6

RATIO AND PROPORTION

INTRODUCTION

You learnt the basic ideas of ratio and proportion in the previous class.

A politician inspecting a school asked the students—'There are 30 students in this class and my monthly salary is ₹90000. So, what is my age?' All students were puzzled how to solve this problem. Then one student said – 'Sir, your age is 40 years.' Now the politician was puzzled. 'How did you find out my correct age?' The student replied – 'Sir, my elder brother is half mad and he is 20 years old.'

Well, that is an example of direct proportion. In this chapter, we shall review and strengthen the ideas learnt in previous class and will add a few tougher problems.

We shall also review and strengthen the unitary method.

RATIO

A ratio is a comparison of measures (or magnitudes) of two or more quantities of the same kind by division.

If a and b are two quantities of the same kind (in same units), then the fraction $\frac{a}{b}$ is called the ratio of a to b. **It is written as $a : b$ (read as 'a' is to 'b').**

Thus, the ratio of a to b = $\frac{a}{b}$ or $a : b$.

The quantities a and b are called the terms of the ratio, a is called the first term (or antecedent) and b is called the second term (or consequent).

For example:

(i) The ratio of ₹9 to ₹16 = $\frac{9}{16}$ or 9 : 16.

(ii) The ratio of 150 cm to 100 cm = $\frac{150}{100} = \frac{3}{2}$ or 3 : 2.

(iii) The ratio of 8 marbles to 12 marbles = $\frac{8}{12} = \frac{2}{3}$ or 2 : 3.

Remarks

- **Since ratio is a fraction, both of its terms (numerator and denominator) can be multiplied or divided by the same (non-zero) number.**

For example:

(*i*) $3 : 5 = \frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20} \Rightarrow 3 : 5 = 12 : 20.$

(*ii*) $16 : 24 = \frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3} \Rightarrow 16 : 24 = 2 : 3.$

Usually, a ratio is expressed in lowest terms (or simplest form).

* *The order of the terms in a ratio is important.*

The ratio 3 : 5 is different from the ratio 5 : 3.

* *Ratio exists only between two quantities of the same kind.*

For example:

(*i*) There is no ratio between number of students in a class and the salary of a politician.

(*ii*) There is no ratio between the weight of one child and the age of another child.

* *Quantities to be compared (by division) must be in the same units.*

For example:

(*i*) Ratio between 150 g and 2 kg = ratio between 150 g and 2000 g

$$= \frac{150}{2000} = \frac{3}{40} = 3 : 40.$$

(*ii*) Ratio between 25 minutes and 45 seconds

= ratio between (25 × 60) sec and 45 sec

$$= \frac{1500}{45} = \frac{100}{3} = 100 : 3.$$

* *Ratio is a number, so it has no units.*
* *If the terms of a ratio are in fractions, convert them to natural numbers by multiplying each term by the LCM of their denominators.*

Equivalent ratios

Two ratios are called equivalent if the fractions corresponding to them are equivalent.

Thus, the ratio 3 : 5 is equivalent to the ratio 9 : 15 or 21 : 35.

Ratio $a : b : c$

Three quantities of the same kind (in same units) are said to be in the ratio $a : b : c$ if the quantities are ak, bk and ck respectively, where k is any positive number.

Similarly, four quantities of the same kind (in same units) are said to be in the ratio $a : b : c : d$ if the quantities are ak, bk, ck and dk respectively, where k is any positive number.

Increase or decrease of a quantity in a given ratio

Let the price of an article increase from ₹ 5 to ₹ 6, we say that the price has increased in the ratio 5 : 6. Thus, new price = $\frac{6}{5}$ of the original price.

Similarly, if the price of an article decreases from ₹ 6 to ₹ 5, we say that the price has decreased in the ratio 6 : 5. Thus, new price = $\frac{5}{6}$ of the original price.

Thus, if a quantity increases or decreases in the ratio a : b, then new quantity = $\frac{b}{a}$ *of the original quantity.*

The fraction by which the original quantity is multiplied to get a new quantity is called **the multiplying ratio (or factor).**

Example 1. *Simplify the following ratios:*

(i) $2\frac{2}{3} : 1\frac{1}{15}$ (ii) $\frac{1}{3} : \frac{1}{6} : \frac{1}{8}$.

Solution. (i) Given ratio = $2\frac{2}{3} : 1\frac{1}{15} = \frac{8}{3} : \frac{16}{15} = \frac{\frac{8}{3}}{\frac{16}{15}} = \frac{8}{3} \times \frac{15}{16} = \frac{5}{2} = 5 : 2.$

(ii) Given ratio = $\frac{1}{3} : \frac{1}{6} : \frac{1}{8}$

$= \frac{1}{3} \times 24 : \frac{1}{6} \times 24 : \frac{1}{8} \times 24 = 8 : 4 : 3.$

LCM of 3, 6 and 8 = 24

Example 2. *If A : B = 3 : 4 and B : C = 6 : 7, then find A : C.*

Solution. Given A : B = 3 : 4 $\Rightarrow \frac{A}{B} = \frac{3}{4}$ ...(i)

and B : C = 6 : 7 $\Rightarrow \frac{B}{C} = \frac{6}{7}$ ...(ii)

Multiplying (i) and (ii), we get

$\frac{A}{B} \times \frac{B}{C} = \frac{3}{4} \times \frac{6}{7} \Rightarrow \frac{A}{C} = \frac{9}{14}.$

Hence, A : C = 9 : 14.

Example 3. *If A : B = 5 : 6 and B : C = 8 : 9, find A : B : C.*

Solution. In A : B = 5 : 6, value of B is 6.

In B : C = 8 : 9, value of B is 8.

To find A : B : C, make B same in both cases.

The LCM of these two values of B *i.e.* 6 and 8 is 24. Thus,

$A : B = 5 : 6 = \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24} = 20 : 24$ and

$B : C = 8 : 9 = \frac{8}{9} = \frac{8 \times 3}{9 \times 3} = \frac{24}{27} = 24 : 27.$

$\therefore$ A : B : C = 20 : 24 : 27

Example 4. *If 3A = 5B = 6C, then find A : B : C.*

Solution. Let 3A = 5B = 6C = k (say), then

$A = \frac{k}{3}, B = \frac{k}{5}, C = \frac{k}{6}$

$\therefore A : B : C = \frac{k}{3} : \frac{k}{5} : \frac{k}{6} = \frac{1}{3} : \frac{1}{5} : \frac{1}{6}$

$= \frac{1}{3} \times 30 : \frac{1}{5} \times 30 : \frac{1}{6} \times 30 = 10 : 6 : 5$

LCM of 3, 5 and 6 = 30

Hence, A : B : C = 10 : 6 : 5

Example 5. *A line segment of 1 metre length is divided into two parts such that the first part is $\frac{2}{3}$ of second part. Find the lengths of two parts in centimetres.*

Solution. Length of line segment = 1 metre = 100 centimetres.

Ratio of lengths of two parts = $\frac{2}{3} : 1 = 2 : 3$.

Sum of the terms of the ratio = 2 + 3 = 5.

$\therefore$ Length of first part = $\frac{2}{5}$ of 100 cm = $\left(\frac{2}{5}\times 100\right)$ cm = 40 cm,

length of second part = $\frac{3}{5}$ of 100 cm = $\left(\frac{3}{5}\times 100\right)$ cm = 60 cm.

Example 6. *Divide ₹260 among three children in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.*

Solution. First we simplify the given ratio.

LCM of 2, 3 and 4 = 12

$$\text{Given ratio} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$$
$$= \frac{1}{2} \times 12 : \frac{1}{3} \times 12 : \frac{1}{4} \times 12$$
$$= 6 : 4 : 3.$$

Thus, we are to divide ₹260 in the ratio 6 : 4 : 3.

Sum of the terms of the ratio = 6 + 4 + 3 = 13.

$\therefore$ Share of first child = $\frac{6}{13}$ of ₹260 = ₹$\left(\frac{6}{13}\times 260\right)$ = ₹120,

share of second child = $\frac{4}{13}$ of ₹260 = ₹$\left(\frac{4}{13}\times 260\right)$ = ₹80,

share of third child = $\frac{3}{13}$ of ₹260 = ₹$\left(\frac{3}{13}\times 260\right)$ = ₹60.

Example 7. *A certain sum of money has been divided into two parts in the ratio 5 : 8. If the first part is ₹250, find the total amount.*

Solution. Let the total amount be ₹x.

The amount has been divided into two parts in the ratio 5 : 8.

Sum of the terms of the ratio = 5 + 8 = 13.

Then first part = $\frac{5}{13}$ of the total amount.

According to given condition, $\frac{5}{13}$ of ₹x = ₹250

$$\Rightarrow \frac{5}{13} \times x = 250 \Rightarrow x = \frac{250 \times 13}{5} = 50 \times 13 = 650.$$

Hence, the total amount = ₹650.

Alternative method

The amount has been divided into two parts in the ratio 5 : 8.

Let the two parts be ₹$5x$ and ₹$8x$.

Then the total amount = ₹$5x$ + ₹$8x$ = ₹$13x$.

According to given condition, $5x = 250 \Rightarrow x = 50$.

$\therefore$ Total amount = ₹$13x$ = ₹(13×50) = ₹650.

Example 8. *A certain sum of money is divided into three parts in the ratio 4 : 7 : 10. If the second part is ₹350, find the total amount and the other two parts.*

Solution. Since the money has been divided into three parts in the ratio 4 : 7 : 10, let the three parts be ₹ $4x$, ₹ $7x$ and ₹ $10x$ respectively.

Then total amount = ₹ $(4x + 7x + 10x)$ = ₹ $21x$

According to the given condition, $7x = 350 \Rightarrow x = 50$

$\therefore$ Total amount = ₹ (21×50) = ₹ 1050

1st part = ₹ (4×50) = ₹ 200 and

3rd part = ₹ (10×50) = ₹ 500.

Example 9. *Divide ₹ 645 into three parts such that the first part is $\frac{2}{5}$ of the second part and the ratio between second and third parts is 4 : 3.*

Solution. Since the ratio between second and third parts is 4 : 3, let second and third parts be ₹ $4x$ and ₹ $3x$ respectively.

Then first part = $\frac{2}{5}$ of ₹ $4x$ = ₹ $\frac{8x}{5}$.

According to the given condition, $\frac{8}{5}x + 4x + 3x = 645$

$$\Rightarrow \frac{8x + 20x + 15x}{5} = 645 \Rightarrow \frac{43}{5}x = 645 \Rightarrow x = \frac{645 \times 5}{43} = 75.$$

$\therefore$ First part = ₹ $\left(\frac{8}{5} \times 75\right)$ = ₹ 120,

second part = ₹(4×75) = ₹ 300 and third part = ₹ (3×75) = ₹ 225.

Example 10. *A natural number has been divided into two parts in the ratio 5 : 9. If the difference of these parts is 24, find the number and the two parts.*

Solution. As the natural number has been divided into two parts in the ratio 5 : 9, let the two parts be $5x$ and $9x$.

Then the difference of these parts = $9x - 5x = 4x$.

According to given, $4x = 24$

$\Rightarrow \quad x = 6$.

$\therefore$ The required natural number = $5x + 9x = 14x = 14 \times 6 = 84$.

The two parts are $5x$ and $9x$ *i.e.* 5×6 and 9×6 *i.e.* 30 and 54.

Hence, the two parts are 30 and 54.

Example 11. *The present ages of Rohan and Divya are in the ratio 4 : 3. Six years hence, their ages will be in the ratio 6 : 5. Find their present ages.*

Solution. As the present ages of Rohan and Divya are in the ratio 4 : 3, let their present ages be $4x$ years and $3x$ years respectively.

After 6 years,

the age of Rohan will be $(4x + 6)$ years and the age of Divya will be $(3x + 6)$ years.

According to given, $\frac{4x + 6}{3x + 6} = \frac{6}{5}$

$\Rightarrow \quad 20x + 30 = 18x + 36$

$\Rightarrow \quad 20x - 18x = 36 - 30$

$\Rightarrow \quad 2x = 6 \Rightarrow x = 3.$

Hence, the present age of Rohan = (4×3) years = 12 years
and the present age of Divya = (3×3) years = 9 years.

Example 12. *The ratio of the number of girls to the number of boys in a school of 720 students is 3 : 5. If 18 new boys are admitted in the school, find how many new girls may be admitted so that the ratio of number of girls to the number of boys may change to 2 : 3.*

Solution. The ratio of the number of girls to the number of boys is 3 : 5.

Sum of the terms of the ratio = 3 + 5 = 8

$\therefore$ The number of girls in the school = $\frac{3}{8} \times 720 = 270$ and

the number of boys in the school = $\frac{5}{8} \times 720 = 450$.

Let the number of new girls admitted be x, then the number of girls become $(270 + x)$.

After admitting 18 new boys, the number of boys become 450 + 18 *i.e.* 468.

According to the given condition, $\frac{270 + x}{468} = \frac{2}{3}$

$\Rightarrow \quad 3(270 + x) = 2 \times 468$

$\Rightarrow \quad 810 + 3x = 936 \Rightarrow 3x = 936 - 810$

$\Rightarrow \quad 3x = 126 \Rightarrow x = 42$

Hence, the number of new girls admitted = 42.

Example 13. *Which ratio is greater — 11 : 21 or 19 : 28?*

Solution. $11 : 21 = \frac{11}{21}$ and $19 : 28 = \frac{19}{28}$.

LCM of 21 and 28 is 84.

$\frac{11}{21} = \frac{11 \times 4}{21 \times 4} = \frac{44}{84}$ and $\frac{19}{28} = \frac{19 \times 3}{28 \times 3} = \frac{57}{84}$.

Convert into equivalent like fractions

As $57 > 44$, $\frac{57}{84} > \frac{44}{84} \Rightarrow \frac{19}{28} > \frac{11}{21}$.

Hence, 19 : 28 is the greater ratio.

Example 14. *(i) Increase ₹ 114 in the ratio 3 : 4.*

(ii) A girl weighs 56 kg. She reduced her weight in the ratio 8 : 7. Find her new weight.

Solution. (*i*) Since ₹ 114 is to be increased in the ratio 3 : 4,

new amount = $\frac{4}{3}$ of ₹ 114 = ₹ $\left(\frac{4}{3} \times 114\right)$

= ₹ (4×38) = ₹ 152.

(*ii*) Weight of girl = 56 kg.

Since she reduced her weight in the ratio 8 : 7,

her new weight = $\frac{7}{8}$ of 56 kg = $\left(\frac{7}{8} \times 56\right)$ kg

= (7×7) kg = 49 kg.

Exercise 6.1

1. Express the following ratios in simplest form:

   (i) $\frac{1}{6} : \frac{1}{9}$ (ii) $4\frac{1}{2} : 1\frac{1}{8}$ (iii) $\frac{1}{5} : \frac{1}{10} : \frac{1}{15}$.

2. Find the ratio of each of the following in simplest form:

   (i) ₹5 to 50 paise (ii) 3km to 300m (iii) 9m to 27cm (iv) 15kg to 210g

   (v) 25 minutes to 1.5 hours (vi) 30 days to 36 hours.

3. If A : B = 3 : 4 and B : C = 8 : 9, then find A : C.

4. If A : B = 5 : 8 and B : C = 18 : 25, then find A : B : C.

5. If 3A = 2B = 5C, then find A : B : C.

6. Out of daily income of ₹120, a labourer spends ₹90 on food and shelter and saves the rest. Find the ratio of his

   (i) spending to income (ii) saving to income (iii) saving to spending.

7. 5 grams of an alloy contains $3\frac{3}{4}$ grams copper and the rest is nickel. Find the ratio by weight of nickel to copper.

8. A pole of height 3 metres is struck by a speeding car and breaks into two pieces such that the first piece is $\frac{1}{2}$ of the second. Find the length of both pieces.

H.W

9. Heights of Anshul and Dhruv are 1.04 m and 78 cm respectively. Divide 35 sweets between them in the ratio of their heights.

   [Hint. Height of Anshul : Height of Dhruv = $\frac{104}{78} = \frac{4}{3} = 4 : 3$.]

H.W

10. ₹180 are to be divided among three children in the ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{6}$. Find the share of each child.

H.W

11. A natural number has been divided into two parts in the ratio 7 : 11. If the difference of two parts is 20, find the number and the two parts.

H.W

12. A certain sum of money has been divided into two parts in the ratio 9 : 13. If the second part is ₹260, find the total amount.

H.W

13. A certain sum of money is divided into three parts in the ratio 5 : 7 : 8. If the first part is ₹ 225, find the total amount and the other two parts.

14. Divide ₹ 1312 into three parts such that first part is $\frac{2}{3}$ of the second and the ratio between second and third parts is 4 : 7.

15. The ratio of the present ages of Anjali and Ashu is 2 : 3. Five years hence, the ratio of their ages will be 3 : 4. Find their present ages.

H.W

16. The present ages of A and B are in the ratio 5 : 6. Three years ago, their ages were in the ratio 4 : 5. Find their present ages.

H.W

17. Two numbers are in the ratio 5 : 6. When 2 is added to first and 3 is added to second, they are in the ratio 4 : 5. Find the numbers.

18. The ratio of number of boys to the number of girls in a school of 1430 students is 7 : 6. If 26 new girls are admitted in the school, find how many new boys may be

admitted so that the ratio of number of boys to the number of girls may change to 8 : 7.

19. Which ratio is greater:
(i) 5 : 6 or 6 : 7 (ii) 13 : 24 or 17 : 32.

20. (i) Increase the number 150 in ratio 5 : 7.
(ii) A man earns ₹ 18,000 per month. His income is increased in the ratio 12 : 13. Find his new monthly income.
(iii) Savita weighs 55 kg. She reduced her weight in the ratio 11 : 9. Find her new weight.

PROPORTION

An equality of two ratios is called a **proportion.**

Four quantities a, b, c and d are said to be in ***proportion*** *if a : b = c : d i.e. if* $\frac{a}{b} = \frac{c}{d}$ *i.e. if ad = bc.*

This proportion is also written as $a : b :: c : d$.

For example : 2, 3, 4, 6 are in proportion since $\frac{2}{3} = \frac{4}{6}$.

Four quantities a, b, c and d are called **terms** of the proportion $a : b :: c : d$ and a, b, c and d are called its first, second, third and fourth terms respectively.

First and fourth terms are called **extremes (or extreme terms).**

Second and third terms are called **means (or middle terms).**

If $a : b = c : d$, then d is called the **fourth proportional.**

If a, b, c and d are in proportion, then $\frac{a}{b} = \frac{c}{d}$ i.e. $ad = bc$ i.e.

product of extreme terms = product of middle terms.

Thus, a, b, c and d are in proportion if

product of extremes = product of means.

This is known as **cross product rule.**

Hence, if $ad \neq bc$, then a, b, c and d are not in proportion.

Remark

In a ratio $a : b$, both quantities must be of the same kind while in a proportion $a : b = c : d$, all the four quantities need not be of the same type. The first two quantities should be of the same kind and the last two quantities should be of same kind.

For example:

₹6 : ₹8 = $\frac{6}{8} = \frac{3}{4}$ = 3 : 4 and

12 toffees : 16 toffees = $\frac{12}{16} = \frac{3}{4}$ = 3 : 4

∴ ₹6 : ₹8 = 12 toffees : 16 toffees.

Continued proportion

Three quantities a, b and c (of the same kind and in same units) are said to be in continued proportion if $a : b = b : c$ i.e. if $\frac{a}{b} = \frac{b}{c}$ i.e. if $b^2 = ac$.

If a, b and c are in continued proportion, then the middle term b is said to be the **mean proportional** between a and c, a is the **first proportional** and c is the **third proportional.**

Example 1. *Check whether the numbers 1.5, 2.5, 3.6, 6.0 are in proportion.*

Solution. Product of extremes $= 1.5 \times 6.0 = 9.0$,

product of means $= 2.5 \times 3.6 = 9.0$

By cross product rule, the numbers 1.5, 2.5, 3.6, 6.0 are in proportion.

Example 2. *Find the value of x in the proportion* $\frac{1}{6} : x = \frac{1}{2} : \frac{1}{5}$.

Solution. Using cross product rule, we get

$$x \times \frac{1}{2} = \frac{1}{6} \times \frac{1}{5}$$

product of means = product of extremes

$$\Rightarrow \quad x = 2 \times \frac{1}{6} \times \frac{1}{5} = \frac{1}{15}.$$

Example 3. *Find the fourth proportional to 1.5, 2.5, 4.5.*

Solution. Let the fourth proportional be x, then 1.5, 2.5, 4.5, x are in proportion.

Using cross product rule, we get

product of extremes = product of means

$$1.5 \times x = 2.5 \times 4.5$$

$$\Rightarrow \quad x = \frac{2.5 \times 4.5}{1.5} = \frac{25}{10} \times \frac{45}{15} = \frac{5}{2} \times 3 = \frac{15}{2} = 7.5$$

Example 4. *Find the third proportional to ₹4, ₹12.*

Solution. Let the third proportional to ₹4, ₹12 be ₹x, then ₹4, ₹12 and ₹x are in continued proportion

$$\Rightarrow \quad \frac{4}{12} = \frac{12}{x} \Rightarrow \frac{1}{3} = \frac{12}{x} \Rightarrow x = 36$$

Hence, the third proportional is ₹36.

Example 5. *In a computer lab, there are 5 computers for every 8 students. How many computers will be needed for 40 students?*

Solution. Let the number of computers required be x, then

$$x : 40 = 5 : 8$$

$$\Rightarrow \quad \frac{x}{40} = \frac{5}{8} \Rightarrow x = \frac{5}{8} \times 40 \Rightarrow x = 25.$$

Hence, the number of computers required = 25.

Example 6. *The ratio between male and female teachers in a school is 5 : 8. If there are 56 female teachers in the school, find*

(i) the number of male teachers in the school.

(ii) the total number of teachers in the school.

Solution. (*i*) Let the number of male teachers in the school be x, then the ratio of number of male teachers to that of female teachers = $x : 56$.

According to given condition,

$$x : 56 = 5 : 8 \Rightarrow x \times 8 = 56 \times 5$$

cross product rule

$$\Rightarrow x = \frac{56 \times 5}{8} = 7 \times 5 = 35.$$

Hence, there are 35 male teachers in the school.

(*ii*) The total number of teachers in the school = 35 + 56 = 91.

Example 7. *At a particular time, the shadow of a pole and a building are 1.7 m and 7.3 m respectively. If the height of the building is 21.9 m. Find the height of the pole.*

Solution. Let the height of the pole be x metres, then ratio of the heights of pole and building = ratio of their lengths of shadow

$$\Rightarrow \quad \frac{x}{21.9} = \frac{1.7}{7.3} \Rightarrow x = 21.9 \times \frac{1.7}{7.3}$$

$$\Rightarrow \quad x = \frac{219}{10} \times \frac{17}{73} = \frac{3 \times 17}{10} = \frac{51}{10} = 5.1$$

$\therefore$ The height of the pole = 5.1 m.

Exercise 6.2

1. Which of the following statements are true?

   (*i*) $2.5 : 1.5 :: 7.0 : 4.2$ (*ii*) $\frac{1}{2} : \frac{1}{3} = \frac{1}{3} : \frac{1}{4}$

   (*iii*) 24 men : 16 men = 33 horses : 22 horses.
2. Check whether the following numbers are in proportion or not:

   (*i*) 18, 10, 9, 5 (*ii*) $3, 3\frac{1}{2}, 4, 4\frac{1}{2}$ (*iii*) 0.1, 0.2, 0.3, 0.6.
3. Find x in the following proportions:

   (*i*) $x : 4 = 9 : 12$ (*ii*) $\frac{1}{13} : x :: \frac{1}{2} : \frac{1}{5}$ (*iii*) $3.6 : 0.4 = x : 0.5$.
4. Find the fourth proportional to

   (*i*) 42, 12, 7 (*ii*) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ (*iii*) 3 kg, 12 kg, 15 kg.
5. Check whether 7, 49, 343 are in continued proportion or not.
6. Find the third proportional to

   (*i*) 36, 18 (*ii*) $5\frac{1}{4}, 7$ (*iii*) 3.2, 0.8.
7. The ratio between the length and width of a rectangular sheet of paper is 7 : 5. If the width of the sheet is 20.5 cm, find its length.
8. The ages of Amit and Archana are in the ratio 4 : 5. If Amit is 4 years 8 months old, find the age of Archana.

UNITARY METHOD

*A method, in which the value of a unit quantity is first obtained to find the value of any required quantity, is called **unitary method.***

Example 1. *If 8 toys cost ₹216, how much would 15 toys cost?*

Solution. Since cost of 8 toys is ₹216,

$\therefore$ cost of 1 toy $= ₹\frac{216}{8} = ₹27$

$\therefore$ cost of 15 toys $= ₹(27 \times 15)$
$= ₹405.$

(Assuming that all toys are alike.)

Less toys, less cost

More toys, more cost

Example 2. *The car that I own can go 150 km with 25 litres of petrol. How far can I go with 30 litres of petrol?*

Solution. With 25 litres of petrol, car goes 150 km,

$\therefore$ with 1 litre of petrol, car goes $\frac{150}{25}$ km = 6 km

$\therefore$ with 30 litres of petrol car goes (30×6) km = 180 km.

Example 3. *Population of Rajasthan is 570 lakhs and population of UP is 1660 lakhs. Area of Rajasthan is 3 lakh km² and area of UP is 2 lakh km².*

(i) How many people are there per km² in both these states?

(ii) Which state is less populated?

Solution. (i) Population of Rajasthan = 570 lakhs and its area = 3 lakh km².

$\therefore$ Number of people in Rajasthan per km² $= \frac{570}{3} = 190.$

Population of UP = 1660 lakhs and its area = 2 lakh km².

$\therefore$ Number of people in UP per km² $= \frac{1660}{2} = 830.$

(ii) As 190 < 830, therefore, Rajasthan is less populated.

Example 4. *Half metre cloth costs ₹30. How much would $2\frac{3}{5}$ metres cost?*

Solution. Since cost of $\frac{1}{2}$ m cloth is ₹30,

$\therefore$ cost of 1 m cloth $= ₹\frac{30}{\frac{1}{2}} = ₹(30 \times 2) = ₹60$

$\therefore$ cost of $2\frac{3}{5}$ m *i.e.* $\frac{13}{5}$ m cloth $= ₹\left(\frac{13}{5} \times 60\right) = ₹156.$

Example 5. *If 2.5 litres of milk cost ₹42.5, how much milk will cost ₹595?*

Solution. Since for ₹42.5, the quantity of milk bought = 2.5 litres,

$\therefore$ for ₹1, the quantity of milk bought $= \frac{2.5}{42.5}$ litre

$= \frac{25}{425}$ litres $= \frac{1}{17}$ litres

Less money, less

$\therefore$ for ₹595, the quantity of milk bought $= \left(\frac{1}{17} \times 595\right)$ litres   More money, more milk

$= 35$ litres.

Example 6. *If 16 women can weave 72 metres of cloth in a day, how many metres of cloth can be woven by 5 women in a day?*

Solution. Since 16 women weave 72 metres cloth in a day,

$\therefore$ 1 woman will weave $= \frac{72}{16}$ m cloth in a day

$\therefore$ 5 women will weave $= \left(\frac{72}{16} \times 5\right)$ m cloth in a day

$= \frac{45}{2}$ m cloth in a day $= 22\frac{1}{2}$ m cloth in a day.

Example 7. *A map is given with a scale of 2 cm = 1000 km. What is the actual distance between two places in km, if the distance in the map is 2.5 cm?*

Solution. Since 2 cm on map represents 1000 km distance,

$\therefore$ 1 cm on map represents $\frac{1000}{2}$ km $= 500$ km distance.

$\therefore$ 2.5 cm on map represents (2.5×500) km distance $= \left(\frac{25}{10} \times 500\right)$ km distance

$= 1250$ km distance.

Alternative method

Let the distance between two places be x km, then

$x : 1000 = 2.5 : 2$

$\Rightarrow \quad \frac{x}{1000} = \frac{2.5}{2} \Rightarrow x = \frac{2.5}{2} \times 1000$

$\Rightarrow \quad x = 2.5 \times 500 = \frac{25}{10} \times 500 = 1250.$

Hence, the distance between two places = 1250 km.

Exercise 6.3

1. 6 bowls cost ₹90. What would be cost of 10 such bowls?
2. Ten pencils cost ₹15. How many pencils can be bought with ₹72?
3. 400 grams cake costs 80 rupees. How much would a 1.5 kg cake cost?
4. A man earns ₹18000 in 3 months.
   (*i*) How much time would he take to earn ₹30000?
   (*ii*) How much money will he earn in 7 months?
5. 12 mangoes weigh 2.4 kg. What is the weight of 8 mangoes?
6. If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weigh $2\frac{1}{2}$ kilograms?

7. A bus consumes 25 litres of diesel in covering a distance of 90 kilometres. How much diesel is needed to cover 288 kilometres?

8. If $\frac{4}{5}$ metre cloth costs ₹ 36, find the cost of $2\frac{1}{5}$ metres of cloth.

9. If 15 men can pack 540 parcels per day, how many men are needed to pack 396 parcels per day?

10. Which is a better buy : 12 kg potatoes for ₹ 132 or 16 kg potatoes for ₹ 168?

SPEED, DISTANCE AND TIME

We know that if a train is going at a steady rate of 60 kilometres an hour, it will cover 120 km in 2 hours, 180 km in 3 hours, 30 km in half an hour and so on.

Speed of an object is the distance covered by it in a unit time.

For example :

(i) If a car covers 210 km in 3 hours, its speed is $\frac{210}{3}$ *i.e.* 70 km per hour.

(ii) If a snail walks 4 cm in 2 hours, its speed is $\frac{4}{2}$ *i.e.* 2 cm per hour.

Thus,

$$speed = \frac{distance}{time}.$$

From this formula, we get

$$distance = speed \times time,\ time = \frac{distance}{speed}.$$

Speed is usually given in km/h (kilometres per hour) or m/sec (metres per second). Note that unit of measuring is important. It is very sloppy to say that a bus is travelling at a speed of 20. You must mention whether it is 20 km/h or 20 m/sec.

Conversion of units of speed

To convert km/h into m/sec, note that

$$1\text{ km/h} = \frac{1\text{ km}}{1\text{ hour}} = \frac{1000\text{ metres}}{(60 \times 60)\text{ seconds}} = \frac{5}{18}\text{ m/sec.}$$

Thus, a speed of 36 km/h = $\left(36 \times \frac{5}{18}\right)$ m/sec = 10 m/sec.

Hence, to convert a speed in km/h to m/sec, multiply by $\frac{5}{18}$.

To convert m/sec into km/h, note that

$$1\text{ m/sec} = \frac{1\text{ m}}{1\text{ sec}} = \frac{\frac{1}{1000}\text{ km}}{\frac{1}{60 \times 60}\text{ hour}} = \frac{3600}{1000}\text{ km/h} = \frac{18}{5}\text{ km/h.}$$

Thus, a speed of 15 m/sec = $\left(15 \times \frac{18}{5}\right)$ km/h = 54 km/h.

Hence, to convert a speed in m/sec to km/h, multiply by $\frac{18}{5}$.

Uniform speed, variable speed

*If an object covers equal distances in equal intervals of time, its speed is said to be **uniform** (or constant); otherwise, its speed is said to be **variable**.*

The above formulae connecting speed, distance and time are based on the assumption that the speed is uniform.

Example 1. *A rabbit runs 400 metres in 50 seconds. Find :*

(i) *its speed* (ii) *distance run by it in 12 seconds*

(iii) *time taken by the rabbit in running* $\frac{3}{10}$ *km.*

Solution. (i) Speed $= \frac{\text{distance}}{\text{time}} = \frac{400 \text{ m}}{50 \text{ sec}} = 8$ m/sec.

(ii) Distance run by the rabbit in 12 sec = speed × time = (8 m/s) × 12 sec = 96 m.

(iii) $\frac{3}{10}$ km $= \left(\frac{3}{10} \times 1000\right)$ m = 300 m.

∴ Time taken by the rabbit in running $\frac{3}{10}$ km *i.e.* 300 m

$$= \frac{\text{distance}}{\text{speed}} = \frac{300 \text{ m}}{8 \text{ m/s}} = \frac{75}{2} \text{ sec} = 37.5 \text{ sec}.$$

Example 2. *An old man walks 200 metres in 20 minutes. Find his speed in km/h as well as in m/sec.*

Solution. Distance = 200 metres $= \frac{200}{1000}$ km $= \frac{1}{5}$ km,

time = 20 minutes $= \frac{20}{60}$ hour $= \frac{1}{3}$ hour.

$$\therefore \quad \text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{\frac{1}{5}\text{ km}}{\frac{1}{3}\text{ h}} = \frac{3}{5} \text{ km/h}.$$

To convert it into m/sec,

$$\text{speed} = \frac{3}{5} \text{ km/h} = \left(\frac{3}{5} \times \frac{5}{18}\right) \text{ m/sec} = \frac{1}{6} \text{ m/sec}.$$

Example 3. *A cyclist covers 600 metres in 5 minutes.*

(i) *How much distance will he travel in half an hour?*

(ii) *How much time will he take to travel 3 km?*

Solution. Here it is easier to work in metres and minutes.

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{600 \text{ metres}}{5 \text{ minutes}} = 120 \text{ m/minute}.$$

(i) Distance travelled in half an hour *i.e.* in 30 minutes

= speed × time = (120 m/minute) × 30 minutes

= (120 × 30) m = 3600 m = 3.6 km.

(ii) Distance = 3 km = 3000 metres.

$$\therefore \quad \text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{3000 \text{ metres}}{120 \text{ m/minute}} = 25 \text{ minutes}.$$

Example 4. *A train going at 60 km/h requires 4 hours to reach Jaipur from Delhi. How much time would it require if speed is 80 km/h? How much time would a passenger train travelling at 40 km/h require to go to Jaipur from Delhi?*

Solution. At a speed of 60 km/h, the train takes 4 hours to go to Jaipur from Delhi,

so, distance between Delhi and Jaipur = speed × time

$= (60 \text{ km/h}) \times 4 \text{ hours} = 240 \text{ km}.$

If speed is 80 km/h, time taken $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{240 \text{ km}}{80 \text{ km/h}} = 3$ hours.

As the speed of passenger train is 40 km/h,

$\therefore$ time taken by it $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{240 \text{ km}}{40 \text{ km/h}} = 6$ hours.

Exercise 6.4

1. Convert the following speeds into m/sec :
   (*i*) 72 km/h (*ii*) 9 km/h (*iii*) 1.2 km/minute (*iv*) 600 m/hour
2. Convert the following speeds into km/h :
   (*i*) 15 m/sec (*ii*) 1.5 m/sec.
3. Which is greater — a speed of 30 m/sec or 30 km/h?
   [Hint. 30 m/sec $= \left(30 \times \dfrac{18}{5}\right)$ km/h = 108 km/h.]
4. An aeroplane is flying at a speed of 720 km/h.
   (*i*) If the aerial distance between two cities is 1800 km, how much time will the aeroplane take in crossing these cities?
   (*ii*) How much distance does the aeroplane cover in 40 minutes?
   (*iii*) How far will it fly in 15 seconds?
5. A dog is walking at a speed of 6 km/h.
   (*i*) How much distance does it cover in 5 minutes?
   (*ii*) How much time would it take to cover 200 metres?
6. A swimming pool is 50 metres long. A boy can swim across the length and then return to his starting position in 5 minutes. What is his swimming speed in km/h?
7. A bus takes 48 minutes to cover a certain distance when travelling at a speed of 50 km/h. How much time will it take to cover the same distance when travelling at a speed of 30 km/h?

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
   (*i*) The simplest form of the ratio $\dfrac{1}{6} : \dfrac{1}{4}$ is ....

(ii) 75 cm : 1.25 m = ...

(iii) If two ratios are equivalent, then the four quantities are said to be in ....

(iv) If 8, x, 48 and 18 are in proportion then the value of x is ....

(v) If the cost of 10 pencils is ₹15, then the cost of 6 pencils is ....

(vi) If a cyclist is travelling at a speed of 15 km/h, then the distance covered by him in 20 minutes is .... .

2. State whether the following statements are true (T) or false (F):

(i) A ratio is always greater than 1.

(ii) Ratio of half an hour to 20 seconds is 30 : 20.

(iii) The ratio 5 : 7 is greater than the ratio 5 : 6.

(iv) If the numbers 3, 5, 12 and x are in proportion then the value of x is 20.

(v) The ratios 3 : 2 and 4 : 5 are equivalent.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 14):

3. A ratio equivalent to 6 : 10 is

(a) 3 : 4 (b) 18 : 30 (c) 12 : 40 (d) 5 : 3

4. A ratio equivalent to the ratio $\frac{2}{3} : \frac{3}{4}$ is

(a) 4 : 6 (b) 8 : 9 (c) 6 : 8 (d) 9 : 8

5. The ratio of 75 mL to 3 litres is

(a) 25 : 1 (b) 40 : 1 (c) 1 : 40 (d) 3 : 200

6. The ratio of the number of sides of a rectangle to the number of edges of a cuboid is

(a) 1 : 2 (b) 1 : 3 (c) 2 : 3 (d) none of these

7. In a class of 35 students, the number of girls is 20. The ratio of number of boys to the number of girls in the class is

(a) 3 : 4 (b) 4 : 3 (c) 7 : 4 (d) 7 : 3

8. The ratio of number of girls to the number of boys in a class is 6 : 7. If there are 21 boys in the class, then the number of girls in the class is

(a) 39 (b) 24 (c) 18 (d) 13

9. Two numbers are in the ratio 3 : 5. If the sum of the numbers is 144, then the larger number is

(a) 48 (b) 54 (c) 72 (d) 90

10. If x, 12, 8 and 32 are in proportion, then x is

(a) 6 (b) 4 (c) 3 (d) 2

11. If 3, 12 and x are in continued proportion, then x is

(a) 4 (b) 6 (c) 16 (d) 48

12. If the weight of 5 bags of sugar is 27 kg, then the weight of one bag of sugar is

(a) 5.04 kg (b) 5.2 kg (c) 5.4 kg (d) 5.6 kg

13. Sonali bought one dozen notebooks for ₹66. What did she pay for one notebook?

(a) ₹6.50 (b) ₹6.60 (c) ₹5.60 (d) ₹5.50

14. The speed of 90 km/h is equal to

(a) 10 m/sec (b) 18 m/sec (c) 25 m/sec (d) none of these

Higher Order Thinking Skills (HOTS)

1. Present ages of Rohit and Mayank are in the ratio 11 : 8. 8 years hence ratio of their ages will be 5 : 4. Find their present ages.
2. Ratio of lenght and breadth of a rectangle is 3 : 2. If the length of rectangle is 5 m more than the breadth, find the perimeter of the rectangle.

Summary

★ Ratio is a comparison of measures (or magnitudes) of two or more quantities of the same kind by division.

★ If a and b are two quantities of the same kind (in same units), then the fraction $\frac{a}{b}$ is called the ratio of a to b and is written as $a : b$. a is called first term (or antecedent) and b is called second term (or consequent).

★ The terms of a ratio can be multiplied or divided by the same (non-zero) number. Usually, the ratio is written in simplest form.

★ Ratio is a number, so it has no units.

★ If the terms of a ratio are in fractions, convert them into natural numbers by multiplying each term by the LCM of their denominators.

★ To compare two ratios, convert them into equivalent like fractions.

★ Three quantities of the same kind (in same units) are said to be in the ratio $a : b : c$ if the quantities are ak, bk and ck respectively, where k is any positive number.

★ If a quantity increases or decreases in the ratio $a : b$, then new quantity $= \frac{b}{a}$ of the original quantity.

The fraction $\frac{b}{a}$ is called the multiplying ratio (or factor).

★ An equality of two ratios is called a proportion.

★ Four quantities a, b, c and d are said to be in proportion if $a : b = c : d$ i.e. if $\frac{a}{b} = \frac{c}{d}$ i.e. if $ad = bc$.

★ If $a : b = c : d$, then a and d are called extremes (or extreme terms) and b and c are called means (or middle terms).

★ If $a : b = c : d$, then d is called the fourth proportional to a, b and c.

★ a, b, c and d are in proportion if $ad = bc$ i.e. if product of extremes = product of means. This is called cross product rule.

★ Three quantities a, b and c (of the same kind and in same units) are said to be in continued proportion if $a : b = b : c$ i.e. if $\frac{a}{b} = \frac{b}{c}$ i.e. if $b^2 = ac$.

★ A method, in which the value of a unit quantity is first obtained to find the value of any required quantity, is called unitary method.

★ Speed of an object is the distance covered by it in a unit time.

★ Speed = $\frac{\text{distance}}{\text{time}}$, time = $\frac{\text{distance}}{\text{speed}}$, and distance = speed × time.

★ To convert units of speed :

❑ 1 km/h = $\frac{5}{18}$ m/sec. ❑ 1 m/sec = $\frac{18}{5}$ km/h.

Check Your Progress

1. A rectangular park is 120 m long and 75 m wide. Find the ratio of:
   (*i*) breadth to its length (*ii*) length to its perimeter.
2. Divide the angles of a triangle in the ratio 2 : 3 : 4.
   [**Hint.** Sum of angles of a triangle is 180°.]
3. Heights of Anshul, Ankita and Dhruv are 1.04 m, 1.30 m and 91 cm respectively. Divide 100 sweets among them in the ratio of their heights.
4. The weights of Divya and Himanshu are in the ratio 5 : 7. If Himanshu weighs 28 kg, find the weight of Divya.
5. The areas of three flats are in the ratio 5 : 6 : 8. If the differences in the areas of flat C and flat A is 180 square metres, find the area of the flat B.
6. The income of a man is increased in the ratio 7 : 8. If the increase in his income is ₹ 4500 per month, find his new income.
7. If 3A = 5B and 4B = 6C, then find A : C.
8. Which ratio is smaller — 9 : 13 or 7 : 11?
9. Find the fourth proportional to
   (*i*) 4, 7, 20 (*ii*) $2\frac{1}{2}, 1\frac{1}{4}, 2.2$
10. A typist types 70 pages in 3 hours 30 minutes. How long will she take to type 300 pages?
11. 12 looms weave 210 m cloth per day. How many metres of cloth will 8 looms weave per day?
12. A journey takes 4 hours 30 minutes at a speed of 60 km/h. How long will the same journey take at a speed of 15 m/sec?
    [**Hint.** 15 m/sec = $\left(15 \times \frac{18}{5}\right)$ km/h = 54 km/h.]