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CompSci 3

Section 3.2

1) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{bmatrix}$

det A = (downward)
$1 \cdot 5 \cdot 9 = 45$
$2 \cdot 6 \cdot 7 = 84$
$3 \cdot 4 \cdot 8 = 96$ +
225

(upward)
$3 \cdot 5 \cdot 7 = 105$
$6 \cdot 8 \cdot 1 = 48$
$9 \cdot 2 \cdot 4 = 72$ +
225

$225 - 225 = 0$

det B = (downward)
$1 \cdot 5 \cdot 0 = 0$
$2 \cdot 6 \cdot 7 = 84$
$3 \cdot 4 \cdot 0 = 0$ +
84

(upward)
$7 \cdot 5 \cdot 3 = 105$
$0 \cdot 6 \cdot 1 = 0$
$0 \cdot 4 \cdot 2 = 0$ +
105

$84 - 105 = -21$

compute = $0 + -21$
$= -21$

2) $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, find $C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

compute AC^T

$C_{11} = 2.2 - -1.-1 = 3$

$C_{12} = -(-1.2 - (-1.0)) = 2$

$C_{13} = (-1.-1) - 2.0 = 1$

$C_{21} = -((-1.2) - (0.-1)) = 2$

$C_{22} = 2.2 - 0.0 = 4$

$C_{23} = -(2.-1) - (-1.0) = 2$

$C_{31} = -1.-1 - 0.2 = 1$

$C_{32} = -(2.-1) - 0.-1 = 2$

$C_{33} = 2.2 - (-1.-1) = 3$

$C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$C^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$AC^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2.3 + -1.2 + 0.1 & 2.2 + -1.4 + 0.2 & 2.1 + -1.2 + 0 \\ -1.3 + 2.2 - 1.1 & -1.2 + 2.4 - 1.2 & -1.1 + 2.2 - 1.3 \\ 0.3 - 1.2 + 2.1 & 0.2 - 1.4 + 2.2 & 0.1 - 1.2 + 2.3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Section 3.3

1) $2x + y = 1$
$x + 2y + z = 0$
$y + 2z = 0$
$\rightarrow A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\det A = 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$

$= 2.(4-1) - 1(2-0) + 0$

$= 6 - 2 + 0 = 4$

Ax = replace first coloumn from matrix A with B

$\hookrightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \det A_x = 1\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix}$

$\hookrightarrow 1.(4-1) - 1(0) + 0$

$= 3$

$x = \frac{\det(A_x)}{\det A} = \frac{3}{4}$

Ay = replace second coloumn from matrix A with B

$\hookrightarrow \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \rightarrow \det A_y = 2\begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$

$\hookrightarrow 2.(0) - 1(2) + 0$

$= -2$

$y = \frac{\det(A_y)}{\det A} = \frac{-2}{4} = -\frac{1}{2}$

Az = replace third coloumn from matrix A with B

$\hookrightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow \det A_z = 2\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$

$\hookrightarrow 2(0) - 1(0) + 1(1)$

$= 1$

$z = \frac{\det(A_z)}{\det A} = \frac{1}{4}$

2) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 7 & 1 \end{bmatrix}$

a. $\det A = 1(3.1 - 0.7) - 2(0.0 - 0.1) + 0.(0-0)$
$= 3$

$C = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 1 & -7 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow C^T = \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & -\frac{7}{3} & 1 \end{bmatrix}$

b. $B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \to \det B = 2(4-1) + 1(-2) + 0(1)$

$= 4$

$C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix} \to C^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/4 & 1 & 1/4 \\ 1/4 & 1/2 & 3/4 \end{bmatrix}$

3) a. area of triangle $(2,1), (3,4), (0,5)$

↳ $\frac{\det}{2} = \frac{1}{2} \begin{vmatrix} 2 & 1 \\ 3 & 4 \\ 0 & 5 \end{vmatrix} \to$ (down) $2 \cdot 4 + 3 \cdot 5 + 0 \cdot 1 = 23$

(up) ↳ $0 \cdot 4 + 3 \cdot 1 + 2 \cdot 5 = 13$ −

10

$= \frac{10}{2} = 5$

b. area of parallelogram, edges $v = 3,2$, $w = 1,4$

$\det = 3 \cdot 4 - 1 \cdot 2 \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

$= 10$

area = determinant

area = 10

c. V of box $(0,0,0)$ $(3,1,1)$ $(1,3,1)$ $(1,1,3)$

$V = \left| \det \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \right|$

↳ $\det = 3 \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + 1 \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}$

$= 3(8) - 1(2) + 1(-2)$

$= 24 - 2 - 2$

$= 20$

Section 4.1

1) $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$

a. $\frac{1}{2} A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ | $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$-A = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix}$

b. Smallest subspace containing A

$tA = \begin{bmatrix} 2t & -2t \\ 2t & -2t \end{bmatrix} \to$ t is a real number

$2t = a \to \begin{bmatrix} a & -a \\ a & -a \end{bmatrix}, a \in \mathbb{R}$

2) a. $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \to C_2 - 4C_1 \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \\ -1 & 0 & -2 \end{bmatrix} \to C_3 - 2C_1 \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \alpha = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \to \left. \begin{matrix} b_1 = \alpha \\ b_2 = 2\alpha \\ b_3 = -\alpha \end{matrix} \right\}$ $\begin{matrix} b_1 = b_1 \\ b_2 = 2b_1 \\ b_3 = -b_1 \end{matrix}$

$(b_1, 2b_1, -b_1)$

b. $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

↳ $c_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ $c_2 = \begin{bmatrix} 4 \\ 9 \\ -4 \end{bmatrix}$

not multiples

$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 9 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$(c_1 \cdot c_2) \cdot b = 0$

$(1,0,1) \cdot (b_1, b_2, b_3) = b_1 + b_3 = 0$

Section 4.2

1) Reduce

$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \to R_2 - R_1 \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 - R_2$

Pivots: Column 1, 3

Free variables: column 2, 4, 5

$B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix} \to R_3 - 2R_2 \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

Pivots: Column 1, 2

Free variable: Column 3

2) $\begin{array}{ccc} v & w & u=? \\ \begin{bmatrix}1\\1\\5\end{bmatrix} & \begin{bmatrix}0\\3\\1\end{bmatrix} & \end{array} \rightarrow$ Column $\qquad \begin{array}{c} x \\ \begin{bmatrix}1\\1\\2\end{bmatrix}\end{array} \rightarrow$ null

$v + w + u = 0$

$$\begin{bmatrix}1\\1\\5\end{bmatrix} + \begin{bmatrix}0\\3\\1\end{bmatrix} = \begin{bmatrix}v\\w\\2u\end{bmatrix}$$

$$\begin{bmatrix}1\\1\\5\end{bmatrix} + \begin{bmatrix}0\\3\\1\end{bmatrix} + 2u = 0$$

$$\begin{bmatrix}1\\4\\6\end{bmatrix} + 2u = \begin{bmatrix}0\\0\\0\end{bmatrix} \rightarrow u = \begin{bmatrix}-1/2\\-2\\-3\end{bmatrix}$$

$$\text{matrix} = \begin{bmatrix}1 & 0 & -1/2\\1 & 3 & -2\\5 & 1 & -3\end{bmatrix}$$

3) $$A = \begin{bmatrix}0&0&0\\0&0&3\\2&4&6\end{bmatrix} = \begin{bmatrix}2&4&6\\0&0&3\\0&0&0\end{bmatrix} \rightarrow \tfrac{1}{2}R_1 \begin{bmatrix}1&2&3\\0&0&3\\0&0&0\end{bmatrix} \rightarrow R_1 - R_2 \begin{bmatrix}1&2&0\\0&0&3\\0&0&0\end{bmatrix}$$

$$\tfrac{1}{3}R_2 \begin{bmatrix}1&2&0\\0&0&1\\0&0&0\end{bmatrix}$$

$$B = \begin{bmatrix}0&0&0&0&0&0\\0&0&3&0&0&6\\2&4&6&4&8&12\end{bmatrix} \quad \begin{bmatrix}2&4&6&4&8&12\\0&0&3&0&0&6\\0&0&0&0&0&0\end{bmatrix}$$

$$= \tfrac{1}{2}R_1 \begin{bmatrix}1&2&3&2&4&6\\0&0&3&0&0&6\\0&0&0&0&0&0\end{bmatrix} \rightarrow R_1 - R_2 \begin{bmatrix}1&2&0&2&4&0\\0&0&3&0&0&6\\0&0&0&0&0&0\end{bmatrix}$$

$$\tfrac{1}{3}R_2 \begin{bmatrix}1&2&0&2&4&0\\0&0&1&0&0&2\\0&0&0&0&0&0\end{bmatrix}$$

Section 4.3

1) $$A = \begin{bmatrix}2&4&6&4\\2&5&7&6\\2&3&5&2\end{bmatrix}$$

$$[A\ b] = \left[\begin{array}{cccc|c}2&4&6&4&4\\2&5&7&6&3\\2&3&5&2&5\end{array}\right] \xrightarrow{R_2-R_1,\ R_3-R_1} \left[\begin{array}{cccc|c}2&4&6&4&4\\0&1&1&2&-1\\0&-1&-1&-2&1\end{array}\right]$$

$$= R_3+R_2 \rightarrow \left[\begin{array}{cccc|c}2&4&6&4&4\\0&1&1&2&-1\\0&0&0&0&0\end{array}\right] \rightarrow \tfrac{1}{2}R_1 \rightarrow \left[\begin{array}{cccc|c}1&2&3&2&2\\0&1&1&2&-1\\0&0&0&0&0\end{array}\right]$$

$$= R_1 - 2R_2 \rightarrow \left[\begin{array}{cccc|c}1&0&1&-2&4\\0&1&1&2&-1\\0&0&0&0&0\end{array}\right]$$

Column A : Span $\left[\begin{pmatrix}2\\2\\2\end{pmatrix}, \begin{pmatrix}4\\5\\3\end{pmatrix}\right]$

the Column space has dimension 2 because there is only 2 Pivots, which lower than 3 rows, thus the Column spaces creates a Plane in $\mathbb{R}^3$. Since vector b forms the final row of reduced echelon so the $Ax = b$ is Created

$x_n \rightarrow x_1 + x_3 - 2x_4 = 0$

$x_2 + x_3 + 2x_4 = -1$

$x_3 = 1 \quad x_1 = -1 \qquad x_3 = 0 \quad x_1 = 2$

$x_2 = -1 \quad x_4 = 0 \qquad x_4 = 1 \quad x_2 = -2$

$$x_n = \begin{bmatrix}-1\\-1\\1\\0\end{bmatrix} x_3 + \begin{bmatrix}2\\-2\\0\\1\end{bmatrix} x_4$$

$$\text{null}(A) = \left[\begin{pmatrix}-1\\-1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\-2\\0\\1\end{pmatrix}\right]$$

the dimension of the null space (nullity) = 2

$x = x_p + x_n$

$x_p \rightarrow x_1 + x_3 - x_4 = 4$

$x_2 + x_3 + 2x_4 = -1$

$x_3 = 0 \quad x_4 = 0$

$x_2 = -1 \quad x_1 = 4$

$$X = \begin{bmatrix}4\\-1\\0\\0\end{bmatrix} + x_3\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix} + x_4\begin{bmatrix}2\\-2\\0\\1\end{bmatrix}$$

2) a. $x + y + z = 4$

$x_p \rightarrow y = 0 \quad z = 0 \quad x = 4 \qquad \begin{bmatrix}4\\0\\0\end{bmatrix}$

$x_n \rightarrow y = -1 \quad z = 0 \quad x = 1 \qquad y = 0 \quad z = 1 \quad x = -1$

$$x = \begin{bmatrix}4\\0\\0\end{bmatrix} + \begin{bmatrix}1\\-1\\0\end{bmatrix} y + \begin{bmatrix}-1\\0\\1\end{bmatrix} z$$

b. $$\left[\begin{array}{ccc|c}1&1&1&4\\1&-1&1&4\end{array}\right] \ R_2 - R_1$$

$$\left[\begin{array}{ccc|c}1&1&1&4\\0&-2&0&0\end{array}\right] \rightarrow -\tfrac{1}{2}R_2 \rightarrow \left[\begin{array}{ccc|c}1&1&1&4\\0&1&0&0\end{array}\right]$$

$$R_1 - R_2 \rightarrow \left[\begin{array}{ccc|c}1&0&1&4\\0&1&0&0\end{array}\right]$$

$$X_p = \begin{bmatrix}4\\0\\0\end{bmatrix} \qquad X_n = \begin{bmatrix}-1\\0\\1\end{bmatrix} z$$

$$X = \begin{bmatrix}4\\0\\0\end{bmatrix} + \begin{bmatrix}-1\\0\\1\end{bmatrix} z$$

3. $Ax = \begin{bmatrix}1\\3\end{bmatrix} \qquad x = \begin{bmatrix}1\\0\end{bmatrix} + c\begin{bmatrix}0\\1\end{bmatrix}$

$x_p = \begin{bmatrix}1\\0\end{bmatrix} \qquad x_n = \begin{bmatrix}0\\1\end{bmatrix}$

$A = \begin{bmatrix}a&c\\b&d\end{bmatrix} \qquad b = \begin{bmatrix}1\\3\end{bmatrix}$

$$A x_n = \begin{bmatrix}a&c\\b&d\end{bmatrix} \cdot \begin{bmatrix}0\\1\end{bmatrix} = 0$$

$$\begin{bmatrix}c\\d\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$A x_p = b$

$$\begin{bmatrix}a&0\\b&0\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$$

$$\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$$

$$\text{Matrix } A = \begin{bmatrix}1&0\\3&0\end{bmatrix}$$