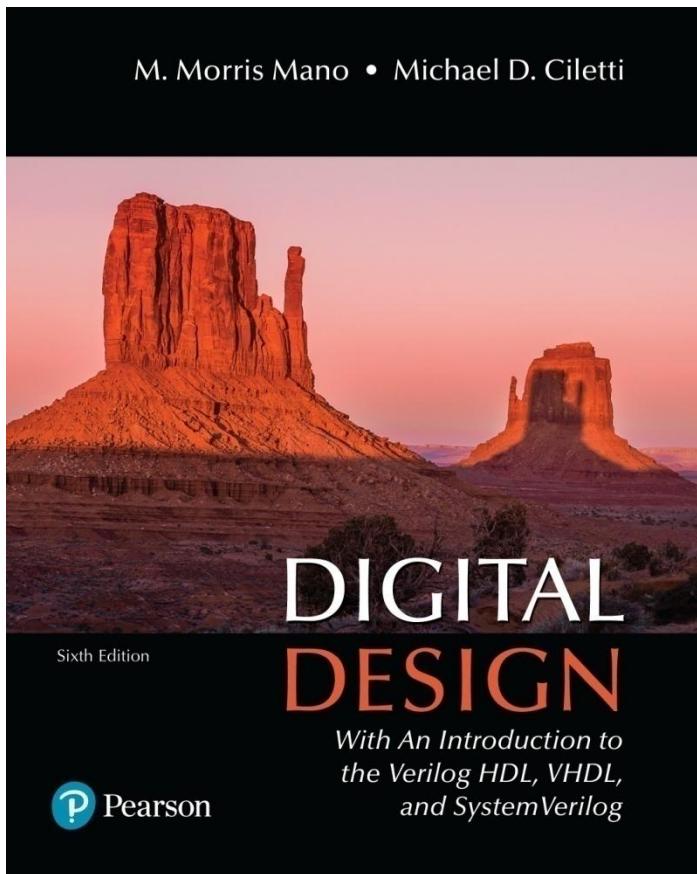


Digital Design

With an Introduction to the Verilog HDL, VHDL, and
SystemVerilog

6th Edition



Chapter 04

Combinational Logic

Figure 4.1
Block diagram of combinational circuit.

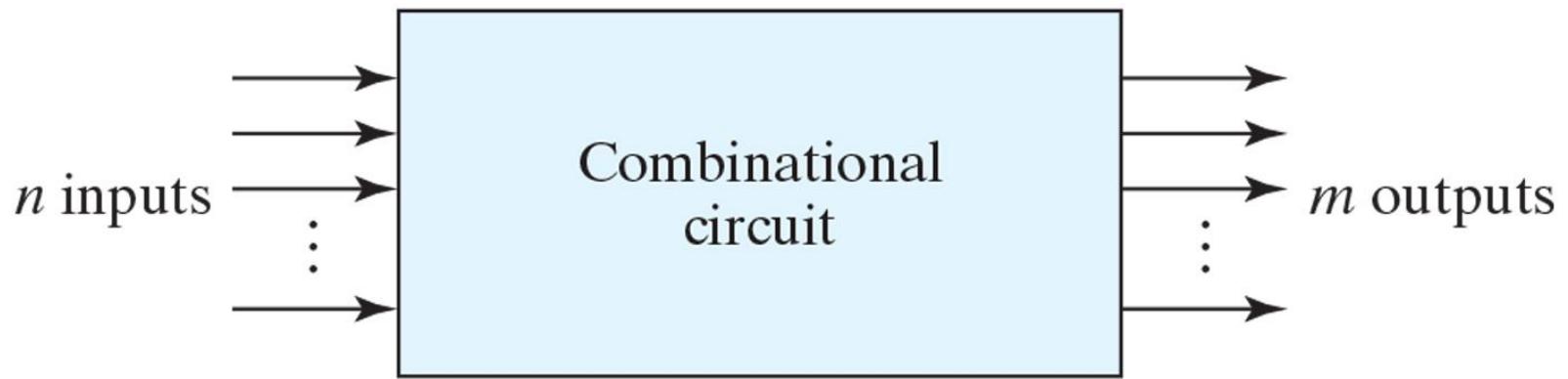


Figure 4.2
Logic diagram for analysis example.

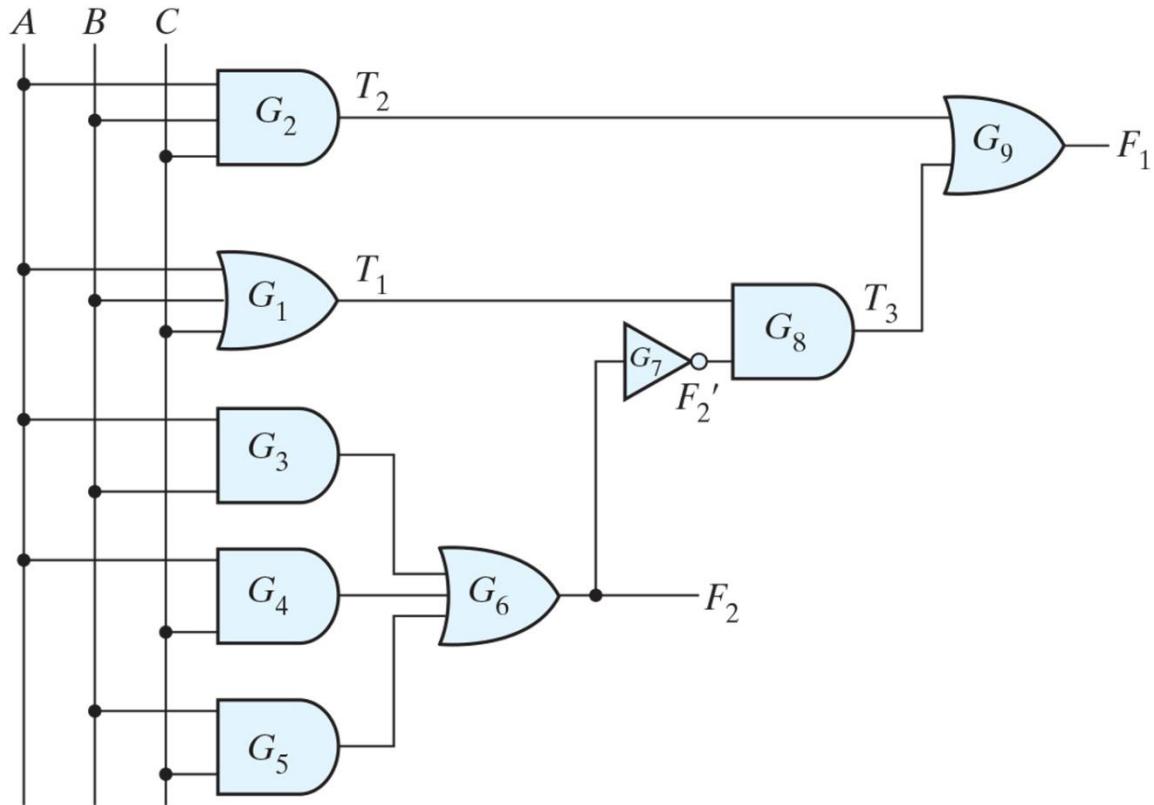


Table 4.1

Truth Table for the Logic Diagram of Fig. 4.2.

<i>A</i>	<i>B</i>	<i>C</i>	<i>F</i>₂	<i>F</i>₂'	<i>T</i>₁	<i>T</i>₂	<i>T</i>₃	<i>F</i>₁
0	0	0	0	1	0	0	0	0
0	0	1	0	1	1	0	1	1
0	1	0	0	1	1	0	1	1
0	1	1	1	0	1	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	1	0	1	0	0	0
1	1	0	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1

Figure PE4.1

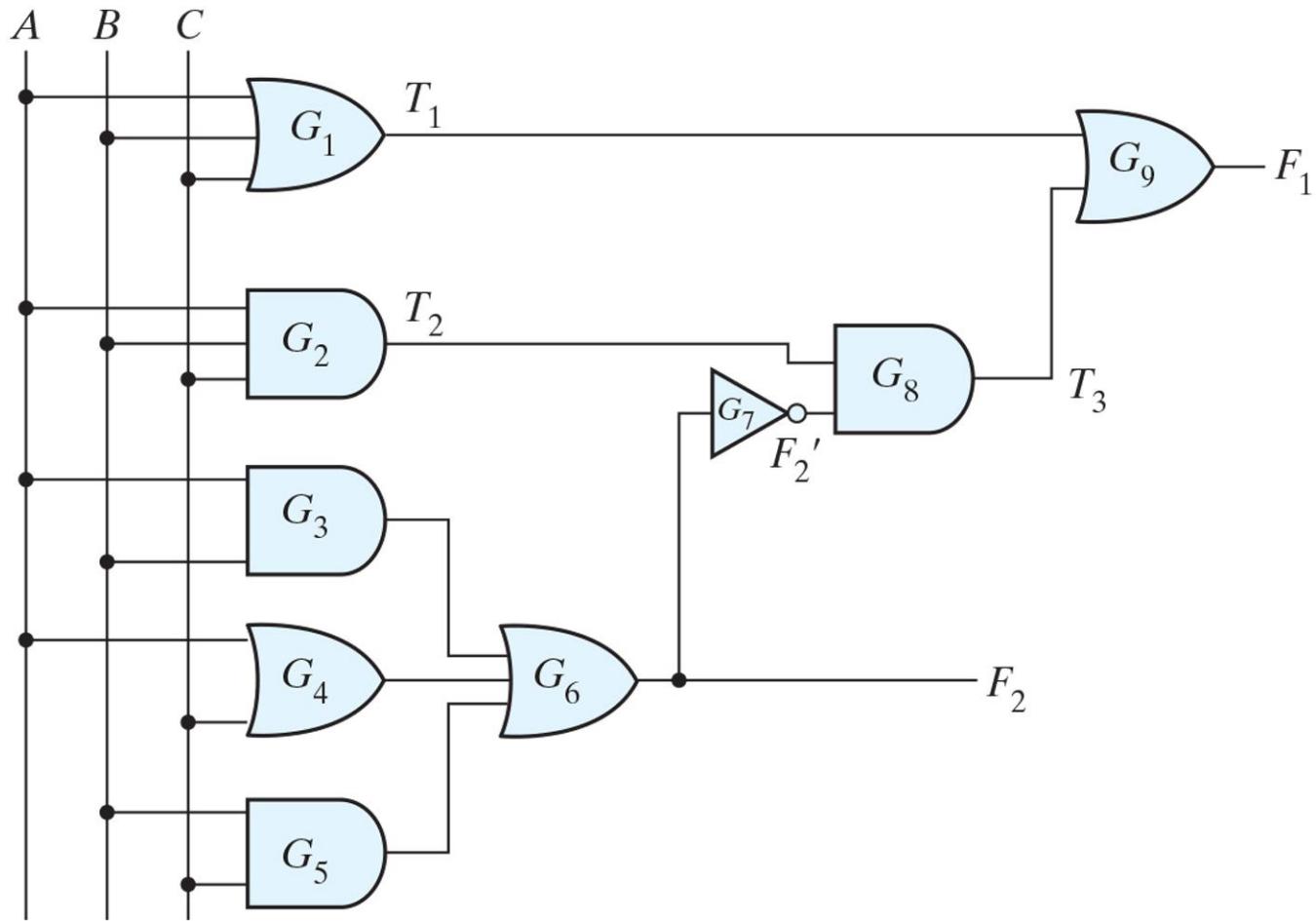


Table 4.2
Truth Table for Code Conversion Example.

Input BCD				Output Excess-3 Code			
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

Figure 4.3
Maps for BCD-to-excess-3 code converter.

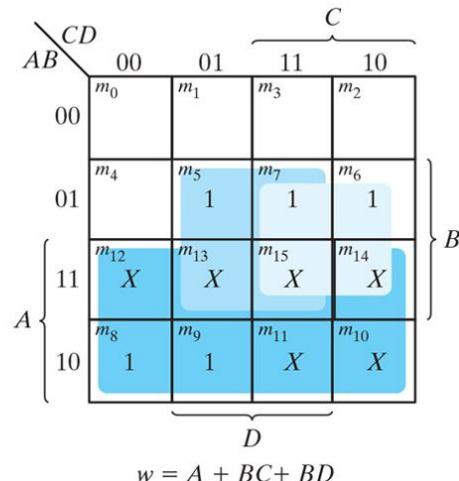
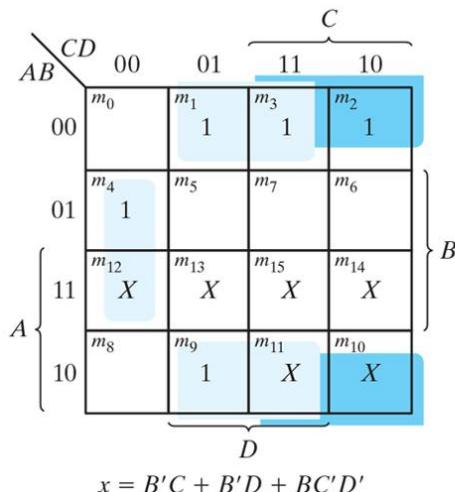
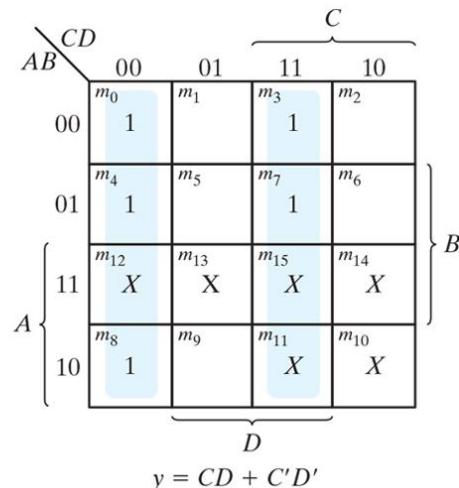
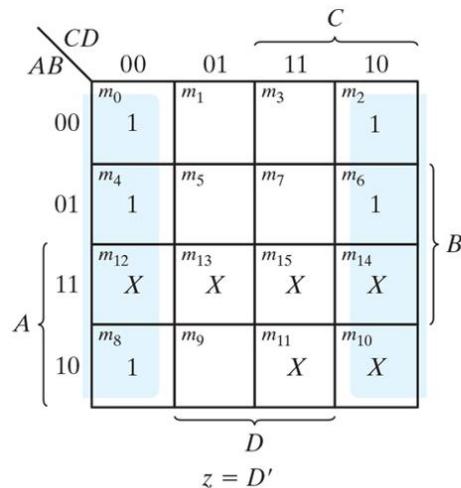


Figure 4.4
Logic diagram for BCD-to-excess-3 code converter.

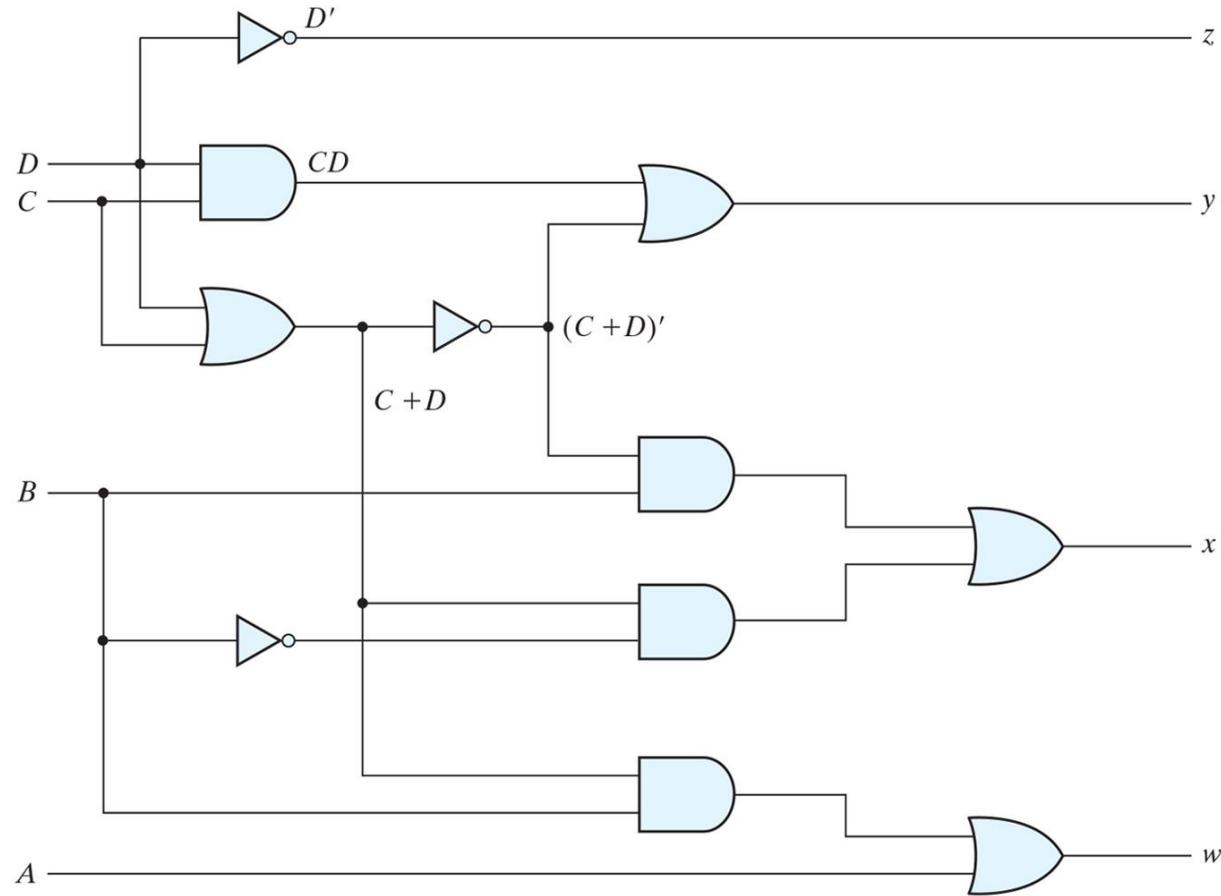


Table 4.3
Half Adder.

x	y	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Figure 4.5
Implementation of half adder.

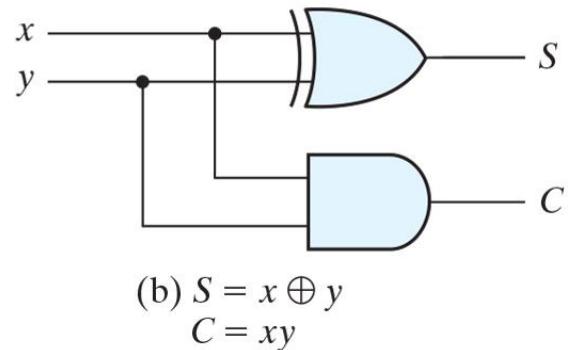
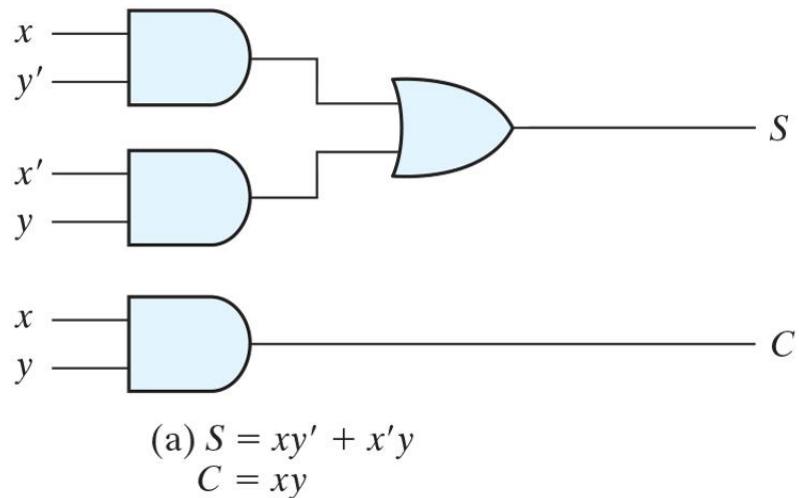
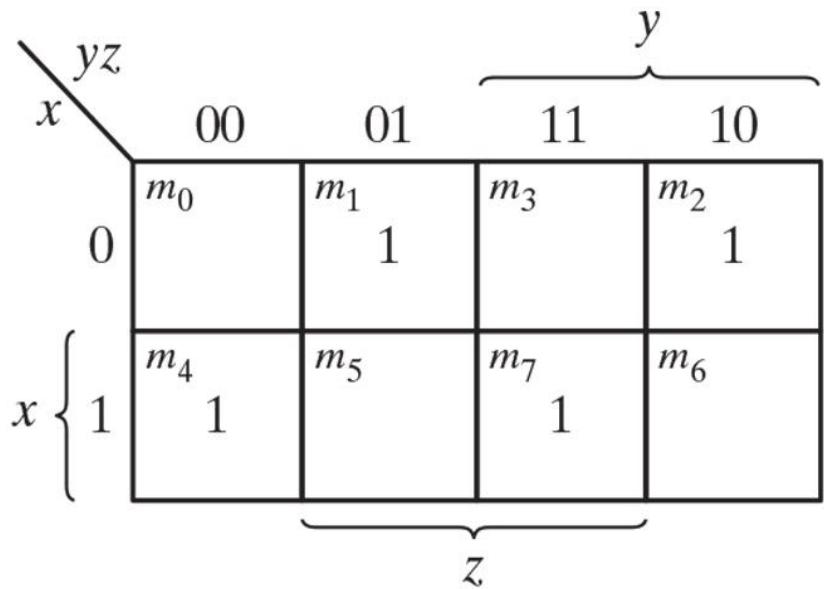


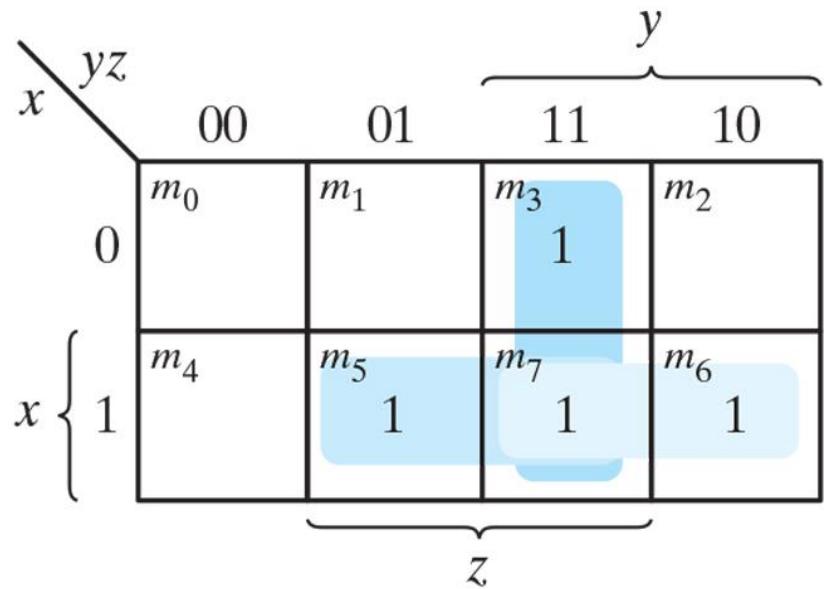
Table 4.4
Full Adder.

x	y	z	c	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Figure 4.6
K-Maps for full adder.

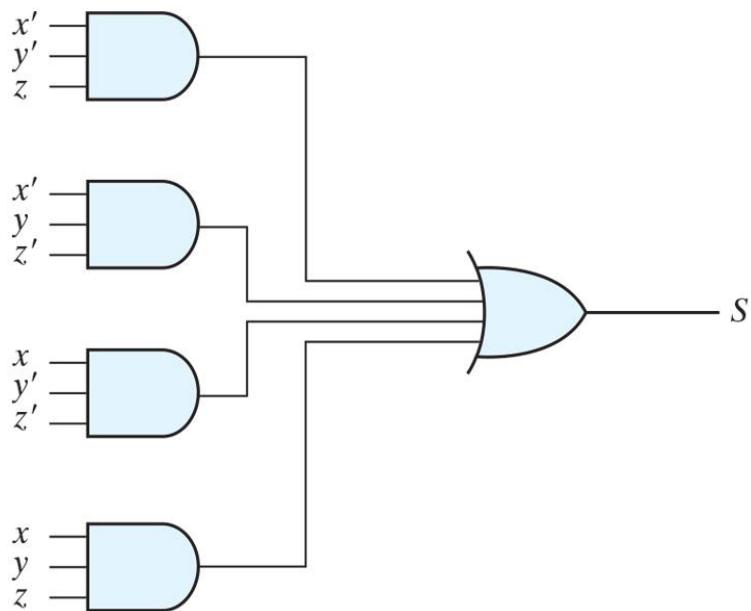


$$(a) S = x'y'z + x'yz' + xy'z' + xyz$$

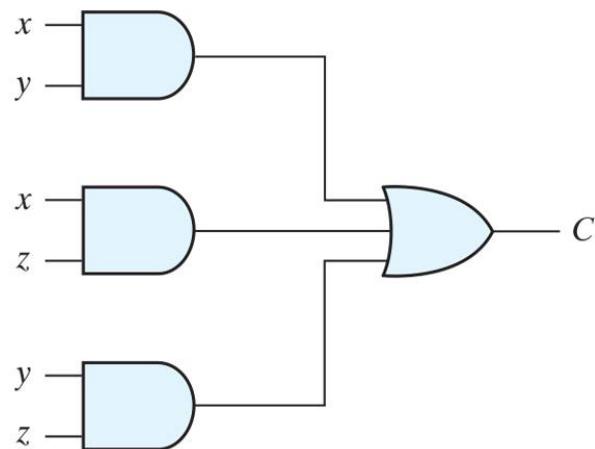


$$(b) C = xy + xz + yz$$

Figure 4.7
Implementation of full adder in sum-of-products form.

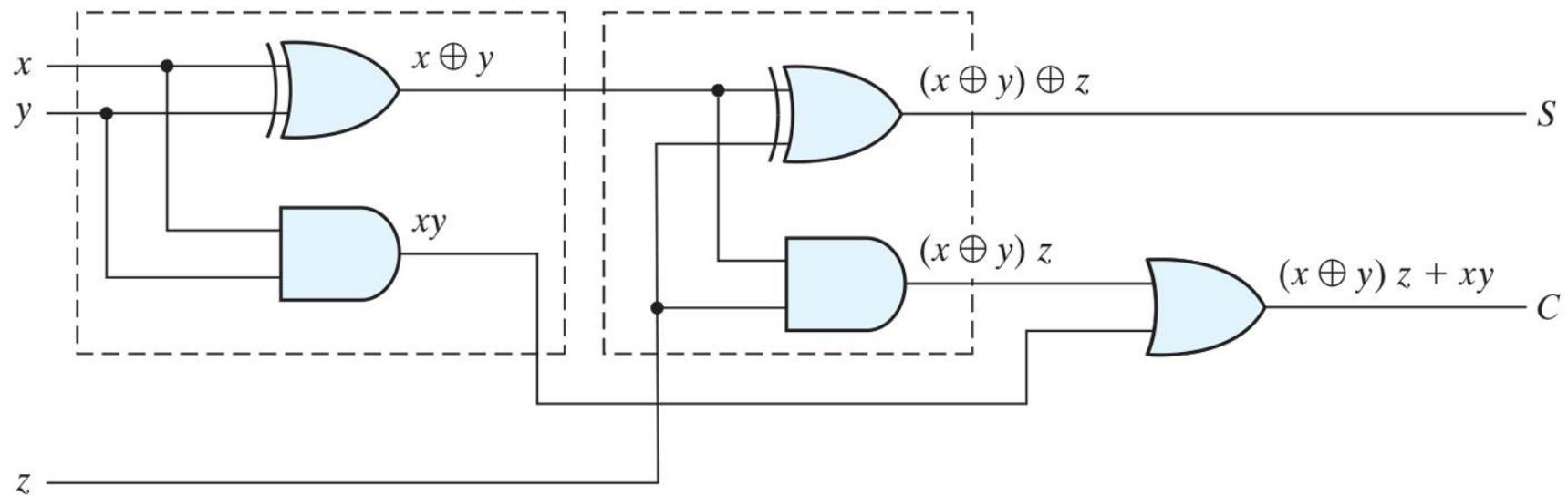


$$\begin{aligned}
 S &= x'y'z + x'yz' + xy'z' + xyz \\
 &= (x'y'z + xyz) + x'yz' + xy'z' \\
 &= (x'y' + xy)z + (x'y + xy')z \\
 &= (x \text{ xor } y)'z + (x \text{ xor } y)z \\
 &= x \text{ xor } y \text{ xor } z
 \end{aligned}$$



$$\begin{aligned}
 C &= xy + yz + xz = xy + xyz + x'y'z + xz \\
 &= xy + xyz + x'y'z + xy'z \\
 &= xy + xyz + (x \text{ XOR } y)z \\
 &= xy + (x \text{ XOR } y)z
 \end{aligned}$$

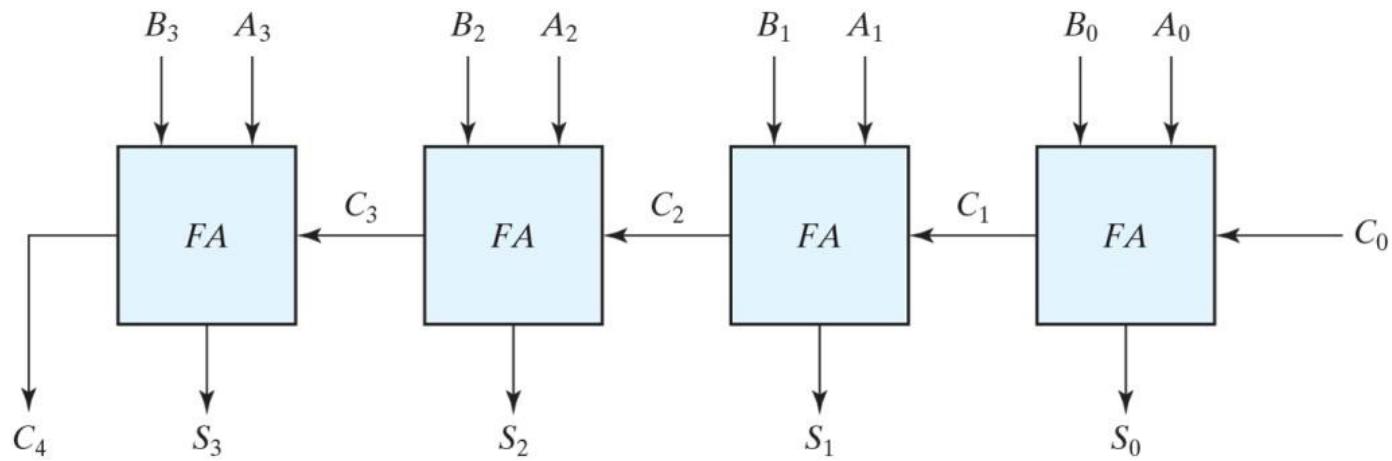
Figure 4.8
Implementation of full adder with two half adders and an OR gate.



$$\begin{aligned}
 S &= x'y'z + x'yz' + xy'z' + xyz \\
 &= (x'y'z + xyz) + x'yz' + xy'z' \\
 &= (x'y' + xy)z + (x'y + xy')z \\
 &= (x \text{ xor } y)z + (x \text{ xor } y)z
 \end{aligned}$$

$$\begin{aligned}
 C &= xy + yz + xz = xy + xyz + x'y'z + xz \\
 &= xy + x'y'z + xyz + xy'z \\
 &= xy + xyz + (x \text{ XOR } y)z \\
 &= xy + (x \text{ XOR } y)z
 \end{aligned}$$

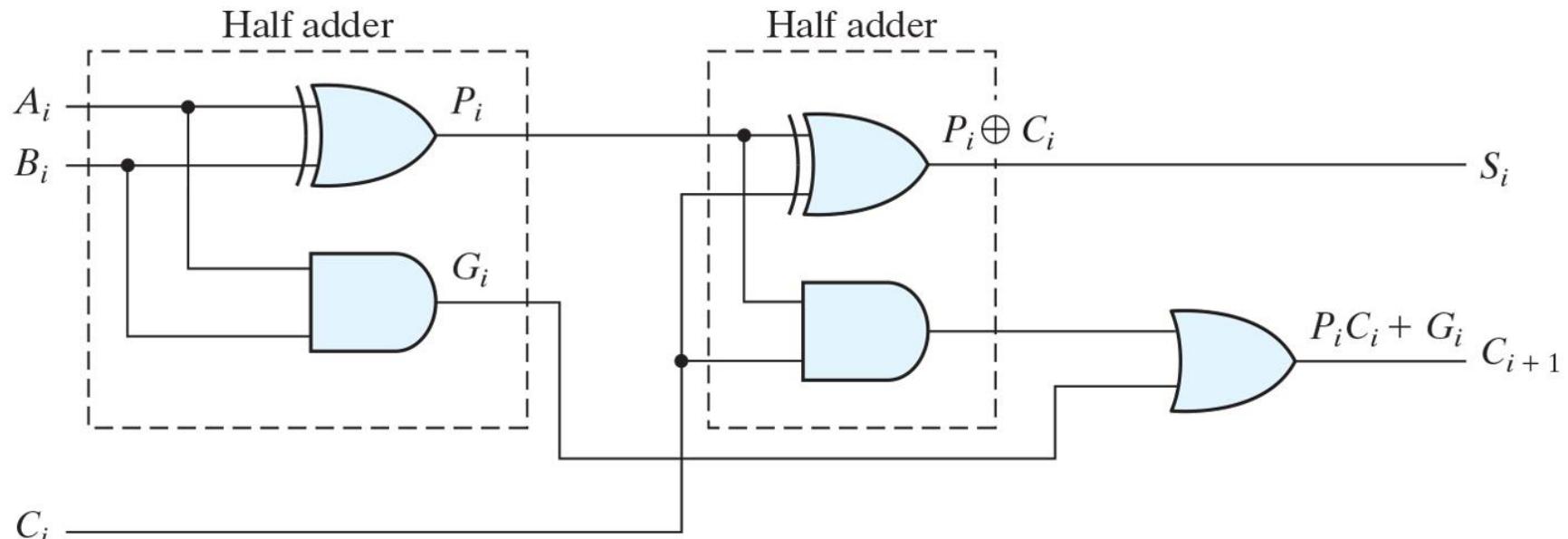
Figure 4.9
Four-bit adder.



Binary Adder

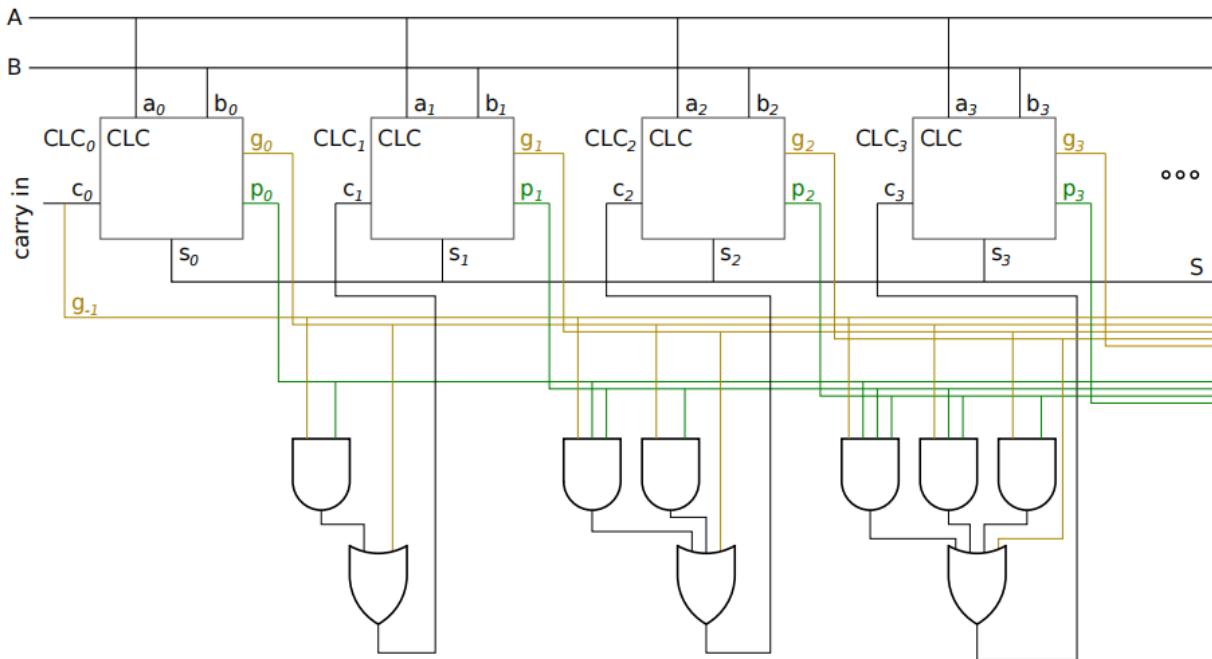
Subscript <i>i</i>:	3	2	1	0	
Input carry	0	1	1	0	C_i
Augend	1	0	1	1	A_i
Addend	0	0	1	1	B_i
Sum	1	1	1	0	S_i
Output carry	0	0	1	1	C_{i+1}

Figure 4.10
Full adder with P and G shown.



$$\begin{aligned}
 C &= xy + yz + xz = xy + xyz + x'y'yz + xz \\
 &= xy + x'y'yz + xyz + xy'z \\
 &= xy + xyz + (x \text{ XOR } y)z \\
 &= xy + (x \text{ XOR } y)z
 \end{aligned}$$

Figure 4.12
Four-bit adder with carry lookahead.



Boolean expressions for carry signals c_0 to c_3 :

$$c_0 = g_{-1}.$$

$$c_1 = g_{-1}p_0 + g_0.$$

$$c_2 = g_{-1}p_0p_1 + g_0p_1 + g_1.$$

$$c_3 = g_{-1}p_0p_1p_2 + g_0p_1p_2 + g_1p_2 + g_2.$$

Figure 4.11
Logic diagram of carry lookahead generator.

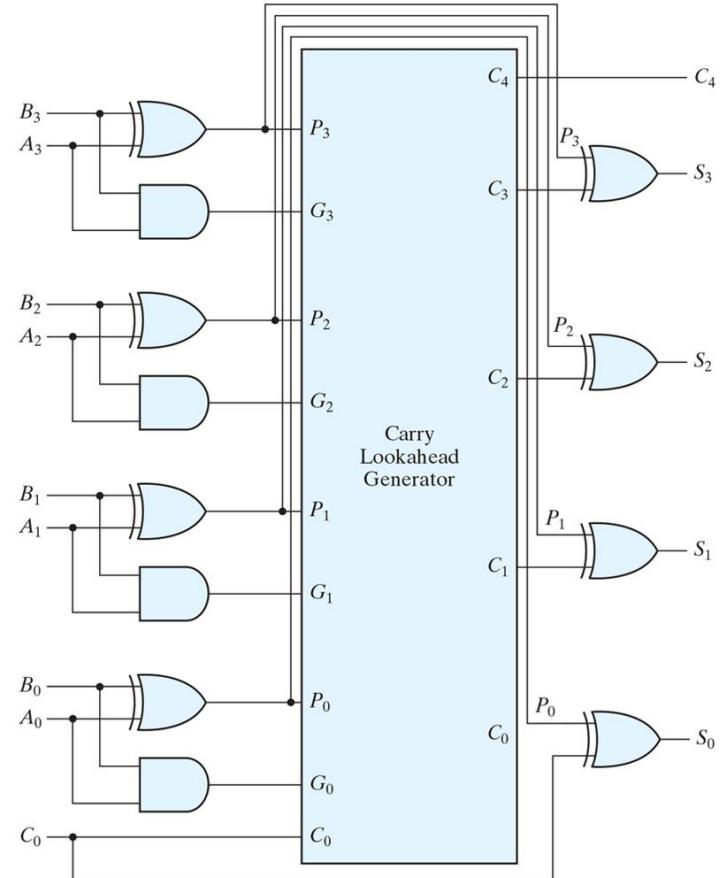
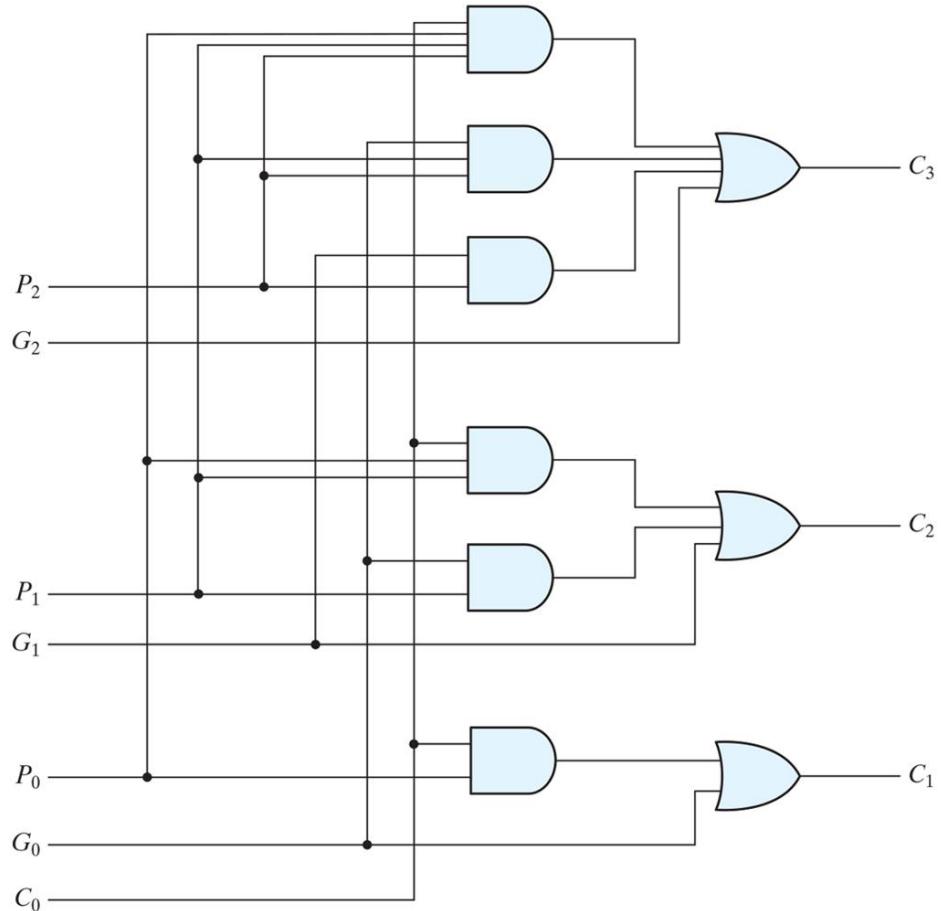


Figure 4.12
Four-bit adder with carry lookahead.

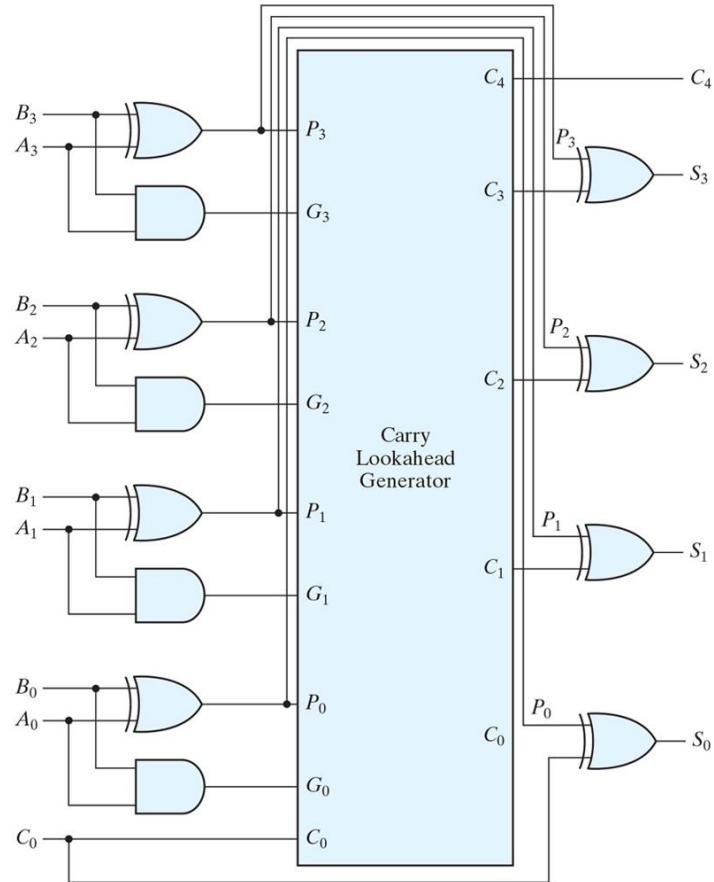


Figure 4.13
Four-bit adder–subtractor (with overflow detection).

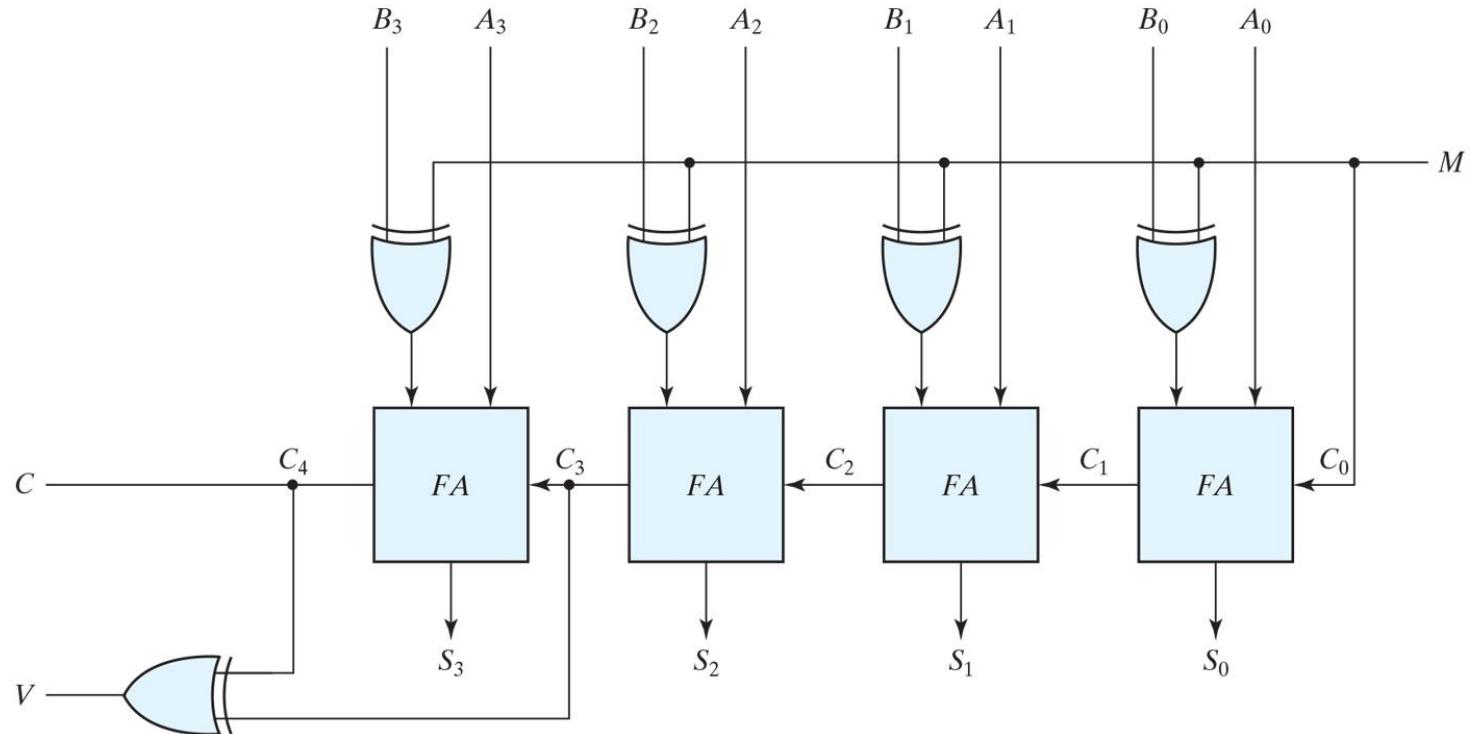


Table 4.5
Derivation of BCD Adder.

K	Binary Sum				BCD Sum					Decimal
	Z₈	Z₄	Z₂	Z₁	C	S₈	S₄	S₂	S₁	
+6	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	1	1
	0	0	0	1	0	0	0	1	0	2
	0	0	0	1	1	0	0	0	1	3
	0	0	1	0	0	0	0	1	0	4
	0	0	1	0	1	0	0	1	0	5
	0	0	1	1	0	0	0	1	0	6
	0	0	1	1	1	0	0	1	1	7
	0	1	0	0	0	0	1	0	0	8
	0	1	0	0	1	0	1	0	1	9
+6	0	1	0	1	0	1	0	0	0	10
	0	1	0	1	1	1	0	0	1	11
	0	1	1	0	0	1	0	0	1	12
	0	1	1	0	1	1	0	0	1	13
	0	1	1	1	0	1	0	1	0	14
	0	1	1	1	1	1	0	1	0	15
	1	0	0	0	0	1	0	1	0	16
	1	0	0	0	1	1	0	1	1	17
	1	0	0	1	0	1	1	0	0	18
	1	0	0	1	1	1	1	0	1	19

Figure 4.14
Block diagram of a BCD adder.

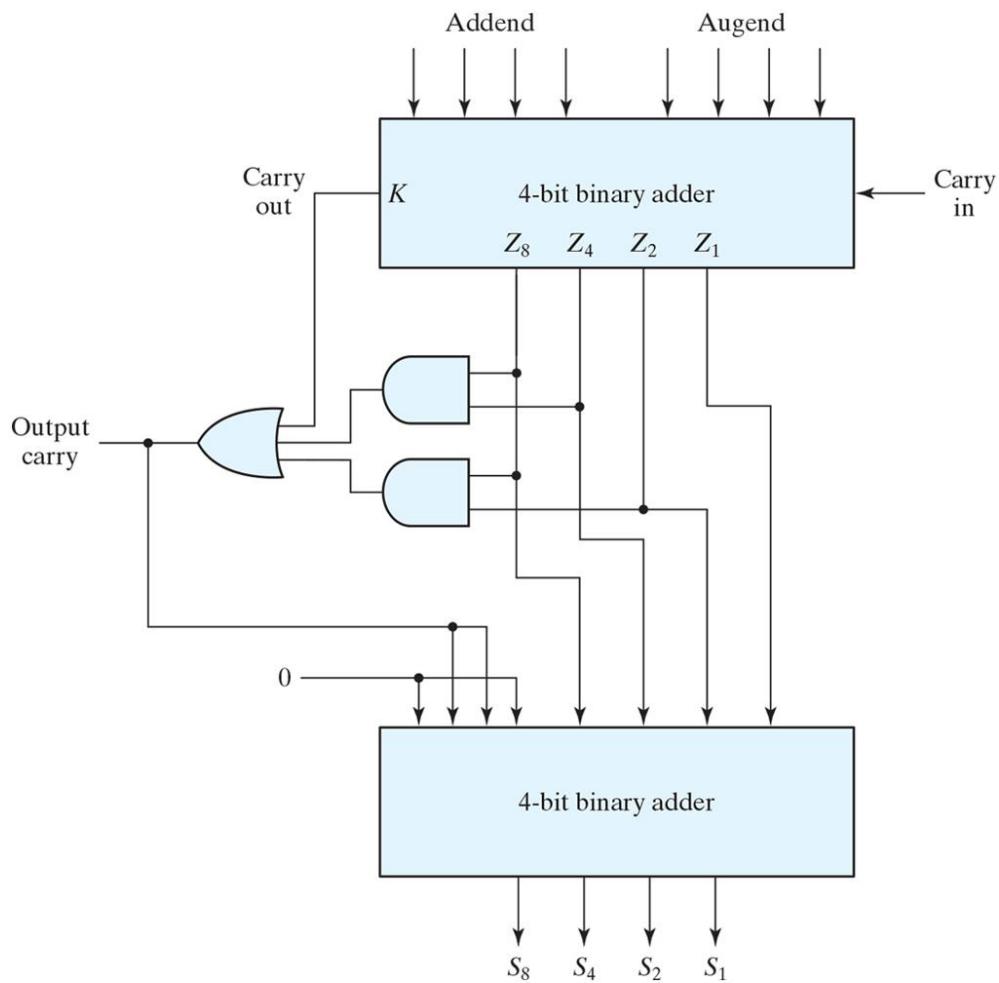


Figure 4.15
Two-bit by two-bit binary multiplier.

$$\begin{array}{r}
 & B_1 & B_0 \\
 & A_1 & A_0 \\
 \hline
 & A_0B_1 & A_0B_0 \\
 \hline
 A_1B_1 & A_1B_0 \\
 \hline
 P_3 & P_2 & P_1 & P_0
 \end{array}$$

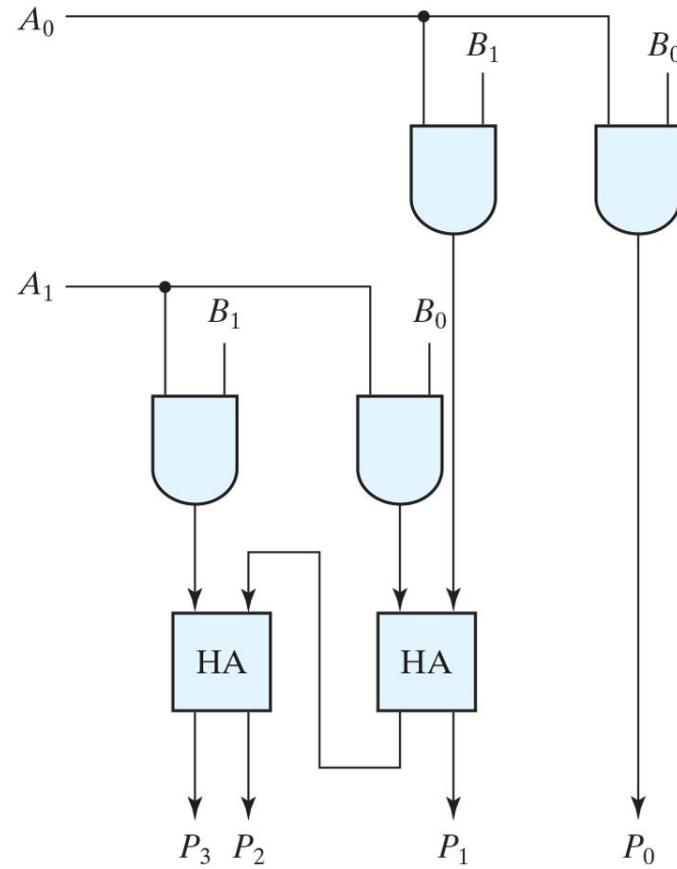


Figure 4.16
Four-bit by three-bit binary multiplier.

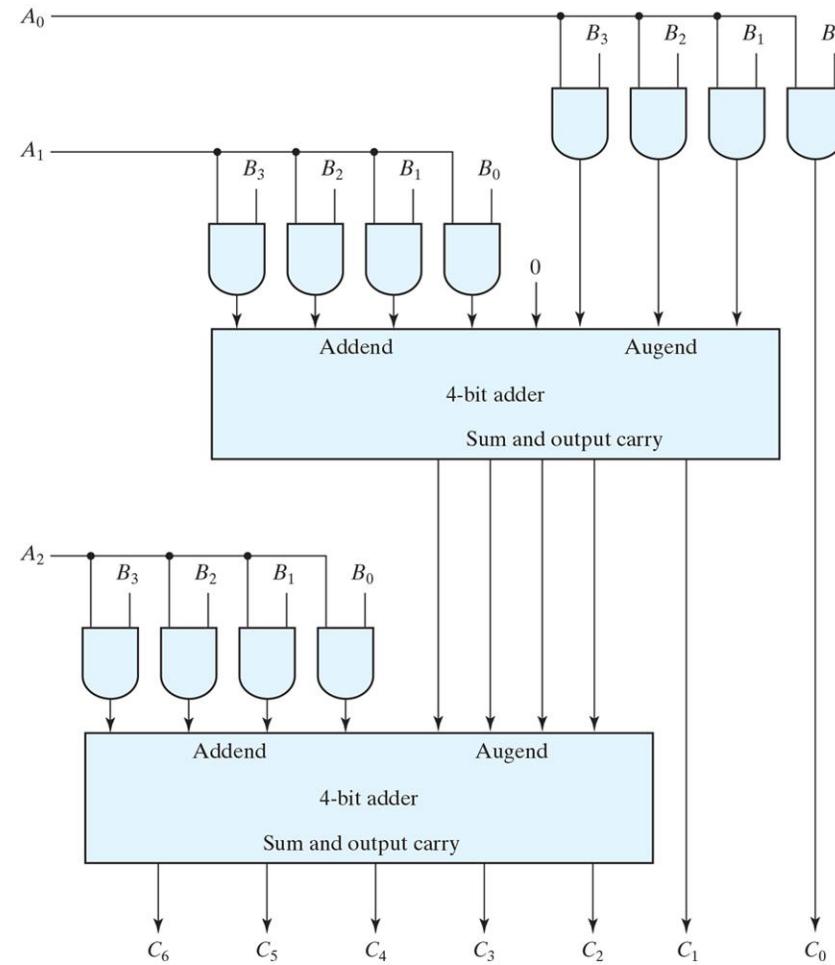


Figure 4.17
Four-bit magnitude comparator.

幅值比较

If the first n bits are equal, then compare the n-1 bit ...

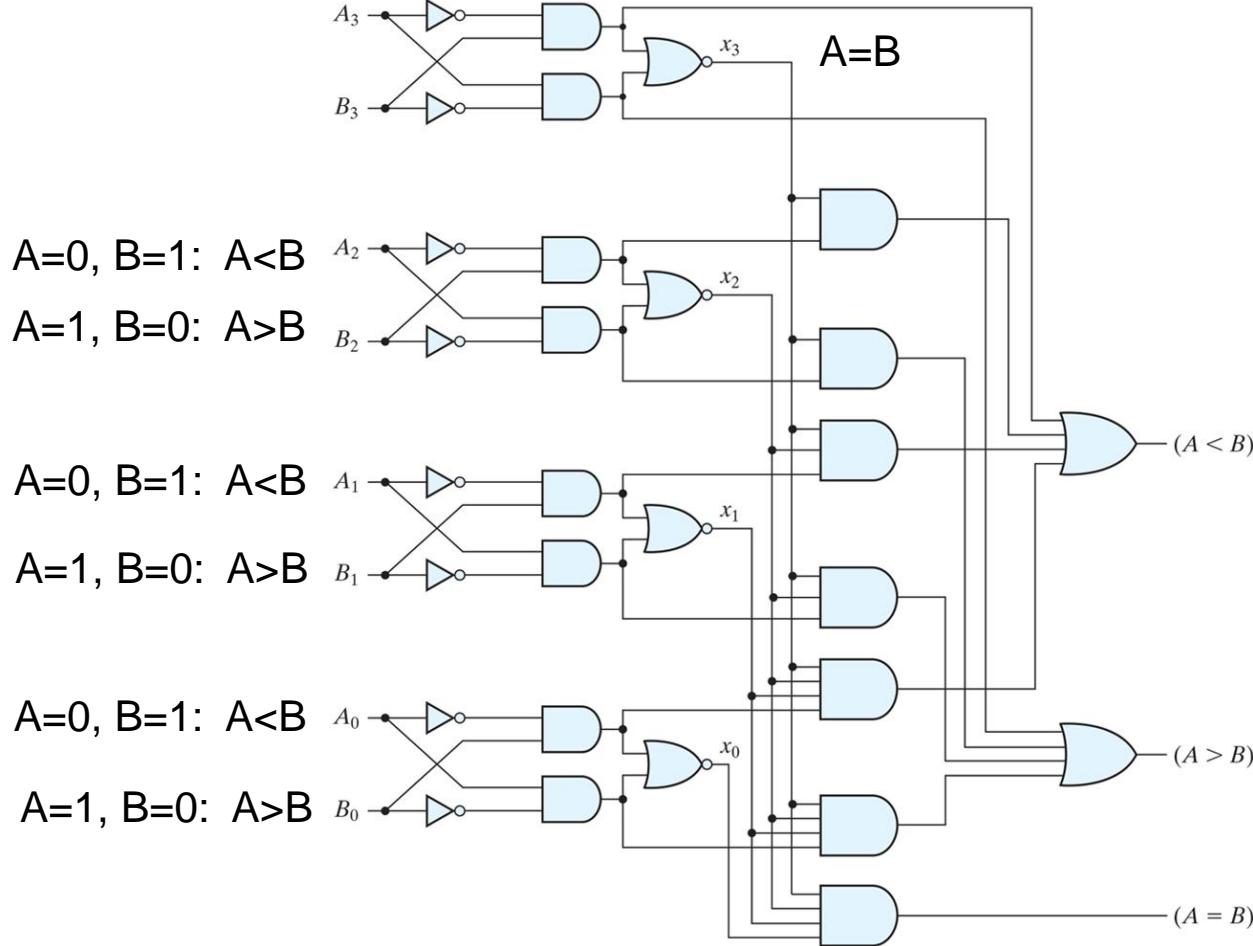


Figure 4.18
Three-to-eight-line decoder.

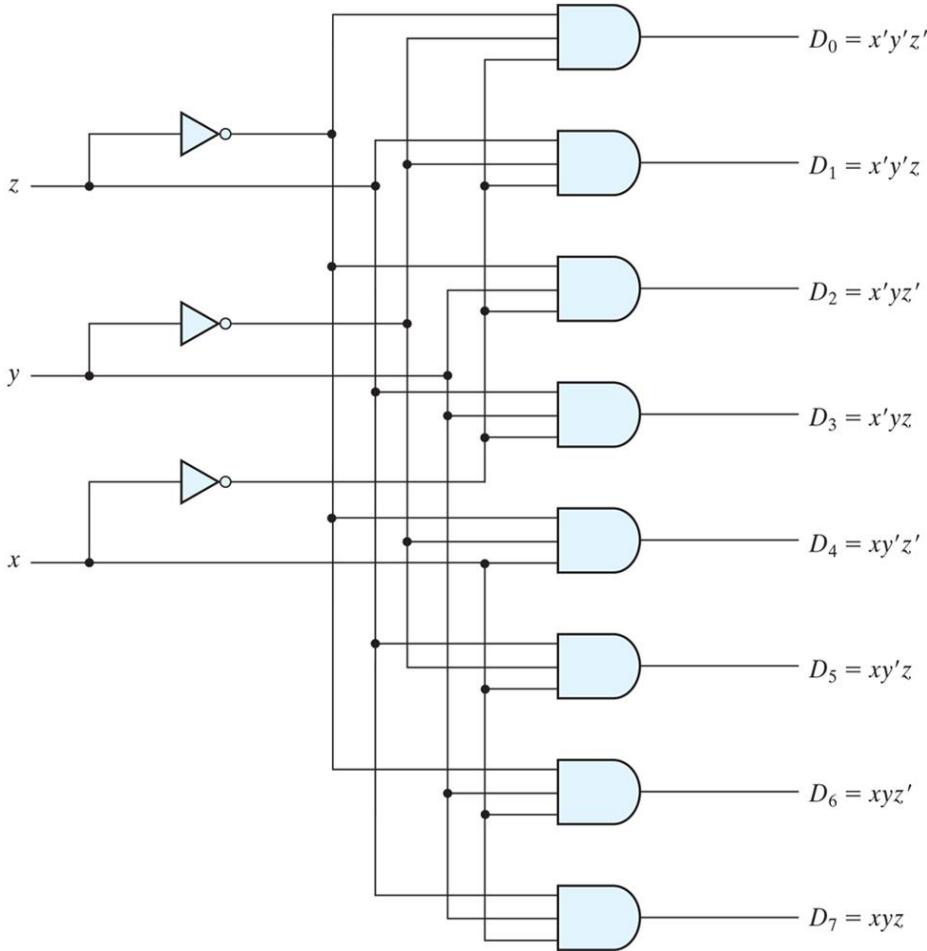


Table 4.6
Truth Table of a Three-to-Eight-Line Decoder.

Inputs			Outputs							
x	y	z	D₀	D₁	D₂	D₃	D₄	D₅	D₆	D₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

x	y	z	c	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

A full adder can be built with 3-8 decoder



I

Figure 4.19
Two-to-four-line decoder with enable input.

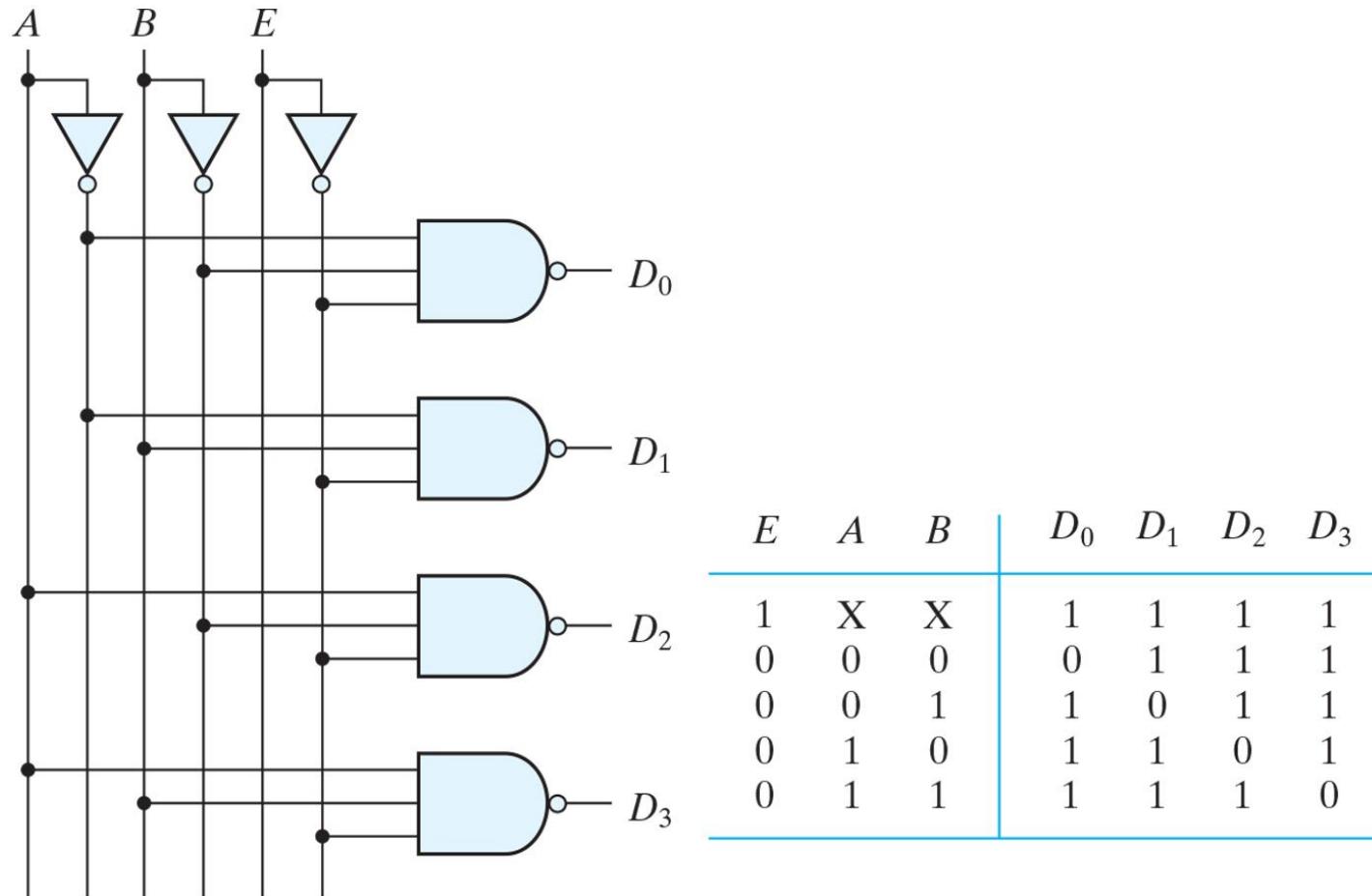


Figure 4.20
4 x 16 decoder constructed with two 3 x 8 decoders.

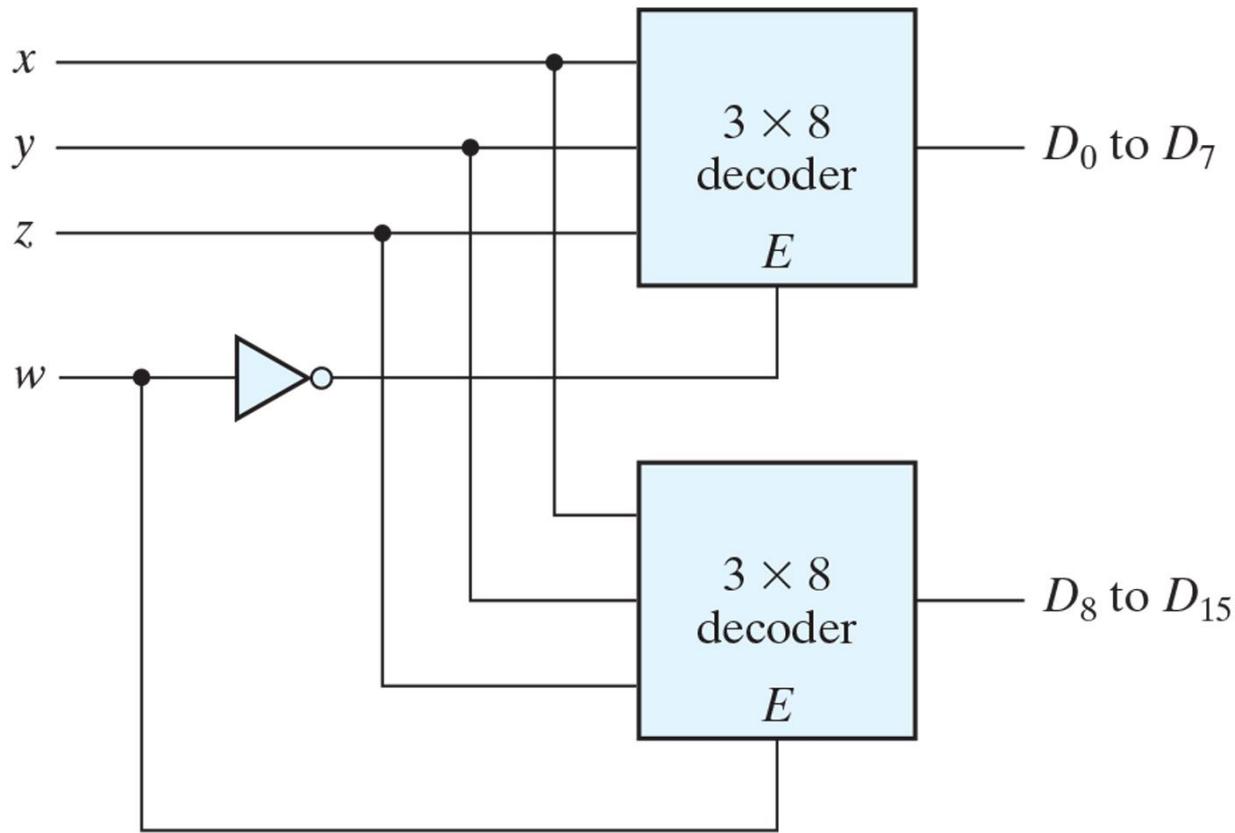


Figure PE4.8

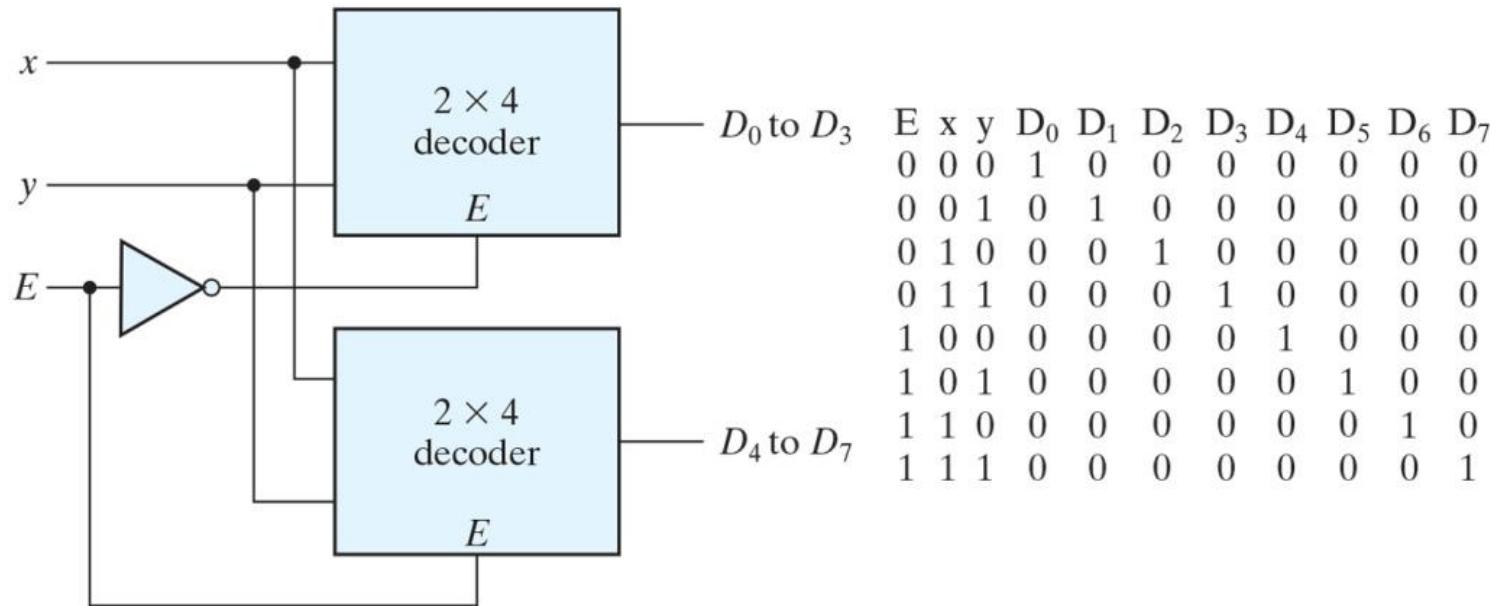
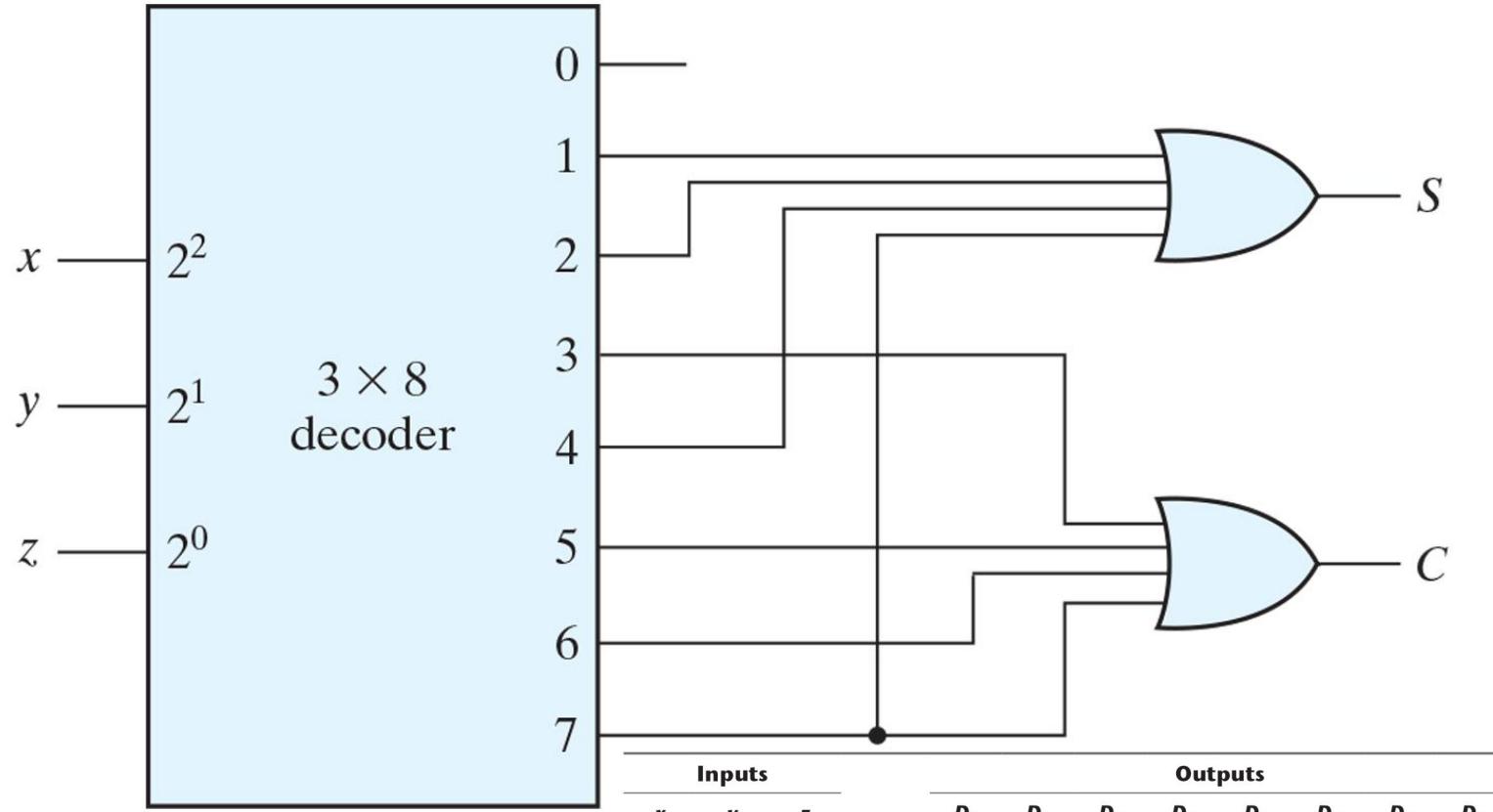


Figure 4.21
Implementation of a full adder with a decoder.



Encoder :Inverse of 3-8 decoder

Table 4.7

Truth Table of an Octal-to-Binary Encoder.

编码器

Inputs								Outputs		
D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	x	y	z
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

Table 4.8
Truth Table of a Priority Encoder.

Inputs				Outputs		
D_0	D_1	D_2	D_3	x	y	v
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

Figure 4.22
Maps for a priority encoder.

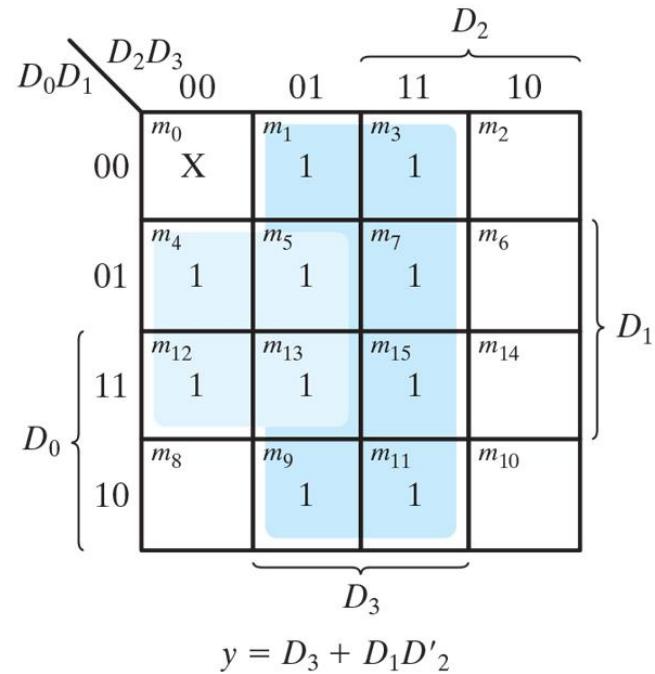
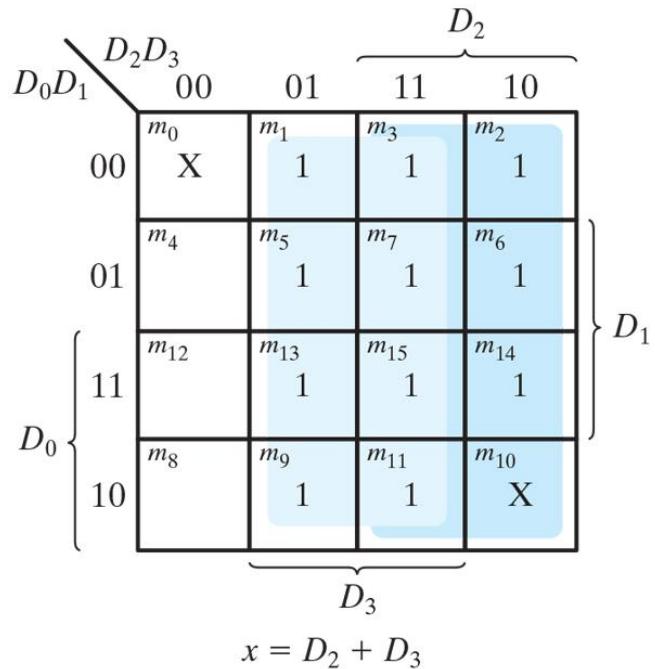


Figure 4.23
Four-input priority encoder.

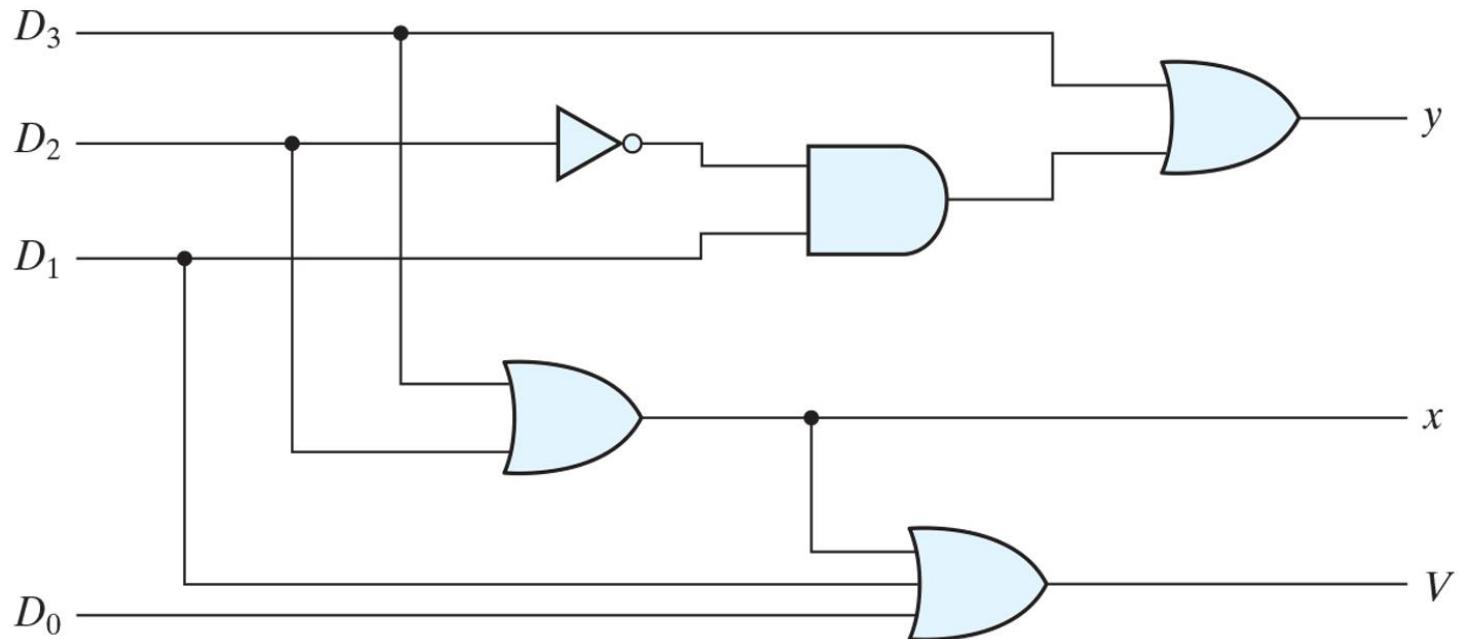
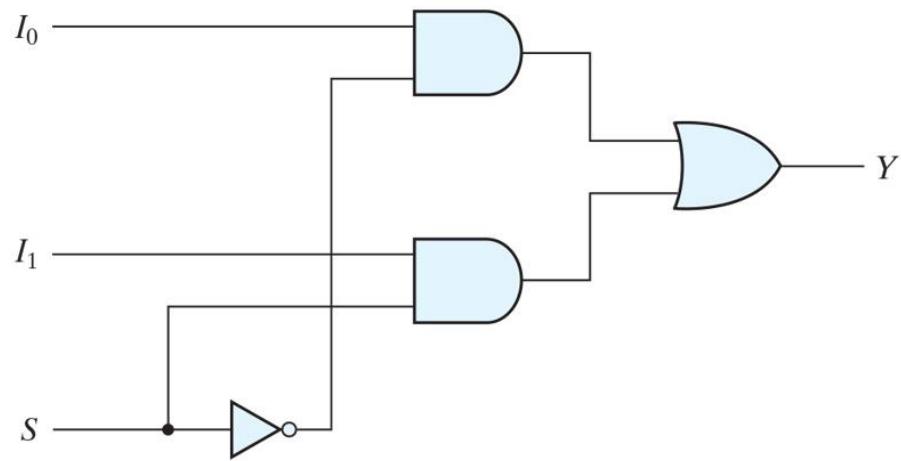
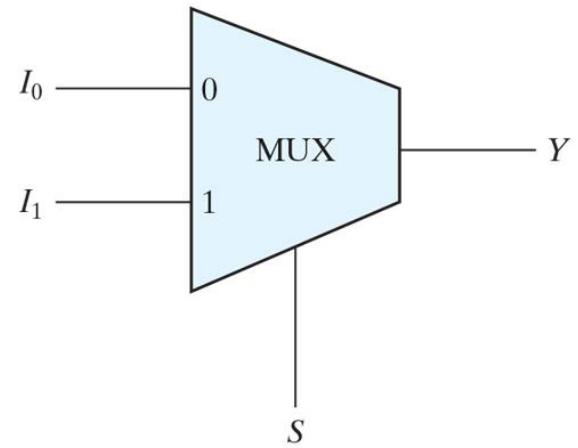


Figure 4.24
Two-to-one-line multiplexer.

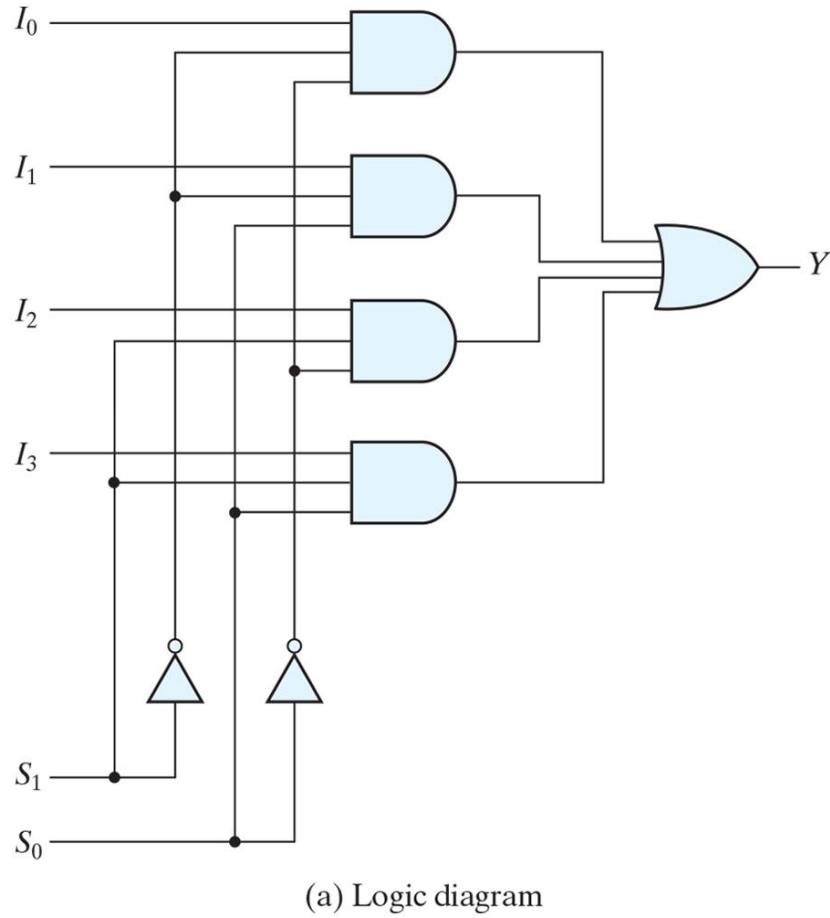


(a) Logic diagram



(b) Block diagram

Figure 4.25
Four-to-one-line multiplexer.



S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

(b) Function table

Figure 4.26
Quadruple two-to-one-line multiplexer.

4X 2-input multiplexer

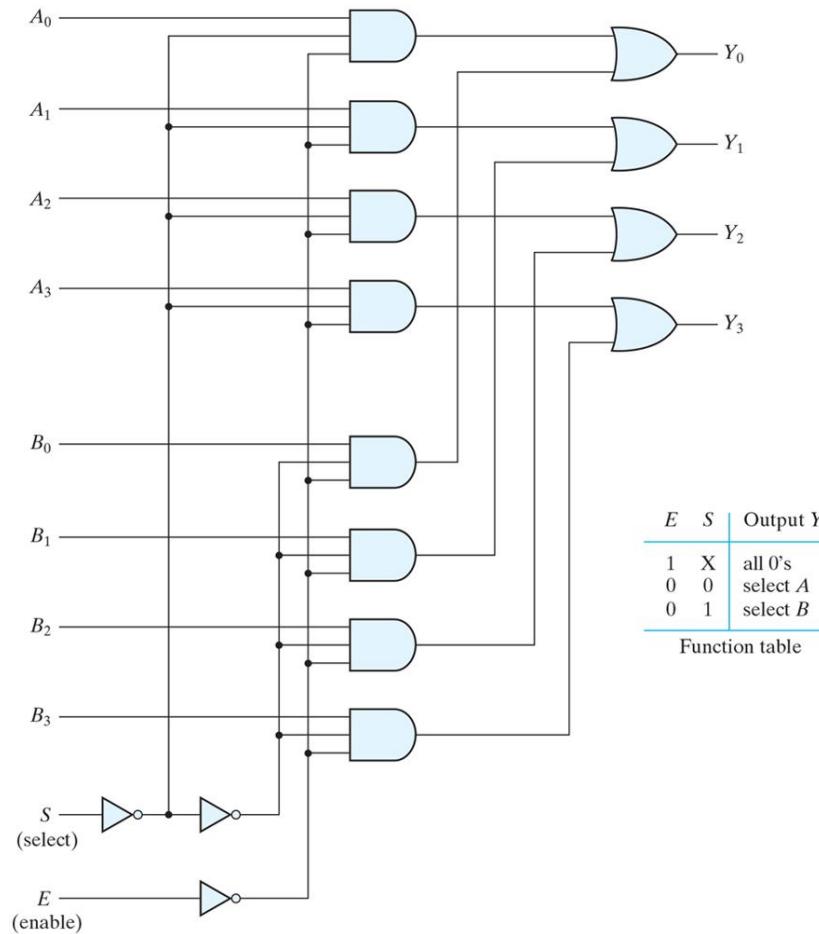
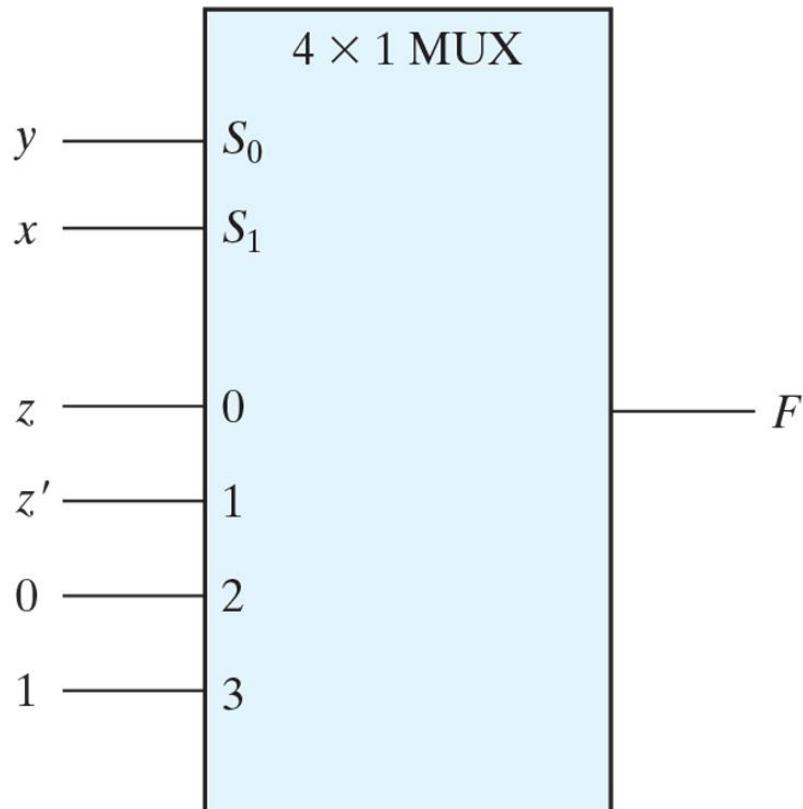


Figure 4.27
Implementing a Boolean function with a multiplexer.

x	y	z	F
0	0	0	0 $F = z$
0	0	1	1
0	1	0	1 $F = z'$
0	1	1	0
1	0	0	0 $F = 0$
1	0	1	0
1	1	0	1 $F = 1$
1	1	1	1

(a) Truth table



(b) Multiplexer implementation

Figure 4.28
Implementing a four-input function with a multiplexer.

A	B	C	D	F
0	0	0	0	0 $F = D$
0	0	0	1	1
0	0	1	0	0 $F = D$
0	0	1	1	1
0	1	0	0	1 $F = D'$
0	1	0	1	0
0	1	1	0	0 $F = 0$
0	1	1	1	0
1	0	0	0	0 $F = 0$
1	0	0	1	0
1	0	1	0	0 $F = D$
1	0	1	1	1
1	1	0	0	1 $F = 1$
1	1	0	1	1
1	1	1	0	1 $F = 1$
1	1	1	1	1

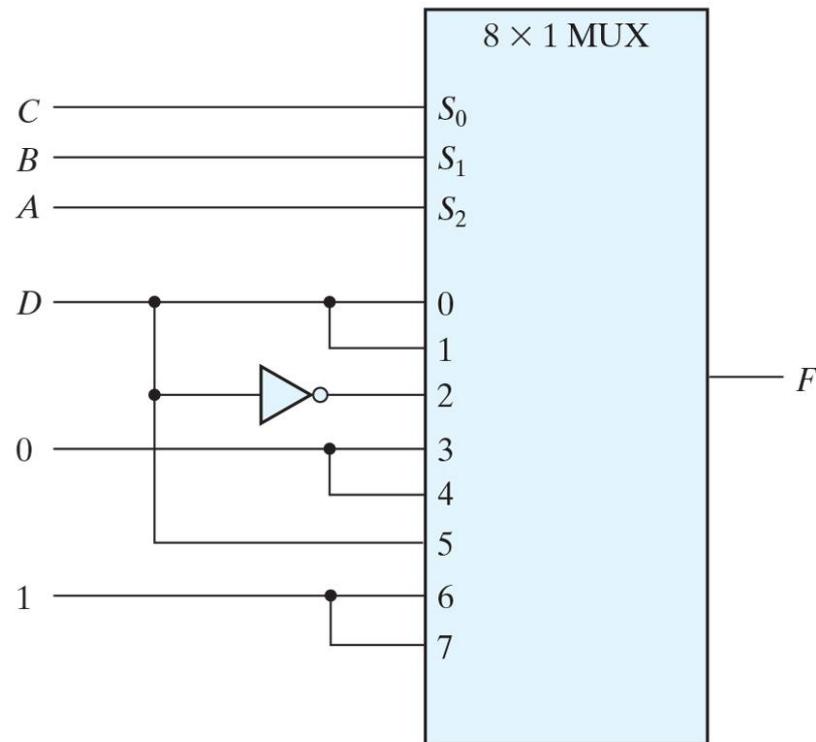


Figure PE4.9

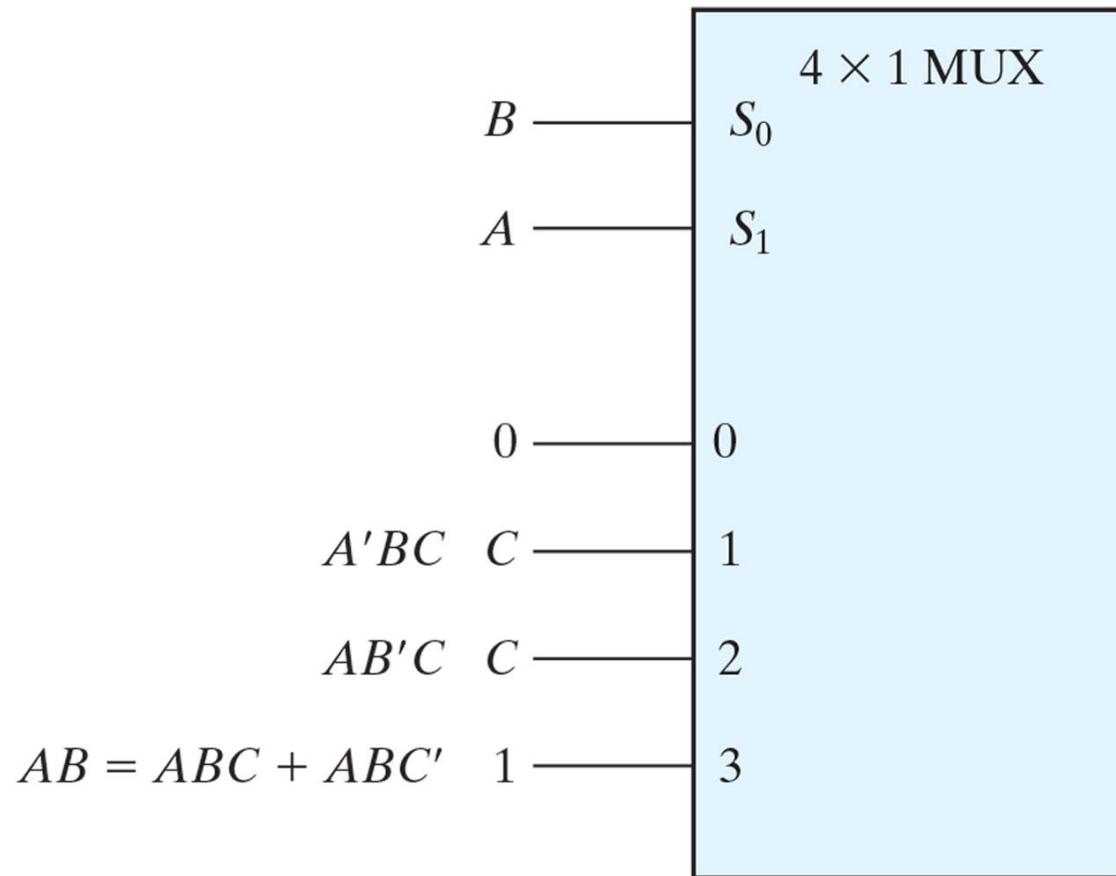


Figure 4.29
Graphic symbol for a three-state buffer.

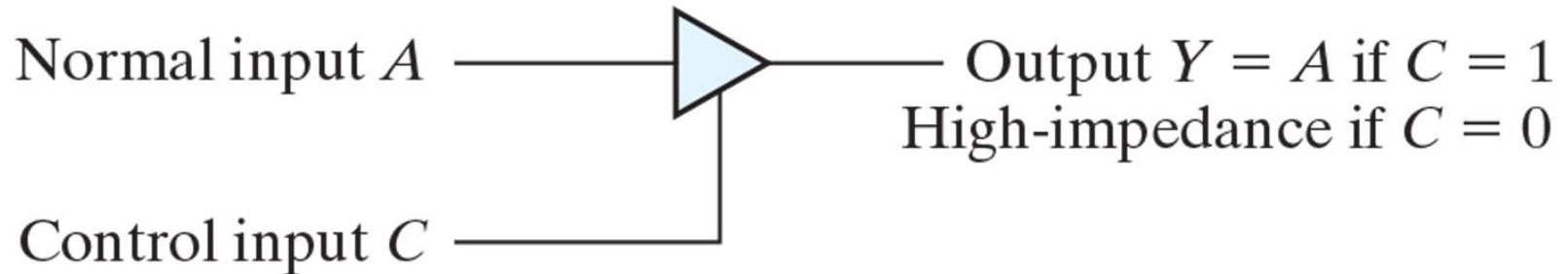
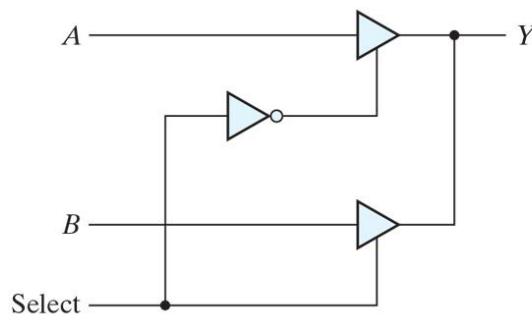
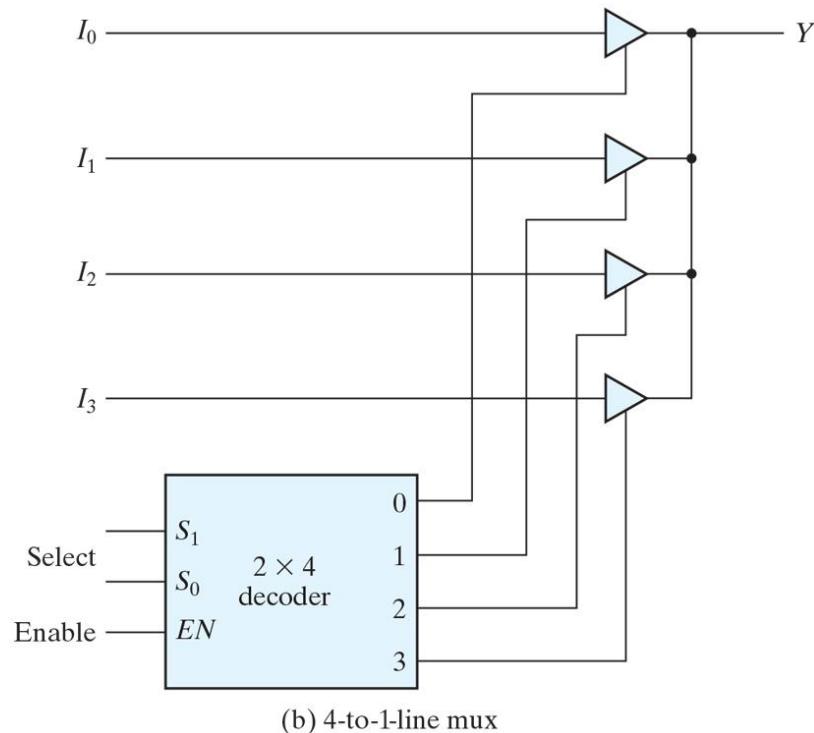


Figure 4.30
Multiplexers with three-state gates.



(a) 2-to-1-line mux



(b) 4-to-1-line mux

Figure 4.31
Relationship of Verilog constructs to truth tables, Boolean equations, and schematics.

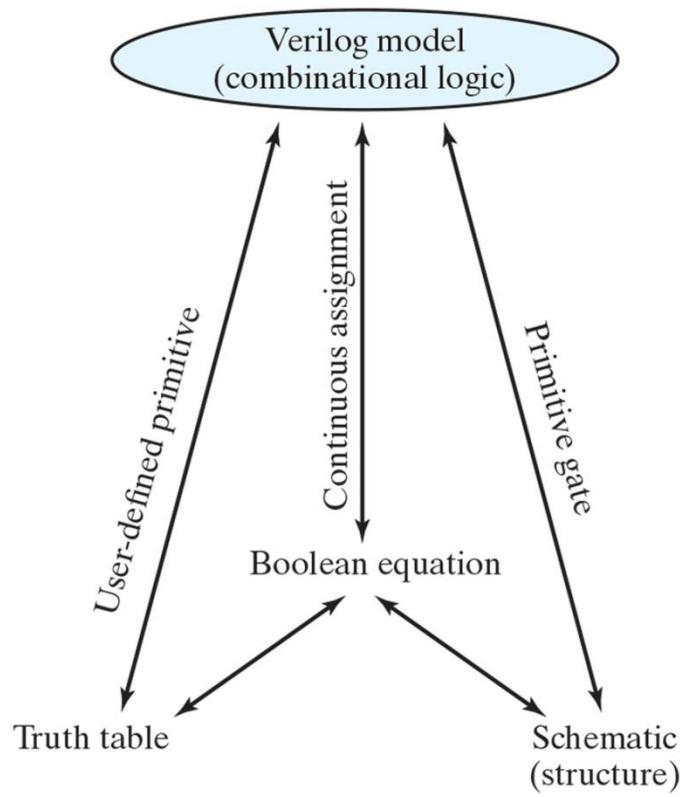


Figure 4.32
Relationship of VHDL constructs to truth tables, Boolean equations,
and schematics three-state gates.

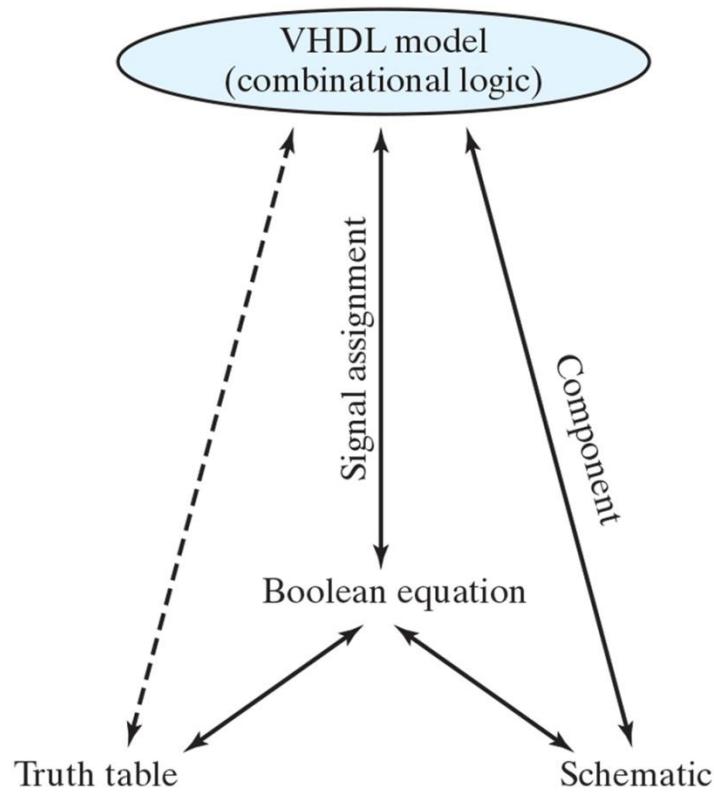


Table 4.9
Truth Table for Predefined Primitive Gates.

and	0	1	x	z	or	0	1	x	z
0	0	0	0	0	0	0	1	x	x
1	0	1	x	x	1	1	1	1	1
x	0	x	x	x	x	x	1	x	x
z	0	x	x	x	z	x	1	x	x
xor	0	1	x	z	not	input	output		
0	0	1	x	x		0	1		
1	1	0	x	x		1	0		
x	x	x	x	x		x	x		
z	x	x	x	x		z	x		

Figure 4.33
Design hierarchy of an 8-bit ripple-carry adder. For simplicity, some blocks are omitted where they replicate what is already shown.

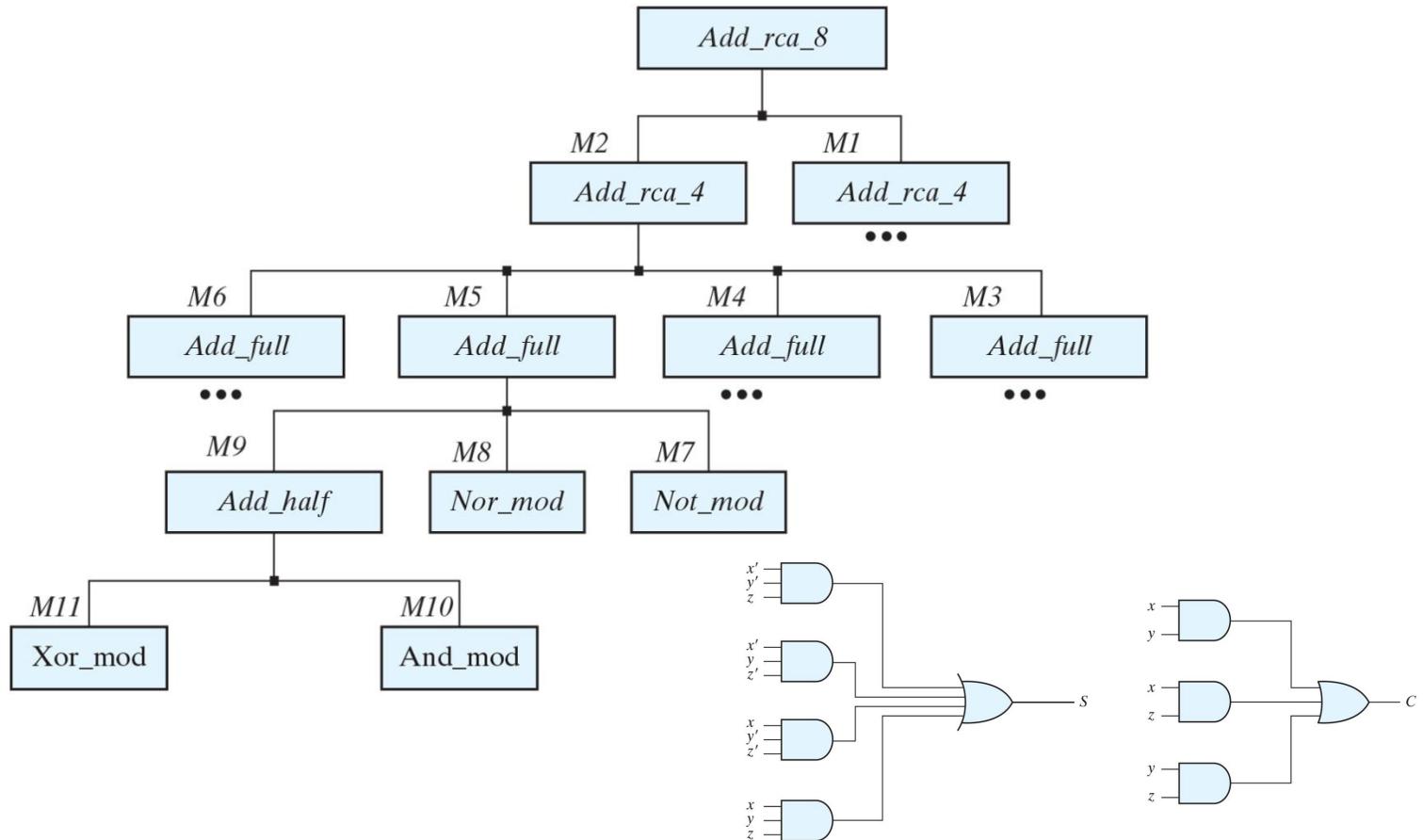


Figure 4.34

Decomposition of an 8-bit ripple carry adder into a chain of two 4-bit adders;⁹ each 4-bit adder consists of a chain of four full adders. The full adders are composed of half adders and one OR gate; the half adders are composed of logic gates.

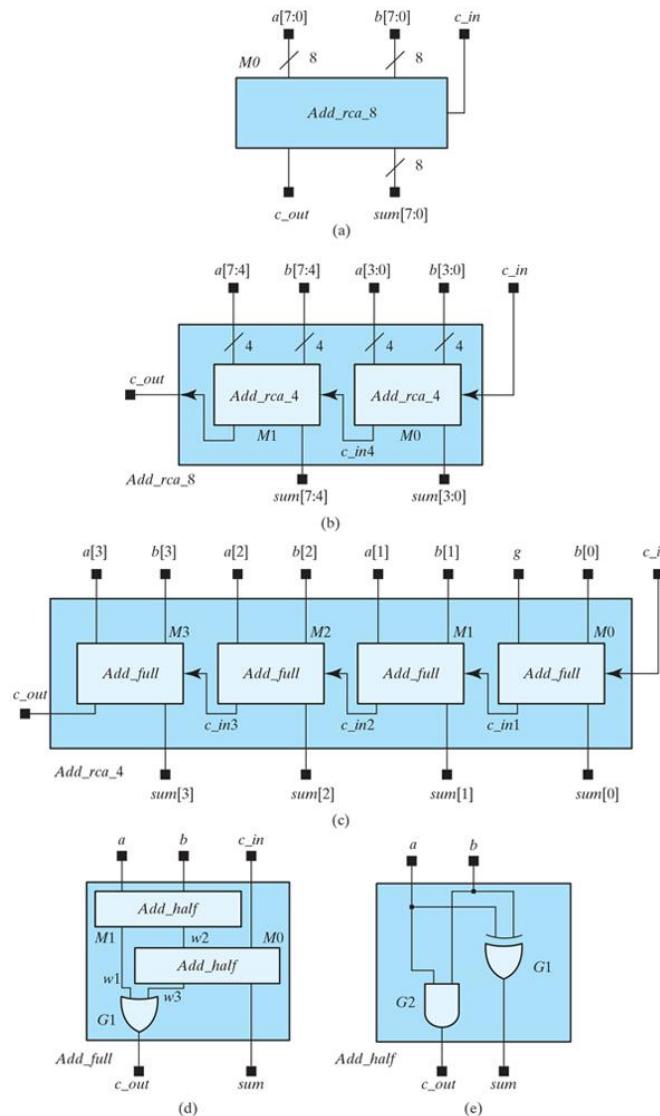


Figure 4.35
Three-state gates.

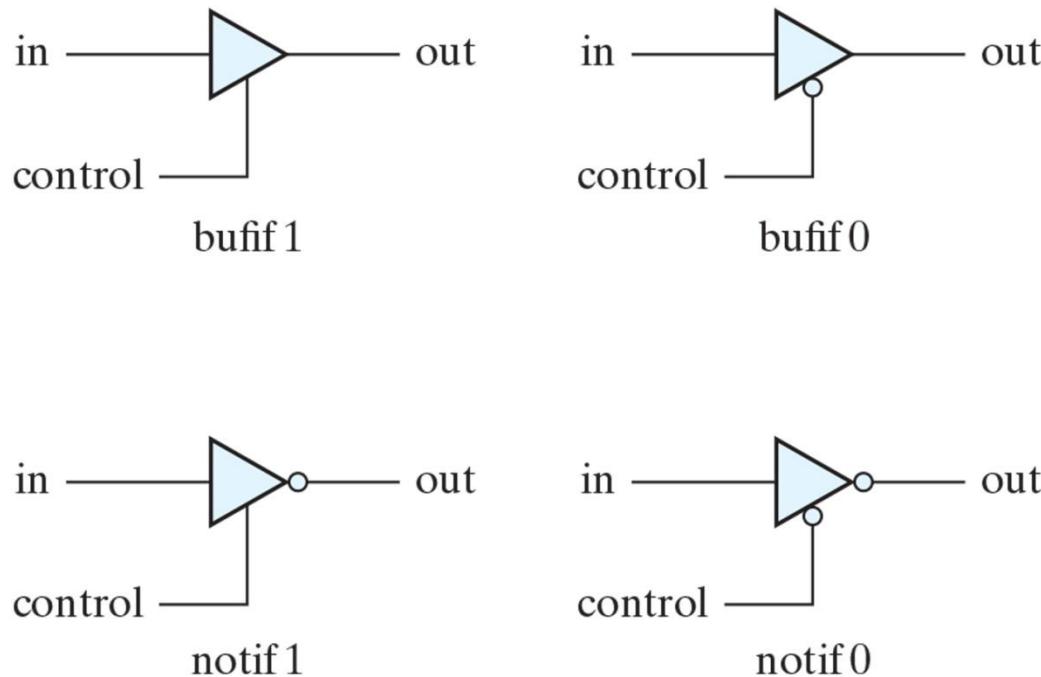


Figure 4.36
Two-to-one-line multiplexer with three-state buffers.

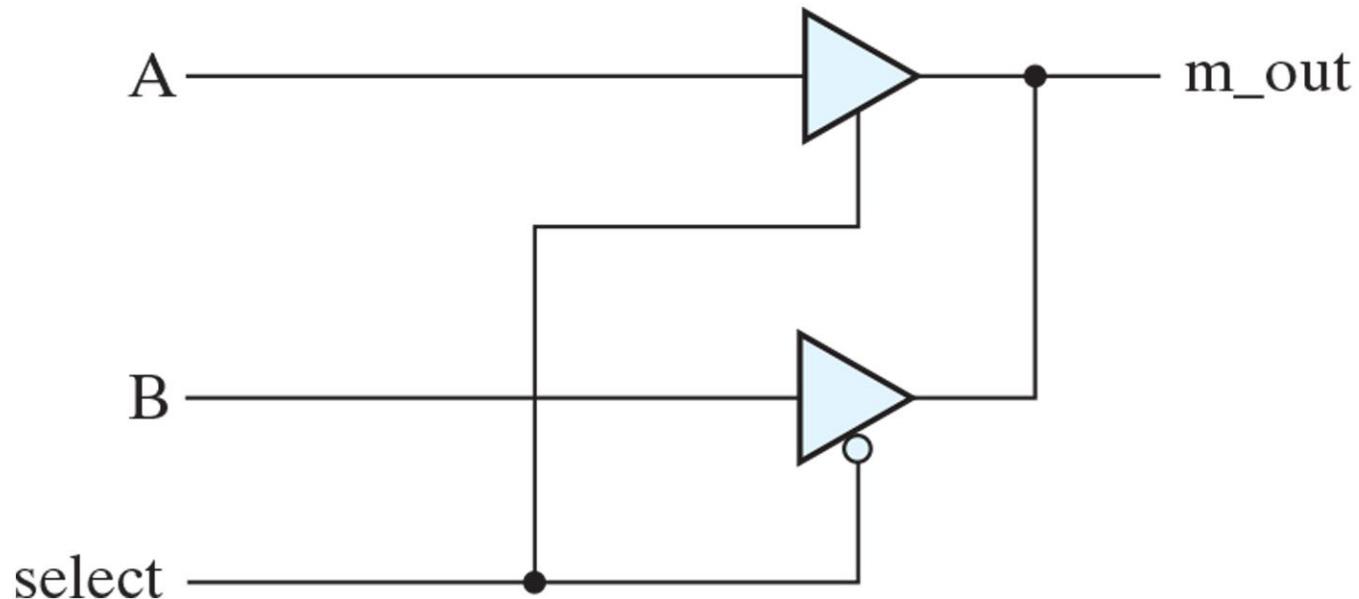


Table 4.10
Some Verilog Operators.

Symbol	Operation	Symbol	Operation
+	binary addition		
-	binary subtraction		
&	bitwise AND	&&	logical AND
	bitwise OR		logical OR
^	bitwise XOR		
~	bitwise NOT	!	logical NOT
==	equality		
>	greater than		
<	less than		
{ }	concatenation		
? :	conditional		

Table 4.11a SystemVerilog Assignment Operators¹⁵.

Operator	Description
<code>+=</code>	Add RHS to LHS and assign
<code>-=</code>	Subtract RHS from LHS and assign
<code>*=</code>	Multiply LHS by RHS and assign
<code>/=</code>	Divide LHS by RHS and assign
<code>%=</code>	Divide LHS by RHS and assign remainder
<code>&=</code>	Bitwise AND RHS with LHS and assign
<code> =</code>	Bitwise OR RHS with LHS and assign
<code>^=</code>	Bitwise exclusive OR RHS with LHS and assign
<code><<=</code>	Bitwise left-shift the LHS by the number of times indicated by the RHS and assign
<code>>>=</code>	Bitwise right-shift the LHS by the number of times indicated by the RHS and assign
<code><<<=</code>	Arithmetic-shift the LHS by the number of times indicated by the RHS and assign
<code>>>>=</code>	Arithmetic-shift the LHS by the number of times indicated by the RHS and assign

Table 4.11b
SystemVerilog Increment/Decrement Operators.

Usage	Operation	Description
$j = i++;$	<i>Postincrement</i>	j gets i , then i is incremented by 1
$j = i--;$	<i>Postdecrement</i>	j gets i , then i is decremented by 1
$j = ++i;$	<i>Preincrement</i>	i is incremented by 1, then j gets i
$j = --i;$	<i>Predecrement</i>	i is decremented by 1, then j gets i

Table 4.12

Predefined VHDL Data Types.

VHDL Data Type	Value
bit	'0' or '1'
boolean	FALSE or TRUE
integer	$-(2^{31} - 1) \leq \text{INTEGER VALUE} \leq (2^{31} - 1)$
positive	$1 \leq \text{INTEGER VALUE} \leq (2^{31} - 1)$
natural	$0 \leq \text{INTEGER VALUE} \leq (2^{31} - 1)$
real	$-1.0\text{e}38 \leq \text{FLOATNG POINT VALUE} \leq +1.0\text{E}38$
character	Alphabetical characters (a . . . z, A . . . Z), digits (0, . . . 9), special characters (e.g., %) each enclosed in single quotes
time	integer with units fs, ps, ns, us, ms, sec, min, or hr

Table 4.13
VHDL Operators.

Operator Type	Symbol	Operand(s)	Result	Precedence
Binary Logical	and or nand nor xor xnor	Bit, boolean, boolean_vector, bit_vector,	Same as operands	
Relational	= / = << = >> =	Two expression matched in type and size	FALSE, TRUE	
Shift Operators	sll srl sla sra rol ror	bit_vector	bit_vector	
Addition Operators	+ -	Integer Real number	Integer Real number	Precedence
Concatenation Operator	&	Vectors	Vectors	
Unary Sign Operator	+ -			
Multiplication Operators	./mod rem			
Miscellaneous Operators	not abs ..	Numerical Numerical Integer, Floating Point	Exponentiated by integer	Highest Precedence

Figure 4.37
Interaction between testbench and Verilog design unit.

```
module t_Design_Unit ();
// Declare stimulus signals
reg t_A, t_B, t_select;
//Declare reponse signals
wire : m_out;
--Instantiate unit under test
Design_Unit UUT
(.A(t_A), .B(t_B), .select(t_select), .m_out(t_m_out));
--Generate stimulus
initial begin
t_a <= '1';
t_B <= '0';
end;

initial begin
select = 1; forever # 10 select = ! select;
end;
end;
endmodule
```

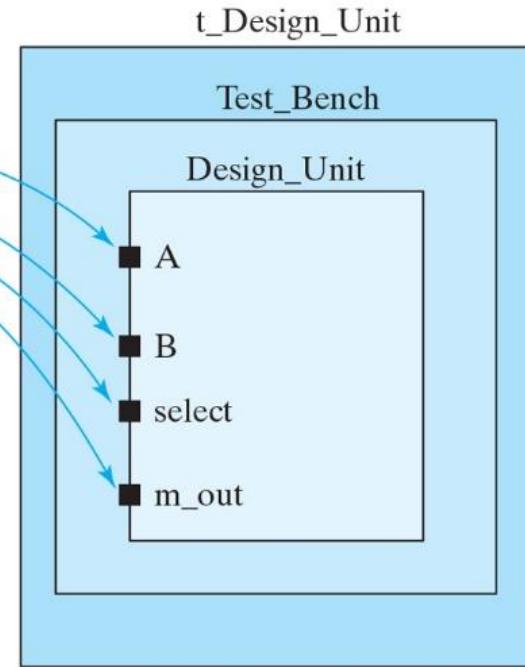


Figure 4.38
Interaction between testbench and VHDL design unit.

```
entity t_Design_Unit is
-- Empty
end;

architecture Test_Bench of t_Design_Unit is
--Declare component of design unit to be tested
component Some_Design_Unit
    port (A, B, select: in bit; m_out: out bit);
end Some_Design_Unit;

--Declare stimulus signals
signal t_A, t_B, t_select: bit;
--Declare response signals
signal t_m_out: bit;
begin
--Instantiate unit under test
UUT: Design_Unit port map (t_A, t_B, t_select, t_m_out);
--Declare process to generate stimulus
process begin
    t_a <='1';
    t_B <='0';
end process;

process begin
select <='1'; wait for 10 ns; select <='0';
end process;
end Test_Bench;
```

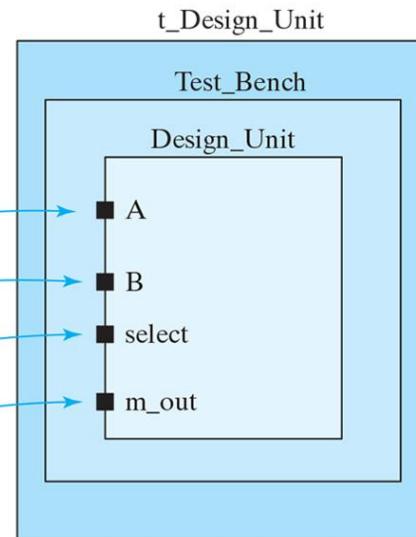


Figure 4.39
Circuit for event-driven simulation.

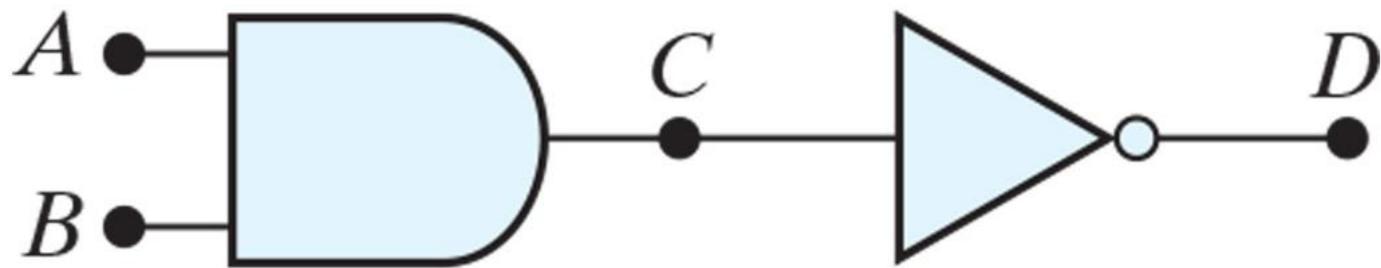


Figure 4.40
Representation of event-driven simulation (with zero delay).

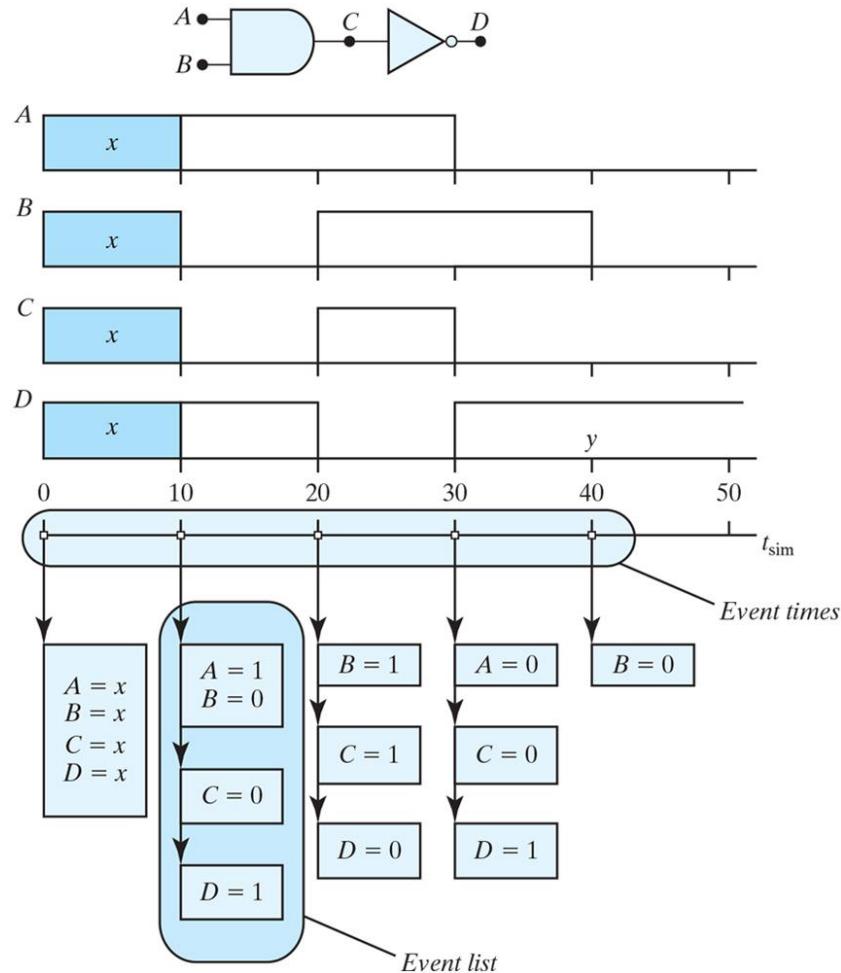


Figure 4.41
Representation of event-driven simulation (with propagation delay).

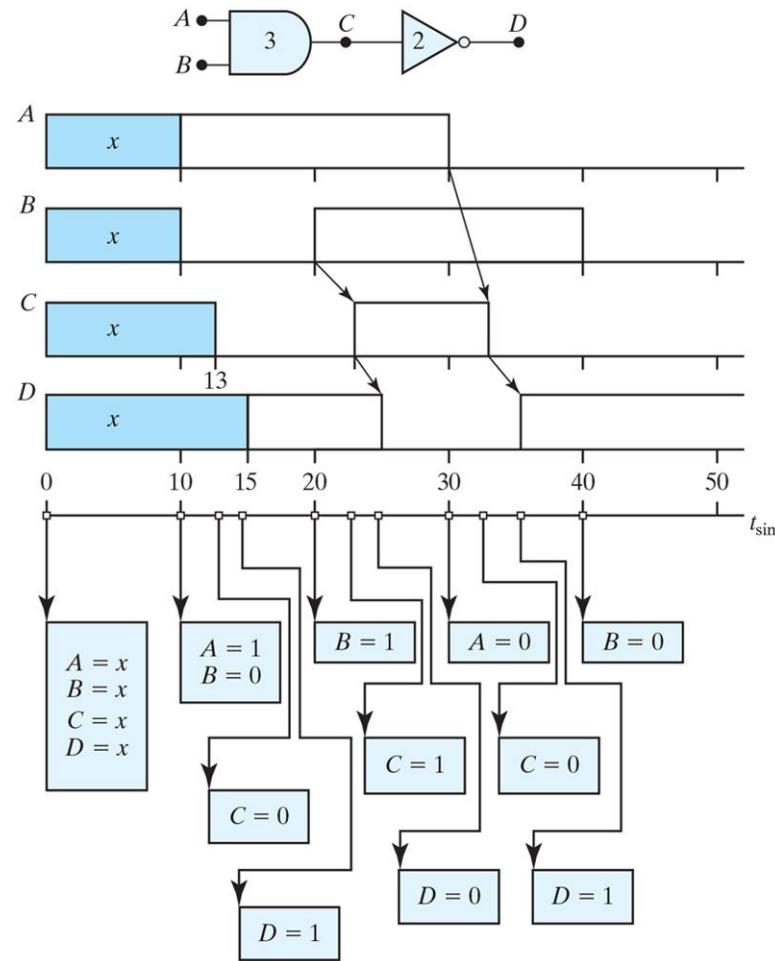


FIGURE P4.1

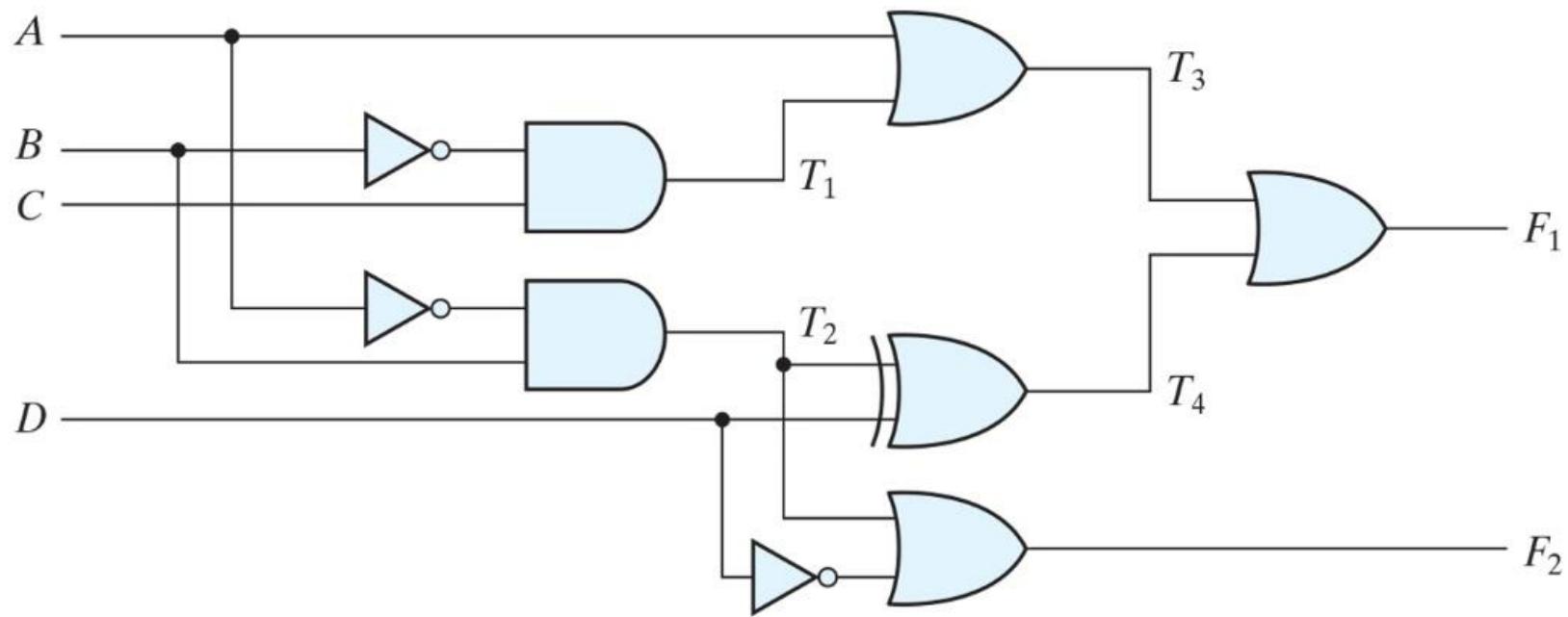


FIGURE P4.2

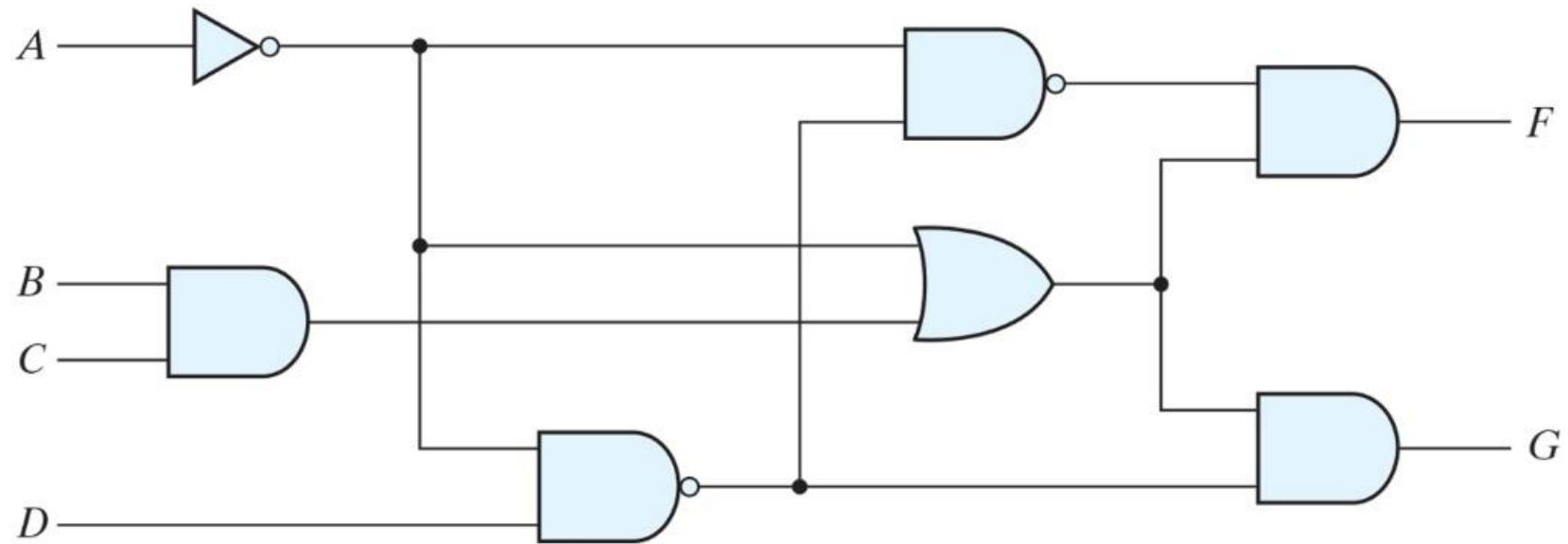
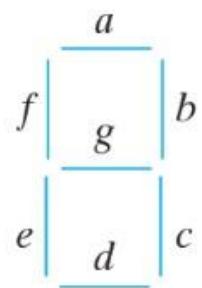


FIGURE P4.9



(a) Segment designation



(b) Numerical designation for display

FIGURE P4.16

