Homework 4

Sampling Distributions and Confidence Intervals

Requirement

- Send soft copies (digital files) to the following email address: 25b358009@stu.hit.edu.cn
- Write your name and student number.
- Submit within a week! Late submission should have a reasonable explanation sent to the TA or lecturer in advance.
- You may discuss with others, but write homework by yourself.

1. Basic concepts

1.1 What does the Central Limit Theorem tell us about the sampling distribution of the sample mean? How large should the sample size be to activate the Central Limit Theorem?

1.2 Describe the effect of increasing the sample size on the population of all possible sample proportions.

1.3 For proportions, how large should the sample size be for being considered large?

1.4 Explain why it is important to calculate a confidence interval.

1.5 Under what conditions is the confidence interval $\left[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$ for μ valid?

1.6 For a fixed sample size, what happens to a confidence interval for μ when we increase the level of confidence?

1.7 For a fixed level of confidence, what happens to a confidence interval for μ when we increase the sample size?

1.8 Explain how each of the following changes as the number of degrees of freedom describing a t curve increases:

a The standard deviation of the t curve. b The points t_{α} and $t_{\alpha/2}$.

Methods and Applications

2. When a pizza restaurant's delivery process is operating effectively, pizzas are delivered in an average of 45 minutes with a standard deviation of 6 minutes. To monitor its delivery process, the restaurant randomly selects five pizzas each night and records their delivery times.
   a For the sake of argument, assume that the population of all delivery times on a given evening is normally distributed with a mean of $\mu = 45$ minutes and a standard deviation of $\sigma = 6$ minutes. (That is, we assume that the delivery process is operating effectively.) Find the mean and the standard deviation of the population of all possible sample means, and calculate an interval containing 99.73 percent of all possible sample means.
   b Suppose that the mean of the five sampled delivery times on a particular evening is $\bar{x} = 55$ minutes. Using the interval that you calculated in *a*, what would you conclude about whether the restaurant's delivery process is operating effectively? Why?

Methods and Applications

3. **7.27** Suppose that we will randomly select a sample of $n = 100$ elements from a population and that we will compute the sample proportion $\hat{p}$ of these elements that fall into a category of interest. If the true population proportion p equals .9:

   a Describe the shape of the sampling distribution of $\hat{p}$. Why can we validly describe the shape?

   b Find the mean and the standard deviation of the sampling distribution of $\hat{p}$.

7.28 For the situation in Exercise 7.27, calculate the following probabilities. In each case sketch the sampling distribution and the probability.

a $P(\hat{p} \geq .96)$

b $P(.855 \leq \hat{p} \leq .945)$

c $P(\hat{p} \leq .915)$

Methods and Applications

4. **THE BANK CUSTOMER WAITING TIME CASE** DS WaitTime

Recall that a bank manager has developed a new system to reduce the time customers spend waiting to be served by tellers during peak business hours. The mean waiting time during peak business hours under the current system is roughly 9 to 10 minutes. The bank manager hopes that the new system will have a mean waiting time that is less than six minutes. The mean of the sample of 100 bank customer waiting times in Table 1.8 is $\bar{x} = 5.46$. If we let μ denote the mean of all possible bank customer waiting times using the new system and assume that σ equals 2.47:

a Calculate 95 percent and 99 percent confidence intervals for μ.

b Using the 95 percent confidence interval, can the bank manager be 95 percent confident that μ is less than six minutes? Explain.

c Using the 99 percent confidence interval, can the bank manager be 99 percent confident that μ is less than six minutes? Explain.

d Based on your answers to parts *b* and *c*, how convinced are you that the new mean waiting time is less than six minutes?

Methods and Applications

5. Suppose that for a sample of $n = 11$ measurements, we find that $\bar{x} = 72$ and $s = 5$. Assuming normality, compute confidence intervals for the population mean μ with the following levels of confidence:

a 95% b 99% c 80% d 90% e 98% f 99.8%

Methods and Applications

6. Consider a population having a standard deviation equal to 10. We wish to estimate the mean of this population.
   - a How large a random sample is needed to construct a 95 percent confidence interval for the mean of this population with a margin of error equal to 1?
   - b Suppose that we now take a random sample of the size we have determined in part *a*. If we obtain a sample mean equal to 295, calculate the 95 percent confidence interval for the population mean. What is the interval's margin of error?

Methods and Applications

7. In a news story distributed by the *Washington Post,* Lew Sichelman reports that a substantial fraction of mortgage loans that go into default within the first year of the mortgage were approved on the basis of falsified applications. For instance, loan applicants often exaggerate their income or fail to declare debts. Suppose that a random sample of 1,000 mortgage loans that were defaulted within the first year reveals that 410 of these loans were approved on the basis of falsified applications.

   a Find a point estimate of and a 95 percent confidence interval for p, the proportion of all first-year defaults that are approved on the basis of falsified applications.

   b Based on your interval, what is a reasonable estimate of the minimum percentage of all first-year defaults that are approved on the basis of falsified applications?