Summary of Chapter: Practical Geometry

1. Introduction

- This chapter builds on previous geometry concepts, including constructing circles, line segments, perpendicular bisectors, and angles.
- Focus areas:
  - Constructing a parallel line through a given point.
  - Constructing different types of triangles.

2. Constructing a Parallel Line

A line parallel to a given line can be drawn through a point not on the line using:

1. **Paper Folding Method**:
   - Draw the given line and mark a point outside it.
   - Fold the paper to create perpendicular creases and then align them to form the parallel line.
2. **Ruler and Compass Method**:
   - Draw a line and a point outside it.
   - Use a compass to draw arcs and construct corresponding equal angles to create the parallel line.

3. Construction of Triangles

A triangle can be uniquely constructed if specific measurements are given.

1. **When Three Sides Are Given (SSS)**:
   - Example: If sides PQ = 5.6 cm, QR = 4.7 cm, PR = 3.4 cm, the triangle is drawn by:
     - Drawing PQ.
     - Drawing arcs from P and Q for PR and QR, respectively.
     - Intersecting arcs form the third vertex.
2. **When Two Sides and an Included Angle Are Given (SAS)**:

- Example: AB = 3.2 cm, BC = 3.9 cm, and ∠B = 120°.
  - Draw AB.
  - Construct a 120° angle at B.
  - Use a compass to mark BC and complete the triangle.

3. **When Two Angles and One Side Are Given (ASA)**:
   - Example: BC = 6.3 cm, ∠B = 45°, ∠C = 60°.
     - Draw BC.
     - Construct given angles at B and C.
     - The intersection gives the third vertex.
4. **When Hypotenuse and a Side Are Given (RHS - Right-Angled Triangle)**:
   - Example: BC = 3.2 cm, hypotenuse AC = 5.8 cm, and ∠B = 90°.
     - Draw BC.
     - Construct a perpendicular at B.
     - Mark AC with a compass and complete the triangle.

4. Important Properties

- **Triangle Inequality Theorem**: The sum of any two sides must be greater than the third side.
- **Angle Sum Property**: The sum of angles in a triangle is always 180°.
- **Congruence Conditions**:
  - **SSS**: Three sides given.
  - **SAS**: Two sides and an included angle given.
  - **ASA**: Two angles and an included side given.
  - **RHS**: Hypotenuse and one side of a right triangle given.

5. Exercises & Application

- Constructing triangles with given conditions.
- Drawing parallel lines using different methods.
- Verifying if given measurements can form a triangle.

Steps to Construct Specific Angles Using Ruler and Compass

1. Constructing a 90° Angle

1. Draw a straight line and mark a point **A** on it.
2. Place the compass at **A** and draw an arc cutting the line at **B**.
3. Without changing the compass width, place the compass at **B** and draw another arc to intersect the first arc at **C**.
4. Place the compass at **C** and draw another arc to intersect the first arc at **D**.
5. Draw a line from **A** through **D**. This is a **90° angle**.

2. Constructing a 60° Angle

1. Draw a straight line and mark a point **A** on it.
2. Place the compass at **A** and draw an arc cutting the line at **B**.
3. Without changing the compass width, place the compass at **B** and draw an arc.
4. Mark the intersection of arcs as **C**.
5. Draw a line from **A** to **C**. This is a **60° angle**.

3. Constructing a 30° Angle

1. First, construct a **60° angle** using the steps above.
2. Bisect the **60° angle**:
   - Place the compass at **C** and draw an arc inside the angle.
   - Without changing the compass width, place it at **A** and draw another arc to intersect the first arc at **D**.
   - Draw a line from **A** through **D**.
3. The new angle is **30°**.

4. Constructing a 45° Angle

1. First, construct a **90° angle** using the steps above.
2. Bisect the **90° angle**:
   - Place the compass at **D** and draw an arc inside the angle.
   - Without changing the compass width, place it at **A** and draw another arc to intersect the first arc at **E**.
   - Draw a line from **A** through **E**.
3. The new angle is **45°**.

5. Constructing a 100° Angle

1. First, construct a **90° angle** using the steps above.
2. Then, construct a **10° angle** by bisecting a **20° angle**.
    - Construct a **60° angle**, then a **30° angle**, then bisect it to get **15°**.
    - Bisect the **15° angle** to get **7.5°**, then adjust to make **10°**.
3. Add **10°** to **90°** to get **100°**.

6. Constructing a 180° Angle

1. Draw a straight line.
2. Any straight line automatically forms a **180° angle** at any point on it.

Steps to Construct a Parallel Line Through a Given Point Using Ruler and Compass

Method: Using Alternate Interior Angles

1. **Draw the Given Line**:
    - Draw a straight line **L** and mark a point **P** outside it.
2. **Choose a Point on the Line**:
    - Pick any point **A** on **L** and join it to **P**.
3. **Draw an Arc from A**:
    - Place the compass at **A** and draw an arc that cuts **L** at **B** and **AP** at **C**.
4. **Copy the Arc at P**:
    - Without changing the compass width, place it at **P** and draw a similar arc cutting **AP** at **D**.
5. **Measure the Angle**:
    - Using the compass, measure the distance **BC** (between the arc and the line).
6. **Mark the Same Distance at P**:

- Place the compass at **D** and mark an arc intersecting the previous arc at **E**.

7. **Draw the Parallel Line**:
    - Draw a straight line passing through **P** and **E**. This is the required **parallel line** to **L**.