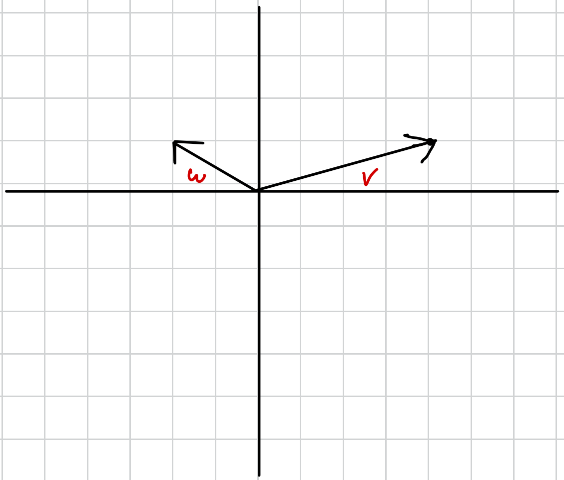
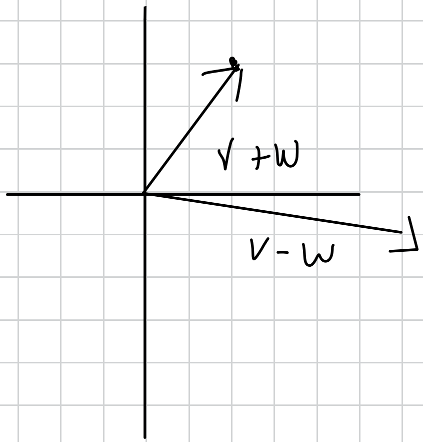
1.1

1. (a) line
(b) Plane
(c) R^3

2.





$$v + w = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$v - w = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

3. $v + w = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$v - w = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

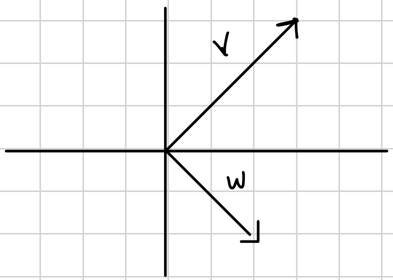
$v = w + \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

$w + w + \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$2w = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

$v + \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$v = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ $w = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$



4. $3\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 7 \\ 5 \end{bmatrix} = 3v + w$

$c\begin{bmatrix} 2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 2c + d \\ c + d \end{bmatrix}$

5. $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$ $w = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$

a. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + \begin{bmatrix} -6 \\ 2 \\ -4 \end{bmatrix} + \begin{bmatrix} 4 \\ -6 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$w = cu + dv$

$\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = c\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + d\begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$

$c = -1 \quad d = -1$

6. $v = (1, -2, 1)$
$w = (0, 1, -1)$

(a) $c\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} c \\ -2c \\ c \end{bmatrix} + \begin{bmatrix} 0 \\ d \\ -d \end{bmatrix}$

(b) $c\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$

(1) Sample exam solve:

1. $\begin{pmatrix} -1 & 4 & 3 & 0 \\ 2 & 3 & -7 & 8 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & -4 & -3 \\ 2 & 0 & 5 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 9 & -3 \\ -6 & 2 \\ 3 & -1 \end{pmatrix}$

$2 \times 4 \quad 4 \times 3$

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$\begin{bmatrix} 5 & -3 & 3 \\ 12 & -22 & -24 \end{bmatrix} \begin{bmatrix} 9 & -3 \\ -6 & 2 \\ 3 & -1 \end{bmatrix}$

$2 \times 3 \quad 3 \times 2$

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$-36 \; -44 \; +24$

$\begin{array}{r} 36 \\ 44 \\ \hline -80 \\ 24 \\ \hline 56 \end{array}$

$\boxed{\begin{bmatrix} 72 & -24 \\ 212 & -56 \end{bmatrix}}$

2 $f(A)$ $f(x) = x^3 + x^2 + x + 1$

$A = \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}^3 + \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}^2 + \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} + 1$

$= \begin{pmatrix} -2 & 6\sqrt{3} \\ -4\sqrt{3} & -8 \end{pmatrix} + \begin{pmatrix} 4 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} + \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} + 1$

$= \begin{pmatrix} 2 & 8\sqrt{3} \\ -6\sqrt{3} & -10 \end{pmatrix} + 1$

(2)

1 $\begin{pmatrix} 2 & -1 & 6 & 5 \\ 3 & 8 & -4 & -2 \\ 1 & -10 & 16 & 12 \end{pmatrix}$

row echelon

$)$

2. $\begin{pmatrix} 13 & -2 & 3 \\ 5 & -1 & 1 \\ 4 & -1 & 1 \end{pmatrix}$

$A^{-1} = \frac{1}{\det A} \text{adj}$ or Gaussian

$$= \left(\begin{array}{ccc|ccc} 13 & -2 & 3 & 1 & 0 & 0 \\ 5 & -1 & 1 & 0 & 1 & 0 \\ 4 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$\longrightarrow -5R_3 + 4R_2$

$$= \left(\begin{array}{ccc|ccc} 13 & -2 & 3 & 1 & 0 & 0 \\ 0 & 9 & -1 & 0 & 4 & -5 \\ 4 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$\longrightarrow -4R_1 + 13R_3$

$$= \left(\begin{array}{ccc|ccc} 13 & -2 & 3 & 1 & 0 & 0 \\ 0 & 9 & -1 & 0 & 4 & -5 \\ 0 & -5 & 1 & -4 & 0 & 13 \end{array}\right)$$

$\longrightarrow 5R_2 + 9R_3$

117
-25
92

$$= \left(\begin{array}{ccc|ccc} 13 & -2 & 3 & 1 & 0 & 0 \\ 0 & 9 & -1 & 0 & 4 & -5 \\ 0 & 0 & 4 & -36 & 20 & 92 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 13 & -2 & 3 & 1 & 0 & 0 \\ 0 & 9 & -1 & 0 & 4 & -5 \\ 0 & 0 & 4 & -36 & 20 & 92 \end{array}\right)$$

$\longrightarrow 2R_2 + 9R_1$

$$= \left(\begin{array}{ccc|ccc} 117 & 0 & 25 & 9 & 8 & -10 \\ 0 & 9 & -1 & 0 & 4 & -5 \\ 0 & 0 & 4 & -36 & 20 & 92 \end{array}\right)$$

$\longrightarrow 25R_2 + R_1$

$$= \left(\begin{array}{ccc|ccc} 117 & 225 & 0 & 9 & 108 & -110 \\ 0 & 9 & -1 & 0 & 4 & -5 \\ 0 & 0 & 4 & -36 & 20 & 92 \end{array}\right)$$

Sample exam 21 practice GPT:

1.
$$\underset{A}{\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 4 \end{bmatrix}} \underset{B}{\begin{bmatrix} 1 & 2 \\ -3 & 4 \\ 2 & 5 \end{bmatrix}} \underset{C}{\begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}}$$

$$\begin{bmatrix} 11 & 15 \\ 9 & 22 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 59 & 23 \\ 58 & 48 \end{bmatrix} = \underline{AB \cdot C}}$$

2.
$$\begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 4 \\ 3 & 0 & -2 \end{bmatrix}$$

3×2 $\quad 2 \times 3$

$$= \boxed{\begin{bmatrix} 11 & -1 & -2 \\ 8 & 2 & -16 \\ 15 & 0 & -10 \end{bmatrix}}$$

$a_{11} = 2+9 = 11$

$a_{12} = -1$

$a_{13} = 4-6 = -2$

$a_{21} = -4+12 = 8$

$a_{22} = 2$

$a_{23} = -16$

$a_{31} = 15$

$a_{32} = 0$

$a_{33} = -10$

3.
$$\begin{bmatrix} -1 & 2 & 0 \\ 3 & -4 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 5 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

3×3 $\quad 3 \times 2$

$$\begin{bmatrix} -4 & 13 \\ 14 & -29 \\ -6 & 10 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -26 & 35 \\ 58 & -73 \\ -20 & 24 \end{bmatrix}}$$

$a_{11} = -26$

$a_{12} = -4+39 = 35$

$a_{21} = 58$

$a_{22} = 14-87 = -73$

$a_{31} = -20$

$a_{32} = -6+30 = 24$

4.
$$\begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

2×3 $\quad 3 \times 2$

12
4
48

$$= \begin{bmatrix} 0 & -10 \\ 11 & 12 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 10 & -40 \\ 21 & 70 \end{bmatrix}}$$

$a_{11} = 2-2-0$

$a_{12} = -6-4$

$a_{21} = 3+8$

$a_{22} = -9+16+5$

$a_{11} = 10$

$a_{12} = -40$

$a_{21} = 33-12$

$a_{22} = 22+48$

5.
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}$$

2×3 $\quad 3 \times 2$

$$\begin{bmatrix} 6 & -3 \\ 2 & -8 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 30 & -15 \\ 24 & -26 \end{bmatrix}}$$

$a_{11} = -1+4+3$

$a_{12} = 3+0-6$

$a_{21} = 0-2+4$

$a_{22} = 0-8$

$a_{11} = 24+6$

$a_{12} = -6-9$

$a_{21} = 8+16$

$a_{22} = -2-24$

Sample Exam 2: Practice GPT:

1

$$A=\begin{bmatrix}2&1\\-1&2\end{bmatrix} \quad f(x)=x^3+2x^2+x+3$$

$$f(A)=A^3+2A^2+A+3I$$

$$A^2=\begin{bmatrix}2&1\\-1&2\end{bmatrix}\begin{bmatrix}2&1\\-1&2\end{bmatrix}=\begin{bmatrix}3&4\\-4&3\end{bmatrix}$$

$$A^3=\begin{bmatrix}3&4\\-4&3\end{bmatrix}\begin{bmatrix}2&1\\-1&2\end{bmatrix}$$

$$\begin{bmatrix}2&11\\-11&2\end{bmatrix}$$

$$=\begin{bmatrix}2&11\\-11&2\end{bmatrix}+2\begin{bmatrix}3&4\\-4&3\end{bmatrix}+\begin{bmatrix}2&1\\-1&2\end{bmatrix}+\begin{bmatrix}3&0\\0&3\end{bmatrix}$$

$$=\begin{bmatrix}4&12\\-12&4\end{bmatrix}+\begin{bmatrix}6&8\\-8&6\end{bmatrix}+\begin{bmatrix}3&0\\0&3\end{bmatrix}$$

$$=\begin{bmatrix}10&20\\-20&10\end{bmatrix}+\begin{bmatrix}3&0\\0&3\end{bmatrix}$$

$$\boxed{=\begin{bmatrix}13&20\\-20&13\end{bmatrix}}$$

2

$$B=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$$

$$f(x)=x^4-2x^2+x+5$$

$$f(B)=B^4-2B^2+B+5$$

$$B^2=\begin{bmatrix}0&1\\-1&0\end{bmatrix}\begin{bmatrix}0&1\\-1&0\end{bmatrix}$$

$$=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$$

$$B^4=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}-2&0\\0&-2\end{bmatrix}+\begin{bmatrix}0&1\\-1&0\end{bmatrix}+\begin{bmatrix}5&0\\0&5\end{bmatrix}$$

$$\begin{bmatrix}-1&0\\0&-1\end{bmatrix}+\begin{bmatrix}5&1\\-1&5\end{bmatrix}=\boxed{\begin{bmatrix}4&1\\-1&4\end{bmatrix}}$$

3

$$\begin{bmatrix}3&2\\0&1\end{bmatrix} \quad C^2-C+2I$$

$$C^2=\begin{bmatrix}3&2\\0&1\end{bmatrix}\begin{bmatrix}3&2\\0&1\end{bmatrix}$$

$a_{11}=9$

$a_{12}=6+2$

$a_{21}=0$

$a_{22}=1$

$$\begin{bmatrix}9&8\\0&1\end{bmatrix}$$

$$\begin{bmatrix}9&8\\0&1\end{bmatrix}-\begin{bmatrix}3&2\\0&1\end{bmatrix}+\begin{bmatrix}2&0\\0&2\end{bmatrix}$$

$$=\begin{bmatrix}6&6\\0&0\end{bmatrix}\begin{bmatrix}2&0\\0&2\end{bmatrix}=\boxed{\begin{bmatrix}8&6\\0&2\end{bmatrix}}$$

4

$$\begin{bmatrix}1&2\\-2&1\end{bmatrix}=D$$

$$D^3-D^2+2D+4I$$

$$D^2=\begin{bmatrix}1&2\\-2&1\end{bmatrix}\begin{bmatrix}1&2\\-2&1\end{bmatrix}$$

$$\begin{bmatrix}-3&4\\-4&-3\end{bmatrix}$$

$$D^3=\begin{bmatrix}-3&4\\-4&-3\end{bmatrix}\begin{bmatrix}1&2\\-2&1\end{bmatrix}$$

$$\begin{bmatrix}-11&-2\\2&-11\end{bmatrix}$$

$$\begin{bmatrix}-11&-2\\2&-11\end{bmatrix}-\begin{bmatrix}-3&4\\-4&-3\end{bmatrix}+\begin{bmatrix}2&4\\-4&2\end{bmatrix}+\begin{bmatrix}4&0\\0&4\end{bmatrix}$$

$$=\begin{bmatrix}14&-6\\6&-8\end{bmatrix}+\begin{bmatrix}6&4\\-4&6\end{bmatrix}$$

$$=\boxed{\begin{bmatrix}20&-2\\2&-2\end{bmatrix}}$$

Q:5

$$\begin{bmatrix}2&\sqrt{2}\\-\sqrt{2}&2\end{bmatrix}$$

$$q(E)=E^4+E^3+E^2+E+I$$

$$E^2=\begin{bmatrix}2&\sqrt{2}\\-\sqrt{2}&2\end{bmatrix}\begin{bmatrix}2&\sqrt{2}\\-\sqrt{2}&2\end{bmatrix}$$

$$\begin{bmatrix}2&4\sqrt{2}\\-4\sqrt{2}&2\end{bmatrix}$$

$$E^3=\begin{bmatrix}2&4\sqrt{2}\\-4\sqrt{2}&2\end{bmatrix}\begin{bmatrix}2&\sqrt{2}\\-\sqrt{2}&2\end{bmatrix}$$

$$=\begin{bmatrix}-4&10\sqrt{2}\\-10\sqrt{2}&-4\end{bmatrix}$$

$$E^4=\begin{bmatrix}2&4\sqrt{2}\\-4\sqrt{2}&2\end{bmatrix}\begin{bmatrix}2&4\sqrt{2}\\-4\sqrt{2}&2\end{bmatrix}$$

$$\begin{bmatrix}-28&16\sqrt{2}\\-16\sqrt{2}&-28\end{bmatrix}+\begin{bmatrix}-4&10\sqrt{2}\\-10\sqrt{2}&-4\end{bmatrix}+\begin{bmatrix}2&4\sqrt{2}\\-4\sqrt{2}&2\end{bmatrix}+\begin{bmatrix}2&\sqrt{2}\\-\sqrt{2}&2\end{bmatrix}$$

$$\begin{bmatrix}-32&26\sqrt{2}\\-26\sqrt{2}&-32\end{bmatrix}+\begin{bmatrix}4&5\sqrt{2}\\-5\sqrt{2}&4\end{bmatrix}$$

$$=\begin{bmatrix}-28&31\sqrt{2}\\-31\sqrt{2}&-28\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}=\boxed{\begin{bmatrix}-27&\cancel{-31\sqrt{2}}\\\cancel{-31\sqrt{2}}&-27\end{bmatrix}}$$

Sample exam 2 3 practice:

1)

$$A=\left[\begin{array}{ccc|c} 3 & 6 & -9 & 12 \\ 6 & 12 & -18 & 24 \\ 9 & 18 & -27 & 36 \end{array}\right]$$

$R_1 \to \frac{R_1}{3}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 6 & 12 & -18 & 24 \\ 9 & 18 & -27 & 36 \end{array}\right]$$

$R_2 \to R_2 - 6R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 9 & 18 & -27 & 36 \end{array}\right]$$

$R_3 \to R_3 - 9R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

2)

$$\left[\begin{array}{ccc|c} 4 & -2 & 1 & 5 \\ 2 & -1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$R_1 \to \frac{R_1}{4}$ 1st Pivot

$$\left[\begin{array}{ccc|c} \boxed{1} & -\frac{1}{2} & \frac{1}{4} & \frac{5}{4} \\ 2 & -1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$R_2 \to -2R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{4} & \frac{5}{4} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$R_3 \to -R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{4} & \frac{5}{4} \\ 0 & 0 & \boxed{-\frac{1}{2}} & \frac{1}{2} \\ 0 & \frac{3}{2} & \frac{3}{4} & -\frac{1}{4} \end{array}\right]$$

second Pivot

$R_2 \to -\frac{1}{2}R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & \frac{1}{4} & \frac{5}{4} \\ 0 & 0 & 1 & -1 \\ 0 & \frac{3}{2} & \frac{3}{4} & -\frac{1}{4} \end{array}\right]$$

eliminate the 3rd column below the pivot

$R_3 \to -\frac{3}{4}R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & \frac{1}{4} & \frac{5}{4} \\ 0 & 0 & 1 & -1 \\ 0 & \boxed{\frac{3}{2}} & 0 & \frac{1}{2} \end{array}\right]$$

3rd Pivot

$R_3 \to \frac{3}{2}R_3$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{4} & \frac{5}{4} \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{3} \end{array}\right]$$

1.

$$\begin{bmatrix} \boxed{2} & 4 & -2 \\ 1 & 2 & 0 \\ 3 & 6 & -6 \end{bmatrix}$$

1st Pivot:

$R_1 \to \frac{R_1}{2}$

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 0 \\ 3 & 6 & -6 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & -1 \\ 3 & 6 & -6 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & \boxed{1} \\ 0 & 0 & -3 \end{bmatrix}$$

$R_3 \to R_3 + 3R_2$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2

$$\begin{bmatrix} \boxed{1} & -1 & 2 & 4 \\ 2 & -2 & 4 & 8 \\ 1 & 1 & -1 & -3 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & \boxed{2} & -3 & -7 \end{bmatrix}$$

$R_3 \to \frac{R_3}{2}$

$$\begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{3}{2} & -\frac{7}{2} \end{bmatrix}$$

5.

$$\left[\begin{array}{ccc|c} \boxed{1} & 3 & 2 & -4 \\ 2 & 6 & 4 & -8 \\ 1 & 2 & 1 & -1 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 3 \end{array}\right]$$

$R_3 \to -R_3$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -3 \end{array}\right]$$

$R = 2$

6. $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 1 \\ 3 & 2 & -3 & 4 \end{bmatrix}$

$R_1 \to \frac{R_1}{2}$

$$\left[\begin{array}{ccc|c} \boxed{1} & \frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ 1 & -1 & 2 & 1 \\ 3 & 2 & -3 & 4 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & \boxed{-\frac{3}{2}} & \frac{5}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} \end{array}\right]$$

$R_2 \to -\frac{3}{2} \div R_2$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{5}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & -\frac{3}{2} & +\frac{1}{2} \end{array}\right]$$

$R_3 \to R_3 - \frac{1}{2} R_2$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -1 & \frac{3}{4} \\ 0 & 0 & & \end{array}\right]$$

$-\frac{3}{2} - \frac{1}{2}\left(-\frac{5}{3}\right)$

$-\frac{3}{2} - \frac{1}{2}\left(-\frac{5}{3}\right) \quad \frac{5}{6} - \frac{3}{2}$

$\frac{10-18}{12} \quad -\cancel{\frac{8}{12}} - \frac{2}{3}$

Model paper Question:

$$\begin{pmatrix} \boxed{2} & -1 & 6 & 5 \\ 3 & 8 & -4 & -2 \\ 1 & 10 & 16 & 12 \end{pmatrix}$$

$R_1 \to \frac{R_1}{2}$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 3 & \frac{5}{2} \\ 3 & 8 & -4 & -2 \\ 1 & 10 & 16 & 12 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 3 & \frac{5}{2} \\ 0 & \frac{19}{2} & -13 & -\frac{19}{2} \\ 0 & \frac{21}{2} & 13 & \frac{19}{2} \end{array}\right]$$

طریقہ سمجھ آگیا -

(1) $Ax = \begin{bmatrix}1&2&4\\-2&3&1\\-4&1&2\end{bmatrix}\begin{bmatrix}2\\2\\3\end{bmatrix}$

(a)
$$\begin{matrix}1\cdot2+2\cdot2+4\cdot3\\-2\cdot2+3\cdot2+1\cdot3\\-4\cdot2+1\cdot2+2\cdot3\end{matrix}=\begin{bmatrix}18\\5\\0\end{bmatrix}$$

(b) $2\begin{bmatrix}1\\-2\\-4\end{bmatrix}+2\begin{bmatrix}2\\3\\1\end{bmatrix}+3\begin{bmatrix}4\\1\\2\end{bmatrix}$

$$\begin{bmatrix}2\\-4\\-8\end{bmatrix}+\begin{bmatrix}4\\6\\2\end{bmatrix}+\begin{bmatrix}12\\3\\1\end{bmatrix}=\begin{bmatrix}18\\5\\0\end{bmatrix}$$

(2)
$$\begin{matrix}3+2+4\\6+0+1\end{matrix}=\begin{bmatrix}10\\7\end{bmatrix}$$

Practice Q:

1: $\begin{bmatrix}3&1&-2\\1&4&0\\-1&2&5\end{bmatrix}\begin{bmatrix}2\\-1\\3\end{bmatrix}$

$3\cdot2-1-6=-1$

$2-4=-2$

$-2-2+15=-11$

2. $2\begin{bmatrix}3\\1\\-1\end{bmatrix}-1\begin{bmatrix}1\\4\\2\end{bmatrix}+3\begin{bmatrix}-2\\0\\5\end{bmatrix}$

$$\begin{bmatrix}6\\2\\-2\end{bmatrix}\begin{bmatrix}-1\\-4\\2\end{bmatrix}\begin{bmatrix}-6\\0\\15\end{bmatrix}=\begin{matrix}-1\\-2\\-11\end{matrix}$$

2.1

2.2 Ch: 2 assignments :

(1)
$2x-4y=6$
$-x+5y=0$

(a) $eq_2-\frac{1}{2}(eq_1)$

$-\frac{1}{2}(2x-4y)=6(-\frac{1}{2})$

$-x+2y=-3$

(b) $-x+5y-(-x+2y)=0-(-3)$

$5y-2y=3$

$\boxed{y=1}$ $\boxed{x=5}$

(c) $3y=-3$

$\boxed{\begin{matrix}y=-1\\x=-5\end{matrix}}$ $2x+4=-6$, $2x=-10$

G.P.T practice

(i) $3x-2y=5$
$-2x+4y=-10$

(a) $eq_2+2(eq_1)$

$6x-4y=10$

(b) $-2x+4y+6x-4y=-10+10$

$x=0$
$y=-\frac{5}{2}$

(C) $6x-4y=-30$

$-2x+4y+6x-4y=-30-10$

$4x=-40$

$\boxed{\begin{matrix}x=-10\\y=-\frac{15}{2}\end{matrix}}$

$6x-y=11$
$3x+4y=2$

(a) $eq_1-2(eq_2)$

$2(3x+4y=2)$

$6x+8y=4$ eq_2

$(6x-y)-(6x+8y)=11-4$

$-9y=7$

$y=-\frac{7}{9}$

$3x+4(-\frac{7}{9})=2$

$3x-\frac{28}{9}=2$

$3x=2+\frac{28}{9}$

$3x=\frac{18+28}{9}$

$3x=\frac{46}{9}$

$x=\frac{46}{27}$

(c) $6x-y=12$
$3x+4y=-4$

$6x+8y=-8$

$(6x-y)-(6x+8y)=12+8$

$-9y=20$

$y=-\frac{20}{9}$

$x=\frac{44}{27}$

2.
$2x-3y=3$
$4x-5y+z=7$
$2x-y-3z=5$

Sol:

$2x=3+3y$

$x=\frac{3(1+y)}{2}$

$4\left[\frac{3(1+y)}{2}\right]-5y+z=7$

$4\left(\frac{3+3y}{2}\right)-5y+z=7$

$6+6y-5y+z=7$

$6+y+z=7$

$z=7-6-y$

$z=1-y$

$2\left(\frac{3+3y}{2}\right)-y-3(1-y)=5$

$3+3y-y-3+3y=5$

$5y=5$

$y=1$

$x=\frac{3+3}{2}$

$=3$

$z=0$

Check:

$6-3=3$

$12-5=7$ ✓

$6-1=5$

$y=0$
$z=0$
$x=\frac{3}{2}$
$t=\frac{5}{2}$

3
$2x+y+0+0=3$
$x+2y+z+0=0$
$y+2z+t=0$
$z+2t=5$

$$\left[\begin{array}{cccc|c}\boxed{2}&1&0&0&3\\1&2&1&0&0\\0&1&2&1&0\\0&0&1&2&5\end{array}\right]$$

$\frac{R_1}{2}$

$$\left[\begin{array}{cccc|c}1&\frac12&0&0&\frac32\\1&2&1&0&0\\0&1&2&1&0\\0&0&1&2&5\end{array}\right]$$

$R_2\to R_2-R_1$

$$\left[\begin{array}{cccc|c}1&\frac12&0&0&\frac32\\0&\frac32&1&0&-\frac32\\0&1&2&1&0\\0&0&1&2&5\end{array}\right]$$

$R_2\to\frac{2}{3}R_2$

$$\left[\begin{array}{cccc|c}1&\frac12&0&0&\frac32\\0&1&\frac23&0&-1\\0&1&2&1&0\\0&0&1&2&5\end{array}\right]$$

$R_3\to R_3-R_2$

$$\left[\begin{array}{cccc|c}1&\frac12&0&0&\frac32\\0&1&\frac23&0&-1\\0&0&\frac43&1&1\\0&0&1&2&5\end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & 0 & -1 \\ 0 & 0 & \frac{4}{3} & 1 & 1 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$R_3 \to \frac{3}{4} R_3$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & 0 & -1 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{3}{4} \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$R_4 \to R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & 0 & -1 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{3}{4} \\ 0 & 0 & 0 & \frac{5}{4} & \frac{17}{4} \end{array}\right]$$

$R_4 \to \frac{4}{5} R_4$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & 0 & -1 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 & \frac{17}{5} \end{array}\right]$$

$R_3 \to R_3 - \frac{3}{4} R_4$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & 0 & -1 \\ 0 & 0 & 1 & \frac{3}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 & \frac{17}{5} \end{array}\right]$$

$t = \frac{17}{5}$, and so on...

G.P.T practice:

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 4 \\ 2 & -1 & 2 & 3 \\ 1 & 1 & 1 & 6 \end{array}\right]$$

$R_1 \to \frac{R_1}{3}$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{1}{3} & \frac{4}{3} \\ 2 & -1 & 2 & 3 \\ 1 & 1 & 1 & 6 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{1}{3} & \frac{4}{3} \\ 0 & -\frac{5}{3} & \frac{8}{3} & \frac{1}{3} \\ 1 & 1 & 1 & 6 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{1}{3} & \frac{4}{3} \\ 0 & -\frac{1}{3} & \frac{8}{3} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{4}{3} & \frac{14}{3} \end{array}\right]$$

$R_2 \to -3R_2$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & -8 & -3 \\ 0 & \frac{2}{3} & \frac{4}{3} & \frac{14}{3} \end{array}\right]$$

$R_3 \to R_3 - \frac{2}{3} R_2$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & -8 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$x = \frac{7}{3}$

$y = -2$

$z = 1$

2.3

(1) (a) $\begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 + 5R_1$

$-5 + 5 = 0$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\downarrow R_1 \Leftrightarrow R_2$

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\downarrow R_2 \Leftrightarrow R_3$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

G.P.T Question

1

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & +7 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\downarrow R_1 \Leftrightarrow R_3$

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} -3 & 2 & 3 \\ -6 & 5 & 6 \\ -9 & 8 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -7 & 3 \\ 4 & -13 & 6 \\ 7 & -19 & 9 \end{bmatrix}$

4

(a) $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

(b) -3 and 2 are together in one matrix

5. a_{21} $\quad -3 \times 1 + 1 \times 0 + 1 \times 0$

2

$$E = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$$

$$EF = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

$$FE = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

3

(a) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 17 \end{bmatrix}$

$2x_1 + 3x_2 = 1$
$4x_1 + x_2 = 17$

(a) $R_2 \rightarrow R_2 - 2(R_1)$
$-4x_1 - 6x_2 = -2$
$\cancel{4}x_1 + x_2 = 17$

(b) $-5x_2 = 15$
$x_2 = -3$

(c) $x_2 = -3$
$x_1 = 2x_1 + 3(-3) = 1$

$2x_1 - 9 = 1$

$2x_1 = 10$
$\boxed{x_1 = 5}$

4 $AB = I \quad BC = I$

$A = C$

$A\cancel{B} = \cancel{B}C$

$(AB)C = A(BC)$
$\cancel{I}C = A\cancel{I}$
$C = A$

$(AB)C = A(BC)$
associative law

GPT Qs

1 $AB = I \quad BA = I$

$AB = BA$
$AA^{-1} = BB^{-1}$

2. $\cancel{A}B = \cancel{A}C \Rightarrow AB - AC = 0 \quad \cancel{A}(B-C) = 0 \quad B = C$

4. $A(B+C) = AB + AC$

6. $A^{-1} = \frac{1}{\det A} \operatorname{adj} A$

7.

2.6

2.1 Repeat (G.P.T)

1. $\begin{bmatrix} 3 & 1 & 0 \\ 0 & -2 & 4 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

(a) $\begin{bmatrix} 3\cdot 1 + 2\cdot 1 \\ -4-4 \\ 1\cdot 1 + 3\cdot 2 + 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 8 \end{bmatrix}$

(b) $1\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} - 1\begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 8 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 4 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

(a) $\begin{bmatrix} 7 \\ 2 \\ 8 \end{bmatrix}$

(b) $3\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} - 1\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 8 \end{bmatrix}$

Q: 2.1

$\underset{2\times 3}{\begin{bmatrix} 2 & 1 & -3 \\ 1 & 4 & 0 \end{bmatrix}} \quad x = \underset{3\times 1}{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

2. 2.2

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$

2.2

1.1

$3x - 2y = 7$

$-x + 4y = 1$

(a) -2 eq 1. or $-\frac{1}{3}$ eq 1

(b) $3x - 2y = 7$

$-3x + 12y = 12$

$0 + 10y = 19$

$y = \frac{19}{10}$

$3x - \frac{38}{10} = 7$

$3x = 7 + \frac{38}{10} \quad \frac{19}{5}$

$= \frac{54}{5}$

(c) $3x - 2y = -5$

$-3x + 12y = 8$

$10y = 3$

$y = \frac{3}{10}$

1.2 $x + 3y = 5$

$4x - y = 2$

$-4x - 12y = -20$

$4x - y = 2$

$0 - 13y = -18$

$y = \frac{18}{13}$

2.3

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 17 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & 1 & 17 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

(a) $$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & -5 & 15 \end{array}\right]$$

$2x_1 + 3x_2 = 1$

(b) $-5x_2 = 15$

$\boxed{x_2 = -3}$

(c)

$2x_1 + 3(-3) = 1$

$2x_1 - 9 = 1$

$\boxed{x_1 = 5}$

2.4

2. $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix}$ $B^2 = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$

$2AB = 2\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 0 \end{bmatrix}$

$(A+B)^2 = A^2 + AB + BA + B^2$

$= \begin{bmatrix} 2 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 4+2 \\ 6 & 6 \end{bmatrix}$

$\begin{bmatrix} 10 & 6 \\ 6 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 10 & 6 \\ 6 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 10 & 6 \\ 6 & 6 \end{bmatrix} \neq \begin{bmatrix} 16 & 4 \\ 3 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$

(a) $R_2 \to R_2 + R_1$

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}}$$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

2.5

1.

(a) Since $r_1 + r_2 = r_3$

$1 + 0 \neq 0$

(b) $b_1 + b_2 = b_3$

like $1 + (-1) = 0$

$(1, -1, 0)$
$b_1\ b_2\ b_3$

(c) eq 3 becomes 0 0 0

A is not invertible because R_3 is dependent which means the determinant is zero.

2. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \begin{bmatrix} ad-bc & 0 \\ 0 & -bc+ad \end{bmatrix}$

$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\quad \frac{1}{ad+bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$ $\quad \begin{bmatrix} \frac{a}{ad+bc} & \frac{b}{ad+bc} \\ \frac{c}{ad+bc} & \frac{d}{ad+bc} \end{bmatrix}$

3.

4.
(a) $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$

$R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$

$R_2 \to R_2 - R_3$
$R_1 \to R_1 - R_3$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 3 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$

$R_1 \to R_1 - R_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$

5

(b) $0 = 2\begin{bmatrix} c & c \\ 7 & c \end{bmatrix} - c\begin{vmatrix} c & c \\ 8 & c \end{vmatrix} + c\begin{vmatrix} c & c \\ 8 & 7 \end{vmatrix}$

$= 2|c^2 - 7c| - c|c^2 - 8c| + c|7c - 8c|$

$= 2c^2 - 14c - c^3 + 8c^2 - c^2$

$0 = 9c^2 - c^3 - 14c$

$= -c^3 + 9c^2 - 14c$

$= -c(c^2 - 9c - 14)$

$\boxed{C = 0 \quad C = 7 \quad c = 2}$

1 -7
1 -2

6. Special Matrix Problem (5 points)

(a) If B is the inverse of A^2, prove that AB is the inverse of A.

(b) For $C = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$, find the values of c for which C is not invertible.

2.6

1 $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$

(a) $R_3 \to R_3 - 3R_1$ $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix} = U$

(b) $E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}$

(c) $E^{-1} = L$

$E^{-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -3 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$R_3 \to R_3 + 3R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 \end{array}\right]$

$0 + 3$

$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

$A = LU$

$\begin{array}{ccc} & L & U \end{array}$

$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}$

~~elementary~~ row ops — E ka same sign

L ka opposite sign

: upper Triangular

$$Ax = b$$

$$Ly = b$$

$$Ux = y$$

3. $Lc = b$

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$$\boxed{\begin{aligned} c_1 &= 4 \\ c_2 &= 1 \\ c_3 &= 1 \end{aligned}}$$

(b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$

$$\boxed{\begin{aligned} x_3 &= 1 \\ x_2 &= 0 \\ x_1 &= 3 \end{aligned}}$$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

2. $\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 9 & 5 & 8 \end{bmatrix}$

(a) $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

U

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix}$$

E

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \quad L} \quad (a)$$

$$A = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}$$

$$= \boxed{\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 9 & 5 & 8 \end{bmatrix}} \quad (b)$$

1.2

(i) $v_1 = (1,3)$ $\quad v_2 = (2,1,2)$

$\vec{v_1} = \sqrt{1^2+3^2} = \sqrt{10}$

Unit vector $= \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} = u_1$

$\vec{v_2} = \sqrt{4+1+4}$
$= 3$

$u_2 = \frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

$u_1 = \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}$ $\quad u_2 = \frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

(ii) $U_1 = -\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}$

$U_2 = \frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$ $= \sqrt{0, \frac{2}{3}, \frac{-1}{3}}$

$1 \quad 0 \quad 0$

$\sqrt{\frac{4}{9} + \frac{1}{9}}$

$0, \frac{2}{3} \div \frac{\sqrt{5}}{3}, \frac{-1}{3} \div \frac{\sqrt{5}}{3},$

$\sqrt{\frac{5}{9}}$

$0, \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}$ $= \frac{\sqrt{5}}{3}$

Step 1:

$i \quad j \quad K$

1 $(1 \quad 0 \quad 0)$

2 $\sqrt{}$ whole th

3 $\div$ each

(x_1, x_2, x_3)

1.3