13

PRACTICAL GEOMETRY

INTRODUCTION

In the previous class, you have learnt to construct a circle of given radius, a line segment of given length, a copy of a line segment, a perpendicular line to a given line at a point on it or outside it, perpendicular bisector of a line segment, an angle of given measure, a copy of an angle, bisector of an angle and angles of some special measures – 60°, 30°, 120°, 90°, 45° etc.

In this chapter, we shall learn:

- construction of a line parallel to a given line through a point not on the line.
- construction of triangles.

CONSTRUCTION OF A LINE PARALLEL TO A GIVEN LINE THROUGH A POINT NOT ON IT

We shall draw a line parallel to a given line through a point not on it by the following methods:

(*i*) by paper folding (activity) (*iii*) by using ruler and compasses.

Activity 8

To draw a line parallel to a given line through a point not on it

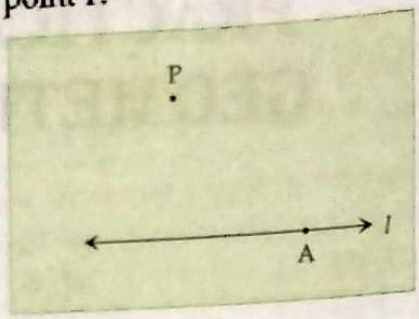
Materials required

(*i*) Tracing paper (*ii*) Coloured ball point pen (*iii*) Ruler.

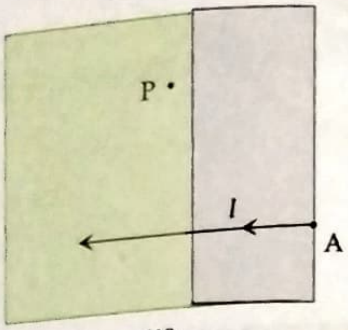
Steps

1. Take a sheet (or piece) of tracing paper and draw a line l on it. Mark a point P not on l and a point A on l as shown in fig. (*i*).
2. Fold the paper through A perpendicular *to* l and form a crease as shown in fig. (*ii*).
3. Unfold the paper and draw a dotted line, say n, on the above fold as shown in fig. (*iii*).

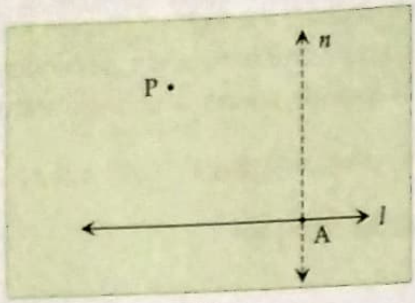
4. Fold the paper perpendicular to line n and move this fold till it passes through P and form a crease as shown in fig. (*iv*).
5. Unfold the paper and draw a line, say m, on this new crease as shown in fig. (*v*). This new line m is the required line parallel to the given line l and passing through point P.



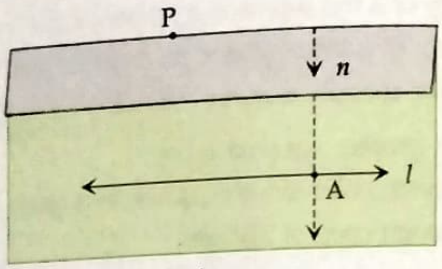
(*i*)



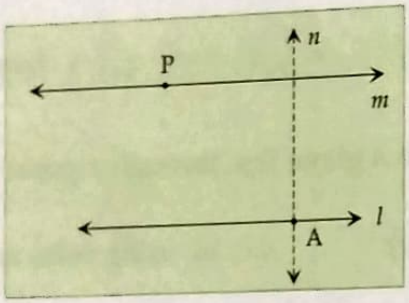
(*ii*)



(*iii*)



(*iv*)



(*v*)

Using ruler and compasses

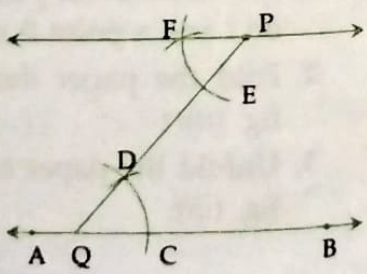
To draw a line parallel to a given line through a point not on it.

Given. Any line AB and a point P outside AB.

Required. To draw a line parallel to AB and passing through the point P.

Steps of construction

1. Take any point Q on AB. Join P and Q.
2. With Q as centre and any suitable radius, draw an arc to meet AB at C and QP at D.
3. With P as centre and same radius (as in step 2), draw an arc to meet PQ at E.
4. Measure the segment CD with compass.
5. With E as centre and radius equal to CD, draw an arc to cut the previous arc at F.



6. Draw a line passing through P and F, then PF is the required line parallel to the line AB and passing through P.

Remark

Note that in the above construction, $\angle PQB$ and $\angle QPF$ are alternate interior angles. Therefore, line AB and FP are parallel. Moreover, we can slightly modify the above construction to use the concept of equal corresponding angles instead of equal alternate interior angles.

CONSTRUCTION OF TRIANGLES

Recall, the sum of lengths of any two sides of a triangle is greater than the length of the third side. Also the sum of measures of all the three angles of a triangle is 180°.

In the chapter on 'Congruence of Triangles', we noticed that a triangle can be uniquely drawn if any of the following sets of measurements is given:

(*i*) the lengths of three sides of a triangle.

(*ii*) the lengths of two sides and measure of included angle between them.

(*iii*) two angles and the length of the included side between them.

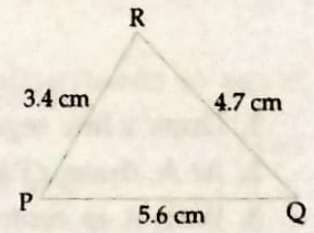
(*iv*) the length of hypotenuse and a side in case of a right angled triangle.

We shall use the above ideas to construct triangles.

Constructing a triangle when the lengths of its three sides are given

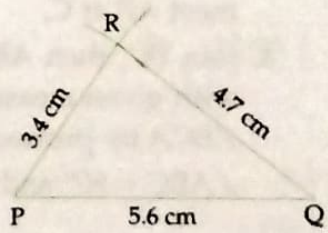
Example 1. *Construct a triangle PQR such that PQ = 5.6 cm, QR = 4.7 cm and PR = 3.4 cm.*

Solution. Draw a rough sketch of ΔPQR and mark the given lengths (this will help us in deciding how to proceed)



Steps of construction

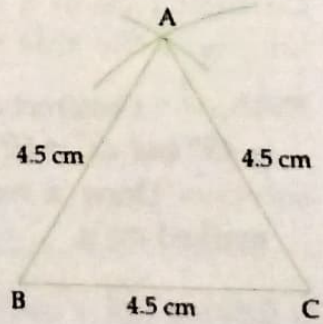
1. Draw a line segment PQ of length 5.6 cm.
2. With Q as centre and radius 4.7 cm (= QR), draw an arc.
3. With P as centre and radius 3.4 cm (= PR), draw an arc to cut the previous arc at R.
4. Join PR and QR. Then PQR is the required triangle.



Example 2. *Construct an equilateral triangle with side 4.5 cm.*

Solution. **Steps of construction**

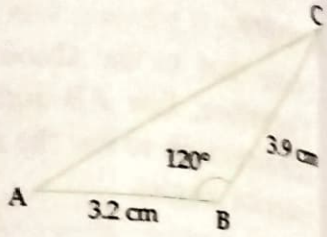
1. Draw a line segment BC of length 4.5 cm.
2. With B as centre and radius 4.5 cm, draw an arc.
3. With C as centre and radius 4.5 cm, draw an arc to cut the previous arc at A.
4. Join AB and AC, then ABC is the required equilateral triangle with side 4.5 cm.



Constructing a triangle when the lengths of its two sides and the measure of the included angle between them are known

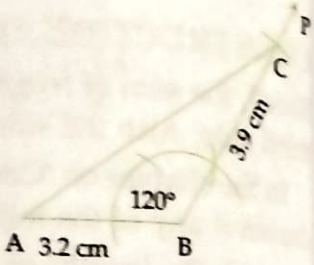
Example 3. *Construct a triangle ABC such that AB = 3.2 cm, BC = 3.9 cm and ∠B = 120°.*

Solution. Draw a rough sketch of ΔABC with measures marked on it.



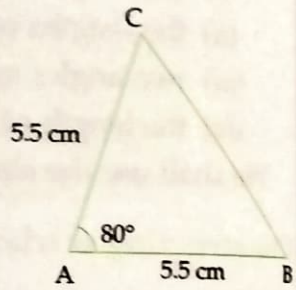
Steps of construction

1. Draw a line segment AB of length 3.2 cm.
2. At B, construct ∠ABP = 120°.
3. With B as centre and radius 3.9 cm, draw an arc to meet BP at C.
4. Join AC, then ABC is the required triangle.



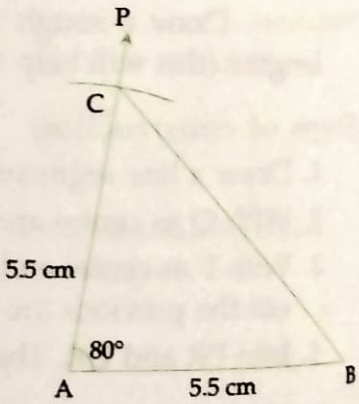
Example 4. *Construct an isosceles triangle in which length of each equal side is 5.5 cm and the angle between them is 80°. Measure other angles.*

Solution. Draw a rough sketch of a triangle with measures marked on it.



Steps of construction

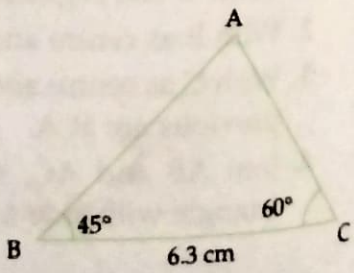
1. Draw a line segment of length 5.5 cm.
2. At A, draw ∠PAB = 80° (by using protractor).
3. With A as centre and radius 5.5 cm draw an arc to meet AP at C.
4. Join BC, then ABC is the required isosceles triangle with given measurements. On measuring ∠ABC and ∠BCA by protractor, we find that:
   ∠ABC = 50° and ∠BCA = 50°.



Construction of a triangle when the measures of two of its angles and the length of the side included between them are given

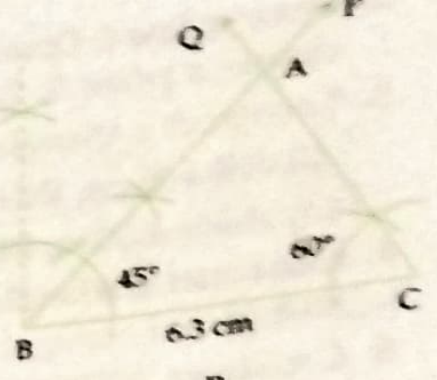
Example 5. *Construct a triangle ABC such that BC = 6.3 cm, ∠B = 45° and ∠C = 60° by using ruler and compasses only.*

Solution. Draw a rough sketch of ΔABC with measures marked on it.



Steps of construction

1. Draw a line segment BC of length 6.3 cm.
2. At B, construct $\angle PBC = 45°$.
3. At C, construct $\angle BCQ = 60°$.
4. Let rays BP and CQ intersect at A, then ABC is the required triangle.



Example 6. *Construct ΔPQR if $PQ = 5$ cm, $m\angle PRQ = 105°$ and $m\angle PQR = 40°$.*

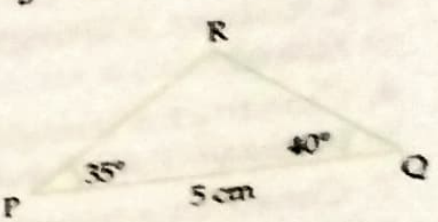
Solution. As the sum of angles of a triangle is 180°,

$\angle RPQ + \angle PQR + \angle QRP = 180°$

$\therefore \quad \angle RPQ + 40° + 105° = 180°$

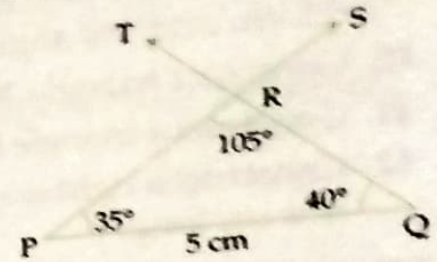
$\Rightarrow \quad \angle RPQ = 180° - 40° - 105° = 35°$.

Draw a rough sketch of ΔPQR with PQ = 5 cm, $\angle RPQ = 35°$ and $\angle PQR = 40°$.



Steps of construction

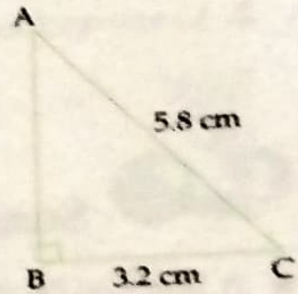
1. Draw a line segment PQ of length 5 cm.
2. At P, draw $\angle SPQ = 35°$ (by using protractor).
3. At Q, draw $\angle PQT = 40°$ (by using protractor).
4. Let rays PS and QT meet at R, then PQR is the required triangle with given data.



Construction of a right angled triangle when the lengths of hypotenuse and one side are given

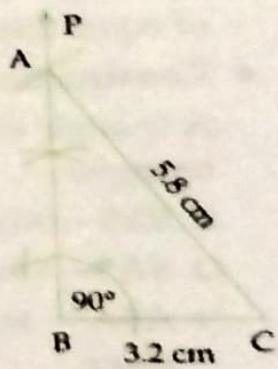
Example 7. *Construct a triangle ABC such that BC = 3.2 cm, $\angle B = 90°$ and hypotenuse AC = 5.8 cm by using ruler and compasses only.*

Solution. Draw a rough sketch of ΔABC with BC = 3.2 cm, $\angle B = 90°$ and hypotenuse AC = 5.8 cm.



Steps of construction

1. Draw a line segment BC of length 3.2 cm.
2. At B, construct $\angle CBP = 90°$.
3. With C as centre and radius 5.8 cm, draw an arc to meet BP at A.
4. Join AC, then ABC is the required triangle.



Exercise 13

1. Draw a line, say *l*, take a point P outside it. Through P, draw a line parallel to *l* using ruler and compasses only.

2. Draw a line l. Draw a perpendicular to l at any point on l. On this perpendicular choose a point A, 3.5 cm away from line l. Through A, draw a line m parallel to l.
3. Construct a triangle ABC, given that
   (*i*) AB = 5 cm, BC = 6 cm and AC = 7 cm
   (*ii*) AB = 4.5 cm, BC = 5 cm and AC = 6 cm.
4. Construct a triangle PQR given that PQ = 5.4 cm, QR = PR = 4.7 cm. Name the triangle.
5. Construct a triangle LMN such that length of each side is 5.3 cm. Name the triangle.
6. Construct a triangle ABC such that AB = 2.5 cm, BC = 6 cm and AC = 6.5 cm. Measure $\angle ABC$ and name the triangle.
7. Construct a triangle PQR, given that PQ = 3 cm, QR = 5.5 cm and $\angle PQR = 60°$.
8. Construct ΔDEF such that DE = 5 cm, DF = 3 cm and $m\angle EDF = 90°$.
9. Construct an isosceles triangle in which the length of each of its equal sides is 6.5 cm and the angle between them is 110°. Measure base angles.
10. Construct triangle XYZ if it is given that XY = 6 cm, $\angle X = 30°$ and $\angle Y = 100°$.
11. Construct a triangle PQR given that PQ = 4.9 cm, $\angle P = 45°$ and $\angle Q = 60°$. Measure $\angle R$.
12. Construct a right angled triangle whose hypotenuse is 6 cm long and one of the legs is 4 cm long.

Higher Order Thinking Skills (HOTS)

1. Construct a triangle ABC such that BC = 5.2 cm, AB = 4.8 cm and median CM = 3.6 cm.
2. Construct an isosceles right angled triangle ABC such that its hypotenuse BC = 6 cm.

Summary

★ A line can be drawn parallel to a given line through a point not on it by using the concept of equal alternate interior angles or by using the concept of equal corresponding angles.

★ **Construction of triangles**

A triangle can uniquely be constructed by using indirectly the concept of congruence of triangles.

- **SSS** – when the lengths of its three sides are given.
- **SAS** – when the lengths of two sides and the measure of the included angle are given.
- **ASA** – when the measures of two angles and the length of the included side are given.

If the measures of two angles and the length of a side are given, then we can find the measure of the third angle by using angle sum property of a triangle and hence the question can be converted to ASA, so triangle can be drawn uniquely.

- **RHS** – when the lengths of hypotenuse and one side of a right angled triangle are given.

Check Your Progress

1. State whether the following statements are true or false. Justify your answer.
   (*i*) A triangle with lengths of sides 2.5 cm, 3 cm and 6 cm can be constructed.
   (*ii*) A triangle DEF with EF = 7.2 cm, $m\angle E = 110°$ and $m\angle F = 80°$ can be constructed.
   (*iii*) If the measure of an acute angle and the length of hypotenuse of a right angled triangle are given, then the triangle can be constructed.
2. Draw a line AB and take a point C outside it. Through C, draw a line parallel to AB by using the concept of equal corresponding angles.
3. Draw a triangle PQR with PQ = 4 cm, QR = 3.5 cm and PR = 4 cm. What type of triangle is this?
4. Construct a triangle ABC with BC = 7.5 cm, AC = 5 cm and $m\angle C = 60°$ by using ruler and compasses only.
5. Construct a triangle ABC, given $m\angle A = 60°$, $m\angle B = 30°$ and AB = 5.8 cm by using ruler and compasses only.
6. Construct an isosceles right angled triangle ABC, with $m\angle ABC = 90°$ and AC = 6 cm.