9

LINEAR EQUATIONS AND LINEAR INEQUALITIES

INTRODUCTION

In class VI, you have read about simple linear equations in one variable and how to solve them. For example, if a number multiplied by 3 is 15, then assuming the number to be x, we obtain an equation $3x = 15$ and then solve this equation to get $x = 5$. In this chapter, we will solve slightly tougher problems than previous class. We shall also learn the uses of linear equations in solving some real life contextual problems. We shall also introduce 'linear inequalities' and find their solution sets.

LINEAR EQUATIONS

An equation containing only one variable (literal) with highest power 1 is called a linear ***equation*** *in that variable.*

For example:

$$3x + 5 = 8, \quad 3 - 2x = 5x + 1 \text{ and } 4n = \frac{2}{3}n - 1$$

are all linear equations in one variable.

In this chapter, we shall take up linear equations in one variable only.

Solving a linear equation in one variable

A number which satisfies the given linear equation is called a ***solution*** *or* ***root*** *of the equation.*

'Satisfying the equation' means that if the variable (literal) involved in the equation is replaced by the number, then both sides of the equation become equal.

The process of finding the particular value of the variable (literal) which makes both sides of the equation equal is called **solving the equation.**

Rules for solving linear equations

- *We can add the same number or expression to both sides of an equation.*
- *We can subtract the same number or expression from both sides of an equation.*
- *We can multiply both sides of an equation by the same non-zero number or expression.*
- *We can divide both sides of an equation by the same non-zero number or expression.*

Rule of transposition

A term may be transposed from one side of the equation to the other side, but its sign will change.

For example :

(i) $7x + 9 = 2 \Rightarrow 7x = 2 - 9$

(Transposing +9 from L.H.S. to R.H.S. and changing its sign.)

(ii) $5y - 3 = 11 - 2y \Rightarrow 5y + 2y - 3 = 11$

(Transposing $-2y$ from R.H.S. to L.H.S. and changing its sign.)

Solving linear equations in one variable

For solving a linear equation in one variable, proceed as under :

- *Simplify both sides by removing group symbols and collecting like terms.*
- *Remove fractions (or decimals) by multiplying both sides by an appropriate factor (LCM of fractions or a power of 10 in case of decimals).*
- *Isolate all variable terms on one side and all constants on the other side. Collect like terms.*
- *Make the coefficient of the variable 1.*

Remark

The solution may be checked (verified) by substituting in the original equation.

Example 1. *Solve the following equations :*

(i) $5x - 3 = 3x + 5$ (ii) $3(y - 1) = y - 11$.

Also represent their solutions graphically.

Solution. (i) Given, $5x - 3 = 3x + 5$

$\Rightarrow 5x = 3x + 5 + 3$ (Transposing -3 to R.H.S.)

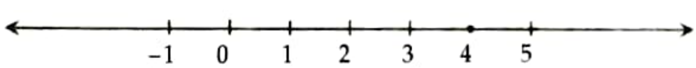
$\Rightarrow 5x = 3x + 8$

$\Rightarrow 5x - 3x = 8$ (Transposing $3x$ to L.H.S.)

$\Rightarrow 2x = 8 \Rightarrow x = \frac{8}{2}$

$\Rightarrow x = 4$, which is the required solution.

The solution is shown by thick dot on the number line.



(ii) Given, $3(y - 1) = y - 11$

$\Rightarrow 3y - 3 = y - 11$ (Removing group symbol)

$\Rightarrow 3y = y - 11 + 3$ (Transposing -3 to R.H.S.)

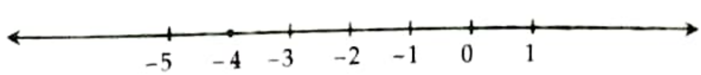
$\Rightarrow 3y = y - 8$

$\Rightarrow 3y - y = -8$ (Transposing y to L.H.S.)

$\Rightarrow 2y = -8 \Rightarrow y = -\frac{8}{2}$

$\Rightarrow y = -4$, which is the required solution.

The solution is shown by thick dot on the number line.



Example 2. *Solve the following equations :*

(*i*) $4x - \frac{1}{5} = 7$ (*ii*) $\frac{4}{5}x - \frac{1}{6} = \frac{9}{2} - 2x.$

Solution. (*i*) Given, $4x - \frac{1}{5} = 7.$

Multiplying both sides by 5, we get

$20x - 1 = 35$

$\Rightarrow 20x = 35 + 1 \Rightarrow 20x = 36$

$\Rightarrow x = \frac{36}{20} \Rightarrow x = \frac{9}{5} \Rightarrow x = 1\frac{4}{5}.$

(*ii*) Given, $\frac{4}{5}x - \frac{1}{6} = \frac{9}{2} - 2x.$

Multiplying both sides by 30, LCM of 5, 6 and 2, we get

$24x - 5 = 135 - 60x$

$\Rightarrow 24x + 60x = 135 + 5$ (Transposing $-60x$ to L.H.S. and -5 to R.H.S.)

$\Rightarrow 84x = 140 \Rightarrow x = \frac{140}{84} = \frac{5}{3} \Rightarrow x = 1\frac{2}{3}.$

Example 3. *Solve the following equations :*

(*i*) $\frac{x+7}{3} - \frac{3x-2}{5} = 3$ (*ii*) $\frac{1-x}{6} + \frac{2x}{3} - \frac{1-7x}{4} = 2\frac{1}{6}.$

Solution. (*i*) Given, $\frac{x+7}{3} - \frac{3x-2}{5} = 3.$

Multiplying both sides by 15, LCM of 3 and 5, we get

$5(x + 7) - 3(3x - 2) = 15 \times 3$

$\Rightarrow 5x + 35 - 9x + 6 = 45$ (Removing group symbols)

$\Rightarrow -4x + 41 = 45 \Rightarrow -4x = 45 - 41$

$\Rightarrow -4x = 4 \Rightarrow x = -1.$

(*ii*) Given, $\frac{1-x}{6} + \frac{2x}{3} - \frac{1-7x}{4} = \frac{13}{6}.$

Multiplying both sides by 12, LCM of 6, 3 and 4, we get

$2(1 - x) + 4(2x) - 3(1 - 7x) = 2 \times 13$

$\Rightarrow 2 - 2x + 8x - 3 + 21x = 26$

$\Rightarrow 27x - 1 = 26 \Rightarrow 27x = 26 + 1$

$\Rightarrow 27x = 27 \Rightarrow x = 1.$

Example 4. *Solve : $2(2p + 1) - 30\%$ of $(5p - 2) = 7.6$*

Solution. Given, $2(2p + 1) - \frac{30}{100}$ of $(5p - 2) = 7.6$

$\Rightarrow 2(2p + 1) - \frac{3}{10} \times (5p - 2) = 7.6$

Multiplying both sides by 10, we get

$20(2p + 1) - 3(5p - 2) = 10 \times 7.6$

$\Rightarrow 40p + 20 - 15p + 6 = 76 \Rightarrow 25p + 26 = 76$

$\Rightarrow 25p = 76 - 26 \Rightarrow 25p = 50 \Rightarrow p = 2.$

Example 5. Find the value of m if the value of the expression $2x^3 - 5x^2 + mx - 7$ is equal to 10 when $x = -1$.

Solution. Given, the value of $2x^3 - 5x^2 + mx - 7$ is 10 when $x = -1$.

$\therefore$ $2(-1)^3 - 5(-1)^2 + m(-1) - 7 = 10$

$\Rightarrow$ $-2 - 5 - m - 7 = 10 \Rightarrow -14 - m = 10$

$\Rightarrow$ $-m = 10 + 14 \Rightarrow -m = 24 \Rightarrow m = -24.$

Exercise 9.1

Solve the following (1 to 9) linear equations :

1. (*i*) $2(3 - 2x) = 13$ H.W (*ii*) $\frac{3}{5}y - 2 = \frac{7}{10}$.
2. (*i*) $\frac{x}{2} = 5 + \frac{x}{3}$ H.W (*ii*) $2\left(x - \frac{3}{2}\right) = 11$.
3. (*i*) $7(x - 2) = 2(2x - 4)$ H.W (*ii*) $21 - 3(x - 7) = x + 20$.
4. (*i*) $3x - \frac{1}{3} = 2\left(x - \frac{1}{2}\right) + 5$ H.W (*ii*) $\frac{2m}{3} - \frac{m}{5} = 7$.
5. (*i*) $\frac{x+1}{5} - \frac{x-7}{2} = 1$ H.W (*ii*) $\frac{3p-2}{7} - \frac{p-2}{4} = 2$.
6. (*i*) $\frac{1}{2}(x + 5) - \frac{1}{3}(x - 2) = 4$ (*ii*) $\frac{2x-3}{6} - \frac{x-5}{2} = \frac{x}{6}$.
7. (*i*) $\frac{x-4}{7} - \frac{x+4}{5} = \frac{x+3}{7}$ H.W (*ii*) $\frac{x-1}{5} + \frac{x-2}{2} = \frac{x}{3} + 1$.
8. (*i*) $y + 1.2y = 4.4$ H.W (*ii*) 15% of $x = 21$.
9. H.W (*i*) $2p$ + 20% of $(2p - 1) = 7$ H.W (*ii*) $3(2x - 1)$ + 25% of $x = 97$.
10. Find the value of p if the value of $x^4 - 3x^3 - px - 5$ is equal to 23 when $x = -2$.

USES OF LINEAR EQUATIONS

Word problems

Problems stated in words are called ***word*** *or* ***applied problems****.*

Success with word (or applied) problems comes with practice. Solving word problem involves two steps. First, translating the words of the problem into an algebraic equation. Second, solving the resulting equation.

Solving word problems

Due to the wide variety of word (or applied) problems, there is no single technique that works in all problems. However, the following general suggestions may prove helpful.

- Read the statement of the problem carefully and determine what quantity must be found.
- Represent the unknown quantity by a letter.
- Determine which expressions are equal and write an equation.
- Solve the resulting equation.

Example 1. *If 5 is added to twice a number, the result is 29. Find the number.*

Solution. Let the required number be x.

Twice the number = $2x$,

5 added to twice the number = $2x + 5$.

According to the problem,

$$2x + 5 = 29$$

$\Rightarrow \quad 2x = 29 - 5 \Rightarrow 2x = 24 \Rightarrow x = 12.$

Hence, the required number is 12.

Example 2. *If thrice a certain number is diminished by 7, the result is 9 more than the number. Find the number.*

Solution. Let the required number be x.

Thrice the number = $3x$.

Thrice the number diminished by 7 = $3x - 7$,

9 more than the number = $x + 9$.

According to the problem,

$$3x - 7 = x + 9$$

$\Rightarrow \quad 3x - x = 9 + 7 \Rightarrow 2x = 16 \Rightarrow x = 8.$

Hence, the required number is 8.

Example 3. *One-fourth of a number exceeds one-fifth of its succeeding number by 3; find the number.*

Solution. Let the required number be x.

Its succeeding number = $x + 1$.

According to the problem,

$$\frac{1}{4}x - \frac{1}{5}(x + 1) = 3$$

$\Rightarrow \quad 5x - 4(x + 1) = 60$ (multiplying by 20)

$\Rightarrow \quad 5x - 4x - 4 = 60$

$\Rightarrow \quad x = 60 + 4 \Rightarrow x = 64.$

Hence, the required number is 64.

Example 4. *Find three consecutive even integers whose sum is 108.*

Solution. Let the smallest of the required consecutive even integers be x.

Then the next two even integers are $x + 2$ and $x + 4$.

According to the problem,

$$x + (x + 2) + (x + 4) = 108$$

$\Rightarrow \quad x + x + 2 + x + 4 = 108$

$\Rightarrow \quad 3x + 6 = 108 \Rightarrow 3x = 108 - 6$

$\Rightarrow \quad 3x = 102 \Rightarrow x = 34.$

$\therefore \quad x + 2 = 34 + 2 = 36$ and $x + 4 = 34 + 4 = 38.$

Hence, the required consecutive even integers are 34, 36 and 38.

Example 5. *The cost of 2 tables and 5 chairs is ₹2300. If a table costs ₹30 more than a chair, find the price of each.*

Solution. Let the price of a chair be ₹x.

As a table costs ₹30 more than a chair, therefore,

the price of a table = ₹$(x + 30)$.

According to the problem,

$5x + 2(x + 30) = 2300$

$5x + 2x + 60 = 2300$

$\Rightarrow \quad 7x = 2300 - 60 \Rightarrow 7x = 2240 \Rightarrow x = 320.$

$\Rightarrow$ The price of a chair = ₹320

$\therefore$ and the price of a table = ₹(320 + 30) = ₹350.

Example 6. *In a lottery, a total of 200 prizes are to be given. A prize is either of ₹500 or ₹100. Find the number of each type of prizes if the total prize money is ₹50000.*

Solution. Let there be x prizes of ₹500 each.

As the total number of prizes = 200, therefore,

the number of prizes of ₹100 each = $200 - x$.

According to the problem,

$500x + 100(200 - x) = 50000$

$\Rightarrow \quad 5x + 200 - x = 500$ (dividing by 100)

$\Rightarrow \quad 4x = 500 - 200 \Rightarrow 4x = 300 \Rightarrow x = 75.$

$\therefore$ The number of prizes of ₹500 each = 75

and the number of prizes of ₹100 each = 200 − 75 = 125.

Example 7. *A son's present age is half the present age of his mother. Ten years ago, the mother was thrice as old as her son. What are their present ages?*

Solution. Let the present age of the mother be x years.

As the son's age is half the present age of his mother, therefore,

the present age of the son = $\frac{1}{2}x$ years.

Ten years ago,

the age of the mother = $(x - 10)$ years

and the age of the son = $\left(\frac{1}{2}x - 10\right)$ years.

According to the problem,

$x - 10 = 3\left(\frac{1}{2}x - 10\right)$

$\Rightarrow \quad x - 10 = \frac{3}{2}x - 30$

$\Rightarrow \quad 2x - 20 = 3x - 60$ (multiplying both sides by 2)

$\Rightarrow \quad 2x - 3x = -60 + 20$

$\Rightarrow \quad -x = -40 \Rightarrow x = 40.$

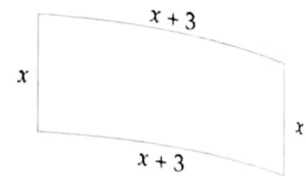
$\therefore$ The present age of the mother = 40 years

and the present age of the son = $\left(\frac{1}{2} \times 40\right)$ years = 20 years.

Example 8. *The length of a rectangle is 3 units more than breadth and the perimeter is 22 units. Find the breadth and length of the rectangle.*

Solution. Let the breadth of the rectangle be x units, so that length is $(x + 3)$ units.

Perimeter = sum of lengths of sides



$\therefore \quad x + x + 3 + x + x + 3 = 22$

$\Rightarrow \quad 4x + 6 = 22$

$\Rightarrow \quad 4x = 22 - 6$

$\Rightarrow \quad 4x = 16 \Rightarrow x = 4.$

Hence, the breadth is 4 units and length is $(4 + 3)$ units *i.e.* 7 units.

Example 9. *The length of each of two equal sides of an isosceles triangle is 4 m less than twice the length of the third side. Find the dimensions of the triangle if its perimeter is 57 m.*

Solution. Let the length of the third side be x metres.

Then the length of each of two equal sides = $(2x - 4)$ metres

$\therefore$ Perimeter of the triangle = $[(2x - 4) + (2x - 4) + x]$ metres.

According to the problem,

$(2x - 4) + (2x - 4) + x = 57$

$\Rightarrow \quad 2x - 4 + 2x - 4 + x = 57$

$\Rightarrow \quad 5x - 8 = 57 \Rightarrow 5x = 57 + 8$

$\Rightarrow \quad 5x = 65 \Rightarrow x = 13.$

$\therefore \quad 2x - 4 = 2 \times 13 - 4 = 26 - 4 = 22.$

$\therefore$ The lengths of the sides of the triangle are 22 m, 22 m and 13 m.

Example 10. *Two supplementary angles differ by 30°. Find the measure of each angle.*

Solution. Since the difference of the angles is 30°, let the angles be $x°$ and $(x + 30)°$.

As the sum of two supplementary angles is 180°, we get

$x + x + 30 = 180$

$\Rightarrow \quad 2x = 180 - 30 = 150 \Rightarrow x = 75.$

Hence, one angle is 75° and the other angle is $(75 + 30)°$ *i.e.* 105°.

Exercise 9.2

1. If 7 is added to five times a number, the result is 57. Find the number.
2. Find a number, such that one-fourth of the number is 3 more than 7.
3. A number is as much greater than 15 as it is less than 51. Find the number.

H.W

4. If $\frac{1}{2}$ is subtracted from a number and the difference is multiplied by 4, the result is 5. What is the number?

H.W

5. The sum of two numbers is 80 and the greater number exceeds twice the smaller by 11. Find the numbers.
6. Find three consecutive odd natural numbers whose sum is 87.
7. In a class of 35 students, the number of girls is two-fifth of the number of boys. Find the number of girls in the class.

8. A chair costs ₹250 and a table costs ₹400. If a housewife purchased a certain number of chairs and two tables for ₹2800, find the number of chairs she purchased.
9. Aparna got ₹ 27840 as her monthly salary and over-time. Her salary exceeds the over-time by ₹ 16560. What is her monthly salary?
Hint. Let Aparna's monthly salary be ₹ x, then payment for over-time = ₹ $(x - 16560)$.
According to problem, $x + x - 16560 = 27840$.
10. Heena has only ₹2 and ₹5 coins in her purse. If in all she has 80 coins in her purse amounting to ₹232, find the number of ₹5 coins.
11. A purse contains ₹550 in notes of denominations of ₹10 and ₹50. If the number of ₹50 notes is one less than that of ₹10 notes, then find the number of ₹50 notes.
12. After 12 years, I shall be 3 times as old as I was 4 years ago. Find my present age.
13. Two equal sides of an isosceles triangle are $(3x - 1)$ units and $(2x + 2)$ units. The third side is $2x$ units. Find x and the perimeter of the triangle.
14. The length of a rectangular plot is 6 m less than thrice its breadth. Find the dimensions of the plot if its perimeter is 148 m.
15. Two complementary angles differ by 20°. Find the measure of each angle.

LINEAR INEQUALITIES IN ONE VARIABLE

Statements such as

$$x < 3, \quad 2x + 1 \leq 5, \quad 3x - 1 > 8, \quad 3 - 5x \geq 2$$

are called **linear inequalities in the variable x.**

In general, a linear inequality in the variable x can be written in any one of the forms :

(*i*) $ax + b < 0$ (*ii*) $ax + b \leq 0$ (*iii*) $ax + b > 0$ (*iv*) $ax + b \geq 0$

where a and b are rational numbers, $a \neq 0$.

Replacement set

*The set from which the values of the variable (involved in the inequality) are choosen is called the **replacement set**.*

Solution set

*A **solution** of an inequality is a number (choosen from the replacement set) which, when substituted for the variable, makes the inequality true. The set of all the solutions of an inequality is called the **solution set** or **truth set** of the inequality.*

For example : Consider the inequation $x < 3$.

Replacement set	Solution set
{1, 2, 3, 4, 5}	{1, 2}
{–1, 0, 2, 4, 6}	{–1, 0, 2}
{0, 3, 6, 9, 10}	{0}
{3, 4, 5, 12}	ϕ
N	{1, 2}
W	{0, 1, 2}

Note that the solution set depends upon the replacement set.

Graphical representation of solution of an inequality

Solution of every inequality can be represented on a number line. See the following examples:

Example 1. *Represent $x < 3$, $x \in \mathbf{N}$, on the number line.*

Solution. Given $x < 3$, $x \in \mathbf{N}$

The solution set = $\{1, 2\}$.

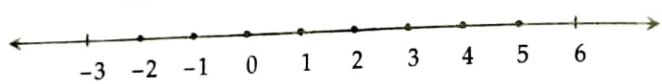
The solution set is shown by thick dots on the number line.

Example 2. *Represent the inequation $-3 < x \leq 5$, $x \in \mathbf{I}$, graphically.*

Solution. Given $-3 < x \leq 5$, $x \in \mathbf{I}$

The solution set = $\{-2, -1, 0, 1, 2, 3, 4, 5\}$.

The solution set is shown by thick dots on the number line.



Solving linear inequalities in one variable

The rules for solving linear inequalities are similar to those for solving equations except for multiplying or dividing by a negative number.

We can do any of the following to an inequality :

- add the same number or expression to both sides.
- subtract the same number or expression from both sides.
- multiply both sides by the same **positive** number.
- divide both sides by the same **positive** number.

However, when we multiply or divide by the same **negative** number, the **symbol of inequality is reversed**.

For example :

(*i*) If $x < 3$ then $-x > -3$ (Multiplying by –1)

(*ii*) If $2x - 3 \geq 7$ then $-3(2x - 3) \leq -21$ (Multiplying by –3)

(*iii*) If $-5x \leq 20$ then $x \geq -4$ (Dividing by –5)

Thus, always reverse the symbol of an inequality when multiplying or dividing by a negative number.

Procedure to solve a linear inequality in one variable :

- *Simplify both sides by removing group symbols and collecting like terms.*
- *Remove fractions (or decimals) by multiplying both sides by an appropriate number (LCM of denominators or a power of 10 in case of decimals).*
- *Isolate all variable terms on one side and all constants on the other side. Collect like terms.*
- *Make the coefficient of the variable 1.*
- *Choose the solution set from the replacement set.*

Example 3. *Solve $2x - 5 < 3$, $x \in$ **N**. Represent its solution on the number line.*

Solution. Given $2x - 5 < 3$

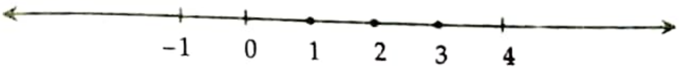
$\Rightarrow \quad 2x - 5 + 5 < 3 + 5$ (Adding 5 to both sides)

$\Rightarrow \quad 2x < 8$

$\Rightarrow \quad x < 4$ (Dividing both sides by 2)

As $x \in$ **N**, the solution set is {1, 2, 3}.

The solution set is shown by thick dots on the number line.



Example 4. *Solve $\frac{3x+1}{4} \leq 3$, $x \in$ **W**. Represent the solution graphically.*

Solution. Given $\frac{3x+1}{4} \leq 3$

$\Rightarrow \quad 3x + 1 \leq 12$ (Multiplying both sides by 4)

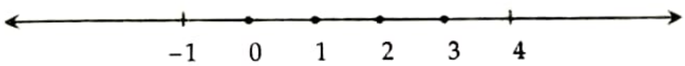
$\Rightarrow \quad 3x + 1 - 1 \leq 12 - 1$ (Subtracting 1 from both sides)

$\Rightarrow \quad 3x \leq 11$

$\Rightarrow \quad x \leq \frac{11}{3}$ (Dividing both sides by 3)

As $x \in$ **W**, the solution set is {0, 1, 2, 3}.

The solution set is shown by thick dots on the number line.



Example 5. *Solve $2 - 3x \geq x - 19$, $x \in$ **W**.*

Solution. Given $2 - 3x \geq x - 19$

$\Rightarrow \quad -2 + 2 - 3x \geq x - 19 - 2$ (Subtracting 2)

$\Rightarrow \quad -3x \geq x - 21$

$\Rightarrow \quad -3x - x \geq -x + x - 21$ (Subtracting x)

$\Rightarrow \quad -4x \geq -21$

$\Rightarrow \quad x \leq \frac{21}{4}$ (Dividing by -4 and reversing the symbol)

As $x \in$ **W**, the solution set is {0, 1, 2, 3, 4, 5}.

Exercise 9.3

1. If the replacement set is {−5, −3, −1, 0, 1, 3, 4}, find the solution set of :
   (*i*) $x < -2$ (*ii*) $x > 1$ (*iii*) $x \geq -1$
   (*iv*) $-5 < x < 3$ (*v*) $-3 \leq x < 4$ (*vi*) $0 \leq x < 7$.
2. Represent the solution set of the following inequalities graphically :
   (*i*) $x \leq 3$, $x \in$ **N** (*ii*) $x < 4$, $x \in$ **W**
   (*iii*) $-2 \leq x < 4$, $x \in$ **I** (*iv*) $-3 \leq x \leq 2$, $x \in$ **I**.

H.W

3. Solve the following inequalities :

   (*i*) $4 - x > -2, x \in \mathbf{N}$ (*ii*) $3x + 1 \leq 8, x \in \mathbf{W}$.

   Also represent their solutions on the number line.

4. Solve $3 - 4x < x - 12, x \in \{-1, 0, 1, 2, 3, 4, 5, 6, 7\}$.

5. Solve $-7 < 4x + 1 \leq 23, x \in \mathbf{I}$.

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:

   (*i*) A linear equation in one variable cannot have more than .... solution.

   (*ii*) If five times a number is 50, then the number is .....

   (*iii*) The number 4 is the .... of the equation $2y - 5 = 3$.

   (*iv*) The equation for the statement '5 less than thrice a number x is 7' is ......

   (*v*) ...... is a solution of the equation $4x + 9 = 5$.

   (*vi*) If $3x + 7 = 1$, then the value of $5x + 13$ is ....

   (*vii*) In natural numbers, $4x + 5 = -7$ has .... solution.

   (*viii*) In integers, $3x - 1 = 4$ has .... solution.

   (*ix*) $5x +$ .... $= 13$ has the solution -3.

   (*x*) If a number is increased by 15, it becomes 50. Then the number is ....

   (*xi*) If 63 exceed another number by 21, then the other number is ....

   (*xii*) If $x \in \mathbf{W}$, then the solution set of $x < 2$ is ....

2. State whether the following statements are true (T) or false (F):

   (*i*) We can add (or subtract) the same number or expression to both sides of an equation.

   (*ii*) We can divide both sides of an equation by the same non-zero number.

   (*iii*) $3x - 5 = 2(x + 3) + 7$ is a linear equation in one variable.

   (*iv*) The solution of the equation $3(x - 4) = 30$ is $x = 6$.

   (*v*) The solution of the equation $3x - 5 = 2$ is $x = \frac{7}{3}$.

   (*vi*) The solution of a linear equation in one variable is always an integer.

   (*vii*) $4x + 5 < 65$ is not an equation.

   (*viii*) $2x + 1 = 7$ and $3x - 5 = 4$ have the same solution.

   (*ix*) $\frac{9}{4}$ is a solution of the equation $5x - 1 = 8$.

   (*x*) If 5 is a solution of variable x in the equation $\frac{5x - 7}{2} = y$, then the value of y is 18.

   (*xi*) One-fourth of a number added to itself gives 10, can be represented as $\frac{x}{4} + 10 = x$.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 17):

3. Which of the following is not a linear equation in one variable?
(*a*) $3x - 1 = 7$ (*b*) $5y - 2 = 3(y + 2)$ (*c*) $2x - 3 = \frac{7}{2}$ (*d*) $7p + q = 3$
4. The solution of the equation $\frac{1}{3}(2y - 1) = 3$ is
(*a*) 5 (*b*) 3 (*c*) 2 (*d*) 1
5. $x = -1$ is a solution of the equation
(*a*) $x - 5 = 6$ (*b*) $2x + 5 = 7$ (*c*) $2(x - 2) + 6 = 0$ (*d*) $3x + 5 = 4$
6. If $3(3n - 10) = 2n + 5$, then the value of n is
(*a*) 12 (*b*) 5 (*c*) 3 (*d*) -5
7. -1 is not a solution of the equation
(*a*) $x + 1 = 0$ (*b*) $3x + 4 = 1$ (*c*) $5x + 7 = 2$ (*d*) $x - 1 = 2$
8. The value of p for which the expressions $p - 13$ and $2p + 1$ become equal is
(*a*) 0 (*b*) 14 (*c*) -14 (*d*) 5
9. The equation which cannot be solved in integers is
(*a*) $5x - 3 = -18$ (*b*) $3y - 5 = y - 1$ (*c*) $3p + 8 = 3 + p$ (*d*) $9z + 8 = 4z - 7$
10. The solution of which of the following equations is neither an integer nor a fraction?
(*a*) $2x + 5 = 1$ (*b*) $3x - 7 = 0$ (*c*) $5x - 7 = x + 1$ (*d*) $4x + 7 = x + 2$
11. If the sum of two consecutive even numbers is 54, then the smaller number is
(*a*) 25 (*b*) 26 (*c*) 27 (*d*) 28
12. If the sum of two consecutive odd numbers is 28, then the bigger number is
(*a*) 19 (*b*) 17 (*c*) 15 (*d*) 13
13. If 5 added to thrice an integer is -7, then the integer is
(*a*) -6 (*b*) -5 (*c*) -4 (*d*) 4
14. If the length of a rectangle is twice its breadth and its perimeter is 120 m, then its length is
(*a*) 20 m (*b*) 40 m (*c*) 60 m (*d*) 30 m
15. If the difference of two complementary angles is 10°, then the smaller angle is
(*a*) 40° (*b*) 50° (*c*) 45° (*d*) 85°
16. If the difference of two supplementary angles is 30°, then the larger angle is
(*a*) 60° (*b*) 75° (*c*) 90° (*d*) 105°
17. If $x \in \mathbf{W}$, the solution set of the inequation $-2 \le x < 3$ is
(*a*) $\{-2, -1, 0, 1, 2\}$ (*b*) $\{-1, 0, 1, 2, 3\}$ (*c*) $\{0, 1, 2, 3\}$ (*d*) $\{0, 1, 2\}$

Higher Order Thinking Skills (HOTS)

1. If $\frac{7m + 2}{3} = -11$, then find the value of $2m^3 + 10m^2 + 4m - 3$.
2. Two persons start moving from two points A and B in opposite directions towards each other. One person start moving from A at the speed of 4 km/h and meets the other person coming from B after 6 hours. If the distance between A and B is 42 km, find the speed of the other person.

3. There are some benches in a classroom. If 4 students sit on each bench then 3 benches remains empty and if 3 students sit on each bench then 3 students remain standing. Find the number of students in the class.

Summary

★ An (algebraic) equation is a statement that two expressions are equal. It may involve one or more than one unknowns (variables or literal numbers).

★ An equation containing only one variable (literal) with highest power 1 is called a linear equation in that variable.

★ A number which satisfies the given equation is called a solution or root of the equation.

★ The process of finding the particular value of the variable (literal) which makes both sides of the equation equal is called solving the equation.

★ An equation may be solved by using the following rules:

❑ The same number or expression may be added to both sides of an equation.

❑ The same number or expression may be subtracted from both sides of an equation.

❑ Both sides of an equation may be multiplied by the same non-zero number or expression.

❑ Both sides of an equation may be divided by the same non-zero number or expression.

❑ A term may be transposed from one side of the equation to the other side, but its sign will change.

★ Solution set of a linear equation can be represented on a number line.

★ While solving simple word (or applied) problems involving one unknown, we first have to write an equation corresponding to the given statement and then solve this equation to find the value of the unknown.

★ A linear inequality in the variable x can be written in any one of the forms :

(i) $ax + b < 0$ (ii) $ax + b \leq 0$ (iii) $ax + b > 0$ (iv) $ax + b \geq 0$

where a and b are rational numbers, $a \neq 0$.

★ The set from which the values of the variable (involved in the inequality) are choosen is called the **replacement set.**

★ A solution of an inequality is a number (choosen from the replacement set) which, when substituted for the variable, makes the inequality true.

★ The set of all solutions of an inequality is called its **solution set**.

★ Rules for solving inequalities are similar to those for solving equations. However, when we multiply or divide both sides of an inequality by a negative number, then the symbol of the inequality is reversed.

★ Solution set of an inequality can be represented on a number line.

Check Your Progress

1. Solve the following equations :

   (i) $2(x-5)+3(x-2)=8+7(x-4)$ (ii) $\frac{3(2x-5)}{4}-\frac{5(7-5x)}{6}=\frac{7x}{3}$.

2. A number exceeds its three-fifth by 22. Find the number.
3. When 9 is added to twice a number, the result is 3 more than thrice the number. Find the number.
4. The ten's digit of a two digit number is twice the unit's digit. The sum of the number and its unit's digit is 66. Find the number.

   [**Hint.** Let unit's digit be x, then ten's digit = $2x$.

   $\therefore$ The number = $2x \times 10 + x = 20x + x$

   According to the problem, $(20x + x) + x = 66$.]
5. A student bought some pens at ₹8 each and some pencils at ₹1.50 each. If the total number of pens and pencils purchased is 16 and their total cost is ₹50, how many pens did he buy?
6. Arvind is eight years older than his sister. In three years, he will be twice as old as his sister. How old are they now?
7. The angels of a triangle are in the ratio 1 : 2 : 3. Find their measure in degrees.
8. Solve the following inequations and represent their solution on a number line :

   (i) $\frac{2x-1}{3} \le 2\frac{1}{2}, x \in \mathbf{W}$ (ii) $-1 < \frac{2x}{3} + 1 \le 4, x \in \mathbf{I}$.