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EXPONENTS AN[D] POWE[RS]

INTRODUCTION

We know that the radius of the earth is approximately 6366000 m, the distance between the sun and the earth is approximately 149,600,000,000 m and the speed of light in vacuum is approximately 299,800,000 m/s.

Such large numbers are difficult to read, understand and compare. To make these numbers easy to rea[d], understand and compare, we use exponents. In this chapter, we shall learn about exponents and the[ir] and also learn how to use them.

EXPONENTS

Repeated multiplication by the same number is very common.

For example:

(i) $2 \times 2 \times 2 \times 2 \times 2$. It is usually written as 2^5.

Thus, $2 \times 2 \times 2 \times 2 \times 2 = 2^5$, where the repeated factor 2 is called the base 5 is called the exponent or index. 2^5 is read as '2 raised to the power 5'

RAMAJUJAN

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(1887 – 1920)

Srinivas Ramanujan was born in erode, a small village about 400 km southwest of Madras (Chennai). He was a remarkable genius in Mathematics. Due to proverty, he could not receive university education in India and was invited to Cambridge by Prof. H.G. Hardy in 1914 where he worked in collaboration with him. He was the first Indian to be selected for the fellowship of Royal Society of England in 1918, and was awarded the distinguished title of F.R.S. at the young age of 31. His researches in the theory of numbers brought worldwise fame to him.

Using one of his identities, mathematicians have been able to calculate the value of π correct to millions of places of decimal.

He was emotionally attached to natural numbers.

His birth centenary was celebrated in 1987.

In his last days, he was sick. Prof. Hardy paid a visit to him. He told Ramanujan that he rode a taxi to residence with a very unlucky number, 1729.

Ramanujan's face lit up with a smile and he said that it was not an unlucky number at all, but a very interest[ing] number. It is the smallest number that can be written as the sum of cubes of two natural numbers in two differ[ent] ways :

$$1729 = 10^3 + 9^3 = 12^3 + 1^3$$

The number 1729 is also the product of 3 prime numbers :

$$1729 = 7 \times 13 \times 19.$$

to the power 5' or simply '2 power 5'. Note that $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ i.e. $32 = 2^5$. 2^5 is called the **exponential form** of 32.

We have:

$$32 = \boxed{2}^{\boxed{5}}$$

5 ← exponent or index

2 ← base

(ii) $3 \times 3 \times 3 \times 3 = 3^4$, where 3 is the base and 4 is the index.

(iii) $(-4) \times (-4) \times (-4) = (-4)^3$, where base = −4 and index = 3.

(iv) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^4$, where base = $\frac{2}{3}$ and index = 4.

This leads to:

If a is any rational number and n is a natural number, then

$a^n = a \times a \times a....$ *multiplied n times,*

where a is called the ***base*** *and n is called the* ***exponent*** *or* ***index*** *and a^n is the* ***exponential*** *form.*

a^n is read as 'a raised to the power n' or 'a to the power n' or simply 'a power n.'

In particular, $a^1 = a$.

For example:

$10^4 = 10 \times 10 \times 10 \times 10$ i.e. $10^4 = 10000$;

here base = 10, exponent (or index) = 4 and 10^4 is the exponential form of the number 10000.

Some powers have special names

If the power of a number is 2 then it is named as square and if the power of a number is 3 then it is named as cube. Thus,

$9 = 3 \times 3 = 3^2$ [read as 3 squared or 3 raised to the power 2]

$100 = 10 \times 10 = 10^2$ [read as 10 squared or 10 raised to power 2]

$125 = 5 \times 5 \times 5 = 5^3$ [read as 5 cubed or 5 raised to power 3]

$1000 = 10 \times 10 \times 10 = 10^3$ [read as 10 cubed or 10 raised to power 3]

Note

$a \times b$ is written as ab, read as a into b.

$a \times a \times a \times b \times b$ is written as a^3b^2, read as a cubed into b squared etc.

Example 1. *Write the base and exponent for the following exponential form:*

(i) 5^3 (ii) 7^5 (iii) $(-3)^6$.

Solution. (i) For 5^3, base = 5 and exponent = 3.

(ii) For 7^5, base = 7 and exponent = 5.

(iii) For $(-3)^6$, base = −3 and exponent = 6.

Example 2. *Find the value of the following:*

(i) 7^4 (ii) $(-3)^5$ (iii) $\left(\frac{2}{3}\right)^3$ (iv) $\left(-\frac{4}{5}\right)^4$

Solution. (i) $7^4 = 7 \times 7 \times 7 \times 7 = 2401$.

(ii) $(-3)^5 = (-3) \times (-3) \times (-3) \times (-3) \times (-3) = -243$.

(iii) $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{8}{27}$.

(iv) $\left(-\frac{4}{5}\right)^4 = \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) = \frac{256}{625}$.

Example 3. *Simplify the following:*

(i) 3×10^3 (ii) $2^6 \times 7$ (iii) $2^3 \times (-3)^2$

Solution. (i) $3 \times 10^3 = 3 \times 10 \times 10 \times 10 = 3000$.

(ii) $2^6 \times 7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 = 448$.

(iii) $2^3 \times (-3)^2 = 2 \times 2 \times 2 \times (-3) \times (-3) = 8 \times 9 = 72$.

Example 4. *Which is the greater number 6^3 or 3^6?*

Solution. $6^3 = 6 \times 6 \times 6 = 216$

and $3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 27 \times 27 = 729$.

Since $729 > 216$, therefore, $3^6 > 6^3$.

Hence, the greater number is 3^6.

Powers of −1

$(-1)^2 = (-1) \times (-1) = 1$,

$(-1)^3 = (-1) \times (-1) \times (-1) = -1$,

$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1$,

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$ and so on.

Thus, we have

$(-1)^{\text{odd natural number}} = -1$ and $(-1)^{\text{even natural number}} = 1$.

Example 5. *Evaluate the following:*

(i) $(-1)^7$ (ii) $(-1)^{15}$ (iii) $(-1)^{100}$

Solution. (i) As 7 is an odd natural number, $(-1)^7 = -1$.

(ii) As 15 is an odd natural number, $(-1)^{15} = -1$.

(iii) As 100 is an even natural number, $(-1)^{100} = 1$.

Example 6. *Write the following numbers as powers of 2:*

(i) 2 (ii) 32 (iii) 512

Solution. (i) $2 = 2^1$

(ii) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

(iii) $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$.

Example 7. *Write the following numbers as powers of −2:*

(i) −2 (ii) −8 (iii) 16 (iv) −32

Solution. (i) $-2 = (-2)^1$

(ii) $-8 = (-2) \times (-2) \times (-2) = (-2)^3$

(iii) $16 = (-2) \times (-2) \times (-2) \times (-2) = (-2)^4$.

(iv) $-32 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = (-2)^5$.

Example 8. *Express the following numbers in exponential form:*

(*i*) 343 (*ii*) 729 (*iii*) 15625

Solution. (*i*) $343 = 7 \times 7 \times 7 = 7^3$

(*ii*) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

(*iii*) $15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

Example 9. *Find the value of x in each of the following:*

(*i*) $3^x = 243$ (*ii*) $(-5)^x = -125$ (*iii*) $\left(-\frac{2}{3}\right)^x = \frac{16}{81}$.

Solution. (*i*) $3^x = 243 \Rightarrow 3^x = 3 \times 3 \times 3 \times 3 \times 3$

$\Rightarrow 3^x = 3^5 \Rightarrow x = 5.$

(*ii*) $(-5)^x = -125 \Rightarrow (-5)^x = (-5) \times (-5) \times (-5)$

$\Rightarrow (-5)^x = (-5)^3 \Rightarrow x = 3.$

(*iii*) $\left(-\frac{2}{3}\right)^x = \frac{16}{81} \Rightarrow \left(-\frac{2}{3}\right)^x = \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right)$

$\Rightarrow \left(-\frac{2}{3}\right)^x = \left(-\frac{2}{3}\right)^4 \Rightarrow x = 4.$

Example 10. *Write the prime factorisation of the following numbers in the exponential form:*

(*i*) 48 (*ii*) 420.

Solution. (*i*) $48 = 2 \times 2 \times 2 \times 2 \times 3$

$= 2^4 \times 3^1.$

2	48
2	24
2	12
2	6
	3

(*ii*) $420 = 2 \times 2 \times 3 \times 5 \times 7$

$= 2^2 \times 3^1 \times 5^1 \times 7^1.$

2	420
2	210
3	105
5	35
	7

Exercise 4.1

1. Fill in the blanks:

(*i*) In the expression 3^7, base = 3. and exponent = ..7

(*ii*) In the expression $(-7)^5$, base = -7. and exponent = 5....

(*iii*) In the expression $\left(\frac{2}{5}\right)^{11}$, base = $\frac{2}{5}$ and exponent = 11...

(*iv*) If base is 6 and exponent is 8, then exponential form = 6^8.

2. Find the value of the following:

(*i*) 2^6 (*ii*) 5^5 (*iii*) $(-6)^4$ (*iv*) $\left(\frac{2}{3}\right)^4$

(*v*) $\left(-\frac{2}{3}\right)^5$ (*vi*) $(-2)^9$.

3. Express the following in the exponential form:
   (i) $6 \times 6 \times 6 \times 6 \times 6$ 6^5
   (ii) $t \times t \times t$ t^3
   (iii) $2 \times 2 \times a \times a \times a \times a$ $2^2 \times a^4$
   (iv) $a \times a \times a \times c \times c \times c \times c \times d$. $a^3 \times c^4 \times d$
4. Simplify the following:
   (i) 7×10^3 (ii) $2^5 \times 9$ (iii) $3^3 \times 10^4$.
5. Simplify the following:
   (i) $(-3) \times (-2)^3$ (ii) $(-3)^2 \times (-5)^2$ (iii) $(-2)^3 \times (-10)^4$
   (iv) $(-1)^9$ (v) $25^2 \times (-1)^{31}$ (vi) $4^2 \times 3^3 \times (-1)^{122}$.
6. Identify the greater number in each of the following:
   (i) 4^3 or 3^4 (ii) 7^3 or 3^7 (iii) 4^5 or 5^4 (iv) 2^{10} or 10^2.
7. Write the following numbers as powers of 2:
   (i) 8 (ii) 128 (iii) 1024.
8. To what power (−2) should be raised to get 16?
9. Write the following numbers as powers of (−3):
   (i) 9 (ii) −27 (iii) 81.
10. Find the value of x in each of the following:
   (i) $7^x = 343$ (ii) $3^x = 729$ (iii) $(-8)^x = -512$
   (iv) $(-4)^x = -1024$ (v) $\left(\frac{2}{5}\right)^x = \frac{32}{3125}$ (vi) $\left(-\frac{3}{4}\right)^x = -\frac{243}{1024}$.

H.W.

11. Write the prime factorization of the following numbers in the exponential form:
   (i) 72 (ii) 360 (iii) 405 (iv) 540
   (v) 2280 (vi) 3600 (vii) 4725 (viii) 8400.

LAWS OF EXPONENTS

We can multiply and divide rational numbers expressed in exponential form.

Multiplying powers with same base

(i) Let us find $2^4 \times 2^3$.

$2^4 \times 2^3 = (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$.

Note that $2^4 \times 2^3 = 2^{4+3} = 2^7$.

(ii) Let us find $(-3)^2 \times (-3)^3$.

$(-3)^2 \times (-3)^3 = [(-3) \times (-3)] \times [(-3) \times (-3) \times (-3)]$
$= (-3) \times (-3) \times (-3) \times (-3) \times (-3) = (-3)^5$.

Note that $(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5$.

In fact, this is true in general. We have:

Law 1. *If a is any rational number and m, n are natural numbers, then* $a^m \times a^n = a^{m+n}$.

Dividing powers with same (non-zero) base

Let us find $5^7 \div 5^4$

$$5^7 \div 5^4 = \frac{5^7}{5^4} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 5 \times 5 \times 5 = 5^3.$$

Note that $5^7 \div 5^4 = \frac{5^7}{5^4} = 5^{7-4} = 5^3$.

In fact, this is true in general. We have:

Law 2. *If a is any (non-zero) rational number and m, n are natural numbers such that $m > n$, then $a^m \div a^n = a^{m-n}$ or $\frac{a^m}{a^n} = a^{m-n}$.*

Zero exponent

Let us find $\frac{3^4}{3^4}$.

$$\frac{3^4}{3^4} = \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = \frac{81}{81} = 1.$$

Also $\frac{3^4}{3^4} = 3^{4-4} = 3^0$.

Note that we have calculated $\frac{3^4}{3^4}$ in two different ways, so answers must be same. It follows that $3^0 = 1$.

In fact, this is true in general. We have:

Law 3. *If a is any (non-zero) rational number, then $a^0 = 1$.*

Taking power of a power

Let us find $(2^3)^2$.

$(2^3)^2 = 2^3 \times 2^3 = 2^{3+3}$ (using law 1)

$= 2^6 = 2^{3 \times 2}$

Thus, $(2^3)^2 = 2^{3 \times 2}$.

In fact, this is true in general. We have:

Law 4. *If a is any rational number and m, n are natural numbers, then $(a^m)^n = a^{m \times n}$.*

Multiplying powers with same exponent

Let us find $3^4 \times 5^4$.

$3^4 \times 5^4 = (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5)$

$= (3 \times 5) \times (3 \times 5) \times (3 \times 5) \times (3 \times 5)$

$= (3 \times 5)^4$.

In fact, this is true in general. We have:

Law 5. *If a, b are any rational numbers and n is a natural number, then $a^n \times b^n = (ab)^n$.*

Dividing powers with same exponent

Let us find $\frac{2^5}{7^5}$.

$$\frac{2^5}{7^5} = \frac{2 \times 2 \times 2 \times 2 \times 2}{7 \times 7 \times 7 \times 7 \times 7} = \frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^5.$$

In fact, this is true in general. We have:

Law 6. *If a, b (≠ 0) are any rational numbers and n is a natural number, then* $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ *or* $a^n \div b^n = \left(\frac{a}{b}\right)^n$.

Negative exponent

For any (non-zero) rational number and natural number n, we have

$\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$. Thus, $a^{-n} = \frac{1}{a^n}$.

Law 7. *If a is any (non-zero) rational number and n is any natural number, then* $a^{-n} = \frac{1}{a^n}$.

In particular, $a^{-1} = \frac{1}{a}$.

Example 1. *Simplify and write in the exponential form:*

(i) $4^3 \times 4^5$ (ii) $(-5)^3 \times (-5)^4$ (iii) $(-4)^{100} \times (-4)^{20}$

(iv) $9^{11} \div 9^7$ (v) $(-7)^{13} \div (-7)^5$ (vi) $\left(\frac{3}{5}\right)^8 \div \left(\frac{3}{5}\right)^5$.

Solution. (i) $4^3 \times 4^5 = 4^{3+5} = 4^8$ (using law 1)

(ii) $(-5)^3 \times (-5)^4 = (-5)^{3+4} = (-5)^7$

(iii) $(-4)^{100} \times (-4)^{20} = (-4)^{100+20} = (-4)^{120}$

(iv) $9^{11} \div 9^7 = 9^{11-7} = 9^4$ (using law 2)

(v) $(-7)^{13} \div (-7)^5 = (-7)^{13-5} = (-7)^8$

(vi) $\left(\frac{3}{5}\right)^8 \div \left(\frac{3}{5}\right)^5 = \left(\frac{3}{5}\right)^{8-5} = \left(\frac{3}{5}\right)^3$.

Example 2. *Simplify and write in the exponential form:*

(i) $3^5 \times 3^6 \times 3^{11}$ (ii) $(5^{20} \div 5^{11}) \times 5^4$

Solution. (i) $3^5 \times 3^6 \times 3^{11} = 3^{5+6+11} = 3^{22}$

(ii) $(5^{20} \div 5^{11}) \times 5^4 = 5^{20-11} \times 5^4 = 5^9 \times 5^4$

$= 5^{9+4} = 5^{13}$.

Example 3. *Simplify and write in the exponential form:*

(i) $(6^2)^4$ (ii) $((-3)^8)^5$ (iii) $\left(\left(\frac{4}{5}\right)^3\right)^7$.

Solution. (i) $(6^2)^4 = 6^{2\times 4} = 6^8$ (using law 3)

(ii) $((-3)^8)^5 = (-3)^{8\times 5} = (-3)^{40}$

(iii) $\left(\left(\frac{4}{5}\right)^3\right)^7 = \left(\frac{4}{5}\right)^{3\times 7} = \left(\frac{4}{5}\right)^{21}$.

Example 4. *Simplify and write in the exponential form:*

(*i*) $3^6 \times 7^6$ (*ii*) $\dfrac{5^8}{7^8}$ (*iii*) $(-5)^5 \times 8^5$.

Solution. (*i*) $3^6 \times 7^6 = (3 \times 7)^6$ (using law 5)

$= (21)^6$

(*ii*) $\dfrac{5^8}{7^8} = \left(\dfrac{5}{7}\right)^8$ (using law 6)

(*iii*) $(-5)^5 \times 8^5 = ((-5) \times 8)^5 = (-40)^5$.

Example 5. *Simplify and express each of the following in the exponential form:*

(*i*) $[(5^2)^3 \times 5^4] \div 5^7$ (*ii*) $125^4 \div 5^3$ (*iii*) $[(2^2)^3 \times 3^6] \times 5^6$

Solution. (*i*) $[(5^2)^3 \times 5^4] \div 5^7 = (5^{2 \times 3} \times 5^4) \div 5^7 = (5^6 \times 5^4) \div 5^7$

$= 5^{6+4} \div 5^7 = 5^{10} \div 5^7 = 5^{10-7} = 5^3$.

(*ii*) As $125 = 5 \times 5 \times 5 = 5^3$,

$\therefore$ $125^4 \div 5^3 = (5^3)^4 \div 5^3 = 5^{3 \times 4} \div 5^3 = 5^{12} \div 5^3$

$= 5^{12-3} = 5^9$.

(*iii*) $[(2^2)^3 \times 3^6] \times 5^6 = (2^{2 \times 3} \times 3^6) \times 5^6 = (2^6 \times 3^6) \times 5^6$

$= (2 \times 3)^6 \times 5^6 = 6^6 \times 5^6 = (6 \times 5)^6 = 30^6$.

Example 6. *Simplify and express each of the following in the exponential form:*

(*i*) $\dfrac{2^3 \times 3^4 \times 4}{3 \times 32}$ (*ii*) $\dfrac{2^8 \times a^8b^3}{4^3 \times a^5b^2}$ (*iii*) $(3^0 + 2^0) \times 5^0$.

Solution. (*i*) As $4 = 2 \times 2 = 2^2$ and $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$,

$$\therefore \quad \frac{2^3 \times 3^4 \times 4}{3 \times 32} = \frac{2^3 \times 3^4 \times 2^2}{3^1 \times 2^5} = \frac{2^{3+2} \times 3^4}{3^1 \times 2^5} = \frac{2^5 \times 3^4}{3^1 \times 2^5}$$

$= 2^{5-5} \times 3^{4-1} = 2^0 \times 3^3 = 1 \times 3^3 = 3^3$.

(*ii*) As $4 = 2 \times 2 = 2^2$,

$$\therefore \quad \frac{2^8 \times a^8b^3}{(4)^3 \times a^5b^2} = \frac{2^8 \times a^8b^3}{(2^2)^3 \times a^5b^2} = \frac{2^8 \times a^8b^3}{2^6 \times a^5b^2} = 2^{8-6} \times a^{8-5} \times b^{3-2}$$

$= 2^2 \times a^3b^1 = 2^2a^3b^1$.

(*iii*) $(3^0 + 2^0) \times 5^0 = (1 + 1) \times 1 = 2 \times 1 = 2 = 2^1$.

Example 7. *Express each of the following rational numbers in the exponential form:*

(*i*) $\dfrac{64}{343}$ (*ii*) $\dfrac{-27}{125}$ (*iii*) $-\dfrac{1}{243}$.

Solution. (*i*) $\dfrac{64}{343} = \dfrac{4^3}{7^3} = \left(\dfrac{4}{7}\right)^3$ (using law 6)

(*ii*) $\dfrac{-27}{243} = \dfrac{(-3)^3}{5^3} = \left(\dfrac{-3}{5}\right)^3$

(*iii*) $-\dfrac{1}{243} = \dfrac{-1}{243} = \dfrac{(-1)^5}{3^5} = \left(\dfrac{-1}{3}\right)^5$.

Example 8. *Simplify the following:*

(i) $(-4m)^3$ (ii) $(-5)^{-4}$ (iii) $\left(\frac{2}{3}\right)^{-3}$ (iv) $\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27}$.

Solution. (i) $(-4m)^3 = ((-4) \times m)^3 = (-4)^3 \times m^3$

$= (-4) \times (-4) \times (-4) \times m^3$

$= -64 \times m^3 = -64m^3$.

(ii) $(-5)^{-4} = \frac{1}{(-5)^4}$ $\left(\because a^{-n} = \frac{1}{a^n}, a \neq 0\right)$

$= \frac{1}{(-5)\times(-5)\times(-5)\times(-5)} = \frac{1}{625}$.

(iii) $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3} = \frac{1}{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}} = \frac{1}{\frac{8}{9}} = 1\times\frac{9}{8} = \frac{9}{8}$.

(iv) $\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2^2\times 3)^4 \times (3^2)^3 \times 2^2}{(2\times 3)^3 \times (2^3)^2 \times 3^3} = \frac{(2^2)^4 \times 3^4 \times 3^6 \times 2^2}{2^3 \times 3^3 \times 2^6 \times 3^3}$

$= \frac{2^8 \times 2^2 \times 3^4 \times 3^6}{2^3 \times 2^6 \times 3^3 \times 3^3} = \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6}$

$= 2^{10-9} \times 3^{10-6} = 2^1 \times 3^4 = 2 \times 81 = 162$.

Example 9. *Simplify the following:*

(i) $\left(\left(\frac{2}{3}\right)^3 \times \left(\frac{-3}{4}\right)^2\right) + \left(\frac{-2}{5}\right)^3$ (ii) $\left[\left(\frac{-2}{3}\right)^3 + \frac{4}{9}\right] + \left(\frac{5}{3}\right)^3$.

Solution. (i) $\left(\left(\frac{2}{3}\right)^3 \times \left(\frac{-3}{4}\right)^2\right) + \left(\frac{-2}{5}\right)^3 = \left(\frac{2^3}{3^3} \times \frac{(-1)^2 \times 3^2}{4^2}\right) + \frac{(-1)^3 \times 2^3}{5^3}$

$= \left(\frac{8}{27} \times \frac{9}{16}\right) + \frac{-8}{125} = \frac{1}{6} + \frac{-8}{125} = \frac{1}{6} \times \left(\frac{125}{-8}\right) = -\frac{125}{48} = -2\frac{29}{48}$.

(ii) $\left[\left(\frac{-2}{3}\right)^3 + \frac{4}{9}\right] + \left(\frac{5}{3}\right)^3 = \left[\frac{(-1)^3 \times 2^3}{3^3} + \frac{4}{9}\right] + \frac{5^3}{3^3}$

$= \left(\frac{-8}{27} + \frac{4}{9}\right) + \frac{125}{27} = \left(\frac{-8+12}{27}\right) + \frac{125}{27}$

$= \frac{4}{27} + \frac{125}{27} = \frac{4}{27} \times \frac{27}{125} = \frac{4}{125}$.

Example 10. Simplify the following:

(i) $\left[\left\{\left(\frac{-2}{3}\right)^2\right\}^{-1}\right]^{-2}$ (ii) $(6^{-1} - 8^{-1}) + (2^{-1} - 3^{-1})^{-1}$.

Solution. (i) $\left[\left\{\left(\frac{-2}{3}\right)^2\right\}^{-1}\right]^{-2} = \left[\left(\frac{-2}{3}\right)^{2\times(-1)}\right]^{-2} = \left[\left(\frac{-2}{3}\right)^{-2}\right]^{-2}$

$= \left(\frac{-2}{3}\right)^{(-2)\times(-2)} = \left(\frac{-2}{3}\right)^4 = \frac{(-1)^4 \times 2^4}{3^4} = \frac{1\times 16}{81} = \frac{16}{81}$.

(ii) $(6^{-1} - 8^{-1})^{-1} \div (2^{-1} - 3^{-1})^{-1} = \left(\frac{1}{6} - \frac{1}{8}\right)^{-1} \div \left(\frac{1}{2} - \frac{1}{3}\right)^{-1}$

$$= \left(\frac{4-3}{24}\right)^{-1} \div \left(\frac{3-2}{6}\right)^{-1} = \left(\frac{1}{24}\right)^{-1} \div \left(\frac{1}{6}\right)^{-1}$$

$$= \frac{24}{1} \div \frac{6}{1} = \frac{24}{1} \times \frac{1}{6} = 4.$$

Example 11. *Simplify and write the following in the exponential form:*

(i) $10^2 \times 7^0 + 3^3 \times 2^2$ (ii) $4^3 \times 5^0 + (-3)^4 - 7^0$

(iii) $2^3 \times 3^2 + (-11)^2 + 2^{-5} \div 2^{-8} - \left(-\frac{2}{5}\right)^0$.

Solution. (i) $10^2 \times 7^0 + 3^3 \times 2^2 = 10 \times 10 \times 1 + 3 \times 3 \times 3 \times 2 \times 2$

$$= 100 + 108 = 208$$

$$= 2 \times 2 \times 2 \times 2 \times 13 = 2^4 \times 13^1$$

(ii) $4^3 \times 5^0 + (-3)^4 - 7^0 = 4 \times 4 \times 4 \times 1 + (-3) \times (-3) \times (-3) \times (-3) - 1$

$$= 64 + 81 - 1 = 144$$

$$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2.$$

(iii) $2^3 \times 3^2 + (-11)^2 + 2^{-5} \div 2^{-8} - \left(-\frac{2}{5}\right)^0$

$$= 8 \times 9 + 121 + 2^{-5-(-8)} - 1$$

$$= 72 + 121 + 2^3 - 1 = 72 + 121 + 8 - 1 = 200$$

$$= 2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2.$$

Example 12. *By what number should we multiply $(-6)^3$ so that the product is 3^5?*

Solution. Let the required number be x, then

$$(-6)^3 \times x = 3^5 \qquad \Rightarrow (-1)^3 \times 6^3 \times x = 3^5$$

$$\Rightarrow \quad -2^3 \times 3^3 \times x = 3^5 \qquad \Rightarrow x = \frac{3^5}{-2^3 \times 3^3}$$

$$\Rightarrow \quad x = -\frac{3^{5-3}}{2^3} = -\frac{3^2}{2^3} = -\frac{9}{8} = -1\frac{1}{8}.$$

Example 13. *Find m so that* $\left(\frac{2}{9}\right)^3 \times \left(\left(\frac{2}{9}\right)^2\right)^4 = \left(\frac{2}{9}\right)^{2m-1}$.

Solution. Given $\left(\frac{2}{9}\right)^3 \times \left(\left(\frac{2}{9}\right)^2\right)^4 = \left(\frac{2}{9}\right)^{2m-1}$

$$\Rightarrow \quad \left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{2\times 4} = \left(\frac{2}{9}\right)^{2m-1}$$

$$\Rightarrow \quad \left(\frac{2}{9}\right)^{2m-1} = \left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^8 = \left(\frac{2}{9}\right)^{3+8} = \left(\frac{2}{9}\right)^{11}$$

$$\Rightarrow \quad 2m - 1 = 11 \quad \Rightarrow \quad 2m = 12 \quad \Rightarrow m = 6.$$

Exercise 4.2

1. Using laws of exponents, simplify and write the following in the exponential form:

   (i) $2^7 \times 2^4$ (ii) $p^5 \times p^3$ (iii) $(-7)^5 \times (-7)^{11}$

   (iv) $\left(\frac{3}{5}\right)^6 \div \left(\frac{3}{5}\right)^2$ (v) $(-6)^7 \div (-6)^3$ (vi) $7^x \times 7^3$.

2. Simplify and write the following in the exponential form:

   H.W (i) $5^3 \times 5^7 \times 5^{12}$ (ii) $a^5 \times a^3 \times a^7$ (iii) $(7^{12} \times 7^3) \div 7^4$

3. Simplify and write the following in the exponential form:

   (i) $(2^2)^{100}$ (ii) $((-7)^6)^5$ (iii) $(3^2)^5 \times (3^4)^7$

4. Simplify and write in the exponential form:

   (i) $\frac{a^3 \times a^5}{(a^3)^2}$ (ii) $(2^3)^4 \div 2^5$ (iii) $[(6^2)^3 \div 6^3] \times 6^5$.

H.W

5. Simplify and write in the exponential form:

   (i) $5^4 \times 8^4$ (ii) $(-3)^6 \times (-5)^6$ (iii) $\left(\frac{3}{10}\right)^5 \times \left(\frac{2}{15}\right)^5$.

6. Simplify and express each of the following in the exponential form:

   (i) $\frac{2^4 \times 2 \times 7^3 \times 7^6}{2^3 \times 7^4}$ H.W (ii) $\frac{(3^2)^3 \times (-2)^5}{(-2)^3}$ (iii) $\frac{2^8 \times a^5}{4^3 \times a^3}$

   H.W (iv) $\frac{3 \times 7^2 \times 11^8}{21 \times 11^3}$ H.W (v) $(2^0 + 3^0)\ 4^0$ H.W (vi) $3^0 \times 4^0 \times 5^0$.

7. Express each of the following rational numbers in the exponential form:

   (i) $\frac{25}{64}$ (ii) $-\frac{125}{216}$ (iii) $-\frac{343}{729}$.

H.W

8. Simplify the following:

   (i) $\frac{(2^5)^2 \times 7^3}{8^3 \times 7}$ (ii) $\frac{25 \times 5^2 \times t^8}{10^3 \times t^4}$ (iii) $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$ (iv) $\left(-\frac{3}{5}\right)^{-3}$.

9. Simplify the following :

   (i) $\left(\frac{-1}{2}\right)^5 \times 2^6 \times \left(\frac{3}{4}\right)^3$ (ii) $\left[\left(\frac{-3}{4}\right)^3 - \left(\frac{-5}{2}\right)^3\right] \times \left(-\frac{2}{3}\right)^4$.

10. Simplify the following :

    (i) $\left(\frac{3}{2}\right)^{-1} + \left(\frac{-2}{5}\right)^{-1}$ (ii) $\left[\left\{\left(\frac{-1}{4}\right)^2\right\}^{-1}\right]^{-2}$.

11. Simplify : $\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{5}\right)^{-2} - \left(\frac{1}{6}\right)^{-2}$.

H.W

12. Express each of the following as a product of prime factors in the exponential form:

    (i) 108×192 (ii) 729×64 H (iii) 384×147

13. Simplify and write the following in the exponential form:

(*i*) $3^3 \times 2^2 + 2^2 \times 5^0$ (*ii*) $9^2 + 11^2 - 2^2 \times 3 \times 17^0$.

14. (*i*) By what number should we multiply 3^4 so that the product is 3^7?

(*ii*) By what number should we multiply $(-6)^{-1}$ so that the product is 10^{-1}?

15. If $\left(\frac{12}{13}\right)^4 \times \left(\frac{13}{12}\right)^{-8} = \left(\frac{12}{13}\right)^{2x}$, then find the value of x.

16. If $(-3)^{x-1} = -243$, then find the value of $(-7)^{x-6}$.

USES OF EXPONENTS

In the beginning of the chapter, we said that large numbers can be conveniently expressed using exponents. Let us see how this can be done.

1. The distance between the sun and the earth is 149,600,000,000 m.
2. The speed of the light in vacuum is 299,800,000 metre per second.
3. The mass of the earth is 5,976,000,000,000,000,000,000,000 kg.

These large numbers are not convenient to read and write. To make it convenient, we use powers of 10.

Look at the following:

$76 = 7.6 \times 10 = 7.6 \times 10^1$

$763 = 7.63 \times 100 = 7.63 \times 10^2$

$7630 = 7.63 \times 1000 = 7.63 \times 10^3$

$76300 = 7.63 \times 10000 = 7.63 \times 10^4$

$763000 = 7.63 \times 100000 = 7.63 \times 10^5$ and so on.

Notice we have expressed all these numbers in a special form. Any number can be expressed as a decimal number between 1 and 10 (including 1 but excluding 10) multiplied by a power of 10. Such a form of a number is called **standard form.**

Scientific notation

Scientific notation is a way of writing numbers that accommodates values too large to be conveniently written in standard form.

Scientific notation has a number of useful properties and is often favoured by scientists, astronomers, mathematicians and engineers who work with such numbers.

In scientific notation all numbers are written as $k \times 10^n$, where k is decimal number such that $1 \le k < 10$ and n is a whole number.

The decimal number k is called **significand.** Scientific notation is also known as **standard form.**

Look at the following table:

Ordinary decimal notation	*Scientific notation*
500	5×10^2
47,000	4.7×10^4
9,830,000,000	9.83×10^9

Converting from decimal notation to scientific notation (or standard form)

Proceed as under:

(i) Move the decimal point to the left till you get just one digit to the left decimal point.

(ii) Write the given number as the product of the number so obtained and 10^n where n is the number of places the decimal point has been moved to the left.

Converting from scientific notation (or standard form) to decimal notation

Proceed as under:

Take the significand and move the decimal to the right by the number of places indicated by the exponent of 10^n adding trailing zeros as necessary.

Example 1. *Write the following numbers in the standard form:*

(i) 763.4 (ii) 83,500 (iii) 573,000 (iv) 9,812,700,000

Solution. (i) $763.4 = 7.634 \times 10^2$

(ii) $83{,}500 = 8.3500 \times 10^4 = 8.35 \times 10^4$

(iii) $573{,}000 = 5.73000 \times 10^5 = 5.73 \times 10^5$

(iv) $9{,}812{,}700{,}000 = 9.812700000 \times 10^9 = 9.8127 \times 10^9$.

Example 2. *Write the following numbers in usual decimal notation:*

(i) 5.37×10^4 (ii) 7.501×10^7 (iii) 2.3049×10^{11}

Solution. (i) $5.37 \times 10^4 = 53700$

(ii) $7.501 \times 10^7 = 75010000$

(iii) $2.3049 \times 10^{11} = 230490000000$.

Example 3. *Express the numbers appearing in the following statements in scientific notation (or standard form):*

(i) *The radius of the earth is 6366000 metres.*

(ii) *The distance between the sun and the earth is 149,600,000,000 m.*

(iii) *The speed of the light in vacuum is 299,800,000 m/sec.*

(iv) *The mass of the earth is 5,976,000,000,000,000,000,000,000 kg.*

Solution. (i) The radius of earth = 6366000 m = 6.366×10^6 m

(ii) The distance between the sun and the earth = 149,600,000,000 m = 1.496×10^{11} m

(iii) The speed of light in vacuum = 299,800,000 m/sec = 2.998×10^8 m/sec

(iv) The mass of earth = 5,976,000,000,000,000,000,000,000 kg = 5.976×10^{24} kg.

Comparing numbers in standard form

To compare numbers which are in standard form, proceed as under:

(i) The number with the greater power of 10 is greater.

(ii) If the powers of 10 are equal in both numbers, then compare their significands. The number with greater significand is greater.

Example 4. *Compare the following numbers:*

(i) 2.7×10^{12}; 1.5×10^8 (ii) 3.547×10^9; 6.02×10^9

Solution. (i) The given numbers are 2.7×10^{12} and 1.5×10^8. Note that both the numbers are in standard form.

Since the power of 10 in 2.7×10^{12} is greater than the power of 10 in 1.5×10^8,

$\therefore \quad 2.7 \times 10^{12} > 1.5 \times 10^8.$

(*ii*) The given numbers are 3.547×10^9 and 6.02×10^9. Note that both the numbers are in standard form. Also we note that both the numbers have equal power of 10. Therefore, we compare their significands.

The significand in 3.547×10^9 is 3.547 and the significand in $6.02 \times 10^9 = 6.02$.

As $6.02 > 3.547$, so $6.02 \times 10^9 > 3.547 \times 10^9$.

Numbers in expanded form

Consider the expansion of the number 753015, we know that

$$753015 = 7 \times 100000 + 5 \times 10000 + 3 \times 1000 + 0 \times 100 + 1 \times 10 + 5 \times 1$$

Using powers of 10 in the exponent, we can write it as

$$\begin{aligned} 753015 &= 7 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 5 \times 10^0 \\ &= 7 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 1 \times 10^1 + 5 \times 10^0 \end{aligned}$$

In fact, the expansion of every number can be written using power of 10 in the exponent.

Example 5. *Write the following numbers in the expanded exponential form:*

(*i*) *30546* (*ii*) *76,09,154*

Solution. (*i*)
$$\begin{aligned} 30546 &= 3 \times 10000 + 0 \times 1000 + 5 \times 100 + 4 \times 10 + 6 \times 1 \\ &= 3 \times 10^4 + 0 \times 10^3 + 5 \times 10^2 + 4 \times 10^1 + 6 \times 10^0 \\ &= 3 \times 10^4 + 5 \times 10^2 + 4 \times 10^1 + 6 \times 10^0 \end{aligned}$$

(*ii*)
$$\begin{aligned} 76{,}09{,}154 &= 7 \times 1000000 + 6 \times 100000 + 0 \times 10000 + 9 \times 1000 + 1 \times 100 + 5 \times 10 + 4 \times 1 \\ &= 7 \times 10^6 + 6 \times 10^5 + 0 \times 10^4 + 9 \times 10^3 + 1 \times 10^2 + 5 \times 10^1 + 4 \times 10^0 \\ &= 7 \times 10^6 + 6 \times 10^5 + 9 \times 10^3 + 1 \times 10^2 + 5 \times 10^1 + 4 \times 10^0. \end{aligned}$$

Example 6. *Find the number from each of the following expanded forms:*

(*i*) $8 \times 10^4 + 6 \times 10^3 + 4 \times 10^1 + 5 \times 10^0$

(*ii*) $9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1$

Solution. (*i*)
$$\begin{aligned} & 8 \times 10^4 + 6 \times 10^3 + 4 \times 10^1 + 5 \times 10^0 \\ &= 8 \times 10000 + 6 \times 1000 + 4 \times 10 + 5 \times 1 \\ &= 80000 + 6000 + 40 + 5 = 86045. \end{aligned}$$

(*ii*)
$$\begin{aligned} 9 \times 10^5 + 2 \times 10^2 + 3 \times 10^1 &= 9 \times 100000 + 2 \times 100 + 3 \times 10 \\ &= 900000 + 200 + 30 = 900230. \end{aligned}$$

Exercise 4.3

1. Write the following numbers in the standard form (or scientific notation):

(*i*) 530.7 (*ii*) 3908.78 (*iii*) 39087.8

(*iv*) 2.35 (*v*) 3,43,000 (*vi*) 70,00,000

(*vii*) 3,18,65,00,000 (*viii*) 893,000,000 (*ix*) 70,040,000,000

2. Write the following numbers in usual decimal notation:
   (i) 4.7×10^3 (ii) 1.205×10^5 (iii) 1.234×10^6
   (iv) 4.87×10^7 (v) 6.05×10^8 (vi) 9.083×10^{11}
3. Express the numbers appearing in the following statements in scientific notation (or standard form):
   (i) The distance between the earth and the moon is 384,000,000 m.
   (ii) The diameter of the sun is 1,400,000,000 m.
   (iii) The universe is estimated to be about 12,000,000,000 years old.
   (iv) In a galaxy there are on an average 100,000,000,000 stars.
4. Compare the following numbers:
   (i) 4.3×10^{14}; 3.01×10^{17} (ii) 1.439×10^{12}; 1.4335×10^{12}.
5. Write the following numbers in the expanded exponential form:
   (i) 279404 (ii) 3006194 (iii) 28061906
6. Find the number from each of the expanded form:
   (i) $3 \times 10^4 + 7 \times 10^2 + 5 \times 10^0$
   (ii) $4 \times 10^5 + 5 \times 10^3 + 3 \times 10^2 + 2 \times 10^0$
   (iii) $8 \times 10^7 + 3 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 8 \times 10^1$

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
   (i) In the expression $(-5)^9$, exponent = ... and base = ...
   (ii) If the base is $-\frac{3}{4}$ and exponent is 5, then exponential form is ...
   (iii) The expression $(x^2y^5)^3$ in the simplest form is ....
   (iv) If $(100)^0 = 10^n$, then the value of n is ....
   (v) $\left(-\frac{1}{2}\right)^0 + (-2)^0 = ...$
   (vi) $(-3)^2 \times (-1)^{2017} = ...$
   (vii) $(-3)^8 \div (-3)^5 = (-3)^{\cdots}$
   (viii) $35070000 = 3.507 \times 10^{\cdots}$
   (ix) If $(-2)^n = -128$, then $n = ...$
2. State whether the following statements are true (T) or false (F):
   (i) If a is a rational number then $a^m \times a^n = a^{m \times n}$.
   (ii) $2^3 \times 3^2 = 6^5$.
   (iii) The value of $(-2)^{-3}$ is $-\frac{1}{8}$.
   (iv) The value of the expression $2^9 \times 2^{91} - 2^{19} \times 2^{81}$ is 1.
   (v) $5^6 \div (-2)^6 = -\frac{5}{2}$.

(vi) $5^0 \times 3^0 = 8^0$.

(vii) $\frac{2^3}{7} < \left(\frac{2}{7}\right)^3$.

(viii) $(10 + 10)^4 = 10^4 + 10^4$

(ix) $x^0 \times x^0 = x^0 \div x^0$, where x is a non-zero rational number.

(x) 4^9 is greater than 16^3.

(xi) $x^m + x^m = x^{2m}$, where x is a non-zero rational number and m is a positive integer.

(xii) $\left(\frac{4}{3}\right)^5 \times \left(\frac{5}{7}\right)^5 = \left(\frac{4}{3} + \frac{5}{7}\right)^5$.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 18):

3. $a \times a \times a \times b \times b \times b$ is equal to

(a) a^3b^2 (b) a^2b^3 (c) $(ab)^3$ (d) a^6b^6

4. $(-2)^3 \times (-3)^2$ is equal to

(a) 6^5 (b) $(-6)^6$ (c) 72 (d) -72.

5. The expression $(pqr)^3$ is equal to

(a) p^3qr (b) pq^3r (c) pqr^3 (d) $p^3q^3r^3$.

6. $\left(-\frac{3}{2}\right)^{-1}$ is equal to

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{4}{9}$.

7. $\left(-\frac{3}{4}\right)^5$ is equal to

(a) $\frac{81}{256}$ (b) $-\frac{81}{256}$ (c) $-\frac{243}{1024}$ (d) $\frac{243}{1024}$.

8. The value of $(5^{30} \times 5^{20}) \div (5^5)^9$ in the exponential form is

(a) 5^{-5} (b) 5^5 (c) 5^{50} (d) 5^{95}.

9. The law $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ does not hold when

(a) $a = 3, b = 2$ (b) $a = -2, b = 3$ (c) $n = 0$ (d) $b = 0$.

10. The value of the expression $\frac{(-1)^{101} \times (8)^5}{(4)^7}$ is equal to

(a) 2 (b) -2 (c) $\frac{1}{16}$ (d) $-\frac{1}{16}$.

11. The value of $\frac{10^{22} + 10^{20}}{10^{20}}$ is

(a) 10 (b) 101 (c) 10^{22} (d) 10^{42}

12. The value of $5^{-1} - 6^{-1}$ is

(a) $\frac{1}{30}$ (b) $-\frac{1}{30}$ (c) 30 (d) -30

13. The value of $(6^{-1} - 8^{-1})^{-1}$ is

(a) $-\frac{1}{2}$ (b) -2 (c) $\frac{1}{24}$ (d) 24

14. The value of $\left(\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}\right) \div \left(\frac{1}{5}\right)^{-2}$ is

(*a*) 0 (*b*) -1 (*c*) 1 (*d*) $\frac{7}{5}$

15. If $2^3 + 1^3 = 3^x$, then the value of x is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3.

16. The standard form of 751.65 is

(*a*) 7.5165×10^2 (*b*) 75.165×10^1 (*c*) 7.5165×10^4 (*d*) 7.51×10^2

17. The usual form of 5.658×10^5 is

(*a*) 5658 (*b*) 56580 (*c*) 565800 (*d*) 5658000.

18. Which of the following numbers is in the standard form?

(*a*) 26.57×10^4 (*b*) 2.657×10^5 (*c*) 265.7×10^3 (*d*) 0.2657×10^6

Higher Order Thinking Skills (HOTS)

1. If $\left(\frac{5}{12}\right)^8 \times \left(\frac{12}{5}\right)^{-4} = \left(\frac{125}{1728}\right)^x$, find x.

2. If $27^{2x-1} = (243)^3$, then find the value of $(-5)^{2x-3}$.

3. If $x = \frac{1}{6} - \frac{5}{3}$ of $\frac{2}{-5} + \frac{4}{9}$, then find the value of $x^3 - 3$.

Summary

★ If a is any rational number and n is a natural number, then

$a^n = a \times a \times a$ ... multiplied n times,

where a is called the **base** and n is called the **exponent** or **index** and a^n is the **exponential form.**

a^n is read as 'a raised to the power n' or 'a to the power n' or simply 'a power n'.

In particular, $a^1 = a$.

★ $(-1)^{\text{odd natural number}} = -1$ and $(-1)^{\text{even natural number}} = 1$.

★ **Laws of exponents**

Law 1 – If a is any rational number and m, n are natural numbers, then $a^m \times a^n = a^{m+n}$.

Law 2 – If a is any (non-zero) rational number and m, n are natural numbers such that $m > n$, then $a^m \div a^n = a^{m-n}$.

Law 3 – If a is any (non-zero) rational number, then $a^0 = 1$.

Law 4 – If a is any rational number and m, n are natural numbers, then $(a^m)^n = a^{m \times n}$.

Law 5 – If a, b are any rational numbers and n is a natural number, then $a^n \times b^n = (ab)^n$.

Law 6 – If a, b ($\neq 0$) are any rational numbers and n is a natural number, then $a^n \div b^n = \left(\frac{a}{b}\right)^n$ or $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$.

Law 7 – If a is any (non-zero) rational number and n is any natural number, then $a^{-n} = \frac{1}{a^n}$.

In particular, $a^{-1} = \frac{1}{a}$.

★ **Standard form or scientific notation**

A number is said to be in the standard form if it is expressed as $k \times 10^n$, where k is a decimal number such that $1 \leq k < 10$ and n is a whole number.

The standard form of a number is also known as scientific notation.

The decimal number k is called significand.

★ **Converting from decimal notation to standard form**

(*i*) Move the decimal point to the left till you get just one digit to the left of decimal place.

(*ii*) Write the given number as the product of the number obtained in step (*i*) and 10^n, where n is the number of places the decimal point has been moved to the left.

★ **Converting from standard form to usual form**

Take the significand and move the decimal point to the right by the number of places indicated by the exponent of 10^n adding trailing zeros as necessary.

★ **Comparing numbers in standard form**

(*i*) The number with greater power of 10 is greater.

(*ii*) If the powers of 10 are equal in both numbers, then compare their significands. The number with greater significand is greater.

Check Your Progress

1. Find the value of each of the following:

(*i*) $(-3)^3 \times 5^2$ (*ii*) $(-1)^{501} \times [(27)^4 \div (9)^5]$ (*iii*) $\left(-3\frac{1}{2}\right)^3$.

2. Simplify the following:

(*i*) $\frac{7^3 \times 11^4 \times 13^0}{7^2 \times 11^2}$ (*ii*) $\frac{(-2)^3 \times (3x)^2 \times (-xy^3)}{3x^2y}$ (*iii*) $\frac{((-5)^3)^4 \times 8^2}{4^3 \times (25)^5}$.

3. Simplify and write the following in exponential form:

(*i*) $\frac{(-3)^5 \times 8^3 \times 2^5}{3^2 \times 4^4}$ (*ii*) $\frac{9^8 \times (x^2)^5}{(27)^4 \times (x^3)^2}$ (*iii*) $\frac{3^2 \times 7^8 \times 13^6}{21^2 \times 91^3}$.

4. If $\left(-\frac{3}{5}\right)^x = -\frac{27}{125}$, then find the value of x.

5. Write the prime factorisation of the following numbers in the exponential form:

(*i*) 24000 (*ii*) 12600 (*iii*) 14157

6. Express the numbers appearing in the following statements in scientific notation:

(*i*) The earth has 1,353,000,000 cubic km of water.

(*ii*) The population of India was about 1,027,000,000 in March, 2001.

7. Compare the following numbers:

(*i*) 5.976×10^{24}; 8.689×10^{23} (*ii*) 3.7662×10^{17}; 3.7671×10^{17}