$$\begin{aligned}\int \frac{cx+d}{x^2+ax+b}\,\mathrm{d}x &= \int \frac{c}{2}\frac{2x+\frac{2d}{c}}{x^2+ax+b}\,\mathrm{d}x \\ &= \frac{c}{2}\int \frac{2x+a-a+\frac{2d}{c}}{x^2+ax+b}\,\mathrm{d}x \\ &= \frac{c}{2}\int \frac{2x+a}{x^2+ax+b}\,\mathrm{d}x + \frac{\frac{2d-ac}{c}}{x^2+ax+b}\,\mathrm{d}x \\ &= \frac{c}{2}\left(\int \frac{2x+a}{x^2+ax+b}\,\mathrm{d}x + \int \frac{\frac{2d-ac}{c}}{x^2+ax+b}\,\mathrm{d}x\right)\end{aligned}$$

Ahora, resolvemos

$$\int \frac{2x+a}{x^2+ax+b}$$

Para ello, aplicamos el cambio de variable

$$u = x^2+ax+b \qquad \mathrm{d}x = \frac{1}{2x+a}\,\mathrm{d}u$$

$$\int \frac{2x+a}{x^2+ax+b}\,\mathrm{d}x = \int \frac{(2x+a)\,\frac{1}{2x+a}}{u}\,\mathrm{d}u = \int \frac{1}{u}\,\mathrm{d}u = \ln|u| = \ln\left|x^2+ax+b\right|$$

Ahora, resolvemos

$$\int \frac{\frac{2d-ac}{c}}{x^2+ax+b}\,\mathrm{d}x = \frac{2d-ac}{c}\int \frac{1}{x^2+ax+b}\,\mathrm{d}x$$

Para ello, completamos cuadrados

$$\int \frac{1}{x^2+ax+b}\,\mathrm{d}x = \int \frac{1}{\left(x+\frac{a}{2}\right)^2+\left(b-\frac{a^2}{4}\right)}\,\mathrm{d}x =$$

$$\int \frac{\frac{1}{b-\frac{a^2}{4}}}{\left(\frac{x+\frac{a}{2}}{\sqrt{b-\frac{a^2}{4}}}\right)^2+1}\,\mathrm{d}x$$

Ahora, aplicamos el cambio de variable

$$u = \frac{x+\frac{a}{2}}{\sqrt{b-\frac{a^2}{4}}} \qquad \mathrm{d}x = \mathrm{d}u\sqrt{b-\frac{a^2}{4}}$$

$$\int \frac{\frac{1}{b-\frac{a^2}{4}}}{\left(\frac{x+\frac{a}{2}}{\sqrt{b-\frac{a^2}{4}}}\right)^2+1}\,\mathrm{d}x = \int \frac{\frac{\sqrt{b-\frac{a^2}{4}}}{b-\frac{a^2}{4}}}{u^2+1}\,\mathrm{d}u = \frac{1}{\sqrt{b-\frac{a^2}{4}}}\int \frac{1}{u^2+1}\,\mathrm{d}u =$$

$$\frac{1}{\sqrt{b-\frac{a^2}{4}}}\arctan(u) = \frac{1}{\sqrt{b-\frac{a^2}{4}}}\arctan\left(\frac{x+\frac{a}{2}}{\sqrt{b-\frac{a^2}{4}}}\right)$$

Y por último substituimos

$$\int \frac{\frac{2d-ac}{c}}{x^2+ax+b}\,\mathrm{d}x = \frac{2d-ac}{c\sqrt{b-\frac{a^2}{4}}}\arctan\left(\frac{x+\frac{a}{2}}{\sqrt{b-\frac{a^2}{4}}}\right)$$

Ahora solo nos queda reemplazar ambas integrales

$$\begin{aligned}\int \frac{cx+d}{x^2+ax+b}\,\mathrm{d}x &= \frac{c}{2}\left(\int \frac{2x+a}{x^2+ax+b}\,\mathrm{d}x + \int \frac{\frac{2d-ac}{c}}{x^2+ax+b}\right) \\ &= \frac{c}{2}\ln\left|x^2+ax+b\right| + \frac{2d-ac}{2\sqrt{b-\frac{a^2}{4}}}\arctan\left(\frac{x+\frac{a}{2}}{\sqrt{b-\frac{a^2}{4}}}\right)\end{aligned}$$