Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

$$\begin{aligned}&AND:\\&\quad\Theta^{(1)}=\begin{bmatrix}-30 & 20 & 20\end{bmatrix}\\&NOR:\\&\quad\Theta^{(1)}=\begin{bmatrix}10 & -20 & -20\end{bmatrix}\\&OR:\\&\quad\Theta^{(1)}=\begin{bmatrix}-10 & 20 & 20\end{bmatrix}\end{aligned}$$

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

$$\begin{bmatrix}x_0\\x_1\\x_2\end{bmatrix}\rightarrow\begin{bmatrix}a_1^{(2)}\\a_2^{(2)}\end{bmatrix}\rightarrow\begin{bmatrix}a^{(3)}\end{bmatrix}\rightarrow h_\Theta(x)$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$\Theta^{(1)}=\begin{bmatrix}-30 & 20 & 2010 & -20 & -20\end{bmatrix}$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

$$\Theta^{(2)}=\begin{bmatrix}-10 & 20 & 20\end{bmatrix}$$

Let's write out the values for all our nodes:

$$\begin{aligned}&a^{(2)}=g(\Theta^{(1)}\cdot x)\\&a^{(3)}=g(\Theta^{(2)}\cdot a^{(2)})\\&h_\Theta(x)=a^{(3)}\end{aligned}$$

And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm: