Prime–Fibonacci Universal Languages: A Mathematical Framework for Technological and Biological Communication Across Intelligent Civilizations

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Abstract

We propose a formal dual-language framework intended as a universal communicative basis for intelligent civilizations. The first component, the Prime Technology Language (PTL), is constructed from axioms where every symbol corresponds to a unique prime number, words correspond to prime products, and grammatical rules are defined as higher-order compositions of primes. We show that this structure naturally supports Gödel-style encodings and is capable of expressing arbitrary computational processes, including representations of a universal Turing machine. The second component, the Fibonacci Biological Language (FBL), models living systems using compositions of Fibonacci numbers and defines biological behaviors as morphisms between Fibonacci sequences. This framework captures recursive growth, branching, and energetic optimization properties observed across life on Earth and potentially universal to life elsewhere. Finally, we demonstrate that PTL and FBL can form a unified Prime–Fibonacci Universal Language (PFUL) capable of expressing both technological and biological information, providing a theoretically minimal substrate for inter-civilizational communication. We discuss mathematical foundations, computational properties, linguistic completeness, and implications for astrobiology and SETI.

1 Introduction

The search for a universal language that can facilitate communication between intelligent civilizations has long been a topic of interest across mathematics, computer science, linguistics, astrobiology, and philosophy. Existing proposals for interstellar messaging have focused primarily on symbolic encodings of arithmetic, simple geometric structures, or physical constants, under the premise that mathematical invariants provide the most robust foundation for cross-civilizational understanding [1,2,5]. While these approaches successfully establish the presence of intelligence, they do not necessarily convey information about the nature of the senders, particularly regarding whether they are biological beings, technological systems, or artificial constructs.

This paper introduces a dual-framework approach to universal communication based on two fundamental numerical structures: prime numbers and Fibonacci numbers. We propose the *Prime Technology Language (PTL)* as a formal linguistic system built entirely upon the properties of prime numbers. In PTL, each symbol corresponds to a unique prime, words correspond to prime products, and grammatical rules are defined as higher-order compositions of primes. This design leverages the uniqueness of prime factorization to create an unambiguous and computationally expressive linguistic substrate. We show that PTL naturally supports Gödel-style encodings and can represent arbitrary computational processes, including the full specification of a universal Turing machine. Thus, PTL serves as a minimal but complete language for expressing technological and computational information.

Complementing PTL, we introduce the *Fibonacci Biological Language (FBL)*, a formal system based on the recursive structure of the Fibonacci sequence. Fibonacci patterns appear widely in biological growth, morphogenesis, and resource optimization across life on Earth [4,6,7]. The ubiquity of Fibonacci-like branching and golden-ratio scaling in biophysical and ecological systems suggests that recursive growth laws may constitute a universal signature of life. In FBL, organisms are represented as compositions of Fibonacci numbers, and behaviors are modeled as morphisms between Fibonacci sequences. We argue that this framework captures fundamental features of living systems, providing a mathematical language that expresses biological organization, development, and interaction.

Together, PTL and FBL form the *Prime–Fibonacci Universal Language (PFUL)*, a candidate framework for communication that is capable of representing both technological computation and biological identity. PFUL is designed to be independent of chemistry, sensory modality, evolutionary history, and symbolic convention. Because it is grounded in numerical invariants derived from number theory and recursive growth, it is accessible to any sufficiently advanced intelligence capable of arithmetic reasoning or empirical observation of natural growth patterns.

The contributions of this paper are as follows:

1. We formalize PTL, a prime-based grammatical system, and prove that it is computationally complete by demonstrating its capacity to encode a universal Turing machine.

2. We formalize FBL, a Fibonacci-based grammatical system, and show that it provides an expressive representation of biological structure and behavior.

3. We propose PFUL, a unified dual-language model, and argue that it constitutes a minimal theoretical substrate for communication between intelligent civilizations that may differ radically in physiology, cognition, and technological development.

4. We discuss implications for interstellar messaging, information theory, astrobiology, and the philosophy of universal languages.

This work aims to lay the foundation for a mathematically rigorous, conceptually minimal, and biologically meaningful universal language, with the potential of serving as an epistemic bridge among technological and living systems across the universe.

2 Related Work

Efforts to design universal languages for communication with non-human intelligences, whether extraterrestrial or artificial, have historically focused on mathematical invariants and symbolic encodings intended to transcend cultural or biological context. Early interstellar messaging proposals, such as those associated with SETI and METI initiatives, typically rely on sequences of prime numbers, geometric figures, or representations of fundamental physical constants as evidence of intentional intelligence [1, 5]. These works provide important foundations but remain limited in scope, as they emphasize the detection of intelligence rather than a full communicative substrate capable of expressing technological capability or biological identity.

2.1 Mathematical and Logic-Based Universal Languages

One of the earliest attempts at constructing a universal interstellar language is Lincos, introduced by Hans Freudenthal (1960), which encodes logical operations, arithmetic, and simple conversational structures using radio pulses. Although highly influential, Lincos presupposes familiarity with symbolic logic and grammar, and it does not provide mechanisms for representing biological or technological structures beyond simple propositional statements. Moreover, Lincos does not leverage the unique arithmetic properties of prime numbers or recursive numerical systems, which limits its ability to serve as a minimal computational foundation.

Gödel's seminal work on formal arithmetic encoding [2] demonstrated that mathematical statements, proofs, and syntactic structures can be systematically encoded using prime numbers. Gödel numbering has since become a central tool in logic, computability, and formal language theory, and it provides a powerful precedent for prime-based encodings in linguistic systems. However, prior literature has not extended Gödel-like encodings into a full grammatical language designed for communication across civilizations, nor has it explored prime factorization as a complete linguistic substrate with computational universality.

2.2 Prime Number Encodings and Computational Representation

Prime-based encodings have appeared in diverse contexts, including cryptography, coding theory, and theoretical computer science. Rivest, Shamir, and Adleman's RSA scheme [9] uses properties of prime factorization for secure communication, while various works in symbolic dynamics and combinatorics investigate the representation of strings or programs using prime products. In formal language theory, prime encodings are occasionally used to demonstrate decidability or undecidability results [3], but no prior work has constructed a full linguistic system where symbols, words, grammar, and computational semantics are all fully grounded in prime arithmetic. The Prime Technology Language (PTL) proposed in this paper builds upon these foundations by providing a complete formal grammar whose operations correspond to meta-compositions of primes, enabling the representation of arbitrary Turing machines.

2.3 Fibonacci Structures in Biology and Universal Growth Laws

The second component of our work builds on extensive literature documenting the pervasiveness of Fibonacci numbers and golden-ratio scaling in biological systems. Applications range from phyllotactic spirals in plants [4,6] to allometric scaling, vascular branching, and morphogenetic pattern formation [7]. These Fibonacci-like structures arise from recursive growth rules that optimize energy distribution, surface area, or packing efficiency. While prior studies have proposed that such patterns might be universal signatures of biological organization, no existing framework formalizes Fibonacci recursion as a linguistic system capable of expressing biological entities, behaviors, or ecological interactions.

The concept of representing biological processes as morphisms between recursive sequences is implicit in work on dynamical systems and theoretical morphology, but has not been integrated into an explicit grammatical model for universal communication. The Fibonacci Biological Language (FBL) introduced in this paper addresses this gap by constructing a formal recursive grammar in which organisms are expressed as compositions of Fibonacci numbers and behaviors are transformations between Fibonacci sequences.

2.4 Toward a Unified Framework for Technological and Biological Communication

To our knowledge, no prior literature has attempted to unify prime-based computational grammar with Fibonacci-based biological grammar to create a dual-language system capable of representing both intelligent technological capability and biological identity. Existing proposals for interstellar communication generally restrict themselves to transmitting basic mathematics, logical operations, physical constants, or pictographic representations, without providing a foundational formalism that is simultaneously computationally universal and biologically meaningful.

The Prime–Fibonacci Universal Language (PFUL) proposed in this work addresses this unfilled space by integrating two mathematically natural number-theoretic structures—prime factorization and Fibonacci recursion—into a coherent linguistic framework. PFUL provides a minimal representational substrate that any sufficiently advanced civilization could, in principle, decode using arithmetic reasoning and empirical biological insight. This positions PFUL as a new candidate for universal communication that is grounded not only in intelligence but also in the deeper structure of life and technology.

3 Preliminaries

This section introduces the mathematical notation and formal concepts required for the construction of the Prime Technology Language (PTL), the Fibonacci Biological Language (FBL), and their unification into the Prime–Fibonacci Universal Language (PFUL). We review basic properties of prime numbers, Fibonacci numbers, formal languages, and computability theory, emphasizing the structures that will serve as the foundations for the axiomatic systems developed in subsequent sections.

3.1 Prime Numbers

Let $\mathbb{P} = \{p_1, p_2, p_3, \dots\}$ denote the set of all prime numbers in increasing order, where $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, and so on. Every integer $n > 1$ has a unique prime factorization

$$n = \prod_{i=1}^{k} p_i^{e_i},$$

where $e_i \in \mathbb{N}$ and only finitely many exponents are nonzero. This uniqueness property, known as the Fundamental Theorem of Arithmetic [8], will play a central role in the structure of PTL, as it ensures that prime compositions provide unambiguous representations of linguistic objects.

We denote by $\mathrm{PF}(n)$ the multiset of prime factors of n, and we use the notation $p_i \mid n$ to indicate that p_i divides n. Throughout this work, primes serve both as atomic linguistic symbols and as algebraic generators of higher-level linguistic constructs.

3.2 Fibonacci Numbers

The Fibonacci sequence $\{F_n\}_{n\geq 0}$ is defined recursively by

$$F_0 = 0, \quad F_1 = 1, \quad F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2.$$

The sequence appears in numerous biological growth patterns, morphogenetic structures, and resource distribution processes [4, 6, 7]. Let $\mathbb{F} = \{F_0, F_1, F_2, \dots\}$ denote the set of Fibonacci numbers.

We define a *Fibonacci composition* as a finite sequence

$$C = (F_{i_1}, F_{i_2}, \dots, F_{i_k}),$$

and a *Fibonacci morphism* as a mapping

$$\phi : \mathbb{F}^* \to \mathbb{F}^*,$$

where $\mathbb{F}^*$ denotes the set of all finite sequences over $\mathbb{F}$. These structures will form the basis for representing organisms and biological behaviors in FBL.

3.3 Formal Languages and Grammars

A formal language L over an alphabet Σ is a subset of Σ^*, where Σ^* denotes the set of all finite strings formed from symbols in Σ [3]. A grammar G is a tuple (Σ, N, S, R) consisting of:

- a set of terminal symbols Σ,
- a set of nonterminal symbols N,
- a distinguished start symbol $S \in N$,
- a set of production rules R.

In PTL and FBL, we modify this classical structure by grounding terminal symbols in numerical invariants (primes or Fibonacci numbers) and defining grammatical rules as algebraic or recursive compositions of these invariants. Thus, our languages are not symbolic in the traditional sense, but numerical and structural.

3.4 Gödel Encoding

Gödel encoding assigns a unique natural number to syntactic objects using prime exponentiation [2]. A sequence $(a_1, a_2, \ldots, a_k)$ is encoded as:

$$\mathrm{GN}(a_1, \ldots, a_k) = 2^{a_1} 3^{a_2} 5^{a_3} \cdots p_k^{a_k}.$$

This establishes a mapping between symbolic structures and arithmetic, ensuring that syntactic operations correspond to arithmetic transformations. The PTL framework generalizes this approach by using prime numbers as primitive linguistic symbols and prime compositions as linguistic constructions, effectively embedding grammar into arithmetic.

3.5 Turing Machines

A Turing machine is a tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ consisting of states, input symbols, tape symbols, a transition function, and designated accept/reject states [10]. The transition function

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

can be encoded arithmetically using Gödel-style methods. We will show in Section 4 that PTL is capable of representing arbitrary Turing machines through prime-based compositions, establishing its computational universality.

3.6 Motivating Numerical Invariance

Both primes and Fibonacci numbers possess structural invariance that makes them suitable for a universal language. Primes are algebraically irreducible and appear in factorization domains across mathematics, while Fibonacci recursion naturally appears in growth processes across biological systems. The Prime Technology Language captures technological and computational structures through prime compositions, whereas the Fibonacci Biological Language expresses biological organization and transformation through recursive relations. Their unification in PFUL provides a minimal numerical substrate for communicating both technological and biological information across civilizations with potentially unknown sensory, cognitive, or symbolic systems.

4 The Prime Technology Language (PTL)

This section introduces the Prime Technology Language (PTL), a formal linguistic system grounded entirely in the arithmetic structure of prime numbers. PTL is designed as a universal technological language: its primitive symbols are prime numbers, its words are prime products, its sentences are hierarchical compositions of such products, and its grammatical rules are meta-compositions over the prime domain. This numerical construction provides a minimal but expressive foundation capable of encoding arbitrary computational processes, including universal Turing machines.

4.1 Axioms of the Prime Technology Language

We define PTL through the following axioms, which formalize the intuitive structure introduced in the conceptual framework of this work.

Axiom 1 (Prime Symbol Axiom)**.** *Each linguistic symbol corresponds to a unique prime number. Formally, the symbol set $\Sigma_{\mathbb{P}}$ is bijective with the set of all primes:*

$$\Sigma_{\mathbb{P}} \cong \mathbb{P}.$$

This ensures that symbols are irreducible atoms in PTL, mirroring their algebraic irreducibility.

Axiom 2 (Prime Word Axiom)**.** *A word in PTL is a finite concatenation of symbols, but its semantic representation is the product of the corresponding primes. For a sequence of symbols*

$$w = (\sigma_{i_1}, \dots, \sigma_{i_k}),$$

the semantic value of w is

$$[\![w]\!] = p_{i_1} p_{i_2} \cdots p_{i_k}.$$

Thus, every word corresponds uniquely to a composite number whose prime factorization recovers the sequence of constituent symbols.

Axiom 3 (Prime Sentence Axiom)**.** *A sentence in PTL is a higher-order composition of words. Formally, a sentence is a finite structured object whose constituents are word-values. If $w_1, w_2, \dots, w_m$ are words, then a sentence S is represented by some composition operator $\circ$ such that*

$$S = \circ([\![w_1]\!], \dots, [\![w_m]\!]),$$

where $\circ$ is defined recursively over prime products. The operator may itself be encoded numerically as a prime composition.

Axiom 4 (Prime Grammar Axiom)**.** *A grammar in PTL is a collection of meta-compositions over prime numbers and their products. Formally, a grammar is a set of rewriting or transformation rules*

$$R = \{r : \mathbb{N}_{>1} \to \mathbb{N}_{>1}\},$$

where each rule maps one prime composition to another. Because each natural number admits unique factorization, grammar rules operate unambiguously on prime structures.

These axioms define a numerical linguistic environment in which all syntactic and semantic constructs are grounded in the arithmetic of primes.

4.2 Prime Words as Arithmetic Objects

Given Axiom 2, any word in PTL maps directly to a natural number $n > 1$ via its prime product representation. We define:

$$\mathrm{Word}(n) \iff n = \prod_{i=1}^{k} p_i^{e_i}, \quad e_i \geq 0.$$

The syntactic structure of the word is preserved through the multiset of its prime factors, and the order of factors (if relevant) can be tracked via exponentiation encoding or sequence encodings analogous to Gödel numbering [2].

Because word identity is equivalent to prime factorization identity, PTL ensures perfect reversibility of decoding: any recipient civilization with knowledge of arithmetic can recover the exact sequence of symbols encoded in a transmitted prime product.

4.3 Prime Sentences as Hierarchical Structures

Sentences combine multiple word-values into structured compositions. Let

$$W = \{[\![w_1]\!], \dots, [\![w_m]\!]\}$$

be the semantic values of the words in a sentence. A sentence is defined recursively by operators from a set $\mathcal{O}$ of allowed compositions:

$$S_1 = [\![w_i]\!], \qquad S_{k+1} = \circ_k(S_k, [\![w_j]\!]).$$

Typical operators include:

- multiplicative composition: $a \circ b = ab$
- exponentiation: $a \circ b = a^b$
- encoding operators using primes greater than any appearing in the constituents

Any operator $\circ$ may itself be encoded using primes, ensuring closure of the system.

4.4 Prime Grammar and Rewriting Rules

A grammar rule in PTL is a transformation of one prime product into another, representing either:

- syntactic rewriting,
- semantic reinterpretation,
- computational state transition,
- or machine instruction.

Formally, a rule is a mapping $r : \mathbb{N}_{>1} \to \mathbb{N}_{>1}$, and a grammar is a set of such rules. Because every integer has a unique prime factorization, grammar rules can operate either on:

1. specific primes (atomic symbols),
2. prime powers (symbol multiplicity),
3. or entire compositions (phrases or constructs).

This allows grammars to encode arbitrary formal systems, including machine instructions of a universal Turing machine.

4.5 Encoding Computation in PTL

We show in Section 7 that PTL is computationally universal. The basic idea is as follows:

1. States of a Turing machine are encoded as primes.
2. Tape symbols are encoded as primes.
3. Instructions $(q, a) \mapsto (q', b, D)$ are encoded as prime compositions.
4. A configuration of the machine is represented by a structured prime product.
5. Grammar rules simulate instruction transitions via prime-to-prime rewritings.

This mirrors Gödel encoding [2] but extends it into a full grammatical system. Because PTL grammars operate over $\mathbb{N}_{>1}$ with unique factorization, they support reversible decoding and unambiguous semantic interpretation.

Thus, PTL furnishes a minimal arithmetic language capable of expressing any computable procedure, making it a robust candidate for a universal technological language.

5 The Fibonacci Biological Language (FBL)

While the Prime Technology Language (PTL) provides a universal arithmetic substrate for expressing technological and computational structures, it does not capture the distinctive properties of biological systems. In this section, we introduce the Fibonacci Biological Language (FBL), a formal linguistic system grounded in the recursive and generative structure of the Fibonacci sequence. FBL is motivated by the ubiquity of Fibonacci-like patterns in biological growth and morphogenesis, and it is designed to express organisms, behaviors, and ecological interactions using numerical constructs that reflect universal growth principles.

5.1 Axioms of the Fibonacci Biological Language

We formulate FBL through the following axioms, which abstract biological organization and behavior into recursive compositions of Fibonacci numbers.

Axiom 5 (Organism Composition Axiom)**.** *Every living organism can be represented as a finite composition of Fibonacci numbers. Formally, an organism O is an element of $\mathbb{F}^*$, i.e.,*

$$O = (F_{i_1}, F_{i_2}, \dots, F_{i_k}), \qquad F_{i_j} \in \mathbb{F}.$$

The sequence encodes structural, morphological, or developmental features, capturing the recursive and modular nature of biological growth [4, 6].

Axiom 6 (Behavioral Mapping Axiom)**.** *A biological behavior is a mapping between Fibonacci compositions. Formally, a behavior B is a morphism*

$$B : \mathbb{F}^* \to \mathbb{F}^*.$$

Thus, behaviors are transformations of organismal states, capturing processes such as growth, reproduction, adaptation, and interaction.

Axiom 7 (Fibonacci Grammar Axiom)**.** *A biological grammar is a set of recursive composition rules operating on Fibonacci sequences. Each rule r is of the form*

$$r : C \mapsto C',$$

where $C, C' \in \mathbb{F}^*$, and the mapping respects Fibonacci recursion. The grammar defines permissible developmental pathways, structural transformations, or ecological interactions.

Axiom 8 (Prime–Fibonacci Interaction Axiom)**.** *The grammar of FBL may be combined with the grammar of PTL through cross-domain composition operators. Formally, if $g_{\mathbb{P}}$ is a prime grammar rule and $g_{\mathbb{F}}$ is a Fibonacci grammar rule, then compositions of the form*

$$g = g_{\mathbb{P}} \bowtie g_{\mathbb{F}}$$

are admissible. These operators enable expression of bio-technological hybrid systems or biological processes that interact with computational structures.

Together, these axioms provide a numerical formalism wherein biological phenomena are expressed through recursive, growth-oriented constructs.

5.2 Organisms as Recursive Fibonacci Structures

The representation of organisms as Fibonacci compositions formalizes the empirical observation that many biological structures are modular and recursive. Examples include:

- phyllotaxis and spiral growth patterns,
- branching structures in vascular and respiratory systems,
- population dynamics that approximate Fibonacci ratios,
- morphological scaling laws governed by golden-ratio relationships [7].

We define the *structure function* of an organism as:

$$\mathrm{Struct}(O) = (F_{i_1}, F_{i_2}, \ldots, F_{i_k}),$$

with the interpretation that each F_{i_j} corresponds to a morphological module or recursive developmental component.

5.3 Behaviors as Fibonacci Morphisms

Behaviors in FBL are morphisms $\phi : \mathbb{F}^* \to \mathbb{F}^*$. Typical examples include:

1. **Growth:**
$$\phi(F_n) = (F_{n+1}, F_n),$$
representing recursive enlargement.

2. **Reproduction:**
$$\phi(F_n) = (F_n, F_{n-1}, F_{n-2}),$$
reflecting branching or lineage expansion.

3. **Adaptation:**
$$\phi(F_{i_1}, \ldots, F_{i_k}) = (F_{i_1+a}, \ldots, F_{i_k+a}),$$
where a encodes environmental pressure.

4. **Ecological interaction:**
$$\phi((F_i), (F_j)) = (F_{i+j}),$$
modeling cooperative or combinatory processes.

These morphisms capture the dynamical essence of biological systems via recursive actions.

5.4 Fibonacci Grammar and Biological Structure

A grammar in FBL is a set of recursive composition rules on Fibonacci sequences. A rule may take one of several forms:

- **Recursive expansion:**
$$(F_n) \mapsto (F_{n-1}, F_{n-2})$$
- **Compositional fusion:**
$$(F_a, F_b) \mapsto (F_{a+b})$$
- **Branching:**
$$(F_n) \mapsto (F_{n-1}, F_{n-3}, F_{n-4})$$
- **Iterative development:**
$$C \mapsto C \parallel \phi(C)$$

Such rules create a generative system that mirrors developmental biology and classical morphogenetic models.

5.5 Biological Expressiveness of FBL

FBL supports expression of biological complexity through:

1. **Hierarchical composition,** modeling multi-level anatomical structures.
2. **Recursive expansion,** corresponding to growth and self-similarity.
3. **Morphisms,** representing dynamical biological processes.
4. **Grammar interaction,** enabling ecological or evolutionary modeling.

Because Fibonacci sequences arise from optimization and resource distribution principles, FBL provides a candidate language for representing life that does not depend on specific biochemistry, morphology, or evolutionary history.

This generality positions FBL as a biologically meaningful substrate for universal communication across living civilizations.

6 The Prime–Fibonacci Universal Language (PFUL)

The Prime Technology Language (PTL) and the Fibonacci Biological Language (FBL) offer complementary representational substrates: PTL expresses technological and computational entities using prime-number arithmetic, while FBL expresses biological structure and behavior using Fibonacci recursion. In this section, we unify these systems into a single framework, the Prime–Fibonacci Universal Language (PFUL), designed to serve as a mathematically minimal basis for communication between intelligent civilizations, independent of their sensory, symbolic, or biochemical constraints.

6.1 Motivation for a Unified Framework

A universal language aimed at cross-civilizational communication must satisfy at least two desiderata:

1. It must express *technological capability*, which is best modeled by computational universality and the ability to encode machines or algorithms.

2. It must express *biological identity*, which requires a representational system capturing growth, morphology, reproduction, and adaptive behavior.

PTL satisfies the first criterion through its prime-based grammatical system, which supports Gödel-style encodings and enables representation of Turing-complete computational processes. FBL satisfies the second criterion by representing organisms and behaviors as recursive Fibonacci structures, reflecting universal principles of biological organization.

PFUL integrates these complementary systems, enabling communication of both the technological and biological signatures of a civilization.

6.2 Domain of PFUL

Let the PFUL domain be defined as:

$$\mathcal{U} = \mathbb{N}_{>1} \cup \mathbb{F}^*.$$

Elements of $\mathbb{N}_{>1}$ correspond to PTL constructs (symbols, words, sentences), while elements of $\mathbb{F}^*$ correspond to FBL constructs (organisms, behaviors, recursive compositions).

We allow composite expressions drawn from both domains, forming mixed objects of the form:

$$X = (n_1, n_2, \ldots, n_k; C_1, C_2, \ldots, C_m),$$

where $n_i \in \mathbb{N}_{>1}$ and $C_j \in \mathbb{F}^*$.

Such mixed objects encode hybrid systems such as:

- organisms interacting with technological constructs,
- technological simulations of biological processes,
- biologically inspired computational architectures,
- or arbitrary relations between computational and biological subsystems.

6.3 Cross-Domain Operators

To unify PTL and FBL, PFUL includes *cross-domain operators* that act simultaneously on prime compositions and Fibonacci compositions. Let $\bowtie$ denote a generic cross-domain operator. Examples include:

1. **Embedding biological structures into technological encodings:**

$$\mathrm{Embed}(C) = \prod_j p_{i_j}, \quad \text{where } C = (F_{i_1}, \ldots, F_{i_k}).$$

   This operator encodes biological sequences into prime-based machine-readable structures.

2. **Lifting technological representations into recursive biological forms:**

$$\mathrm{Lift}(n) = (F_{e_1}, F_{e_2}, \ldots, F_{e_k}), \quad \text{where } n = \prod p_i^{e_i}.$$

   This interprets prime exponents as growth indices.

3. **Interaction operator:**

$$\bowtie (n, C) = \phi_C(n) \parallel \psi_n(C),$$

   where ϕ_C and ψ_n are context-dependent transformations.

These operators allow PFUL to express relations between computational and biological entities in a structured, interpretable fashion.

6.4 Unified Grammar

A PFUL grammar is defined as:

$$G_{\mathcal{U}} = (R_{\mathbb{P}}, R_{\mathbb{F}}, R_{\bowtie}),$$

where:

- $R_{\mathbb{P}}$ consists of PTL prime-composition rules,
- $R_{\mathbb{F}}$ consists of FBL Fibonacci-recursive rules,
- $R_{\bowtie}$ consists of cross-domain rules combining both.

A derivation in PFUL may interleave rules from all three sets, yielding hybrid expressions. This joint grammar enables representation of complex systems involving both life and technology, such as:

- evolutionary robotics,
- bio-computational systems,
- technological artifacts built by living organisms,
- biological signaling interpreted through computational processes.

6.5 Expressive Completeness of PFUL

PFUL inherits expressive capabilities from its component languages:

1. From PTL: the ability to encode arbitrary computations, algorithms, and machine architectures.
2. From FBL: the ability to encode biological structures and transformations.
3. From cross-domain rules: the ability to represent interactions between biological and technological entities.

Thus, PFUL is capable of expressing:

- the biological nature of a communicating species,
- their cognitive architecture (via computational modeling),
- the technological sophistication of their civilization,
- and relational knowledge spanning living and engineered systems.

PFUL therefore constitutes a unified language capable of encoding both life and intelligence.

6.6 PFUL as a Candidate Universal Language

Because PFUL is grounded in numerical invariants that are independent of culture, chemistry, or symbolic convention, any mathematically capable civilization could, in principle, decode PFUL expressions. The reliance on:

- prime-number irreducibility, and
- Fibonacci recursive universality

makes PFUL accessible to intelligences that evolved independently but share fundamental mathematical reasoning abilities.

PFUL thus provides a theoretically minimal foundation for cross-civilizational communication, integrating the universal signatures of both life and technology into a single coherent linguistic system.

7 Computational Properties of PTL and PFUL

This section establishes the computational universality of the Prime Technology Language (PTL) and analyzes the computational properties of the unified Prime–Fibonacci Universal Language (PFUL). We show that PTL is capable of encoding arbitrary Turing machines using prime-based grammatical rules, thereby proving its expressive completeness. We also demonstrate that PFUL inherits this universality and extends it by incorporating biological recursion into a computationally expressive framework.

7.1 Encoding Turing Machines in PTL

A Turing machine M is defined by the tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}}),$$

where the transition function is given by

$$\delta(q, a) = (q', b, D), \qquad D \in \{L, R\}.$$

To encode M in PTL, we assign:

1. a unique prime p_q to each state $q \in Q$,
2. a unique prime p_a to each tape symbol $a \in \Gamma$,
3. a unique prime p_L, p_R for head movement directions.

A transition rule

$$(q, a) \to (q', b, D)$$

is encoded as the prime product:

$$T = p_q \cdot p_a \cdot p_{q'} \cdot p_b \cdot p_D.$$

Since prime factorization is unique, any receiver can decode T into the original instruction.

7.2 Encoding Configurations

A machine configuration consists of:

- the tape contents (a sequence of symbols),
- the current state,
- the head position.

We encode a tape cell containing symbol a in position i using:

$$C_i = p_a^{i+2},$$

ensuring positional disambiguation. The full configuration is:

$$\text{Conf} = p_q \cdot \prod_i C_i.$$

This representation is reversible because the exponents uniquely encode position, and the prime bases encode tape symbols.

7.3 Simulating Computation using Grammar Rules

A PTL grammar rule $r : \mathbb{N}_{>1} \to \mathbb{N}_{>1}$ simulates a machine step by transforming:

$$\mathrm{Conf}_k \mapsto \mathrm{Conf}_{k+1}.$$

The rule for instruction T is:

$$r_T(\mathrm{Conf}) = \mathrm{Conf}',$$

where the arithmetic transformation:

1. replaces p_q with $p_{q'}$,
2. replaces the symbol prime at the head position,
3. shifts the head by multiplying or dividing by p_L or p_R encodings.

Thus, every Turing computation corresponds to a sequence of prime-rewriting grammar rules.

7.4 Theorem: PTL is Turing-Complete

Theorem 1. *Every Turing machine M can be encoded as a PTL grammar, and every computation of M corresponds to a derivation sequence in PTL. Therefore, the Prime Technology Language is Turing-complete.*

Proof. (Outline.) Assign distinct primes to all states, symbols, and head movements. Encode instructions as prime products. Encode configurations as prime compositions with exponent-indexed positions. Define rewriting rules for each transition that map configuration encodings according to the machine semantics.

By uniqueness of prime factorization, each encoding is reversible, and by closure under prime composition, each rewriting rule is well-defined.

Thus, every computational step of M is simulated by a PTL rule. □

This construction parallels Gödel encodings [2] but extends them to a full generative grammatical system.

7.5 Computational Power of PFUL

PFUL contains PTL as a sub-language:

$$\mathbb{N}_{>1} \subset \mathcal{U}.$$

Thus, PFUL inherits Turing-completeness.

Moreover, PFUL includes recursive Fibonacci structures, enabling representation of:

- complex biological growth,
- dynamical processes,
- ecological interactions,

- hybrid bio-technological systems.

Since FBL grammars can be expressed as morphisms on $\mathbb{F}^*$, and cross-domain operators allow bidirectional translation between prime and Fibonacci compositions, every PFUL expression can incorporate computational structure.

Therefore:

Theorem 2. *PFUL is computationally universal and strictly more expressive than PTL alone, due to its ability to represent biological recursion and cross-domain hybrid systems.*

7.6 Decidability and Closure Properties

PTL inherits many closure properties of arithmetic languages, including closure under:

- prime product,
- exponentiation,
- compositional rewriting,
- substitution and concatenation (via prime multiplication).

FBL is closed under:

- Fibonacci recursion,
- sequence morphisms,
- compositional growth operators.

PFUL is closed under the union:

$$\mathrm{Closure}(\mathcal{U}) = \mathrm{Closure}(R_{\mathbb{P}} \cup R_{\mathbb{F}} \cup R_{\bowtie}).$$

Certain decision problems in PFUL may be undecidable (e.g., halting-like biological behaviors), but this is expected given its computational universality.

7.7 Computational Interpretation of Biological Processes

Biological recursion (Fibonacci-based) and technological computation (prime-based) become interoperable. For example:

- growth processes correspond to iterative functions,
- development corresponds to recursive expansion,
- ecological interactions correspond to compositional algebra,
- gene regulatory networks correspond to morphism grammars.

Thus, PFUL provides a unified computational view of both biological and technological information.

8 Discussion

The Prime–Fibonacci Universal Language (PFUL) provides an integrated numerical framework for communicating both technological and biological information across civilizations. Its foundations in prime arithmetic and Fibonacci recursion permit representation of computational structures and biological systems without requiring shared symbolic conventions, sensory modalities, or biochemical substrates. In this section, we discuss the implications of PFUL for universal communication, its relationship to existing proposals in interstellar messaging and mathematical linguistics, and its potential applications in theoretical biology, artificial intelligence, and the study of complex systems.

8.1 Implications for Interstellar Communication

Most interstellar communication proposals, including those associated with SETI and METI, have emphasized the transmission of minimal mathematical or geometric structures to demonstrate intelligence [1, 5]. Examples include sequences of prime numbers, simple diagrams, or encodings of physical constants. While effective at signaling intentionality, these approaches provide limited means of conveying the nature of the communicating civilization.

PFUL advances beyond these methods by offering dual expressive capabilities:

1. **Technological expressiveness** through PTL's ability to encode arbitrary computations, algorithms, and machine architectures.

2. **Biological expressiveness** through FBL's ability to encode organismal structure, growth, behavior, and ecological relations.

A PFUL transmission could therefore communicate not only that a civilization exists, but also:

- the computational sophistication of its technological systems,
- the recursive structure of its biological forms,
- relationships between its living and technological entities,
- or the presence of hybrid systems such as bio-engineered machines.

This dual nature provides a more comprehensive signature of life and intelligence than any purely mathematical or purely pictorial approach.

8.2 Relation to Universal Languages and Mathematical Logic

Prior universal languages such as Freudenthal's Lincos aimed to communicate descriptions of logical operations, arithmetic, and conversational constructs. However, these languages rely on symbolic conventions that must be inferred by the receiver and do not provide intrinsic grounding in numerical or biological invariants.

PFUL differs fundamentally in that:

1. PTL symbols are irreducible by definition, as they correspond to prime numbers.

2. FBL constructs derive from a ubiquitous recursive structure found throughout biological systems.

3. Grammar rules are encoded numerically rather than symbolically, enabling direct computational interpretation.

4. The language is fully grounded in arithmetic, not in symbolic or cultural conventions.

In this sense, PFUL occupies a unique position between:

- **formal languages** (with computational universality),
- **biological generative systems** (with recursive expressiveness),
- **numerical invariants** accessible to any mathematically capable intelligence.

8.3 Biological and Philosophical Implications

The use of Fibonacci recursion as a basis for expressing biological structure has several conceptual implications:

1. It suggests that the mathematics of growth, rather than the chemistry of life, may constitute a universal signature of living systems.

2. It aligns with theories that biological forms are generated by simple recursive rules that optimize resource use, structural stability, or spatial distribution.

3. It proposes that universal biology may be understood more through dynamical and structural principles than through Earth-specific molecular details.

This perspective resonates with recent research in theoretical morphology, complexity science, and mathematical biology, which emphasize universal generative principles over molecular specifics.

8.4 Applications in Artificial Intelligence and Synthetic Biology

A formal language that unifies computational and biological structures may have applications beyond interstellar communication. Potential uses include:

- **AI architectures** based on recursive growth and modular compositionality.
- **Bio-inspired computation** where algorithms mimic biological recursion.
- **Synthetic biology languages** for encoding gene networks or developmental programs.
- **Hybrid bio-technological systems** where machinery interfaces with living tissue.
- **Formal verification of bio-computational processes** using prime encodings.

PFUL may therefore serve as a foundation for interdisciplinary work in computational biology, biologically grounded AI, or universal models of cognitive evolution.

8.5 Limitations and Open Questions

Despite its expressive power, PFUL raises several open questions:

1. **Decoding complexity:** While prime factorization is theoretically reversible, practical decoding of large composite numbers may be difficult for civilizations lacking efficient algorithms.

2. **Universality of Fibonacci recursion:** Although widespread in life on Earth, it is unknown whether Fibonacci-like structures are ubiquitous in extraterrestrial biology.

3. **Cross-domain ambiguity:** Hybrid PFUL expressions may require contextual inference to distinguish computational from biological semantics.

4. **Semantic richness:** PFUL expresses structural and dynamical information but may require extensions for aesthetic, cultural, or experiential content.

These considerations highlight opportunities for future research into optimizing PFUL for robustness, interpretability, and cosmological applicability.

8.6 PFUL as an Epistemic Bridge between Life and Technology

By integrating prime arithmetic and Fibonacci recursion into a unified grammatical system, PFUL provides a language that simultaneously expresses two universal signatures:

- the irreducible structure of computation, and
- the recursive generative structure of biological life.

This dual nature positions PFUL as not merely a communication protocol but as a conceptual bridge, linking:

- mathematics and biology,
- life and intelligence,
- natural evolution and technological evolution,
- biological cognition and artificial computation.

PFUL thus offers a unified epistemic framework for understanding the relationship between living systems and their technological counterparts across the universe.

9 Conclusion

This work has introduced the Prime–Fibonacci Universal Language (PFUL), a mathematical framework designed to enable communication across intelligent civilizations by integrating two numerical structures of fundamental significance: prime arithmetic and Fibonacci recursion. The Prime Technology Language (PTL) provides a minimal and computationally universal substrate for expressing technological information, grounded in the irreducibility and unique factorization properties of prime numbers. The Fibonacci Biological Language (FBL) complements this by offering a formal system for representing biological structure, growth, and behavior through recursive compositions of Fibonacci numbers.

The unification of PTL and FBL into PFUL yields a dual-language model capable of simultaneously expressing both technological and biological identity. Unlike prior proposals for universal languages that rely on symbolic conventions, pictorial representations, or low-level arithmetic sequences, PFUL is grounded entirely in numerical invariants accessible to any civilization capable of arithmetic reasoning or empirical observation of natural growth patterns. PTL ensures computational completeness by enabling the encoding of arbitrary Turing machines, while FBL captures the recursive generative principles that underlie a wide range of biological systems.

The computational universality of PFUL positions it not only as a candidate for interstellar communication but also as a foundational framework for interdisciplinary research spanning theoretical biology, artificial intelligence, synthetic biology, complexity theory, and the philosophy of universal languages. The dual nature of PFUL—bridging life and computation—provides a conceptual tool for understanding both the origins and the evolution of intelligent systems.

Several avenues for future research arise from this work. These include:

- the development of optimized encoding schemes for practical transmission,
- the exploration of alternative biological recursion systems beyond Fibonacci structures,
- the investigation of semantic layers built upon PFUL grammar rules,
- applications in the design of hybrid bio-technological systems,
- and the study of PFUL-like frameworks as models for cognitive evolution across species.

By grounding communication in two universal mathematical structures, PFUL offers a pathway toward a language that may be intelligible not only to technologically advanced intelligences but also to any form of life capable of recognizing growth, recursion, and arithmetic irreducibility. As such, PFUL represents a step toward a universal epistemic bridge—a language that transcends the boundaries between biology and technology, between computation and life, and potentially between civilizations separated by vast cosmic distances.

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A Proofs

This appendix presents formal proofs and detailed justifications for the theorems and constructions introduced in the main text. We focus primarily on the computational properties of the Prime Technology Language (PTL) and the expressive completeness of the Prime–Fibonacci Universal Language (PFUL).

A.1 Proof of Turing-Completeness of PTL

We restate the principal theorem.

Theorem 3. *Every Turing machine M can be encoded by a PTL grammar, and every computation of M can be simulated by a sequence of PTL derivations. Therefore, the Prime Technology Language is Turing-complete.*

Proof. Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ be an arbitrary Turing machine.

1. Prime Assignments. Assign distinct primes to each component:

$$q \mapsto p_q, \quad a \mapsto p_a, \quad L \mapsto p_L, \quad R \mapsto p_R.$$

These assignments are injective, ensuring unique representation.

2. Encoding of Transitions. Every transition

$$(q, a) \to (q', b, D)$$

is encoded as the composite number:

$$T = p_q \cdot p_a \cdot p_{q'} \cdot p_b \cdot p_D.$$

By the Fundamental Theorem of Arithmetic, T uniquely identifies the transition.

3. Encoding of Configurations. Tape cells are indexed by position. Let the symbol at position i be a_i. Encode cell i as:

$$C_i = p_{a_i}^{i+2}.$$

A configuration in state q is:

$$\mathrm{Conf} = p_q \cdot \prod_i C_i.$$

Reversibility is guaranteed because:

- base primes encode symbols,
- exponents encode positions,
- the state is encoded separately.

4. Grammar Rules Simulate Transitions. For each transition encoding T, define a PTL grammar rule:

$$r_T : \mathrm{Conf}_k \mapsto \mathrm{Conf}_{k+1}.$$

The rule performs the following arithmetic operations:

1. Replace p_q with $p_{q'}$ (state update).
2. Modify the prime at the head position (symbol update).
3. Multiply or divide by p_L or p_R encodings to shift the head.

These operations correspond exactly to the semantics of δ.

5. Soundness. If M transitions from Conf_k to Conf_{k+1}, then $r_T(\mathrm{Conf}_k) = \mathrm{Conf}_{k+1}$ by construction.

6. Completeness. If r_T applies to Conf_k, then by unique factorization of T and Conf_k, the corresponding Turing transition must be exactly $(q, a) \to (q', b, D)$. Thus, every PTL derivation corresponds to a valid Turing computation.

Therefore, PTL can simulate any Turing machine, proving Turing-completeness. □

A.2 Proof that PFUL Inherits Turing-Completeness

Theorem 4. *PFUL is computationally universal.*

Proof. PFUL includes all natural numbers $n > 1$ as valid expressions:

$$\mathbb{N}_{>1} \subset \mathcal{U}.$$

Thus, every PTL expression is also a PFUL expression, and every PTL grammar rule is also a PFUL rule.

Since PTL is Turing-complete, PFUL inherits computational universality.

Addition of Fibonacci sequences and cross-domain operators cannot reduce computational power, because PFUL contains PTL as a sub-algebra.

Therefore, PFUL is computationally universal. □

A.3 Proof of Closure Under Cross-Domain Operators

Theorem 5. *The PFUL domain $\mathcal{U}$ is closed under all cross-domain operators defined in Section 6.*

Proof. Let $n \in \mathbb{N}_{>1}$ and $C \in \mathbb{F}^*$.

1. Embedding Operator.

$$\mathrm{Embed}(C) = \prod_j p_{i_j} \in \mathbb{N}_{>1}.$$

Thus the result lies in PTL's domain.

2. Lifting Operator.

$$\mathrm{Lift}(n) = (F_{e_1}, F_{e_2}, \dots, F_{e_k}) \in \mathbb{F}^*.$$

Thus the result lies in FBL's domain.

3. Interaction Operator.

$$\bowtie (n, C) = \phi_C(n) \parallel \psi_n(C)$$

where both $\phi_C(n)$ and $\psi_n(C)$ lie in the union

$$\mathbb{N}_{>1} \cup \mathbb{F}^* = \mathcal{U}.$$

Concatenation $\parallel$ preserves membership in $\mathcal{U}$.

Therefore every operator maps $\mathcal{U} \times \mathcal{U}$ into $\mathcal{U}$, and closure is established. □

A.4 Lemma: Structural Reversibility of PTL Representations

Lemma 1. *Every PTL construct is reversible under prime factorization.*

Proof. Immediate from the Fundamental Theorem of Arithmetic and the injectivity of symbol–prime assignments. □

A.5 Lemma: Structural Reversibility of FBL Representations

Lemma 2. *Every FBL construct is reversible under sequence decoding.*

Proof. Since FBL expressions are finite sequences in $\mathbb{F}^*$, recovery of constituent Fibonacci numbers is trivial. $\square$

A.6 Theorem: PFUL Supports Mixed Biological–Technological Derivations

Theorem 6. *For any sequence of PTL expressions and FBL expressions, a PFUL derivation exists that interleaves transformations across domains.*

Proof. Since $R_{\mathbb{P}}$, $R_{\mathbb{F}}$, and $R_{\bowtie}$ are all subsets of the PFUL grammar, and since $\mathcal{U}$ is closed under the cross-domain operators, all mixed sequences of derivations remain within PFUL's domain.

Thus, hybrid derivations exist for all combinations of biological and technological constructs. $\square$

B Extended Examples

This appendix provides illustrative examples of constructions in the Prime Technology Language (PTL), the Fibonacci Biological Language (FBL), and the unified Prime–Fibonacci Universal Language (PFUL). These examples are intended to demonstrate the practical interpretability of each component language and to show how biological and technological information may be encoded numerically.

B.1 Examples in the Prime Technology Language (PTL)

B.1.1 Example 1: Prime-Based Symbol Encoding

Let the first five primes represent five basic symbols:

$$\sigma_A \mapsto 2, \quad \sigma_B \mapsto 3, \quad \sigma_C \mapsto 5, \quad \sigma_D \mapsto 7, \quad \sigma_E \mapsto 11.$$

These assignments are injective and form the primitive alphabet of PTL.

B.1.2 Example 2: Words as Prime Products

Consider the word

$$w = (\sigma_A, \sigma_C, \sigma_A, \sigma_D).$$

Its semantic value is:

$$[\![w]\!] = 2 \cdot 5 \cdot 2 \cdot 7 = 140.$$

A receiver decodes this by prime factorizing:

$$140 = 2^2 \cdot 5 \cdot 7.$$

B.1.3 Example 3: Sentence Construction

Let words w_1 and w_2 have semantic values:

$$[\![w_1]\!] = 30 = 2 \cdot 3 \cdot 5,$$

$$[\![w_2]\!] = 77 = 7 \cdot 11.$$

A sentence using multiplicative composition is:

$$S = 30 \circ 77 = 30 \cdot 77 = 2310.$$

Prime factorization recovers both words.

B.1.4 Example 4: Encoding a Turing Machine Instruction

Consider the instruction:

$$(q_1, a_0) \to (q_2, a_1, R).$$

Assign primes:

$$q_1 \mapsto 13, \quad q_2 \mapsto 17, \quad a_0 \mapsto 19, \quad a_1 \mapsto 23, \quad R \mapsto 29.$$

The PTL instruction encoding is:

$$T = 13 \cdot 19 \cdot 17 \cdot 23 \cdot 29.$$

A civilization receiving T can decode the transition exactly.

B.2 Examples in the Fibonacci Biological Language (FBL)

B.2.1 Example 1: Organism Representation

Let an organism be represented by the Fibonacci composition:

$$O = (F_2, F_3, F_5, F_3) = (1, 2, 5, 2).$$

This may encode:

- recursive branching levels,
- developmental modules,
- or a simplified morphological structure.

B.2.2 Example 2: Behavior as a Morphism

A growth behavior ϕ is defined by:

$$\phi(F_n) = (F_{n+1}, F_n).$$

Applied to the organism above:

$$\phi(O) = ((2, 1), (3, 2), (8, 5), (3, 2)).$$

B.2.3 Example 3: Ecological Interaction

If two organisms

$$O_1 = (F_2, F_4), \quad O_2 = (F_1, F_3)$$

interact cooperatively, a rule may be:

$$(F_a, F_b), (F_c, F_d) \mapsto (F_{a+c}, F_{b+d}).$$

Thus:

$$O_1, O_2 \mapsto (F_{2+1}, F_{3+2}) = (F_3, F_5) = (2, 5).$$

B.2.4 Example 4: Adaptive Transformation

Environmental pressure shifting all modules upward by $+2$:

$$\psi(F_{i_j}) = F_{i_j+2}.$$

B.3 Examples in PFUL

B.3.1 Example 1: Embedding a Biological Sequence in PTL

Let

$$C = (F_2, F_3, F_4).$$

Embedding yields:

$$\mathrm{Embed}(C) = 2 \cdot 3 \cdot 5 = 30.$$

This produces a technologically interpretable prime composition.

B.3.2 Example 2: Lifting a Prime Composition into FBL

Let

$$n = 2^3 \cdot 3^1 \cdot 5^2.$$

Lifting gives:

$$\mathrm{Lift}(n) = (F_3, F_1, F_2).$$

B.3.3 Example 3: Mixed Biological–Technological Expression

A hybrid PFUL message:

$$X = (210; (F_2, F_3, F_1)),$$

where $210 = 2 \cdot 3 \cdot 5 \cdot 7$ encodes a technological structure and (F_2, F_3, F_1) encodes a biological component.

B.3.4 Example 4: Simple Turing Machine Simulation in PTL

Consider a machine that toggles a single binary cell:

$$0 \to 1, \quad 1 \to 0.$$

Assign:

$$0 \mapsto 2, \quad 1 \mapsto 3.$$

Configuration for cell=0:

$$\mathrm{Conf}_0 = 2^2.$$

Rule:

$$2^2 \mapsto 3^2.$$

Reverse rule:

$$3^2 \mapsto 2^2.$$

This minimal example illustrates PTL simulation of a computational system.

B.3.5 Example 5: PFUL Cosmic Transmission Example

A civilization may transmit the following sequence:

$$(2,3,5,7,11) \quad \text{(prime symbols)}$$
$$(1,1,2,3,5,8) \quad \text{(Fibonacci life signature)}$$
$$2310 \quad \text{(encoded PTL sentence)}$$
$$(5,8,13) \mapsto (13,21) \quad \text{(FBL growth rule)}$$
$$(2310;(1,2,3)) \quad \text{(PFUL hybrid identity)}$$

Such a message conveys:

- intelligence (via primes),
- life (via Fibonacci growth),
- computation (via PTL encoding),
- biology (via FBL rule),
- an integrated identity (via PFUL).