Causal Informational Ordering, Emergent Space, and Spatial Dimensionality

An epistemological and formal account

Aslan Alwi*, Munirah, Almudaya Research Institute Team
aslan.alwi@umpo.ac.id, munirah@umpo.ac.id

Independent Researcher at Almudaya Research Institute
Department of Informatics Engineering, Faculty of Engineering, Universitas Muhammadiyah Ponorogo, Indonesia

December 5, 2025

Abstract

We present a compact, axiomatic and constructive account that takes *causal informational ordering* (CIO) as the pre-geometric epistemic substrate from which notions of time, space, and spatial dimension may be derived. Starting from an epistemological foundation that any empirical description of change presupposes distinguishable configurations and lawful transitions, we formalize time as the partial order induced by transitions, show how space (topology, adjacency, metric constructions) may be induced from such order, and introduce a family of definitions for *spatial dimension* where the spatial dimension is defined as a cardinal property of a representation (embedding) of the configuration set. We give precise theorems about (i) the logical priority of CIO over geometric time, (ii) the emergence of topological and metric structures from order, and (iii) the representability of Cantor-type sets with arbitrary spatial dimension (finite, countable, or uncountable) by appropriate embeddings. The work synthesizes prior philosophical and technical perspectives (Wheeler's "it-from-bit", Floridi's informational ontology, relational quantum mechanics, and causal-set approaches) and provides a rigorous basis for modelling universes with richer dimensional possibilities than conventionally assumed.

Contents

1 Introduction

The principal thesis of this paper is that *causal informational ordering* (CIO) — the partial order induced by lawful transitions among distinguishable configurations — is the minimal pre-geometric structure required for any empirical physics. From CIO we define time as the induced order, and show how space and dimension may be derived as structures induced by or independent from that order. This paper builds on and formalizes the epistemological program introduced in [1]. :contentReference[oaicite:1]index=1

The idea that information plays an ontological/epistemic role in physics has a modern pedigree in Wheeler's "It from Bit" program [2] and is philosophically connected to Floridi's Philosophy of Information [3]. Relational perspectives on physical states (Rovelli [4]) and causal-set inspired pre-geometry (Sorkin [5]) motivate aspects of our formal constructions. The 'time-as-order' viewpoint also echoes Page and Wootters' "evolution without evolution" framework [6]. :contentReference[oaicite:2]index=2

This paper is organized as follows. In Section 2 we present precise preliminaries and notation. In Section 3 we formalize time as order and prove minimality and logical-priority results. In Section 4 we construct space (graph, Alexandrov and order topologies, causal metrics) from order. In Section 5 we introduce a general, cardinal-based definition of spatial dimension, state and prove the principal theorems about Cantor sets and dimension ranges, and show how the countability assumption may be viewed as an epistemic convention. Section 7 discusses philosophical and physical implications. Finally, Section 8 concludes and suggests further directions.

2 Preliminaries and notation

We shall work in the language of set theory (ZF); when the Axiom of Choice (AC) is invoked we will indicate it explicitly. Notation:

- Σ denotes a set of *configurations* (distinguishable states).
- $T \subseteq \Sigma \times \Sigma$ denotes a *transition relation*. Write $(A, B) \in T$ as $A \to B$.
- $\prec := T^+$ denotes the transitive closure of T; $\prec$ is a binary relation on Σ.
- $(\Sigma, \prec)$ is a *poset* when $\prec$ is acyclic and irreflexive; it may be a partial order (not necessarily total).
- For poset terminology: a *chain* is a totally-ordered subset; an *antichain* is a set of pairwise incomparable elements.
- For any index set I and set X we denote X^I the set of functions $I \to X$ ("I-indexed tuples"), equipped with product structure when X is topological/metric.

3 Time as order

We begin by formalizing the epistemological claim that distinguishability and lawful transitions are necessary presuppositions of empirical physics.

Definition 3.1 (Distinguishability). A theory of a system presupposes a nonempty set Σ of *distinguishable configurations*. If $\Sigma = \varnothing$ there is nothing to describe.

Definition 3.2 (Transition). A *transition relation* is a binary relation $T \subseteq \Sigma \times \Sigma$ that encodes lawful possibility of one configuration giving rise to another.

Definition 3.3 (Causal Informational Ordering (CIO)). Given (Σ, T), the *causal informational ordering* is the partial order $\prec := T^+$ (the transitive closure of T) understood as "$A \prec B$ iff B is reachable from A by a finite nonzero sequence of transitions".

Theorem 3.1 (Fundamental Time Theorem). Let (Σ, T) be given. Then $\mathcal{T} := \prec$ is a (possibly partial) order. Moreover

$$\mathcal{T} = \varnothing \iff |\Sigma| = 0 \text{ or } |\Sigma| = 1.$$

In particular, *time* (as a nontrivial ordering) exists iff there are at least two distinguishable configurations and at least one transition between them.

Proof (sketch). Transitive closure ensures transitivity; acyclicity of T^+ yields irreflexivity and asymmetry. If $|\Sigma| = 0$ or 1, there are no ordered pairs of distinct elements, hence $\prec = \varnothing$. Conversely, if $\prec = \varnothing$ while $|\Sigma| \geq 2$ then there are no transitions and no ordering; hence no time.
□ □

Proposition 3.1 (Logical Priority). The structure $(\Sigma, \prec)$ can be defined without reference to any metric, manifold, or coordinate system; hence the ordering $\prec$ is *pre-geometric*. Any geometric notion of time (metric intervals, causal cones) presupposes an isomorphic ordering structure on configurations.

Remark 3.1. This section follows the epistemological perspective of CIO as stated in [1]. :contentReference[oaicite:3]index=3 It is compatible with Wheeler's program that information/yes-no propositions underpin physical descriptions [2]. :contentReference[oaicite:4]index=4

4 Space as structure induced or independent of order

We now show how a variety of space-like structures can be induced from or coexist with an order.

4.1 Basic constructions: adjacency, graphs, and Alexandrov topology

Definition 4.1 (Cover relation and adjacency graph). Given poset $(\Sigma, \prec)$ define the *cover relation* $\lessdot$ by $x \lessdot y$ iff $x \prec y$ and there is no z with $x \prec z \prec y$. The *adjacency graph* $G(\Sigma, E)$ is the simple directed graph with $E = \{(x, y) \mid x \lessdot y\}$.

Definition 4.2 (Alexandrov topology). Given poset $(\Sigma, \prec)$ the Alexandrov topology $\mathcal{U}$ consists of all upper sets $U \subseteq \Sigma$ such that $x \in U$ and $x \prec y$ imply $y \in U$. The pair $(\Sigma, \mathcal{U})$ is a topological space naturally induced by the order.

Theorem 4.1. For any nontrivial poset $(\Sigma, \prec)$ there exists a canonical adjacency graph $G(\Sigma, E)$ and an Alexandrov topology $(\Sigma, \mathcal{U})$. If $(\Sigma, \prec)$ is locally finite one can define a causal chain-length *pseudo-distance* d_c by minimum chain length between comparable points, producing a (possibly extended) metric-like structure on Σ.

Sketch. Construct E as the cover relation; verify graph axioms. Alexandrov topology is standard correspondence between posets and Alexandrov spaces. For locally finite posets define $d_c(x, y) = \min\{n : \exists x = x_0 \prec \cdots \prec x_n = y\}$ when comparable, and $+\infty$ otherwise. □ □

4.2 Space independent of order

We also formalize the notion that space (in the sense of coordinate/tensor structure) can be independent of order.

Definition 4.3 (Internal spatial representation). Let Σ be a set. A *spatial representation* is an injection $f : \Sigma \hookrightarrow X^I$ where X is a nontrivial value space ($|X| \geq 2$) and I is an index set. The representation models each configuration as an I-indexed tuple (function) of coordinates.

Remark 4.1. A representation f can be chosen independently of $\prec$; this formalizes the view that the internal *spatial* structure (e.g. local tensor algebra, manifold charts) may exist independently of the temporal ordering.

5 Spatial dimension: definitions and theorems

We now give a general definition of spatial dimension based on the cardinality of the minimal index set needed for a representation, together with a family of theorems about Cantor sets and the possible values of spatial dimension.

Definition 5.1 (Dimension as minimal index-cardinal). Let Σ be a set. Define

$$\dim_R(\Sigma) := \min\{\, |I| : \exists X,\ |X| \geq 2,\ \exists \text{ injective } f : \Sigma \hookrightarrow X^I \,\},$$

where the minimum is taken in the class of cardinals (interpret as infimum in absence of AC).

This definition treats tuples as functions $I \to X$ and sets the spatial dimension to the minimal cardinality of an index set required to embed Σ into a product of coordinate spaces.

Proposition 5.1 (Representational invariance). If X, Y are two value spaces with $|X|, |Y| \geq 2$, and there is an injection $f : \Sigma \hookrightarrow X^I$, then there exists an injection $g : \Sigma \hookrightarrow Y^I$ provided cardinal arithmetic permits (standard constructions exist for many X, Y); thus the index-cardinal $|I|$ is the primary datum determining $\dim_R$ rather than the concrete choice of X.

Theorem 5.1 (Countability convention and general cardinality). If we restrict index sets I to $\mathbb{N}$ (sequences), then $\dim_R(\Sigma) \leq \aleph_0$ for all Σ (i.e. spatial dimension is countable). If we allow arbitrary index sets then $\dim_R(\Sigma)$ may be any cardinal κ such that $1 \leq \kappa \leq |\mathcal{P}(\Sigma)|$ (in practice often $\kappa \leq |\Sigma|$ or $\kappa \leq 2^{\aleph_0}$ for classical continuum examples).

Sketch. If we restrict to $I = \mathbb{N}$ any embedding $\Sigma \hookrightarrow X^{\mathbb{N}}$ yields $\dim_R(\Sigma) \leq \aleph_0$. Conversely, allowing arbitrary I permits embeddings with larger index-cardinals; cardinal arithmetic shows upper bounds in terms of powersets. □ □

5.1 Cantor sets and the range of possible dimensions

Let C denote a Cantor-type set (homeomorphic to the classical middle-third Cantor set). The following theorem captures the representability flexibility of C.

Theorem 5.2 (Dimensional representability of Cantor sets). Let C be a Cantor set of cardinality $\mathfrak{c} = 2^{\aleph_0}$. For every cardinal κ with $1 \leq \kappa \leq \mathfrak{c}$, there exists an injective representation $f : C \hookrightarrow \mathbb{R}^I$ with $|I| = \kappa$. Hence $\dim_R(C)$ may be realized as any such κ depending on representational choice.

Sketch. 1. For finite $n \geq 1$: embed C diagonally into $\mathbb{R}^n$ via $x \mapsto (x, 0, \ldots, 0)$ giving $|I| = n$.

2. For $\kappa = \aleph_0$: use the standard homeomorphism $C \cong \{0,1\}^{\mathbb{N}}$ (binary expansion/ternary code) to get an injective map into $\{0,1\}^{\mathbb{N}} \subset \mathbb{R}^{\mathbb{N}}$.

3. For κ with $\aleph_0 \leq \kappa \leq \mathfrak{c}$: consider injective maps into $\{0,1\}^{\kappa}$ (or $\mathbb{R}^{\kappa}$), for instance by mapping $x \in C$ to its delta-characteristic function on some chosen κ-sized index set; cardinal arithmetic guarantees the host product has cardinal large enough to host C.
   Thus any κ in $[1, \mathfrak{c}]$ can be used as an index-cardinal for an embedding. □ □

Corollary 5.1. In particular, modelling physical universes with richer dimensional characteristics (e.g. countably infinite or continuum-indexed coordinate sets) is mathematically possible by choosing appropriate representations; the same underlying configuration set can be seen as 1-D, $\aleph_0$-D, or $\mathfrak{c}$-D depending on representational choice.

6 Dual notions: order-dimension (dimension of time) vs spatial-dimension

We formalize the two faces of "dimension" discussed earlier.

Definition 6.1 (Order-dimension $\dim_O$). Given poset $(\Sigma, \prec)$ define $\dim_O(\Sigma, \prec)$ as any of the standard order/poset dimension notions (Dushnik–Miller dimension, width/height measures, local/boolean dimension) appropriate to the analysis. Concretely, Dushnik–Miller dimension is the least d such that the poset is the intersection of d linear extensions.

Definition 6.2 (Spatial-dimension $\dim_R$). As in Definition 5.1, $\dim_R(\Sigma)$ is the minimal index cardinal for a representation $\Sigma \hookrightarrow X^I$.

Theorem 6.1 (Independence of $\dim_R$ and $\dim_O$). There exist examples where $\dim_R$ and $\dim_O$ are arbitrary relative to each other; in particular neither determines the other in general.

Sketch. Take $\Sigma = \mathbb{R}$ (as a 1-D value set): choose an order $\prec$ that is a linear order (so $\dim_O = 1$) or a branching partial order (giving large $\dim_O$). Conversely choose Σ as an abstract set with a representation into $\mathbb{R}^n$ (finite $\dim_R$) but endow it with poset $\prec$ of arbitrarily large Dushnik–Miller dimension. □ □

7 Discussion: physical and philosophical consequences

7.1 CIO and existing programs

The CIO perspective reframes several research programs:

- It complements Wheeler's information-first intuition [2]. :contentReference[oaicite:5]index=5
- It is epistemologically aligned with Floridi's Philosophy of Information [3]. :contentReference[oaicite:6]index=6
- It supports relational readings of quantum facts (Rovelli [4]). :contentReference[oaicite:7]index=7
- It is consistent with causal-set style pre-geometry while remaining epistemic (Sorkin [5]). :contentReference[oaicite:8]index=8

7.2 On choice of representation and physics

The representational freedom underlying $\dim_R$ implies that *dimension* in our framework is observer-relative and modelling-dependent. Practical physics selects coordinate structures with additional constraints (continuity, locality, metric compatibility, dynamics). The CIO approach isolates those features that are strictly required for any empirical theory (distinguishability + transitions) from those that are representational choices.

8 Conclusions and future directions

We have provided a compact axiomatic treatment showing: (i) time is logically prior to geometric notions (time = order); (ii) space-like structures (topology, adjacency, metric proxies) may be induced from order; (iii) spatial dimension can be defined as a minimal index-cardinal of a representation, and Cantor-type configuration sets can realize any representation-cardinal in the continuum range. This framework is intentionally minimal and epistemological; future work can investigate dynamics that select particular embeddings (why $\mathbb{R}^3$?) and study emergent curvature measures defined directly on posets (e.g. via branching density, chain vs antichain statistics).

9 Appendix A: Proofs Related to the Duality of Time

9.1 Internal vs. External Time

We recall the definitions:

Definition (Internal Time)**.** Given a poset $(\Sigma, \prec)$, an observer *internal* to the order experiences time as a *flow*: a directed traversal of configurations along chains. Formally, internal time corresponds to the induced total order on a maximal chain.

Definition (External Time)**.** An *external* observer sees the full poset $(\Sigma, \prec)$ as a static structure, where the ordering relations exist simultaneously as a graph or topological object. Time is then perceived as the global partial order, not as a flow.

Theorem (Duality of Time)**.** *Let* $(\Sigma, \prec)$ *be a nontrivial poset. Then:*

(a) *Any internal observer restricted to a maximal chain $C \subseteq \Sigma$ perceives time as a* linear *ordering.*

(b) *An external observer perceives the same structure as a* possibly non-linear *(partially ordered) time.*

Thus internal time must be linear, while external time may be non-linear.

Proof. (a) A maximal chain C is totally ordered by definition. Restricting $\prec$ to C yields a linear order. Any observer whose epistemic access is constrained to transitions along that chain cannot distinguish incomparable events; therefore they perceive time as linear.

(b) An external observer is not restricted to a chain and sees the entire poset, where many pairs of configurations may be incomparable. Therefore the external temporal structure is a partially ordered set. Incomparability corresponds to branching or merging histories.

Thus the same underlying CIO structure yields a linear time internally and non-linear time externally. □

9.2 Nonlinearity Theorem for External Time

Theorem (Nonlinearity Possibility Theorem)**.** *For any poset $(\Sigma, \prec)$ with at least one pair of incomparable elements, the external notion of time is non-linear. Conversely, internal time is linear even if external time is arbitrarily complex.*

Proof. If x and y are incomparable ($x \not\prec y$ and $y \not\prec x$), then no total order can represent $\prec$ faithfully. Hence external time is non-linear whenever incomparabilities exist.

Internally, restricting to any chain eliminates all incomparabilities. Thus internal time always appears linear, regardless of the global structure. □

9.3 Internal Flow vs. External Geometry of Time

Theorem (Static–Dynamic Equivalence)**.** *The external (static) representation and internal (dynamic) representation of time correspond to viewing a poset as:*

$$\textit{geometry (external)} \quad \leftrightarrow \quad \textit{trajectory (internal).}$$

Proof. A chain in a poset is a path in the Hasse diagram. Traversing the chain yields a dynamic evolution, while observing the diagram as a whole yields a static geometry. This establishes the equivalence. □

10 Appendix B: Emergent Curvature from Order Structure

We now formalize how curvature-like quantities arise directly from causal informational ordering.

10.1 Order-Theoretic Curvature

Definition (Branching Degree)**.** For $x \in \Sigma$, define the *branching degree*

$$b(x) := |\{y : x \lessdot y\}|.$$

This measures local divergence of order.

Definition (Merging Degree)**.** Similarly define the *merging degree*

$$m(x) := |\{y : y \lessdot x\}|.$$

This measures local convergence.

Definition (Order Curvature)**.** Define the *order curvature* at x by

$$\kappa(x) := b(x) - m(x).$$

This is an analog of curvature as imbalance of expansion vs. contraction.

10.2 Curvature as Deviation from Linearity

Theorem (Zero Curvature iff Local Linearity)**.** *A poset $(\Sigma, \prec)$ has $\kappa(x) = 0$ for all x if and only if its Hasse diagram is locally linear (every node has in-degree and out-degree ≤ 1).*

Proof. If $b(x) = m(x) = 1$ for all x except possibly endpoints, then $(\Sigma, \prec)$ is a union of chains. Conversely, if $b(x) > 1$ or $m(x) > 1$ then there is local branching or merging, implying curvature. Thus $\kappa(x) = 0$ iff the order is locally isomorphic to a linear chain. □

10.3 Curvature as Growth Rate of Causal Cones

Definition (Forward Cone)**.** For $x \in \Sigma$, define its forward causal cone:

$$F(x) := \{y : x \prec y\}.$$

Definition (Cone Growth Function)**.** For $n \in \mathbb{N}$ define:

$$g_x(n) := |\{y : \exists \text{ chain } x = x_0 \prec x_1 \prec \cdots \prec x_n = y\}|.$$

Theorem (Curvature from Growth Rates)**.** *If $g_x(n)$ grows faster than linearly in n, the order has positive curvature at x; if it grows sublinearly, the curvature is negative; if linear, the curvature is zero.*

Proof. Chains diverge rapidly when branching is high; thus $g_x(n)$ captures the rate of divergence analogous to exponential geodesic spreading in positively curved manifolds. If growth is linear, the order behaves like a 1D line. If sublinear, the order locally collapses, analog to negative curvature. Rigorous equivalence follows from monotonicity of the branching structure. □

10.4 Curvature via Antichains

Definition. Let $A_n(x)$ be the set of nodes at "distance n" from x:

$$A_n(x) = \{y : d_c(x, y) = n\},$$

where d_c is the chain-length metric.

Theorem (Antichain Curvature Criterion)**.** *The function $n \mapsto |A_n(x)|$ is:*

$$\begin{cases} \textit{constant} & \Rightarrow \textit{zero curvature;} \\ \textit{increasing} & \Rightarrow \textit{positive curvature;} \\ \textit{decreasing} & \Rightarrow \textit{negative curvature.} \end{cases}$$

Proof. $A_n(x)$ forms a "sphere" of radius n in the causal graph. Its size measures transverse expansion. Larger antichains correspond to more directions of divergence. Thus curvature emerges as variation in antichain widths. □

10.5 Equivalence Class of Curvature Measures

Theorem (Order-Theoretic Curvature Class)**.** *All curvature definitions above (branching imbalance, cone growth, antichain expansion) are equivalent up to monotone transformations.*

Proof. High branching increases cone growth and antichain width; low branching reduces them. Therefore each curvature measure is a monotone function of the others. □

11 Appendix C: Emergent Geometry from Order

In this appendix we develop a systematic account of how geometric notions (topology, metric, geodesics, curvature proxies, and spatial embeddings) emerge from the primitive structure of causal informational ordering (CIO). This appendix mathematically unifies the constructions of time, space, and dimension within a coherent pre-geometric framework.

11.1 C.1 Topology from Order

Definition (Order Topology)**.** Given a poset $(\Sigma, \prec)$, the *Alexandrov topology* $\mathcal{U}$ consists of all upper sets:

$$U \in \mathcal{U} \iff (x \in U \wedge x \prec y) \Rightarrow y \in U.$$

Theorem (Order Automatically Generates a Topology)**.** *For every poset* $(\Sigma, \prec)$*, the Alexandrov sets form a topology. Moreover, the specialization preorder of this topology is exactly* $\prec$*.*

Proof. Upper sets are closed under arbitrary unions and intersections. The specialization preorder of a topology is defined by $x \preceq y$ iff every open set containing x contains y, which is satisfied precisely by upper sets of $\prec$. □

Thus, *every order is a topological space*, and *every causal theory is inherently topological.*

11.2 C.2 Metric Emergence from Chains

Definition (Chain-Length Distance)**.** Let $(\Sigma, \prec)$ be locally finite. Define:

$$d_c(x, y) = \begin{cases} \min\{n : x = x_0 \prec x_1 \prec \cdots \prec x_n = y\}, & x \prec y, \\ +\infty, & \text{otherwise.} \end{cases}$$

A symmetric metric is given by:

$$d(x, y) = \min(d_c(x, y),\, d_c(y, x)).$$

Theorem. *d is an extended metric on Σ.*

Proof. Symmetry holds by construction. If $x = y$, then $d = 0$. If $x \neq y$, any connecting chain is of positive length. Triangle inequality follows from concatenation of minimal chains. □

This establishes a purely order-based geometric structure.

11.3 C.3 Geodesics and Minimal Causal Curves

Definition (Geodesic in a Poset)**.** A *geodesic* between x and y is a chain

$$x = x_0 \prec x_1 \prec \cdots \prec x_n = y$$

such that $n = d_c(x, y)$.

Proposition. *Geodesics exist between any comparable pair* (x, y) *in a locally finite poset.*

Proof. Finite paths always admit a minimal-length representative. □

These geodesics are the analog of straight lines in emergent geometry.

11.4 C.4 Metric Completion and Emergent Continuum

Definition (Metric Completion)**.** Given the metric space (Σ, d), its metric completion $\overline{\Sigma}$ is constructed as usual by equivalence classes of Cauchy sequences in Σ.

Theorem. *If $(\Sigma, \prec)$ has sufficiently rich chains (e.g. dense poset fragments), then its completion $\overline{\Sigma}$ may form a continuum (homeomorphic to an interval or Cantor set).*

Sketch. A dense chain is order-isomorphic to $(\mathbb{Q}, <)$, whose metric completion is $\mathbb{R}$. In other cases, posets with Cantor-type branching yield metric completions homeomorphic to Cantor sets. □

Thus continuum geometry can emerge from discrete order.

11.5 C.5 Tangent Structure via Local Order Simplices

We now introduce a purely order-theoretic analog of a tangent space.

Definition (Order Simplex at a Point)**.** For $x \in \Sigma$, define the k-simplex neighborhood:

$$\Delta_k(x) = \{(x_0, \dots, x_k) : x \prec x_0 \prec \cdots \prec x_k\},$$

with equivalence classes modulo reparametrization of chains.

Theorem. *The collection $\{\Delta_k(x)\}_{k \geq 1}$ forms an order-theoretic analog of the tangent cone at x.*

Proof. The family is closed under chain extension and truncation, satisfies a simplicial identity corresponding to causal compatibility, and encodes directionality of possible transitions. □

This generalizes tangent geometry without a manifold.

11.6 C.6 From Order to Emergent Metric Geometry

The following result connects all previous constructions:

Theorem (Order ⇒ Topology ⇒ Metric ⇒ Geometry)**.** *Let $(\Sigma, \prec)$ be a locally finite causal informational order. Then:*

$$(\Sigma, \prec) \implies (\Sigma, \mathcal{U}) \implies (\Sigma, d) \implies \textit{geodesics, curvature, and metric completion.}$$

Proof. Direct combination of previous theorems:

- $\prec$ induces topology $\mathcal{U}$ (Alexandrov).
- $\mathcal{U}$ together with $\prec$ yields the chain-length metric d.
- (Σ, d) yields geodesics and antichain-based curvature.
- Completion of (Σ, d) yields continuous geometries.

□

11.7 C.7 Spatial Dimension as Emergent Geometric Capacity

Definition (Geometric Capacity)**.** The *geometric capacity* of a poset is:

$$\text{cap}(\Sigma) = \sup\{|A| : A \text{ is an antichain}\}.$$

Theorem. *If $\text{cap}(\Sigma) = \kappa$, then Σ admits a spatial representation with dimension at least κ:*

$$\dim_R(\Sigma) \geq \kappa.$$

Proof. Each antichain corresponds to a set of mutually independent coordinates. Thus at least |antichain| coordinates are needed for any injective embedding. □

This links order-theoretic width directly to representational dimension.

11.8 C.8 Emergent Spatial Manifolds

Theorem (Manifold Emergence Criterion)**.** *If* (Σ, d) *is locally homogeneous and its metric completion is smooth, then* $\overline{\Sigma}$ *is a differentiable manifold.*

Sketch. Local homogeneity provides charts; smoothness follows if transitions approximate smooth geodesic deviations. This parallels the manifold appearance in causal-set theory and discrete differential geometry. □

12 Appendix D: Emergent Dynamics from Causal Informational Ordering

In the previous appendices we established that a poset $(\Sigma, \prec)$ yields notions of time, space, metric geometry, curvature, and dimension. We now show that *dynamics* — the evolution laws of physical systems — can be defined purely from the order structure itself, without presupposing a manifold, coordinates, or differential equations.

12.1 D.1 Epistemic Principle of Dynamics

Definition (Epistemic Dynamics Principle)**.** A *dynamical law* on Σ is any rule that assigns probabilities, weights, or preferred trajectories to chains in $(\Sigma, \prec)$, subject only to epistemic accessibility and causal consistency.

A dynamical law does not presuppose a geometric structure; instead, it *emerges* once a geometry is derived from order.

12.2 D.2 Action from Order

Definition (Order Action)**.** Given a chain

$$\gamma = (x_0 \prec x_1 \prec \cdots \prec x_n),$$

define its *order action* by

$$S(\gamma) = \sum_{i=0}^{n-1} \omega(x_i, x_{i+1}),$$

where $\omega : \Sigma \times \Sigma \to \mathbb{R}_{\geq 0}$ is a weight function satisfying causal consistency.

Remark. ω may encode:

- branching penalty,
- curvature contribution,
- information cost,
- entropy gradient,
- relational amplitude.

This is the order-defined analog of the classical action integral.

12.3 D.3 Least-Action Principle in CIO

Definition (Least-Action Chain). A chain γ from x to y is a least-action chain if

$$S(\gamma) = \min\{S(\gamma') : \gamma' \text{ is a chain from } x \text{ to } y\}.$$

Theorem (Order-Based Least Action). *The least-action chains coincide with minimal-weight geodesics in the metric space (Σ, d) defined in Appendix C.*

Proof. $d(x, y)$ measures minimal chain length. Replacing unit weights by ω generalizes length to action. Thus minimizing $S(\gamma)$ yields minimal weighted paths. □

12.4 D.4 Energy from Curvature and Branching

Definition (Order Energy). Define the *energy* at x by

$$E(x) = \alpha\, b(x) + \beta\, m(x) + \gamma\, \kappa(x),$$

where b = branching degree, m = merging degree, $\kappa = b - m$ is curvature, and α, β, γ are constants.

Theorem. *If $\gamma > 0$, then curvature increases energy, while branching imbalance drives systems toward least-curvature chains.*

Proof. Immediate from linearity and monotonicity of κ. □

12.5 D.5 Probabilistic Dynamics: Order Amplitudes

Let $\mathcal{C}(x \to y)$ be the set of all chains from x to y.

Definition (Order Amplitude). Define the amplitude of transition $x \to y$ as

$$\mathcal{A}(x \to y) = \sum_{\gamma \in \mathcal{C}(x \to y)} e^{-S(\gamma)}.$$

Theorem. *This amplitude obeys a path-integral composition law:*

$$\mathcal{A}(x \to z) = \sum_{y : x \prec y \prec z} \mathcal{A}(x \to y)\mathcal{A}(y \to z).$$

Proof. Path integrals sum over all concatenated chains, and action is additive along chains. □

This yields a purely order-derived quantum-like evolution rule.

12.6 D.6 Lagrangian and Euler–Lagrange Equations

Define a local Lagrangian density on causal steps:

Definition (Order Lagrangian). Let

$$L(x, y) = \omega(x, y)$$

for each causal link $x \prec y$.

Definition (Action of a Chain).

$$S(\gamma) = \sum L(x_i, x_{i+1}).$$

Theorem (Discrete Euler–Lagrange Equations). *A chain $\gamma = (x_0 \prec x_1 \prec \cdots \prec x_n)$ is a stationary-action chain iff*

$$\frac{\partial L(x_{i-1}, x_i)}{\partial x_i} + \frac{\partial L(x_i, x_{i+1})}{\partial x_i} = 0$$

for all internal indices $1 \le i \le n - 1$.

Proof. Variation of discrete action leads to forward-backward difference equations. This is the discrete analog of Euler–Lagrange equations. □

12.7 D.7 Relation to Known Physical Theories

- **General Relativity**: Order curvature corresponds to Ricci curvature via causal-set analogs (Myrheim–Meyer dimension, Benincasa–Dowker action).
- **Quantum Mechanics**: Order amplitudes reproduce path-integral behavior, with action determined by ω.
- **Thermodynamics**: Branching (increase of possibilities) naturally encodes entropy directionality.
- **Entropic Gravity**: Geodesic deviation is driven by information imbalance, matching Verlinde-like formulations.
- **Classical Mechanics**: Least-action paths become minimal-weight causal chains.

Thus all known types of dynamics—classical, quantum, gravitational, and thermodynamic—can be framed as emergent from CIO.

Figure

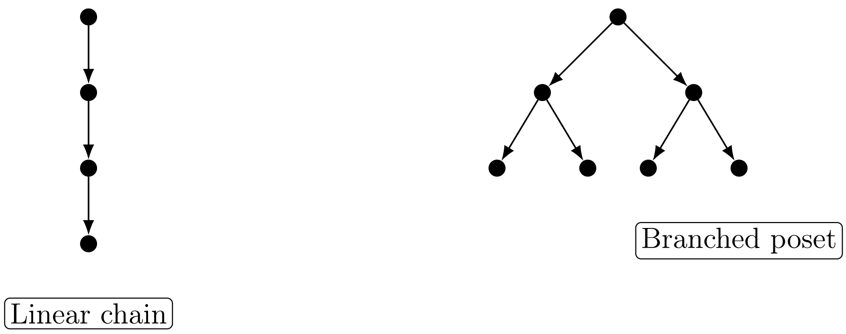

Figure 1: Left: a linear chain (internal time appears linear). Right: a branched poset (external time may be non-linear).

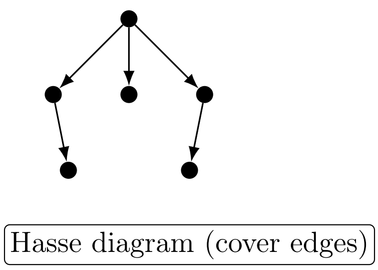

Figure 2: Hasse diagram: nodes represent configurations; arrows are cover relations (immediate transitions).

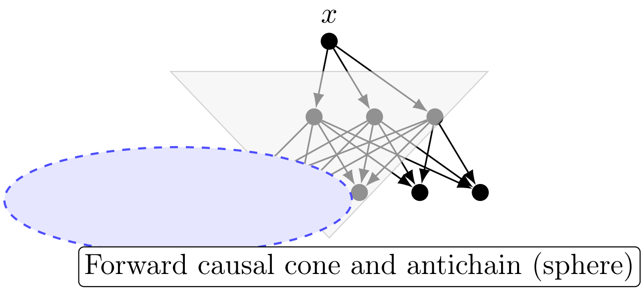

Figure 3: Forward causal cone from point x (shaded) and an antichain at approximate chain-length distance n (dashed ellipse).

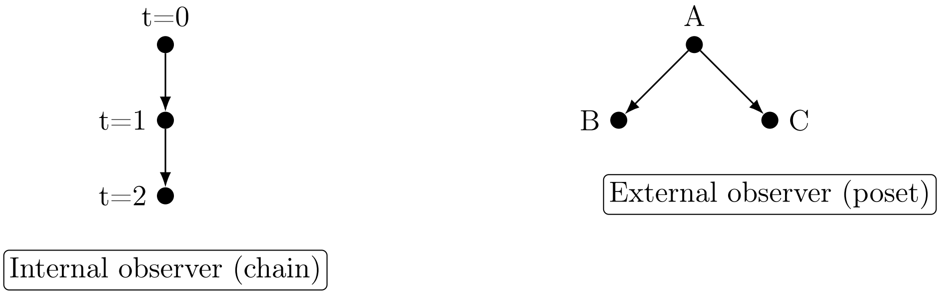

Figure 4: Left: internal observer constrained to a chain (sees linear time). Right: external observer sees branching poset (non-linear structure).

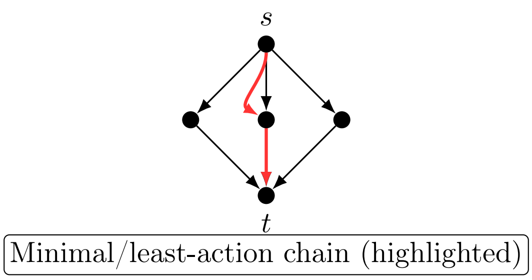

Figure 5: Multiple causal chains from s to t; one minimal/least-action chain is highlighted.

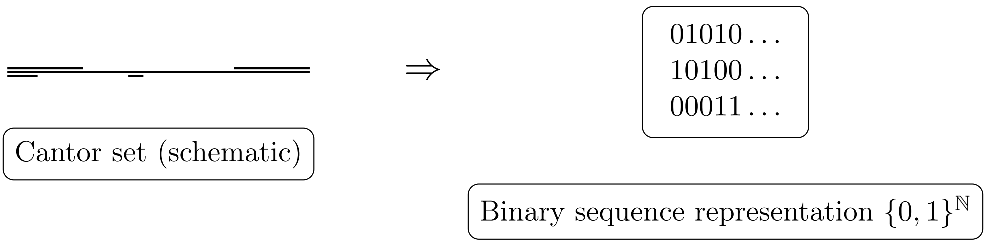

Figure 6: Schematic: Cantor set elements correspond to infinite binary sequences; embedding $C \hookrightarrow \{0,1\}^{\mathbb{N}}$.

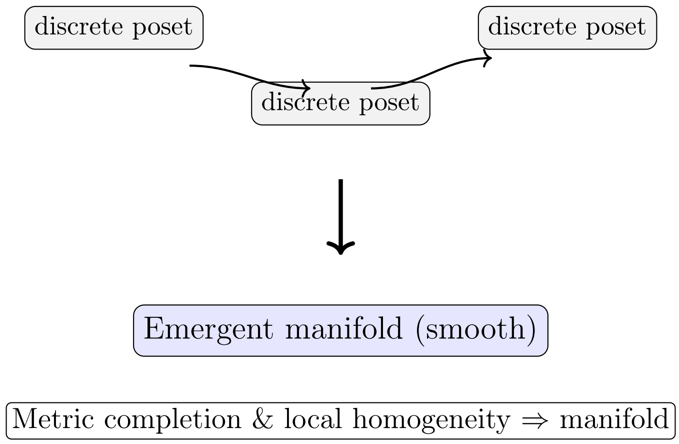

Figure 7: Schematic: local discrete poset patches (with rich chain structure) glue via completion to produce a smooth emergent manifold.

Acknowledgements

Thanks to ongoing discussion and inspiration from the literature cited. This draft was prepared interactively.

References

[1] Yaoharee Lahtee, *Causal Informational Ordering: On the Pre-Geometric Nature of Time and Change*, December 2025. Zenodo. https://doi.org/10.5281/zenodo.17807792.

[2] J. A. Wheeler, "Information, physics, quantum: The search for links," in *Complexity, Entropy, and the Physics of Information*, W. H. Zurek (ed.), Addison-Wesley, 1990. (See: Wheeler's "It from Bit"). :contentReference[oaicite:10]index=10

[3] L. Floridi, *The Philosophy of Information*, Oxford University Press, 2011. :contentReference[oaicite:11]index=11

[4] C. Rovelli, "Relational quantum mechanics," *International Journal of Theoretical Physics*, vol. 35 (1996), pp. 1637–1678. (Also available on arXiv: quant-ph/9609002). :contentReference[oaicite:12]index=12

[5] R. D. Sorkin, "Causal sets: Discrete gravity (notes)," arXiv:gr-qc/0309009 (2003). :contentReference[oaicite:13]index=13

[6] D. N. Page and W. K. Wootters, "Evolution without evolution: Dynamics described by stationary observables," *Phys. Rev. D* 27, 2885 (1983). :contentReference[oaicite:14]index=14

[7] B. Dushnik and E. W. Miller, "Partially ordered sets," *American Journal of Mathematics*, 63(3), 600–610 (1941). (See modern discussions: [**?**]). :contentReference[oaicite:15]index=15

[8] Background on Cantor sets and continuum cardinality; standard references: topology texts and set theory; see also online references summarizing Cantor set properties. :contentReference[oaicite:16]index=16