10

LINES AND ANGLES

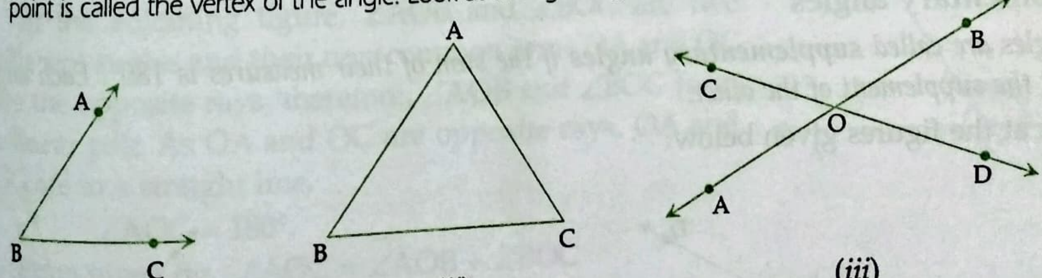
INTRODUCTION

In class VI, you have learnt some basic concepts and terms of geometry – point, line, plane, line segment, ray, angle and types of angles. In this chapter, we shall learn about some pairs of related angles – complementary angles, supplementary angles, adjacent angles, linear pair, sum of angles at a point, sum of angles at a point on one side of a straight line and vertically opposite angles. We shall also learn about pairs of intersecting and parallel lines, transversal, angles made by a transversal and properties of angles associated with a pair of parallel lines.

Recall, the basic concept of a line is its straightness and it extends indefinitely in both directions i.e. it has no definite length and it has no end points. A line segment is a portion of a line. It has two end points and a definite length. A ray is a part of a line that extends only in one direction. It has one end point (called initial point) and has no definite length.

line   line segment   ray

We know that when two rays or two line segments or two lines meet, an angle is formed. The common point is called the vertex of the angle. Look at the figures given below:



(i)   (ii)   (iii)

In fig. (i), two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ meet at B, they form angle ABC. Point B is the vertex and two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are the arms (or sides) of the angle. The angle ABC is denoted by ∠ABC.

In fig. (ii), the pairs of line segments AB, BC; BC, CA; CA, AB meet at the points B, C and A respectively. Three angles formed by these pairs of line segments are ∠ABC, ∠BCA and ∠CAB respectively.

In fig. (iii), the pair of lines AB and CD meet at the point O. Four angles formed are $\angle AOD$, $\angle DOB$, $\angle BOC$ and $\angle COA$.

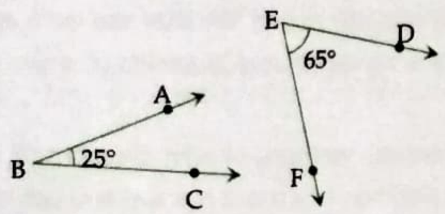
The **size** (magnitude or measure) of an angle is the amount by which one of the arms needs to be rotated about the vertex so that it lies on the top of the other arm. To measure an angle, one complete turn is divided into 360 equal parts and each part is called one degree and is written as 1°. We shall measure angles in degrees. Usually, the measure of $\angle ABC$ i.e. $m\angle ABC$ is written as $\angle ABC$. Two angles are called **equal** if they have same measure.

RELATED ANGLES

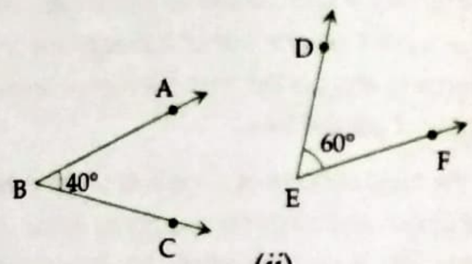
Complementary angles

Two angles are called ***complementary angles*** *if the sum of their measures is 90°. Each angle is called the complement of the other.*

Look at the figures given below:



(i)



(ii)

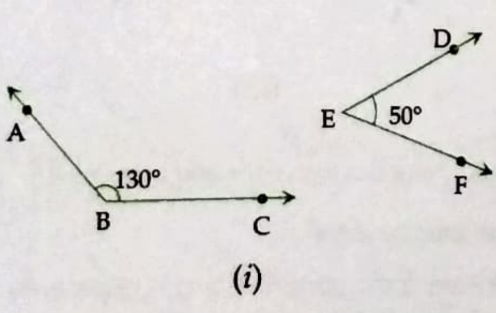
In fig. (i), $\angle ABC + \angle DEF = 25° + 65° = 90°$ *i.e.* sum of their measures is 90°, therefore, $\angle ABC$ and $\angle DEF$ are complementary.

In fig. (ii), $\angle ABC + \angle DEF = 40° + 60° = 100°$ *i.e.* sum of their measures is not 90°, therefore, $\angle ABC$ and $\angle DEF$ are not complementary.

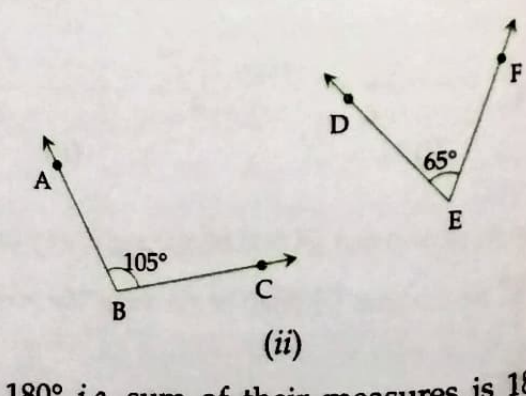
Supplementary angles

Two angles are called ***supplementary angles*** *if the sum of their measures is 180°. Each angle is called the supplement of the other.*

Look at the figures given below:



(i)



(ii)

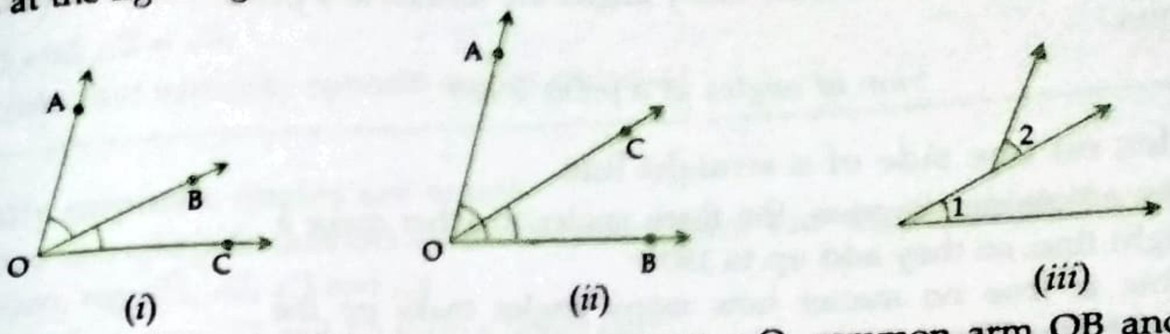
In fig. (i), $\angle ABC + \angle DEF = 130° + 50° = 180°$ *i.e.* sum of their measures is 180°, therefore, $\angle ABC$ and $\angle DEF$ are supplementary.

In fig. (ii), $\angle ABC + \angle DEF = 105° + 65° = 170°$ *i.e.* sum of their measures is not 180°, therefore, $\angle ABC$ and $\angle DEF$ are not supplementary.

Adjacent angles

Two angles are called ***adjacent angles*** *if they have a common vertex, a common arm and their non-common arms lie on either side of the common arm.*

Look at the figures given below:



In fig. (*i*), ∠AOB and ∠BOC have a common vertex O, common arm OB and the non-common arms OA and OC lie on either (opposite) side of the common arm OB, therefore, ∠AOB and ∠BOC are adjacent angles.

In fig. (*ii*), ∠AOB and ∠BOC have a common vertex O, common arm OB but the non-common arms OA and OC lie on the same side of the common arm OB, therefore, ∠AOB and ∠BOC are not adjacent angles.

In fig. (*iii*), ∠1 and ∠2 do not have a common vertex, therefore, ∠1 and ∠2 are not adjacent angles.

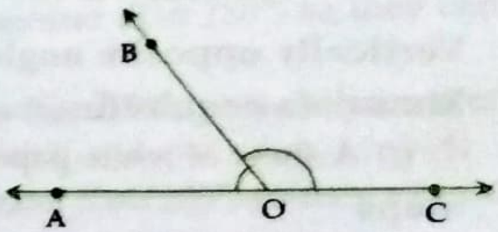
Note

Two adjacent angles have no common interior points.

Linear pair

Two adjacent angles are said to form a ***linear pair*** *if their non-common arms are opposite rays i.e. they are in a straight line.*

In the adjoining figure, ∠AOB and ∠BOC are two adjacent angles and their non-common arms OA and OC are the opposite rays, therefore, ∠AOB and ∠BOC form a linear pair. As OA and OC are opposite rays, OA and OC are in a straight line.



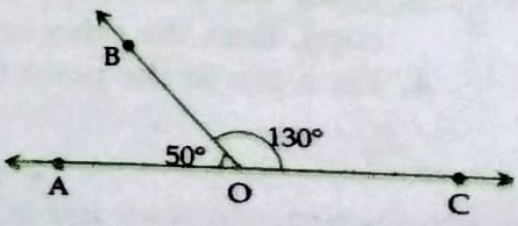
$\therefore \quad \angle AOC = 180°$.

From given fig., $\angle AOC = \angle AOB + \angle BOC$

$\Rightarrow \quad \angle AOB + \angle BOC = 180°$.

Thus, two angles in a linear pair are supplementary. Conversely, if two adjacent angles are supplementary *i.e.* if the sum of their measures is 180°, then the non-common arms are in a straight line and hence they form a linear pair.

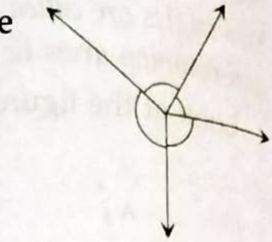
In the adjoining figure, ∠AOB and ∠BOC are two adjacent angles such that $\angle AOB + \angle BOC = 50° + 130° = 180°$, therefore, ∠AOB and ∠BOC form a linear pair.



Note

If a pair of supplementary angles are placed adjacent to each other, then they form a linear pair.

Angles at a point



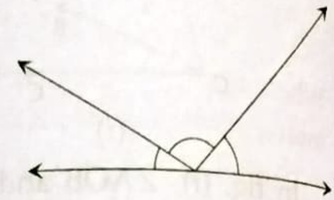
In the adjoining diagram, the four angles together make one complete turn, so they add up to 360°.

This is true no matter how many angles are formed at a point.

Thus:

Sum of angles at a point = 360°.

Angles on one side of a straight line

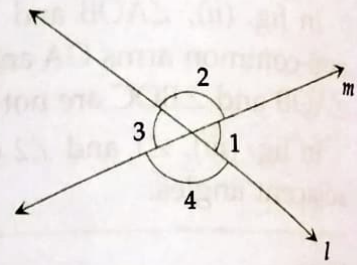


In the adjoining diagram, the three angles together make a straight line, so they add up to 180°.

This is true no matter how many angles make up the straight line. Thus:

Sum of angles at a point on one side of a straight line = 180°.

Vertically opposite angles



When two straight lines intersect each other, they form four angles at their point of intersection say $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

$\angle 1$ and $\angle 3$ are called **vertically opposite angles** to each other and so are $\angle 2$ and $\angle 4$.

They are called vertically opposite angles because they have the same vertex and are opposite to each other. In fact, vertically opposite angles are formed by the non-common arms.

Activity 3

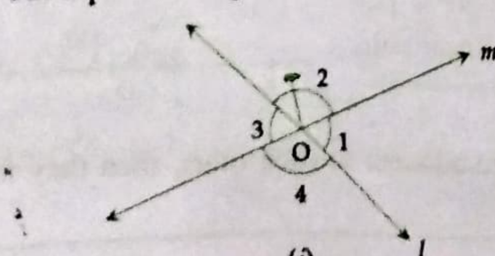
Vertically opposite angles are equal

Materials required

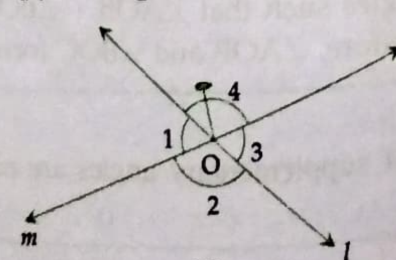
(*i*) A sheet of white paper (*ii*) Ruler (*iii*) Tracing paper (*iv*) Pin

Steps

1. On a sheet of paper, draw two straight lines *l* and *m* intersecting at the point O. Four angles are formed at the point O, say $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. $\angle 1$, $\angle 3$ form one pair of vertically opposite angles and $\angle 2$, $\angle 4$ form another pair of vertically opposite angles.
2. Make a trace copy (replica) of the fig. (*i*) on a tracing paper.
3. Place the trace copy on the original figure such that one of the angles match its copy, then the other angles will match the copy.
4. Fix a pin at the point O and rotate the copy through 180°.



(*i*)



(*ii*)

Observation

The lines of the copy will coincide with the original figure (as shown in fig. (*ii*)).

We note that $\angle 1$ coincides with $\angle 3$ and $\angle 2$ coincides with $\angle 4$.

Result

$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

If follows that vertically opposite angles are equal.

Vertically opposite angles are equal

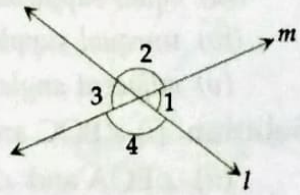
When two straight line intersect each other, they form four angles at their point of intersection, say $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Look at the figure, $\angle 1$ and $\angle 2$ form a linear pair.

$\therefore \quad \angle 1 + \angle 2 = 180°$ ...(*i*)

Again from figure, $\angle 3$ and $\angle 2$ form a linear pair.

$\therefore \quad \angle 3 + \angle 2 = 180°$ ...(*ii*)



From (*i*) and (*ii*), we get

$\angle 1 + \angle 2 = \angle 3 + \angle 2 \Rightarrow \angle 1 = \angle 3.$

In the same way, we can show that $\angle 2 = \angle 4$.

Hence, vertically opposite angles are equal.

Example 1. (*i*) *Can two acute angles be complement to each other?*

(*ii*) *Can two obtuse angles be complement to each other?*

(*iii*) *Can two acute angles be supplementary?*

(*iv*) *Can two adjacent obtuse angles form a linear pair?*

Solution. (*i*) Yes; pairs of angles like 30° and 60°; 25° and 65° are complements of each others.

(*ii*) No; as the sum of two obtuse angles is always greater than 180°, so they can never be complement of each other.

(*iii*) No; as the sum of two acute angles is always less than 180°, so they can never be supplementary angles.

(*iv*) No; as the sum of two obtuse angles is always greater than 180°, so they cannot form a linear pair.

Example 2. *In the given figure, straight lines AB and CD intersect each other at O.*

(*i*) *Is $\angle 1$ adjacent to $\angle 2$?*

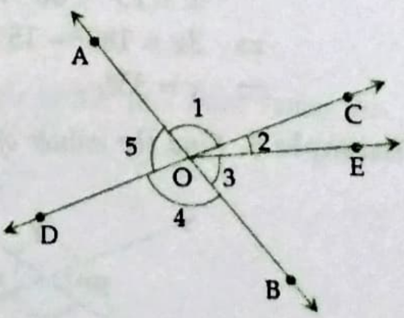
(*ii*) *Is $\angle AOC$ adjacent to $\angle AOE$?*

(*iii*) *Do $\angle COE$ and $\angle EOD$ form a linear pair?*

(*iv*) *Are $\angle BOD$ and $\angle DOA$ supplementary?*

(*v*) *Is $\angle 1$ vertically opposite to $\angle 4$?*

(*vi*) *What is the vertically opposite angle of $\angle 5$?*



Solution. (*i*) Yes; it is clear from figure.

(*ii*) No; OA is the common arm of $\angle AOC$ and $\angle AOE$ but their non-common arms OC and OE lie on the same side of the common arm OA, therefore, $\angle AOC$ and $\angle AOE$ are not adjacent angles.

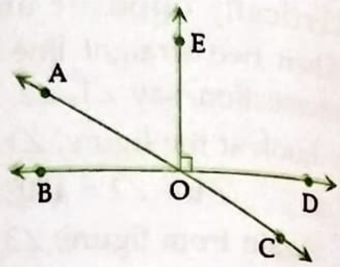
(*iii*) Yes; because ∠COE and ∠EOD are adjacent angles and their non-common arms are in a straight line.

(*iv*) Yes; as AB is a straight line, ∠BOD + ∠DOA = 180°, therefore these angles are supplementary.

(*v*) Yes, it is clear from figure.

(*vi*) ∠BOC is vertically opposite to ∠5.

Example 3. *In the adjoining figure, straight lines AC and BD intersect each other at O and* $\overrightarrow{OE} \perp \overleftrightarrow{BD}$. *Name the following pairs of angles:*



(*i*) *obtuse vertically opposite angles*

(*ii*) *adjacent complementary angles*

(*iii*) *equal supplementary angles*

(*iv*) *unequal supplementary angles*

(*v*) *adjacent angles that do not form a linear pair.*

Solution. (*i*) ∠BOC and ∠AOD

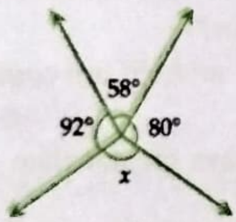
(*ii*) ∠EOA and ∠AOB

(*iii*) ∠BOE and ∠EOD

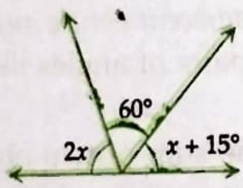
(*iv*) Pairs of unequal supplementary angles are: ∠BOA, ∠AOD; ∠AOD, ∠DOC; ∠DOC, ∠COB; ∠COB, ∠BOA, ∠AOE, ∠EOC

(*v*) Pair of adjacent angles that do not form a linear pair are: ∠AOB, ∠AOE; ∠AOE, ∠EOD; ∠EOD, ∠DOC.

Example 4. *Find the value of x in each of the following diagrams:*



(*i*)



(*ii*)

Solution. (*i*) As the sum of angles at a point = 360°,

$x + 80° + 58° + 92° = 360°$

$\Rightarrow x = 360° - 80° - 58° - 92° = 130°$

State which geometrical fact you are using to find the angle

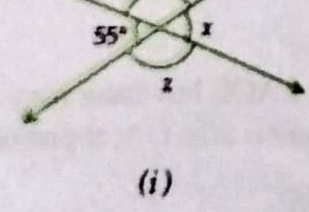
(*ii*) As the sum of angles at a point on one side of a straight line is 180°,

$x + 15° + 60° + 2x = 180°$

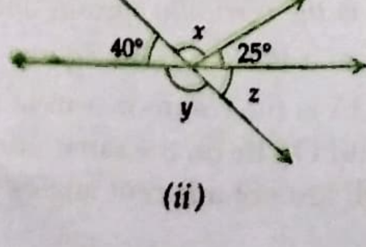
$\Rightarrow 3x = 180° - 15° - 60° = 105°$

$\Rightarrow x = 35°.$

Example 5. *Find the values of x, y and z in each of the following diagrams:*



(*i*)



(*ii*)

Solution. (i) $x = 55°$ (vertically opposite angles)

$55° + y = 180°$ (linear pair)

$\Rightarrow$ $y = 180° - 55° \Rightarrow y = 125°$

$z = y$ (vertically opposite angles)

$\Rightarrow$ $z = 125°$.

(ii) As the sum of angles at a point on one side of a straight line is 180°,

$40° + x + 25° = 180° \Rightarrow x = 180° - 40° - 25°$

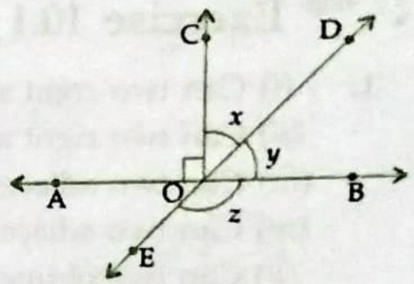
$\Rightarrow$ $x = 115°$

$40° + y = 180°$ (linear pair)

$\Rightarrow$ $y = 180° - 40° \Rightarrow y = 140°$

$z = 40°$ (vertically opposite angles)

Example 6. *In the adjoining figure, straight line AB and DE intersect each other at O and OC ⊥ AB. If $x : y = 3 : 2$, find the values of x, y and z.*



Solution. As $x : y = 3 : 2$, let $x = 3k$ and $y = 2k$.

Since the sum of angles at a point on one side of a straight line is 180°,

$x + y + 90° = 180°$ $\Rightarrow$ $3k + 2k = 180° - 90°$

$\Rightarrow$ $5k = 90°$ $\Rightarrow$ $k = 18°$.

$\therefore$ $x = (3 \times 18)° = 54°$ and $y = (2 \times 18)° = 36°$.

As ∠DOB and ∠BOE form a linear pair,

$\angle DOB + \angle BOE = 180°$

$\Rightarrow$ $y + z = 180°$

$\Rightarrow$ $36° + z = 180°$ $(\because y = 36°)$

$\Rightarrow$ $z = 180° - 36° \Rightarrow z = 144°$

Hence, $x = 54°$, $y = 36°$ and $z = 144°$.

Example 7. *Two complementary angles are in the ratio 2 : 3, find these angles.*

Solution. Since the given angles are in the ratio 2 : 3, let the angles be $2x$ and $3x$.

As the given angles are complementary angles,

$2x + 3x = 90° \Rightarrow 5x = 90°$

$\Rightarrow$ $x = 18°$

$\Rightarrow$ $2x = 36°$ and $3x = 54°$.

Hence, the required angles are 36° and 54°.

Example 8. *If two angles are supplementary angles and one angle is 30° less than twice the other, find the angles.*

Solution. Let one angle be $x°$, then the other angle $= (2x - 30)°$.

As the given angles are supplementary angles,

$x + (2x - 30) = 180$

$\Rightarrow$ $3x = 180 + 30 = 210$

$\Rightarrow$ $x = 70$ and $2x - 30 = 2 \times 70 - 30 = 110$.

Hence, the required angles are 70° and 110°.

Example 9. *An angle is 30° more than one-half of its complement. Find the angle in degrees.*

Solution. Let the angle be $x°$, then its complement $= (90 - x)°$.

According to given,

$$x° = \frac{1}{2}(90 - x)° + 30°$$

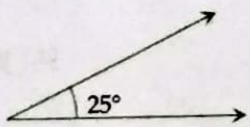
$\Rightarrow \quad 2x = 90 - x + 60$

$\Rightarrow \quad 2x + x = 150 \quad \Rightarrow \quad 3x = 150 \quad \Rightarrow \quad x = 50.$

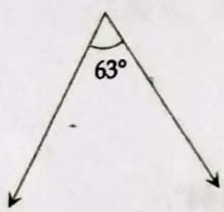
Hence, the required angle = 50°.

Exercise 10.1

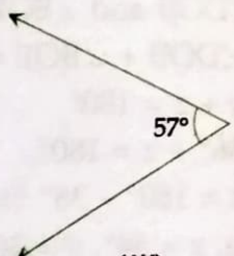
1. (*i*) Can two right angles be complementary?
   (*ii*) Can two right angles be supplementary?
   (*iii*) Can two adjacent angles be complementary?
   (*iv*) Can two adjacent angles be supplementary?
   (*v*) Can two obtuse angles be adjacent?
   (*vi*) Can an acute angle be adjacent to an obtuse angle?
   (*vii*) Can two right angles form a linear pair?
2. Find the complement of each of the following angles:



(*i*)

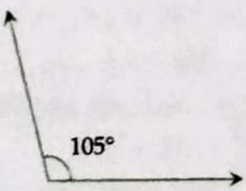


(*ii*)

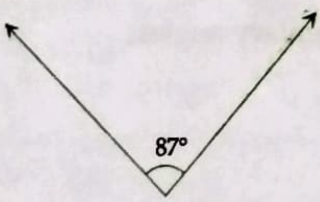


(*iii*)

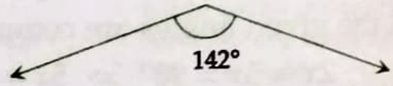
3. Find the supplement of each of the following angles:



(*i*)



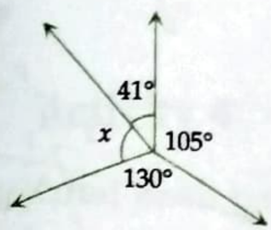
(*ii*)



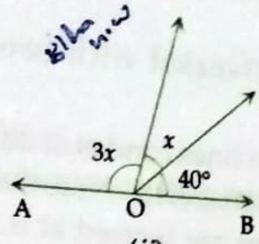
(*iii*)

4. Identify which of the following pairs of angles are complementary and which are supplementary:
   (*i*) 55°, 125° (*ii*) 34°, 56° (*iii*) 137°, 43°
   (*iv*) 112°, 68° (*v*) 45°, 45° (*vi*) 72°, 18°.
5. (*i*) Find the angle which is equal to its complement.
   (*ii*) Find the angle which is equal to its supplement.
6. Two complementary angles are $(x + 4)°$ and $(2x - 7)°$, find the value of x.
7. Two supplementary angles are in the ratio of 2 : 7, find the angles.

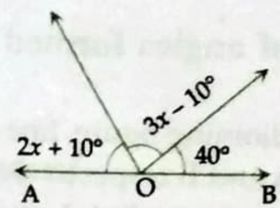
8. Among two supplementary angles, the measure of the longer angle is 44° more than the measure of the smaller angle. Find their measures.
9. If an angle is half of its complement, find the measure of angles.
10. Two adjacent angles are in the ratio $5:3$ and they together form an angle of 128°, find these angles.
11. Find the value of x in each of the following diagrams:



(i)

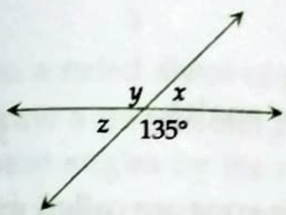


(ii)

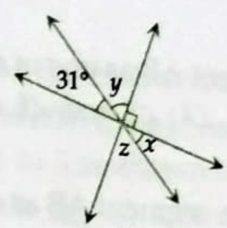


(iii)

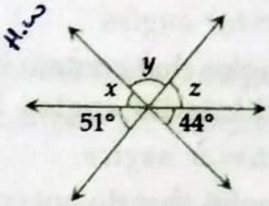
12. Find the values of x, y and z in each of the following diagrams:



(i)

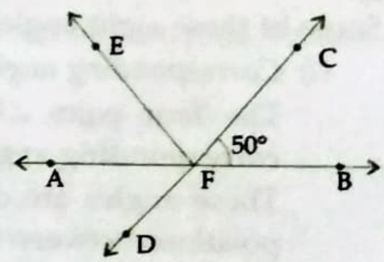


(ii)



(iii)

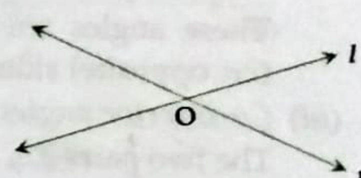
13. In the adjoining figure, lines AB and CD intersect at F. If $\angle EFA = \angle AFD$ and $\angle CFB = 50°$, find $\angle EFC$.



PAIRS OF LINES

Intersecting lines

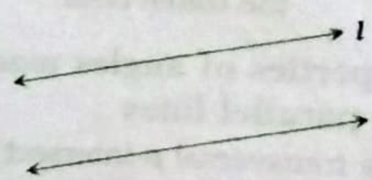
Two lines l and m are intersecting lines if they have a point in common.



In the adjoining figure, lines l and m intersect each other at the point O. The point O is called the **point of intersection.**

Parallel lines

Two lines drawn in a plane are parallel lines if they have no common point i.e. if they do not meet.

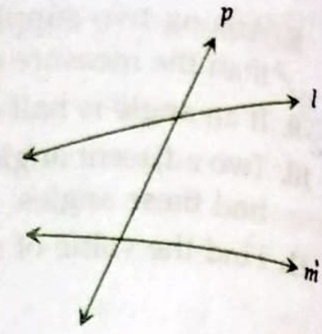


In the adjoining figure, lines l and m have no point in common, so these lines are parallel. The symbol '||' is used to denote parallel lines. Thus, l is parallel to m is written as $l \parallel m$. Sometimes parallel lines are denoted by matching arrows.

Note that the distance between the parallel lines remains the same.

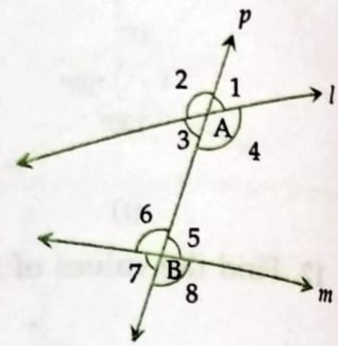
Transversal

*A line that intersects two (or more) lines in a plane at different points is called a **transversal**.*



In the adjoining figure, line p intersects two lines l and m at different points, therefore, line p is a transversal.

Types of angles formed by a transversal with two lines



In the adjoining figure, line p meets two lines l and m at different points A and B respectively, so p is a transversal. Note that four angles are formed at A and four angles are formed at B. Label them by the numerals 1 to 8 for convenience. These angles are given some special names.

(i) Interior angles

The angles that contain line segment AB as one of their arms are called **interior angles**. Here, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are interior angles.

(ii) Exterior angles

The angles that do not contain line segment AB as one of their arms are called **exterior angles**. Here, $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are exterior angles.

Some of these eight angles can be paired.

(i) *Corresponding angles*

The four pairs $\angle 1$, $\angle 5$; $\angle 2$, $\angle 6$; $\angle 3$, $\angle 7$ and $\angle 4$, $\angle 8$ are called pairs of **corresponding angles.**

These angles are called corresponding angles because they are in matching positions between the transversal and the two lines.

(ii) *Alternate angles*

The two pairs $\angle 3$, $\angle 5$ and $\angle 4$, $\angle 6$ are called **alternate interior angles** or simply **the alternate angles.**

The two pairs $\angle 1$, $\angle 7$ and $\angle 2$, $\angle 8$ are called **alternate exterior angles.**

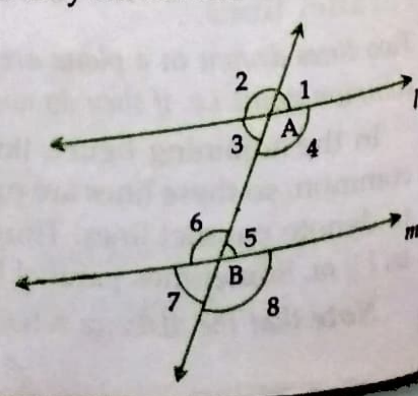
These angles are called alternate angles because they are on the alternate (*i.e.* opposite) sides of the transversal.

(iii) *Co-interior angles*

The two pairs $\angle 3$, $\angle 6$ and $\angle 4$, $\angle 5$ are called **co-interior** (or **consecutive interior**) **angles.**

These angles are called co-interior angles because they are on the same side of the transversal.

Properties of angles made by a transversal with two parallel lines



Let a transversal p intersect two parallel lines l and m at the points A and B respectively. Four angles are formed at A and four angles are formed at B. For convenience, label these angles by the numerals 1 to 8.

Then:

- *Each pair of corresponding angles are equal.*

In the given diagram, $\angle 1 = \angle 5$ and $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$.

Measure a pair of corresponding angles, say $\angle 1$ and $\angle 5$. We shall find that their measures are equal *i.e.* $\angle 1 = \angle 5$.

Similarly, we find that $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$.

Activity 4

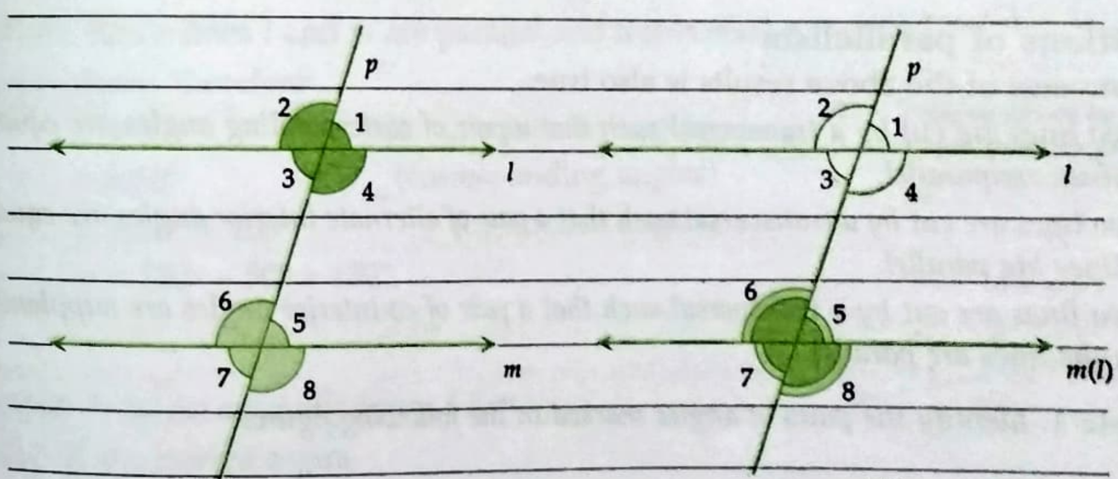
To verify that each pair of corresponding angles are equal

Materials required

(*i*) Ruled sheet of paper (*ii*) Ruler (*iii*) Tracing paper

Steps

1. On a ruled sheet of paper, draw (in thick colour) two parallel lines *l* and *m*.
2. Draw a transversal *p* to meet parallel lines *l* and *m*. Eight angles are formed. Label these angles by the numerals 1 to 8 as shown in fig. (*i*).
3. Place the tracing paper over the fig. (*i*) and make a trace copy (replica) of the fig. (*i*).
4. Hold the ruled paper firmly and slide the trace copy along the line *p* till the line *l* coincides with the line *m* as shown in fig. (*ii*).



(*i*) (*ii*)

Observation

We note that $\angle 1$ coincides with $\angle 5$, $\angle 2$ coincides with $\angle 6$, $\angle 3$ coincides with $\angle 7$ and $\angle 4$ coincides with $\angle 8$.

Result

$\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$.

It follows that each pair of corresponding angles are equal.

- *Each pair of alternate interior angles are equal.*

In the above diagram, $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$.

This property can be proved by using the above property of corresponding angles.

$\angle 1 = \angle 5$ (corresponding angles)

$\angle 1 = \angle 3$ (vertically opposite angles)

$\therefore \quad \angle 3 = \angle 5.$

Similarly, $\angle 4 = \angle 6$.

It follows that each pair of alternate interior angles are equal.

- *Each pair of co-interior angles are supplementary.*

In the above diagram, $\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$.

This property can be proved by using the above property of alternate interior angles.

$\angle 3 = \angle 5$ (alternate interior angles)

$\angle 4 + \angle 3 = 180°$ (linear pair)

$\therefore \quad \angle 4 + \angle 5 = 180°.$

Similarly, $\angle 3 + \angle 6 = 180°$.

It follows that each pair of co-interior angles are supplementary.

Thus, we have:

If a transversal cuts two parallel lines, then

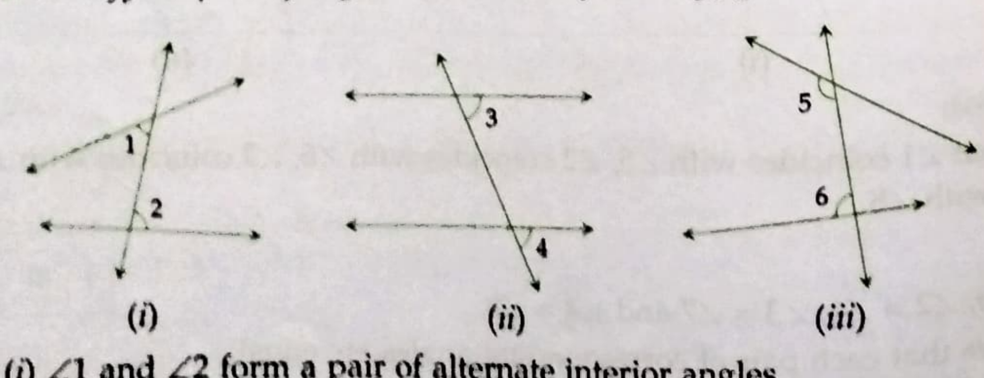
- each pair of corresponding angles are equal
- each pair of alternate interior angles are equal
- each pair of co-interior angles are supplementary *i.e.* the sum of measures of each pair of co-interior angles is 180°.

Conditions of parallelism

The converse of the above results is also true.

- *If two lines are cut by a transversal such that a pair of corresponding angles are equal, then the lines are parallel.*
- *If two lines are cut by a transversal such that a pair of alternate interior angles are equal, then the lines are parallel.*
- *If two lines are cut by a transversal such that a pair of co-interior angles are supplementary, then the lines are parallel.*

Example 1. *Identify the pairs of angles marked in the following figures:*



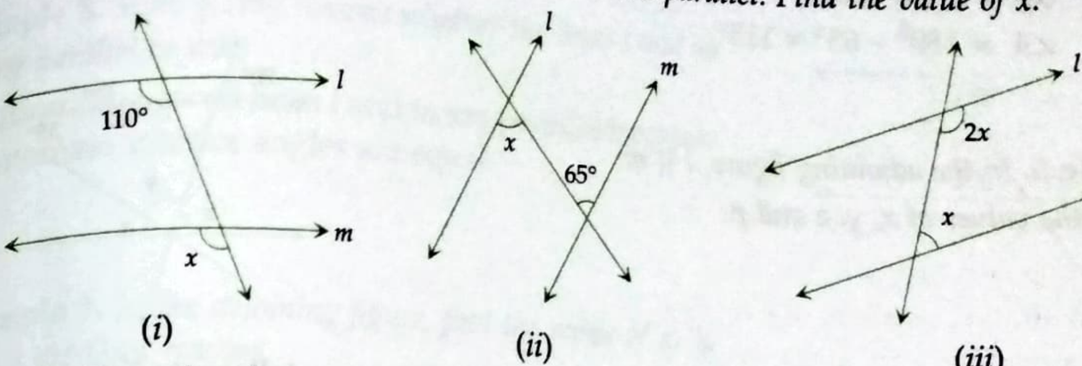
(*i*) (*ii*) (*iii*)

Solution. (*i*) $\angle 1$ and $\angle 2$ form a pair of alternate interior angles.

(*ii*) $\angle 3$ and $\angle 4$ form a pair of corresponding angles.

(*iii*) $\angle 5$ and $\angle 6$ form a pair of co-interior angles.

Example 2. *In the following figures, lines l and m are parallel. Find the value of x:*



Solution. Given l and m are parallel and a transversal meets them.

(i) $x = 110°$ (corresponding angles)

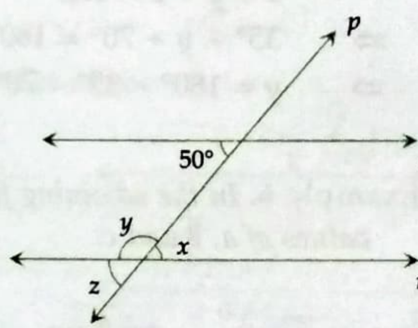
(ii) $x = 65°$ (alternate interior angles)

(iii) $x + 2x = 180°$ (co-interior angles)

$\Rightarrow$ $3x = 180° \Rightarrow x = 60°$.

> **State which geometrical fact you are using to find the angle**

Example 3. *In the adjoining figure, lines l and m are parallel. Find the values of x, y and z.*



Solution. Since lines l and m are parallel and transversal p meets them, therefore

$x = 50°$ (alternate interior angles)

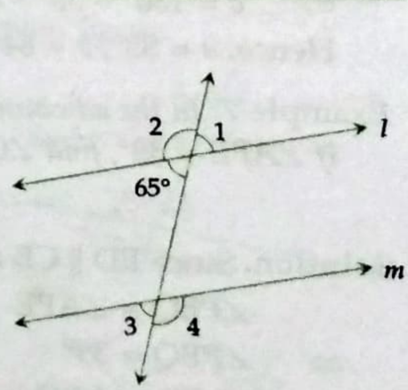
$z = 50°$ (corresponding angles)

$y + 50° = 180°$ (co-interior angles)

$\Rightarrow$ $y = 180° - 50° = 130°$.

> **State which geometrical fact you are using to find the angle**

Example 4. *In the adjoining figure, $l \parallel m$. Find all the marked angles.*



Solution. Since lines l and m are parallel and a transversal meets them, therefore

$\angle 3 = 65°$ (corresponding angles)

$\angle 1 = 65°$ (vertically opposite angles)

$\angle 2 + 65° = 180°$ (linear pair)

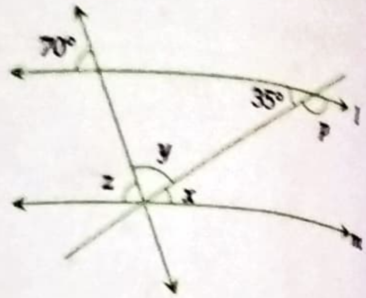
$\Rightarrow$ $\angle 2 = 180° - 65° = 115°$.

$\angle 3 + \angle 4 = 180°$ (linear pair)

$\Rightarrow \quad 65° + \angle 4 = 180°$

$\Rightarrow \quad \angle 4 = 180° - 65° = 115°.$

Example 5. *In the adjoining figure, l ∥ m. Find the values of x, y, z and p.*



Solution. $x = 35°$ (alternate interior angles)

$z = 70°$ (corresponding angles)

$p + 35° = 180°$ (linear pair)

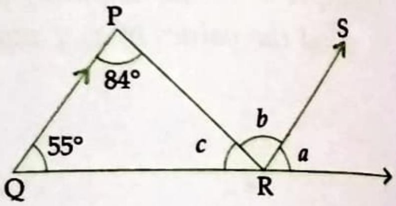
$\Rightarrow \quad p = 180° - 35° = 145°.$

$x + y + z = 180°$ (angles at a point on a straight line)

$\Rightarrow \quad 35° + y + 70° = 180°$

$\Rightarrow \quad y = 180° - 35° - 70° = 75°.$

Example 6. *In the adjoining figure, QP ∥ RS. Find the values of a, b and c.*



Solution. Given QP ∥ RS,

$\therefore \quad a = 55°$ (corresponding angles)

$b = 84°$ (alternate angles)

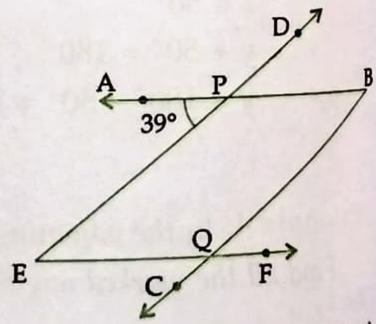
$a + b + c = 180°$ (angles at a point on a straight line)

$\Rightarrow \quad 55° + 84° + c = 180°$

$\Rightarrow \quad c = 180° - 55° - 84° = 41°$

Hence, $a = 55°$, $b = 84°$ and $c = 41°$.

Example 7. *In the adjoining figure, AB ∥ EF and ED ∥ CB. If ∠APE = 39°, find ∠CQF.*



Solution. Since ED ∥ CB and AB is a transversal,

$\angle PBQ = \angle APE$ (corresponding angles)

$\Rightarrow \quad \angle PBQ = 39°$ ($\because \angle APE = 39°$)

As AB ∥ EF and BQ is a transversal,

$\angle FQB = \angle PBQ$ (alternate angles)

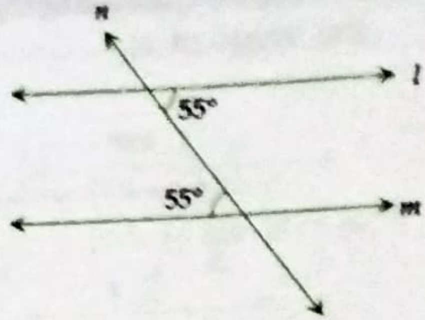
$\Rightarrow \quad \angle FQB = 39°$ ($\because \angle PBQ = 39°$, proved above)

Also $\angle CQF + \angle FQB = 180°$ (linear pair)

$\Rightarrow \quad \angle CQF + 39° = 180°$

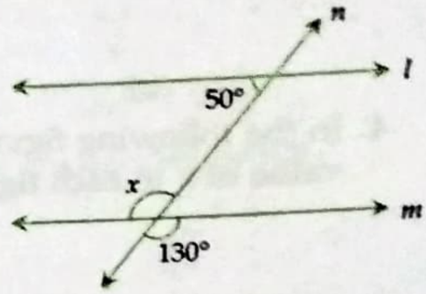
$\Rightarrow \quad \angle CQF = 180° - 39° = 141°.$

Example 8. *State giving reasons whether the lines l and m are parallel or not.*



Solution. The given lines l and m are parallel because alternate interior angles are equal.

Example 9. *In the adjoining figure, find the value of x. Is $l \parallel m$? Give reasons.*



Solution. $x = 130°$. (vertically opposite angles)

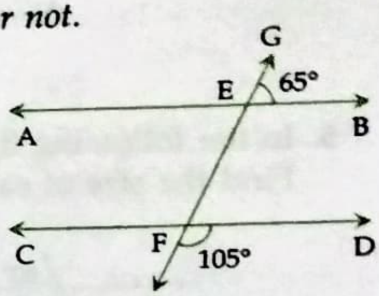
Sum of co-interior angles

$= x + 50° = 130° + 50° = 180°$

$\Rightarrow$ co-interior angles are supplementary.

Therefore, the lines l and m are parallel.

Example 10. *State giving reasons whether AB and CD are parallel or not.*



Solution. $\angle EFD + 105° = 180°$ (linear pair)

$\Rightarrow$ $\angle EFD = 180° - 105° = 75°$

$\angle GEB = 65°$ (given)

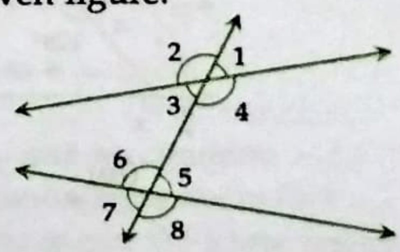
$\Rightarrow$ $\angle GEB \neq \angle EFD$

$\Rightarrow$ corresponding angles are not equal.

Hence, AB and CD are not parallel lines.

Exercise 10.2

1. Identify each of the given pair of angles as alternate interior angles, co-interior angles or corresponding angles or none of these in the given figure:



   (*i*) $\angle 2, \angle 6$

   (*ii*) $\angle 1, \angle 6$

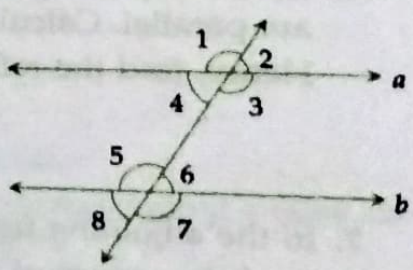
   (*iii*) $\angle 3, \angle 5$

   (*iv*) $\angle 2, \angle 7$

   (*v*) $\angle 3, \angle 6$

   (*vi*) $\angle 4, \angle 8$.

2. State the property that is used in each of the following statements:

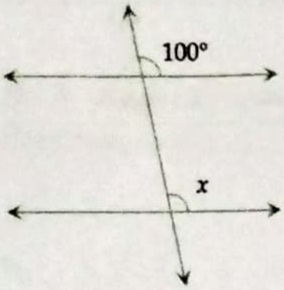


   (*i*) If $a \parallel b$, then $\angle 1 = \angle 5$.

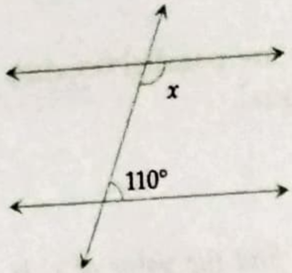
   (*ii*) If $\angle 4 = \angle 6$, then $a \parallel b$.

   (*iii*) If $\angle 4 + \angle 5 = 180°$, then $a \parallel b$.

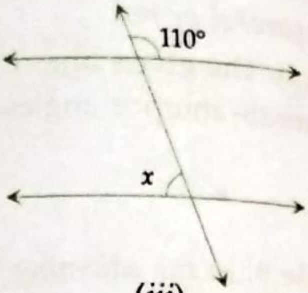
3. In each of the following figures, a pair of parallel lines is cut by a transversal. Find the value of x:



(i)

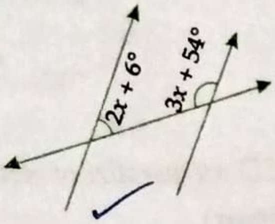


(ii)

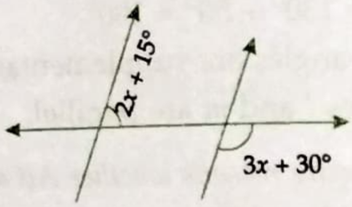


(iii)

4. In the following figures, a pair of parallel lines are cut by a transversal . Find the value of x in each figure.

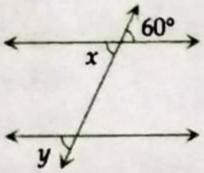


(i)

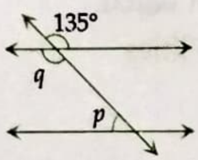


(ii)

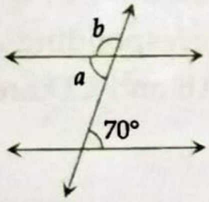
5. In the following figures (i) to (vi), a pair of parallel lines are cut by a transversal. Find the size of each lettered angle.



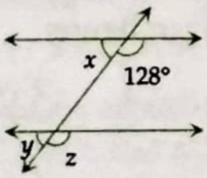
(i)



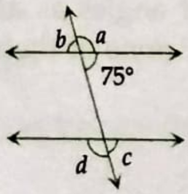
(ii)



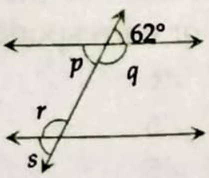
(iii)



(iv)

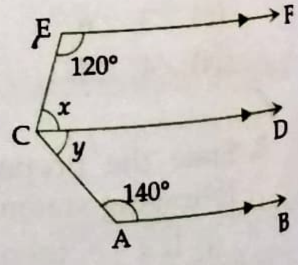


(v)

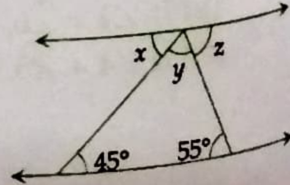


(vi)

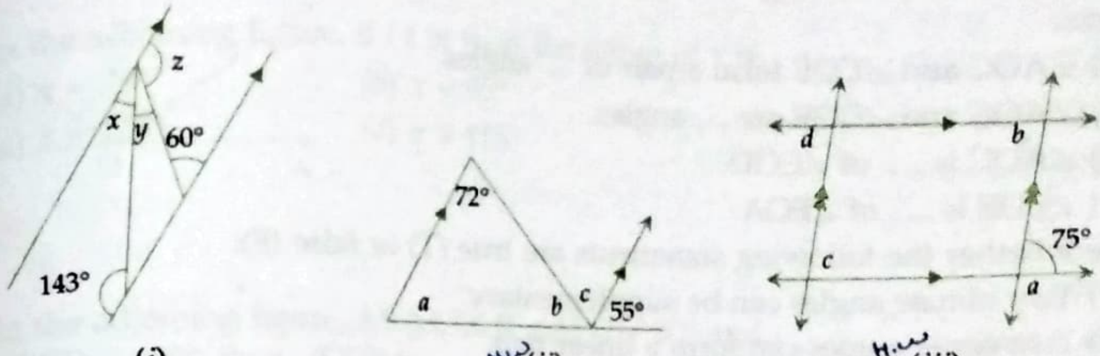
6. In the adjoining diagram, lines AB, CD and EF are parallel. Calculate the values of x and y. Hence, find the reflex angle ECA.



7. In the adjoining figure, $l \parallel m$. Find the values of x, y and z.



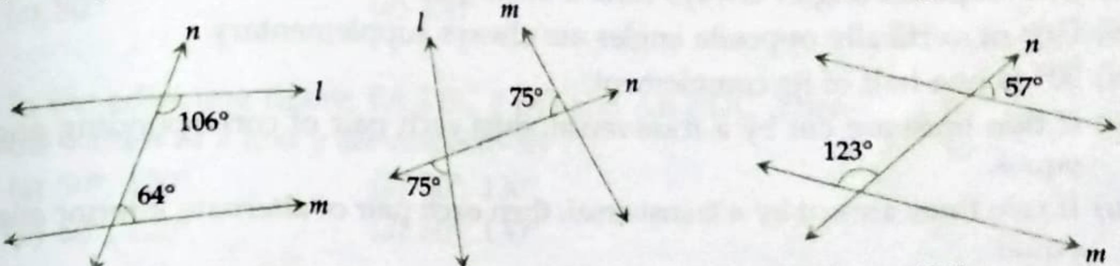
8. Calculate the measure of each lettered angle in the following figure (parallel lines, segment or rays are denoted by thick matching arrows) :



(i) (ii) H.W (iii) H.W

H.W

9. In the figures given below, state whether the lines l and m are parallel or not.



(i) ✗ (ii) ✗ (iii) ✓

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:

(i) If two angles are complementary, then the sum of their measures is ....

(ii) If two angles are supplementary, then the sum of their measures is ....

(iii) Supplement of an obtuse angle is ....

(iv) Two angles forming a linear pair are ....

(v) If two adjacent angles are supplementary, then they form a ....

(vi) Angles of a linear pair are ..... as well as ..... .

(vii) Adjacent angles have a common vertex, a common ..... and no common ..... .

(viii) Angles formed by two intersecting lines having no common arms are called .....

(ix) If two lines intersect and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are ....

(x) Two lines in a plane which never meet are called ......

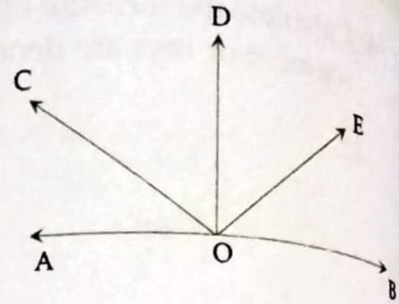
(xi) Alternate interior angles have one common ..... .

(xii) Corresponding angles are on the ..... side of transversal.

(xiii) Alternate interior angles are on the ...... side of transversal.

(xiv) If two lines are cut by a transversal such that a pair of corresponding angles are not equal, then the lines are ....

2. In the adjoining figure, AB is a straight line and OD ⊥ AB. Observe the figure and fill in the following blanks:



(*i*) ∠AOC and ∠COE form a pair of .... angles.
(*ii*) ∠AOC and ∠COB are .... angles.
(*iii*) ∠AOC is ..... of ∠COD.
(*iv*) ∠BOE is ..... of ∠EOA.

3. State whether the following statements are true (T) or false (F):
(*i*) Two obtuse angles can be supplementary.
(*ii*) Two acute angles can form a linear pair.
(*iii*) Two obtuse angles can form a linear pair.
(*iv*) Two adjacent angles always form a linear pair.
(*v*) Pair of vertically opposite angles are always supplementary.
(*vi*) 30° is one-half of its complement.
(*vii*) If two lines are cut by a transversal, then each pair of corresponding angles are equal.
(*viii*) If two lines are cut by a transversal, then each pair of alternate interior angles are equal.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (4 to 14):

4. A pair of complementary angles is
(*a*) 130°, 50° (*b*) 35°, 55° (*c*) 25°, 75° (*d*) 27°, 53°

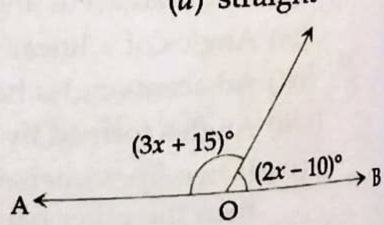
5. A pair of supplementary angles is
(*a*) 55°, 115° (*b*) 65°, 125° (*c*) 47°, 133° (*d*) 40°, 50°

6. If an angle is one-third of its supplement, then the measure of the angle is
(*a*) 45° (*b*) 30° (*c*) 135° (*d*) 150°

7. If an angle measures 10° more than its complement, then the measure of the angle is
(*a*) 40° (*b*) 55° (*c*) 35° (*d*) 50°

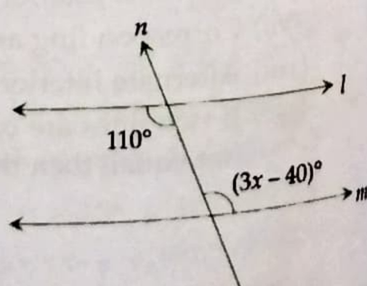
8. If one angle of a linear pair is acute, then the other angle is
(*a*) acute (*b*) obtuse (*c*) right (*d*) straight

9. In the adjoining figure, the value of x that will make AOB a straight line is



(*a*) $x = 40$ (*b*) $x = 35$
(*c*) $x = 30$ (*d*) $x = 25$

10. If two lines are intersected by a transversal, then the number of pairs of interior angles on the same side of transversal is
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

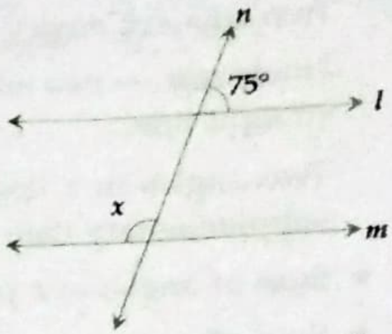


11. In the adjoining figure, if $l \parallel m$ then the value of x is
(*a*) $x = 50$ (*b*) $x = 60$
(*c*) $x = 70$ (*d*) $x = 45$

12. In the adjoining figure, if $l \parallel m$ then the value of x is

(a) $x = 75°$ (b) $x = 95°$

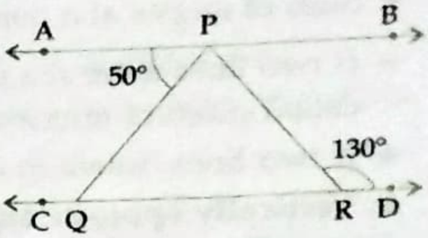
(c) $x = 105°$ (d) $x = 115°$



13. In the adjoining figure, AB $\parallel$ CD. If $\angle APQ = 50°$ and $\angle PRD = 130°$, then $\angle QPR$ is

(a) 30° (b) 50°

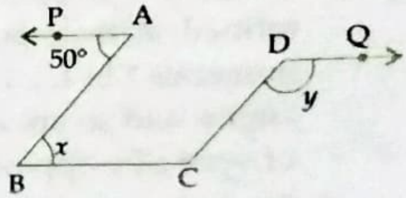
(c) 80° (d) 130°



14. In the adjoining figure, PA $\parallel$ BC $\parallel$ DQ and AB $\parallel$ DC. Then the values of x and y are respectively :

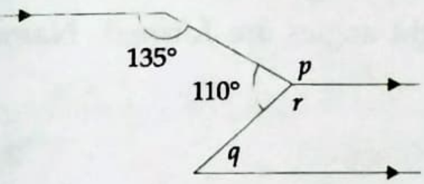
(a) 50°, 120° (b) 50°, 130°

(c) 60°, 120° (d) 60°, 130°

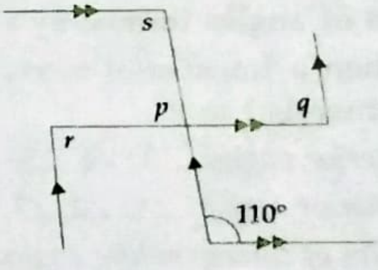


Higher Order Thinking Skills (HOTS)

Calculate the measure of each lettered angle in the following figures (parallel line segments/rays are denoted by thick matching arrows):



(i)



(ii)

Summary

★ An *angle* is formed when two rays or two line segments or two lines meet. The common point is called the vertex of the angle.

★ The *measure* (or *magnitude*) of an angle is the amount by which one of the arms needs to be rotated about the vertex so that it lies on the top of the other arm.

★ Two angles are called *equal* if they have same measure.

★ **Types of angles**

Complementary angles — two angles whose sum of measures is 90°.

Supplementary angles — two angles whose sum of measures is 180°.

Adjacent angles — two angles that have a common vertex, a common arm and their non-common arms lie on either side of the common arm.