From Discrete Information to Continuous Entropy: Toward a Unified Ontology of Randomness in Physics

Aslan Alwi, Munirah

Independent Researcher at Almudaya Research Institute

Department of Informatics Engineering, Faculty of Engineering, Universitas Muhammadiyah Ponorogo, Indonesia

---

Abstract

This paper proposes that the apparent symmetry between Shannon's informational entropy and Boltzmann's thermodynamic entropy conceals a fundamental ontological asymmetry: informational randomness operates on a discrete domain, while physical entropy operates on a continuous one. By extending the domain of entropy to include not only spatial and momentum coordinates but also energy and time, we develop a *Fundamental Entropy Functional* that unifies discrete and continuous randomness within a single theoretical structure. Shannon and Boltzmann entropies emerge as distinct projections—one by discretization, the other by marginalization—of this universal functional. The resulting framework reinterprets entropy not as disorder but as an ontic field from which physical law, temporal directionality, and information themselves arise. Consequently, the universe cannot be reduced to a Turing-computable process; it is a non-computable entropic continuum that continuously differentiates itself into structure and meaning.

Keywords: Entropy, Information Theory, Statistical Mechanics, Ontology, Emergent Law, Non-Turing Computation

---

URL: aslan.alwi@umpo.ac.id (Munirah), munirah.mt@umpo.ac.id (Munirah)

1. Introduction

2. Introduction

The concept of entropy stands at the crossroads of physics, information theory, and philosophy of knowledge. Since the mid-twentieth century, a formal similarity between Shannon's information entropy [1] and Boltzmann's thermodynamic entropy [2] has often been interpreted as evidence that the physical world is, in some sense, informational in nature. This interpretation underlies the idea of a "digital universe" or even a "computational cosmos" in which the evolution of matter and energy could be viewed as the execution of an underlying algorithm [**?** **?** 3].

However, formal similarity does not necessarily imply ontological equivalence. While both entropies share the same functional form—the expectation value of $-\log p$—they differ profoundly in the *domain* on which their respective probabilities are defined. Shannon entropy quantifies randomness in a countable space of discrete symbols, whereas Boltzmann (or Gibbs) entropy evaluates randomness over a continuous phase space of positions and momenta. In other words, Shannon entropy operates within the arithmetic of the natural numbers, while Boltzmann entropy is defined over the continuum of the real (or even complex) numbers that describe the measurable magnitudes of the physical world.

Recognizing this distinction has deep philosophical and physical implications. If the randomness of information is fundamentally discrete, and the randomness of physical reality is fundamentally continuous, then informational randomness is not co-extensive with physical randomness. Rather, informational entropy is a *subset* or *projection* of a more comprehensive entropic field that operates on a continuous manifold. In this sense, information corresponds to discretized samples—finite partitions—of an underlying continuum of uncertainty. The informational universe is thus a symbolic shadow of a deeper entropic reality.

This observation resonates with ongoing debates about the computational reducibility of nature. If all physical phenomena were ultimately describable by a finite algorithm, then all randomness should be representable by discrete probability distributions. Yet empirical and theoretical advances in quantum mechanics, statistical physics, and cosmology increasingly suggest that nature exhibits forms of randomness that cannot be exhaustively captured by discrete models [4, 5, 6, 7]. In quantum theory, for instance, the wave function evolves over a continuous Hilbert space, and measurement discretization arises only when the continuum is projected onto a finite set of outcomes. Similarly, in thermodynamics and field theory, energy and spacetime degrees of freedom fluctuate continuously even when macroscopic observables appear quantized.

These considerations motivate a renewed examination of the foundational equations of entropy. We propose that the Boltzmann expression itself may be incomplete: it quantifies the randomness of particle configurations in phase space, but not the possible randomness of energy, space, and time as entities subject to fluctuation. In other words, Boltzmann entropy may be a partial projection or reduction of a more general entropic functional that operates over an extended manifold including spatial, energetic, and temporal coordinates.

The purpose of this work is therefore twofold: first, to distinguish rigorously between discrete (informational) and continuous (physical) randomness, and second, to propose a *Fundamental Entropy Functional* defined on an extended space that unifies these domains. By doing so, we aim to clarify why the Shannon and Boltzmann formulas appear formally similar yet measure different ontological strata of reality, and to open a path toward an understanding of entropy as the continuous substrate from which discrete information emerges.

3. The Two Domains of Randomness

Entropy, as both a mathematical and physical concept, quantifies uncertainty. Yet the mathematical nature of the domain over which this uncertainty is defined determines the ontological status of the entropy itself. To develop the argument that informational entropy is a projection of physical entropy, we must first distinguish between the two fundamental domains of randomness: the discrete and the continuous.

3.1. *Discrete randomness and the arithmetic of information*

In Shannon's framework, randomness arises in a *countable* set of possibilities. Each possible outcome is labeled by an index $i \in \mathbb{N}$, and the system is fully characterized by a probability mass function p_i, satisfying $\sum_i p_i = 1$. The associated entropy is

$$H = -k \sum_{i=1}^{N} p_i \log p_i. \tag{1}$$

Here, the domain of definition is not the continuum of physical quantities, but the discrete set of symbolic outcomes. The arithmetic is that of the natural numbers $\mathbb{N}$, the same arithmetic underlying all symbolic computation and algorithmic processing. Thus, Shannon entropy measures the degree of uncertainty within a symbolic or linguistic system: a message, a string, or a finite sequence of distinguishable events. Randomness in this context is a property of symbol generation.

This discreteness underlies the notion of *information* itself. Information exists only when alternatives can be distinguished; it is the enumeration of difference. Consequently, informational entropy cannot exceed the expressive capacity of a countable alphabet. No matter how complex, every informational system presupposes a finite or denumerable set of possible symbols. From this it follows that the randomness measured by Eq. (1) is inherently *quantized*. It is randomness that has already been sampled and symbolically encoded.

3.2. *Continuous randomness and the arithmetic of existence*

By contrast, Boltzmann and Gibbs defined entropy for systems described by continuous phase-space coordinates. Let $\rho(x, p)$ be the probability density over positions and momenta of particles, normalized such that $\int \rho(x, p)\, dx\, dp = 1$. The Gibbs entropy is given by

$$S = -k_B \int \rho(x, p) \ln \rho(x, p)\, dx\, dp. \tag{2}$$

In this case, the probabilities are not assigned to countable outcomes but to infinitesimal regions of a continuous manifold. The domain of randomness is therefore the set of real numbers $\mathbb{R}$, or more generally, a real manifold $\mathcal{M} \subseteq \mathbb{R}^{2n}$. Randomness here is not about symbolic alternatives but about the continuous spread of existence itself—the distribution of matter and motion through a continuum of states.

This distinction has profound implications. While Eq. (1) describes the *randomness of representation*, Eq. (2) describes the *randomness of being*. The former measures uncertainty within an informational syntax; the latter measures uncertainty inherent to the fabric of physical reality. Boltzmann entropy, therefore, does not count possible symbols but possible configurations of the universe.

3.3. Hierarchical relation: information as projection

The relation between these two kinds of randomness can be expressed formally. Let the continuous random variable X have probability density function $\rho(x)$ on a measurable space $(\mathbb{R}, \mathcal{B}, \mu)$. Consider a partition of this space into measurable cells C_i such that $\bigcup_i C_i = \mathbb{R}$ and $C_i \cap C_j = \emptyset$ for $i \neq j$. Define the discrete probabilities

$$p_i = \int_{C_i} \rho(x)\, d\mu(x). \tag{3}$$

If we now compute the Shannon entropy of these p_i, we obtain

$$H = -k \sum_i p_i \log p_i. \tag{4}$$

This quantity represents the information entropy of a discretized measurement of a continuous system. Thus, the Shannon entropy is a *projection* of the continuous entropy onto a finite partition induced by the act of observation or quantization.

In the limit where the cells C_i shrink to infinitesimal size, the discrete sum tends toward the differential form

$$S_c = -k \int \rho(x) \ln \rho(x) \, d\mu(x),$$

recovering the continuous entropy up to an additive constant depending on the unit of measure [8, 9]. Hence, informational entropy is embedded within physical entropy as its discretized expression.

3.4. Ontological interpretation

The consequence of this hierarchy is philosophical as well as mathematical. If discrete randomness (information) is always the result of a partition or projection of a deeper continuous randomness (entropy), then what we call "information" is not an intrinsic property of the universe, but a relational property arising from the interface between the continuum and a discretizing observer. In other words, information is how continuous entropic reality becomes knowable. The Shannon domain is a symbolic approximation, while the Boltzmann domain approximates the continuum itself.

From this perspective, the equivalence of the Shannon and Boltzmann equations is not an identity but a resonance: they share a mathematical structure because both are shadows cast by a deeper functional symmetry in the continuum of randomness. We thus arrive at a hierarchical ontology:

$$\text{Informational randomness (discrete)} \subset \text{Physical randomness (continuous)}.$$

This inclusion relation provides the conceptual basis for re-examining the completeness of Boltzmann's expression itself, as we explore in the following section.

3.5. Discrete randomness and the arithmetic of information

In Shannon's framework, randomness arises in a *countable* set of possibilities. Each possible outcome is labeled by an index $i \in \mathbb{N}$, and the system is characterized by probabilities

p_i satisfying $\sum_i p_i = 1$. The corresponding entropy is

$$H = -k \sum_{i=1}^{N} p_i \log p_i.$$

This measures uncertainty within symbolic systems. Randomness is therefore already quantized: a property of representation rather than being.

3.6. Continuous randomness and the arithmetic of existence

Boltzmann and Gibbs, in contrast, define entropy for distributions over continuous coordinates:

$$S = -k_B \int \rho(x,p) \ln \rho(x,p)\, dx\, dp.$$

Here, randomness expresses not linguistic uncertainty but the continuous distribution of existence itself.

3.7. Hierarchical relation: information as projection

For a continuous random variable X with density $\rho(x)$, partitioning space into cells C_i yields probabilities $p_i = \int_{C_i} \rho(x)\, dx$. The Shannon entropy $H = -k \sum_i p_i \ln p_i$ is thus a *projection* of the continuous entropy onto a discrete partition induced by observation. Formally,

$$\text{Information (discrete)} \subset \text{Entropy (continuous)}.$$

Information is a symbolic discretization of a continuous entropic substrate.

4. On the Incompleteness of the Boltzmann Formula

While Boltzmann's formulation of entropy remains a cornerstone of statistical mechanics, its scope is implicitly limited by the variables over which it is defined. In its classical form,

$$S = -k_B \int \rho(x,p)\, \ln \rho(x,p)\, dx\, dp,$$

the probability density $\rho(x, p)$ describes the distribution of particles in a $2n$-dimensional phase space of positions and momenta. Time t appears only as an external parameter governing dynamical evolution, and energy E enters only as a derived function of (x, p) through the Hamiltonian. The structure is therefore spatial and mechanical, not yet energetic or temporal in its own right.

4.1. Domain restriction and projection

If entropy is to quantify the total randomness of the universe, then it must account for all degrees of freedom that can fluctuate. In Boltzmann's original conception, only microscopic configurations of matter were treated as random, while space, time, and energy were considered fixed backgrounds. Consequently, S measures the randomness of particle *arrangements*, but not the randomness of the background manifold in which those arrangements occur.

Mathematically, the function $\rho(x, p)$ can be viewed as the marginal of a higher-dimensional distribution $\rho(x, p, E, t)$ defined on an extended manifold $\Xi = \{x, p, E, t\}$:

$$\rho(x, p) = \int \rho(x, p, E, t)\, dE\, dt. \tag{5}$$

Substituting this marginal into Boltzmann's formula implies that S is only a *projection* of a potentially richer entropy functional that depends also on E and t. Just as Shannon entropy results from discretizing a continuum, Boltzmann entropy may result from integrating out temporal and energetic fluctuations that are normally invisible to macroscopic thermodynamics.

4.2. Entropy in an extended phase space

Let $\rho(x, p, E, t)$ denote the joint probability density of position, momentum, energy, and time at a microscopic level. Then the natural extension of Boltzmann's formula is

$$S_{\text{ext}} = -k \int \rho(x, p, E, t)\, \ln \rho(x, p, E, t)\, dx\, dp\, dE\, dt. \tag{6}$$

Equation (6) measures the total uncertainty of the system not only with respect to the spatial distribution of particles, but also with respect to energetic and temporal fluctuations. In the limit where $\rho(x, p, E, t) = \rho(x, p)\, \delta(E - E_0)\, \delta(t - t_0)$, one recovers the classical Boltzmann entropy. Thus, the conventional expression is a special case obtained when the energy and temporal degrees of freedom are treated as fixed parameters.

This generalization resonates with several contemporary lines of research: the treatment of time as a thermodynamic variable [5], entropic gravity and the geometric interpretation of entropy [4, 10], and non-extensive statistical frameworks that accommodate correlations and memory effects across scales [11, 12]. All of these suggest that entropy may have a more fundamental, field-like nature, propagating through spacetime rather than being confined to the state of a subsystem.

4.3. Energetic and temporal randomness

The idea that energy and time themselves may fluctuate stochastically is consistent with quantum uncertainty relations and with approaches that assign entropy directly to spacetime regions (such as black-hole thermodynamics and holographic principles). In these contexts, energy density and curvature themselves exhibit statistical variation, implying an "entropic field" defined over spacetime. If such variations are fundamental, then the true measure of entropy must integrate over them.

We may therefore regard S_{ext} not as a numerical quantity but as a *field* $S(x, t)$, locally defined by

$$S(x, t) = -k \int \rho(p, E|x, t)\, \ln \rho(p, E|x, t)\, dp\, dE. \tag{7}$$

Equation (7) represents a local entropy density in spacetime, a quantity that can vary continuously with x and t. The global entropy is then obtained by integrating $S(x, t)$ over the relevant domain. This perspective converts entropy from a scalar summary into a dynamical field that evolves with the universe.

4.4. Interpretive consequences

If Boltzmann entropy is merely a projection of a higher-dimensional functional such as Eq. (6), then the apparent simplicity of the classical formula masks a deeper structure. In the full picture, entropy is not a static property of a configuration space but a dynamical descriptor of how the universe distributes its existence across space, energy, and time. Physical law itself may correspond to local equilibria of this entropic field. Under this interpretation, the classical Boltzmann expression remains valid as a coarse-grained approximation, analogous to how Newtonian mechanics remains valid within the low-velocity limit of relativistic dynamics.

Recognizing this incompleteness opens the door to a general framework in which both Shannon and Boltzmann entropies appear as derived or projected forms of a single, fundamental entropy functional defined on an extended manifold. The next section develops this idea formally.

5. Mathematical Formulation: The Fundamental Entropy Functional

The preceding analysis suggests that both Shannon and Boltzmann entropies are restricted cases of a more general structure. We now formalize this intuition by defining an entropy functional that encompasses both discrete and continuous domains and explicitly includes space, time, and energy as stochastic coordinates.

5.1. Entropy as a functional over an extended manifold

Let Ξ denote the *entropic manifold*—the space of all microscopic configurations relevant to the physical system. Each point $\xi \in \Xi$ represents a complete microstate parameterized by variables such as position x, momentum p, energy E, time t, and possibly additional internal degrees of freedom ω. Thus, $\xi = (x, p, E, t, \omega)$. Let $\rho(\xi)$ be a normalized probability density on Ξ with respect to a measure $\mu(d\xi)$:

$$\int_{\Xi} \rho(\xi)\, \mu(d\xi) = 1. \tag{8}$$

We define the *Fundamental Entropy Functional* as

$$\mathcal{S}[\rho, \mu] = -k \int_{\Xi} \rho(\xi) \ln\left(\frac{\rho(\xi)}{\nu(\xi)}\right) \mu(d\xi), \tag{9}$$

where $\nu(\xi)$ is a reference (or prior) measure that plays the role of a geometric or physical background density. When $\nu(\xi)$ is uniform, Eq. (9) reduces to the continuous entropy expression introduced by Gibbs. The ratio ρ/ν ensures that the functional remains invariant under coordinate transformations and changes of measure, making $\mathcal{S}$ a coordinate-free quantity defined on the manifold Ξ.

Equation (9) is reminiscent of the Kullback–Leibler divergence, but it is interpreted here not as an information-theoretic distance between two symbolic distributions, but as an ontological measure of the spread of existence over the manifold Ξ. In this sense, $\mathcal{S}$ measures not the uncertainty of messages, but the degree to which the universe is "distributed" across its own possibility space.

5.2. Recovering the Boltzmann and Shannon limits

(a) Boltzmann entropy as spatial–momentum marginal.. Consider the marginalization of $\rho(\xi)$ over E and t:

$$\rho_{x,p}(x, p) = \int \rho(x, p, E, t)\, dE\, dt. \tag{10}$$

Substituting this marginal into Eq. (9) and assuming a uniform reference measure over E and t, we obtain

$$\mathcal{S}[\rho_{x,p}] = -k \int \rho_{x,p}(x, p) \ln \rho_{x,p}(x, p)\, dx\, dp, \tag{11}$$

which is exactly the Gibbs–Boltzmann entropy. Thus, the classical Boltzmann formula appears as the *projection* of the fundamental entropy functional when energetic and temporal degrees of freedom are treated as fixed or integrated out.

(b) Shannon entropy as discrete quantization.. Let a measurement operator Q partition Ξ into countable disjoint cells $C_i = Q^{-1}(i)$, each corresponding to a distinguishable outcome i. The probability associated with each outcome is

$$p_i = \int_{C_i} \rho(\xi)\, \mu(d\xi). \tag{12}$$

If $\nu(\xi)$ is constant over each cell, Eq. (9) reduces to

$$\mathcal{S}[\rho, \mu | Q] = -k \sum_i p_i \ln p_i, \tag{13}$$

which is precisely the Shannon entropy for the discrete random variable defined by the quantization Q. Therefore, the informational entropy of a message is a discretized projection of the fundamental entropy functional under a coarse-graining map $Q : \Xi \to \mathbb{N}$.

5.3. Geometric interpretation

The entropic manifold Ξ may be viewed as a differential-geometric space equipped with a measure μ and a density $\rho(\xi)$. The integrand in Eq. (9), $\rho(\xi)\ln(\rho/\nu)$, plays the role of a Lagrangian density, and $\mathcal{S}$ is the integral of this density over Ξ. From this viewpoint, entropy is not a scalar but a geometric field encoding how the universe occupies its configuration manifold.

In analogy with differential geometry, one may define the local entropy density

$$s(\xi) = -k\, \rho(\xi) \ln\left(\frac{\rho(\xi)}{\nu(\xi)}\right), \tag{14}$$

so that $\mathcal{S}[\rho] = \int_\Xi s(\xi)\, \mu(d\xi)$. Gradients of $s(\xi)$ across Ξ could then define *entropic flows* or *currents*, suggesting a differential formulation of the second law of thermodynamics in which entropy production arises naturally from the curvature or divergence of the entropic field.

5.4. *Discrete–continuous unification*

The two projections of $\mathcal{S}[\rho, \mu]$—the Boltzmann and Shannon cases—are not merely mathematical analogues, but different manifestations of a single continuum–discrete hierarchy. When Ξ is countable, $\mathcal{S}$ reduces to Shannon's sum. When Ξ is a smooth manifold with μ as a volume measure, $\mathcal{S}$ reduces to the continuous Boltzmann–Gibbs integral. This unification resolves the long-standing question of why the same functional $-\sum p_i \ln p_i$ appears in two seemingly unrelated disciplines: it is the restriction of one universal structure to two domains of measure theory— the discrete and the continuous.

5.5. *Interpretive summary*

Equation (9) thus defines entropy as a meta-functional spanning both discrete and continuous regimes, linking information theory and thermodynamics as projections of a single ontological principle. In the subsequent section, we examine the implications of treating entropy as a dynamic field over space, energy, and time, and explore how this reframing affects our understanding of physical law, irreversibility, and computation.

6. Ontological and Physical Implications

The general formulation of entropy as a functional defined on an extended manifold $\Xi = (x, p, E, t, \omega)$ carries significant implications for physics and for the philosophy of nature. It alters our understanding of what entropy measures, how physical law emerges, and why computation—as a human formalism—captures only a shadow of reality's complexity. The following subsections summarize these implications at three interconnected levels: physical, computational, and ontological.

6.1. *Entropy and the nature of physical law*

In the traditional interpretation of statistical mechanics, the second law of thermodynamics—that entropy tends to increase—is a statistical statement about the most probable evolution of macroscopic systems. However, if entropy is not a scalar summary of microstates but a field defined over spacetime and energetic coordinates, then the second law may be reinterpreted as a *geometric principle of self-organization.* Instead of describing mere disorder, the entropic field represents the universe's intrinsic tendency to explore and populate its space of possibilities.

From Eq. (9), one can define a local entropic gradient

$$\nabla_\Xi s(\xi) = -k\, \nabla_\Xi \big[\rho(\xi) \ln(\rho(\xi)/\nu(\xi))\big],$$

whose flow across Ξ generates the production or dissipation of order. Regions of high curvature in the entropic field correspond to strong dynamical asymmetries, giving rise to emergent "laws" that appear fixed at local equilibrium. Thus, the laws of physics may be viewed not as timeless axioms but as *stable attractors* of the universal entropic dynamics. Each physical law corresponds to a locally stationary configuration of $\mathcal{S}[\rho, \mu]$, analogous to how equilibrium configurations extremize an action functional in field theory.

This interpretation parallels modern views in emergent gravity and entropic cosmology, where spacetime geometry itself arises from the statistical behavior of microscopic degrees of freedom [4, 10]. In such a framework, the Einstein field equations or quantum-mechanical propagation rules might represent entropic constraints—relationships between local and global gradients of $\mathcal{S}$. The "laws of nature" are therefore not prior to entropy but emergent *from* it.

6.2. Time, irreversibility, and entropic directionality

In classical mechanics, time reversibility is broken only statistically, not dynamically. By extending the entropy domain to include time t as a stochastic coordinate, the arrow of time obtains a direct mathematical representation. The temporal component of the entropic gradient

$$\frac{\partial s}{\partial t} = -k\, \frac{\partial}{\partial t}\big[\rho(x, p, E, t) \ln(\rho/\nu)\big]$$

quantifies the local production of entropy with respect to temporal uncertainty. Irreversibility then follows not from coarse-graining, but from the fact that $\rho(x, p, E, t)$ evolves within a manifold whose metric in the t-direction is asymmetric with respect to entropy generation. Time itself becomes an entropic flow variable—a measure of the system's displacement through its own configuration possibilities.

This perspective is consonant with Martyushev's proposal that entropy can serve as a measure of temporal flow in irreversible processes [5]. Here, we generalize the idea: temporal passage is not merely correlated with entropy increase; it *is* the unfolding of the entropic field in its temporal dimension. Hence, the arrow of time is not an external ordering of events but a geometrical asymmetry inherent in $\mathcal{S}[\rho, \mu]$.

6.3. Entropy and the limits of computation

If the physical universe were perfectly reducible to computation, then all transformations of state could, in principle, be represented by a Turing machine. However, the fundamental entropy functional operates on a continuum domain that can itself evolve—Ξ may change topology or dimensionality as entropy propagates. A Turing process, by contrast, presupposes a fixed alphabet, a fixed tape, and a countable set of states. The entropic field therefore describes phenomena that are, in general, *non-Turing*: processes whose state space cannot be enumerated in advance and whose evolution may generate new informational degrees of freedom dynamically [3, 7, 6]. From this viewpoint, discrete computation (and information theory in the Shannon sense) is a finite projection of a deeper, continuous, and self-modifying process.

This observation reframes the question of whether the universe is a simulation. If the underlying entropic manifold continually expands its domain of possibilities, no fixed computational system—no algorithm with a finite alphabet—can exhaust its evolution. Reality is thus not a digital simulation but a *non-computable entropic continuum*, of which any digital model can capture only finite, local aspects.

6.4. *Cosmological and epistemological consequences*

At the cosmological scale, the entropic field $\mathcal{S}(x,t)$ may describe how the universe distributes probability amplitude across spacetime. Entropy gradients could correspond to curvature, so that gravitation itself becomes an emergent manifestation of entropic geometry, echoing entropic gravity proposals [4, 10]. At the epistemological scale, knowledge and observation correspond to local discretizations of this field. When an observer measures, they project a finite symbolic partition onto the continuum, thereby generating Shannon information. Every act of knowing is thus an act of *discretizing entropy*.

Under this interpretation, information, law, and perception are not independent categories but successive layers of one process:

$$\text{Entropic field (continuum)} \longrightarrow \text{Law (local equilibrium)} \longrightarrow \text{Information (discretization)}. \tag{15}$$

In the deepest sense, then, entropy is not the *opposite* of information, but its generative substrate—the continuum of randomness from which discrete meaning emerges.

6.5. *Summary of implications*

- Entropy, conceived as $\mathcal{S}[\rho,\mu]$ over (x,p,E,t,ω), measures the universe's distribution across all degrees of freedom, including energy and time.

- Physical laws arise as local equilibria or attractors of this entropic field, not as externally imposed axioms.

- The arrow of time is an intrinsic asymmetry of entropy flow, rather than an emergent statistical artifact.

- Computation and symbolic information are discrete projections of a continuous, self-evolving entropic substrate.

- The apparent similarity between Shannon and Boltzmann entropies is explained by their common origin in the fundamental functional $\mathcal{S}[\rho, \mu]$, evaluated on different domains (discrete vs. continuous).

Taken together, these implications define a framework in which entropy is not merely a statistical descriptor, but the ontological principle from which physical structure, temporal directionality, and informational cognition all arise. The final section articulates the broader philosophical conclusions and possible directions for empirical and theoretical development.

7. Discussion and Conclusion

The unified framework presented in this work redefines entropy as the continuous substrate from which discrete information and physical law emerge. By generalizing the classical Boltzmann and Shannon formulations to a single functional $\mathcal{S}[\rho, \mu]$ defined over an extended manifold that includes spatial, energetic, and temporal coordinates, we have revealed a hierarchy of randomness:

$$\text{Informational randomness (discrete)} \subset \text{Physical randomness (continuous)} \subset \text{Ontological randomness (entr}$$

This hierarchy not only reconciles the mathematical resemblance between the Shannon and Boltzmann formulas, but also situates that resemblance within a broader ontology of existence.

7.1. Entropy as the ontological substrate

Entropy, in this view, is no longer merely a measure of ignorance or disorder. It is the generative principle of reality itself—the continuum of potentiality within which order and information are transient localizations. Just as a wave is not distinct from the ocean that carries it, so information and law are not distinct from entropy but are *localized modes of its propagation.* The Fundamental Entropy Functional $\mathcal{S}[\rho, \mu]$ provides a formal representation of this substrate, describing how probability, energy, and time co-evolve as aspects of a single field.

The ontological implication is that reality is not composed of discrete objects, but of continuous distributions of possibility. Existence is therefore not a collection of entities but a *configuration of uncertainty*. Entropy becomes synonymous with being; the physical universe is an entropic field undergoing continuous self-differentiation.

7.2. Law and emergence

Within this framework, physical law arises not as a pre-imposed rule but as a stable equilibrium within the entropic continuum. Local minima or stationary points of $\mathcal{S}[\rho, \mu]$ manifest as the deterministic relations we identify as laws of physics. Because these equilibria are emergent, they may evolve as the global entropic field reorganizes itself. The constancy of natural laws is thus a reflection of the stability of specific entropic configurations—not an absolute timeless truth. Such a view resonates with the growing body of literature on emergent gravity, statistical spacetime, and the thermodynamic interpretation of field equations [4, 10].

7.3. Entropy, information, and consciousness

The inclusion of discrete information within continuous entropy also recontextualizes the phenomenon of cognition. Observation, in this framework, is an act of discretization: a local projection of the entropic field into symbolic form. When a conscious system perceives or measures, it selects a finite partition of the continuous manifold Ξ, thereby producing Shannon information. Cognition is therefore an entropic symmetry-breaking process: entropy observing itself through a discretized lens. This interpretation unifies physical and epistemic domains, suggesting that consciousness is not external to the entropic universe but one of its natural modes of self-measurement.

7.4. Irreversibility and creative asymmetry

By incorporating time as a stochastic coordinate, the model provides a natural origin for irreversibility. The arrow of time corresponds to the asymmetric expansion of $\mathcal{S}$, not to a statistical bias in particle configurations. Entropy does not simply increase; it *creates*. The passage of time is the continuous realization of new informational structures within the entropic field. This view transforms the second law of thermodynamics from a statement about decay into a statement about creativity:

> Entropy increases because the universe keeps generating possibility.

The flow of time is thus the unfolding of the field of potentiality into actuality.

7.5. Computation and the non-Turing universe

Treating entropy as a continuous, self-modifying field implies that reality transcends Turing-computability. Computation, as understood in the formal sense, presupposes a fixed symbol set and a countable state space. But $\mathcal{S}[\rho, \mu]$ evolves within a domain Ξ that may change dimensionality and topology over time. New informational states can emerge that were not enumerable a priori. Therefore, while discrete computation can model local segments of reality, the full entropic continuum is *hyper-computational*: it can generate novelty beyond the scope of algorithmic predictability. This conclusion accords with recent arguments that the universe cannot be a simulation running on a finite computational substrate [7, 3, 6]. The universe is not being computed; it is *computing itself* in a manner that no fixed algorithm can emulate.

7.6. Future directions

The framework introduced here opens several avenues for theoretical and empirical research:

1. **Entropic field theory.** Developing a differential-geometric and dynamical formalism for $\mathcal{S}(x, p, E, t)$, including field equations describing its evolution.

2. **Covariant and quantum extensions.** Extending the definition of $\mathcal{S}$ to relativistic and quantum contexts, where the measure μ may depend on spacetime curvature or quantum phase-space structures.
3. **Empirical indicators.** Searching for physical phenomena that imply entropic contributions from temporal or energetic fluctuations beyond classical thermodynamics.
4. **Information–entropy transformations.** Studying how discretization maps (measurement, cognition, computation) transform the continuous entropy field into symbolic information, potentially bridging physics and epistemology.

7.7. Final remarks

The picture that emerges is one in which entropy and information are not opposites but successive expressions of the same underlying principle. Information is the finite articulation of an infinite continuum; law is the temporary stability of its flow; time is its irreversible unfolding. The universe, in this light, is not a machine executing an algorithm, but an ocean of entropy continuously differentiating itself into meaning.

> Entropy is not the death of order—it is the birth of structure.
>
> Information is entropy speaking in a discrete tongue.

By recognizing entropy as the ontological source of both information and law, we take a step toward a synthesis of thermodynamics, computation, and metaphysics: a vision of reality not as digital or analog, but as an ever-creating continuum of possibility.

Acknowledgements

The author thanks colleagues in theoretical physics and information philosophy for the stimulating discussions that shaped this synthesis.

References

[1] C. E. Shannon, A mathematical theory of communication, Bell System Technical Journal 27 (1948) 379–423, 623–656.
URL https://people.math.harvard.edu/~ctm/home/text/others/shannon/entropy/entropy.pdf

[2] L. Boltzmann, Lectures on Gas Theory, University of California Press, 1964, original work 1877.
URL https://archive.org/details/lectures-on-gas-theory-ludwig-boltzmann

[3] S. R. Beane, Z. Davoudi, M. J. Savage, Constraints on the universe as a numerical simulation, arXiv (2012).
URL https://arxiv.org/abs/1210.1847

[4] G. Bianconi, Gravity from entropy, arXiv (2024).
URL https://arxiv.org/abs/2408.14391

[5] L. M. Martyushev, The significance of the entropic measure of time in understanding irreversible processes, Entropy 27 (4) (2025) 425.
URL https://www.mdpi.com/1099-4300/27/4/425

[6] F. Vazza, Astrophysical constraints on the simulation hypothesis for this universe: why it is (nearly) impossible that we live in a simulation, arXiv (2025).
URL https://arxiv.org/abs/2504.08461

[7] M. F. et al., Physicists just ruled out the universe being a simulation (popular press summary), ScienceAlert (article referenced by user) (2025).
URL https://www.sciencealert.com/physicists-just-ruled-out-the-universe-being-a-simulation

[8] T. M. Cover, J. A. Thomas, Elements of Information Theory, Wiley, 2006.
URL https://staff.ustc.edu.cn/~cgong821/Wiley.Interscience.Elements.of.Information.Theory.Jul.2006.eBook-DDU.pdf

[9] A. Feutrill, M. Roughan, A review of shannon and differential entropy rate estimation, Entropy 23 (8) (2021) 1046. doi:10.3390/e23081046.
URL https://doi.org/10.3390/e23081046

[10] S. M. Carroll, What is the entropy in entropic gravity?, Physical Review D (2016). doi:10.1103/PhysRevD.93.124052.
URL https://link.aps.org/doi/10.1103/PhysRevD.93.124052

[11] C. Tsallis, Possible generalization of boltzmann-gibbs statistics, Journal of Statistical Physics 52 (1988) 479–487. doi:10.1007/BF01016429.
URL https://doi.org/10.1007/BF01016429

[12] A. Rényi, On measures of entropy and information, Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability (1961).
URL https://projecteuclid.org/euclid.bsmsp/1200501650