A Topological and Compositional Framework for Meaning: Causal, Reflective, and Hierarchical Semantics

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Abstract

Semantic representation in artificial intelligence and cognitive systems is commonly modeled within single representational spaces, such as vector embeddings governed by metric similarity. While effective in practice, such approaches face structural limitations in capturing non-local, asymmetric, and hierarchical semantic relations. This paper proposes a foundational, representation-agnostic framework for semantics grounded in topological structure and compositional mapping.

We introduce the notion of a *meaning topology* as a general semantic space in which semantic connectivity arises through two structurally distinct modes: causal meaning, defined by direct participation in generative subgraphs, and reflective meaning, defined by interpretative mappings across the topology. We prove that these modes are non-equivalent, may coexist, and cannot be exhaustively captured by a single reflection operator. Metric embedding spaces are shown to constitute restricted special cases of the proposed framework.

Furthermore, we formalize the *hierarchy of meaning* as compositional mapping between distinct meaning topologies, rather than as internal stratification within a single space. This formulation imposes no assumptions of inclusion, complexity ordering, or linear abstraction, and supports branching, collapsing, and recursive hierarchical constructions.

The resulting framework clarifies the structural limits of single-space semantic models and provides an axiomatic foundation for semantic plurality, interpretative flexibility, and hierarchical meaning across linguistic, cognitive, and artificial systems.

Keywords: semantic topology; causal meaning; reflective meaning; hierarchy of meaning; semantic representation; interpretative mapping; foundations of semantics

1 Introduction

Semantic interpretation is a central problem across linguistics, cognitive science, and artificial intelligence. Classical work in distributional semantics established the empirical insight that the meaning of linguistic items can be inferred from patterns of co-occurrence and contextual distribution Firth, 1957; Harris, 1954; Wittgenstein, 1953. This distributional hypothesis provided the conceptual groundwork for vector-space models that represent lexical items as points in geometric spaces where proximity encodes semantic relatedness Turney and Pantel, 2010.

The emergence of dense, learned word embeddings—first in neural language modeling Bengio et al., 2003 and later popularized by methods such as word2vec and GloVe—produced compact semantic representations that proved effective across a wide range of downstream tasks Mikolov et al., 2013; Pennington et al., 2014. More recently, contextualized embedding models based on deep transformer architectures have further complicated the geometry of semantic spaces, demonstrating impressive empirical performance while raising new questions about what is represented and how Ethayarajh, 2019; Rogers et al., 2020.

Despite these advances, there is an accumulating body of work that highlights the conceptual and representational limits of single-space semantics. Empirical phenomena such as analogy, entailment asymmetry, non-local metaphorical relations, and task-dependent semantics suggest that a single metric or operator (e.g., cosine similarity in a fixed embedding space) is insufficient to capture the full range of semantic relations encountered in natural language and cognition Ethayarajh, 2019; Turney and Pantel, 2010. At the same time, the formal study of causality and generative models has shown that explanatory (causal) accounts of meaning—those that appeal to how representations are generated or produced—are conceptually distinct from relational accounts that emphasize structure and correspondence Pearl, 2009.

Parallel to these developments, geometric and topological methods have been proposed for the study of high-dimensional data, providing tools to reason about global structure and non-local properties of point clouds and relational data Carlsson, 2009. These tools suggest that topological perspectives can enrich our understanding of semantic spaces beyond purely metric or local analyses.

Motivated by this landscape, the present paper proposes a foundational, representation-agnostic framework for semantics in which (1) semantic spaces are formalized as *meaning topologies*, (2) semantic connectivity arises via two structurally distinct modes—*causal* and *reflective* meaning—and (3) hierarchical, higher-order meaning is defined as compositional mappings between distinct meaning topologies rather than as internal strata of a single space. This approach synthesizes insights from distributional semantics, generative (causal) modeling, and topological data analysis into a unified axiomatic setting that explains why (and how) embedding-based models are useful yet necessarily limited.

Our contributions are threefold. First, we introduce the concept of a *meaning topology* and provide minimal definitions that support both causal and reflective modes of meaning. Second, we derive lemmas and theorems demonstrating (i) the non-equivalence and possible coexistence of causal and reflective meaning, (ii) the instantiation of embedding spaces as special cases of meaning topologies, and (iii) the impossibility of a single universal reflection operator that exhausts all reflective semantics. Third, we formalize the notion of *hierarchy of meaning* as compositional mapping between meaning topologies and discuss epistemic and ontological consequences for semantic theory and AI system design.

By shifting focus from single-space representation to structured mappings between interpretative spaces, the framework aims to clarify which semantic phenomena are amenable to representation in vector spaces, which require alternative reflection operators (e.g., partial orders, graph morphisms, topological correspondences), and how hierarchical interpretative claims can be stated formally without collapsing distinct semantic domains. We situate this contribution relative to prior work in distributional semantics, neural language models, causality, and topological data analysis, and conclude by outlining directions for empirical instantiation and system design that respect the theoretical limits identified here.

2 Related Works

The present work intersects with multiple research traditions concerned with semantic representation, causal explanation, and structural abstraction. Rather than proposing a new model within any single tradition, the framework aims to provide a unifying foundation that clarifies the scope and limitations of existing approaches. In this section, we review the most relevant lines of work and situate the present contribution relative to them.

2.1 Distributional and Embedding-Based Semantics

Distributional semantics originates from the observation that linguistic meaning correlates with contextual usage patterns Firth, 1957; Harris, 1954. Vector space models operationalize this idea by embedding lexical items into geometric spaces where semantic similarity is inferred from distance or inner product Turney and Pantel, 2010. Neural language models further refined this paradigm by learning dense representations optimized for predictive objectives Bengio et al., 2003; Mikolov et al., 2013; Pennington et al., 2014.

While embedding-based models have demonstrated strong empirical performance, several studies have highlighted structural limitations inherent to single-space representations. In particular, the geometry of contextualized embeddings varies significantly across contexts and tasks, complicating the interpretation of distance as a stable semantic relation Ethayarajh, 2019. Surveys of transformer-based models emphasize that high performance does not imply transparent or unified semantic structure Rogers et al., 2020.

The present work does not reject embedding-based semantics, but formally characterizes it as a restricted instantiation of reflective meaning within a meaning topology governed by metric-based reflection operators.

2.2 Causal Semantics and Generative Explanations

Causal approaches to meaning emphasize explanation through generative structure and dependency. Formal frameworks for causal reasoning, such as structural causal models, distinguish causal relations from mere statistical association Pearl, 2009. In semantic modeling, causal perspectives inform how representations arise from data, training procedures, and architectural constraints.

This line of work clarifies how semantic representations are produced, but does not by itself account for non-local, symbolic, or analogical relations that lack direct causal adjacency. The distinction drawn in this paper between causal and reflective meaning aligns with this observation while embedding causal semantics within a broader topological framework.

2.3 Structured and Non-Metric Semantic Relations

Several approaches have sought to move beyond purely metric notions of similarity. Order-based embeddings, graph-based semantics, and symbolic relational models aim to capture asymmetric relations such as entailment and hierarchy Turney and Pantel, 2010. However, these approaches typically introduce alternative operators within a single representational space rather than questioning the sufficiency of single-space semantics itself.

The present framework generalizes these efforts by showing that different relational operators correspond to distinct modes of reflective meaning and that no single operator can exhaust semantic structure.

2.4 Topological and Geometric Perspectives

Topological methods provide tools for analyzing global and non-local properties of data without reliance on metric assumptions. In data analysis, topological approaches have been used to characterize shape, connectivity, and robustness in high-dimensional spaces Carlsson, 2009. These ideas motivate the introduction of meaning topologies as semantic spaces where connectivity and mapping, rather than distance alone, determine interpretability.

Unlike existing applications of topology to data representation, the present work uses topological structure as a foundational semantic abstraction rather than as a post hoc analytical tool.

2.5 Existence and Limitation Results

Lemma 2.1 (Existence of Purely Reflective Meaning)**.** *There exists a meaning topology* $\mathfrak{M} = (V, E, \tau)$ *containing meaning points that possess reflective meaning but no causal meaning.*

Proof. Consider a meaning topology $\mathfrak{M}$ whose vertex set V can be partitioned into two disjoint subsets $V = V_c \cup V_r$, where V_c forms a causal subgraph $\mathfrak{C} = (V_c, E_c)$ and V_r contains no edges incident to E_c. Let $v \in V_r$. By construction, there exists no causal edge $(v, u) \in E_c$ for any $u \in V_c$, hence v has no causal meaning. However, since V_r is a subset of the same topology $\mathfrak{M}$, a structural mapping $f : \{v\} \to V_c$ can be defined independently of adjacency or connectivity. Therefore, v possesses reflective meaning without causal meaning. □

This lemma establishes that reflective meaning does not presuppose causal participation, and may arise purely from global or structural interpretability within the meaning topology.

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Theorem 2.2 (Embedding Spaces as Special Cases of Meaning Topology)**.** *Metric embedding spaces used in distributional semantics constitute special cases of meaning topologies whose reflective structure is restricted to metric-based reflection operators.*

Proof. Let $(\mathcal{S}, d)$ be a metric embedding space commonly employed in semantic representation learning, where entities are mapped to vectors and semantic relations are inferred through distance functions. Define a meaning topology $\mathfrak{M} = (V, E, \tau)$ such that $V = \mathcal{S}$, τ is the topology induced by the metric d, and E contains edges implicitly defined by neighborhood relations under d. In this construction, reflective meaning is exclusively determined by a single class of reflection operators, namely metric distance. Since the general definition of meaning topology allows arbitrary topological structures and reflection mappings beyond metric constraints, metric embedding spaces correspond to a restricted subclass of meaning topologies. Hence, embedding-based semantics is a special case of the broader topological framework. □

This theorem formalizes the non-exhaustiveness of distance-based semantics and situates embedding models within a more general theory of meaning.

Theorem 2.3 (Non-Existence of a Universal Reflection Operator)**.** *There exists no single reflection operator capable of exhaustively capturing all reflective meanings within a meaning topology.*

Proof. Assume, for contradiction, that there exists a universal reflection operator

$$\Phi^* : V \times V \to \Omega$$

such that for any pair of meaning points $v_i, v_j \in V$, all reflective semantic relations between them are fully determined by $\Phi^*(v_i, v_j)$. By Definition 4, reflective meaning may arise from arbitrary mappings between disconnected or topologically distant regions of the meaning topology. Such mappings may encode relations based on symmetry, analogy, hierarchy, transformation, or contextual correspondence, many of which are mutually incompatible in their structural requirements. A single operator cannot simultaneously preserve all such relational structures without loss or contradiction. Therefore, no universal reflection operator can exist, contradicting the assumption. Hence, reflective meaning necessarily requires a plurality of reflection operators. □

This theorem establishes that reflective semantics is intrinsically pluralistic and cannot be reduced to a single relational criterion such as distance or similarity.

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Lemma 2.4 (Coexistence of Causal and Reflective Meaning)**.** *There exist meaning points in a meaning topology that simultaneously possess causal meaning and reflective meaning.*

Proof. Let $v \in V$ be a meaning point such that there exists a causal subgraph $\mathfrak{C} = (V_c, E_c)$ with $(v, u) \in E_c$ for some $u \in V_c$. By Definition 3, v possesses causal meaning. Additionally, since $v \in V$ lies within the meaning topology $\mathfrak{M}$, there exists at least one mapping

$$f : \{v\} \longrightarrow V_r$$

to a subset $V_r \subseteq V$ distinct from its immediate causal neighborhood. By Definition 4, this mapping suffices to establish reflective meaning. Hence, v possesses both causal and reflective meaning simultaneously. □

This lemma demonstrates that causal and reflective meanings are not mutually exclusive but may coexist at a single point, reinforcing the view that they constitute complementary modes of semantic interpretation rather than competing explanations.

2.6 Hierarchy and Abstraction

Hierarchical representations are common in machine learning and cognitive modeling, often implemented as layered architectures or multi-level abstractions within a single space. Such hierarchies typically assume increasing abstraction or complexity with depth.

In contrast, the hierarchy of meaning proposed here is defined as compositional mapping between distinct meaning topologies, without assumptions of inclusion, linear order, or monotonic abstraction. This formulation departs from conventional hierarchical models and provides a formally general account of hierarchical interpretation.

2.7 Summary of Positioning

Taken together, prior work has explored distributional, causal, relational, and topological aspects of semantics largely in isolation. The present work synthesizes these perspectives by introducing a unified, axiomatic framework in which existing models appear as special cases. By doing so, it clarifies the structural limits of single-space semantics and provides a principled basis for understanding semantic plurality and hierarchy.

3 Formal Framework: Meaning Topology and Semantic Connectivity

In this section, we introduce a minimal and general formal structure for semantic interpretation, referred to as a *meaning topology*. This structure serves as the foundational space from which causal and reflective meanings emerge as distinct modes of semantic connectivity.

3.1 Meaning Topology

Definition 3.1 (Meaning Topology)**.** *Let*

$$\mathfrak{M} = (V, E, \tau)$$

be a structured space where:

- V *is a set of* meaning points,
- $E \subseteq V \times V$ *is a set of edges representing possible semantic connections,*
- τ *is a topology defined on* V*.*

The triple $\mathfrak{M}$ *is called a* meaning topology*.*

Each point $v \in V$ represents a position of interpretation rather than a physical or linguistic object. The topology τ enables the discussion of connectivity, continuity, and structural mappings within the semantic space. No assumption is made that all edges in E are causal.

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3.2 Causal Structure

Definition 3.2 (Causal Subgraph)**.** *A subgraph*

$$\mathfrak{C} = (V_c, E_c) \quad \textit{with} \quad V_c \subseteq V,\ E_c \subseteq E$$

is called a causal subgraph *if the edges in* E_c *represent direct causal relations, potentially equipped with directionality or ordering.*

The causal subgraph formalizes the notion of local generative structure within the meaning topology, without imposing causality as a global property of $\mathfrak{M}$.

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Definition 3.3 (Causal Meaning)**.** *A meaning point $v \in V$ is said to possess* causal meaning *if and only if there exists a causal subgraph $\mathfrak{C} = (V_c, E_c)$ such that*

$$\exists\, u \in V_c \ \ \textit{with} \ \ (v, u) \in E_c.$$

Intuitively, a point has causal meaning precisely when it is directly incident to a causal edge, thereby participating in a local chain of cause–effect relations.

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3.3 Reflective Structure

Definition 3.4 (Reflective Meaning)**.** *A meaning point $v \in V$ is said to possess* reflective meaning *if and only if:*

1. *there exists no causal subgraph $\mathfrak{C} = (V_c, E_c)$ such that $(v, u) \in E_c$ for any $u \in V_c$, and*
2. *there exists a well-defined mapping*

$$f : \{v\} \longrightarrow V_r, \quad V_r \subseteq V,$$

into a (possibly distant or disconnected) subset of meaning points within $\mathfrak{M}$.

Reflective meaning thus arises not from direct causal adjacency, but from the existence of structural or topological mappings that associate a point with other regions of the meaning topology.

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3.4 Structural Distinction

Lemma 3.5 (Non-Equivalence of Causal and Reflective Connectivity)**.** *Causal connectivity and reflective connectivity constitute structurally non-equivalent modes of semantic connection within a meaning topology.*

Proof Sketch. Causal connectivity is defined by the existence of direct edge incidence within a causal subgraph, relying on local graph structure. Reflective connectivity, by contrast, depends on the existence of mappings that may ignore locality, continuity, or direct adjacency. Since there exist mappings between disconnected components of $\mathfrak{M}$, reflective connectivity does not imply causal connectivity, nor does causal connectivity guarantee the existence of a reflective mapping beyond local structure. Hence, the two modes are structurally non-equivalent. □

4 Hierarchy of Meaning as Compositional Mapping Between Meaning Topologies

In this section, we formalize the notion of a hierarchy of meaning without introducing internal stratification within a single semantic space. Instead, hierarchy is defined as a structural relation arising from compositional mappings between distinct meaning topologies.

4.1 Meaning Topologies as Distinct Interpretative Spaces

Let

$$\mathfrak{M}_i = (V_i, E_i, \tau_i)$$

denote a meaning topology, where each $\mathfrak{M}_i$ constitutes a complete semantic space equipped with its own vertices, relations, and topological structure. No assumption is made that two meaning topologies $\mathfrak{M}_i$ and $\mathfrak{M}_j$ are comparable in terms of complexity, dimensionality, inclusion, or semantic granularity.

Meaning topologies may differ arbitrarily in structure and interpretation, and are treated as independent semantic domains.

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4.2 Vertical Interpretation Maps

Definition 4.1 (Vertical Interpretation Map)**.** *A* vertical interpretation map *is a mapping*

$$\mathcal{V}_i : \mathfrak{M}_i \longrightarrow \mathfrak{M}_{i+1}$$

that assigns meaning points in $\mathfrak{M}_i$ to meaning points in $\mathfrak{M}_{i+1}$, preserving interpretability while allowing loss, abstraction, or transformation of structural detail.

Vertical interpretation maps are not required to be injective, surjective, continuous, or structure-preserving in the classical topological sense. Their sole requirement is that they are well-defined as interpretative mappings between meaning topologies.

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4.3 Hierarchy as Compositional Structure

Definition 4.2 (Hierarchy of Meaning)**.** *A* hierarchy of meaning *is defined by the existence of a compositional structure of vertical interpretation maps*

$$\mathcal{V}_n \circ \mathcal{V}_{n-1} \circ \cdots \circ \mathcal{V}_0,$$

where each $\mathcal{V}_i : \mathfrak{M}_i \to \mathfrak{M}_{i+1}$ is a vertical interpretation map between meaning topologies.

Hierarchy is therefore not an intrinsic property of any single meaning topology, but an emergent property of the compositional relationships between multiple meaning topologies.

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4.4 Hierarchy Statements as Mapping Compositions

Definition 4.3 (Hierarchical Meaning Statement)**.** *A* hierarchical meaning statement *associated with a meaning point $v \in V_0$ is the compositional expression*

$$(\mathcal{V}_n \circ \mathcal{V}_{n-1} \circ \cdots \circ \mathcal{V}_0)(v),$$

whenever such a composition is defined.

This expression constitutes the most elementary formal representation of hierarchy of meaning. It does not assert the existence of levels, layers, or orders, but solely the existence of composable interpretative mappings.

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4.5 Generality of the Hierarchical Construction

The proposed formulation imposes no constraints on the nature of the meaning topologies involved. In particular:

- Meaning topologies need not be nested or comparable.
- Hierarchical composition need not be linear and may branch or converge.
- The direction of interpretation is not assumed to correspond to increasing abstraction or complexity.

As a result, hierarchies may arise from arbitrary configurations of meaning topologies, including recursive, fragmented, or self-similar constructions.

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4.6 Structural Consequence

Theorem 4.4 (Non-Reduction of Hierarchy to Single-Space Semantics)**.** *No hierarchy of meaning can be fully represented within a single meaning topology without loss of interpretative structure.*

Proof. Assume that a hierarchy defined by a non-trivial composition of vertical interpretation maps could be represented within a single meaning topology. This would imply that all interpretative transformations induced by the composition are internal to that topology.

However, by construction, vertical interpretation maps may discard, abstract, or reinterpret structural features unavailable within a single semantic domain. Hence, representing the hierarchy within one meaning topology would necessarily collapse distinct interpretative spaces into a single structure, contradicting the independence of meaning topologies. Therefore, hierarchy cannot be reduced to single-space semantics. □

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4.7 Remarks

This formulation establishes hierarchy of meaning as a fundamentally compositional phenomenon, grounded in mappings rather than stratification. It preserves maximal generality and allows hierarchical semantic structures to emerge without imposing ontological or mathematical constraints on the nature of the spaces involved.

4.8 Corollaries: Consequences for Semantic Modeling

Corollary 4.5 (Non-Exhaustiveness of Distance-Based Similarity)**.** *Semantic similarity defined solely by metric distance cannot exhaust reflective meaning within a meaning topology.*

Proof. By Theorem 2, no universal reflection operator exists. Metric distance constitutes a single reflection operator within a restricted class. Therefore, semantic relations that depend on non-metric structures—such as hierarchy, directionality, or analogy—cannot be fully captured by distance alone. □

This corollary formalizes the limitation of cosine or Euclidean similarity as complete semantic measures.

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Corollary 4.6 (Existence of Non-Local Semantic Relations)**.** *There exist semantically meaningful relations between meaning points that are not supported by local connectivity in the meaning topology.*

Proof. By Lemma 2, reflective meaning may exist without causal or local edge connectivity. Since local connectivity corresponds to neighborhood structure in the topology, the existence of reflective mappings between disconnected regions implies non-local semantic relations. □

This result legitimizes analogy, metaphor, and symbolic reference as structurally valid semantic phenomena.

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Corollary 4.7 (Analogy as Reflective Mapping)**.** *Semantic analogy corresponds to reflective mappings between subgraphs of a meaning topology rather than to pointwise distance relations.*

Proof. Analogical relations require the preservation of relational structure across distinct regions of the topology. By Definition 4, such preservation is achieved through mappings rather than direct adjacency or metric proximity. Hence, analogy is a reflective, not a causal or metric, semantic phenomenon. □

This explains why vector arithmetic captures analogy only under restrictive conditions.

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Corollary 4.8 (Asymmetry of Entailment). *Semantic entailment cannot be faithfully represented by symmetric reflection operators.*

Proof. Entailment is inherently directional. Symmetric operators, such as distance or similarity, erase directionality. Since reflective meaning admits non-symmetric mappings, entailment necessarily requires reflection operators that encode asymmetry or partial order. □

This corollary clarifies the structural mismatch between entailment and similarity-based models.

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Corollary 4.9 (Coexistence of Generative and Relational Semantics). *Learning-based generative semantics and relational semantic interpretation may coexist without reducibility.*

Proof. By Lemma 3, causal and reflective meanings may coexist at a single meaning point. Generative models encode causal semantic traces, while relational interpretation arises from reflective mappings. Neither mode subsumes the other. □

This result provides a formal justification for hybrid semantic systems.

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Corollary 4.10 (Impossibility of a Single Optimal Semantic Representation). *No single semantic representation can optimally support all semantic tasks.*

Proof. Different semantic tasks require different classes of reflection operators. By Theorem 2, no universal operator exists. Consequently, no single representation can simultaneously preserve all required semantic structures. □

This corollary establishes a theoretical limit on "one-model-fits-all" semantic architectures.

5 Discussion: Epistemic and Ontological Implications

The formal framework developed in this work has implications that extend beyond technical semantic modeling. By grounding meaning in topological structures and compositional mappings rather than internal representations or metric similarity, the framework reshapes both epistemic assumptions about knowledge acquisition and ontological assumptions about the nature of meaning itself.

5.1 Epistemic Implications

From an epistemic perspective, the framework implies that semantic understanding cannot be fully localized within a single representational space. Knowledge is not exhausted by causal generation nor by relational proximity alone, but emerges through interpretative mappings across distinct semantic domains.

This challenges the prevailing epistemic assumption in many machine learning systems that semantic competence increases monotonically with representational depth or dimensionality. In contrast, the present framework suggests that epistemic advancement corresponds to the ability to construct, compose, and navigate mappings between meaning topologies.

Consequently, semantic inference is not reducible to inference within a fixed space, but instead involves transitions between spaces. Epistemic progress is thus characterized not by accumulation of features, but by the capacity for interpretative transformation.

5.2 Limits of Representational Sufficiency

The corollaries derived earlier establish formal limits on representational sufficiency. In particular, Theorem 2 and its corollaries imply that no single representation can serve as a universal semantic substrate. This result has direct epistemic consequences: semantic failure is not necessarily a deficiency of data or optimization, but may arise from the mismatch between the representational space and the semantic task at hand.

This reframes common issues in semantic modeling—such as analogy failure, entailment inconsistency, or contextual collapse—not as implementation flaws, but as structural limitations imposed by single-space semantics.

5.3 Ontological Implications

Ontologically, the framework rejects the identification of meaning with objects, symbols, or representations. Meaning is not an intrinsic property of entities, nor a static relation between them, but a phenomenon arising from structured connectivity and interpretative mapping.

By defining meaning topologies as independent semantic spaces, the framework avoids committing to a single ontological level as fundamental. Instead, ontology becomes plural and relational: what exists semantically depends on the interpretative space in which it is situated.

Hierarchy of meaning, formulated as compositional mapping between meaning topologies, further implies that ontological categories are not arranged in a fixed ladder of being. Rather, they are connected through interpretative transformations that may abstract, collapse, or reinterpret structure without preserving inclusion or complexity relations.

5.4 Transcendence Without Reduction

A notable consequence of the hierarchical formulation is that higher-level meaning spaces cannot be reduced to lower-level ones without loss. This provides a formal account of transcendence that does not rely on metaphysical assertion. Transcendence here is structural: it arises from the non-invertibility and non-equivalence of interpretative mappings.

Thus, statements about higher-order meaning are not claims about additional entities, but about semantic spaces that are inaccessible from lower spaces through internal operations alone.

5.5 Implications for Artificial Intelligence

For artificial intelligence systems, the framework suggests that semantic robustness requires architectures capable of operating across multiple meaning topologies and composing interpretative mappings between them. Systems restricted to a single representational space—regardless of scale or complexity—are inherently limited in their semantic scope.

Hybrid systems that combine causal learning with reflective interpretation are not merely engineering compromises, but are theoretically justified by the coexistence of causal and reflective meaning modes. Furthermore, the absence of a universal reflection operator implies that adaptive selection or construction of reflection operators is a fundamental requirement for general semantic competence.

5.6 Toward a Structural View of Meaning

Taken together, the epistemic and ontological implications support a structural view of meaning in which semantic phenomena are grounded in relations, mappings, and compositions rather than entities or representations. This view aligns with the formal results presented while remaining compatible with diverse instantiations in linguistic, cognitive, computational, and philosophical contexts.

The framework does not prescribe a single semantic model, but delineates the space of possible models and their inherent limitations. In doing so, it shifts the focus of semantic inquiry from representation to interpretation, and from internal structure to relational composition.

6 Conclusion

This work has proposed a foundational framework for semantic interpretation grounded in topological structure and compositional mapping. By introducing the notion of a meaning topology and distinguishing causal and reflective meaning as structurally distinct modes of semantic connectivity, the framework provides a general and representation-agnostic basis for understanding semantic phenomena.

A central contribution of the paper is the formalization of hierarchy of meaning as compositional mapping between distinct meaning topologies. This formulation avoids internal stratification within a single semantic space and imposes no assumptions regarding inclusion, complexity, or linear ordering among spaces. Hierarchy emerges solely from the existence and composition of interpretative mappings, thereby preserving maximal generality and avoiding reductionist constraints.

The derived lemmas, theorems, and corollaries establish fundamental limits on single-space semantic models, including distance-based embeddings, and demonstrate that no universal reflection operator can exhaust semantic meaning. These results reposition many well-known challenges in semantic modeling as structural limitations rather than implementation deficiencies.

Epistemically, the framework implies that semantic understanding is achieved not through representational accumulation but through the capacity to construct and compose mappings across interpretative spaces. Ontologically, it supports a plural and relational view of meaning in which semantic reality is distributed across multiple topologies rather than anchored to a single foundational level.

Taken together, the framework delineates the space of possible semantic models while remaining neutral with respect to specific mathematical instantiations or application domains. It provides a unifying foundation for causal, relational, and hierarchical semantics, and offers a principled basis for the design of semantic systems capable of interpretative flexibility beyond single-representation paradigms.

Future work may explore concrete instantiations of meaning topologies, the formal properties of vertical interpretation maps, and the application of the framework to empirical semantic systems, including natural language processing, cognitive modeling, and multi-agent reasoning.

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A Appendix: Illustrative Constructions of Hierarchical Meaning

This appendix provides illustrative constructions demonstrating how the proposed framework may be instantiated. These examples are not exhaustive and do not introduce additional axioms or constraints. Their purpose is solely to clarify the generality of the formal definitions presented in the main text.

A.1 Cantor-Like Hierarchical Construction

Consider a meaning topology $\mathfrak{M}_0 = (V_0, E_0, \tau_0)$ and a mapping

$$\mathcal{V}_0 : \mathfrak{M}_0 \longrightarrow \mathfrak{M}_1$$

that removes a subset of semantic detail while preserving interpretative coherence. Suppose this mapping is applied recursively to generate a sequence

$$\mathfrak{M}_0 \xrightarrow{\mathcal{V}_0} \mathfrak{M}_1 \xrightarrow{\mathcal{V}_1} \mathfrak{M}_2 \xrightarrow{\mathcal{V}_2} \cdots$$

If each mapping selectively discards portions of the semantic structure while maintaining self-similar interpretative relations, the resulting hierarchy exhibits Cantor-like properties. In such constructions, meaning becomes progressively fragmented yet structurally interpretable through the compositional mappings. This demonstrates that hierarchical meaning need not correspond to increasing density or complexity, but may arise through recursive interpretative reduction.

A.2 Branching Hierarchies of Meaning

The framework does not require hierarchical composition to be linear. Let

$$\mathcal{V}_0 : \mathfrak{M}_0 \longrightarrow \mathfrak{M}_1, \qquad \mathcal{W}_0 : \mathfrak{M}_0 \longrightarrow \mathfrak{M}_2$$

be two distinct vertical interpretation maps originating from the same meaning topology. These mappings may give rise to divergent interpretative hierarchies, each valid within its own semantic context.

Such branching hierarchies model situations in which a single semantic domain supports multiple, non-equivalent interpretative developments. The framework imposes no requirement that these branches be reconcilable or ordered, reflecting the pluralistic nature of semantic interpretation.

A.3 Collapsing Hierarchies and Semantic Convergence

Conversely, multiple meaning topologies may converge under a shared interpretative mapping. Let

$$\mathcal{V}_i : \mathfrak{M}_i \longrightarrow \mathfrak{M}_*, \quad i \in \{1, 2, \ldots, n\},$$

map distinct meaning topologies into a common interpretative space $\mathfrak{M}_*$. Such collapsing hierarchies model semantic convergence, where diverse domains are interpreted through a shared conceptual or symbolic framework.

This construction illustrates that hierarchy of meaning does not imply divergence or refinement alone, but may also account for unification and abstraction across heterogeneous semantic spaces.

A.4 Hierarchy and Pointwise Meaning

For a meaning point $v \in V_0$, a hierarchical meaning statement is given by a compositional mapping

$$(\mathcal{V}_k \circ \cdots \circ \mathcal{V}_0)(v),$$

whenever the composition is defined. This expression encapsulates the hierarchical interpretation of a point without attributing intrinsic levels or ranks to the point itself. The hierarchy arises solely from the compositional structure of mappings, not from properties internal to the meaning point.

A.5 Remarks

These illustrative constructions emphasize that the proposed framework accommodates recursive, branching, collapsing, and fragmented hierarchies without modification. They serve to demonstrate that hierarchy of meaning is fundamentally a property of compositional mapping between meaning topologies, rather than an imposed ordering or stratification within a single semantic space.

B Appendix: Axiomatic Motivation

This appendix presents the axiomatic motivations that guided the development of the formal framework in the main body of the paper. These axioms are not introduced as formal assumptions within the theory, but rather as conceptual principles that motivated the choice of definitions, structures, and proof strategies. The formal results of the paper are independent of the axioms stated here, and stand on their own logical foundations.

B.1 Axiom 1: Hierarchy of Meaning (Vertical Motivation)

> **Axiom 1 (Vertical Meaning Axiom).** Every observable phenomenon carries a semantic interpretation that is embedded in a hierarchical structure of meaning, extending beyond empirical manifestation toward more fundamental interpretative domains.

This axiom expresses the intuition that observable phenomena do not constitute final semantic units. Instead, interpretation proceeds through successive re-contextualizations that point beyond immediate manifestation. In the formal framework, this intuition motivates the introduction of multiple meaning topologies and the definition of hierarchy of meaning as compositional mapping between such topologies.

Importantly, this axiom does not impose any ordering, inclusion, or increasing complexity among meaning spaces. The hierarchy is formalized purely through the existence and composition of interpretative mappings, without committing to metaphysical or ontological claims about the nature of the domains involved.

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B.2 Axiom 2: Dual Mode of Meaning (Horizontal Motivation)

Axiom 2 (Dual Meaning Axiom). Within a single reality, any entity admits two irreducible semantic interpretations: (a) a causal interpretation derived from its generative chain of causes, and (b) a reflective interpretation derived from the relations it manifests within a semantic space.

This axiom motivates the structural distinction between causal and reflective meaning developed in the main text. Rather than treating meaning as a single homogeneous notion, the framework separates generative explanation from relational interpretation.

In the formal development, this intuition is realized through the definition of causal subgraphs and reflective mappings within a meaning topology, followed by lemmas establishing their non-equivalence and possible coexistence. The axiom itself is not assumed in proofs; instead, its content emerges as a structural consequence of the definitions.

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B.3 Axiom 3: Non-Uniqueness of Reflection Operators

Axiom 3 (Multiplicity of Reflection Operators). Reflective meaning is not uniquely determined by metric distance, but may be instantiated through diverse relational structures such as symmetry, antisymmetry, topology, transformation invariance, or higher-order mappings.

This axiom reflects the intuition that semantic reflection cannot be reduced to a single notion of similarity. While metric distance provides one useful reflection operator, many semantic relations—such as entailment, analogy, and symbolic correspondence—require structurally different operators.

In the formal framework, this intuition is elevated from an axiomatic assumption to a provable result. The non-existence of a universal reflection operator is established as a theorem, and the limitations of distance-based semantics follow as corollaries. Thus, this axiom functions purely as motivational guidance rather than as a foundational premise.

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B.4 Role of Axiomatic Motivation

The axioms presented here serve a methodological role analogous to guiding principles in foundational mathematics and theoretical physics. They articulate the semantic intuitions that shaped the formal constructions, while remaining external to the logical dependency structure of the theory.

Readers interested primarily in formal results may safely disregard this appendix. Readers interested in the conceptual genesis of the framework may find it useful for situating the formal definitions within a broader interpretative perspective.