A Deductive Proof that Information is Not Identical to Entropy

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Abstract

This paper presents a deductive argument demonstrating that information cannot be identified with entropy. Starting from a single empirical postulate—that the universe cannot, in principle, be simulated—we derive a theorem showing that information is only a proper subset of entropy. If information were equivalent to entropy, all natural laws would be reducible to informational computation, implying that the universe is a Turing-simulable process. However, this contradicts the postulate of non-simulability. Therefore, entropy must contain aspects that exceed the representational capacity of information. The proof establishes that information represents the discretized, computable projection of a deeper entropic continuum, which is ontologically non-computable.

Keywords: Entropy, Information, Non-Simulability, Ontology, Computability Theory

1. Axiomatic Framework

We present the argument in an axiomatic structure. The reasoning proceeds from one empirical postulate to a deductive theorem.

Postulat 1 (Empirical Premise)

Statement: The universe cannot be simulated, nor can it be completely represented as a computational process. This postulate reflects recent physical analyses suggesting that

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empirical reality cannot correspond to a Turing-simulable model of a physical universe [1, 2, 3].

Formally,

$$\neg\exists T : T(\text{rules of nature}) = \text{Universe},$$

where T denotes any Turing-computable function representing informational rules.

Definition 1: Information and Entropy

Let:

- $\mathcal{I}$ denote the total set of informational structures representable by a Turing machine (discrete computable states);
- $\mathcal{E}$ denote the total set of entropic configurations of the universe (continuous stochastic states).

Common interpretations equate information and entropy:

$$\mathcal{I} = \mathcal{E}.$$

The theorem will demonstrate that this equality cannot hold.

2. Theorem and Proof

Theorem

Information is a proper subset of entropy:

$$\mathcal{I} \subsetneq \mathcal{E}.$$

Equivalently, entropy contains non-informational components that cannot be represented by any finite symbolic computation.

Proof (Deductive Argument)

Step 1. Assume for contradiction that information equals entropy.. Let

$$\mathcal{I} = \mathcal{E}.$$

Then all rules, laws, and processes of the universe are informational in nature, and each law of physics corresponds to a computable cluster of information.

Step 2. Therefore, the totality of natural laws constitutes an informational Turing machine.. Because information is by definition discrete and computable, if $\mathcal{I} = \mathcal{E}$, then every entropic evolution of the universe can be expressed as a computable process. Hence, there exists a Turing machine T such that

$$T(\text{rules of nature}) = \text{Universe}.$$

This corresponds to the idea that the universe is a perfect simulation of itself.

Step 3. Contradiction with Postulat 1.. Postulat 1 asserts that the universe cannot be simulated:

$$\neg\exists T : T(\text{rules of nature}) = \text{Universe}.$$

Therefore, assuming $\mathcal{I} = \mathcal{E}$ implies the existence of a Turing simulation T that contradicts Postulat 1.

Step 4. Therefore, $\mathcal{I} \neq \mathcal{E}$.. Since the assumption $\mathcal{I} = \mathcal{E}$ leads to contradiction, the identity must be false. We conclude that information and entropy are not equivalent.

Step 5. Existence of non-informational entropy.. Because the universe continues to evolve according to entropic dynamics that cannot be simulated, there must exist entropic configurations that are not expressible in informational (computable) form. Hence, there exists a

non-empty set

$$\mathcal{E} \setminus \mathcal{I} \neq \emptyset,$$

which represents the domain of non-computable entropy.

Step 6. Conclusion..

$$\boxed{\mathcal{I} \subsetneq \mathcal{E}.}$$

Information is a computable, discrete projection of entropy, but entropy as a continuum exceeds informational representation.

3. Interpretation

The theorem establishes an ontological distinction between information and entropy. Information corresponds to discrete, symbolic, and computable structures. Entropy encompasses continuous, stochastic, and non-computable configurations. If information were identical to entropy, the universe would be a fully computable entity—a simulation of itself. Because physical evidence and theoretical arguments deny this possibility, entropy must contain trans-computational aspects that information cannot capture.

Therefore, entropy is not the absence of information but its superseding field: the continuum of potentiality within which information manifests as a discrete subset.

4. Philosophical Consequence

This proof situates information as an epistemic derivative of entropy. Information is the part of entropy that has become symbolically accessible to observers. Entropy, in contrast, represents the full ontological field of potential states, including those beyond computational representation. Reality thus cannot be fully informational; it is entropic in essence and informational only in appearance.

Conclusion

From the empirical postulate that the universe cannot be simulated, we derived that information cannot be identical to entropy. Information constitutes the discrete computable subset of a deeper, continuous, non-Turing entropic continuum. The universe, therefore, is not informational in substance but entropic in being. Information is the discretized reflection of an uncomputable continuum.

If the universe cannot be simulated,
then reality cannot be reduced to information.
Entropy is the continuum that computes itself.

References

[1] M. Starr, Physicists just ruled out the universe being a simulation, scienceAlert. Available at: https://www.sciencealert.com/physicists-just-ruled-out-the-universe-being-a-simulation (Accessed: 6 November 2025) (2025).

[2] S. R. Beane, Z. Davoudi, M. J. Savage, Constraints on the universe as a numerical simulation, European Physical Journal A 50 (148) (2012). doi:10.1140/epja/i2012-12148-8.
URL https://arxiv.org/abs/1210.1847

[3] F. Vazza, Astrophysical constraints on the simulation hypothesis for this universe: Why it is (nearly) impossible that we live in a simulation, arXiv preprint (2025).
URL https://arxiv.org/abs/2504.08461