

2.) Resolved la recurrencia:

$$a_{n+1} - \alpha a_n = \beta^n, \quad a_0 = \lambda,$$

con α, β, λ numeros reales.

Primero resolvemos la recurrencia general:

$$x - \alpha = 0 \implies x = \alpha.$$

Si $\alpha \neq \beta$, tenemos:

$$a_n = A\alpha^n + B\beta^n,$$

con $a_0 = \lambda$ y $a_1 = 1 + \alpha\lambda$. Y por lo tanto

$$\begin{cases} \lambda = A + B \\ 1 + \alpha\lambda = A\alpha + B\beta \end{cases} \implies \begin{cases} A = \lambda - \frac{1}{\beta - \alpha} = \frac{\lambda\beta - \lambda\alpha - 1}{\beta - \alpha} \\ B = \frac{1}{\beta - \alpha} \end{cases}.$$

Si $\alpha = \beta$, entonces:

$$a_n = A\alpha^n + Bn\alpha^n,$$

con $a_0 = \lambda$ y $a_1 = 1 + \alpha\lambda$. Por lo tanto:

$$\begin{cases} \lambda = A \\ 1 + \alpha\lambda = A\alpha + B\alpha \end{cases} \implies \begin{cases} A = \lambda \\ B = \frac{1}{\alpha} \end{cases}.$$