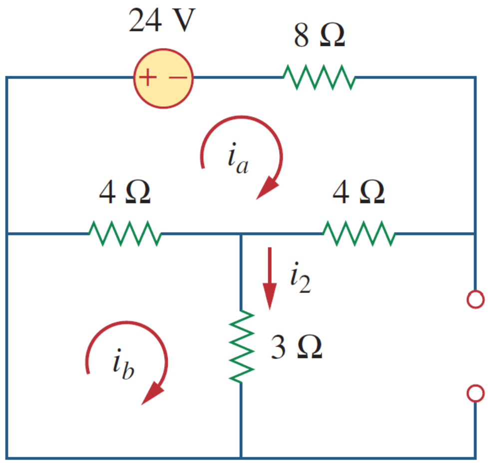


Figure 4.12
For Example 4.5.

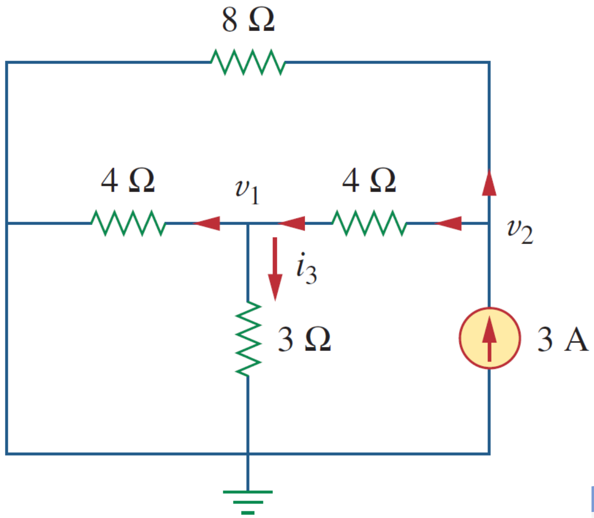
$$i = i_1 + i_2 + i_3$$



(a)



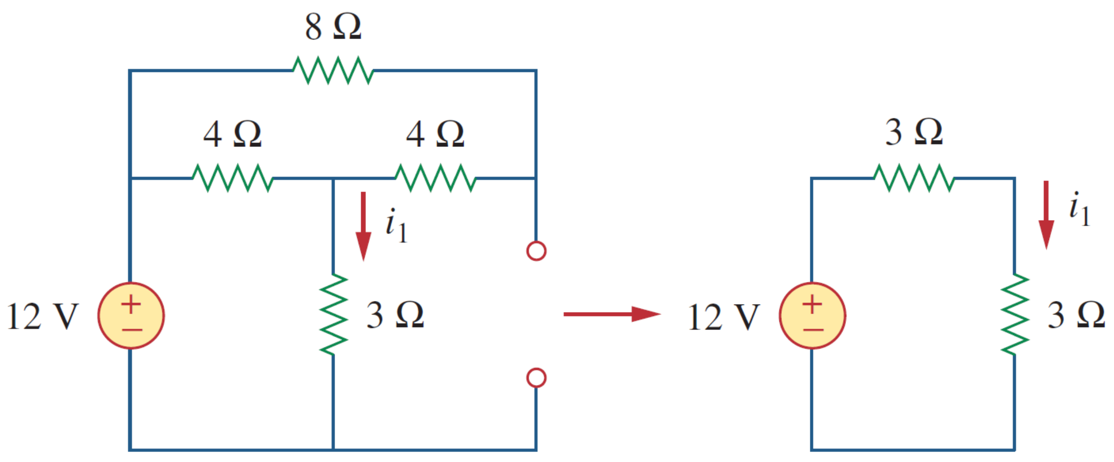
(b)



(c)

Example

Example 4.5 For the circuit in Fig. 4.12, use the superposition theorem to find i.

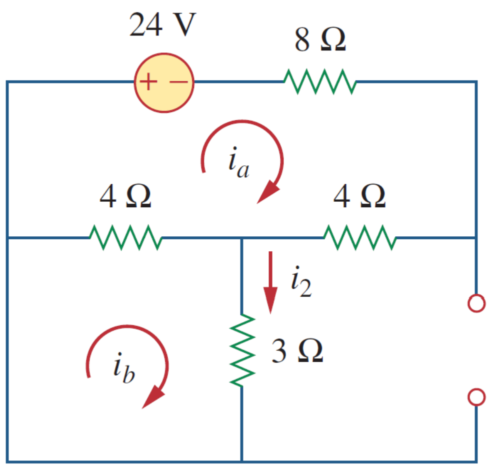


(a)

$$i_1 = \frac{12}{6} = 2 \text{ A}$$

Example

Example 4.5 For the circuit in Fig. 4.12, use the superposition theorem to find i.



(b)

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis gives

$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \qquad \textbf{(4.5.1)}$$

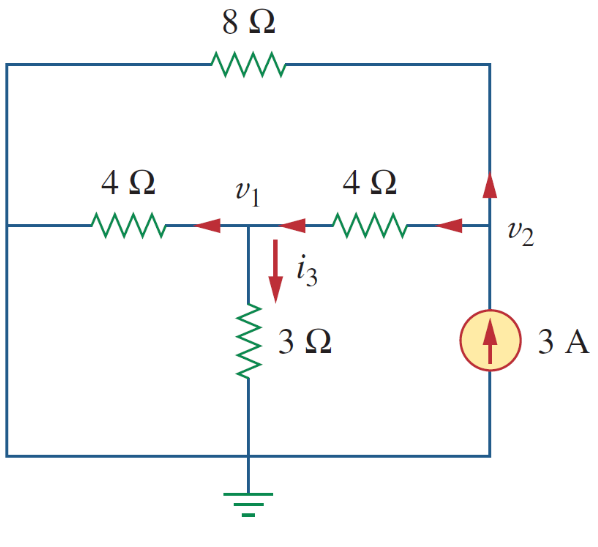
$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b \qquad \textbf{(4.5.2)}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

Example

Example 4.5 For the circuit in Fig. 4.12, use the superposition theorem to find i.



(c)

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis gives

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1 \qquad \textbf{(4.5.3)}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1 \qquad \textbf{(4.5.4)}$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

Example

Example 4.5 For the circuit in Fig. 4.12, use the superposition theorem to find i.

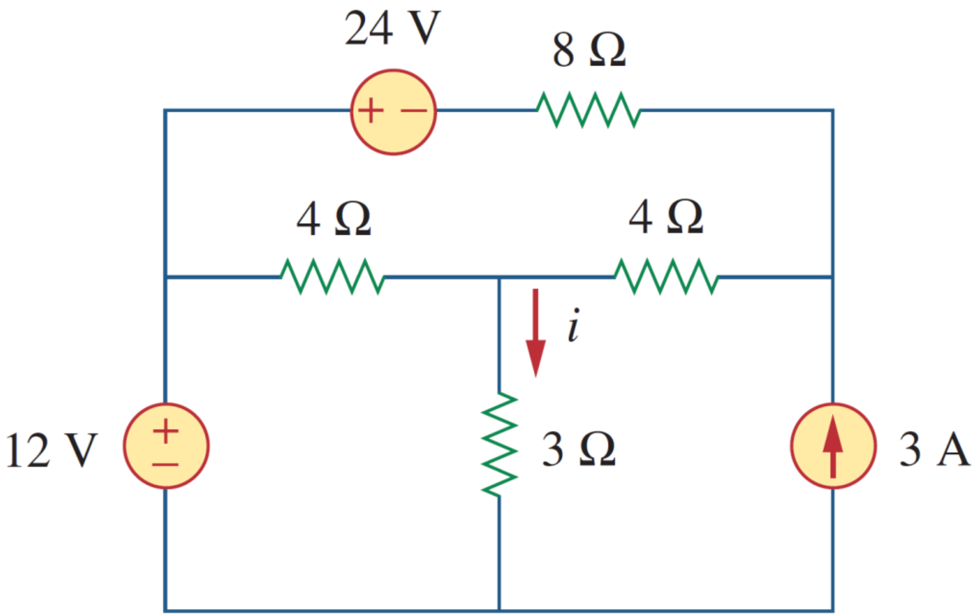


Figure 4.12
For Example 4.5.

The sources can be divided into several groups

Example

Find I in the circuit of Fig. 4.14 using the superposition principle.

Practice Problem 4.5

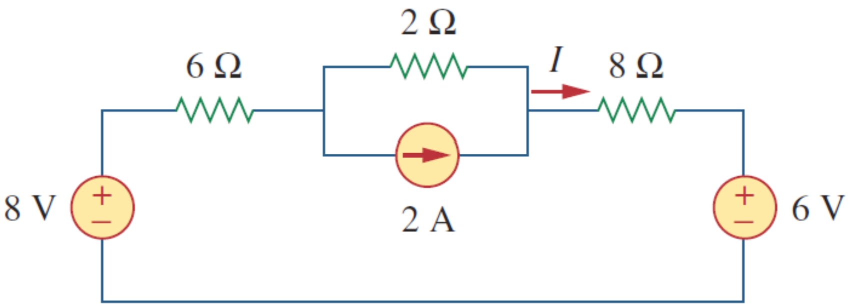


Figure 4.14
For Practice Prob. 4.5.

Answer: 375 mA.

4.4 Source Transformation

- Much like the delta-wye transformation, it is possible to transform a source from one form to another, which can be useful for simplifying circuits.
- The principle behind all of these transformations is ***equivalence.***
- ***An equivalent circuit is one whose v-i characteristics are identical with the original circuit.***

4.4 Source Transformation

- A source transformation is the process of replacing a voltage source $\boldsymbol{v_s}$ in series with a resistor $\boldsymbol{R}$ by a current source $\boldsymbol{i_s}$ in parallel with a resistor $\boldsymbol{R}$, or vice versa.

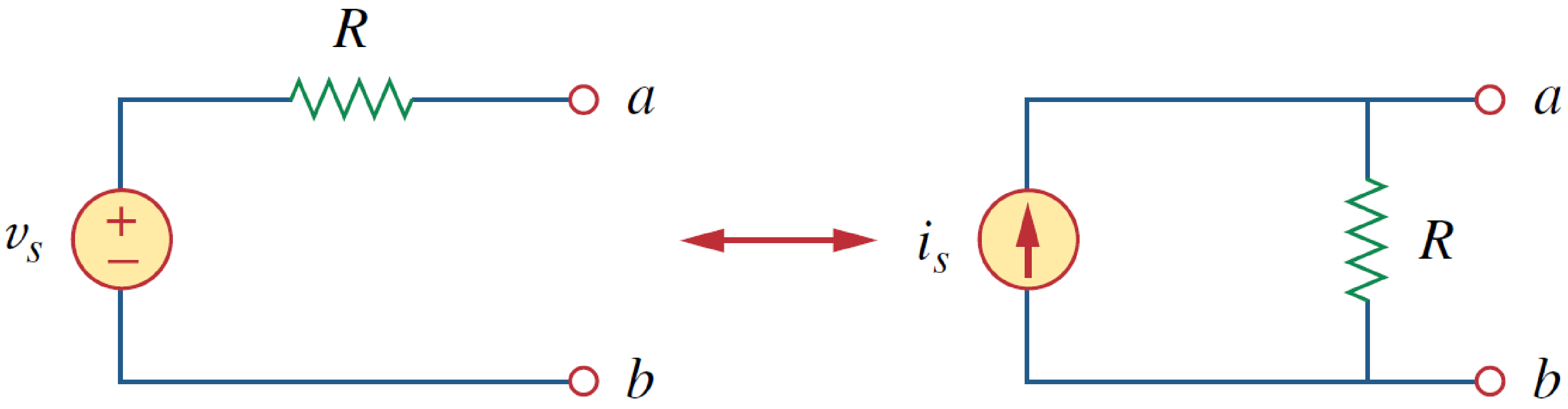


Figure 4.15 ***The same v-i relation at terminal a-b***
Transformation of independent sources.

The same *v-i* relation at terminal *a-b*

- If the sources are turned off, the equivalent resistances at terminal a-b are both *R*
- If the terminals are short-circuited, the short-circuit currents need to be the same
- From this we get the following requirement: $v_s = i_s R$ or $i_s = \frac{v_s}{R}$

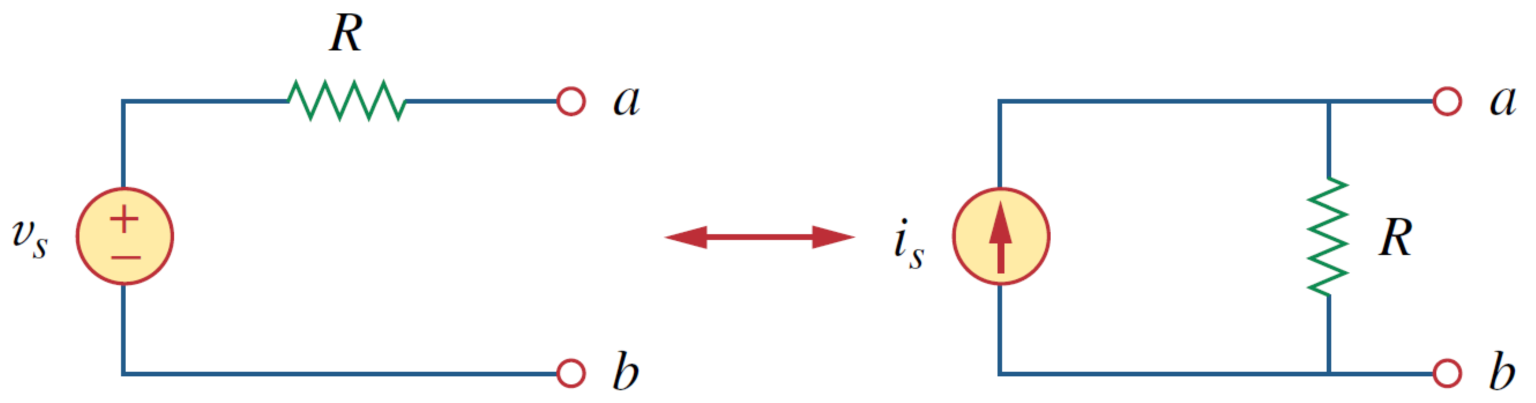


Figure 4.15
Transformation of independent sources.

Dependent Sources

- Source transformation also applies to dependent sources.
- Dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor.
- The same relationship between the voltage and current holds here:

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

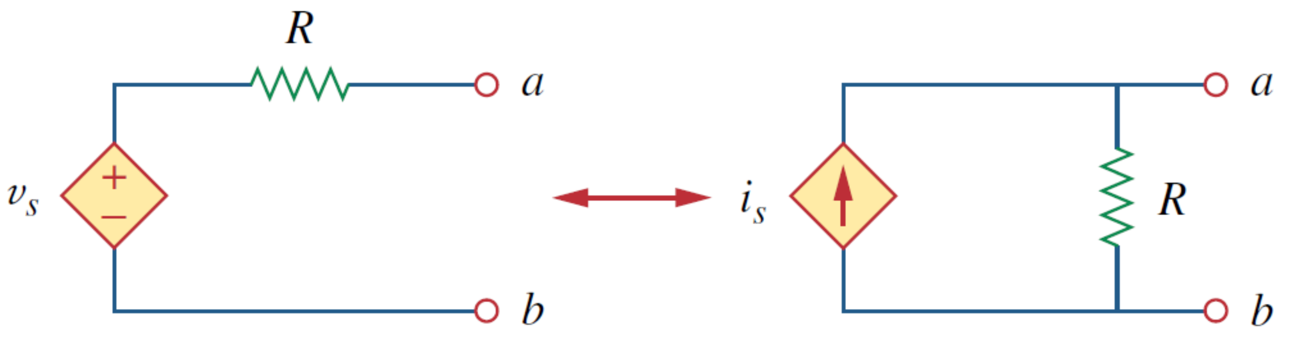


Figure 4.16
Transformation of dependent sources.

Source transformation rules

- A source transformation does not affect the remaining part of the circuit.
- Note that the arrow of the current source is directed towards the positive terminal of the voltage source.
- Source transformation is not possible when R=0 for an ideal voltage source.
- A ideal current source with $R = \infty$ cannot be replaced by a voltage source.

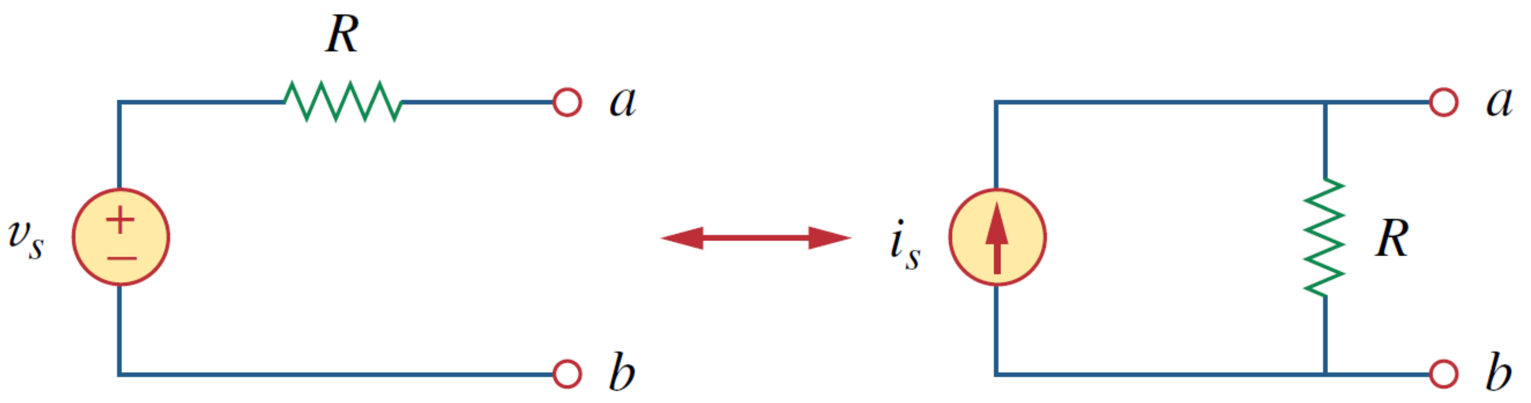


Figure 4.15
Transformation of independent sources.

Example

Use source transformation to find v_o in the circuit of Fig. 4.17.

Example 4.6

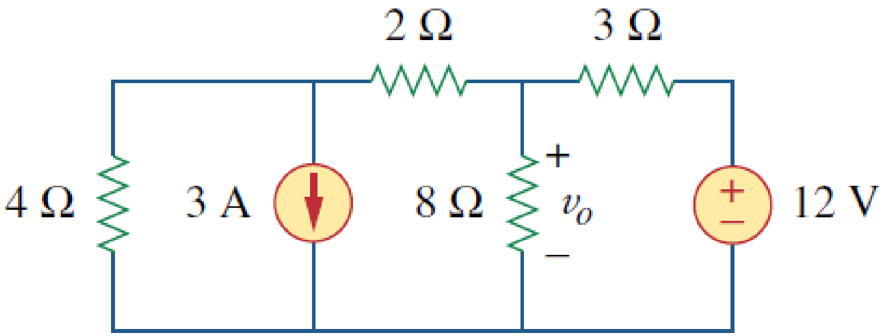
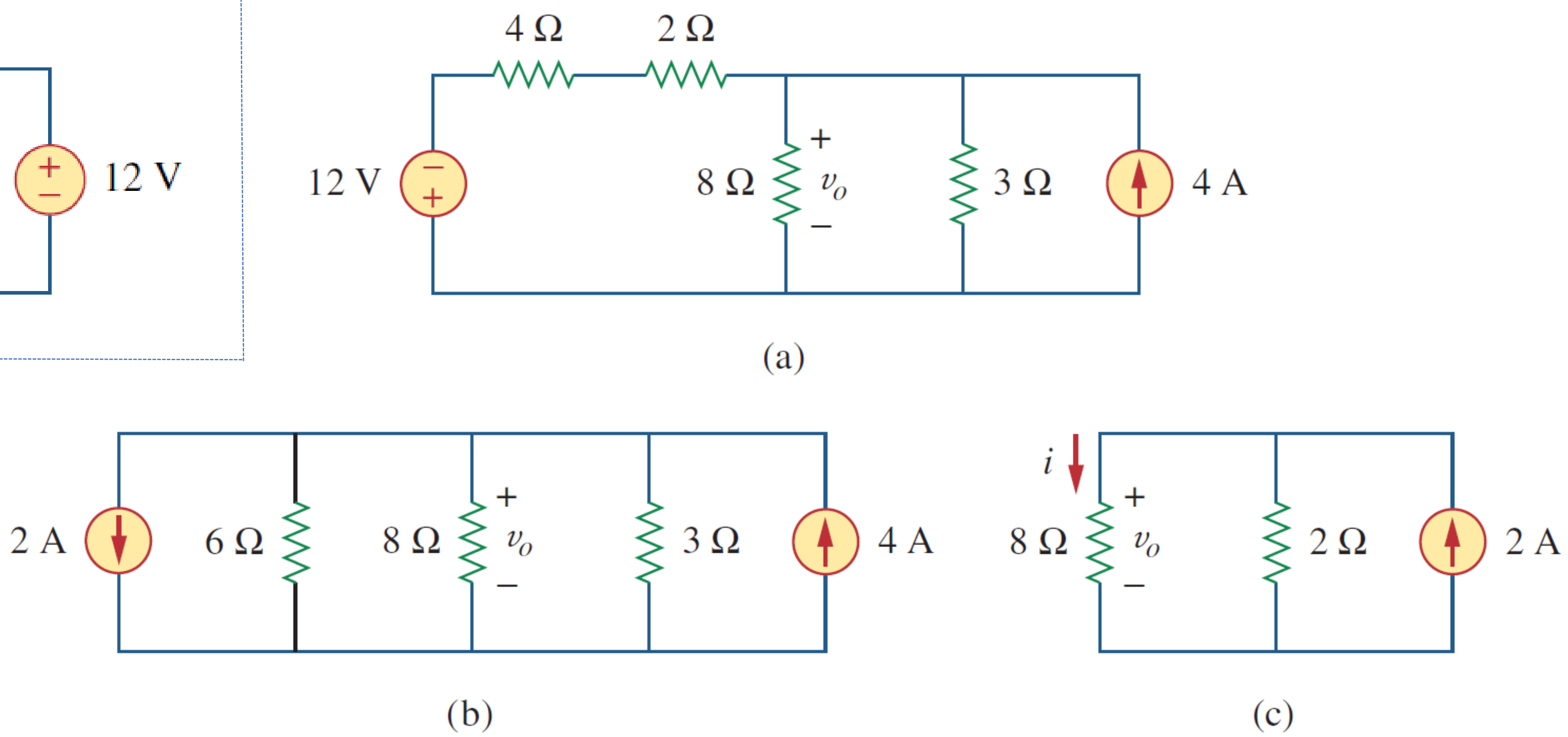


Figure 4.17
For Example 4.6.



Example

Find i_o in the circuit of Fig. 4.19 using source transformation.

Practice Problem 4.6

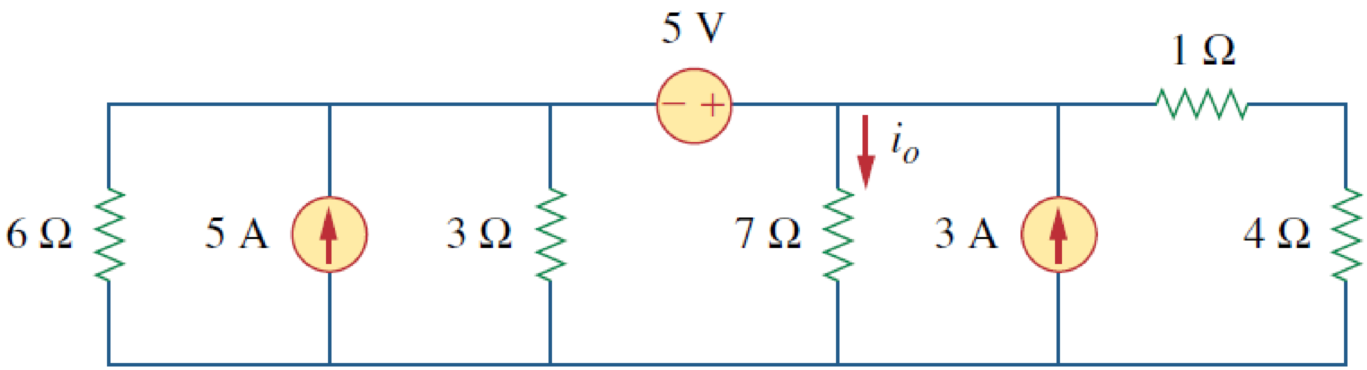


Figure 4.19
For Practice Prob. 4.6.

Answer: 1.78 A.

Example 4.7

Find v_x in Fig. 4.20 using source transformation.

Example

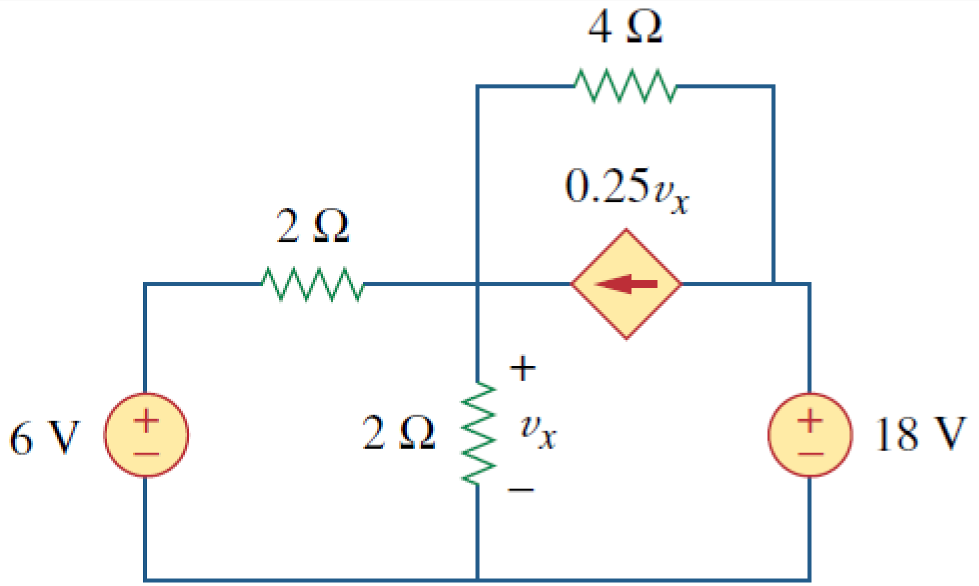
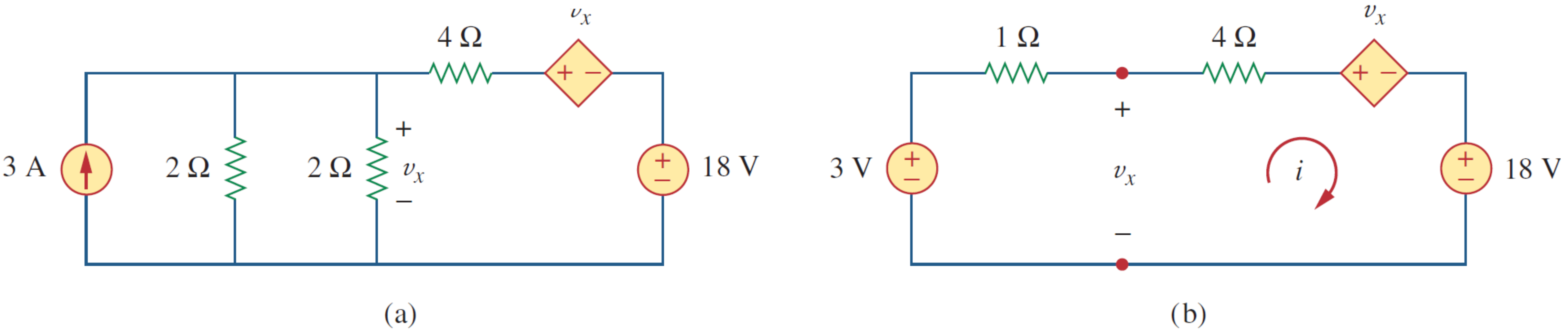


Figure 4.20
For Example 4.7.



Example

Practice Problem 4.7

Use source transformation to find i_x in the circuit shown in Fig. 4.22.

Answer: 7.059 mA.

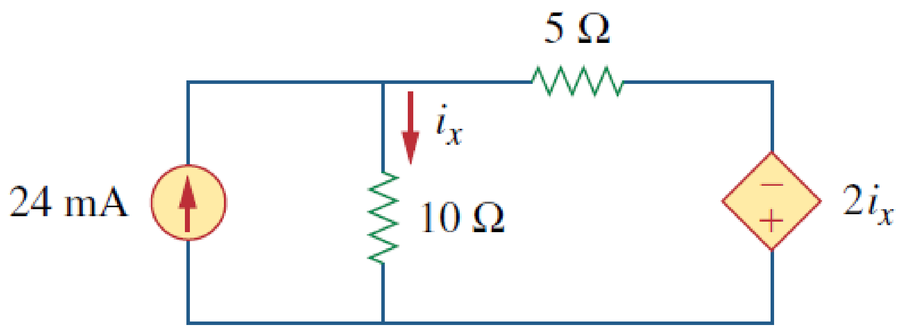


Figure 4.22
For Practice Prob. 4.7.

4.5 Thevenin’s Theorem

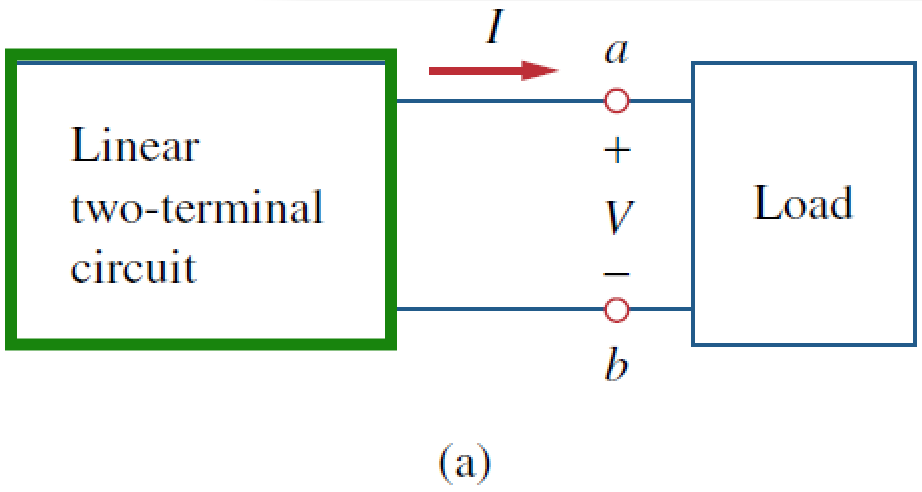
- In many circuits, a particular element in a circuit is variable (usually called the load) while other elements are fixed
- An example of this is mains power; many different appliances may be plugged into the outlet, which presents as a variable load
- Each time the load is changed, the circuit has to be analyzed over again.
- To avoid the problem, in Thevenin’s theorem, the fixed part of the circuit is replaced by an equivalent circuit.

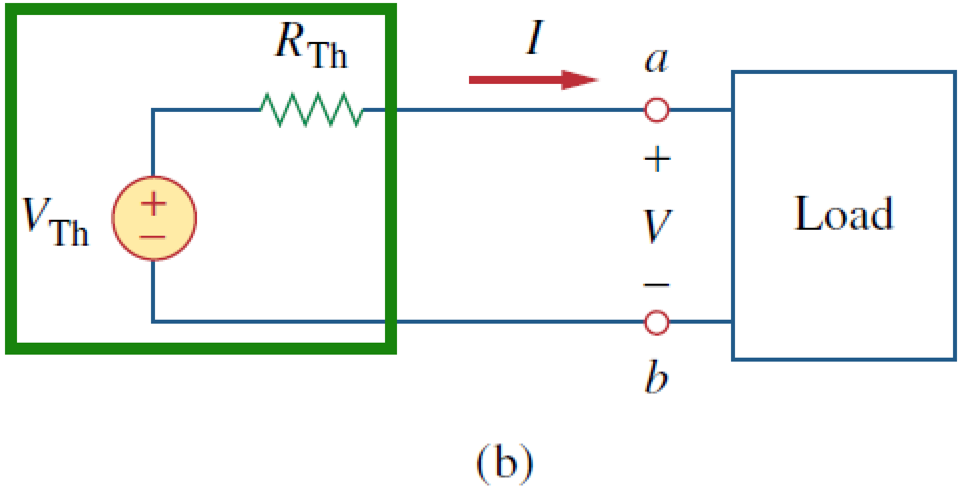
4.5 Thevenin’s Theorem

- Thevenin’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $\boldsymbol{V_{th}}$ in series with a resistor $\boldsymbol{R_{th}}$

- $\boldsymbol{V_{th}}$ is the open-circuit voltage at the terminals

- $\boldsymbol{R_{th}}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.





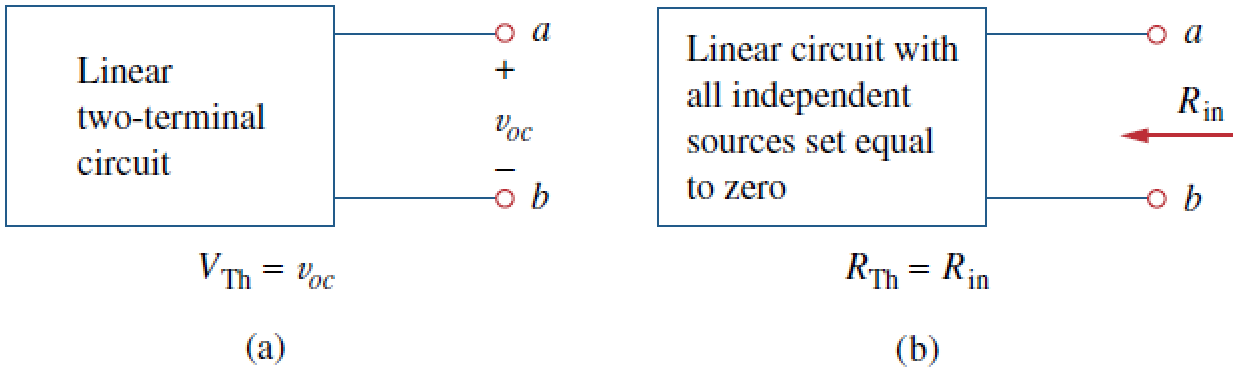
Thevenin’s equivalent circuit

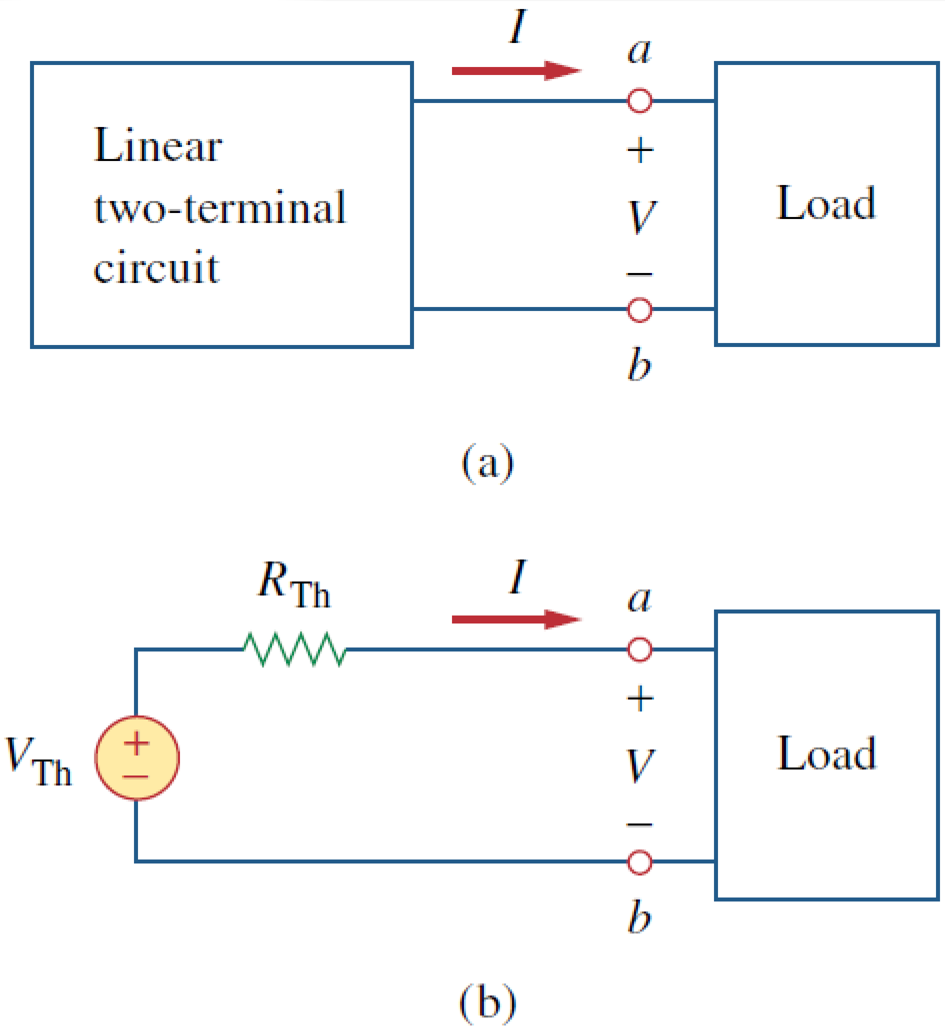
Finding V_{th} and R_{th}

Equivalent: the same voltage-current relation at the terminals

V_{th} is the open-circuit voltage at the terminals

R_{th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

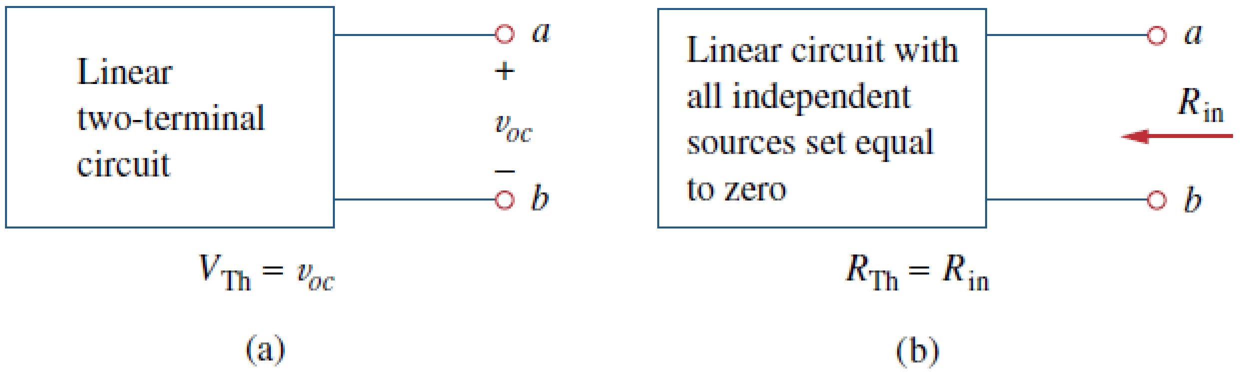




Thevenin's equivalent circuit

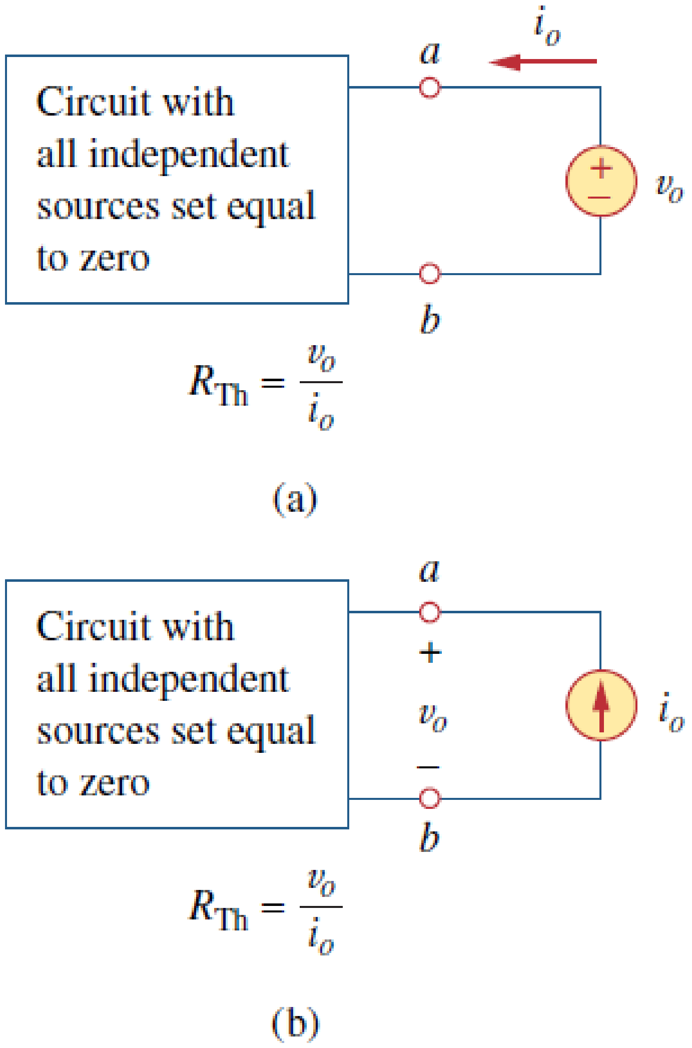
Finding R_{th}

- There are two cases to consider when finding the equivalent resistance
- **Case 1**: If there are no dependent sources, then the resistance is found by simply turning off all the independent sources. The $\boldsymbol{R_{th}}$ is the input resistance of the network looking between terminals *a* and *b*.



Finding R_{th}

- **Case 2**: If there are dependent sources, we still turn off all the independent sources. Dependent sources are not turned off because they are controlled by circuit variables.
- Apply a voltage source v_0 (or current source i_0) at terminals *a* and *b*, and determine the resulting current i_0 (voltage v_0).
- In a circuit with dependent sources, it is possible to find the value of R_{th} negative, which implies the circuit is supplying power



With Thevenin's Equivalent Circuit

- A linear circuit with a variable load can be replaced by the Thevenin's equivalent, exclusive of the load.
- The equivalent circuit behaves externally exactly the same as the original circuit.
- Load current can be easily calculated
- Load voltages can be determined by the voltage divider rule.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

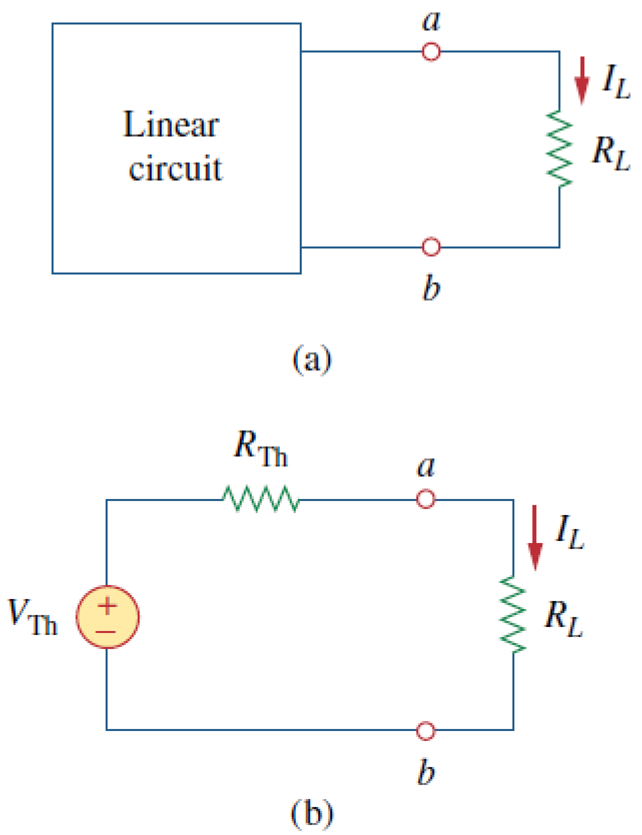


Figure 4.26
A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

Example

Example 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6$, 16, and 36 Ω.

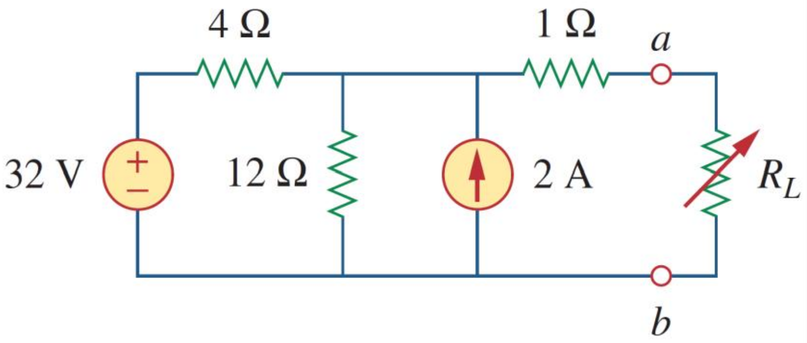
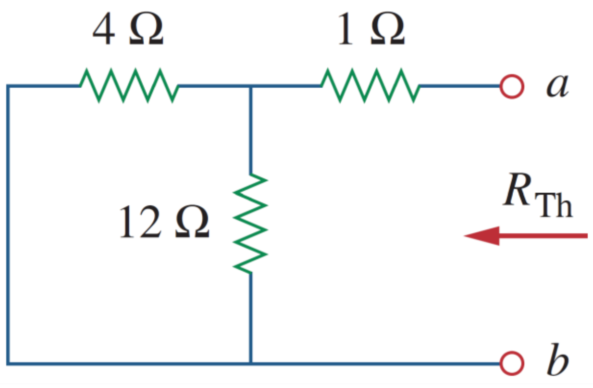


Figure 4.27
For Example 4.8.

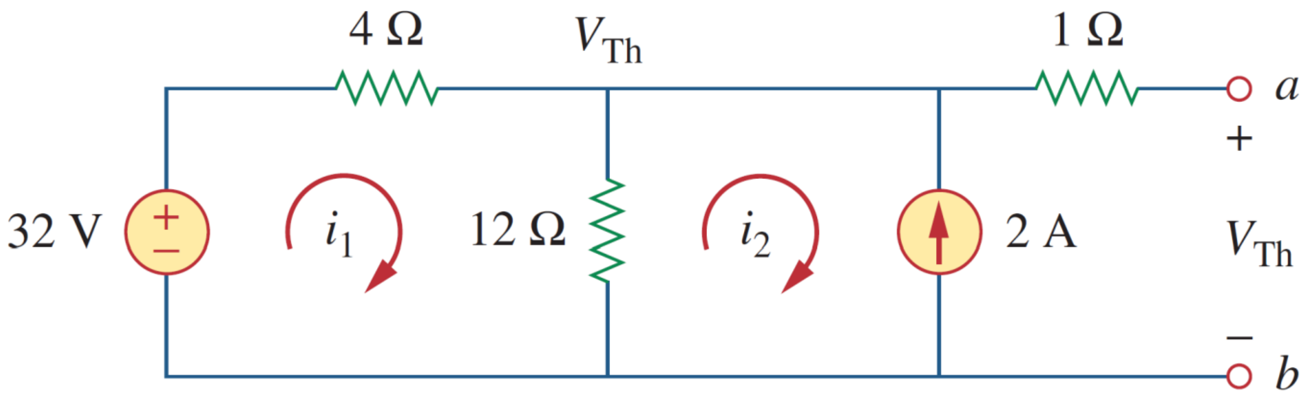
Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

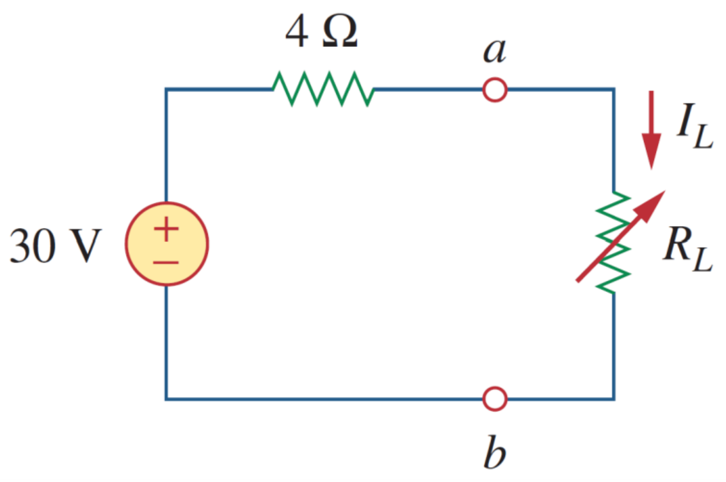


$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

Finding V_{th} ?



(b)



The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3 \text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

Example

Practice Problem 4.8

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit of Fig. 4.30. Then find I.

Answer: $V_{Th} = 6$ V, $R_{Th} = 3\ \Omega$, $I = 1.5$ A.

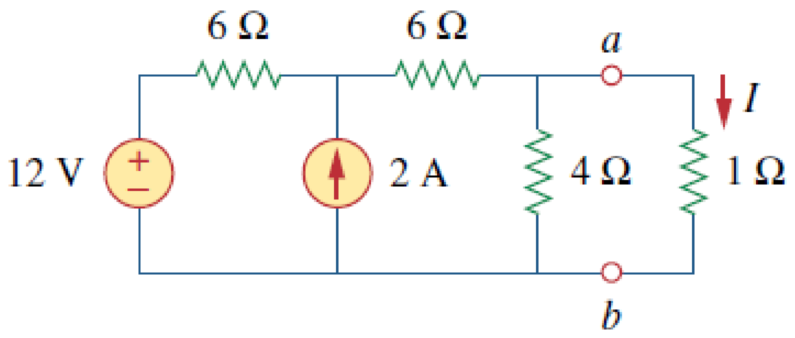


Figure 4.30
For Practice Prob. 4.8.

Example

Example 4.9

Find the Thevenin equivalent of the circuit in Fig. 4.31 at terminals *a*-*b*.

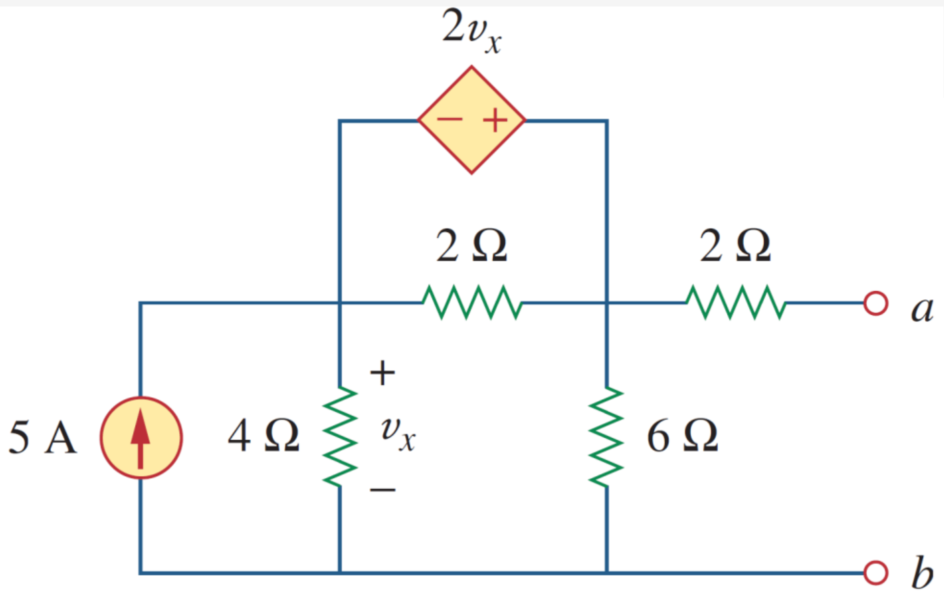
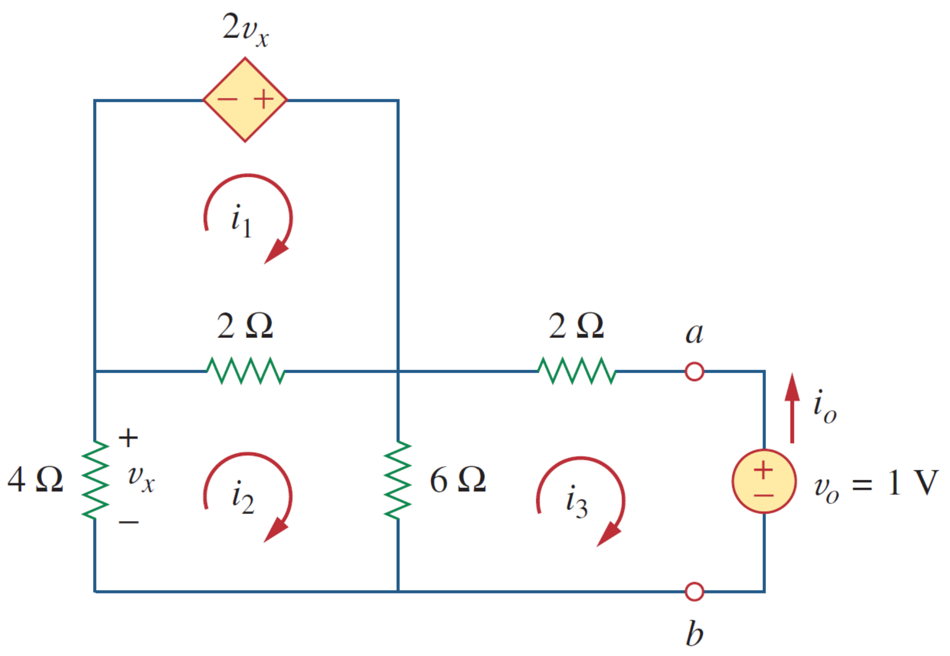


Figure 4.31
For Example 4.9.



(a)

Applying mesh analysis to loop 1 in the circuit of Fig. 4.32(a) results in

$$-2v_x + 2(i_1 - i_2) = 0 \qquad \text{or} \qquad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2 \tag{4.9.1}$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \tag{4.9.2}$$

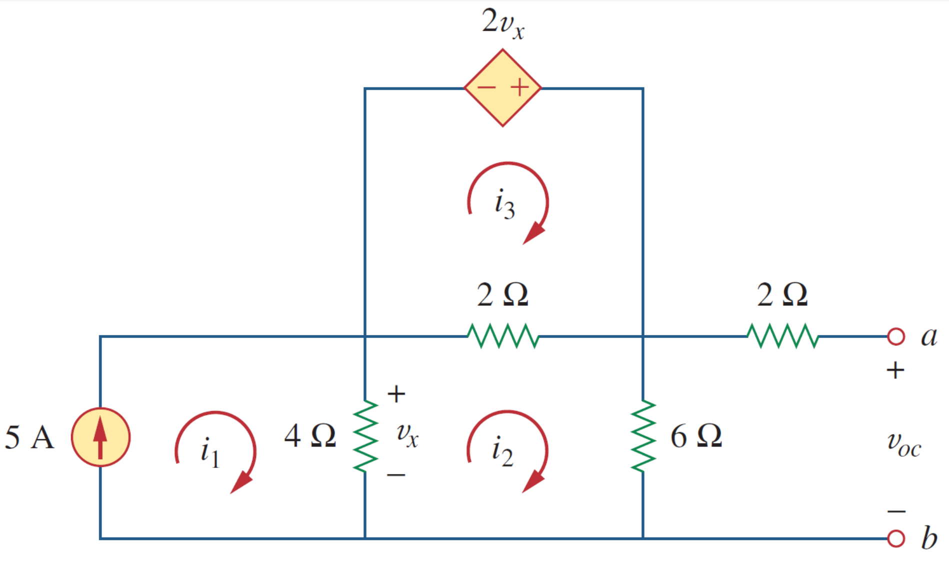
$$6(i_3 - i_2) + 2i_3 + 1 = 0 \tag{4.9.3}$$

Solving these equations gives

$$i_3 = -\frac{1}{6}\text{ A}$$

But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{\text{Th}} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$



(b)

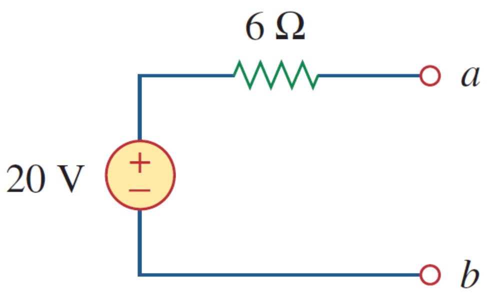


Figure 4.33
The Thevenin equivalent of the circuit in Fig. 4.31.

To get V_{Th}, we find v_{oc} in the circuit of Fig. 4.32(b). Applying mesh analysis, we get

$$i_1 = 5 \tag{4.9.4}$$

$$-2v_x + 2(i_3 - i_2) = 0 \quad \Rightarrow \quad v_x = i_3 - i_2 \tag{4.9.5}$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

or

$$12i_2 - 4i_1 - 2i_3 = 0 \tag{4.9.6}$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{\text{Th}} = v_{oc} = 6i_2 = 20 \text{ V}$$

The Thevenin equivalent is as shown in Fig. 4.33.

Example

Practice Problem 4.9

Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

Answer: $V_{\mathrm{Th}} = 5.333$ V, $R_{\mathrm{Th}} = 444.4$ mΩ.

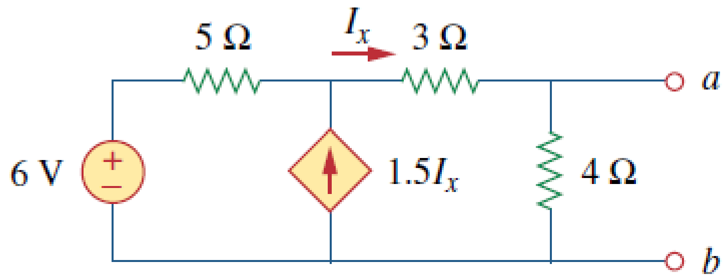
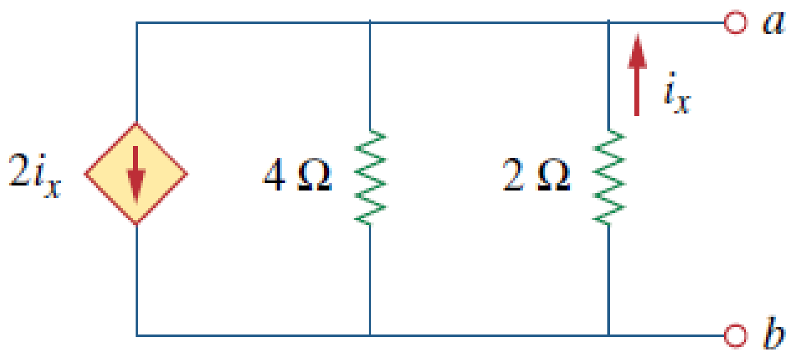


Figure 4.34
For Practice Prob. 4.9.

Example

Determine the Thevenin equivalent of the circuit in Fig. 4.35(a) at terminals *a-b*.

Example 4.10



(a)

Example

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Practice Problem 4.10

Answer: $V_{Th} = 0$ V, $R_{Th} = -7.5\ \Omega$.

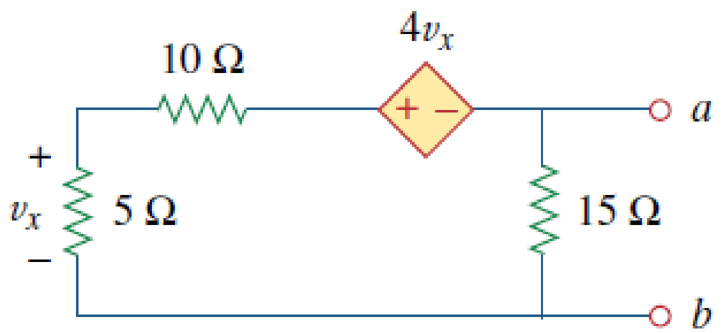


Figure 4.36
For Practice Prob. 4.10.

4.6 Norton's Theorem

- Norton's theorem states that a linear two- terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N
- I_N is the short-circuit current through the terminals
- R_N is the input or equivalent resistance at the terminals when the independent sources are turned off
- According to source transformation, the Norton resistance R_N and the Thevenin resistance R_{th} are equal

$$\boxed{R_N = R_{\text{Th}}}$$

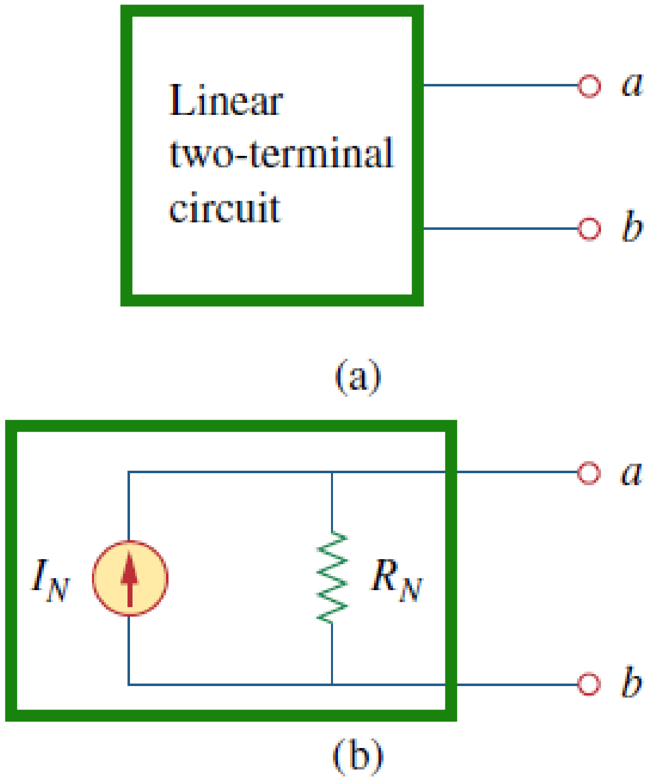


Figure 4.37
(a) Original circuit, (b) Norton equivalent circuit.

Finding the Norton current I_N

The Norton current I_N is found by short circuiting the circuit's terminals and measuring the resulting current

$$I_N = i_{sc}$$

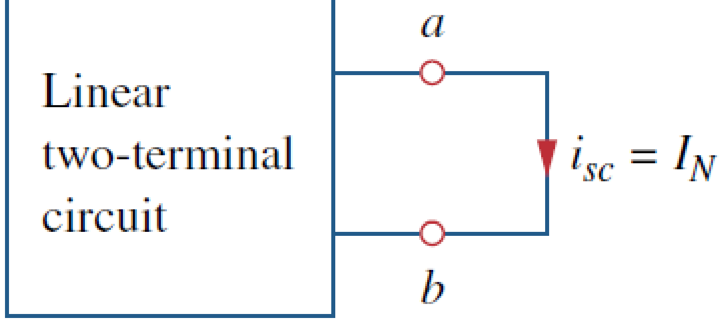


Figure 4.38
Finding Norton current I_N.

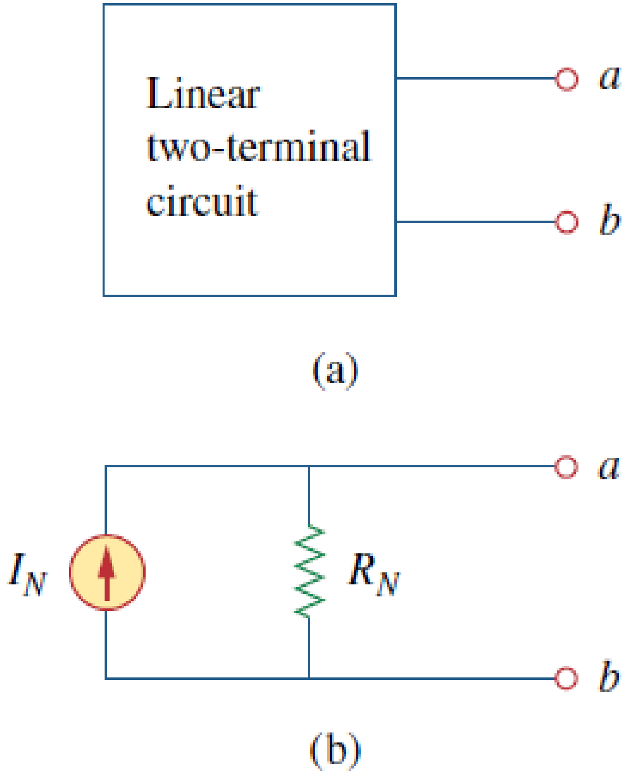


Figure 4.37
(a) Original circuit, (b) Norton equivalent circuit.

Norton vs. Thevenin

The Norton current and Thevenin voltage are related to each other as follows:

$$I_N = \frac{V_{Th}}{R_{Th}}$$

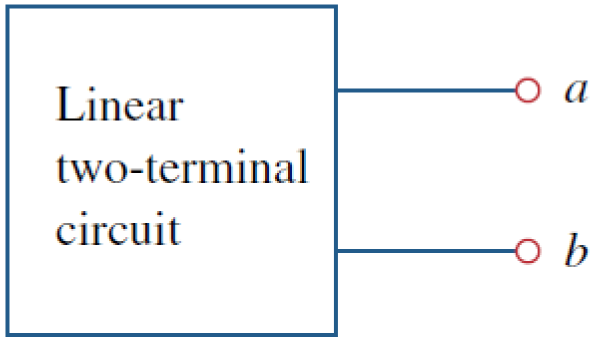
With $\boldsymbol{V}_{\mathbf{TH}}$, $\boldsymbol{I}_{\mathbf{N}}$, and $(\boldsymbol{R}_{\mathbf{TH}} = \boldsymbol{R}_{\mathbf{N}})$ related, to determine the Thevenin or Norton equivalent circuit requires that we find any two of the three:

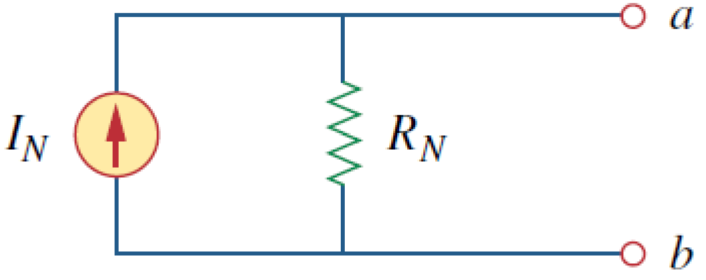
- The open-circuit voltage v_{oc} across terminals *a and b.*
- The short-circuit current i_{sc} at terminals *a and b.*
- The equivalent or input resistance R_{in} at terminals *a and b* when all independent sources are turned off.

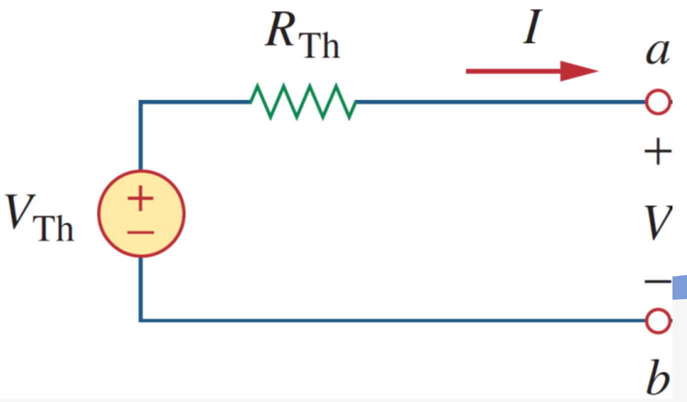
$$V_{\text{Th}} = v_{oc}$$

$$I_N = i_{sc}$$

$$R_{\text{Th}} = \frac{v_{oc}}{i_{sc}} = R_N$$







Example

Example 4.11

Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals *a*-*b*.

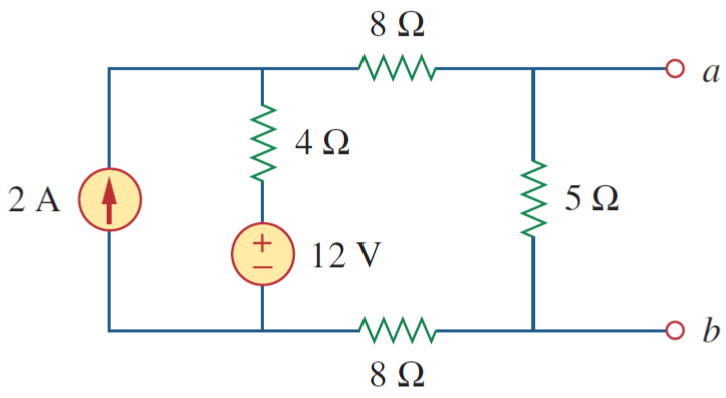
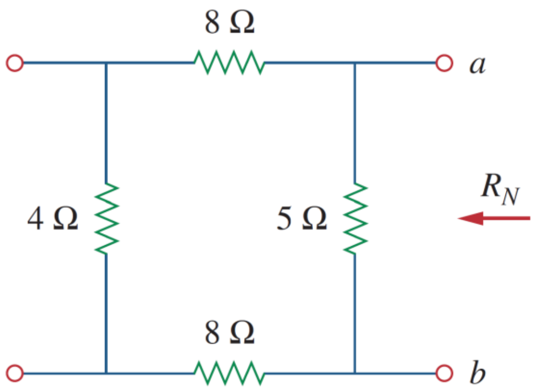


Figure 4.39
For Example 4.11.



(1) Set the independent sources equal to zero.

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

Example

Example 4.11 Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals *a*-*b*.

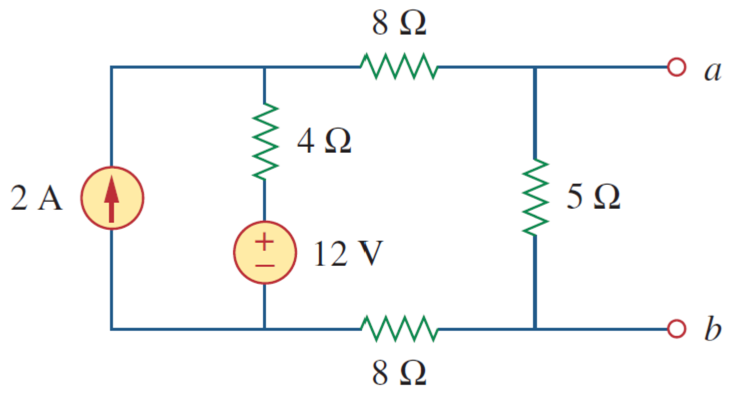
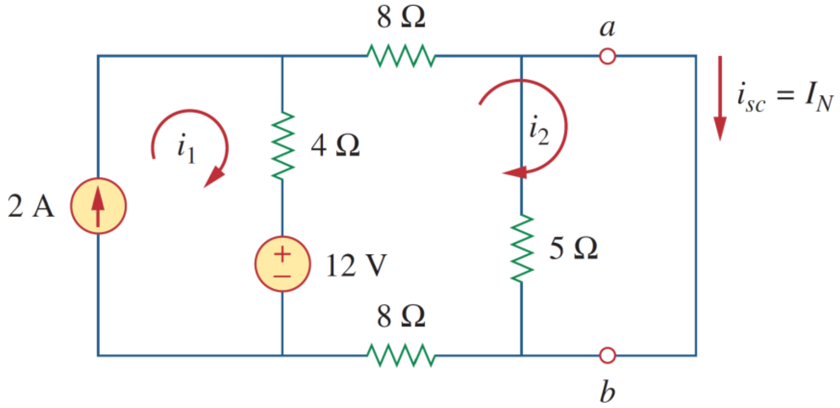


Figure 4.39
For Example 4.11.



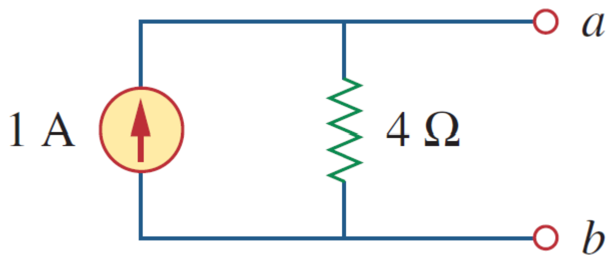
(2) To find I_N, we short-circuit terminals *a* and *b*

Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$



Example

Example 4.11 Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals *a-b*.

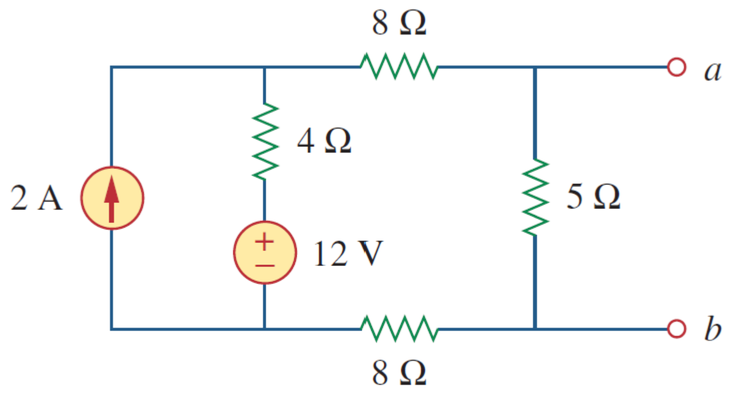
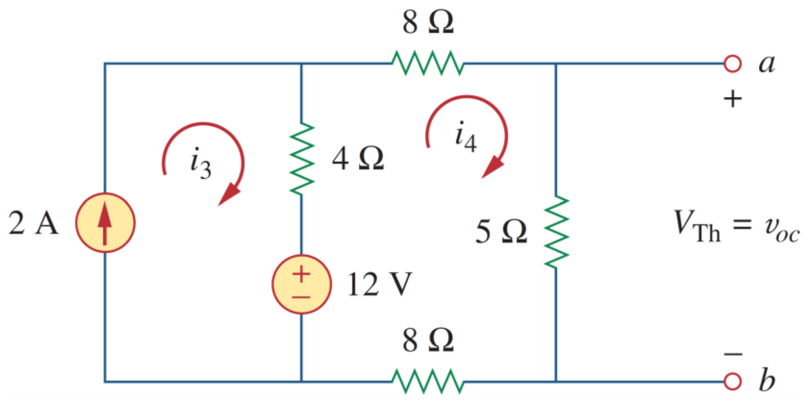


Figure 4.39
For Example 4.11.



May determine the Norton current from Thevenin voltage !

Using mesh analysis, we obtain

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

and

$$v_{oc} = V_{\text{Th}} = 5i_4 = 4 \text{ V}$$

Hence,

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}} = \frac{4}{4} = 1 \text{ A}$$

Example

Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals *a*-*b*.

Practice Problem 4.11

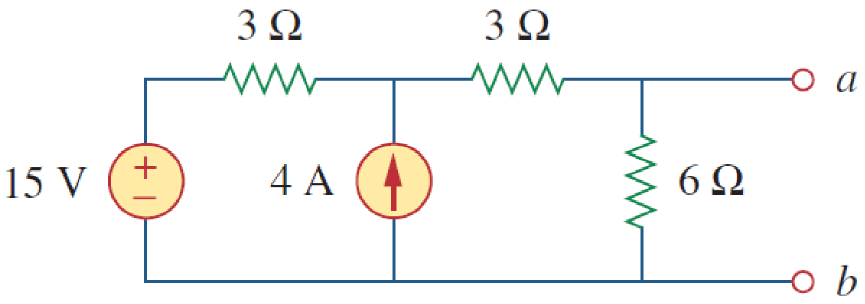


Figure 4.42
For Practice Prob. 4.11.

Answer: $R_N = 3\ \Omega$, $I_N = 4.5$ A.

Example

Example 4.12

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals *a*-*b*.

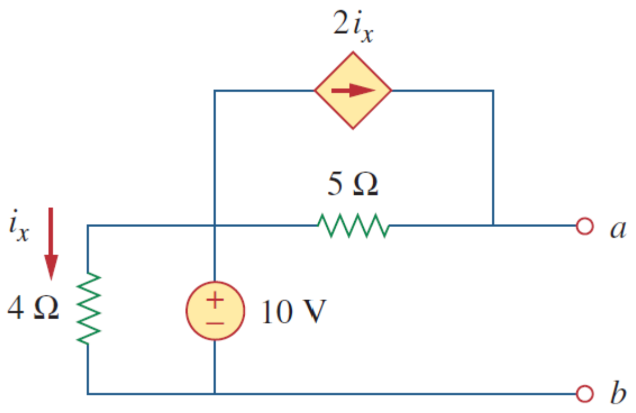
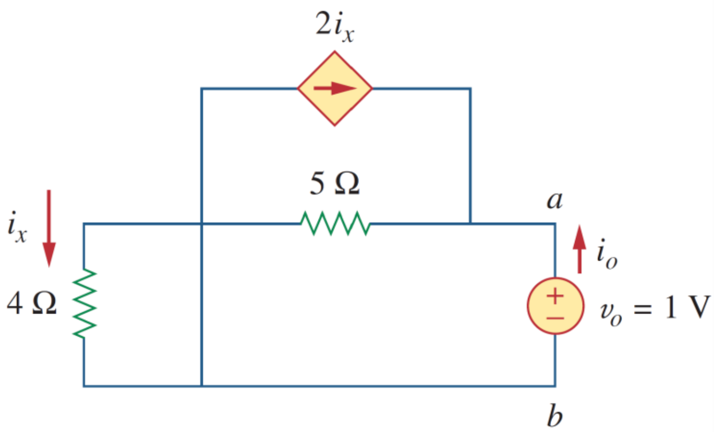


Figure 4.43
For Example 4.12.

(1) Finding R_N



$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$

Example

Example 4.12 Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals a-b.

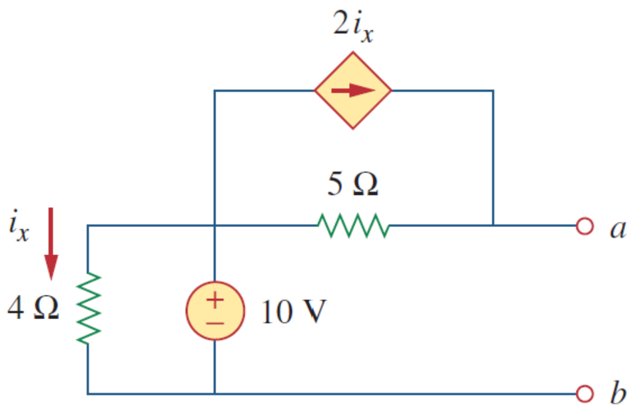
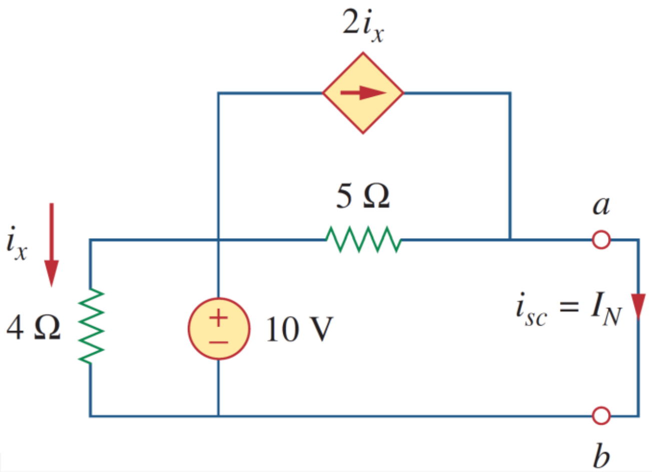


Figure 4.43
For Example 4.12.



(2) Finding I_N

$$i_x = \frac{10}{4} = 2.5 \text{ A}$$

At node a, KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7 \text{ A}$$

Example

Practice Problem 4.12 Find the Norton equivalent circuit of the circuit in Fig. 4.45 at terminals *a*-*b*.

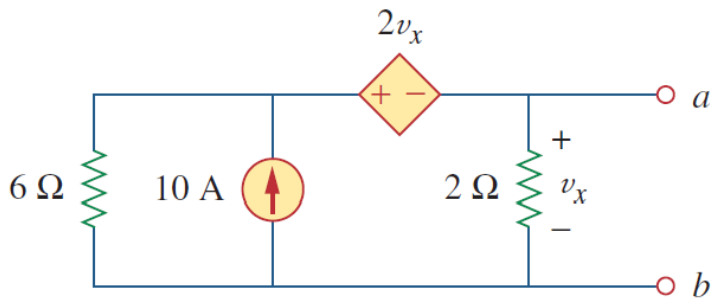


Figure 4.45
For Practice Prob. 4.12.

Answer: $R_N = 1\ \Omega$, $I_N = 10$ A.

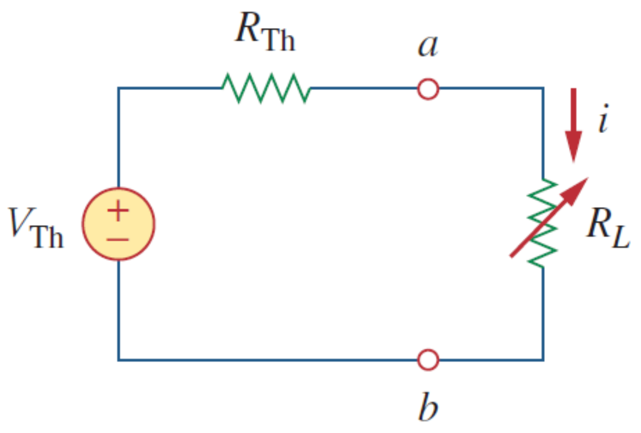
4.8 Maximum Power Transfer

- In many applications, a circuit is designed to power a load
- Among those applications there are many cases where we wish to maximize the power transferred to the load
- Given a system with internal resistance, the power delivered to the load is restricted.

How to find the maximum Power?

- Replace the entire circuit by the Thevenin equivalent circuit except for the load
- Assume the load resistance can be varied
- The power delivered to the load is

$$p = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$



How to find the maximum Power?

- For a given circuit, $\mathbf{V_{TH}}$ and $\mathbf{R_{TH}}$ are fixed. By varying the load resistance $\mathbf{R_L}$, the power delivered to the load varies as shown

- You can see that when $\mathbf{R_L}$ approaches to 0 and ∞, the transferred power goes to zero.

- In fact the maximum power occurs when $\mathbf{R_L = R_{TH}}$.

$$p = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

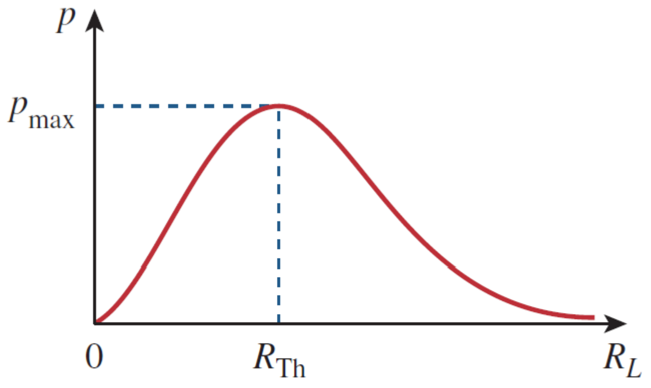


Figure 4.49
Power delivered to the load as a function of R_L.

$$\frac{dp}{dR_L} = V_{Th}^2\left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

$$= V_{Th}^2\left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0$$

The maximum power theorem

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{th}$)

The source and load are said to be ***matched*** when $R_L = R_{th}$

The maximum power transferred is obtained

$$p = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \quad \Rightarrow \quad \boxed{p_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}}}$$

Example

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

Example 4.13

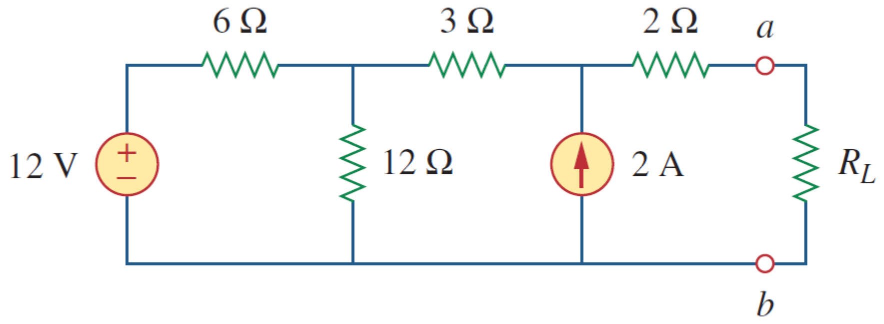
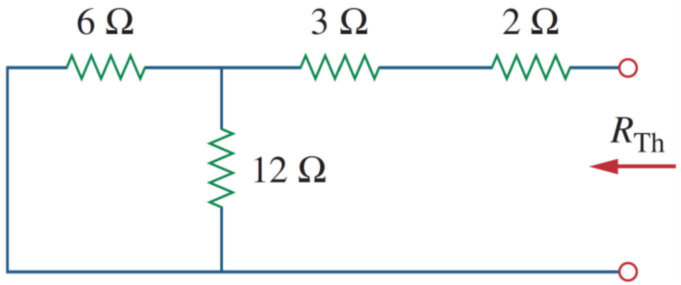


Figure 4.50
For Example 4.13.



Finding R_N

$$R_{Th} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

Example

Example 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

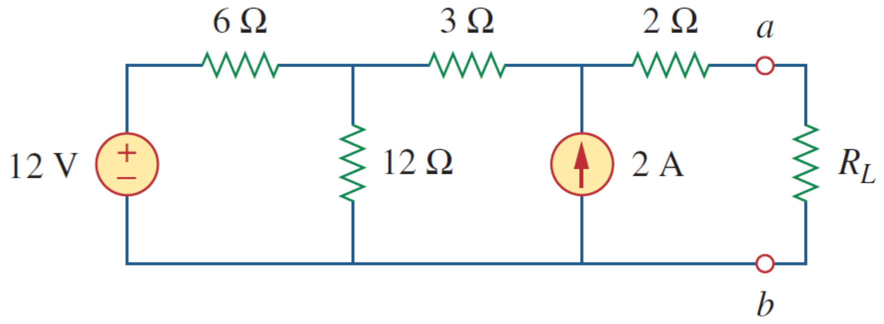
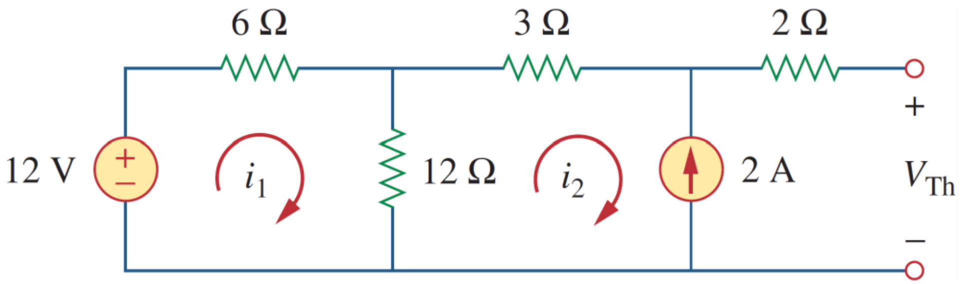


Figure 4.50
For Example 4.13.



To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2 \text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \qquad \Rightarrow \qquad V_{Th} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

Example

Practice Problem 4.13

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

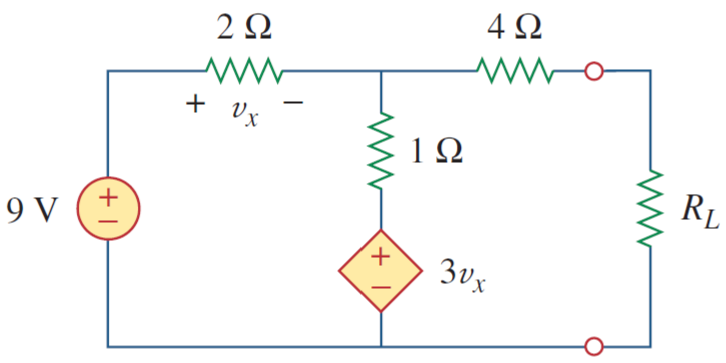


Figure 4.52
For Practice Prob. 4.13.

Answer: 4.222 Ω, 2.901 W.