An Axiomatic Variational Framework for the Emergence of Time, Space, and Topology

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Abstract

The origin of temporal orientation, spatial geometry, dimensionality, and topology remains a foundational open problem in physics. Existing approaches typically assume these structures or explain them through statistical or boundary-condition-based arguments. In this paper, we present an axiomatic variational framework in which physically realizable universes correspond to stable extremal histories of a global action functional. No temporal direction, spatial metric, dimensionality, or topology is assumed a priori. Instead, these structures emerge as deductive consequences of global variational stability. Temporal orientation arises through the exclusion of time-symmetric unstable extrema, spatial geometry emerges via Euler–Lagrange structure, dimensionality is constrained by stability requirements, and topology is selected by the absence of unstable global deformation modes. Established physical theories are interpreted as effective intra-history descriptions operating within a variationally selected spacetime.

Keywords: Variational principles; Axiomatic foundations; Arrow of time; Emergent spacetime; Dimensional selection; Topology

Highlights

- Introduces an axiomatic variational framework for spacetime emergence.

- Derives temporal orientation from global stability of extremal histories.
- Shows how spatial geometry arises from Euler–Lagrange structure.
- Establishes dimensionality and topology as variationally selected properties.
- Positions known physical theories as emergent intra-history descriptions.

1 Introduction

Contemporary fundamental physics relies extensively on variational principles. From classical mechanics to quantum field theory and general relativity, the equations governing physical dynamics and spacetime structure are obtained through the extremization of action functionals Landau and Lifshitz, 1975; Wald, 1984. Despite this universality, the role of the action principle is typically confined to a methodological level: it is employed to derive equations of motion within a pre-existing spacetime, while the origin of spacetime structure itself—its temporal orientation, dimensionality, and topology—is taken as given.

In a companion conceptual paper, we argued that this hierarchy may be inverted. Rather than treating spacetime as a prior arena in which the action principle operates, we proposed that spacetime itself should be regarded as an outcome of a global variational selection. Within this perspective, the principle of action functions as a pre-physical organizing rule that determines which histories are physically realizable. The present paper develops this proposal in a formal and axiomatic manner.

The central objective of this work is to formulate an axiomatic variational framework in which time orientation, spatial geometry, dimensionality, and global topology emerge as consequences of global action extremization. No temporal direction, spatial dimension, or topological structure is assumed at the outset. Instead, these features are derived as properties of stable extremal solutions defined over a space of admissible histories. The framework is intentionally minimal: it introduces only those postulates required to support a coherent deductive structure, while avoiding commitments to specific dynamical models or microscopic degrees of freedom.

A key methodological departure of this work lies in its global perspective. Standard dynamical formulations describe evolution locally, prescribing how systems change infinitesimally in time. In contrast, the variational framework adopted here treats entire histories as the fundamental objects of analysis. Physical realizability is determined not by local evolution alone, but by the existence and stability of global extrema of the action functional. This shift in perspective allows temporal orientation to be addressed without presupposing an external time parameter, and spatial structure to be derived without assuming a fixed background geometry.

Within this framework, time-reversal symmetry may remain intact at the level of the action functional, while being broken at the level of realized solutions. Temporal orientation arises as a stability condition on extremal histories rather than as a consequence of asymmetric dynamical laws or probabilistic bias. This approach differs from standard entropy-based accounts of time asymmetry, which typically appeal to statistical considerations or special boundary conditions Albert, 2000; Price, 1996. Similarly, spatial geometry and geodesic structure emerge as solutions to variational conditions, and dimensionality is constrained by the requirement that extremal histories admit stable and globally consistent realizations. Topological features are likewise subject to selection, insofar as only certain manifolds support admissible extremal configurations.

The scope of this paper is deliberately restricted. We do not propose a specific form of the fundamental action, nor do we attempt to quantize the resulting framework. Instead, we focus on establishing a general axiomatic structure and deriving its qualitative consequences. Questions concerning the numerical values of physical constants, detailed cosmological dynamics, and quantum fluctuations lie beyond the present analysis and are left for future investigation.

The paper is organized as follows. Section 2 introduces the mathematical setting and defines the space of admissible histories and action functionals. Section 3 presents the fundamental postulates of the framework. Section 4 derives the emergence of temporal orientation from stability conditions on extremal histories. Section 5 develops the emergence of spatial geometry and geodesic structure. Section 6 addresses dimensional selection, and Section 7 extends the analysis to global topological constraints. Section 8 discusses the relation of the present framework to established physical theories. The paper concludes with a summary of results and an outline of open problems.

By formalizing the principle of action as a global criterion of physical realizability, this work aims to provide a unified and deductive account of spacetime structure. Time, space, dimensionality, and topology are treated not as primitive inputs, but as outputs of a single variational selection mechanism. In doing so, the framework offers a systematic foundation for further exploration of the origins of spacetime within a purely action-based approach.

2 Preliminaries and Mathematical Setting

This section introduces the mathematical structures and definitions required for the axiomatic development that follows. The purpose is not to specify a concrete physical model, but to establish a general and sufficiently flexible setting in which global variational selection can be formulated rigorously.

2.1 Histories and Configuration Space

We begin by introducing the notion of a *history*. A history is understood as a complete specification of all physically relevant degrees of freedom over an entire domain of realization. Importantly, no prior temporal orientation or spacetime structure is assumed at this stage.

Let $\mathcal{H}$ denote the space of admissible histories. Elements $h \in \mathcal{H}$ represent global configurations, which may include geometric, topological, and matter-like degrees of freedom. The precise nature of these degrees of freedom is left unspecified, reflecting the pre-geometric character of the framework.

The space $\mathcal{H}$ is assumed to possess sufficient structure to support the definition of functionals and variations. In particular, we assume that $\mathcal{H}$ is endowed with a topology that allows for continuous deformations of histories and the definition of infinitesimal variations.

2.2 Action Functionals

An *action functional* is defined as a real-valued functional

$$\mathcal{S} : \mathcal{H} \to \mathbb{R}, \tag{1}$$

which assigns to each admissible history $h \in \mathcal{H}$ a real number $\mathcal{S}[h]$. The action functional is taken to be global in nature, meaning that it depends on the history as a whole rather than on local data defined at individual points.

No specific form of $\mathcal{S}$ is assumed. In particular, $\mathcal{S}$ need not be expressible as an integral over a predefined spacetime manifold. This generality reflects the fact that spacetime structure itself is intended to emerge from the variational framework rather than being presupposed.

2.3 Variations and Extremal Histories

To define variational principles on $\mathcal{H}$, we introduce the notion of a variation. Let $h \in \mathcal{H}$ be a history and let δh denote an infinitesimal deformation of h within $\mathcal{H}$. The corresponding variation of the action is given by

$$\delta\mathcal{S}[h] = \frac{d}{d\epsilon}\mathcal{S}[h + \epsilon\,\delta h]\bigg|_{\epsilon=0}, \tag{2}$$

whenever this expression is well-defined.

A history $h^* \in \mathcal{H}$ is said to be an *extremal history* if

$$\delta\mathcal{S}[h^*] = 0 \tag{3}$$

for all admissible variations δh. Extremal histories are candidates for physical realization within the framework.

We emphasize that extremality does not necessarily correspond to minimization. Depending on the structure of $\mathcal{H}$ and $\mathcal{S}$, an extremal history may correspond to a minimum, maximum, or saddle point of the action functional.

2.4 Stability of Extremal Histories

Not all extremal histories are physically admissible. To capture the notion of robustness under perturbations, we introduce a stability criterion.

An extremal history h^* is said to be *stable* if, for all sufficiently small variations δh, the second variation of the action satisfies

$$\delta^2\mathcal{S}[h^*] \geq 0, \tag{4}$$

or an analogous condition appropriate to the structure of $\mathcal{H}$. Stability is understood here in a global sense, referring to variations of the entire history rather than to local dynamical perturbations.

Stability plays a central role in the framework, as only stable extremal histories will be considered physically realizable in subsequent sections.

2.5 Admissibility and Equivalence Classes

Finally, we introduce the notion of admissibility. Two histories $h_1, h_2 \in \mathcal{H}$ are said to be equivalent if they differ only by transformations that leave the action invariant. Such transformations may include reparametrizations, gauge transformations, or other symmetries of the action functional.

The space of physically distinct histories is thus given by the quotient

$$\widetilde{\mathcal{H}} = \mathcal{H}/\sim, \tag{5}$$

where $\sim$ denotes equivalence under action-invariant transformations.

In the following sections, all statements concerning extremality, stability, and realizability are understood to apply to equivalence classes of histories in $\widetilde{\mathcal{H}}$ rather than to individual representatives.

3 Fundamental Postulates

In this section we state the fundamental postulates of the framework. These postulates define the conditions under which a history is physically realizable and fix the logical structure from which subsequent results are derived. No reference is made to a predefined spacetime, temporal orientation, spatial dimensionality, or topology. All such structures are treated as emergent.

3.1 Postulate I: Existence of a Global Action

Postulate I (Global Action). There exists a real-valued action functional

$$\mathcal{S} : \mathcal{H} \to \mathbb{R}, \tag{6}$$

defined on the space of admissible histories $\mathcal{H}$, such that $\mathcal{S}$ depends on each history as a whole and is invariant under action-preserving transformations.

Remark. This postulate establishes the action functional as the sole primitive structure of the framework. No decomposition into local densities or integration over a predefined manifold is assumed. The invariance requirement ensures that physically equivalent descriptions do not lead to distinct realizations.

3.2 Postulate II: Variational Realizability

Postulate II (Variational Realizability). A history $h \in \mathcal{H}$ corresponds to a physically realizable universe if and only if h is an extremal history of the action, i.e.,

$$\delta\mathcal{S}[h] = 0 \tag{7}$$

for all admissible variations δh.

Remark. This postulate excludes non-extremal histories from physical consideration. It shifts the notion of physical law from local evolution rules to a global selection criterion acting on entire histories.

3.3 Postulate III: Stability of Realized Histories

Postulate III (Stability). Among all extremal histories, only those that satisfy a global stability condition are physically realized. Specifically, a realizable history h^* must satisfy

$$\delta^2\mathcal{S}[h^*] \geq 0, \tag{8}$$

or an equivalent condition ensuring robustness under infinitesimal global variations.

Remark. The stability requirement distinguishes physically realizable histories from mathematically admissible but unstable extrema. This postulate plays a central role in the emergence of temporal orientation and dimensional selection in later sections.

3.4 Postulate IV: Pre-Geometric Neutrality

Postulate IV (Pre-Geometric Neutrality). No temporal orientation, spatial metric, dimensionality, or global topology is assumed prior to the application of the variational principle. Any such structures arise only as properties of stable extremal histories selected by Postulates I–III.

Remark. This postulate prevents the introduction of hidden background structures. In particular, it allows the action functional itself to be invariant under time reversal and spatial symmetries, while permitting their breaking at the level of realized solutions.

3.5 Postulate V: Equivalence under Action-Preserving Transformations

Postulate V (Equivalence). Histories related by transformations that leave the action invariant represent the same physical realization. Physical statements therefore concern equivalence classes of histories in

$$\widetilde{\mathcal{H}} = \mathcal{H}/\sim, \tag{9}$$

where $\sim$ denotes equivalence under action-preserving transformations.

Remark. This postulate ensures that redundancies arising from gauge freedom, reparametrization, or other symmetries do not lead to spurious multiplicities of physical realizations.

3.6 Postulate VI: Emergence of Physical Law

Postulate VI (Emergence of Physical Law). All dynamical laws, statistical regularities, and effective descriptions operate only within a realized history and are emergent consequences of the variationally selected structure. No such laws participate in the selection of realizable histories.

Remark. This postulate formally separates pre-physical selection criteria from intra-history dynamics. It clarifies that thermodynamic, statistical, and effective field-theoretic laws presuppose the existence of a realized spacetime and do not determine its fundamental structure.

Taken together, these postulates define a closed axiomatic system. Subsequent sections explore their deductive consequences, including the emergence of temporal orientation, spatial geometry, dimensionality, and global topology.

4 Emergence of Temporal Orientation

In this section we derive the emergence of a temporal orientation as a consequence of the axiomatic framework introduced in Sections 2 and 3. No temporal direction is assumed a priori. Instead, temporal orientation arises as a structural property of stable extremal histories.

4.1 Time-Reversal Symmetry at the Level of the Action

Let $\mathcal{T}$ denote a time-reversal transformation acting on histories in $\mathcal{H}$. At the pre-geometric level, $\mathcal{T}$ is defined abstractly as an involutive mapping,

$$\mathcal{T} : \mathcal{H} \to \mathcal{H}, \qquad \mathcal{T}^2 = \mathrm{id}, \tag{10}$$

which reverses the ordering of events within a history without presupposing a temporal metric or parameter.

By Postulate IV (Pre-Geometric Neutrality), the action functional $\mathcal{S}$ is permitted to be invariant under $\mathcal{T}$,

$$\mathcal{S}[h] = \mathcal{S}[\mathcal{T}h], \tag{11}$$

for all admissible histories $h \in \mathcal{H}$. Time-reversal symmetry therefore holds at the level of admissible histories and the action functional.

4.2 Lemma: Instability of Time-Symmetric Extremal Histories

We now establish a key result concerning the stability of time-symmetric extrema.

Lemma 4.1 (Instability of Time-Symmetric Extremal Histories). Let $h^* \in \mathcal{H}$ be an extremal history satisfying

$$\mathcal{T}h^* = h^*. \tag{12}$$

If the space of admissible histories admits nontrivial time-antisymmetric variations δh such that

$$\mathcal{T}(\delta h) = -\delta h, \tag{13}$$

then h^* is generically unstable under global variations.

Proof (sketch). Since h^* is extremal, the first variation vanishes:

$$\delta\mathcal{S}[h^*] = 0.$$

Consider the second variation along a time-antisymmetric direction δh. By time-reversal invariance of $\mathcal{S}$, contributions to $\delta^2\mathcal{S}$ from forward- and backward-oriented components of the variation enter with opposite sign. Unless constrained by additional symmetry or degeneracy, these contributions fail to be positive definite. Consequently, the second variation admits directions of negative curvature, rendering the extremum unstable. □

4.3 Theorem: Emergence of Temporal Orientation

The lemma above leads directly to the emergence of a preferred temporal orientation.

Theorem 4.2 (Temporal Orientation from Stability). Under Postulates I–V, any physically realizable history admits a unique temporal orientation up to global reversal.

Proof. By Postulate II, physically realizable histories must be extremal. By Postulate III, they must additionally be stable. Lemma 4.1 implies that time-symmetric extremal histories are generically unstable whenever nontrivial time-antisymmetric variations exist. Such histories are therefore excluded from physical realization.

It follows that any stable extremal history h^* must satisfy

$$\mathcal{T}h^* \neq h^*.$$

The pair $\{h^*, \mathcal{T}h^*\}$ represents two distinct but action-equivalent histories related by global time reversal. By Postulate V, these histories correspond to distinct physical realizations distinguished solely by temporal orientation. Hence, each realizable history admits a unique intrinsic direction of time, determined by the choice of stable extremal branch. □

4.4 Consequences

Theorem 4.2 establishes temporal orientation as a structural consequence of global variational selection. The arrow of time is not imposed by asymmetric dynamical laws, nor derived from probabilistic or thermodynamic considerations. Instead, it arises because only temporally oriented histories satisfy the global stability requirement of the action principle.

Once a temporally oriented history is realized, all subsequent physical processes occur within this orientation. Statistical irreversibility and entropy production are then understood as emergent phenomena internal to the realized history, rather than as mechanisms responsible for fixing temporal direction.

This result provides the formal foundation for treating temporal asymmetry as prior to thermodynamic descriptions and prepares the ground for the derivation of spatial geometry and dimensionality in subsequent sections.

5 Emergence of Spatial Geometry

In this section we derive the emergence of spatial geometry as a consequence of the global variational framework. No metric, connection, or notion of distance is assumed a priori. Instead, geometric structure arises from the extremal and stability properties of realizable histories.

5.1 Decomposition of Realized Histories

Let $h^* \in \mathcal{H}$ denote a stable extremal history selected by Postulates I–V. By Theorem 4.2, h^* admits an intrinsic temporal orientation. This allows the introduction of an ordered foliation of h^* into equivalence classes of configurations, interpreted as spatial sections, without assuming a background time parameter.

Let Σ denote a representative configuration within such a foliation. The collection of all admissible configurations forms a configuration space $\mathcal{C}$, which is induced by the structure of h^* rather than assumed independently.

5.2 Effective Action on Configurations

Restricting attention to variations that preserve the temporal ordering of h^*, the global action $\mathcal{S}$ induces an effective action on curves in configuration space,

$$\mathcal{S}_{\text{eff}}[\gamma] = \mathcal{S}[h^* + \delta h(\gamma)], \tag{14}$$

where $\gamma : I \to \mathcal{C}$ denotes a continuous path in configuration space, and $\delta h(\gamma)$ represents variations compatible with the realized temporal orientation.

The existence of $\mathcal{S}_{\text{eff}}$ follows from the global nature of the action functional and does not presuppose locality or a predefined metric structure.

5.3 Lemma: Geodesics as Euler–Lagrange Solutions

We now establish the central result linking variational selection to geometric structure.

Lemma 5.1 (Geodesics as Euler–Lagrange Solutions). Let $\gamma^* : I \to \mathcal{C}$ be a path in configuration space corresponding to a physically realizable history h^*. Then γ^* satisfies the Euler–Lagrange equations associated with $\mathcal{S}_{\text{eff}}$. The solutions of these equations define geodesic curves with respect to an emergent geometric structure on $\mathcal{C}$.

Proof (sketch). By Postulate II, h^* extremizes the global action $\mathcal{S}$. Variations of h^* that respect the realized temporal orientation correspond to variations of the induced path γ in configuration space. The vanishing of the first variation of $\mathcal{S}$ under such variations implies

$$\delta\mathcal{S}_{\text{eff}}[\gamma^*] = 0,$$

which yields the Euler–Lagrange equations for $\mathcal{S}_{\text{eff}}$.

The solutions γ^* of these equations extremize the effective action among nearby paths and therefore define preferred curves in $\mathcal{C}$. By definition, such curves are geodesics with respect to the emergent geometric structure determined by $\mathcal{S}_{\text{eff}}$. □

5.4 Emergent Metric Structure

The Euler–Lagrange equations derived from $\mathcal{S}_{\text{eff}}$ induce a notion of infinitesimal distance along configuration-space paths. When the effective action admits a quadratic form in generalized velocities, this structure can be identified with a metric tensor on $\mathcal{C}$. Importantly, this metric is not postulated but inferred from the variational properties of realizable histories.

In this sense, spatial geometry is an emergent property of stable extremal solutions. Geodesic motion is not imposed by geometric axioms but arises as a consequence of variational consistency. The geometry of space is therefore secondary to, and determined by, the principle of action.

5.5 Consequences

The results of this section establish that spatial geometry emerges from the same global variational selection mechanism that fixes temporal orientation. Once a temporally oriented extremal history is realized, the induced configuration space inherits a geometric structure for which physically realized paths are geodesics.

This conclusion inverts the standard explanatory order: rather than assuming geometry to define geodesics, the existence of geodesic motion defines geometry. The framework thus provides a unified variational origin for both temporal and spatial structure, setting the stage for the analysis of dimensionality and topology in subsequent sections.

6 Dimensional Selection Theorem

In this section we address the emergence and selection of spatial dimensionality within the axiomatic variational framework. Dimensionality is not assumed as a primitive attribute of space. Instead, it is treated as a property of stable extremal histories selected by the global action principle.

6.1 Dimensional Degrees of Freedom

Let h^* be a stable extremal history admitting a temporal orientation and an emergent configuration space $\mathcal{C}$, as established in Sections 4 and 5. We denote by d the effective dimensionality of the spatial configurations associated with h^*, understood as the minimal number of independent parameters required to locally characterize configurations in $\mathcal{C}$.

At the pre-geometric level, the space of admissible histories $\mathcal{H}$ permits histories whose induced configuration spaces have varying dimensionality. No restriction on d is imposed a priori.

6.2 Variational Stability and Dimensional Dependence

The stability of an extremal history depends on the behavior of the second variation of the action under admissible perturbations. Importantly, the number and structure of admissible variations generally increase with the dimensionality of the induced configuration space.

Let δh_d denote variations of a history whose induced configuration space has effective dimension d. The second variation of the action can be formally written as

$$\delta^2 \mathcal{S}[h^*] = \langle \delta h_d, \mathcal{K}_d \, \delta h_d \rangle, \tag{15}$$

where $\mathcal{K}_d$ is a stability operator whose spectral properties depend on d.

6.3 Lemma: Dimensional Growth of Unstable Modes

Lemma 6.1 (Dimensional Growth of Unstable Modes). For a generic class of global action functionals satisfying Postulates I–IV, the number of independent directions in which the second variation $\delta^2 \mathcal{S}$ admits non-positive contributions is a non-decreasing function of the effective spatial dimensionality d.

Proof (sketch). As the dimensionality d increases, the space of admissible variations δh_d acquires additional independent components corresponding to deformations along distinct spatial directions. Unless the action functional exhibits special degeneracies or fine-tuned symmetries, these additional degrees of freedom introduce new directions in variation space. Genericity implies that not all such directions can be stabilized simultaneously by the same variational structure, leading to an increase in the number of modes for which $\delta^2 \mathcal{S}$ fails to be positive definite. □

6.4 Theorem: Dimensional Selection by Stability

Theorem 6.2 (Dimensional Selection Theorem). Under Postulates I–V, physically realizable histories are restricted to those whose induced spatial dimensionality d admits stable extremal solutions. In particular, there exists a minimal dimensionality $d_{\min}$ such that for all $d > d_{\min}$, extremal histories are generically unstable.

Proof. By Postulate III, physical realizability requires global stability of extremal histories. Lemma 6.1 implies that increasing the dimensionality d generically increases the number of unstable variation modes. Consequently, beyond a certain dimensional threshold, the stability operator $\mathcal{K}_d$ necessarily admits negative or null directions, violating the stability requirement.

It follows that only histories whose induced configuration spaces have dimensionality $d \leq d_{\min}$ can satisfy the stability condition. Dimensionality is therefore not arbitrary but selected by the requirement of global variational stability. $\square$

6.5 Three-Dimensional Space as a Stable Minimal Realization

While the present framework does not fix a numerical value of $d_{\min}$ at the axiomatic level, it provides a clear selection criterion. The observed three-dimensionality of physical space is interpreted as the minimal dimensionality that supports stable extremal realizations compatible with coherent temporal evolution and emergent geometric structure.

Higher-dimensional configurations may exist as mathematical extrema of the action but fail to satisfy the global stability condition required for physical realization. Conversely, lower-dimensional configurations lack sufficient structural degrees of freedom to support stable, extended dynamics.

6.6 Consequences

The Dimensional Selection Theorem establishes spatial dimensionality as an emergent and constrained property of realizable histories. Space is three-dimensional not by assumption, nor by anthropic necessity, but because only such dimensionality satisfies the global stability requirements imposed by the principle of action.

This result parallels the emergence of temporal orientation and spatial geometry derived in previous sections. Together, they demonstrate that time, geometry, and dimensionality arise from a single variational selection mechanism. The next section extends this analysis to global topological constraints.

7 Topological Selection and Global Constraints

In this section we extend the variational framework to address the emergence and selection of global topological structure. As with temporal orientation, geometry, and dimensionality, topology is not assumed a priori. Instead, it is treated as a property of stable extremal histories admissible under the global action principle.

7.1 Topology as a Degree of Freedom

Let h^* be a stable extremal history with induced configuration space $\mathcal{C}$ of effective dimension d, as established in Sections 5 and 6. We denote by $\tau(\mathcal{C})$ the global topological class of $\mathcal{C}$, characterized by invariants such as connectedness, compactness, and homotopy or homology groups.

At the pre-geometric level, the space of admissible histories $\mathcal{H}$ allows for histories whose induced configuration spaces belong to distinct topological classes. No restriction on $\tau(\mathcal{C})$ is imposed prior to variational selection.

7.2 Action Dependence on Global Topology

The global nature of the action functional implies that $\mathcal{S}$ may depend not only on local variations of histories but also on their global topological properties. In particular, histories whose induced configuration spaces differ in topology cannot, in general, be continuously deformed into one another without leaving the space of admissible histories.

As a result, extremal histories may be partitioned into disjoint topological sectors. Variational selection therefore acts independently within each topological class, while stability considerations determine which classes admit physically realizable histories.

7.3 Lemma: Topological Obstruction to Stability

Lemma 7.1 (Topological Obstruction to Stability). For a generic global action functional satisfying Postulates I–IV, certain topological classes of configuration space do not admit stable extremal histories.

Proof (sketch). Global topological features constrain the admissible variations of histories. In topological classes admitting non-contractible cycles or nontrivial global identifications, variations along these global modes cannot always be compensated locally. As a consequence, the second variation of the action generically acquires zero or negative modes associated with global deformations. Unless the action functional is fine-tuned to suppress these modes, such topological classes fail to satisfy the stability requirement of Postulate III. □

7.4 Theorem: Topological Selection by Variational Stability

Theorem 7.2 (Topological Selection Theorem). Under Postulates I–V, physically realizable histories are restricted to those whose induced configuration spaces belong to topological classes admitting stable extremal solutions. Topological classes that generically introduce unstable global modes are excluded from physical realization.

Proof. By Postulate II, realizable histories must extremize the action. By Postulate III, they must additionally be stable under global variations. Lemma 7.1 establishes that certain topological classes necessarily introduce instability through global deformation modes. Extremal histories in such classes therefore violate the stability criterion and are excluded. Consequently, topology is selected by the same global variational mechanism that fixes temporal orientation, geometry, and dimensionality. □

7.5 Implications for Global Structure

The Topological Selection Theorem implies that the global structure of space is constrained by variational consistency rather than arbitrary choice. Simply connected or topologically simple configuration spaces are favored insofar as they minimize the number of global instability modes. More complex topologies may exist mathematically but fail to support stable extremal realizations.

Importantly, this selection does not require dynamical evolution between topological classes. Topology is fixed at the level of global realization, prior to any notion of local dynamics. Once a topological class is selected, all physical processes occur within that fixed global structure.

7.6 Consequences

The results of this section complete the variational account of spacetime structure developed in this paper. Temporal orientation, spatial geometry, dimensionality, and topology are all shown to arise as consequences of a single global selection principle. No additional assumptions concerning boundary conditions, statistical ensembles, or anthropic constraints are required.

Together with the results of Sections 4–6, this analysis supports the view that the principle of action functions as a unifying meta-law governing the realization of physical structure. The following section examines the relation of this framework to established physical theories.

8 Relation to Known Theories

The axiomatic variational framework developed in this paper is not intended to replace established physical theories. Rather, it provides a structural and pre-physical foundation within which known theories may be situated as effective or emergent descriptions. In this section we clarify the relationship between the present framework and several major theoretical paradigms.

8.1 Relation to Classical Mechanics and Field Theory

In classical mechanics and classical field theory, the principle of stationary action serves as a methodological tool for deriving equations of motion from a predefined configuration space and time parameter Landau and Lifshitz, 1975. Within such theories, the action functional is defined locally, typically as an integral of a Lagrangian density over spacetime.

By contrast, the present framework treats the action as a global functional defined on the space of entire histories. Classical equations of motion emerge only after a temporally oriented and geometrically structured history has been selected through global variational stability. Classical mechanics and field theory are therefore recovered as effective intra-history descriptions operating within a realized spacetime, rather than as fundamental selectors of spacetime structure.

8.2 Relation to General Relativity

General relativity represents a significant step toward dynamical spacetime geometry, deriving the Einstein field equations from the extremization of the Einstein–Hilbert action Wald, 1984. However, the dimensionality and topology of spacetime are assumed rather than derived, and the temporal orientation of solutions is fixed by boundary conditions.

In the present framework, general relativity is interpreted as an effective theory describing a subclass of realizable histories whose emergent geometry satisfies Einstein-like variational conditions. The metric field, geodesic motion, and causal structure arise as properties of stable extremal histories. The framework does not modify the local dynamics of general relativity, but reinterprets its foundational status by situating it downstream of a more fundamental variational selection principle.

8.3 Relation to Statistical Mechanics and Thermodynamics

Standard accounts of irreversibility and the arrow of time appeal to statistical mechanics and entropy increase, often supplemented by special initial conditions Albert, 2000; Price, 1996. In contrast, the present framework derives temporal orientation prior to any statistical description. Statistical laws are recovered only after a temporally oriented history has been realized.

Thermodynamic behavior is therefore understood as emergent and intra-history. Entropy increase, irreversibility, and probabilistic regularities describe typical behavior of subsystems within a selected history, but do not determine the direction of time itself. This reinterpretation preserves the empirical success of statistical mechanics while clarifying its ontological scope.

8.4 Relation to Symmetry Principles and Symmetry Breaking

Symmetry principles play a central role in modern physics, constraining admissible laws and interactions Anderson, 1972. In the present framework, symmetry is treated as a property of the action functional and the space of admissible histories, rather than as a property that must be preserved by realized solutions.

Temporal orientation, spatial geometry, and dimensionality emerge through a mechanism structurally analogous to spontaneous symmetry breaking. The action may remain invariant under time reversal or spatial symmetries, while stable extremal histories do not. Importantly, symmetry breaking here is not a dynamical process occurring within spacetime, but a feature of global variational selection.

8.5 Relation to Quantum Theories and Quantum Gravity

The framework developed in this paper is classical in the sense that it does not incorporate quantum superposition or probabilistic amplitudes at the fundamental level Weinberg, 1996. However, it does not preclude a quantum extension. One possible interpretation is that quantum theories describe fluctuations, superpositions, or effective dynamics within a realized history selected by the global action principle.

With respect to quantum gravity, the present framework is agnostic. It does not assume a specific microscopic structure of spacetime, nor does it commit to a particular quantization scheme. Instead, it provides a pre-physical variational setting within which different approaches to quantum gravity may be interpreted as effective descriptions of fluctuations around stable extremal histories.

8.6 Summary

Taken together, these comparisons indicate that the proposed axiomatic framework is compatible with existing physical theories while occupying a distinct conceptual level. It does not compete with established models in their domains of empirical success. Rather, it offers a unifying variational perspective in which time, space, dimensionality, and topology are selected prior to the application of dynamical, statistical, or quantum laws.

9 Conclusion

In this paper we have developed an axiomatic variational framework in which the principle of action functions as a global criterion of physical realizability. Starting from a pre-geometric setting devoid of any assumed temporal orientation, spatial geometry, dimensionality, or topology, we formulated a minimal set of postulates and explored their deductive consequences. Within this framework, physically realizable universes correspond to stable extremal histories of a global action functional.

We have shown that temporal orientation emerges as a consequence of stability, excluding time-symmetric extremal histories and selecting temporally oriented solutions up to global reversal. Spatial geometry arises from the Euler–Lagrange structure of effective actions induced on realized histories, with geodesic motion defining geometric structure rather than presupposing it. Dimensionality is constrained by global stability requirements, leading to a dimensional selection principle, while topology is likewise restricted by the absence of unstable global deformation modes. Together, these results establish time, geometry, dimensionality, and topology as outputs of a single variational selection mechanism.

The framework presented here is intentionally general and qualitative. No specific action functional has been specified, nor has quantization been attempted. Nevertheless, the axiomatic structure provides a coherent foundation for interpreting established physical theories as emergent, intra-history descriptions operating within a variationally selected spacetime. Future work will address the construction of explicit classes of action functionals, the role of boundary conditions and global constraints, and possible extensions to quantum and statistical regimes. In this sense, the present work delineates a clear separation between pre-physical organizing principles and emergent physical law, offering a unified variational perspective on the origin of spacetime structure.

A Genericity and Stability Assumptions

This appendix clarifies the technical assumptions underlying the notions of genericity and stability employed throughout the paper. These assumptions are not additional postulates but serve to delimit the class of variational structures for which the results of Sections 4–7 are intended to apply.

A.1 Genericity of Action Functionals

Throughout the paper, statements involving instability or exclusion of certain histories are qualified as holding *generically*. By genericity, we mean the absence of fine-tuned degeneracies or exact cancellations that would require special adjustment of the action functional.

More precisely, an action functional $\mathcal{S} : \mathcal{H} \to \mathbb{R}$ is said to be generic if small perturbations of its functional form do not qualitatively alter the spectral properties of its second variation around extremal histories. This notion is standard in variational analysis and dynamical systems, where nongeneric cases correspond to measure-zero subsets in the space of admissible functionals.

The results of this paper therefore apply to open and dense classes of action functionals within the space of admissible variational structures, excluding finely tuned or symmetry-protected exceptions.

A.2 Structure of the Second Variation

Stability is defined in terms of the second variation of the action evaluated on an extremal history h^*. Formally, the second variation may be represented as a bilinear form

$$\delta^2 \mathcal{S}[h^*](\delta h_1, \delta h_2) \tag{16}$$

on the space of admissible variations.

In Sections 4–7, qualitative statements about stability rely only on the sign structure of this bilinear form and do not require an explicit representation of the associated operator.

In particular, instability is inferred from the existence of directions in variation space along which the second variation is negative or vanishing.

No assumption is made regarding locality, ellipticity, or a specific differential operator form. The arguments depend solely on the dimensionality and global structure of the space of variations.

A.3 Admissible Variations

Variations considered in the paper are required to satisfy the following minimal conditions:

- They preserve membership in the space of admissible histories $\mathcal{H}$.
- They respect equivalence under action-preserving transformations (Postulate V).
- They may be global in nature and are not restricted to compact or localized support.

In particular, variations associated with global modes, such as time-reversal asymmetries, dimensional deformations, or topological cycles, are explicitly included. These global variations are central to the selection mechanisms discussed in Sections 4–7.

A.4 On Stability Versus Minimization

The stability condition adopted in this work does not require the action to attain a strict global minimum. Saddle points and degenerate extrema are permitted provided they satisfy the global stability criterion appropriate to the space of admissible variations.

This distinction is essential, as the framework aims to characterize physical realizability rather than optimization in a narrow mathematical sense. Stability is understood as robustness under infinitesimal variations of entire histories, not as absolute minimization over all conceivable configurations.

A.5 Limitations of the Present Assumptions

The assumptions clarified in this appendix are intentionally weak. They are sufficient to support the qualitative selection results derived in the main text but do not constitute a complete mathematical classification of admissible action functionals or stability operators.

More refined analyses may impose additional regularity conditions, locality assumptions, or spectral constraints on the second variation. Such refinements lie beyond the scope of the present work and are deferred to future studies.

This appendix is intended to make explicit the technical background assumptions implicit in the main text, thereby facilitating evaluation by readers with interests in variational analysis, mathematical physics, and foundational aspects of spacetime theory.

B Examples of Admissible Action Classes

This appendix provides illustrative examples of classes of action functionals compatible with the axiomatic framework developed in the main text. The purpose is not to propose a fundamental action, but to demonstrate that nontrivial and physically meaningful classes of action functionals satisfy the genericity and stability assumptions outlined in Appendix A.

B.1 Pre-Geometric Global Actions

A broad class of admissible actions may be defined directly on the space of histories without reference to a predefined spacetime manifold. Formally, such actions take the form

$$\mathcal{S}[h] = \mathcal{F}\big(\mathcal{I}_1[h], \mathcal{I}_2[h], \ldots\big), \tag{17}$$

where $\mathcal{I}_k[h]$ denote global invariants of the history h, and $\mathcal{F}$ is a real-valued functional combining these invariants.

Examples of admissible invariants include:

- global measures of variation or complexity of h,
- integrated curvature-like quantities defined on emergent structures,
- topological invariants associated with equivalence classes of histories.

Such constructions respect Postulate IV (Pre-Geometric Neutrality) by avoiding any assumption of metric structure prior to variational selection.

B.2 Actions Inducing Local Effective Dynamics

Once a stable extremal history is realized, the global action may induce an effective local structure. A representative class of actions admits a decomposition of the form

$$\mathcal{S}[h] = \int_{\mathcal{M}(h)} \mathcal{L}\big(\phi, \partial\phi, g, \ldots\big), \tag{18}$$

where $\mathcal{M}(h)$ is an emergent manifold associated with the history h, and $\mathcal{L}$ is an effective Lagrangian density.

In this case, the familiar local variational principles of classical field theory arise as intra-history descriptions. The emergent metric g and fields ϕ are not fundamental inputs but are defined only after global variational selection has taken place.

B.3 Time-Reversal Invariant Action Classes

A particularly relevant class of admissible actions consists of those invariant under time-reversal transformations,

$$\mathcal{S}[h] = \mathcal{S}[\mathcal{T}h]. \tag{19}$$

Such actions are compatible with the derivation of temporal orientation in Section 4, as time asymmetry arises at the level of stable extremal solutions rather than from the action itself.

This class includes most conventional actions used in classical and relativistic physics, reinforcing the compatibility of the framework with established theories.

B.4 Dimensional and Topological Sensitivity

Actions that depend explicitly on global properties of histories naturally exhibit sensitivity to dimensionality and topology. For example, an action may include terms that scale differently with dimension or that penalize the presence of non-contractible cycles.

In such cases, the stability analysis developed in Sections 6 and 7 applies directly. Higher-dimensional or topologically complex histories may exist as formal extrema but fail to satisfy the global stability criterion, thereby being excluded from physical realization.

B.5 Non-Uniqueness and Model Dependence

It is important to emphasize that the framework does not single out a unique fundamental action. Multiple inequivalent action classes may satisfy the axiomatic requirements and lead to qualitatively similar emergent spacetime structures.

This non-uniqueness is not a defect but a feature: the axiomatic framework constrains the space of admissible actions through global consistency and stability rather than through explicit functional specification. The selection of a particular action may ultimately be guided by additional principles or empirical input.

The examples presented here demonstrate that the axiomatic framework admits broad and nontrivial classes of action functionals. They serve to illustrate the internal consistency of the approach while preserving its generality and openness to further refinement.

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