8

ALGEBRAIC EXPRESSIONS

INTRODUCTION

In class VI, you have already learnt some basic concepts of algebra, constants and variables, uses of variables in forming rules from geometry and arithmetic. We also came across with some simple algebraic expressions like $x + 3$, $2x - y$, $7 - y$, $4y + 9$ etc. and have seen the use of such expressions in forming word problems and simple equations.

In this chapter, we shall learn more about algebraic expressions and how they are formed and developed by using one or more variables. We shall study – terms of an algebraic expression, factors of a term, coefficient of a term, like and unlike terms, type of algebraic expressions, polynomials, addition and subtraction of algebraic expressions and value of an algebraic expression.

ALGEBRAIC EXPRESSION

In algebra, we use two types of symbols – constants and variables. A symbol which has a fixed value is called a **constant**. For example, each of 9, –5, 0, $\frac{3}{5}, -\frac{7}{11}, 3\frac{5}{8}$ etc. is a constant.

A symbol which can be given various numerical values is called a **variable**. Thus, a variable is a number which does not have a fixed value *i.e.* it can take different values. So variables are generalized numbers or unknown numbers. We use the letters x, y, z, m, n, ... etc. to denote a variable. Variables are also known as **literals** (or **literal numbers**).

Since the variables (literals) are numbers, so they obey all the rules of operations of addition, subtraction, multiplication and division of numbers.

For any variables (literals) x, y and z, we have:

$x + y = y + x$ (commutative law)

$(x + y) + z = x + (y + z)$ (associative law)

$x + 0 = x = 0 + x$ (additive identity)

$x + (-x) = 0 = (-x) + x$ (additive inverse)

$xy = yx$ (commutative law)

$(xy)\,z = x\,(yz)$ (associative law)

$1x = x = x1$ (multiplicative identity)

$x(y + z) = xy + xz$ and $(y + z)\,x = yx + zx$ (distributive laws)

$0x = 0 = x0$.

Powers of a variable

Let x be any variable (literal), then

(i) $x \times x$ is written as x^2 and is read as x square.

(ii) $x \times x \times x$ is written as x^3 and is read as x cube etc.

This leads to:

If x is any literal and n is a natural number, then

$x^n = x \times x \times x$ ... multiplied n times,

where x is called the **base** and n is called the **exponent** or **index** and x^n is the **exponential form**.

x^n is read as x raised to the power n or x to the power n or simply x power n.

In particular, $x^1 = x$ and $x^0 = 1$.

Algebraic expression

A collection of constants and variables connected by one or more of the operations of addition, subtraction, multiplication and division is called an ***algebraic expression.***

For example, $4x + 5$, $9y - 13$ are algebraic expressions. The expression $4x + 5$ is formed, first multiplying the variable x by the constant 4 and then adding the constant 5 to the product. Similarly, the expression $9y - 13$ is formed, first multiplying the variable y by the constant 9 and then subtracting the constant 13 from the product.

Let us see how the following algebraic expressions are formed:

$3x^2$, $5y^3 - 7$, $2xy + 11$

(i) The expression $3x^2$ is formed, first multiplying the variable x with itself to get x^2 and then multiply it by the constant 3. Thus $3x^2 = 3 \times x \times x$.

(ii) The expression $5y^3 - 7$ is formed, first multiplying the variable y three times to get y^3 *i.e.* $y^3 = y \times y \times y$ and multiply it by the constant 5 to get $5y^3$ and then subtract the constant 7 from $5y^3$ to finally arrive at $5y^3 - 7$.

(iii) In $2xy + 11$, first obtain xy by multiplying the variable x with the variable y *i.e.* $xy = x \times y$ and then multiply it by the constant 2 to get $2xy$ *i.e.* $2xy = 2 \times x \times y$ and then add the constant 11 to $2xy$ to finally arrive at $2xy + 11$.

Terms of an algebraic expression

The various parts of an algebraic expression separated by + or − sign are called the ***terms*** *of the algebraic expression.*

For example:

	Algebraic expression	*Number of terms*	*Terms*
(i)	$3x$	1	$3x$
(ii)	$3x + 7y^2$	2	$3x, 7y^2$
(iii)	$2x - 5y$	2	$2x, -5y$
(iv)	$5 + 3x^2 - 8x^3$	3	$5, 3x^2, -8x^3$
(v)	$3x^2 - 5x^2y + 8xy^2 - 7$	4	$3x^2, -5x^2y, 8xy^2, -7$

Remark

Multiplication and division do not separate the terms of an algebraic expression. Thus, $5xy$ is one term. Also $3x^2yz$ is one term, while $5x + y$ has two terms $5x$ and y.

Constant term

The term of an algebraic expression having no variable(s) is called its ***constant term****.*

For example:

(*i*) In the algebraic expression $2x^3 - 3x + 5$, the constant term is 5.

(*ii*) In the algebraic expression $7xy - 2x - 3z^2 - \frac{2}{3}$, the constant term is $-\frac{2}{3}$.

(*iii*) The algebraic expression $9x^2y - 5x$ has no constant term.

Product

When two or more constants or literals (or both) are multiplied, then the result so obtained is called the ***product****.*

For example:

$3xy$ is the product of 3, x and y.

Factors

Each of the quantity (constant or literal) multiplied together to form a product is called a ***factor*** *of the product.*

A constant factor is called a **numerical factor** and other factors are called **variable** (or **literal**) **factors**.

For example:

(*i*) In $3xy$; the numerical factor is 3 and the variable factors are x and y.

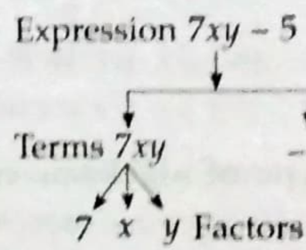
(*ii*) In $-5x^2y$, the numerical factor is -5 and the literal factors are x, x and y.

Tree diagrams

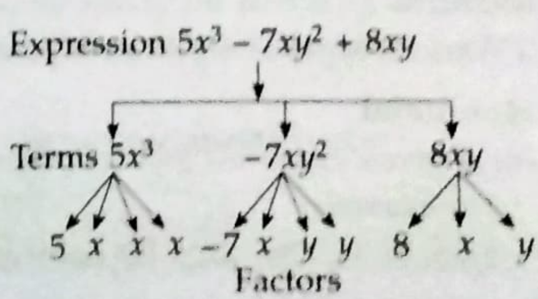
The terms and the factors of each term of an algebraic expression can very conveniently be shown by a **tree diagram**.

For example:

(*i*) The expression $7xy - 5$ has two terms, $7xy$ and -5. The tree diagram for the expression $7xy - 5$ is shown in the adjacent figure.



(*ii*) The expression $5x^3 - 7xy^2 + 8xy$ has three terms, $5x^3$, $-7xy^2$ and $8xy$. The tree diagram for the given expression is shown in the adjacent figure.



Coefficients

Any factor of a (non-constant) term of an algebraic expression is called the ***coefficient*** *of the remaining factor of the term.*

In particular, the constant part is called the **numerical coefficient** or simply the **coefficient** of the term and the remaining part is called the **literal coefficient** of the term.

For example:

Consider the expression $3xy - 5x^2y + 7$

(*i*) In the term $3xy$,
the numerical coefficient = 3,
the literal coefficient = xy,
the coefficient of $x = 3y$,

(*ii*) In the term $-5x^2y$,
the numerical coefficient = -5,
the literal coefficient = x^2y,
the coefficient of $x = -5xy$,
the coefficient of $5x = -xy$,
the coefficient of $-5x^2 = y$,
the coefficient of $xy = -5x$ and so on.

When the coefficient is +1, it is usually omitted. For example, $1x$ is written as x, $1xy^2$ is written as xy^2 etc. Also the coefficient -1 is indicated only by the minus sign. For example, $(-1)x$ is written as $-x$, $(-1)x^2y^3$ is written as $-x^2y^3$ etc.

Like and unlike terms

The terms having same variable factors are called ***like terms*** *and the terms having different variable factors are called* ***unlike terms****. In other words, the terms having same literal coefficients are called* ***like terms****; otherwise, the terms are called* ***unlike terms****.*

For example:

(*i*) $3xy, 6xy, -2yx, -\frac{5}{2}xy$ are like terms.

(*ii*) $7x^2y^2z, \frac{3}{2}x^2y^2z, -5x^2y^2z$ are like terms.

(*iii*) $3a, 3ab, 5a^2b^2$ are unlike terms.

(*iv*) xy, x^2y are unlike terms.

Type of algebraic expressions

Some algebraic expressions have only one term while some expression have more than one term.

Various types of algebraic expressions are:

Monomial

An algebraic expression having only one term is called a ***monomial****.*

For example:

Each of $5x, -3y, 15xy, 7x^4$ and $9xy^2z$ is a monomial.

Binomial

An algebraic expression having two (unlike) terms is called a ***binomial****.*

For example:

Each of $5x - 3y, 7xy - 9z^2, 2x - 3x^2$ is a binomial.

Trinomial

An algebraic expression having three (unlike) terms is called a ***trinomial****.*

For example:

Each of $5a - 3b^2 + \frac{7}{2}c^2$, $9x - 5x^3 + 7$, $3x - 2xy + 5x^2y^2$ is a trinomial.

Multinomial

An algebraic expression having two or more (unlike) terms is called a ***multinomial.***

For example:

(i) $5x^2 - 2x$ is a multinomial having 2 terms.

(ii) $5x^3 - 2xy + 7y^2$ is a multinomial having 3 terms.

(iii) $5xy - 9yz + 6zx - 7$ is a multinomial having 4 terms.

(iv) $15xy^3 - 23y^3z^2 - 9z^3 + 5xyz - \frac{11}{2}$ is a multinomial having 5 terms.

Polynomials in one variable

An algebraic expression containing only one variable is called a ***polynomial*** *in that variable if the powers of the variable in each term are whole numbers.*

Thus, an algebraic expression of the form $a + bx + cx^2 + dx^3 + \ldots$, where a, b, c, d, ... are constants and x is a variable, is called a ***polynomial*** *in the variable x.*

The greatest power of x present in the polynomial is called the ***degree of the polynomial.***

For example:

(i) $5 + 7x$ is a polynomial in x of degree 1.

(ii) $3 - 7y + 8y^2$ is a polynomial in y of degree 2.

(iii) $-6 + 7t^2 - 8t^3$ is a polynomial in t of degree 3.

(iv) $5x - 8x^2 + \frac{3}{2}x^3 - 6x^4$ is a polynomial in x of degree 4.

(v) $3 + 2x - 7x^3 + \frac{2}{x}$ is not a polynomial because in $\frac{2}{x}$ *i.e.* $2x^{-1}$, the power of x is -1 which is not a whole number.

(vi) 5 can be written as $5x^0$, so it is a polynomial in x of degree 0.

Polynomials in two or more variables

An algebraic expression in two or more variables is called a ***polynomial*** *if the powers of every variable in each term is a whole number.*

In case of a polynomial in two or more variables, the sum of the powers of the variables in each term is taken and the greatest sum is the ***degree of the polynomial.***

For example:

(i) $3 + 5x - 7y + 6xy - 9x^2$ is a polynomial in two variables x and y.
The degrees of its terms are 0, 1, 1, 1 + 1, 2.
So the degree of the polynomial is 2.

(ii) $7a^3 - 9a^2b^3 + 11b^2 - \frac{3}{2}a + 13$ is a polynomial in two variables a and b.
The degrees of its terms are 3, 2 + 3, 2, 1, 0.
So the degree of the polynomial is 5.

(iii) $5xy - 7yz + 9xyz - x^3 + 3y$ is a polynomial in three variables x, y and z.
The degrees of its terms are 1 + 1, 1 + 1, 1 + 1 + 1, 3, 1.
So the degree of the polynomial is 3.

Example 1. *Form the algebraic expressions using variables, constants and arithmetic operations.*

(i) 2 more than 5 times a number x.

(ii) One-fourth of the product of y and z.

(iii) Three times x is subtracted from 10.

(iv) The number x is multiplied by itself and added to the thrice of y.

Solution. (i) 2 more than 5 times of a number $x = 5x + 2$.

(ii) One-fourth of the product of y and $z = \frac{1}{4}yz$.

(iii) Three times x is subtracted from $10 = 10 - 3x$.

(iv) The number x is multiplied by itself and added to the thrice of $y = x^2 + 3y$.

Example 2. *Write down the algebraic expressions whose terms are*

(i) $3a, -4b, 7c$ (ii) $5xy, 2y, -3x, 8$ (iii) $3pq, 5pq^2, -7p^2q, 3$.

Solution. (i) $3a - 4b + 7c$ (ii) $5xy + 2y - 3x + 8$ (iii) $3pq + 5pq^2 - 7p^2q + 3$.

Example 3. *Write all the terms of each of the following algebraic expressions:*

(i) $5x - 9y + 3xy$ (ii) $3ab - 9ab^3 + 5c^2 - 3$

(iii) $-7pq^4 - 9p^2q + 3pq - 2p^2 + 11p + 13q - 3$.

Solution. (i) $5x, -9y, 3xy$ (ii) $3ab, -9ab^3, 5c^2, -3$

(iii) $-7pq^4, -9p^2q, 3pq, -2p^2, 11p, 13q, -3$.

Example 4. *Identify the terms and their factors in the following expressions by showing tree diagram:*

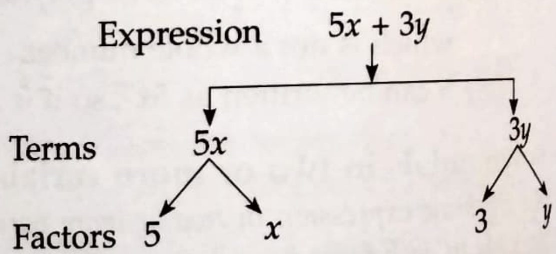
(i) $5x + 3y$ (ii) $3xy - 7x$ (iii) $x^2y + xy^2 - 5$

Solution.

(i) Given expression is $5x + 3y$

Terms of expression are $5x, 3y$

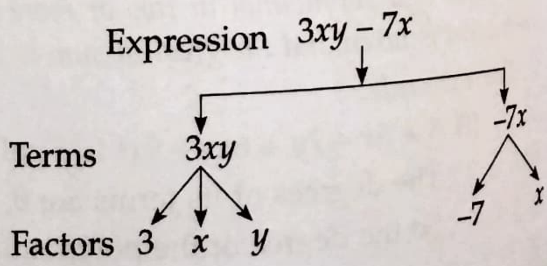
Tree diagram is shown in the adjacent figure.



(ii) Given expression is $3xy - 7x$

Terms of expression are $3xy, -7x$

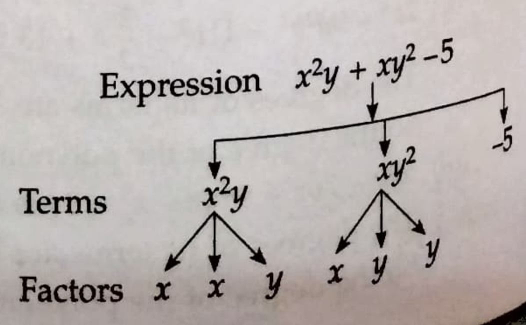
Tree diagram is shown in the adjacent figure.



(iii) Given expression is $x^2y + xy^2 - 5$

Terms of expression are $x^2y, xy^2, -5$

Tree diagram is shown in the adjacent figure.



Example 5. *Consider the algebraic expression $4x^3y - 2x^2y^2 + \frac{3}{2}xy^2 - 6x$.*

(i) *What is the number of terms? List all the terms.*

(ii) *What is the numerical coefficient of the term $-2x^2y^2$?*

(iii) *What is the coefficient of x^2 in the term $4x^3y$?*

(iv) *What is the coefficient of y^2 in the term $\frac{3}{2}xy^2$?*

Solution. (i) There are four terms, namely $4x^3y$, $-2x^2y^2$, $\frac{3}{2}xy^2$ and $-6x$.

(ii) The numerical coefficient of the term $-2x^2y^2$ is -2.

(iii) The coefficient of x^2 in the term $4x^3y$ is $4xy$.

(iv) The coefficient of y^2 in the term $\frac{3}{2}xy^2$ is $\frac{3}{2}x$.

Example 6. *Identify the term containing x and write the coefficient of x in each of the following expressions:*

$$5x - 3y,\ 3 - 2x + y^2,\ 3z - 5xyz + 8y^3$$

Solution. In each expression, we look for a term with x as a factor. The remaining part of the term is the coefficient of x.

Expression	Term containing x	Coefficient of x
$5x - 3y$	$5x$	5
$3 - 2x + y^2$	$-2x$	-2
$3z - 5xyz + 8y^3$	$-5xyz$	$-5yz$

Example 7. *Identify the terms (other than constants) and write their numerical coefficients in each of the following algebraic expressions:*

(i) $5 - 3t^2$ (ii) $1 + y + 2y^2 - 5y^3$

(iii) $-p^2q^2 + 7pq$ (iv) $5a - \frac{3}{7}b^2 + 9$.

Solution. In each expression, we look for algebraic (non-constant) terms and note their numerical coefficients:

Expression	Algebraic term(s)	Numerical coefficient
(i) $5 - 3t^2$	$-3t^2$	-3
(ii) $1 + y + 2y^2 - 5y^3$	y	1
	$2y^2$	2
	$-5y^3$	-5
(iii) $-p^2q^2 + 7pq$	$-p^2q^2$	-1
	$7pq$	7
(iv) $5a - \frac{3}{7}b^2 + 9$	$5a$	5
	$-\frac{3}{7}b^2$	$-\frac{3}{7}$

Example 8. *Identify monomials, binomials and trinomials from the following algebraic expressions:*

(i) $-3a^2b^2c^2$ (ii) $9a^2 - 5b^2 + 7$ (iii) $2x - 3$

(iv) $pq + rst$ (v) $-5 + ab + bc + ca$ (vi) $x^2 - 2y + 3z + 5x + 1$

Solution. (*i*) As the given expression has only one term, it is a monomial.

(*ii*) As the given expression has 3 terms, it is a trinomial.

(*iii*) As the given expression has 2 terms, it is a binomial.

(*iv*) As the given expression has 2 terms, it is a binomial.

(*v*) As the given expression has 4 terms, it is a multinomial having 4 terms.

(*vi*) As the given expression has 5 terms, it is a multinomial having 5 terms.

Example 9. *State with reasons which of the following pairs of terms are of like terms and which are of unlike terms:*

(*i*) $5x, -12x$ (*ii*) $7x, 25y$ (*iii*) $-7ab, 3ba$

(*iv*) $6xy^2, -8x^2y$ (*v*) $pq^2, -4pq^2$ (*vi*) $3mn^2, -10mn$.

Solution. In each pair, we look for variable factors and check that the algebraic factors are same or different.

Pair	Variable factors	Variable factors same or different	Like/unlike terms
(*i*) $5x$ $-12x$	x x	Same	Like
(*ii*) $7x$ $25y$	x y	Different	Unlike
(*iii*) $-7ab$ $3ba$	a, b b, a	Same	Like
(*iv*) $6xy^2$ $-8x^2y$	x, y, y x, x, y	Different	Unlike
(*v*) pq^2 $-4pq^2$	p, q, q p, q, q	Same	Like
(*vi*) $3mn^2$ $-10mn$	m, n, n m, n	Different	Unlike

Example 10. *Identify the like terms in the following:*

$-xy^2, -4yx^2, 8x^2, 2xy^2, 7y, -11x^2, -100x, -11yx, 20x^2y, -6x^2, y, 2xy, 3x$

Solution. Like terms are:

$-xy^2, 2xy^2; -4yx^2, 20x^2y; 8x^2, -11x^2, -6x^2; 7y, y; -100x, 3x; -11yx, 2xy$.

Exercise 8.1

1. Form the algebraic expressions using variables, constants and arithmetic operations:

(*i*) 6 more than thrice a number x.

(*ii*) 5 times x is subtracted from 13.

(*iii*) The numbers x and y both squared and added.

(*iv*) Number 7 is added to 3 times the product of p and q.

(*v*) Three times of x is subtracted from the product of x with itself.

(*vi*) Sum of the numbers m and n is subtracted from their product.

2. A taxi charges ₹ 9 per km and a fixed charges of ₹ 50. If the taxi is hired for x km, write an algebraic expression for this situation.

3. Write down the algebraic expressions whose terms are:

(i) $5a, -3b, c$ (ii) $x^2, -5x, 6$ (iii) $x^2y, xy, -xy^2$.

4. Write all the terms of each of the following algebraic expressions:

(i) $3 - 7x$ (ii) $2 - 5a + \frac{3}{2}b$ (iii) $3x^5 + 4y^3 - 7xy^2 + 3$.

5. Identify the terms and their factors in the algebraic expressions given below:

(i) $-4x + 5y$ (ii) $xy + 2x^2y^2$ (iii) $1.2ab - 2.4b + 3.6a$

6. Show the terms and their factors by tree diagrams of the following algebraic expressions:

(i) $8x + 3y^2$ (ii) $y - y^3$ (iii) $5xy^2 + 7x^2y$ (iv) $-ab + 2b^2 - 3a^2$.

7. Write down the numerical coefficient of each of the following:

(i) $-7x$ -7 (ii) $-2x^3y^2$ -2 (iii) $6abcd^2$ 6 (iv) $\frac{2}{3}pq^2$ $\frac{2}{3}$

8. Write down the coefficient of x in the following:

(i) $-4bx$ $-4b$ (ii) $5xyz$ $5yz$ (iii) $-x$ -1 (iv) $-3x^2y$. $3xy$

H.W 9. In $-7xy^2z^3$, write down the coefficient of:

(i) $7x$ (ii) $-xy^2$ (iii) xyz (iv) $7yz^2$.

10. Identify the terms (other than constants) and write their numerical coefficients in each of the following algebraic expressions:

H.W (i) $3 - 7x$ -7 (ii) $1 + 2x - 3x^2$ 2, 3x (iii) $1.2a + 0.8b$ 1.2, 0.8

11. Identify the terms which contain x and write the coefficient of x in each of the following expressions:

H.W (i) $13y^2 - 8xy$ (ii) $7x - xy^2$ (iii) $5 - 7xyz + 4x^2y$.

12. Identify the term which contain y^2 and write the coefficient of y^2 in each of the following expressions:

H.W (i) $8 - xy^2$ (ii) $5y^2 + 7x - 3xy^2$ (iii) $2x^2y - 15xy^2 + 7y^2$.

13. Classify into monomials, binomials and trinomials:

(i) $4y - 7z$ (ii) $-5xy^2$ mono (iii) $x + y - xy$

(iv) $ab^2 - 5b - 3a$ (v) $4p^2q - 5pq^2$ (vi) 2017

(vii) $1 + x + x^2$ (viii) $5x^2 - 7 + 3x + 4$. multi

[Hint. (viii) Note that $-7 + 4 = -3$.]

14. State whether the given pair of terms is of like or unlike terms:

(i) $-7x, \frac{5}{2}x$ Like (ii) $-29x, -29y$ unlike (iii) $2xy, 2xyz$ unlike

H.W (iv) $4m^2p, 4mp^2$ unlike (v) $12xz, 12x^2z^2$ unlike (vi) $-5pq, 7qp$. like

15. Identify like terms in the following:

(i) $x^2y, 3xy^2, -2x^2y, 4x^2y^2$ (ii) $3a^2b, 2abc, -6a^2b, 4abc$

(iii) $10pq, 7p, 8q, -p^2q^2, -7qp, -100q, -23, 12q^2p^2, -5p^2, 41, 2405p, 78qp, 13p^2q, qp^2, 701p^2$.

16. Write down the degree of following polynomials in x:

(i) $x^2 - 6x^7 + x^8$ 8 (ii) $3 - 2x$ 1 (iii) -2 0 (iv) $1 - x^2$. 2

17. Write the degree of the following polynomials:

(i) $3x^2 - 5xy^2 + 7$ 3 (ii) $xy^2 - y^3 + 3y^4 - 2$ 4 (iii) $7 - 2x^3 - 5xy^3 + 9y^5$. 5

H.W

18. State true or false:

(i) If 5 is constant and y is variable, then $5y$ and $5 + y$ are variables false

(ii) $7x$ has two terms, 7 and x true

(iii) $5 + xy$ is a trinomial false

(iv) $7a \times bc$ is a binomial true

(v) $7x^3 + 2x^2 + 3x - 5$ is a polynomial true

(vi) $2x^2 - \frac{3}{x}$ is a polynomial false

(vii) coefficient of x in $-3xy$ is -3. false

ADDITION/SUBTRACTION OF ALGEBRAIC EXPRESSIONS

Addition of like terms

You know that:

$$2x = x + x,\ 3x = x + x + x, \text{ so}$$
$$2x + 3x = x + x + x + x + x = 5x = (2 + 3)\,x$$

Similarly,

$$4x + 3x = (4 + 3)\,x = 7x,$$
$$3x^2 + 5x^2 = (3 + 5)\,x^2 = 8x^2,$$
$$6xy + (-4)\,xy = [6 + (-4)]xy = (6 - 4)xy = 2xy,$$
$$2a + 4a + 7a = (2 + 4 + 7)a = 13a,$$
$$7ab + (-2)ab + (-3)\,ab = [7 + (-2) + (-3)]ab$$
$$= (7 - 2 - 3)ab = 2ab \text{ etc.}$$

This leads to:

The sum of two or more like terms is a like term whose coefficient is the sum of coefficients of given like terms.

Note

You should recognise that you cannot simplify $2x + 3y$ or $2x^2 + 3x + 5$ to single term because they do not contain like terms.

Example 1. *Add $7x^2yz$, $-2x^2yz$ and $4x^2yz$.*

Solution. Given three terms are like terms, their coefficients are 7, -2 and 4 respectively.

So, their sum is a like term whose coefficient = sum of coefficients of given terms

$= 7 + (-2) + 4 = 7 - 2 + 4 = 9.$

Hence, $7x^2yz + (-2)\,x^2yz + 4x^2yz = 9x^2yz$.

Alternatively

Sum $= 7x^2yz + (-2)x^2yz + 4x^2yz = [7 + (-2) + 4]x^2yz$

$= (7 - 2 + 4)x^2yz = 9x^2yz.$

Example 2. *Add $5ab$, $-3ab$, $-2ab$, $6ab$, $-ab$.*

Solution. Required sum $= 5ab + (-3)ab + (-2)ab + 6ab + (-1)ab$

$= [5 + (-3) + (-2) + 6 + (-1)]ab$

$= (5 - 3 - 2 + 6 - 1)ab = (11 - 6)ab = 5ab.$

Example 3. *Simplify: $7xy - 3xy - 6xy + xy - 5xy$.*

Solution. $7xy - 3xy - 6xy + xy - 5xy = (7 - 3 - 6 + 1 - 5)\,xy$

$= (8 - 14)\,xy = -6\,xy.$

Addition of algebraic expressions

Till now, we have added like terms. To add algebraic expressions, we have to group like terms and then carry out addition on them. This can be done by two methods:

(i) *Horizontal method.* Write all expressions in a horizontal line and group like terms together. Take care of signs and carry out the addition.

(ii) *Column method.* Write each expression in a separate row and arrange their like terms in such a way that they are one below the other in a column. Take care of signs and carry out the addition.

Example 4. *Add the expressions: $3a - 2b + 5c$ and $4a + 7b - 9c$.*

Solution. *Horizontal method:*

$$\begin{aligned}(3a - 2b + 5c) + (4a + 7b - 9c) &= 3a - 2b + 5c + 4a + 7b - 9c\\ &= 3a + 4a - 2b + 7b + 5c - 9c\\ &= (3 + 4)a + (-2 + 7)b + (5 - 9)c\\ &= 7a + 5b - 4c.\end{aligned}$$

Column method:

$$\begin{array}{r} 3a - 2b + 5c \\ 4a + 7b - 9c \\ \hline 7a + 5b - 4c \end{array}$$

Arrange like terms in such a way that they are one below the other

$\therefore$ Sum $= 7a + 5b - 4c$.

Example 5. *Add the following algebraic expressions:*

(i) $5x^3 - 3x + 7$, $2x^2 - 11$ *and* $7x^3 - 11x^2 + 4x - 3$

(ii) $2a - 3b + 4c$, $-3a + 2b - 5c$, $7a - c$ *and* $3b + 6c$.

Solution.

(i)

$$\begin{array}{rrrr} 5x^3 & & -\ 3x & +\ 7 \\ & 2x^2 & & -\ 11 \\ 7x^3 & -\ 11x^2 & +\ 4x & -\ 3 \\ \hline 12x^3 & -\ 9x^2 & +\ x & -\ 7 \end{array}$$

Sum $= 12x^3 - 9x^2 + x - 7$.

(ii)

$$\begin{array}{rrr} 2a & -\ 3b & +\ 4c \\ -\ 3a & +\ 2b & -\ 5c \\ 7a & & -\ c \\ & 3b & +\ 6c \\ \hline 6a & +\ 2b & +\ 4c \end{array}$$

Sum $= 6a + 2b + 4c$.

Subtraction of like terms

Subtraction of like terms can be done in a manner exactly similar to that of integers.

If a and b are any two integers, then $a - b = a + (-b)$.

In other words, change the sign of the integer to be subtracted and then add. Thus, we have the following rule for the subtraction of like terms:

Change the sign of the term to be subtracted and then add.

Note

You should recognise that you cannot simplify $4x - 3y$ to a single term because it does not contain like terms.

Example 6. *Subtract:*

(i) $-7x$ from $12x$ (ii) $12a$ from $-7a$

(iii) $-12p$ from $-7p$ (iv) $-\frac{1}{2}ab$ from $-\frac{2}{3}ab$.

Solution. (i) $12x-(-7x) = 12x + 7x = (12+7)\,x = 19x$ Change the sign of $-7x$ and add

(ii) $-7a - 12a = (-7-12)\,a = -19a$

(iii) $-7p-(-12p) = -7p + 12p = (-7+12)\,p = 5p$

(iv) $-\frac{2}{3}ab - \left(-\frac{1}{2}ab\right) = -\frac{2}{3}ab + \frac{1}{2}ab = \left(-\frac{2}{3}+\frac{1}{2}\right)ab = \frac{-4+3}{6}ab = -\frac{1}{6}ab$.

Subtraction of algebraic expressions

To subtract one expression from the other, we have to group like terms. Subtraction can be done by two methods:

(i) **Horizontal method.** Change the sign of each term of the expression to be subtracted and then add.

(ii) **Column method.** Write the expression to be subtracted below the other expression. Arrange their like terms in such a way that they are one below the other. Change the sign of each term of the expression to be subtracted and then add.

Note

Just as $-(7-3) = -7+3$, $-(a-b) = -a+b$.
The signs of algebraic terms are handled in the same way as the signs of numbers.

Example 7. *Subtract $x - z + 2y$ from $3x + y - 2z$.*

Solution. *Horizontal method:*

$$\begin{aligned}(3x+y-2z)-(x-z+2y) &= 3x+y-2z-x+z-2y\\ &= 3x-x+y-2y-2z+z\\ &= (3-1)\,x+(1-2)\,y+(-2+1)\,z\\ &= 2x-y-z.\end{aligned}$$

Column method:

$$\begin{array}{rrrr} 3x & +\;y & -\;2z \\ x & +\;2y & -\;z \\ - & - & + \\ \hline 2x & -\;y & -\;z \end{array}$$

Change the sign of each term to be subtracted and then add

Note

Usually, we prefer column method.

Example 8. *Subtract $3x^3 - 5x^2 - 9x + 6$ from $2x^3 + 9x^2 - 7x - 8$.*

Solution.

$$\begin{array}{rrrr} 2x^3 & +\;9x^2 & -\;7x & -\;8 \\ 3x^3 & -\;5x^2 & -\;9x & +\;6 \\ - & + & + & - \\ \hline -x^3 & +\;14x^2 & +\;2x & -\;14 \end{array}$$

Change the sign of each term to be subtracted and then add

Example 9. *Subtract $4x^2 - z^2$ from the sum of $2x^2 + 3y^2 - 4z^2$ and $x^2 - 2y^2 + z^2$.*

Solution.

$$
\begin{array}{lrrr}
 & 2x^2 & +\,3y^2 & -\,4z^2 \\
\text{Add} & x^2 & -\,2y^2 & +\,z^2 \\
\hline
 & 3x^2 & +\,y^2 & -\,3z^2 \\
\text{Sub} & 4x^2 & & -\,z^2 \\
 & - & & + \\
\hline
 & -\,x^2 & +\,y^2 & -\,2z^2
\end{array}
$$

Example 10. *What must be added to $5a - 3b + 2c$ to get $3a - 4b + 7c$?*

Solution. The sum is $3a - 4b + 7c$, so we have to subtract $5a - 3b + 2c$ from $3a - 4b + 7c$.

$$
\begin{array}{rrr}
3a & -\,4b & +\,7c \\
5a & -\,3b & +\,2c \\
- & + & - \\
\hline
-2a & -\,b & +\,5c
\end{array}
$$

Example 11. *What must be subtracted from $3x^3 - 5x^2 - x + 2$ to get $4x^3 + 3x - 5$?*

Solution. The required expression is the difference between $3x^3 - 5x^2 - x + 2$ and $4x^3 + 3x - 5$.

$$
\begin{array}{rrrr}
3x^3 & -\,5x^2 & -\,x & +\,2 \\
4x^3 & & +\,3x & -\,5 \\
- & & - & + \\
\hline
-\,x^3 & -\,5x^2 & -\,4x & +\,7
\end{array}
$$

Exercise 8.2

1. Add:
   (i) $7x, -3x$ (ii) $6x, -11x$ (iii) $5x^2, -9x^2$
   (iv) $3ab^2, -5ab^2$ (v) $\frac{1}{2}pq, -\frac{1}{3}pq$ (vi) $5x^3y, -\frac{2}{3}x^3y$.
2. Add:
   (i) $3x, -5x, 7x$ (ii) $7xy, 2xy, -8xy$ (iii) $-2abc, 3abc, abc$
   (iv) $3mn, -5mn, 8mn, -4mn$ (v) $2x^3, 3x^3, -4x^3, -5x^3$
3. Simplify the following combining like terms:
   (i) $21b - 32 + 7b - 20b$
   (ii) $12m^2 - 9m + 5m - 4m^2 - 7m + 10$
   (iii) $-z^2 + 13z^2 - 5z + 7z^3 - 15z$
   (iv) $5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$
   (v) $p - (p - q) - (q - p) - q$
   (vi) $3a - 2b - ab - (a - b + ab) + 3ab + b - a$
   (vii) $(3y^2 + 5y - 4) - (8y - y^2 - 4)$
4. Find the sum of the following algebraic expressions:
   (i) $5xy, -7xy, 3x^2$ (ii) $4x^2y, -3xy^2, -5xy^2, 5x^2y$
   (iii) $-7mn + 5, 12mn + 2, 8mn - 8, -2mn - 3$ (iv) $a + b - 3, b - a + 3, a - b + 3$

(v) $14x + 10y - 12xy - 13$, $18 - 7x - 10y + 8xy$, $4xy$
(vi) $5m - 7n$, $3n - 4m + 2$, $2m - 3mn - 5$
(vii) $3x^3 - 5x^2 + 2x + 1$, $3x - 2x^2 - x^3$, $2x^2 - 7x + 9$
(viii) $7a^2 - 5a + 2$, $3a^2 - 7$, $2a + 9$, $1 + 2a - 5a^2$.

5. Simplify the following :
(i) $2x^2 + 3y^2 - 5xy + 5x^2 - y^2 + 6xy - 3x^2$
(ii) $3xy^2 - 5x^2y + 7xy - 8xy^2 - 4xy + 6x^2y$
(iii) $5x^4 - 7x^2 + 8x - 1 + 3x^3 - 9x^2 + 7 - 3x^4 + 11x - 2 + 8x^2$.

6. Subtract:
(i) $-5y^2$ from y^2
(ii) $-7xy$ from $-2xy$
(iii) $a(b - 5)$ from $b(5 - a)$
(iv) $-m^2 + 5mn$ from $4m^2 - 3mn + 8$
(v) $5a^2 - 7ab + 5b^2$ from $3ab - 2a^2 - 2b^2$
(vi) $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$
(vii) $7xy + 5x^2 - 7y^2 + 3$ from $7x^2 - 8xy + 3y^2 - 5$
(viii) $2x^4 - 7x^2 + 5x + 3$ from $x^4 - 3x^3 - 2x^2 + 3$.

7. Subtract $p - 2q + r$ from the sum of $10p - r$ and $5p + 2q$.
8. From the sum of $4 + 3x$ and $5 - 4x + 2x^2$, subtract the sum of $3x^2 - 5x$ and $-x^2 + 2x + 5$.
9. What should be added to $x^2 - y^2 + 2xy$ to obtain $x^2 + y^2 + 5xy$?
10. What should be subtracted from $-7mn + 2m^2 + 3n^2$ to get $m^2 + 2mn + n^2$?
11. How much is $y^4 - 12y^2 + y + 14$ greater than $17y^3 + 34y^2 - 51y + 68$?
12. How much does $93p^2 - 55p + 4$ exceed $13p^3 - 5p^2 + 17p - 90$?
13. What should be taken away from $3x^2 - 4y^2 + 5xy + 20$ to obtain $-x^2 - y^2 + 6xy + 20$?
14. From the sum of $2y^2 + 3yz$, $-y^2 - yz - z^2$ and $yz + 2z^2$, subtract the sum of $3y^2 - z^2$ and $-y^2 + yz + z^2$.

VALUE OF AN ALGEBRAIC EXPRESSION

An algebraic expression contains one or more variables (literals) which represent numbers. The value of the expression depends upon the values of the variables. In order to obtain the value of the expression, we simply replace each variable (literal) by its value and get a numerical expression and then evaluate it by usual method of arithmetic.

The process of finding the value of an algebraic expression by replacing the variables (literals) by their particular values is called **substitution.**

To learn the working, study the following examples:

Example 1. *Find the value of the following expressions for $x = 2$:*
(i) $x + 4$
(ii) $4x - 3$
(iii) $19 - 5x^2$
(iv) $100 - 10x^3$.

Solution. Putting $x = 2$ in each of the given expression, we get
(i) $x + 4 = 2 + 4 = 6$.
(ii) $4x - 3 = 4 \times 2 - 3 = 8 - 3 = 5$.
(iii) $19 - 5x^2 = 19 - 5 \times 2^2 = 19 - 5 \times 4 = 19 - 20 = -1$.
(iv) $100 - 10x^3 = 100 - 10 \times 2^3 = 100 - 10 \times 8 = 100 - 80 = 20$.

Example 2. *Find the value of the following expressions for $n = -2$:*

(i) $5n - 2$ (ii) $5n^2 + 5n - 2$ (iii) $n^3 + 5n^2 + 5n - 2$

Solution. Substituting $n = -2$ in each of the given expression, we get

(i) $5n - 2 = 5 \times (-2) - 2 = -10 - 2 = -12.$

(ii) $5n^2 + 5n - 2 = 5 \times (-2)^2 + 5 \times (-2) - 2 = 5 \times 4 - 10 - 2$
$= 20 - 12 - 2 = 8.$

(iii) $n^3 + 5n^2 + 5n - 2 = (-2)^3 + 5 \times (-2)^2 + 5 \times (-2) - 2$
$= -8 + 5 \times 4 - 10 - 2 = -8 + 20 - 10 - 2 = 0.$

Example 3. *Find the value of the following expressions for $a = 3$, $b = 2$:*

(i) $7a - 4b$ (ii) $a^2 + 2ab + b^2$ (iii) $a^3 - b^3$.

Solution. Substituting $a = 3$ and $b = 2$ in each of the given expression, we get

(i) $7a - 4b = 7 \times 3 - 4 \times 2 = 21 - 8 = 13.$

(ii) $a^2 + 2ab + b^2 = 3^2 + 2 \times 3 \times 2 + 2^2 = 9 + 12 + 4 = 25.$

(iii) $a^3 - b^3 = 3^3 - 2^3 = 27 - 8 = 19.$

Example 4. *If $a = 3$, $b = 2$ and $c = -4$, find the values of:*

(i) $c^2 - a^2$ (ii) $2a + 3b - 5c$ (iii) $3ab - 2b^2 + 4abc$.

Solution. (i) $c^2 - a^2 = (-4)^2 - 3^2 = (-4) \times (-4) - 3 \times 3$
$= 16 - 9 = 7.$

(ii) $2a + 3b - 5c = 2 \times 3 + 3 \times 2 - 5 \times (-4)$
$= 6 + 6 + 20 = 32.$

(iii) $3ab - 2b^2 + 4abc = 3 \times 3 \times 2 - 2 \times 2^2 + 4 \times 3 \times 2 \times (-4)$
$= 18 - 8 - 96 = 18 - 104 = -86.$

Example 5. *Simplify the following expressions and find their value when $x = -2$:*

(i) $5(2 - 3x) + 7x - 11$ (ii) $2(x^2 - 3x) - 5(7x - 4)$

Solution. (i) $5(2 - 3x) + 7x - 11 = 10 - 15x + 7x - 11$
$= (10 - 11) + (-15 + 7)x = -1 - 8x.$

Value $= -1 - 8 \times (-2) = -1 - (-16) = -1 + 16 = 15.$

(ii) $2(x^2 - 3x) - 5(7x - 4) = 2x^2 - 6x - 35x + 20$
$= 2x^2 + (-6 - 35)x + 20 = 2x^2 - 41x + 20.$

Value $= 2 \times (-2)^2 - 41 \times (-2) + 20 = 2 \times 4 - (-82) + 20$
$= 8 + 82 + 20 = 110.$

Exercise 8.3

1. If $m = 2$, find the value of:

   (i) $3m - 5$ (ii) $9 - 5m$ (iii) $3m^2 - 2m - 7$ (iv) $\frac{5}{2}m - 4$

2. If $p = -2$, find the value of:

   (i) $4p + 7$ (ii) $-3p^2 + 4p + 7$ (iii) $-2p^3 - 3p^2 + 4p + 7$

3. If $a = 2$, $b = -2$, find the value of:

   (i) $a^2 + b^2$ (ii) $a^2 + ab + b^2$ (iii) $a^2 - b^2$

4. When $a = 0, b = -1$, find the value of the given expressions:
   (i) $2a^2 + b^2 + 1$ (ii) $a^2 + ab + 2$ (iii) $2a^2b + 2ab^2 + ab$
5. If $p = -10$, find the value of $p^2 - 2p - 100$.
6. If $z = 10$, find the value of $z^3 - 3(z - 10)$.
7. Simplify the following expressions and find their values when $x = 2$:
   (i) $x + 7 + 4(x - 5)$ (ii) $3(x + 2) + 5x - 7$
   (iii) $6x + 5(x - 2)$ (iv) $4(2x - 1) + 3x + 11$
8. Simplify the following expressions and find their values when $a = -1, b = -2$:
   (i) $2a - 2b - 4 - 5 + a$ (ii) $2(a^2 + ab) + 3 - ab$.

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
   (i) The terms with different algebraic factors are called ....
   (ii) The number of terms in a monomial is ....
   (iii) An algebraic expression having two unlike terms is called a .... .
   (iv) $3a^2b$ and $-7ba^2$ are ...... terms.
   (v) $-6a^2b$ and $-6ab^2$ are ...... terms.
   (vi) The number of unlike terms in the algebraic expression $3x^2 - 2xy + 5x^2$ is ....
   (vii) The factors of the term $-3p^2q^2$ are ....
   (viii) The perimeter of a triangle whose sides measure $2a$, b and $a + b$ is ....
   (ix) The value of the expression $2x^3 - 7x^2 + 5x - 3$ when $x = 1$ is ....
   (x) In the term $-7a^2bc$, the coefficient of a is .....
   (xi) The degree of the polynomial $3 - 5x^2 + 7x^3 - x^4$ is ....
   (xii) The degree of the polynomial $3x^2 - 2xy^2 + 5$ is ....
2. State whether the following statements are true (T) or false (F):
   (i) The expression $5x + 7 - 2x$ is a trinomial.
   (ii) $(7x - 10) - (3x - 5) = 4x - 15$.
   (iii) The coefficient of $3x$ in $-3x^3y$ is $-xy$.
   (iv) The constant term in the expression $2x^2 - 3xy - 7$ is 7.
   (v) If $x = 3$ and $y = \frac{1}{3}$ then the value of $xy(x^2 + y^2)$ is $9\frac{1}{9}$.
   (vi) $(3x - y + 5) - (x + y)$ is a binomial.
   (vii) Sum of 2 and p is $2p$.
   (viii) Sum of $x^2 + x$ and $y^2 + y$ is $2x^2 + 2y^2$.
   (ix) In like terms, variables and their powers are same.
   (x) Every polynomial is a monomial.
   (xi) If we add a monomial and a binomial, then answer can never be a monomial.
   (xii) If we subtract a monomial from a binomial, then the answer is atleast a binomial.

(*xiii*) If we add a monomial and a trinomial, then the answer can be a monomial.

(*xiv*) If we add a monomial and a binomial, then the answer can be a trinomial.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 14):

3. The algebraic expression for the statement 'Thrice square of a number x subtracted from five times the sum of y and 2' is
   (*a*) $5y + 2 - 3x^2$ (*b*) $3x^2 - (5y + 2)$
   (*c*) $5(y + 2) - 3x^2$ (*d*) $5(y + 2) - (3x)^2$
4. The expression $7x - 5(x^2 + y^2)$ is a
   (*a*) monomial (*b*) binomial (*c*) trinomial (*d*) none of these
5. The coefficient of $5a^2$ in $-5a^3bc$ is
   (*a*) $-bc$ (*b*) a^2bc (*c*) $-a^2bc$ (*d*) $-abc$
6. Which of the following is a pair of like terms?
   (*a*) $-5xy$, $5x$ (*b*) $-5xy$, $3yz$ (*c*) $-5xy$, $-5y$ (*d*) $-5xy$, $7yx$
7. The like terms in the expressions $3x(3 - 2y)$ and $2(xy + x^2)$ are
   (*a*) $9x$ and $2x^2$ (*b*) $-6xy$ and $2xy$ (*c*) $9x$ and $2xy$ (*d*) $-6xy$ and $2x^2$
8. Identify the binomial out of the following :
   (*a*) $3xy^2 + 5y - x^2y$ (*b*) $2x^2y - 5y - 2x^2y$ (*c*) $3xy^2 + 5y - xy^2$ (*d*) $xy + yz + zx$
9. The number of (unlike) terms in the expression $3xy^2 + 2y^2z - y^2x + y(xz + yz) - 5$ is
   (*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6
10. The value of the expression $x^3 + y^3$ when $x = 2$ and $y = -2$ is
    (*a*) 0 (*b*) 8 (*c*) 16 (*d*) -16
11. $-xy - (-5xy)$ is equal to
    (*a*) $-6xy$ (*b*) $6xy$ (*c*) $-4xy$ (*d*) $4xy$
12. On subtracting $7x + 5y - 3$ from $5y - 3x - 9$, we get
    (*a*) $10x + 6$ (*b*) $-10x - 6$ (*c*) $10x + 10y - 12$ (*d*) $-10x - 12$
13. The value of the expression $\frac{5}{3}x^2 + 1$ when $x = -2$ is
    (*a*) $-\frac{17}{3}$ (*b*) $-\frac{7}{3}$ (*c*) $\frac{21}{3}$ (*d*) $\frac{23}{3}$
14. The degree of the polynomial $3x^3y - 5xy^4 - 2x + 1$ is
    (*a*) 5 (*b*) 4 (*c*) 3 (*d*) 2.

Higher Order Thinking Skills (HOTS)

1. The length of a rectangle is $3x - 4y + 6z$ and the perimeter is $7x + 8y + 17z$, find the breadth of the rectangle.
2. Simplify: $\frac{3x}{5} + \frac{2x}{3} - \left(\frac{x}{2} + \frac{2x}{5}\right)$.
3. If $a = 3$, $b = -1$, then find the value of each of the following:
   (*i*) a^b (*ii*) b^a (*iii*) $(ab)^b$
   (*iv*) $(a + b)^b$ (*v*) $\left(\frac{b}{a}\right)^b$ (*vi*) $\left(\frac{a}{b} + \frac{b}{a}\right)^b$.