Generalization Edge Detection Operators and the Laplace operator in any dimension and any resolution image

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Abstract—**This study attempted to rewrite some of the concepts of digital image processing in the form of formal redefinition to build some basic terminology in a consistent way, then from the set of formal definitions are presented, researchers deduce a few basic ideas which one is the general form of edge detection operators, where we are expect that from the general form We can conclude all the edge detection operator previously known, and from the general form we can derive some variation of edge detection operators never before. Also based on a set of formal definitions, we build the generalization form of laplace operator so that we can derive an original laplace operator and a variety of laplace operator which may be infinite. This study also expands the concept of edge detection to the infinite image dimensions and resolution, by first expanding the general form of edge detection operator and laplace operator to the infinite image dimensions and resolution.**

Keywords-component; edge detectio,laplace operator, image processing, general form of edge detection operator, general form of laplace operator

I. INTRODUCTION

This paper is proposed to the exploration and extending of the idea of edge detection in digital image processing. This paper proposed new idea where we build another way to interpret of image processing.

On this research is focused on the generalization of all forms of edge detection operator to the one form of equation and generalized to the entire second derivative operators including laplace operator into a single form of the equation.

This research has been preceded by the reconstruction effort over some basic terminology in digital image processing, where we redefine some basic terms until then the generalization of edge detection and Laplace operator. All in 2-space or two-dimensional image. The report on this research has pointed out in a paper in the proceedings of the Seminar Nasional Fisika Terapan III di UNAIR (SNAFT III 2012), Indonesia [2].

This paper is a paper at the seminar ICCSE 2012 make improvements some deficiencies in the statement as well as expand the equation and all equations to arbitrary n-dimensional space and to the resolution of the image of any of the previous papers on the seminar SNAFT III 2012.

All definitions and postulates in this paper is new, and the summary from our previous paper is new, we proposed all as the new set interpretation of image processing, not referenced from other papers before.

Below is a summary of the definition set that we proposed as the new definitions in image processing, out in previous papers as an introduction to this paper, which is basically presented as a redefinition of some basic terms:

Definition 1:
Pixel is a 1-1 mapping of a two-dimensional point into a three-dimensional vector where each entry is a positive integer or zero.

Definition 2 (generalization to a set of point):
Pixel is a 1-1 mapping of a 3-dimensional convex point set into a three-dimensional vector where each entry is a positive integer or zero.

Definition 3:
Voxels is a 1-1 mapping of a 3-dimensional convex point set into a three-dimensional vector where each entry is a positive integer or zero.

Definition 4 (generalization of pixel and voxel):
Polysel-n,m is a 1-1 mapping of a set of n-dimensional convex point to an m-dimensional vector.

Definition 5 (generalization of pixel and voxel):
Polysel-n, m is a 1-1 mapping of a polytope-n into an m-dimensional vector.

Definition 6 (color):
A Color Polysel-n,m is polysel-n, m where each entry vectors are positive integers or zero

Definition 7 (gray):
A Gray Polysel-n,m is The Color Polysel-n,m where each entry is the same each other in the vector.

Definition 8 (black-white):
A Black-White Polysel-n,m is The Gray Polysel-n,m where the entry vector is zero or maximum value.

Definition 9 (digital image generalization):
A Digital Image-n,m digital is The set of polysel-n,m.

Definition 10 (gradient pixel):
A gradient pixel is the difference of two adjacent pixel (with no pixel between them) divide by they distance..

Definition 10.a (gradient pixel)
A gradient pixel is the difference of two adjacent pixel (with no pixel between them).

Definition 11 (edge pixel):

An edge pixel is the sum of any gradient pixel in the neighborhood.If g(x,y) is an edge pixel, then it is write as:

$$g(x,y) = \sum_{i,j,k,t} f(x+i, y+j) - f(x+k, y+t)$$

where $|i-k| = 1$ or $|i-k| = 0$

$|j-t| = 1$ or $|j-t| = 0$

(we proposed this equation as a new equation for representation of edge pixel, this is proposed on our previous paper [2]).

Definition 12 (edge pixel generalization)

Suppose $g_{N(4+p4)}(x,y)$ is an edge pixel taken from the set of points N(4+p4) neighborhood, then the edge pixel write as:

$$g_{N(4+p4)}(x,y) = \sum_{i_1,\dots,i_4} f(x+i_1, y+i_2) - f(x+i_3, y+i_4)$$

where:

$p = 1, 2, 3, \dots$

$|i_k - i_{2+k}| = 1$ or $|i_k - i_{2+k}| = 0$

$i_j = -p, -(p-1), -(p-2), \dots, -1, 0, 1, 2, \dots, p-2, p-1, p$

$k = 1, 2.$

$j = 1, 2, 3, 4.$

(we proposed this equation as a new equation for representation of edge pixel generalization, this is proposed on our previous paper [2]).

Definition 13 (The definition of a derivative pixels):
Suppose f(x,y) is a pixel, then the derivative of a pixel is stated as:

$$\frac{\partial f(x,y)}{\partial x} = \frac{f(x+1,y)-f(x,y)}{\Delta x} \quad \text{in x-direction}$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{f(x,y+1)-f(x,y)}{\Delta y} \quad \text{in y-direction}$$

or:

$$\frac{\partial f(x,y)}{\partial y} = f(x, y+1) - f(x,y)$$

$$\frac{\partial f(x,y)}{\partial x} = f(x+1, y) - f(x,y)$$

.

Definition 14 (Laplace operator generalization)

Generalization of Laplace operator is the sum of derivative of edge pixel in x-direction and y-direction.

$$L\big(f(x,y)\big) = \frac{\partial g_{N(4+p4)}}{\partial x} + \frac{\partial g_{N(4+p4)}}{\partial y}$$

(we proposed this equation as a new equation for representation of edge pixel generalization, this is proposed on our previous paper [2]).

There are flaws in the equation presented in Definition 14 in the previous paper (SNAFT III 2012), the remainder of this paper, the research is directed in an effort to improve the equation, and to expand the equation in definition 12.

II. IMPROVEMENT AND EXPANSION

A. *Generalization of Definition 12.*

Definition 15.

Suppose (x1,x2,x3,…,xn) is a point in a n-dimensional space, A is neighborhood of a polysel-n,m, and ∂A is a set of distance (chess board distance) of two any point in A, then an edge polysel-n,m at point (x1,x2,x3,…,xn) stated as:

$$g_A(x_1, x_2, x_3 \dots, x_n) = \sum_{i_1,i_2,i_3,\dots,i_{2n}} f(x_1+i_1, \dots, x_n+i_n) - (x_1+i_{1+n}, \dots, x_n+i_{2n})$$

where :

$$i_1, i_2, i_3, \dots, i_{2n} = -\frac{1}{2}Max(\partial A), -\frac{1}{2}Max(\partial A)+1, \dots, 0, \dots, \frac{1}{2}Max(\partial A)-1, \frac{1}{2}Max(\partial A)$$

$$|i_k - i_{k+n}| = \begin{cases} 1, i_k \neq i_{k+n} \\ 0, i_k = i_{k+n} \end{cases}$$

$$k = 1,2,3,\dots,n$$

(we proposed this equation as a new equation for generalization of definition 12, this not referenced from other papers before).

B. *Improvement and expansion of Definition 14 (Operator Laplace)*

Improvement begins with propose the concept of direction to any changing of a value of pixel, as follows:

Definition 16:

The direction of a pixel change in N8 neighborhood is stated as variation from numbers, -1,0,1, in ordered 2-tupel.

Suppose given a pixel:

changing in positive x-direction, is (1,0)

changing in positive y-direction, is (0,1)

changing in positive xy-direction, is (1,1)

changing in negative x-direction, is (-1,0)

Definition 17:

The direction of a pixel change in N(4+p4) neighborhood is stated as variation from numbers, p=-p,-p+1,...,-2,-1,0,1,2,…,p-1, in ordered 2-tupel.

Further definition is generalized to voxels and polysel-n,m as follows:

Definition 17:

Suppose A is a neighborhood of a polysel-n,m, and ∂A is a set of integer distance (chess board distance) from any polysel-n,m in A, and {p} is a sequence of – ½max(∂A), –½max(∂A)+1, …,-2,-1,0,1,2,…, ½max(∂A)-1, ½max(∂A), nad $Var(\{p\})^n=\{x|x=(i_1,i_2,i_3,\dots,i_n), i_j=\{p\} \ \ j=1,2,3,\dots n\}$, a 1-1 mapping T is which states the direction of change polysel-n,m in the A, write as:

$$T: A \longrightarrow Var(\{p\})^n$$

(we proposed this function as a new function for representation of direction, this not referenced from other papers before).

Example:

A polysel-2, 4 (pixels), in the neighborhood N8 (in this case set A = N8), then the direction is expressed as:

$T: N8 \longrightarrow Var(-1,0,1)^n$

where :

$Var(-1,0,1)^n = \{(-1,0),(0,-1),(-1,-1),(-1,1),(1,-1), (1,1), (0,1),(1,0),(0,0)\}$

Example:

Suppose an n-dimensional vector space with the vector basis e1, e2, e3, ..., en, if polysel-n,m just changed in the positive direction of e1, the direction is written (1,0,0, ..., 0). If the change in the direction of e1, e2, e3, ..., en is positive, then the direction is written (1,1,1, ..., 1) and so on.

Furthermore, in this paper, the negative and the positive direction are united only in the word "direction", ie the positive x direction and the negative x direction is expressed only as a direction x.

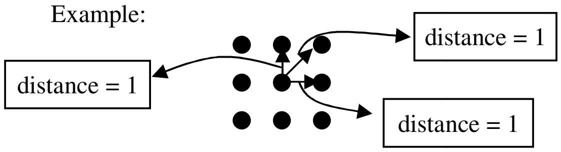
Furthermore, a postulate is presented as follows:

Postulat 1:

Distance of any two points polysel-n, m are adjacent (with no polysel-n, m between the two) in all directions is one.

For example in any space with the vector basis e1, e2, e3, ..., en, a distance of two points in the direction of any adjacent pixels ie $(i_1,i_2,i_3,\ldots,i_n) \in Var(\{p\})^n$ is one.

Example:



Postulate is only intended to make the idea of the distance of the hypotenuse Pythagoras does not apply to the nearest distance of two pixels, or the polysel-n,m in any dimension. Postulate is also to facilitate declare an edge polysel-n,m that g_A as in Definition 15 as the decomposition according to all direction.

Furthermore, the concept of pixel gradient presented in Definition 10 previously enriched with the concept of pixel gradient in the x-direction, y-direction and the xy diagonal direction, only three groups such as the direction of pixels squeezed by Definition 10 only exists in the neighborhood N8, this concept as the following:

Definition 18:

1. Gradient pixel in the x-direction is the difference of two adjacent pixels is divided by the distance of them in x-direction.
2. Gradient pixel in the y-direction is the difference of two adjacent pixels is divided by the distance of them in y-direction.
3. Gradient pixel in the diagonal xy-direction is the difference of two adjacent pixels is divided by the distance of them in xy-direction.

Based on the postulate 1, the distance of two adjacent pixels in all directions is one, then Definition 18 can be simplified into:

1. Gradient pixel in x-direction is the difference of two adjacent pixel in x-direction.
2. Gradient pixel in y-direction is the difference of two adjacent pixel in y-direction.
3. Gradient pixel in diagonal xy-direction is the difference of two adjacent pixel in x-direction.

Further Definition 11 on the edge pixel are interpreted as decomposition number gradient in the x-direction, y-direction and the xy-direction, namely that the edge pixel in the direction of x is the sum of gradient in the x direction, the edge pixel in the y direction is the sum of the gradient in the y direction, and the edge pixel in diagonal direction is the sum of the gradient in xy-diagonal direction. Similarly, the expansion of the concept of edge pixel defined as follows

Definition 19:

1. The edge pixel in the x-direction is written as follows:

$$g(x,y)|_x = \sum_{i,k} f(x+i,y) - (x+k,y)$$

2. The edge pixel in the y-direction is written as follows:

$$g(x,y)|_y = \sum_{j,t} f(x,y+j) - (x,y+t)$$

3. The edge pixel in the xy-direction is written as follows:

$$g(x,y)|_{xy} = \sum_{i,j,k,t} f(x+i,y+j) - (x+k,y+t)$$

Dimana $|i-k| = 1$ atau $|i-k| = 0$
$|j-t| = 1$ atau $|j-t| = 0$

(we proposed this equations as some new equations for edge pixel in the x-direction, y-direction, and xy-direction, this not referenced from other papers before).

Further definition 11 can be rewritten as follows:

$$g(x,y) = g(x,y)|_x + g(x,y)|_y + g(x,y)|_{xy}$$

And further definition 12 can be rewritten as follows:

$$g_{N(4+p4)}(x,y) = g_{N(4+p4)}\big|_x + g_{N(4+p4)}\big|_y + g_{N(4+p4)}\big|_{xy}$$

And further to Definition 15, a similar thing can be done.

At this point, Definition 14 can be rewritten as follows:

Definition 20: (Improvement to definition 14)

Generalization of Laplace operator is the sum of derivative of edge pixel in x-direction and derivative edge pixel in y-direction

$$L(f(x,y)) = \frac{\partial g_{N(4+p4)}|_x}{\partial x} + \frac{\partial g_{N(4+p4)}|_y}{\partial y}$$

(we proposed this equation as a new equation for generalization of laplace operator, this not referenced from other papers before).

Definition 21: (The generalization of the Laplace operator for polysel)

Laplace operator for polysel-n,m is expressed as follows:

$$L(f(x_1, x_2, .., x_3)) = \frac{\partial g_A|_{x_1}}{\partial x_1} + \frac{\partial g_A|_{x_2}}{\partial x_2} + \cdots + \frac{\partial g_A|_{x_n}}{\partial x_n}$$

(we proposed this equation as a new equation for generalization of laplace operator to higher dimension, this not referenced from other papers before).

This paper has put forward the idea in the form of an extension of the method of edge detection and laplace operator. With the first reconstruct some basic concepts in digital image processing, but this is just a point of view. Maybe there are some ideas for formulating digital image processing especially edge detection in the form of representation to another. Hopefully, this point of view can be useful in building a better knowledge.

III. CONCLUSION

We just finish this idea, about reconstruction of image processing fundamental, and about generalization of edge detection operator and Laplace operator.

All of definitions and postulate, and equations that we propose in this paper are new, and until finish, we conclude the research in this paper that is:

1. Generalization of pixel and voxel terms in higher dimension are polysel n,m. where n is constitute the dimension of convex set point in space and m is the dimension of value of convex set point.

2. Generalization of edge polysel n,m is :

$$g_A(x_1, x_2, x_3 \ldots, x_n) = \sum_{i_1, i_2, i_3, \ldots, i_{2n}} f(x_1 + i_1, \ldots, x_n + i_n) - (x_1 + i_{1+n}, \ldots, x_n + i_{2n})$$

where:

$$i_1, i_2, i_3, \ldots, i_{2n} = -\frac{1}{2}Max(\partial A), -\frac{1}{2}Max(\partial A) + 1, \ldots, 0, \ldots, \frac{1}{2}Max(\partial A) - 1, \frac{1}{2}Max(\partial A)$$

$$|i_k - i_{k+n}| = \begin{cases} 1, i_k \neq i_{k+n} \\ 0, i_k = i_{k+n} \end{cases}$$

$$k = 1,2,3, \ldots, n$$

3. Generalization of Laplace operator is:

$$L(f(x,y)) = \frac{\partial g_{N(4+p4)}|_x}{\partial x} + \frac{\partial g_{N(4+p4)}|_y}{\partial y}$$

4. Generalization of Laplace operator on polysel n,m is:

$$L(f(x_1, x_2, .., x_3)) = \frac{\partial g_A|_{x_1}}{\partial x_1} + \frac{\partial g_A|_{x_2}}{\partial x_2} + \cdots + \frac{\partial g_A|_{x_n}}{\partial x_n}$$

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