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CONGRUENCE OF TRIANGLES

INTRODUCTION

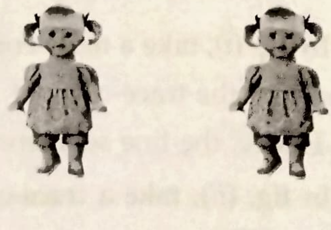
In our daily life, we see many objects around us which have same shape and size. Objects that have exactly same shape and same size are congruent to each other.

For example:

(i) Two copies of a photograph of the same size are congruent.

(ii) Two coins of ₹5 minted at the same time are congruent.

(iii) Two toys made of same mould are congruent.



Two objects are called **congruent** if and only if they have exactly the same shape and the same size. The relation of two objects being congruent is called **congruence**.

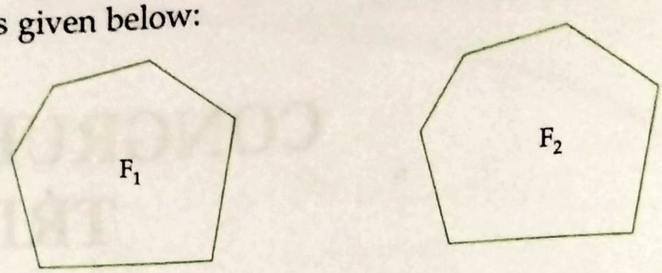
In the present chapter, we shall be dealing only with plane figures, although congruence is a general concept applicable to three-dimensional shapes also. In particular, we shall study congruence of triangles.

CONGRUENCE OF PLANE FIGURES

Two plane figures are called ***congruent*** *if and only if they have exactly the same shape and the same size.*

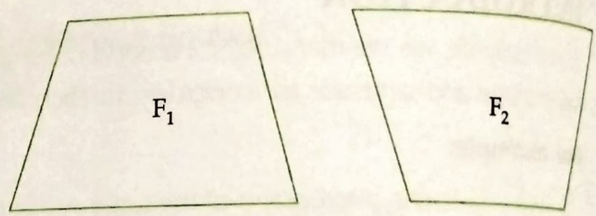
To check for congruency, take a trace-copy of one of the two figures and place it over the other figure. The two figures will be congruent if this trace-copy exactly fits the other figure.

Alternatively, we may cut out one figure and place it over the other.
This method of comparing two figures is called **method of superposition.**
Look at the figures given below:



Take a trace-copy of figure F_1 and place it over figure F_2. We note that this trace-copy of F_1 exactly covers the figure F_2. Therefore, F_1 and F_2 are congruent. We write it as $F_1 \cong F_2$. The symbol '$\cong$' (read as 'congruent to') is used to indicate the congruency of two figures.

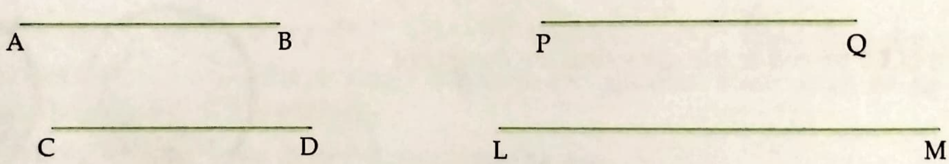
Sometimes, it may be necessary to rotate or flip the figure to check whether they are congruent.



The adjoining figures F_1 and F_2 are congruent. Note that we have to rotate figure F_1 through 180°.

Congruence of line segments

Look at the figures given below:



(*i*) (*ii*)

In fig. (*i*), take a trace-copy of line segment AB and place it over line segment CD. We find that the trace-copy of $\overline{AB}$ exactly covers $\overline{CD}$ with A falling on C and B falling on D. Hence, the line segments AB and CD are congruent. We write $\overline{AB} \cong \overline{CD}$.

In fig. (*ii*), take a trace-copy of $\overline{PQ}$ and place it over $\overline{LM}$. We find that it does not cover $\overline{LM}$ completely. In fact, it falls short of $\overline{LM}$. So, line segments PQ and LM are not congruent.

In fig. (*i*), the pair of line segments $\overline{AB}$ and $\overline{CD}$ match with each other because they had same length; and this was not in fig. (*ii*).

Thus, we have:

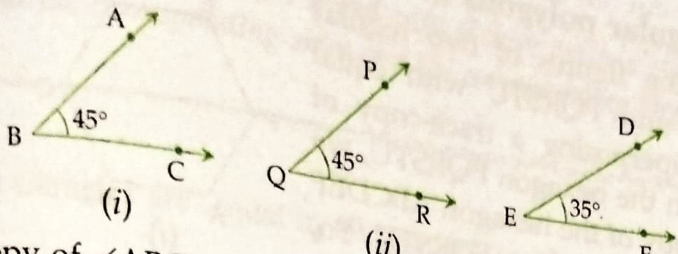
If two line segments have same length then they are congruent. Conversely, if two line segments are congruent then they have same length. Hence:

Two line segments are congruent if and only if they have equal length.

If two line segments are congruent, then we also say that they are equal. We write AB = CD to mean $\overline{AB} \cong \overline{CD}$.

gruence of angles

ok at the figures given below:



(i) (ii) (iii)

ake a trace-copy of $\angle ABC$ and try to superpose it on $\angle PQR$. For this, first place B Q and then $\overline{BC}$ on $\overline{QR}$. We shall find that $\overline{BA}$ falls along $\overline{QP}$. Thus, $\angle ABC$ matches ctly with $\angle PQR$.

$\angle ABC$ and $\angle PQR$ are congruent (Note that measurements of $\angle ABC$ and $\angle PQR$ equals.)

We write it as $\angle ABC \cong \angle PQR$ or $m\,\angle ABC = m\,\angle PQR$. Now try to superpose the ce-copy of $\angle ABC$ on $\angle DEF$. First place B on E and then $\overline{BC}$ along $\overline{EF}$. We note that A does not fall on $\overline{ED}$ *i.e.* $\angle ABC$ does not cover $\angle DEF$ exactly, therefore, $\angle ABC$ and DEF are not congruent.

(Note that the measurements of $\angle ABC$ and $\angle DEF$ are not equal.)

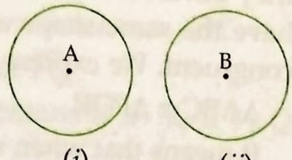
Thus, if two angles have same measurement then they are congruent. Conversely, if o angles are congruent then they have same measurement. Hence:

Two angles are congruent if and only if they have same measurement.

If two angles are congruent, then we also say that they are equal. We write ABC = $\angle PQR$ to mean $\angle ABC \cong \angle PQR$.

ngruence of circles

ok at the adjoining figures of two circles A and B with equal dii. On superposing a trace-copy of circle A on circle B, we d that the trace-copy of circle A exactly fits the circle B. o, the circles A and B are congruent *i.e.* circle A $\cong$ circle B. nversely, if two circles A and B are congruent then they ve equal radii. Hence:

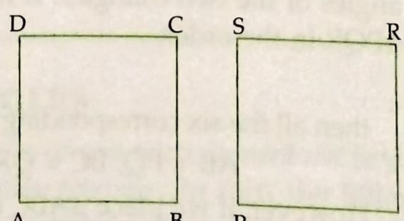


(i) (ii)

wo circles are congruent if and only if they have equal radii.

ngruence of squares

ok at the adjoining figures of two squares ABCD d PQRS with equal side lengths. On superposing a ce-copy of square ABCD on the square PQRS, we d that the trace-copy of square ABCD exactly fits vers) the square PQRS.



(i) (ii)

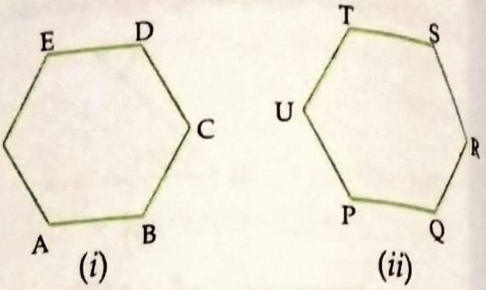
o, square ABCD $\cong$ square PQRS.

onversely, if two squares are congruent then they have equal side lengths. Hence:

wo squares are congruent if and only if they have equal side lengths.

Congruence of regular polygons of equal number of sides

Look at the adjoining figures of two regular hexagons ABCDEF and PQRSTU with equal side lengths. On superposing a trace-copy of hexagon ABCDEF on the hexagon PQRSTU, we find that the trace-copy of the hexagon ABCDEF exactly fits (covers) the hexagon PQRSTU. So, hexagon ABCDEF ≅ hexagon PQRSTU.



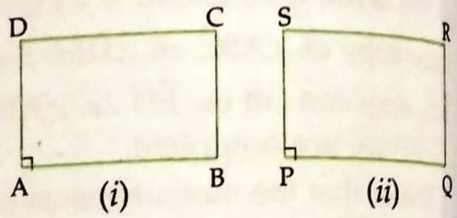
Conversely, if two regular hexagons are congruent then they have equal side lengths. Hence:

Two regular hexagons are congruent if and only if they have equal side lengths.

In fact, two regular polygons with equal number of sides are congruent if and only if they have equal side lengths.

Congruence of two rectangles

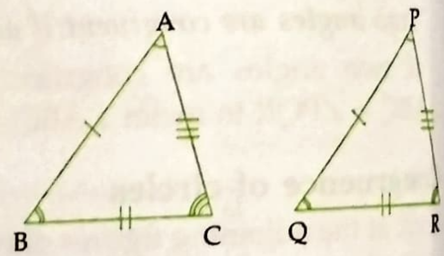
Two rectangles are congruent if and only if they have equal lengths and equal breadths.



CONGRUENCE OF TRIANGLES

Two triangles are called congruent if and only if they have exactly the same shape and the same size.

In the adjoining diagram, two triangles ABC and PQR are such that if the trace-copy of one triangle is placed onto the other they fit each other exactly *i.e.* when one triangle is superposed on the other, they cover each other exactly. Thus, these triangles have the same shape and the same size, so they are congruent. We express this as



$\Delta ABC \cong \Delta PQR$

It means that when we place a trace-copy of ΔABC on ΔPQR, vertex A falls on vertex P, vertex B on vertex Q and vertex C on vertex R. Then side AB falls on PQ, BC on QR and CA on RP. Also ∠A falls on ∠P, ∠B on ∠Q and ∠C. Thus, the order in which the vertices match, automatically determines a correspondence between the sides and the angles of the two triangles. It follows that if the vertices of ΔABC match the vertices of ΔPQR in the order:

$$A \leftrightarrow P, B \leftrightarrow Q, C \leftrightarrow R$$

then all the six corresponding parts (3 sides and 3 angles) of two triangles are equal *i.e.*

$$AB = PQ, BC = QR, CA = RP, \angle A = \angle P, \angle B = \angle Q \text{ and } \angle C = \angle R.$$

However, if we place ΔABC on ΔPQR such that A falls on Q, then other vertices may not correspond suitably. Take a trace-copy of ΔABC and place vertex A on vertex Q and try to find out!

This shows that while talking about congruence of triangles, not only the measures of angles and the lengths of sides matter, but also the matching of vertices matter. In the above triangles ABC and PQR, the correspondence is

$$A \leftrightarrow P, B \leftrightarrow Q, C \leftrightarrow R,$$

we may write this as ABC $\leftrightarrow$ PQR.

Thus, two triangles are congruent if and only if there exists a correspondence between their vertices such that the corresponding sides and the corresponding angles of the two triangles are equal.

Remarks

- Congruent triangles are 'equal in all respects' *i.e.* they are the exact *duplicate* of each other.
- If two triangles are congruent, then any one can be *superposed* on the other to cover it exactly.
- In congruent triangles, the sides and the angles which coincide by superposition are called *corresponding sides* and *corresponding angles.*
- The corresponding sides lie opposite to the equal angles and the corresponding angles lie opposite to the equal sides.

  In the above diagram, $\angle A = \angle P$, therefore, the corresponding sides BC and QR are equal. Also BC = QR, therefore, the corresponding angles A and P are equal.
- The abbreviation 'CPCT' or 'c.p.c.t.' will be used for corresponding parts of congruent triangles.
- The order of the letters in the names of congruent triangles displays the corresponding relationship between the two triangles.

  Thus, when we write $\Delta ABC \cong \Delta PQR$, it means that A lies on P, B lies on Q and C lies on R *i.e.* $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and BC = QR, CA = RP, AB = PQ.

  Writing any other correspondence *i.e.* $\Delta ABC \cong \Delta PRQ$, $\Delta ABC \cong \Delta RPQ$ etc. will be incorrect.

Example 1. *If ΔABC and ΔPQR are congruent under the correspondence: $ABC \leftrightarrow RQP$ then write parts of ΔABC that correspond to $\angle P$, $\angle Q$, $\overline{RP}$ and $\overline{PQ}$.*

Solution. The given correspondence is ABC $\leftrightarrow$ RQP. This means A $\leftrightarrow$ R, B $\leftrightarrow$ Q and C $\leftrightarrow$ P.

$\therefore \quad \angle P \leftrightarrow \angle C$, $\angle Q \leftrightarrow \angle B$, $\overline{RP} \leftrightarrow \overline{AC}$ and $\overline{PQ} \leftrightarrow \overline{CB}$.

Note

In fact, $\Delta ABC \cong \Delta RQP$.

CRITERIA FOR CONGRUENCE OF TRIANGLES

For any two triangles to be congruent, the *six elements* of one triangle *need not be proved* equal to the corresponding six elements of the other triangle. In fact, the following conditions are sufficient to ensure the congruency of two triangles:

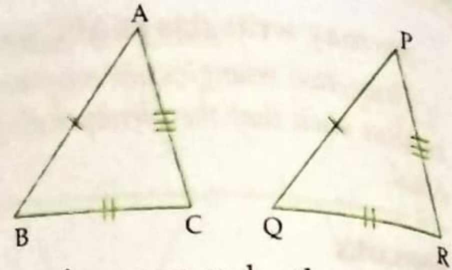
SSS (Side – Side – Side) congruence criterion

Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.

In the adjoining diagram,

$AB = PQ$, $BC = QR$ and $AC = PR$

$\therefore$ $\Delta ABC \cong \Delta PQR$



(Take a trace-copy of ΔABC and superpose it on ΔPQR such that vertex A falls on vertex P and side AB falls on side PQ. We shall find that vertex B falls on vertex Q and vertex C falls on vertex R. The two triangles cover each other exactly, so these triangles are congruent.)

Note

If three angles of one triangle are equal to three angles of another triangle, the two triangles may not be congruent.

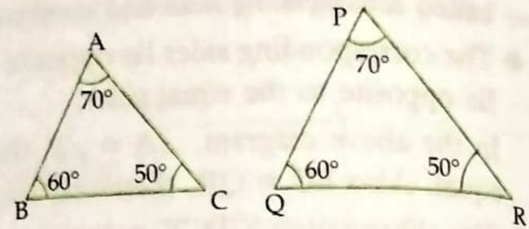
Look at the adjoining figure:

In ΔABC and ΔPQR,

$\angle A = \angle P$ (= 70° each)

$\angle B = \angle Q$ (= 60° each)

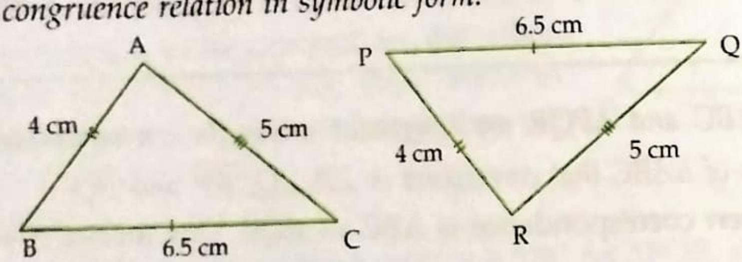
$\angle C = \angle R$ (= 50° each)



We note that these triangles have the same shape but their sizes are different, therefore, these triangles are not congruent.

Hence, AAA (Angle – Angle – Angle) does not ensure the congruency of two triangles.

Example 2. *In triangles ABC and PQR, AB = 4 cm, BC = 6.5 cm, AC = 5 cm, PQ = 6.5 cm, QR = 5 cm and PR = 4 cm. Examine whether the two triangles are congruent or not. If yes, then write the congruence relation in symbolic form.*



Solution. Here, $AB = PR$ (= 4 cm each)

$BC = PQ$ (= 6.5 cm each)

$AC = QR$ (= 5 cm each)

This shows that the three sides of one triangle are equal to three sides of the other triangle, therefore, by SSS congruence criterion, the two triangles are congruent.

It means that if we superpose one triangle on other triangle (by rotation/sliding etc.), then the two triangles cover each other exactly. In this process, we note that A falls on R, B on P and C on Q *i.e.* $A \leftrightarrow R$, $B \leftrightarrow P$ and $C \leftrightarrow Q$.

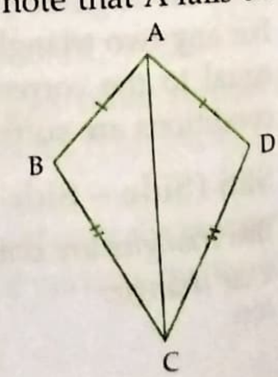
So, we have $\Delta ABC \cong \Delta RPQ$.

Example 3. *In the adjoining figure, AB = AD and BC = DC.*

(i) State three pairs of equal parts in ΔABC and ΔADC.

(ii) Is $\Delta ABC \cong \Delta ADC$? Give reasons.

(iii) Does AC bisect $\angle BAD$? Give reasons.



Solution. (*i*) In ΔABC and ΔADC, three pairs of equal parts are given below:

$AB = AD$ (given)

$BC = DC$ (given)

$AC = AC$ (common in both)

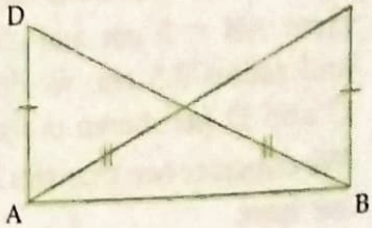
(*ii*) From part (*i*), $\Delta ABC \cong \Delta ADC$ (SSS rule of congruency)

(Note that $A \leftrightarrow A$, $B \leftrightarrow D$ and $C \leftrightarrow C$)

(*iii*) $\angle BAC = \angle DAC$ (corresponding parts of congruent triangles)

$\Rightarrow$ AC bisects $\angle BAD$.

Example 4. *In the adjoining figure, AC = BD and AD = BC. Which of the following statements is meaningful?*



(*i*) $\Delta ABC \cong \Delta ABD$

(*ii*) $\Delta ABC \cong \Delta BAD$

Solution. In ΔABC and ΔABD, we have

$AC = BD$ (given)

$BC = AD$ (given)

$AB = BA$ (common)

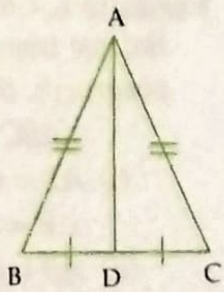
Therefore, by SSS congruence criterion, the two triangles are congruent.

If we superpose ΔABC on ΔABD (by flipping) then the two triangles cover each other exactly. In this process, we note that A falls on B, B on A and C on D *i.e.* $A \leftrightarrow B$, $B \leftrightarrow A$ and $C \leftrightarrow D$.

$\therefore \quad \Delta ABC \cong \Delta BAD$

Hence, the statement (*ii*) is meaningful.

Example 5. *In the adjoining figure, AB = AC and BD = DC. Prove that*



(*i*) $\angle BAD = \angle CAD$

(*ii*) *AD is perpendicular to BC.*

Solution. In ΔABD and ΔACD,

$AB = AC$ (given)

$BD = DC$ (given)

$AD = AD$ (common)

$\therefore \quad \Delta ABD \cong \Delta ACD$ (SSS rule of congruency)

(*i*) $\angle BAD = \angle CAD$ (corresponding parts of congruent triangles)

(*ii*) $\angle ADB = \angle ADC$ (corresponding parts of congruent triangles)

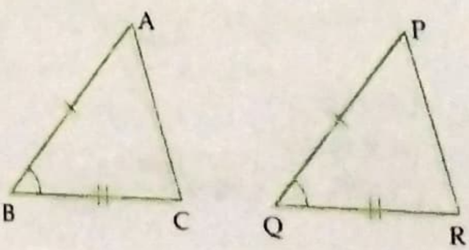
But $\angle ADB + \angle ADC = 180°$ (linear pair)

$\Rightarrow \angle ADC + \angle ADC = 180° \Rightarrow 2\angle ADC = 180° \Rightarrow \angle ADC = 90°$.

Hence, AD is perpendicular to BC.

SAS (Side – Angle – Side) congruence criterion

Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle.



In the adjoining diagram,

$AB = PQ$, $BC = QR$ and $\angle B = \angle Q$

$\therefore$ $\Delta ABC \cong \Delta PQR$.

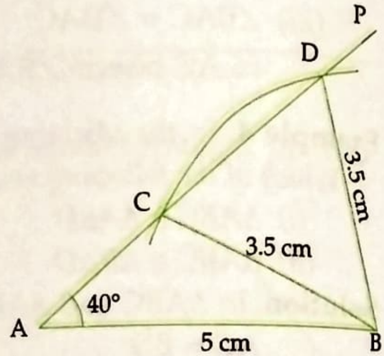
(Take a trace-copy of ΔABC and superpose it on ΔPQR such that vertex A falls on vertex P and side AB falls on side PQ. We shall find that vertex B falls on vertex Q and vertex C falls on vertex R. The two triangles cover each other exactly, so these triangles are congruent.)

Note

The equality of the 'included angle' is essential.

Look at the adjoining figure:

Here AB = 5 cm and $\angle PAB = 40°$. Taking B as centre and radius 3.5 cm, we draw an arc to meet AP at points C and D (as shown in figure).



We consider two triangles ABC and ABD. In these triangles, we have

$$AB = AB,\ BC = BD \text{ and}$$
$$\angle CAB = \angle DAB \qquad \text{(each = 40°)}$$

Thus, two sides and one angle of one triangle are equal to two sides and one angle of the other triangle but the triangles are not congruent which is clear from the figure because ΔABC is a part of ΔABD. In fact, two vertices A and B are same for both triangles but their third vertices C and D do not coincide. Here, note that $\angle CAB$ and $\angle DAB$ are not **included angles.**

Hence, for SAS congruence criterion the equality of the included angle is essential.

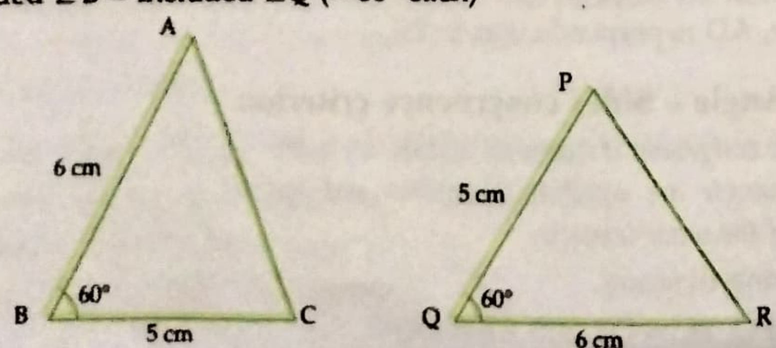
Example 6. *Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by using SAS rule of congruency. If the triangles are congruent, then write them in symbolic form.*

ΔABC	ΔPQR
(i) $AB = 6$ cm, $BC = 5$ cm, $\angle B = 60°$	$PQ = 5$ cm, $QR = 6$ cm, $\angle Q = 60°$.
(ii) $AB = 4$ cm, $AC = 4.5$ cm, $\angle A = 60°$	$QR = 4.5$ cm, $QP = 4$ cm, $\angle Q = 55°$.
(iii) $AB = 5$ cm, $BC = 4.5$ cm, $\angle B = 50°$	$PQ = 5$ cm, $QR = 4.5$ cm, $\angle R = 50°$.

Solution. (i) Here, AB = QR (= 6 cm each)

BC = PQ (= 5 cm each)

included $\angle B$ = included $\angle Q$ (= 60° each)

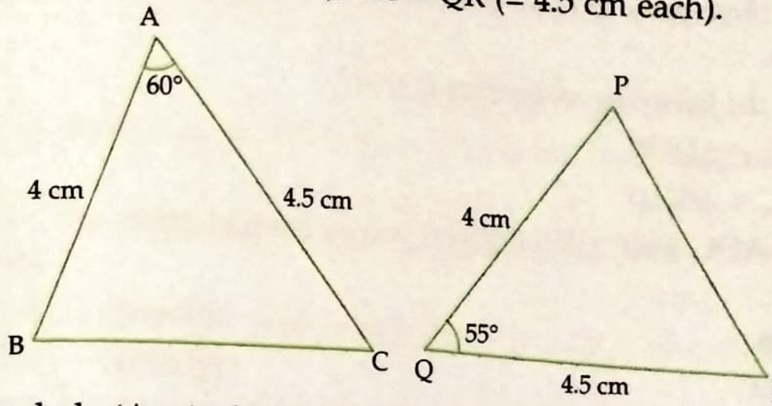


This shows that the two sides and included angle of one triangle are equal to two sides and the included angle of the other triangle, therefore, by SAS rule of congruency, the two triangles are congruent.

Also, we note that A ↔ R, B ↔ Q and C ↔ P.

∴ $\Delta ABC \cong \Delta RQP$.

(ii) Here, AB = QP (= 4 cm each), AC = QR (= 4.5 cm each).



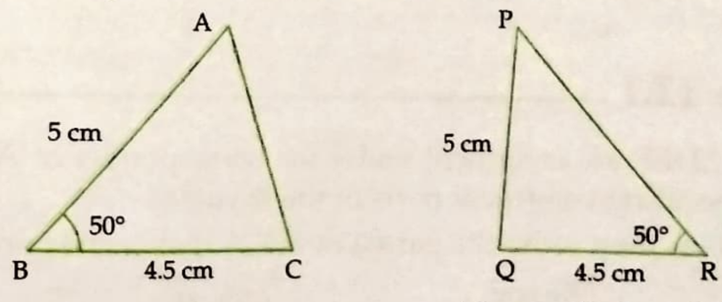
But included ∠A ≠ included ∠Q. Therefore, SAS rule of congruency cannot be applied. So, we cannot say that the two triangles are congruent. In fact, the two triangles are not congruent as the trace-copy of one triangle does not cover the other triangle exactly (check it).

(iii) Here, AB = PQ (= 5 cm each)

BC = QR (= 4.5 cm each)

and ∠B = ∠R (= 50° each)

Here we note that ∠B is included angle between sides AB and BC but ∠R is not included angle between sides PQ and QR.



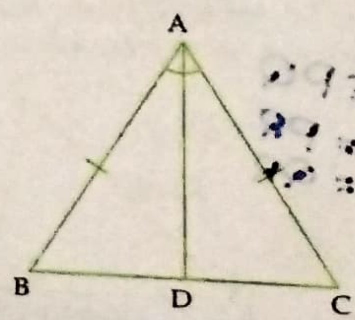
Therefore, SAS rule of congruency cannot be applied and we cannot conclude that the two triangles are congruent. In fact, the two triangles are not congruent as these triangles do not match in shape and size.

Example 7. *In the adjoining figure, AB = AC and AD is the bisector of ∠BAC.*

(i) State three pairs of equal parts in triangles ADB and ADC.

(ii) Is $\Delta ADB \cong \Delta ADC$? Give reasons.

(iii) Is ∠B = ∠C? Give reasons.



Solution. (i) In ΔADB and ΔADC, three pairs of equal parts are:

AB = AC (given)

AD = AD (common)

∠BAD = ∠CAD (∵ AD is bisector of ∠BAC)

(SAS rule of congruency)

(ii) From part (i), $\Delta ADB \cong \Delta ADC$

(Note that $A \leftrightarrow A$, $D \leftrightarrow D$ and $B \leftrightarrow C$)

(iii) $\angle B = \angle C$ (corresponding parts of congruent triangles)

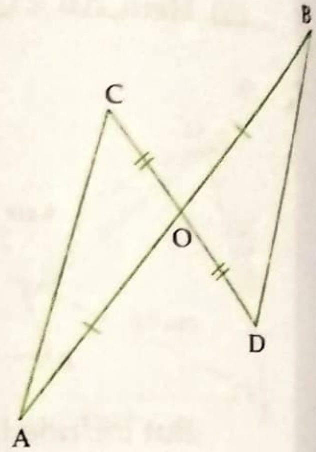
Example 8. *In the adjoining figure, $\overline{AB}$ and $\overline{CD}$ bisect each other at O.*

(i) State the three pairs of equal parts in two triangles AOC and BOD.

(ii) Which of the following statements is true?

(a) $\Delta AOC \cong \Delta DOB$

(b) $\Delta AOC \cong \Delta BOD$.



Solution. (i) In ΔAOC and ΔBOD, three pairs of equal parts are:

$OA = OB$ (given)

$OC = OD$ (given)

$\angle AOC = \angle BOD$ (vertically opposite angles)

(ii) From part (i), by SAS rule of congruency, the given triangles are congruent.

If we superpose trace-copy of ΔAOC on ΔBOD (by rotating), then the two triangles cover each other exactly. In this process, we note that A falls on B, C falls on D and O falls on O *i.e.* $A \leftrightarrow B$, $C \leftrightarrow D$ and $O \leftrightarrow O$.

$\therefore \quad \Delta AOC \cong \Delta BOD$.

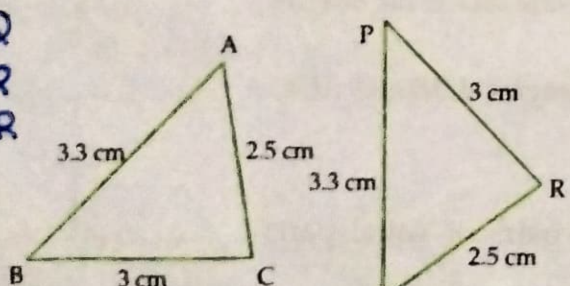
Hence, the statement (b) is true.

Exercise 12.1

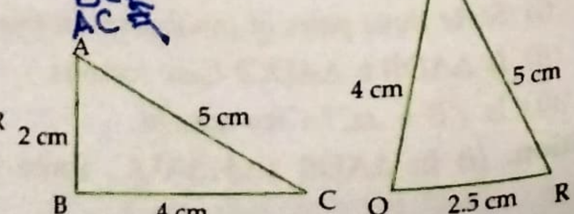
1. If ΔABC and ΔDEF are congruent under the correspondence $ABC \leftrightarrow FED$, write all the corresponding congruent parts of the triangles.
2. If $\Delta DEF \cong \Delta BCA$, then write the part(s) of ΔBCA that correspond to

   (i) $\angle E$ (ii) $\overline{EF}$ (iii) $\angle F$ (iv) $\overline{DF}$
3. In the figures given below, the lengths of the sides of the triangles are indicated. By using SSS congruency rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form:

$\overline{AB} \cong PQ$
$\overline{BC} = PR$
$AC = QR$

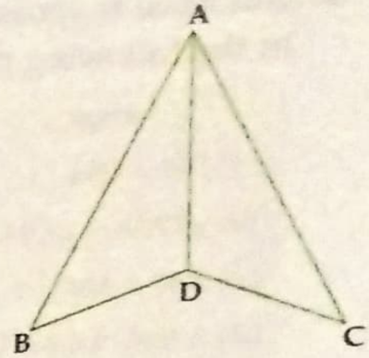


(i)



(ii)

4. In the adjoining figure, AB = 5 cm, AC = 5 cm, BD = 2.5 cm and CD = 2.5 cm.

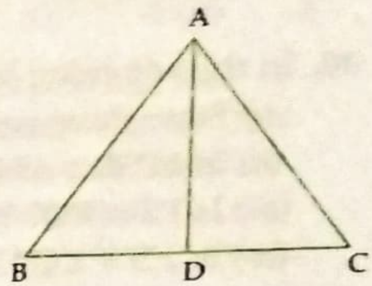


(i) State the three pairs of equal parts in ΔADB and ΔADC.

(ii) Is $\Delta ADB \cong \Delta ADC$? Give reasons.

(iii) Is $\angle B = \angle C$? Why?

5. In the adjoining figure, AB = AC and D is the mid-point of $\overline{BC}$.

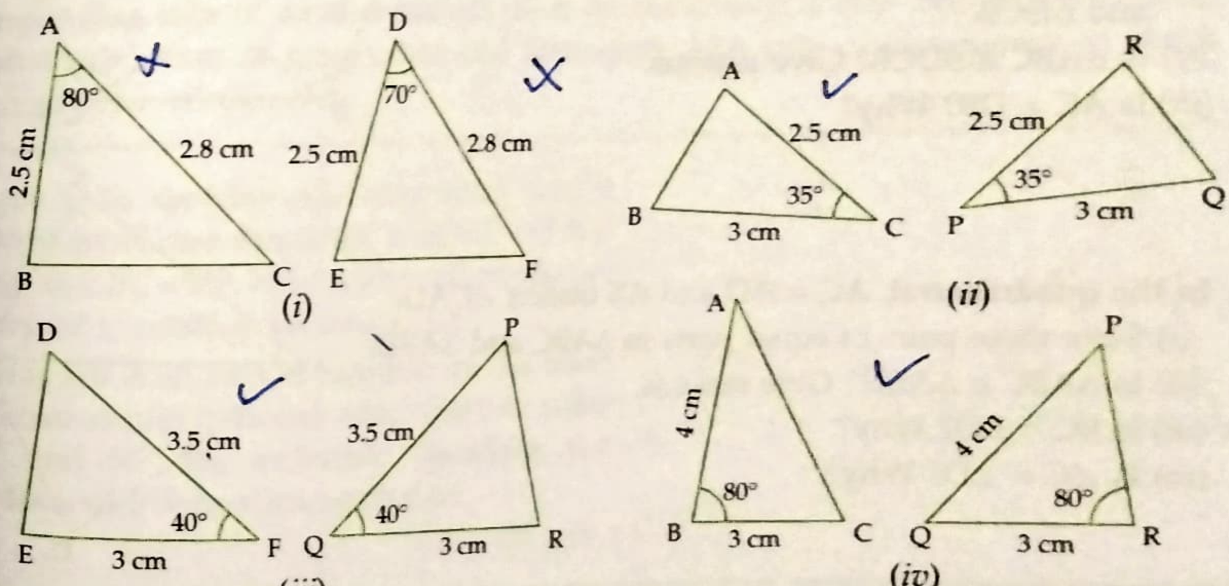


(i) State the three pairs of equal parts in ΔADB and ΔADC.

(ii) Is $\Delta ADB \cong \Delta ADC$? Give reasons.

(iii) Is $\angle B = \angle C$? Why?

6. In the figures given below, the measures of some parts of the triangles are indicated. By using SAS rule of congruency, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



7. By applying SAS congruence rule, you want to establish that $\Delta PQR \cong \Delta FED$. It is given that PQ = FE and RP = DF. What additional information is needed to establish the congruence?

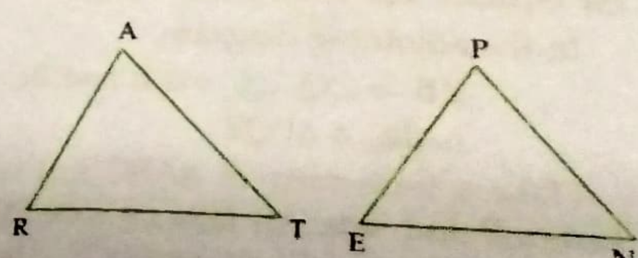
8. You want to show that $\Delta ART \cong \Delta PEN$

(a) If you have to use SSS criterion, then you need to show

(i) AR = (ii) RT = (iii) AT =

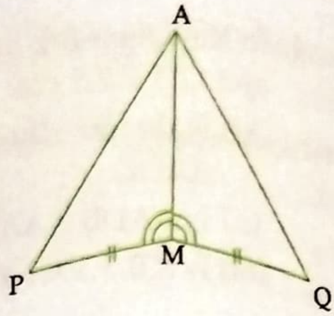
(b) If it is given that $\angle T = \angle N$ and you are to use SAS criterion, you need to have

(i) RT = and (ii) PN =

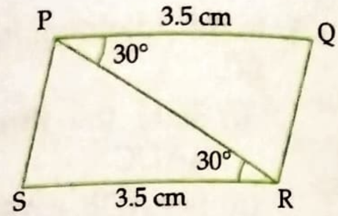


9. You have to show that $\Delta AMP \cong \Delta AMQ$.

In the following proof, supply the missing reasons.



Steps	*Reasons*
(*i*) PM = QM	(*i*) ...
(*ii*) $\angle PMA = \angle QMA$	(*ii*) ...
(*iii*) AM = AM	(*iii*) ...
(*iv*) $\Delta AMP \cong \Delta AMQ$	(*iv*) ...



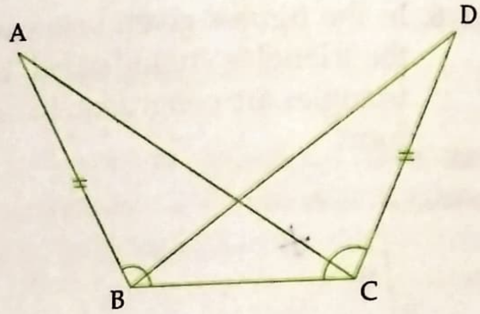
10. In the adjoining figure:

   (*i*) State three pairs of equal parts in ΔPSR and ΔRQP.

   (*ii*) Is $\Delta PSR \cong \Delta RQP$? Give reasons.

   (*iii*) Is PS = RQ? Why?

   (*iv*) Is $\angle S = \angle Q$? Why?

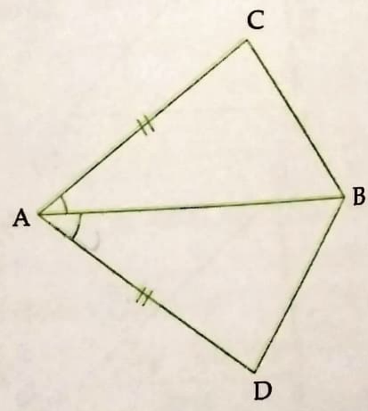


11. In the adjoining figure, AB = DC and $\angle ABC = \angle DCB$.

   (*i*) State three pairs of equal parts in ΔABC and ΔDCB.

   (*ii*) Is $\Delta ABC \cong \Delta DCB$? Give reasons.

   (*iii*) Is AC = DB? Why?



12. In the quadrilateral, AC = AD and AB bisects $\angle CAD$.

   (*i*) State three pairs of equal parts in ΔABC and ΔABD.

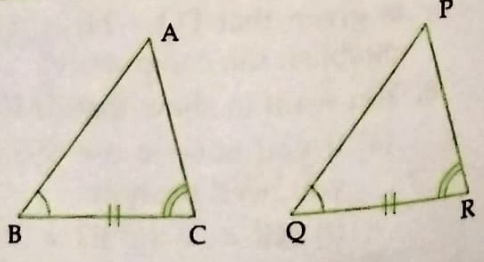
   (*ii*) Is $\Delta ABC \cong \Delta ABD$? Give reasons.

   (*iii*) Is BC = BD? Why?

   (*iv*) Is $\angle C = \angle D$? Why?

ASA (Angle – Side – Angle) congruence criterion

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.



In the adjoining diagram,

$\angle B = \angle Q$, $\angle C = \angle R$ and BC = QR

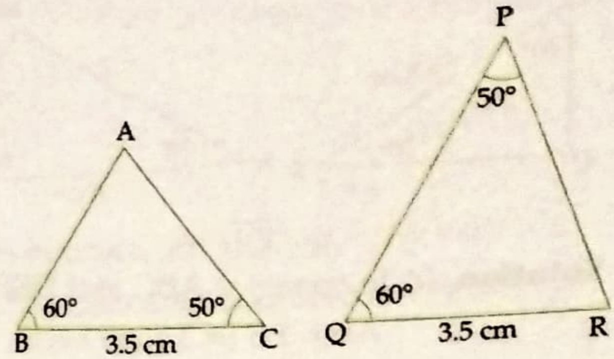
$\therefore$ $\Delta ABC \cong \Delta PQR$.

(Take a trace-copy of ΔABC and superpose it on ΔPQR such that vertex A falls on vertex P and side AB falls on side PQ. We shall find that the vertex B falls on vertex Q and vertex C falls on vertex R. The two triangles cover each other exactly, so these triangles are congruent).

Note

The equality of the 'included side' is essential.

Look at the given figure:



Consider two triangles ABC and PQR. In these triangles, we have

$BC = QR$ (= 3.5 cm each)

$\angle B = \angle Q$ (= 60° each)

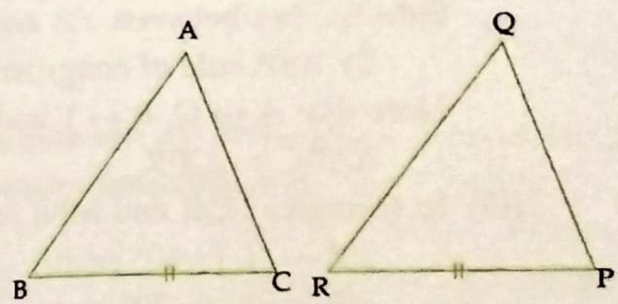
$\angle C = \angle P$ (= 50° each)

Thus, two angles and one side of one triangle are equal to two angles and one side of the other triangle but the triangles are not congruent which is clear from the figure. They have different shape and size. Here, note that the side QR of ΔPQR is not included. Hence, for ASA congruence criterion the equality of included side is essential.

Remark

We know that the sum of three angles of a triangle is 180°. So, if two pairs of angles are equal, then the third pair of angles is also equal (180° – sum of two angles). Thus, whenever, two angles and one side of one triangle are equal to two angles and the **corresponding side** of another triangle, then we can convert it into 'two angles and the included side' form of congruence and then apply ASA rule of congruency to check the congruence of triangles.

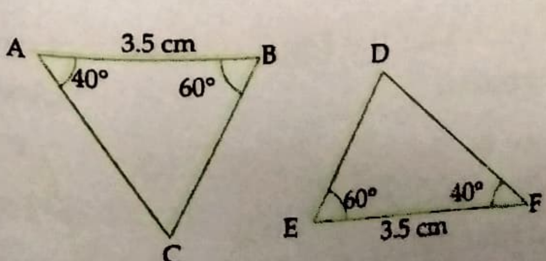
Example 1. *By applying ASA congruence rule, it is to be established that $\Delta ABC \cong \Delta QRP$ and it is given that $BC = RP$. What additional information is needed to establish the congruence?*



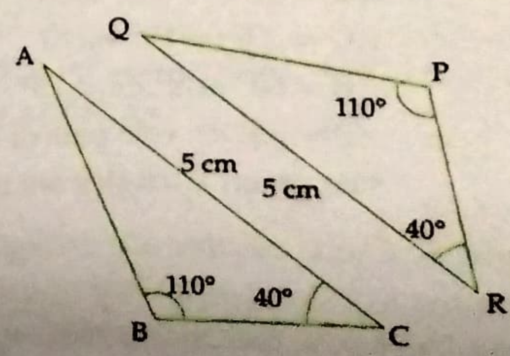
Solution. For ASA rule of congruence, we need the two angles between which the two sides BC and RP are included. Therefore, the additional information needed is:

$$\angle B = \angle R \text{ and } \angle C = \angle P.$$

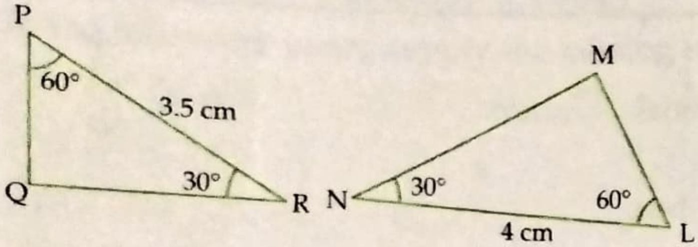
Example 2. *In the figures given below, measures of some parts of triangles are indicated. By applying ASA congruence rule, state which pairs of triangles are congruent. In case of congruence, write the result in symbolic form:*

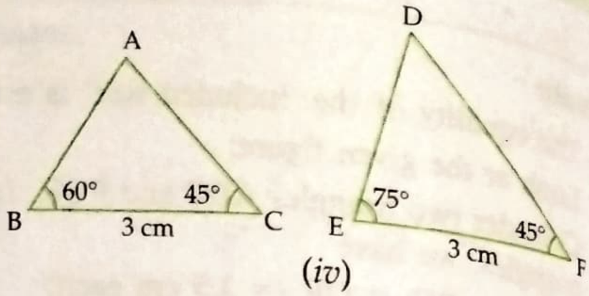


(*i*)



(*ii*)





(*iii*) (*iv*)

Solution. (*i*) In triangles ABC and DEF, we have

AB = EF (= 3.5 cm each)

∠A = ∠F (= 40° each)

∠B = ∠E (= 60° each)

Side AB lies between ∠A and ∠B; side FE lies between ∠F and ∠E.

∴ By ASA rule of congruence, the two triangles are congruent.

Note that A ↔ F, B ↔ E and C ↔ D.

∴ ΔABC ≅ ΔFED.

(*ii*) In ΔABC, ∠A = 180° – (110° + 40°) = 30°; in ΔPQR, ∠Q = 180° – (110° + 40°) = 30°.

In triangles ABC and PQR, we have

AC = QR (= 5 cm each)

∠A = ∠Q (= 30° each)

∠C = ∠R (= 40° each)

Side AC lies between ∠A and ∠C; side QR lies between ∠Q and ∠R.

∴ By ASA rule of congruency, the two triangles are congruent.

Note that A ↔ Q, B ↔ P and C ↔ Q.

∴ ΔABC ≅ ΔQPR.

(*iii*) In triangles PQR and MNL, we have

∠P = ∠L (= 60° each)

∠R = ∠N (= 30° each)

PR = 3.5 cm and NL = 4 cm

⇒ PR ≠ NL *i.e.* included sides are not equal.

∴ The given triangles are not congruent (ASA rule)

(*iv*) In triangles ABC and DEF, we have

BC = EF (= 3 cm each)

∠C = ∠F (= 45° each)

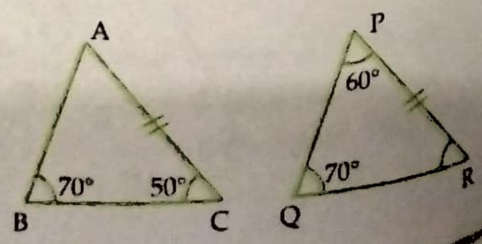
∠B = 60° and ∠E = 75°

⇒ ∠B ≠ ∠E *i.e.* one pair of angles including the equal sides are not equal.

∴ The given triangles are not congruent. (ASA rule)

Example 3. *Check whether the triangles shown in the adjoining figure, are congruent or not.*

Solution. We know that the sum of angles of a triangle is 180°.

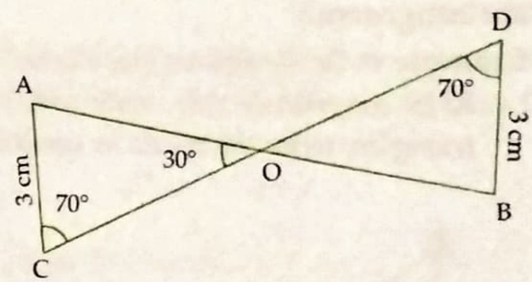


In ΔABC, $\angle A = 180° - (70° + 50°) = 60°$,
in ΔPQR, $\angle R = 180° - (70° + 60°) = 50°$.
In triangles ABC and PQR, we have

$\angle A = \angle P$ (= 60° each)
$\angle C = \angle R$ (= 50° each)
AC = PR (given)

Side AC lies between $\angle A$ and $\angle C$; side PR lies between $\angle P$ and $\angle R$.
∴ By ASA rule of congruency, the two given triangles are congruent.
Note that $A \leftrightarrow P$, $B \leftrightarrow Q$ and $C \leftrightarrow R$.
∴ $\Delta ABC \cong \Delta PQR$.

Example 4. *In the adjoining figure, can you use ASA congruence rule and conclude that $\Delta AOC \cong \Delta BOD$?*



Solution. In ΔAOC,

$\angle A = 180° - (70° + 30°) = 80°$.

In ΔBOD, $\angle BOD = \angle AOC$ (vertically opposite angles)

⇒ $\angle BOD = 30°$ (∵ $\angle AOC = 30°$ given)

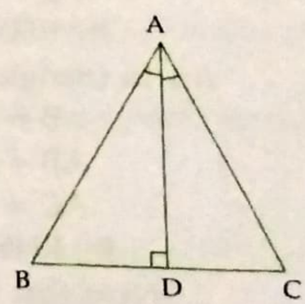
∴ $\angle B = 180° - (70° + 30°) = 80°$.

In two triangles AOC and BOD, we have

$\angle A = \angle B$ (= 80° each)
$\angle C = \angle D$ (= 70° each)
AC = BD (= 3 cm each)

Side AC lies between $\angle A$ and $\angle C$; side BD lies between $\angle B$ and $\angle D$.
∴ By ASA congruence rule, the two given triangles are congruent.
Note that $A \leftrightarrow B$, $C \leftrightarrow D$ and $O \leftrightarrow O$.
∴ $\Delta AOC \cong \Delta BOD$.

Example 5. *In triangle ABC (shown in the adjoining figure), the bisector AD of $\angle A$ is perpendicular to the side BC.*



(i) State three pairs of equal parts in ΔABD and ΔACD.
(ii) Is $\Delta ABD \cong \Delta ACD$? Give reasons.
(iii) Is AB = AC? Why?

Solution. As AD is bisector of $\angle A$, so $\angle BAD = \angle CAD$.
Also AD is perpendicular to side BC, so $\angle ADB = 90°$ and $\angle ADC = 90°$.

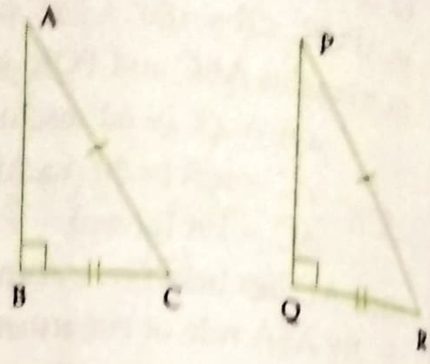
(i) Three pairs of equal parts in ΔABD and ΔADC are

$\angle BAD = \angle CAD$
$\angle ADB = \angle ADC$ (= 90° each)
AD = AD (common)

(ii) From part (i), $\Delta ABD \cong \Delta ACD$ (by ASA congruence rule)
(iii) AB = AC (corresponding parts of congruent triangles)

RHS (Right angle - Hypotenuse - Side) congruence criterion

Two right angled triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle. In the adjoining diagram:



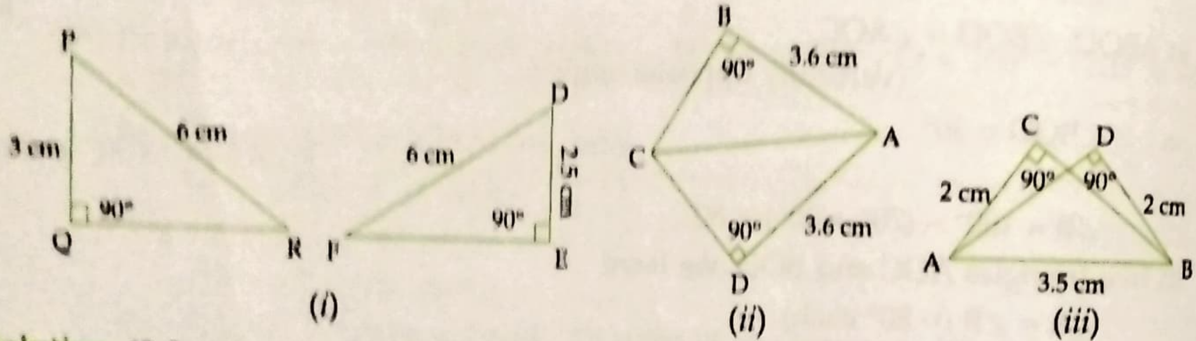
∠B = a right angle = ∠Q,

AC = PR and BC = QR

∴ ΔABC ≅ ΔPQR

(Superpose ΔABC on ΔPQR such that vertex A falls on vertex P and side AB falls on PQ, we shall find that vertex B falls on vertex Q and vertex C falls on vertex R. The two triangles cover each other exactly, so these triangles are congruent.)

Example 6. *In the figures given below, measures of some parts of triangles are given. By using RHS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form:*



Solution. (*i*) In triangles PQR and DEF, we have

∠Q = ∠E (= 90° each)

PR = DF (= 6 cm each, hypotenuse)

PQ = 3 cm and DE = 2.5 cm

⇒ PQ ≠ DE, so sides are not equal.

∴ By RHS rule of congruence, given triangles are not congruent.

(*ii*) In triangles ABC and ADC, we have

∠B = ∠D (= 90° each, right angle)

AB = AD (= 3.6 cm each, side)

AC = AC (common, hypotenuse)

∴ By RHS rule of congruence, these triangles are congruent.

Note that B ↔ D, A ↔ A and C ↔ C.

∴ ΔABC ≅ ΔADC.

(*iii*) In triangles ABC and ADC, we have

∠C = ∠D (= 90° each, right angle)

AC = BD (= 2 cm each, side)

AB = BA (common, hypotenuse)

∴ By RHS rule of congruence, these triangles are congruent.

Note that A ↔ B, B ↔ A and C ↔ D.

∴ ΔABC ≅ ΔBAD.

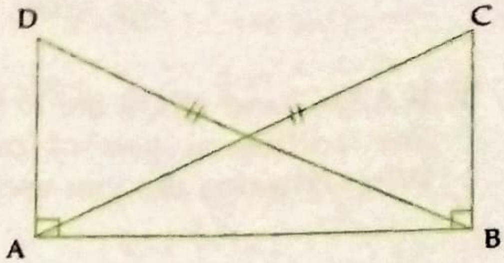
Example 7. *In the adjoining figure, DA ⊥ AB, CB ⊥ AB and AC = BD.*

(i) State three pairs of equal parts in ΔABC and ΔDAB.

(ii) Which of the following statements is meaningful?

(a) ΔABC ≅ ΔBAD

(b) ΔABC ≅ ΔABD.



Solution. (*i*) In ΔABC and ΔDAB, three pairs of equal parts are:

∠ABC = ∠BAD (= 90° each, right angle)

AC = BD (given)

AB = BA (common)

(*ii*) From part (*i*), by RHS rule of congruence, the given triangles are congruent. By superposition of ΔABC on ΔDAB, we note that A ↔ B, B ↔ A and C ↔ D.

∴ ΔABC ≅ ΔBAD.

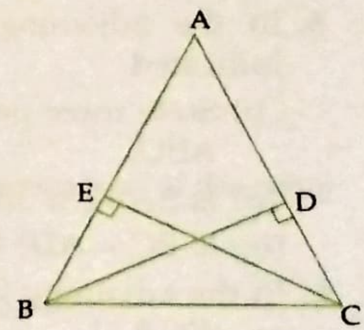
Hence, the statement (*a*) is meaningful.

Example 8. *In the adjoining figure, BD and CE are altitudes of ΔABC such that BD = CE.*

(i) State three pairs of equal parts in ΔCBD and ΔBCE.

(ii) Is ΔCBD ≅ ΔBCE? Give reasons.

(iii) Is ∠DCB = ∠EBC? Why?



Solution.

(*i*) In ΔCBD and ΔBCE, three pairs of equal parts are:

∠BDC = ∠CEB (= 90° each, as BD and CE are altitudes)

BD = CE (given)

BC = CB (common)

(*ii*) From part (*i*), ΔCBD ≅ ΔBCE (by RHS rule of congruency)

(Note that B ↔ C, C ↔ B and D ↔ E)

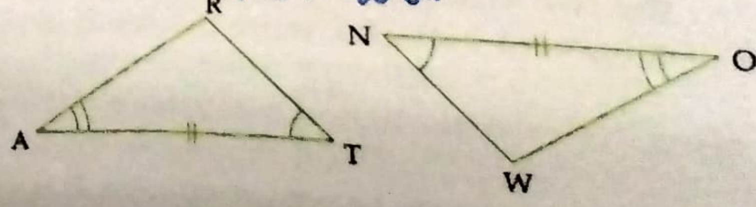
(*iii*) ∠DCB = ∠EBC (corresponding parts of congruent triangles)

Exercise 12.2

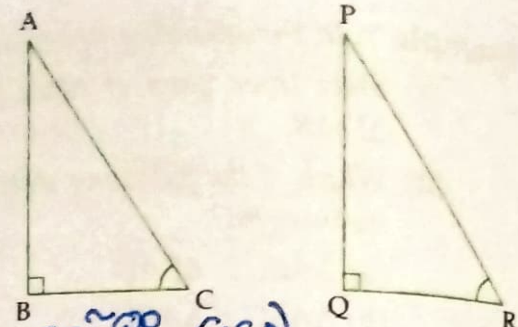
1. You want to establish ΔDEF ≅ ΔMNP, using ASA rule of congruence. You are given that ∠D = ∠M and ∠F = ∠P. What additional information is needed to establish the congruence?

2. In the adjoining figure, two triangles are congruent. The corresponding parts are marked. We can write ΔRAT ≅ ?

rat = won



3. If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?

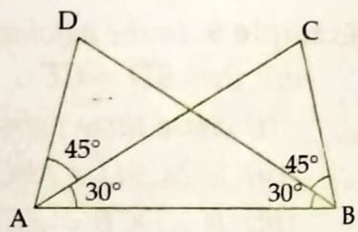


BC≅QR (ASA)

4. Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by ASA congruence rule. In case of congruence, write it in symbolic form.

ΔDEF	ΔPQR
(*i*) $\angle D = 60°$, $\angle F = 80°$, DF = 5 cm	$\angle Q = 60°$, $\angle R = 80°$, QR = 5 cm
(*ii*) $\angle D = 60°$, $\angle F = 80°$, DF = 6 cm	$\angle Q = 60°$, $\angle R = 80°$, QP = 6 cm
(*iii*) $\angle E = 80°$, $\angle F = 30°$, EF = 5 cm	$\angle P = 80°$, PQ = 5 cm, $\angle R = 30°$

5. In the adjoining figure, measures of some parts are indicated.

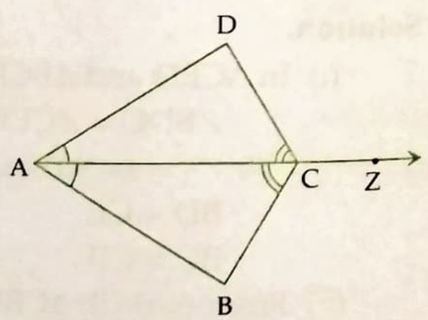


(*i*) State three pairs of equal parts in triangles ABC and ABD.

(*ii*) Is $\Delta ABC \cong \Delta BAD$? Give reasons.

(*iii*) Is BC = AD? Why?

6. In the adjoining figure, ray AZ bisects $\angle DAB$ as well as $\angle DCB$.



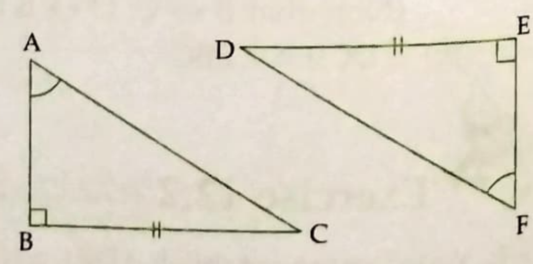
(*i*) State the three pairs of equal parts in triangles BAC and DAC.

(*ii*) Is $\Delta BAC \cong \Delta DAC$? Give reasons.

(*iii*) Is AB = AD? Justify your answer.

(*iv*) Is CD = CB? Give reasons.

7. Explain why $\Delta ABC \cong \Delta FED$?



[**Hint.** $\angle C = 180° - \angle B - \angle A$

$= 180° - 90° - \angle A = 90° - \angle A,$

$\angle D = 180° - \angle E - \angle F$

$= 180° - 90° - \angle A$ $\quad$ ($\because \angle A = \angle F$)

$= 90° - \angle A$

$\Rightarrow \angle C = \angle D$.]

8. Given below are the measurements of some parts of triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In case of congruent triangles, write the result in symbolic form:

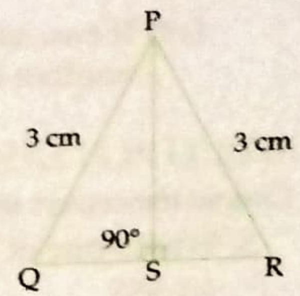
ΔABC	ΔPQR
(*i*) $\angle B = 90°$, AC = 8 cm, AB = 3 cm	(*ii*) $\angle P = 90°$, PR = 3 cm, QR = 8 cm
(*ii*) $\angle A = 90°$, AC = 5 cm, BC = 9 cm	(*ii*) $\angle Q = 90°$, PR = 8 cm, PQ = 5 cm

9. In the adjoining figure, measurements of some parts are given.

(i) State the three pairs of equal parts in ΔPQS and ΔPRS.

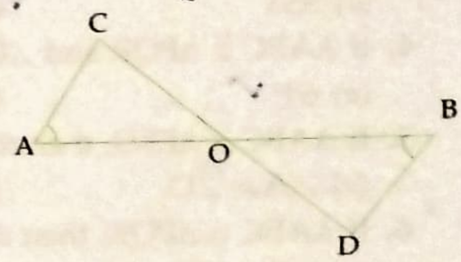
(ii) Is $\Delta PQS \cong \Delta PRS$? Give reasons.

(iii) Is S mid-point of $\overline{QR}$? Why?



10. In the adjoining figure, O is mid-point of $\overline{AB}$ and $\angle A = \angle B$. Show that $\Delta AOC \cong \Delta BOD$.

[Hint. $\angle AOC = \angle BOD$ (vertically opposite angles) Also $AO = OB$. Use ASA congruence rule.]



Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:

(i) Two line segments are congruent if ....

(ii) Among two congruent angles, one has a measure of 63°; the measure of the other angle is ....

(iii) When we write $\angle A = \angle B$, we actually mean ....

(iv) The side included between $\angle M$ and $\angle N$ of ΔMNP is ....

(v) The side QR of ΔPQR is included between angles ....

(vi) It two triangles ABC and PQR are congruent under the correspondence $A \leftrightarrow R$, $B \leftrightarrow P$ and $C \leftrightarrow Q$, then in symbolic form it can be written as $\Delta ABC \cong$ ....

(vii) If $\Delta DEF \cong \Delta SRT$, then the correspondence between vertices is ....

2. State whether the following statements are true (T) or false (F):

(i) All circles are congruent.

(ii) Circles having equal radii are congruent.

(iii) Two congruent triangles have equal areas and equal perimeters.

(iv) Two triangles having equal areas are congruent.

(v) Two squares having equal areas are congruent.

(vi) Two rectangles having equal areas are congruent.

(vii) All acute angles are congruent.

(viii) All right angles are congruent.

(ix) Two figures are congruent if they have same shape.

(x) A two rupee coin is congruent to a five rupee coin.

(xi) All equilateral triangles are congruent.

(xii) Two equilateral triangles having equal perimeters are congruent.

(xiii) If two legs of one right angle triangle are equal to two legs of another right angle triangle, then the two triangles are congruent by SAS rule.

(xiv) If three angles of two triangles are equal, then triangles are congruent.