Alpha-Stable Distributions and Saccadic Foraging

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Abstract—

Index Terms—Alpha-Stable, Levy, Foraging, HVS, Search, Visual Saliency

1 Introduction

Alpha-stable distributions are a family of four parameter heavy-tail distributions that generalize the normal distribution and have been shown to be advantageous when searching for randomly and sparsely distributed resources. Research remains limited because closed form analytic expressions for these non-Gaussian distributions are not available. Recently these curves have characterized in the frequency domain and efficient algorithms involving Fast Fourier Transform (FFT) have been developed to numerically approximate these expressions. An optimal foraging strategy must balance intensification with diversification, exploration with exploitation, searching around the current solutions while making sure to explore the space efficiently. Research gathering animal behaviors of many species show that foragers try to minimize the distance travelled between targets to maximize their energy gain. Gaze shifts in the human visual system (HVS) can be thought of as the visual system foraging for areas rich in visual saliency. The visual system must decide when an image region has undergone sufficient processing to move to a new location. Given the limited perceptive range and informational capacity of the visual system, what is an optimal eye-movement strategy? While previous research has considered which image locations are informationally rich, much less research has considered general factors of saccadic movements, such as how often and how far the eyes should move under an optimal information-gathering strategy. Statistical models show overlap between simple animal foraging and gaze relocation. In this work, we develop new graphics processing units (GPU) techniques for the estimation of alpha-stable distribution parameters. There are many methods for fitting alpha-stable distributions and mixtures of alpha-stable distributions.The FFT provides a convent and fast method for approximating alpha-stable distributions. For Maximum Likelihood Estimation Methods (MLE) many FFT would be required thus justifying the use of the GPU. These high speed numerical methods allow of for the fitting alpha-stable parameters to a variety of HVS measurements including the focus here: saccadic distributions. Eye tracking data is typically first characterized by a thresholding into fixations and saccades and the two data subsets are then analyzed separately. Here we employ recently developed numerical techniques that allow for the characterization of distributions with heavy tails. This modern characterization allows the entire dataset to be analyzed as a whole. The primary parameter of interest is the first parameter alpha $alpha$. This parameter determines how heavy the tail of the distribution will be. For example for $\alpha = 2$ we have the special case of the Gaussian curve, with this normal distribution we would not expect many samples more than 3 sigma from the mean, with a heavy tail distribution $\alpha < 2$ there will be a nonzero chance of finding a sample many times the standard deviation from the mean. Intuitively this means that most samples will be close zero and everyone once in a while there is a very large sample drawn. Many natural systems behave in this manner with a many small changes punctuated infrequently by very large events.Data suggests that eye movement distributions are well described by alpha-stable distributions and thus do not have closed form solutions and should be characterized by numerical approximations of stable distributions. With a higher resolution in the fovea and limited processing resources we can think of visual saccades as a searching mechanism. Visual search is analogous to animal foraging in the sense that it is not feasible to sample the entire search space and limited resources constrain the behavior. The HVS is foraging for information rich or salient regions and because resources are limited the sensing mechanism must compress the data stream as it is sampled. Human scanpath behavior can thus be thought of as a search strategy or infotaxis mechanism. Heuristic search is a branch of soft computing in the sense that rigorous proofs are not available to guarantee that the algorithm will terminate with a solution. Often in many real world

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problems exact solutions are often not necessary and feasible or potentials solution are desired. For many interesting problems a potential solution can be validated in polynomial time but the generation of a optimal solution can be exponential in its running time. Even for fitness functions that will run in polynomial time it is still prohibitive to brute force the design space to find the optimal solution. For even many trivial tasks, the curse of dimensionality ensures that systematic Cartesian search will never pan out. Modern heuristic search algorithms rely on the injection of outside noise for the generation of potential solutions. Could it be that the HVS also inject noise into the search procedure? If so, what type of noise would be ideal? In the mathematical optimization algorithms alpha-stable distributions have been shown to be advantageous in n-dimensional search. This work is an attempt to characterize human saccade and scan path behavior and identify task specific and subject differences. The HVS is able to traverse larges swaths of the fitness (saliency) landscape quickly and settle on a solution.

2 MMS

The Methodology, Measurement, and Statistics (MMS) Program is an interdisciplinary program in the Social, Behavioral, and Economic Sciences that supports the development of innovative analytical and statistical methods and models for those sciences.

MMS seeks proposals that are methodologically innovative, grounded in theory, and have potential utility for multiple fields within the social and behavioral sciences.

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3 Alpha-Stable Distributions

Index of Stability or Characteristic Exponent $\alpha \in (0, 2]$
Skewness Parameter $\beta \in [-1, 1]$
Scale Parameter $\gamma > 0$
Location Parameter $\delta \in \mathbb{R}$

3.1 Alpha Stable Code

```
function [x,y]=Hahn_stable1(alpha,beta,gama,delta


mult = 4;
n = 8;
xmax = 15;

xmax = xmax*(2^mult);

n = n + mult;
M = 2^n;
R = pi/xmax;
dt = 1/(R*M);

xx = (-2^(n-1)+.5:(2^(n-1)-.5))/(2^n*dt);

piover2 = (pi/2);


yy = exp( -(gama.*abs(xx)).^alpha.*( 1+i*beta.*sign(xx)
.*tan(alpha*piover2).*( (gama.*abs(xx)).^(1-alpha)-1 ) ) + i*delta*xx );


yy1 = [yy((2^(n-1)+1):2^n), yy(1:2^(n-1))];
z = real( fft(yy1) )/(2*pi)*R;

x = (2*pi)*((0:1:(M-1))/(M*R)-1/(2*R));
y = [z((2^(n-1)+1):2^n), z(1:2^(n-1))];

T = find((x<=xmax/(2^mult)) & (x>=-xmax/(2^mult)));
x = x(T);
x = x(:);
y = y(T);
y = y(:);
```

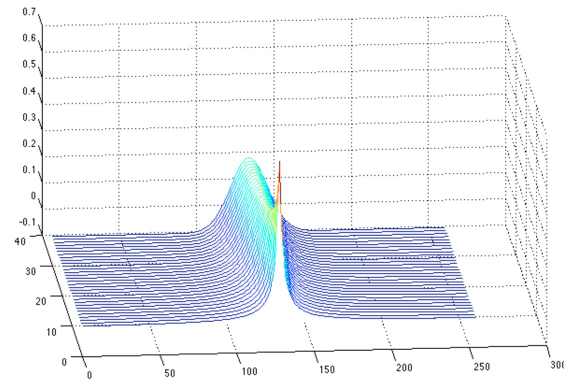

Fig. 2. Alpha-Stable Distributions as a function of Alpha

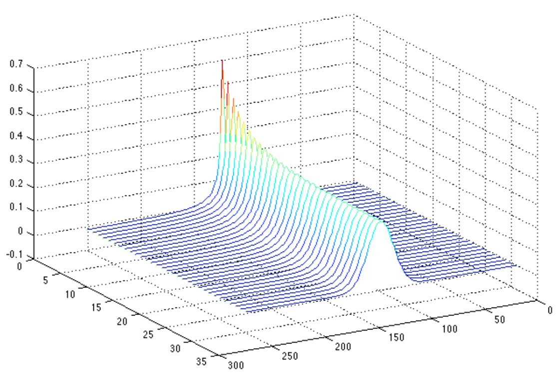

Fig. 1. Alpha-Stable Distributions as a function of Alpha

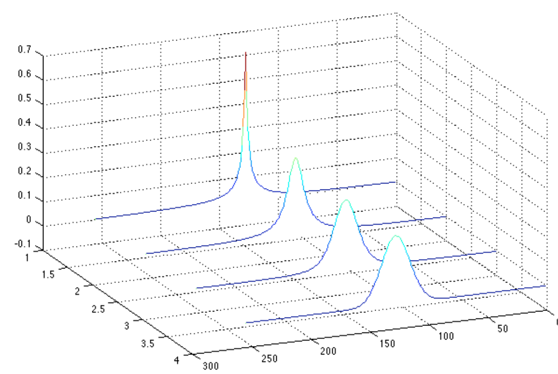

Fig. 3. Alpha-Stable Distributions as a function of Alpha

APPENDIX A NOTES

information gathering
compressive sensing
scanpath
search strategy
forest vs meadow
gaussian vs alpha stable
superclass of functions
no longer limited to special cases
analytical tools limit
abstract search mechanisms
injection of noise into search algorithms
what type of noise is optimal
classically assumed that noise was a bug
maybe a feature
mathematical search
search as random walk
markov chain
optimization
meta heuristic algorithms
nature uses noise and uncertainty to compute
now we can measure
without arbitrary thresholds
characterize numerically
applications
characterizing human visual search behaviors
compression of images and video
characterizing human and animal foraging and movement behaviors
optimization and search algorithms
unmanned vehicles
financial markets
knowing where humans are looking important for human computer interaction shared attention
HVS
human visual system
MLE
fft
gpu
saccadic mechanism
motor system implementation of an active random sampling strategy that the HVS has evolved in order to efficiently and effectively infer properties of the surrounding world.
real time saliency fft on gpu demo
logo
Further these can be thought of Markov chains
CS has fewer parameter than PSO or Genetic Algorithms (GA)
network
multi graph
soft computing
neural network np-complete
corn field vector
boids model
collective intelligence
Boosting
which goose known how to get to canada?
can a set of weak learners form a strong learner
wisdom of the crowds
natural parallelism
scales to large number of processors each computing the fitness of a single particle location and communicating update to the global best to a common server asynchronously
grand canyon
fat tail distribution of noise
most proposed changes will be small
similar to previous design
at times large scale changes in parameters to shake up system dynamics and move out of local minima
cooling schedule
typically momentum term is lowered monotonically according to a described cooling schedule
analog to the cooling of solids crystals glasses metals
glass blowing
metal working
ceramics
explore potential energy landscape
temperature provides random noise that effectively proposes novel permutations of the system
cooling forces a choice
cooling slowly improves chance ending up in global minima
advantage over metropolis hasting
very long cooling times
random markov selection of new candidate
advantage to use existing knowledge of energy landscape to constrain search
orbit of dynamical system
learning rule is a dynamical system
attractors
fixed point at global minima
basin of attraction
stochastic gradient decent
flocking behavior
swarm algorithms
social algorithm
genetic algorithm
geospatial metaphor
flock towards optimal solution in n-dimensional space
perceptron
neural network
non-linear threshold
weighted sum of inputs
neuron fires if weighted sum excedes bias
output set to one
activation function
required to be differential for back propagation
graph theory
set of vertex or node
set of edges
directed weighted acyclic graph
space of all possible graphs

fixed topology determines space of network weights

evaluation of proposed weight set is implemented though matrix multiplications and a nonlinear threshold operation

elegant implementation in vector language such as Matlab

single line of code

wrapper scripts to calculate error

difference of network output with desired output

supervised learning

training set

testing set

validation

mapping of input and output pairs

network convergence

avoid over fitting

multimodal optimization

multiple feasible solutions

find both corn fields

matrix operations can be implemented on GPU for fast operation

distribute swarm over many GPU/workstations and have separate central dedicated unit to rank fitness scores and distribute global best updates

two stage search

back propagation can be used at each particle location before fitness evaluation or back propagation can be preformed only on the global best to save resources.

even relatively large parameter space

transmission costs over a network for strings of n-bits

transmit global best over ethernet

generate alpha-stable distribution

no closed form solution exists

numerical methods

fast Fourier transform

four parameter model

different than method of moments

gaussian distribution

levy

cauchy

special case of alpha-stable

more general distributions allow heavy tails

infinite variance

meta optimization

find alpha value that optimizes utility

single gene genome

particle swarm on one dimensional landscape

swarm parameters can also determine network topology

number of hidden units

variety of feature units

final weight indicate which features of the input provide the most information

random walk

levy flights

levy distribution

optimization as markov chain

intensification

diversification

multiobjective

each node in the network

related to many similar techniques

genetic algorithms

differential evolution

ant colony

stigmergy

krill herd

firefly algorithm

extreme learning machine

only train output layer

Optimization as Markov Chain

backpropagation swarm

The neural network

Particle swarm optimization

evaluations of fitness function are costly in time and space

resources

search

cooperative search

explore energy landscape

share information

neighborhood

limited perceptual range

applications in real world search

search and rescue

multi-swarm

random restart

APSO does not use velocity

swarm intelligence

animal herds

bird flocking

ant colonies

bacterial growth biofilm

fish schooling

local minima

social interaction to problem solving

1995 by James Kennedy (social-psychologist) and Russell Eberhart (electrical engineer)

It uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution

Each particle is treated as a point in a N-dimensional space

according to its own experience as well as the experience of other particles

personal best

global best

position

velocity

current searching point

new searching point

current velocity

new velocity

distance between current position and personal best

distance between current position and global best

selection mechanism

crossover operation
Stochastic Inputs

William Hahn Received his undergraduate degree in Mathematics / Physics from Guilford College in North Carolina in 2008, with a research focus in flocking and swarm behavior before joining the Center for Complex Systems and Brain Sciences at Florida Atlantic University in 2011.

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