Exploring Braess' Paradox in Repeated Congestion Games

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Small network based on https://en.wikipedia.org/wiki/File:Braess_paradox_road_example.svg was created to explore the effect of repeated trials and different learning models on Braess' paradox. Variables explored include learning algorithms, number of agents, and addition/removal of superhighway.

KEYWORDS

Congestion games, Braess' paradox, Repeated games, Fictitious play, ϵ-greedy, UCB1

1 INTRODUCTION

In class we learned about Braess' paradox which states that adding new edges/paths to a network system may have the reverse effect as intended, and may actually slow down the system rather than improve it. This purposes of this experiment are to explore the effects of running repeated congestion games on Braess' paradox with the hypothesis that repeated trials will allow the system to converge on an equilibrium, effectively overcoming the paradox experienced in single-shot games.

2 EXPERIMENTAL AND COMPUTATIONAL DETAILS

2.1 Superhighways

The primary method used to test the existence of the paradox involves running two simulations for each of the changes described below, one without the presence of a superhighway and one with. This superhighway allows agents to move from point A to point B without incurring any cost and, from an equilibrium standpoint, should allow agents to avoid large fixed costs when the total number of agents is relatively low (X/1000 < 45). In a single-shot game we know that the addition of a theoretically beneficial new path will cause self-serving agents to all flock to the new path, decreasing the efficiency of the overall system. With the varying number of agents and different learning models explained below, we expect to see the addition of the superhighway improve efficiency in the long run after a sufficient number of trials.

2.2 Number of Agents

One main independent variable we introduce is a varying number of agents traversing through the system. As illustrated in the network diagram, the costs of certain paths (Start -> A, B -> End) are dependent on the total number of agents traversing that path. At a sufficiently low number of agents (< 4,500), these variable paths will be preferable to the fixed cost paths. Further, the presence or absence of the superhighway will affect the number of variable cost edges agents are able to utilize.

2.3 Learning Models

2.3.1 Fictitious Play. This is the most naïve of the learning models, and isn't truly a learning model itself. Fictitious play involves the knowledge that all other agents will be choosing a path randomly and, in effect, you should too. This essentially simulates a random path choice in order to establish a base line against which to test the actual learning methods.

2.3.2 ϵ-greedy. This is the first real learning model and is in the category of multi-armed bandit algorithms. In this learning model, each agent has a specified probability, ϵ, of choosing a random path, and a $1 - \epsilon$ probability of picking his/her best historical path.

2.3.3 UCB1. This learning model is most complex and we expect it will give us the best results. In this model, each agent chooses that path that minimizes his historical average return for the path plus the square root of 2 times the log of the total number of times the car has driven in the game divided by the number of times the car has tried that particular path.

3 RESULTS AND DISCUSSION

3.1 Two Paths vs. Three Paths (Superhighway Presence)

Across all three of our simulation models, we saw a reduction in average cost across similar numbers of agents between the absence of the superhighway to the presence of the superhighway. It seems that when agents are either acting randomly or are learning from past trials, they are generally able to overcome Braess' paradox in a long run, repeated game. That said, we do notice significantly different results across the three different simulations models described previously.

3.2 Analysis and Comparison of Learning Models

As stated, the fictitious play strategy is the most basic of the three models tested and is meant to serve as our base line when comparing the other models. In this model, all agents choose their paths randomly. Not surprisingly, we do see average agent cost decrease when the superhighway is added as this strategy will distribute equal amounts of agents among each of the paths, effectively ignoring the problem of self-serving agents described in the paradox. As a result, the intent of improving network efficiency by expanding path options is realized when players act randomly. Specifically, average per-agent cost is about 10-20 units lower when the superhighway is available.

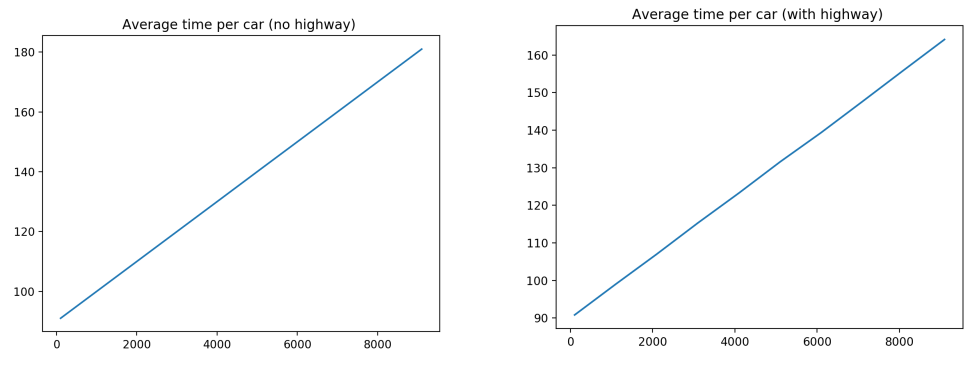

Fig. 1. Fictitious: Total number of agents (x-axis) vs. average per-agent cost (y-axis).

Next, we explored the ϵ-greedy model as a way to model agent behavior and demonstrate some learning over time. For our trials, we used $\epsilon = 0.1$. In general, this model performed much better than fictitious play, independent of the superhighway. However, this model truly shines when comparing its performance on no highway vs. highway. Specifically, we notice a 50-80 unit reduction in average per-agent cost when the superhighway path is added as a potential option. This is expected as this model uses past performance to inform current decisions. As more and more agents utilize the superhighway, they remember the performance benefits and are better able to make the optimal path decision. An interesting note about this model is that while cost vs. number of agents is linear with all other model and highway combinations, ϵ-greedy with the presence of a superhighway experiences a large reduction in how much cost increases with the number of agents in the system. Specifically, when the total number of agents reaches the previously mentioned "break point" of 4,500, the increasing cost with agents largely levels out and the slope decreases substantially. Presumably, this new slope is based on the fixed cost paths that agents are now incentivized to take.

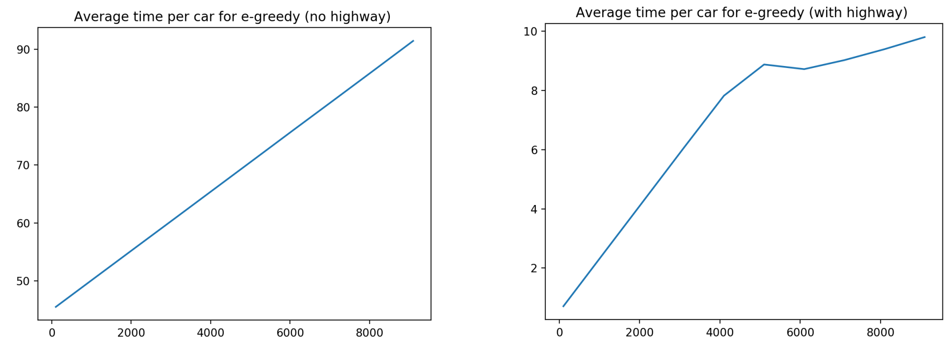

Fig. 2. ϵ-greedy: Total number of agents (x-axis) vs. average per-agent cost (y-axis).

Though we expected UCB1 to perform the best, it was actually outperformed by ϵ-greedy by a large margin, especially with the presence of the superhighway. It seems that the UCB1 algorithm, like ϵ-greedy, incorporates some aspect of regret into the equation and we are therefore not surprised to see better performance than fictitious play. It encourages exploration of other paths, but not as heavily as our -greedy apparently. In our case, we can see that the results aren't as good as ϵ-greedy. This is likely because the algorithm could be essentially choosing the best historical average without exploring enough. If we altered the weights in the algorithm, better results might be attained. However, ϵ-greedy seems much more able to learn from past decisions in order to land on the optimal path given the number of agents.

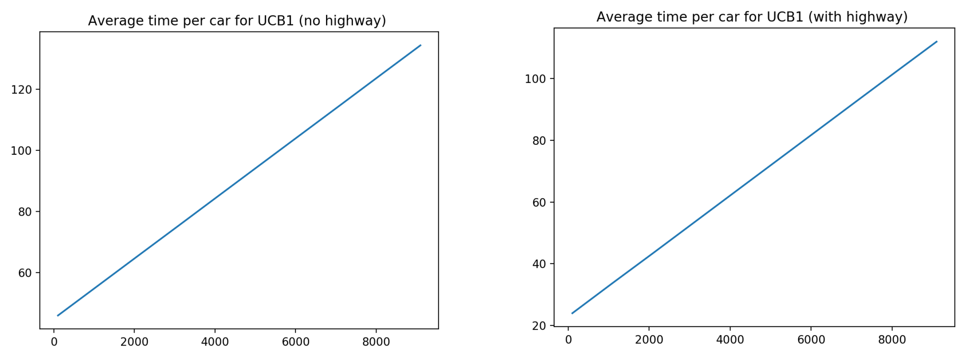

Fig. 3. UCB1: Total number of agents (x-axis) vs. average per-agent cost (y-axis).

4 CONCLUSIONS

Under fictitious play, the addition of the superhighway doesn't substantially affect travel cost as the agents are choosing their paths randomly, so they don't follow the unhealthy Nash equilibrium of using the path. Also, the highway doesn't seem to have a large effect using the other algorithms. These algorithms seem to prevent agents from fully committing to the Nash equilibrium as they stick with best tried paths and only switch with a small probability. If the agents began by using the Nash equilibrium path, though, these results could be much worse. Furthermore, if we altered the value of epsilon, the ϵ-greedy algorithm might lead to worse results as more agents might try the Nash equilibrium and end up getting stuck there. In short though, these models have demonstrated that there are learning algorithms that can prevent Braess' paradox from occurring.

Received May 2018