11

TRIANGLE AND ITS PROPERTIES

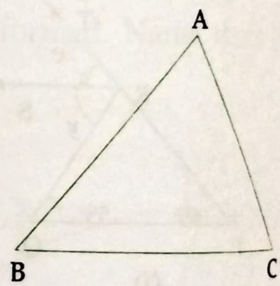
INTRODUCTION

In class VI, you have already learnt about triangles and the types of triangles on the basis of sides and also on the basis of angles. In this chapter, we shall review, revise and strengthen these. We shall also learn the following properties of a triangle:

- Exterior angle of a triangle and its property
- Angle sum property of a triangle
- Angle property of special triangles
- Sum of lengths of two sides of a triangle
- Pythagoras property of a right angled triangle.

TRIANGLE

A **triangle** *is a simple closed curve made of three line segments.*



In the adjoining figure, ABC is a triangle. Usually, triangle ABC is written as ΔABC. It has three sides – $\overline{AB}, \overline{BC}, \overline{CA}$. It has three angles – $\angle BAC$, $\angle ABC$, $\angle ACB$. It has three vertices – A, B, C.

Thus, a triangle has three sides and three angles, and all the three sides and all the three angles are called **six elements** of the triangle ABC. In the above figure, look at the vertex A. It is the point of intersection of the sides AB and AC, BC is the remaining side. We say that vertex A and side BC are opposite to each other. Also $\angle A$ and side BC are opposite to each other.

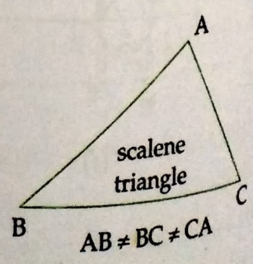
Similarly, vertex B and side CA are opposite to each other; $\angle B$ and side CA are opposite to each other. Same can be said about vertex C, $\angle C$ and side AB.

Types of triangles on the basis of sides

(i) Scalene triangle

If all the sides of a triangle are unequal, it is called a ***scalene triangle.***

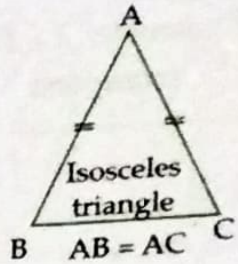
In the adjoining diagram, $AB \neq BC \neq CA$, so ΔABC is a scalene triangle.



(ii) **Isosceles triangle**

If any two sides of a triangle are equal, it is called an ***isosceles triangle.***

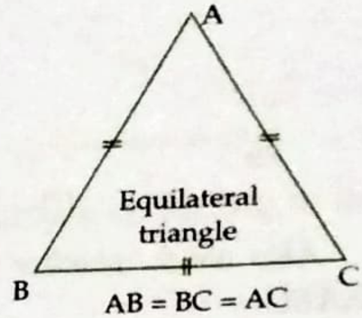
In the adjoining diagram, AB = AC, so ΔABC is an isosceles triangle. Usually, equal sides are indicated by putting marks on each of them.



(iii) **Equilateral triangle**

If all the three sides of a triangle are equal, it is called an ***equilateral triangle.***

In the adjoining diagram, AB = BC = AC, so ΔABC is an equilateral triangle.

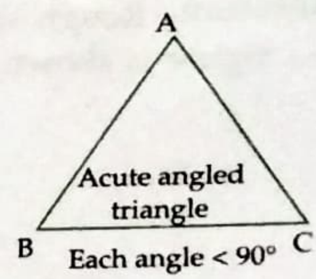


Types of triangles on the basis of angles

(i) **Acute angled triangle**

If all the three angles of a triangle are acute (less than 90°), it is called an ***acute angled triangle.***

In the adjoining diagram, each angle is less than 90°, so ΔABC is an acute angled triangle.

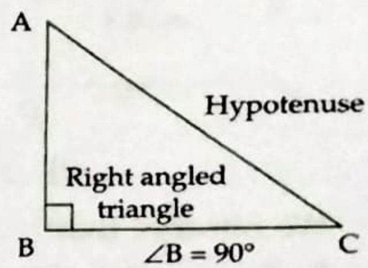


(ii) **Right angled triangle**

If one angle of a triangle is right angle (= 90°), it is called a ***right angled triangle.***

In a right angled triangle, the side opposite to right angle is called **hypotenuse.**

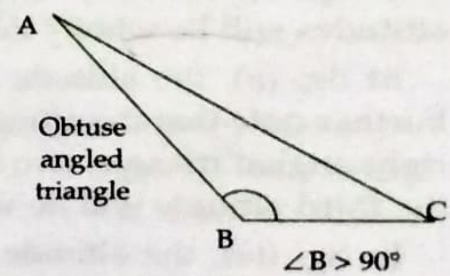
In the adjoining diagram, $\angle B = 90°$, so ΔABC is a right angled triangle and side AC is the hypotenuse.



(iii) **Obtuse angled triangle**

If one angle of a triangle is obtuse (greater than 90°), it is called an ***obtuse angled triangle.***

In the adjoining diagram, $\angle B$ is obtuse (greater than 90°), so ΔABC is an obtuse angled triangle.

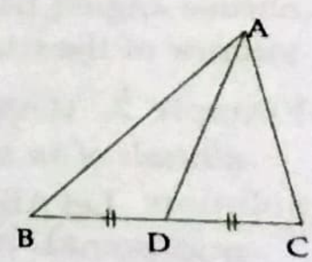


Medians of a triangle

The line segment joining a vertex of a triangle to the mid-point of the opposite side is called a ***median*** *of the triangle.*

In adjoining diagram, AD is median from A to the side BC. A median lies wholly in the interior of a triangle.

A triangle has three medians.

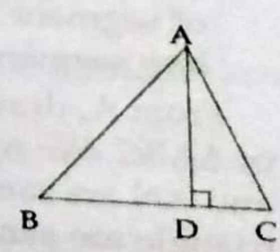


Altitudes of a triangle

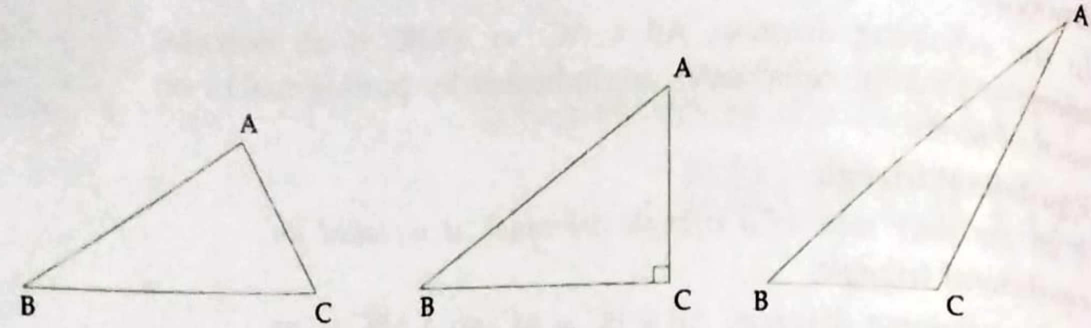
The perpendicular line segment from a vertex of a triangle to the opposite side is called an ***altitude*** *of the triangle.*

In adjoining diagram, AD is an altitude from A to the side BC.

A triangle has three altitudes.



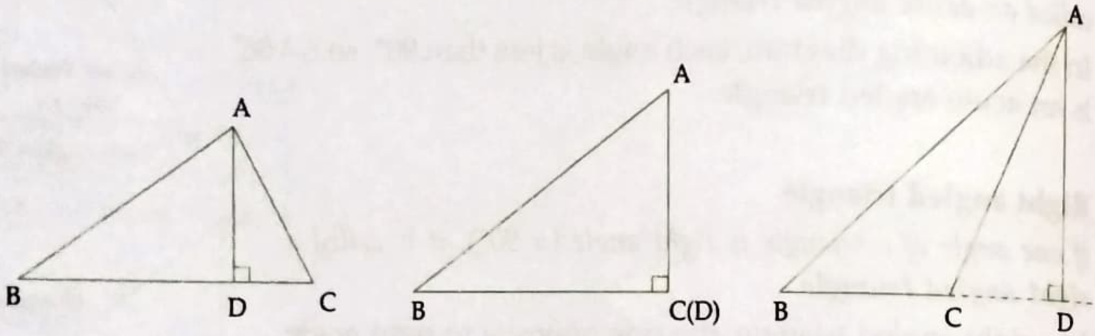
Example 1. *Draw rough sketches of altitudes from vertex A to side BC for the following triangles:*



(*i*) (*ii*) (*iii*)

Also check whether the altitude lies in the interior or the exterior or it is a side itself of ΔABC.

Solution. Rough sketches of altitude AD from vertex A to side BC of ΔABC in each figure is shown below:



(*i*) (*ii*) (*iii*)

We observe that:

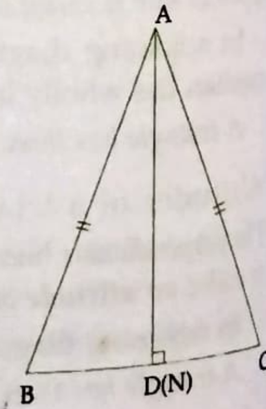
In fig. (*i*), the altitude AD lies wholly in the interior of ΔABC. In fact, all the three altitudes will lie wholly in the interior of an acute angled triangle.

In fig. (*ii*), the altitude AD coincides with the side AC *i.e.* it is the side AC itself. Further note that the altitude from vertex B to side AC is the side BC itself. In fact, in a right angled triangle, two sides containing the right angle are themselves altitudes and the third altitude will lie wholly in the triangle.

In fig. (*iii*), the altitude AD lies completely in the exterior of ΔABC. In fact, in an obtuse angled triangle, two altitudes lie in the exterior and one altitude will lie in the interior of the triangle.

Example 2. *Verify by drawing a diagram whether a median and an altitude of an isosceles triangle can be same.*

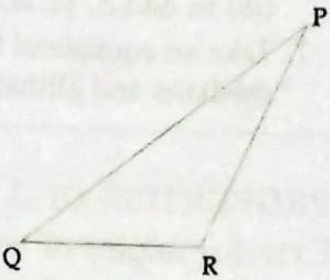
Solution. Let ABC be an isosceles triangle with AB = AC. Find the mid-point D of side BC (by locating the perpendicular bisector of segment BC by paper folding or by construction). Then the line segment joining points A and D is a median of ΔABC.



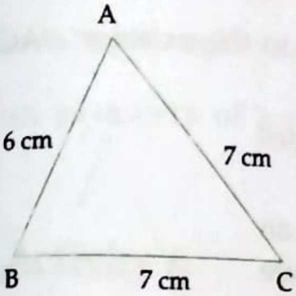
From A, draw AN perpendicular to BC. Then AN is an altitude of ΔABC. We note that the point N coincides with point D *i.e.* AD and AN are same. Hence, a median and an altitude of an isosceles triangle are same.

Exercise 11.1

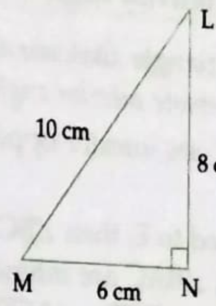
1. In the adjoining figure:
   (i) Name the vertex opposite to side PQ.
   (ii) Name the side opposite to vertex Q.
   (iii) Name the angle opposite to side QR.
   (iv) Name the side opposite to ∠R.



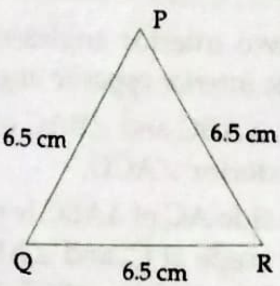
2. Look at the figures given below and classify each of the triangle according to its
   (a) sides (b) angles
   (You may judge the nature of the angle by observation):



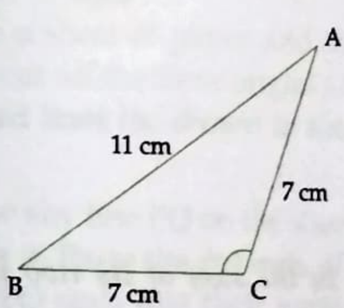
(i)



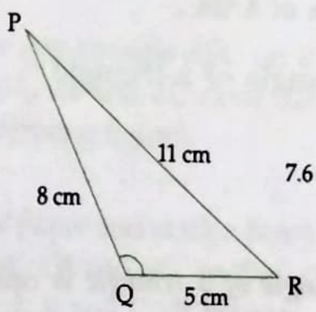
(ii)



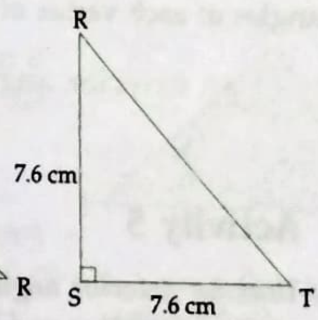
(iii)



(iv)

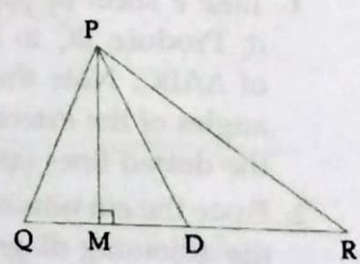


(v)



(vi)

3. In the adjoining ΔPQR, if D is the mid-point of $\overline{QR}$, then
   (i) $\overline{PM}$ is ..... (ii) $\overline{PD}$ is .....
   Is QM = MR?

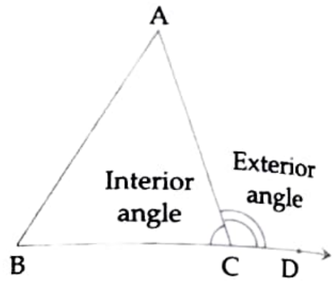


4. Will an altitude always lie in the interior of a triangle? If no, draw a rough sketch to show such a case.
5. Can you think of a triangle in which two altitudes of the triangle are its sides? If yes, draw a rough sketch to show such a case.

6. Draw rough sketches for the following:
   (i) In ΔABC, BE is a median of the triangle.
   (ii) In ΔPQR, PQ and PR are altitudes of the triangle.
   (iii) In ΔXYZ, YL is an altitude in the exterior of the triangle.
7. Take an equilateral triangle and draw its medians and altitudes and check that the medians and altitudes are same.

PROPERTIES OF A TRIANGLE

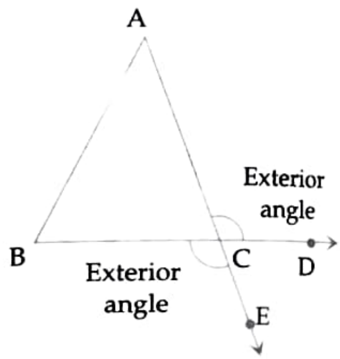
Exterior angles of a triangle



Let ABC be a triangle and its side BC be produced to D, then $\angle ACD$ is called an **exterior angle** at C. Notice that $\angle BCA$ and $\angle ACD$ are adjacent supplementary angles. Thus:

An exterior angle + adjacent interior angle = 180°.

The two interior angles of the triangle that are opposite to the exterior $\angle ACD$ are called its *interior opposite angles or remote interior angles.*



Thus, $\angle ABC$ and $\angle BAC$ of ΔABC are interior opposite angles of the exterior $\angle ACD$.

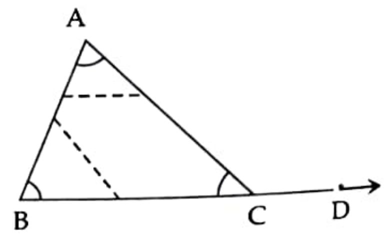
If the side AC of ΔABC is produced to E, then $\angle BCE$ is also an exterior angle at C, and $\angle ABC$ and $\angle BAC$ are interior opposite angles of the exterior $\angle BCE$. Note that $\angle BCE = \angle ACD$ (vertically opposite angle). In fact, six exterior angles are formed–two exterior angles at each vertex of ΔABC.

Property of an exterior angle of a triangle

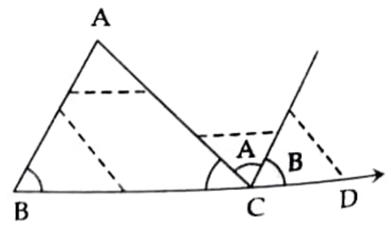
Activity 5

To verify that an exterior angle of a triangle is equal to the sum of its two interior opposite angles by cutting and pasting.

Steps

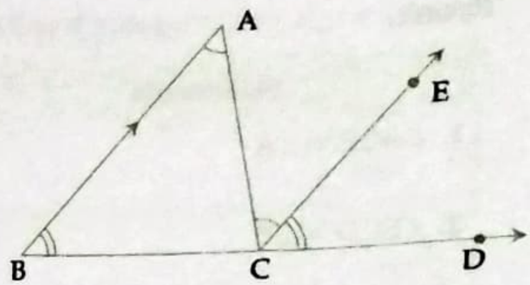


1. Take a sheet of paper and draw any triangle ABC on it. Produce BC to D, then $\angle ACD$ is an exterior angle of ΔABC. Note that $\angle A$ and $\angle B$ are interior opposite angles of the exterior $\angle ACD$. Cut off $\angle A$ and $\angle B$ along the dotted lines (as shown in the adjoining diagram).



2. Paste the cut outs of $\angle A$ and $\angle B$ on $\angle ACD$ as shown in the adjoining diagram. We note that the cut out of $\angle A$ and $\angle B$ exactly cover $\angle ACD$. It follows that
   $\angle ACD = \angle A + \angle B$.
   Hence, an exterior angle of a triangle = sum of its two interior opposite angles.

- *An exterior angle of a triangle is equal to sum of its two interior opposite angles.*



Given. A triangle ABC and ∠ACD is an exterior angle.

To prove. ∠ACD = ∠A + ∠B.

Construction. Through C, draw CE parallel to BA.

Proof.

Statements	Reasons
1. ∠ACE = ∠A	1. CE ∥ BA and AC is a transversal, alternate interior angles are equal.
2. ∠ECD = ∠B	2. CE ∥ BA and BC is a transversal, corresponding angles are equal.
3. ∠ACD = ∠ACE + ∠CED	3. From figure.
4. ∠ACD = ∠A + ∠B	4. Using 1 and 2

Thus, an exterior angle = sum of two interior opposite angles,

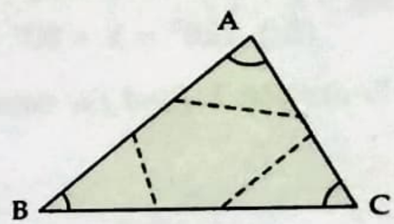
Angle sum property of a triangle

Activity 6

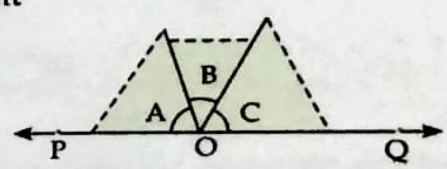
To verify that sum of angles of a triangle is 180° by cutting and pasting.

Steps

1. Take a sheet of paper and draw any triangle ABC on it and cut off the three angles *i.e.* ∠A, ∠B and ∠C along the dotted lines (as shown in the adjoining figure).



2. Draw any line PQ on the sheet of paper and mark a point O on it. Paste the cut outs of ∠A, ∠B and ∠C on the line PQ such that their vertices A, B and C all fall at the point O (as shown in the adjoining figure).
   Note that the outer arms of ∠A and ∠C coincide with the line PQ. The three angles now constitute one angle *i.e.* ∠POQ.
   But ∠POQ is a straight angle,
   ∴ ∠A + ∠B + ∠C = 180°.

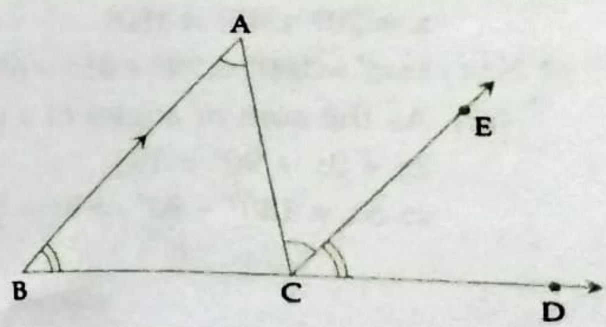


- *Sum of angles of a triangle is 180°.*

Given. A triangle ABC.

To prove. ∠A + ∠B + ∠C = 180°.

Construction. Produce BC to D and through C, draw CE parallel to BA.

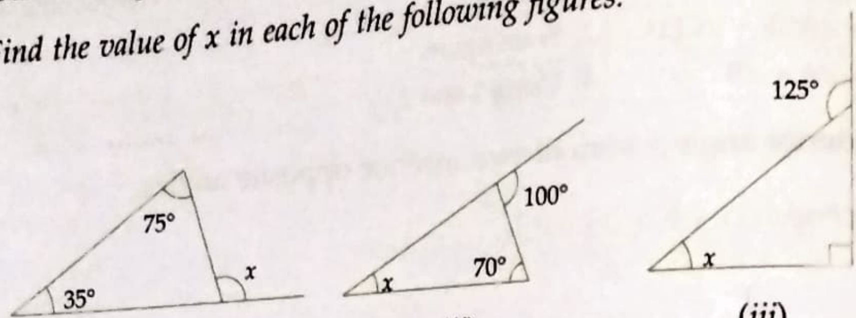


Proof.

Statements	Reasons
1. $\angle ACE = \angle A$	1. CE ∥ BA and AC is a transversal, alternate interior angles are equal.
2. $\angle ECD = \angle B$	2. CE ∥ BA and BC is a transversal, corresponding angles are equal.
3. $\angle C + \angle ACE + \angle ECD = 180°$	3. BCD is a straight line, sum of angles at a point on one side of a line = 180°.
4. $\angle C + \angle A + \angle B = 180°$	4. Using 1 and 2.

Thus, the sum of angles of a triangle is 180°.

Example 1. *Find the value of x in each of the following figures:*



(i) (ii) (iii)

Solution. An exterior angle of a triangle = sum of its two interior opposite angles.

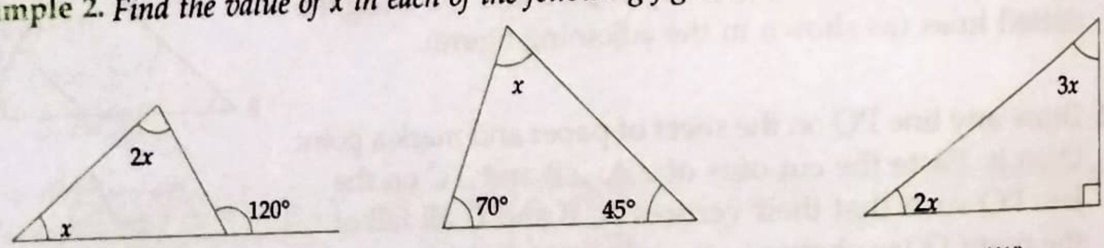
(i) $x = 35° + 75° \Rightarrow x = 110°$.

(ii) $100° = x + 70° \Rightarrow x = 100° - 70° = 30°$.

(iii) $125° = x + 90° \Rightarrow x = 125° - 90° = 35°$.

State which geometrical fact is used to find angle

Example 2. *Find the value of x in each of the following figures:*



(i) (ii) (iii)

Solution. (i) As an exterior angle of a triangle = sum of its two interior opposite angles,

$x + 2x = 120° \Rightarrow 3x = 120° \Rightarrow x = 40°$.

(ii) As the sum of angles of a triangle is 180°,

$x + 70° + 45° = 180°$

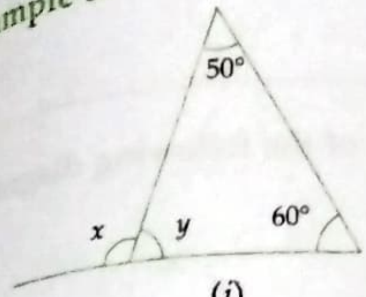
$\Rightarrow x = 180° - 70° - 45° = 65°$.

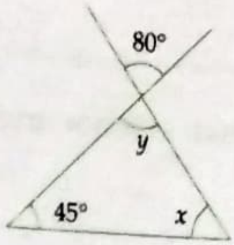
(iii) As the sum of angles of a triangle is 180°,

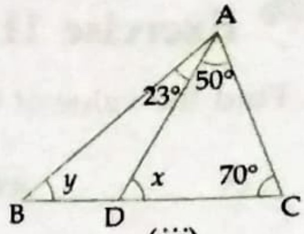
$2x + 3x + 90° = 180°$

$\Rightarrow 5x = 180° - 90° \Rightarrow 5x = 90° \Rightarrow x = 18°$.

Example 3. *Find the values of x and y in each of the following figures:*







Solution. (*i*) As an exterior angle = sum of two interior opposite angles,

$x = 50° + 60° \Rightarrow x = 110°$.

Also $x + y = 180°$ (linear pair)

$\Rightarrow$ $110° + y = 180° \Rightarrow y = 180° - 110° = 70°$.

(*ii*) $y = 80°$ (vertically opposite angles are equal)

Also $x + y + 45° = 180°$ (sum of angles of a triangle = 180°)

$\Rightarrow$ $x + 80° + 45° = 180°$

$\Rightarrow$ $x = 180° - 80° - 45° = 55°$.

(*iii*) In ΔADC, the sum of angles = 180°,

$\therefore$ $x + 50° + 70° = 180°$

$\Rightarrow$ $x = 180° - 50° - 70° = 60°$

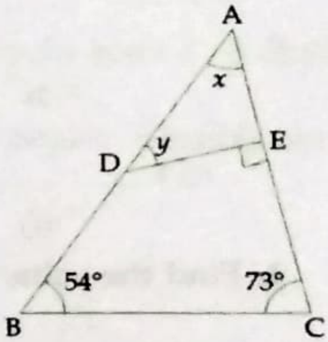
For ΔABD, $\angle x$ is an exterior angle

$\therefore$ $x = 23° + y$ ($\because$ an exterior angle = sum of two interior opposite angles)

$\Rightarrow$ $60° = 23° + y$ ($\because x = 60°$)

$\Rightarrow$ $y = 60° - 23° = 37°$.

Example 4. *In the adjoining figure, find the values of x and y.*



Solution. In ΔABC, sum of angles = 180°,

$\therefore$ $x + 54° + 73° = 180°$

$\Rightarrow$ $x = 180° - 54° - 73° = 53°$.

For ΔADE, $\angle E$ is an exterior angle

$\therefore$ $x + y = 90°$ ($\because$ an exterior angle = sum of two interior opposite angles)

$\Rightarrow$ $53° + y = 90°$ ($\because x = 53°$)

$\Rightarrow$ $y = 90° - 53° = 37°$

Hence, $x = 53°$ and $y = 37°$.

Example 5. *If the angles of a triangle are in the ratio 2 : 3 : 4, find all the angles.*

Solution. As the angles of a triangle are in the ratio 2 : 3 : 4, let the angles be $2x$, $3x$ and $4x$.

Since the sum of angles of a triangle is 180°,

$2x + 3x + 4x = 180°$

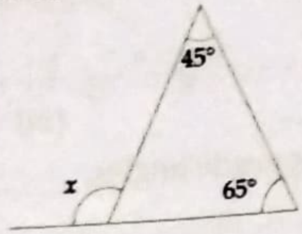
$\Rightarrow$ $9x = 180° \Rightarrow x = 20°$.

$\therefore$ $2x = 2 \times 20° = 40°$, $3x = 3 \times 20° = 60°$ and $4x = 4 \times 20° = 80°$.

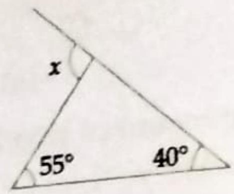
Hence, the angles of the triangle are 40°, 60° and 80°.

Exercise 11.2

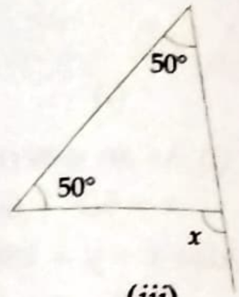
1. Find the value of the unknown exterior angle x in each of the following diagrams:



(i)

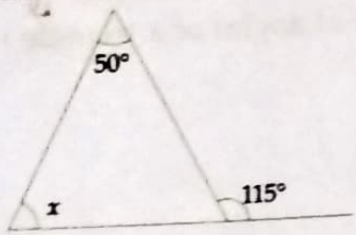


(ii)

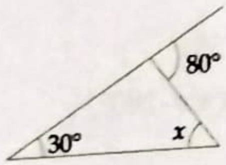


(iii)

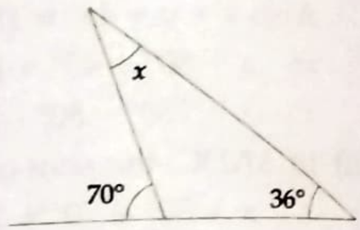
2. Find the value of the unknown interior angle x in each of the following diagrams:



(i)

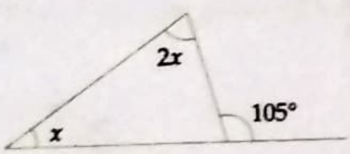


(ii)

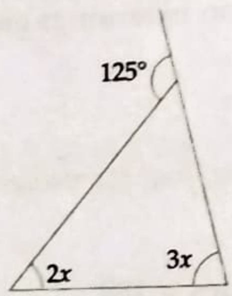


(iii)

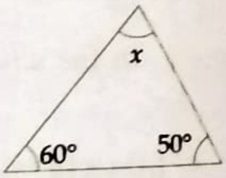
3. Find the value of x in each of the following diagrams:



(i)

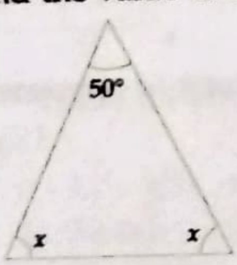


(ii)

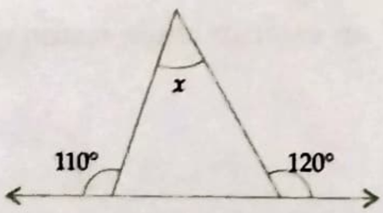


(iii)

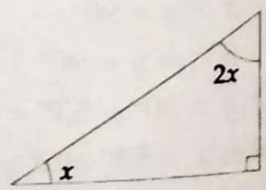
4. Find the value of unknown x in each of the following diagrams:



(i)



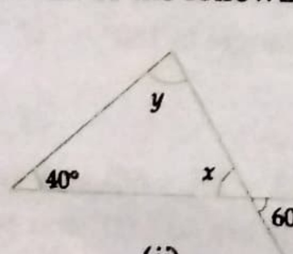
(ii)



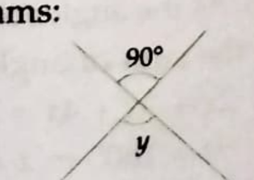
(iii)

5. Find the values of x and y in each of the following diagrams:

(i)

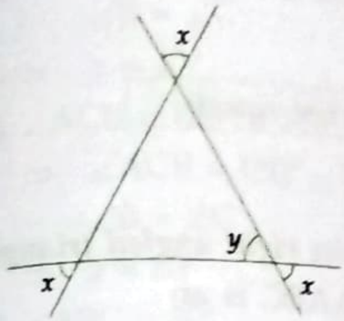


(ii)

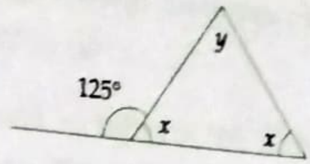


(iii)

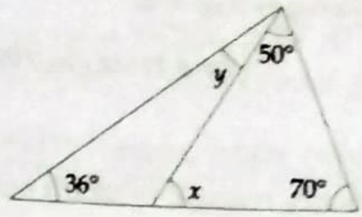
6. Find the values of x and y in each of the following diagrams:



(i)



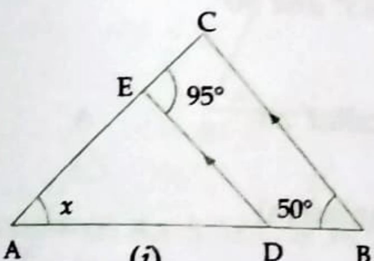
(ii)



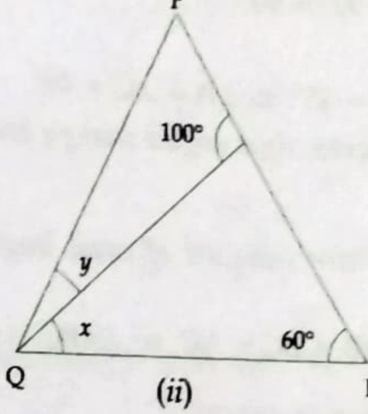
(iii)

√

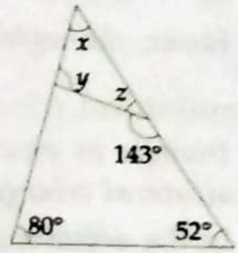
7. (a) In the fig. (i) given below, DE ∥ BC. Find the value of x.
   (b) In the fig. (ii) given below, $\angle P = 3y$. Find the values of x and y.
   (c) In the fig. (iii) given below, find the size of each lettered angle.



(i)



(ii)



(iii)

8. One of the angles of a triangle measures 80° and the other two angles are equal. Find the measure of each of the equal angles.
9. If one angle of a triangle is 60° and the other two angles are in the ratio 2 : 3, find these angles.
10. If the angles of a triangle are in the ratio 1 : 2 : 3, find the angles. Classify the triangle in two different ways.
11. Can a triangle have three angles whose measures are

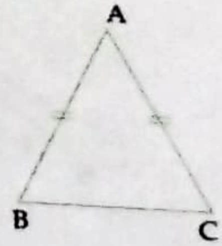
    (i) 65°, 74°, 39°? (ii) $\frac{1}{3}$ right angle, 1 right angle, 60°?

    [**Hint.** Find the sum of angles.]

Angle property of special triangles

Isosceles triangle

A triangle in which lengths of two sides are equal is called an ***isosceles triangle.***



In the adjoining figure, AB = AC, so ΔABC is an isosceles triangle. The angles which are opposite equal sides are called *base angles* and ∠A is called *vertical angle*.

Make a replica of ∠B and place it over ∠C. We observe that replica of ∠B exactly covers ∠C. It follows that ∠B = ∠C *i.e.* the base angles are equal. It is an important result to remember.

In a triangle, the angles opposite equal sides are equal.

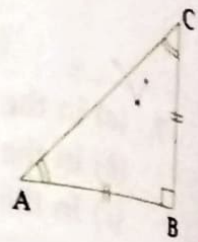
Converse is also true.

In a triangle, the sides opposite equal angles are equal.

Isosceles right angled triangle

An isosceles triangle in which one angle is 90° is called an isosceles ***right angled triangle.***

In the adjoining diagram, AB = BC and $\angle B = 90°$, so ΔABC is an isosceles right angled triangle.



As AB = BC, $\angle A = \angle C$ (angles opposite equal sides are equal)

Since sum of angles of a triangle = 180°,

$$\angle A + \angle C + 90° = 180°$$

$$\Rightarrow \quad \angle A + \angle C = 180° - 90° = 90°$$

$$\Rightarrow \quad \angle A + \angle A = 90° \qquad (\because \angle A = \angle C)$$

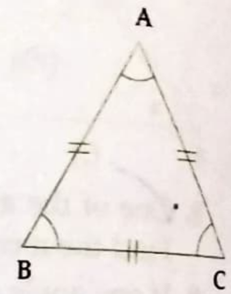
$$\Rightarrow \quad 2\angle A = 90° \Rightarrow \angle A = 45° \Rightarrow \angle A = \angle C = 45°.$$

Hence, the angles of an isosceles right angled triangle are 45°, 45° and 90°.

Equilateral triangle

A triangle in which all the three sides are of equal length is called an ***equilateral triangle.***

In the adjoining figure, AB = BC = AC, so ΔABC is an equilateral triangle.



As AB = BC = AC,

$\angle A = \angle B = \angle C$ (angles opposite equal sides are equal)

Since sum of angles of a triangle = 180°,

$$\angle A + \angle B + \angle C = 180°$$

$$\Rightarrow \quad \angle A + \angle A + \angle A = 180° \qquad (\because \angle A = \angle B = \angle C)$$

$$\Rightarrow \quad 3\angle A = 180° \Rightarrow \angle A = 60°$$

$$\Rightarrow \quad \angle A = \angle B = \angle C = 60°.$$

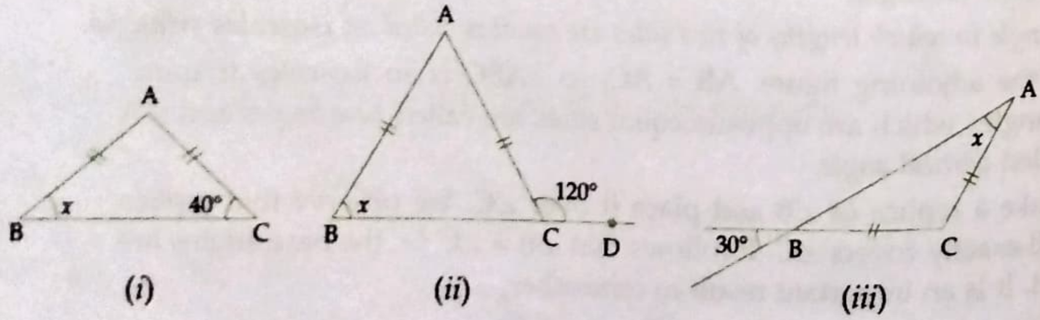
Thus, we have:

In an equilateral triangle, all the three angles are equal and each angle = 60°.

Converse is also true.

If in a triangle, all the three angles are equal (each = 60°), then it is an equilateral triangle.

Example 1. *Find the value of x in each of following figures:*



(i) (ii) (iii)

Solution. (*i*) As ABC is an isosceles triangle with AB = AC,

$\angle B = \angle C$ (angles opposite equal sides are equal)

$\Rightarrow \; x = 40°$ ($\because \angle C = 40°$ given)

(*ii*) $\angle ACB + 120° = 180°$ (linear pair)

$\Rightarrow \; \angle ACB = 180° - 120° = 60°$ *i.e.* $\angle C = 60°$

$\angle B = \angle C$ (angles opposite equal sides are equal)

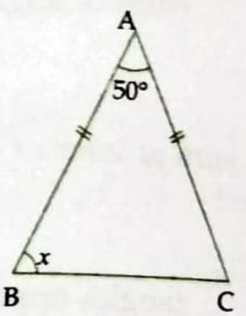
$\Rightarrow \; x = 60°$ ($\because \angle C = 60°$)

(*iii*) $\angle ABC = 30°$ (vertically opposite angles are equal)

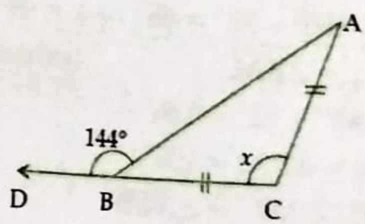
$\angle A = \angle ABC$ (angles opposite equal sides are equal)

$\Rightarrow \; x = 30°$ ($\because \angle ABC = 30°$)

Example 2. *Find the value of x in each of the following figures:*



(*i*)



(*ii*)

Solution. (*i*) As ΔABC is an isosceles triangle with AB = AC,

$\angle C = \angle B$ (angles opposite equal sides are equal)

$\Rightarrow \; \angle C = x$

$x + x + 50° = 180°$ (sum of angles in a triangle)

$\Rightarrow \; 2x = 180° - 50° = 130°$

$\Rightarrow \; x = 65°.$

(*ii*) $144° + \angle ABC = 180°$ (linear pair)

$\Rightarrow \; \angle ABC = 180° - 144° = 36°.$

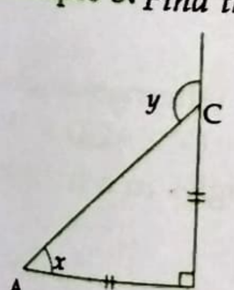
$\angle BAC = \angle ABC$ (angles opposite equal sides are equal)

$\Rightarrow \; \angle BAC = 36°$ ($\because \angle ABC = 36°$)

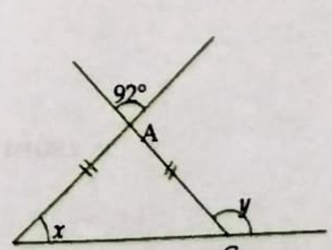
$x + 36° + 36° = 180°$ (sum of angles in a triangle)

$\Rightarrow \; x = 180° - 36° - 36° = 108°.$

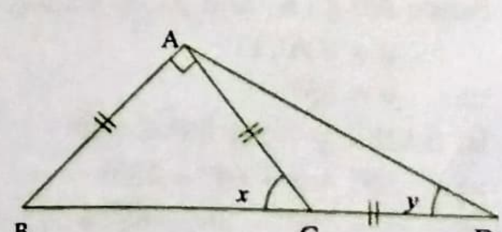
Example 3. *Find the values of x and y in each of the following figures:*



(*i*)



(*ii*)



(*iii*)

Solution. (*i*) $\angle ACB = \angle CAB$ (angles opposite equal sides are equal)

$\Rightarrow$ $\angle ACB = x$ ($\because \angle CAB = x$)

$\angle CAB + \angle ACB + \angle ABC = 180°$. (sum of angles of a triangle)

$\Rightarrow$ $x + x + 90° = 180°$

$\Rightarrow$ $2x = 180° - 90° = 90° \Rightarrow x = 45°$.

$\Rightarrow$ $y = x + 90°$ (exterior angle = sum of interior opposite angles)

$\Rightarrow$ $y = 45° + 90° = 135°$.

Hence, $x = 45°$, $y = 135°$.

(*ii*) $\angle BAC = 92°$ (vertically opposite angles are equal)

$\angle ACB = \angle ABC$ (angles opposite equal sides are equal)

$\Rightarrow$ $\angle ACB = x$ ($\because \angle ABC = x$)

$\angle ABC + \angle ACB + \angle BAC = 180°$ (sum of angles of a triangle)

$\Rightarrow$ $x + x + 92° = 180°$

$\Rightarrow$ $2x = 180° - 92° = 88° \Rightarrow x = 44°$.

$\Rightarrow$ $y = x + \angle BAC$ (exterior angle = sum of interior opposite angles)

$\Rightarrow$ $y = 44° + 92° = 136°$.

Hence, $x = 44°$, $y = 136°$.

(*iii*) In ΔABC, $\angle ABC = \angle ACB$ (angles opposite equal sides)

$\Rightarrow$ $\angle ABC = x$ ($\because \angle ACB = x$, given)

Also, $x + x + 90° = 180°$ (sum of angles in a triangle)

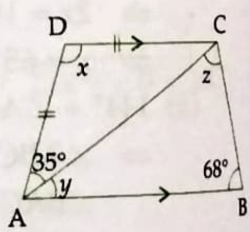
$\Rightarrow$ $2x = 180° - 90° = 90° \Rightarrow x = 45°$

In ΔACD, $\angle CAD = y$ (angles opposite equal sides)

$x = y + y$ (exterior angle = sum of interior opposite angles)

$\Rightarrow$ $45° = 2y \Rightarrow y = 22\frac{1}{2}°$.

Example 4. *In the adjoining figure, AB ∥ DC. If $\angle ABC = 68°$ and $\angle DAC = 35°$, find the values of x, y and z.*



Solution. In ΔACD, $\angle ACD = \angle DAC$

($\because$ angles opposite equal sides are equal)

$\Rightarrow$ $\angle ACD = 35°$ ($\because \angle DAC = 35°$, given)

$\therefore$ $x + \angle ACD + \angle DAC = 180°$ (sum of angles in a triangle)

$\Rightarrow$ $x + 35° + 35° = 180°$

$\Rightarrow$ $x = 180° - 35° - 35° = 110°$.

Since AB ∥ DC and AC is transversal,

$y = \angle ACD$ (alternate angles)

$\Rightarrow$ $y = 35°$ ($\because \angle ACD = 35°$)

In ΔABC, $y + z + 68° = 180°$ (sum of angles in a triangle)

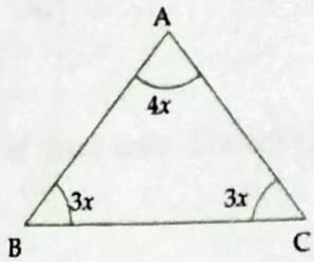
$\Rightarrow$ $35° + z + 68° = 180°$

$\Rightarrow$ $z = 180° - 35° - 68° = 77°$

Hence, $x = 110°$, $y = 35°$ and $z = 77°$.

Example 5. *The ratio between the vertical angle and a base angle of an isosceles triangle is 4 : 3. Find all the angles of the triangle.*

Solution. We know that the base angles of an isosceles triangle are equal.



Since the ratio between the vertical angle and a base angle of an isosceles triangle is 4 : 3, let the vertical angle be $4x$, then each base angle is $3x$.

$\therefore \quad 4x + 3x + 3x = 180°$ (sum of angles in a triangle)

$\Rightarrow \quad 10x = 180°$

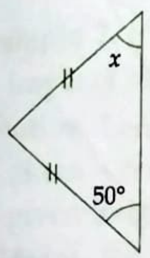
$\Rightarrow \quad x = 18°$

$\Rightarrow \quad 4x = 72°$ and $3x = 54°$.

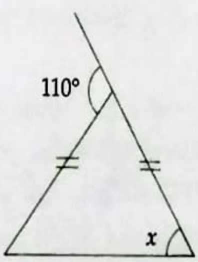
Hence, the angles of the triangle are 72°, 54°, 54°.

Exercise 11.3

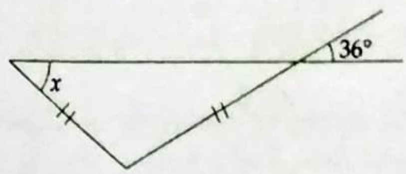
1. Find the value of x in each of the following figures:



(i)

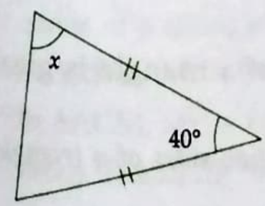


(ii)

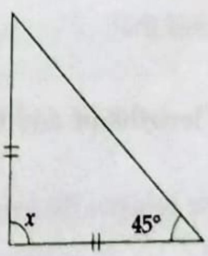


(iii)

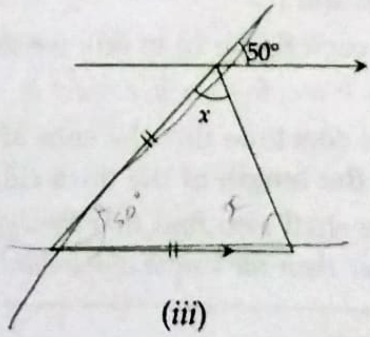
2. Find the value of x in each of the following figures:



(i)

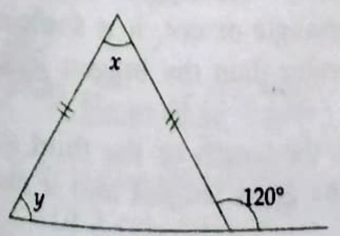


(ii)

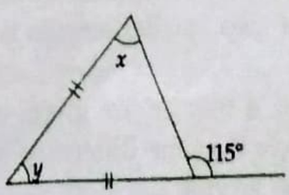


(iii)

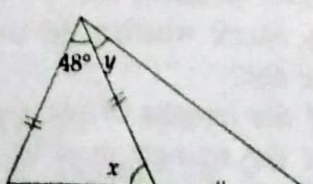
3. Find the values of x and y in each of the following figures:



(i)

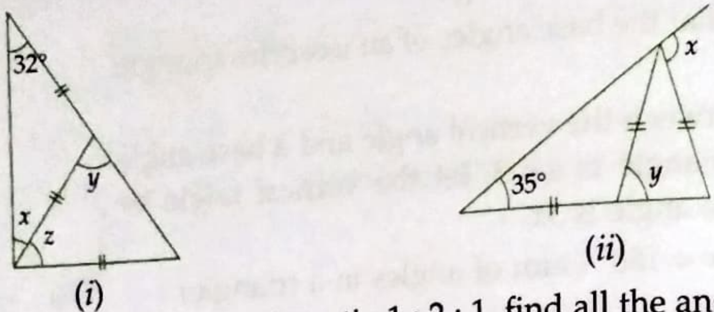


(ii)



(iii)

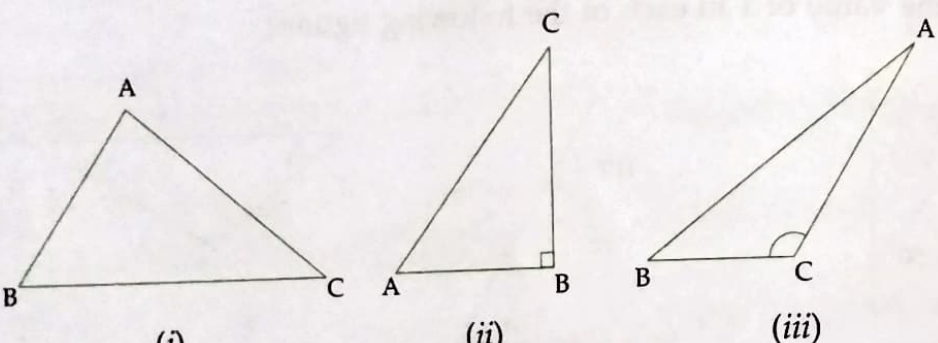
4. Calculate the size of each lettered angle in the following figures :



5. If the angles of a triangle are in the ratio 1 : 2 : 1, find all the angles of the triangle. Classify the triangle in two different ways.
6. In an isosceles triangle, a base angle is four times its vertical angle. Find all the angles of the triangle.

Sum of lengths of two sides of a triangle

Draw any three triangles (say, acute, right angled and obtuse) as shown in the figures below:



Let BC = a, CA = b and AB = c.

Measure the lengths of all the three sides of triangle ABC in each figure *i.e.* the lengths of a, b and c.

In each figure (*i*) to (*iii*), we shall find that:

$a + b > c$, $b + c > a$, $c + a > b$.

We conclude that the **sum of the lengths of any two sides of a triangle is greater than the length of the third side.**

We shall also find that *the difference between the lengths of any two sides of a triangle is smaller than the length of the third side.*

Remarks

- If the lengths of three line segments is given, then identify the biggest length. In order to check whether the given line segments form a triangle or not, it is sufficient to check whether the sum of two smaller lengths is greater than the biggest length or not.
- If the lengths of two sides of a triangle are given, then the length of the third side of the triangle must be greater than the difference of the given lengths and it must be less than the sum of given lengths.

Example 1. *Is it possible to have a triangle with the following sides?*

(i) 3 cm, 6 cm, 7 cm *(ii)* 11 cm, 4 cm, 6 cm.

Solution. *(i)* 3 cm + 6 cm = 9 cm > 7 cm,

6 cm + 7 cm = 13 cm > 3 cm,

7 cm + 3 cm = 10 cm > 6 cm.

Thus, the sum of lengths of any two sides > the length of third side. Therefore, the triangle is possible.

Alternatively

Given lengths are 3 cm, 6 cm and 7 cm.

Note that the biggest length is 7 cm.

Sum of two smaller lengths = 3 cm + 6 cm = 9 cm > the biggest length.

Therefore, the triangle is possible.

(ii) 4 cm + 6 cm = 10 cm < 11 cm.

Thus, the sum of lengths of two sides is not greater than the length of third side.

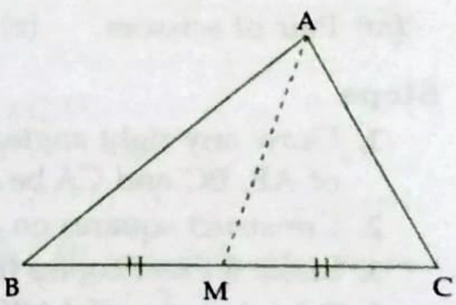
Therefore, the triangle is not possible.

Example 2. *If the lengths of two sides of a triangle are 6 cm and 8 cm, then what can be the length of the third side?*

Solution. Length of third side must be greater than the difference of given length *i.e.* (8 cm – 6 cm) *i.e.* 2 cm. Also the length of third side must be less than the sum of given lengths *i.e.* (6 cm + 8 cm) *i.e.* 14 cm.

Hence, the length of the third side of the triangle must greater than 2 cm and less than 14 cm.

Example 3. *In the given figure, AM is a median of a triangle ABC. Show that AB + BC + CA > 2 AM.*



Solution. We know that the sum of the lengths of two sides of a triangle > the length of third side.

In ΔABM, AB + BM > AM ...*(i)*

In ΔACM, MC + CA > AM ...*(ii)*

From *(i)* and *(ii)*, we get

AB + BM + MC + CA > 2 AM

⇒ AB + (BM + MC) + CA > 2 AM

⇒ AB + BC + CA > 2 AM. (∵ BM + MC = BC)

Exercise 11.4

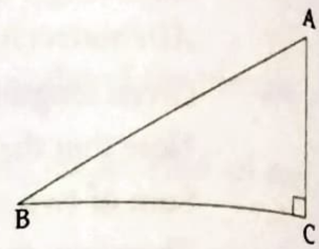
1. Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm no *(ii)* 2.5 cm, 4.5 cm, 8 cm no

(iii) 10.2 cm, 5.8 cm, 4.5 cm yes *(iv)* 3.4 cm, 4.7 cm, 6.2 cm. yes

2. If the lengths of two sides of a triangle are 7 cm and 10 cm, then what can be the length of the third side?
3. We know that in a triangle, the sum of lengths of any two sides is greater than the length of the third side. Is the sum of any angles of a triangle also greater than the third angle? If no, draw a rough sketch to show such a case.

Pythagoras property of a right angled triangle



In a right angled triangle, the sides have special names. The side opposite to the right angle is called **hypotenuse** and the other two sides are called the **legs** of the right angled triangle. Hypotenuse is the longest side. In the adjoining triangle ABC, $\angle C = 90°$. So, AB is its hypotenuse and BC and CA are the two legs.

Pythagoras property

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

or

In a right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

Activity 7

To verify Pythagoras property of a right angled triangle

Materials required

(*i*) Chart paper (*ii*) White sheet of paper (*iii*) Geometry box
(*iv*) Pair of scissors (*v*) Fevistic/gum.

Steps

1. Draw any right angled ΔABC, right angled at C, on a chart paper. Let the lengths of AB, BC and CA be c, a and b units respectively as shown in fig. (*i*).
2. Construct squares on AB, BC, CA respectively as shown in fig. (*i*).
3. Make 8 exact copies (replicas) of ΔABC.
4. Take 4 copies of ΔABC and the squares on side a and b. Paste all these figures on a drawing sheet as shown in fig. (*ii*).
5. Take the remaining 4 copies of ΔABC and the square on side c. Paste all these figures on a drawing sheet as shown in fig. (*iii*).

PYTHAGORAS

Pythagoras, a Greek philospher of sixth century B.C. discovered a very important and useful property of right angled triangles, named Pythagoras property.

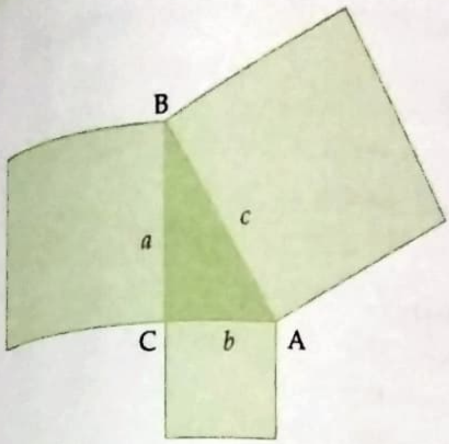


Fig. (*i*)

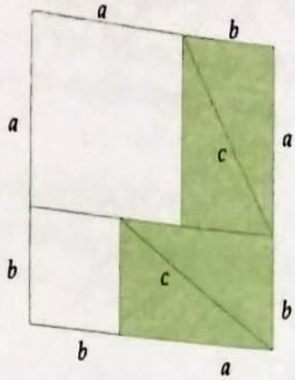


Fig. (*ii*)

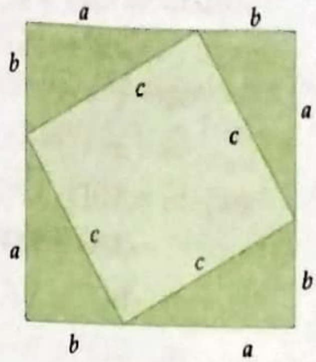


Fig. (*iii*)

Result

We observe that each of the figures as shown in fig. (*ii*) and fig. (*iii*), is a square of side $(a + b)$ units.

$\therefore$ Area of square in fig. (*iii*) = area of square in fig. (*ii*).

Now remove the four triangles from fig. (*iii*) and the four triangles from fig. (*ii*).

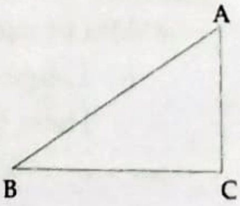
Since the areas of the triangles removed from both figures is the same, therefore, the remaining areas of both figures are equal *i.e.* area of the square on side c = area of square on side a + area of square on side b

$\Rightarrow \quad c^2 = a^2 + b^2.$

It verifies the Pythagoras property of a right angled triangle.

Converse is also true i.e. if Pythagoras property holds for some triangle then the triangle will be a right angled triangle.

Thus, if ABC is a triangle such that $AB^2 = BC^2 + CA^2$, then triangle ABC is a right angled triangle at C.



For example:

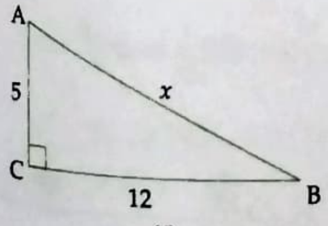
Let AB = 5 cm, BC = 4 cm and AC = 3 cm.

Note that $5^2 = 4^2 + 3^2$ *i.e.* $25 = 16 + 9$ is true, so $AB^2 = BC^2 + CA^2$.

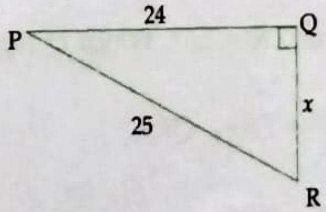
Measure $\angle C$ and check that its measure is 90°.

Hence, ABC is a right angled triangle at C.

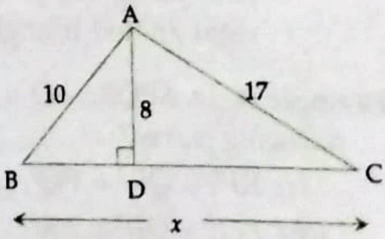
Example 1. *Find the value of x in each of the following figures. All measurements are in centimetres.*



(*i*)



(*ii*)



(*iii*)

Solution. (*i*) In ΔABC, $\angle C = 90°$. By Pythagoras property,

$$AB^2 = BC^2 + CA^2 \Rightarrow x^2 = 12^2 + 5^2$$

$\Rightarrow \quad x^2 = 144 + 25 = 169 \Rightarrow x = 13.$

(*ii*) In ΔPQR, $\angle Q = 90°$. By Pythagoras property,

$PR^2 = PQ^2 + QR^2 \Rightarrow 25^2 = 24^2 + x^2$

$\Rightarrow \quad x^2 = 25^2 - 24^2 \Rightarrow x^2 = 625 - 576$

$\Rightarrow \quad x^2 = 49 \Rightarrow x = 7.$

(*iii*) In ΔABD, $\angle ADB = 90°$. By Pythagoras property,

$AB^2 = BD^2 + AD^2 \Rightarrow 10^2 = BD^2 + 8^2$

$\Rightarrow \quad BD^2 = 10^2 - 8^2 \Rightarrow BD^2 = 100 - 64$

$\Rightarrow \quad BD^2 = 36 \Rightarrow BD = 6.$

As $\angle ADB$ and $\angle ADC$ form a linear pair,

$\angle ADC + \angle ADB = 180° \Rightarrow \angle ADC + 90° = 180° \Rightarrow \angle ADC = 90°.$

In ΔADC, $\angle ADC = 90°$. By Pythagoras property

$AC^2 = AD^2 + DC^2 \Rightarrow 17^2 = 8^2 + DC^2$

$\Rightarrow \quad DC^2 = 17^2 - 8^2 \Rightarrow DC^2 = 289 - 64$

$\Rightarrow \quad DC^2 = 225 \Rightarrow DC = 15.$

$\therefore \quad x = BD + DC = 6 + 15 = 21.$

Example 2. *Which of the following can be the sides of a right angled triangle?*

(*i*) *2.5 cm, 6.5 cm, 6 cm* (*ii*) *4 cm, 5 cm, 6 cm*

In case of a right angled triangle, identify the right angle.

Solution. We check whether Pythagoras property is satisfied or not *i.e.* (longest side)^2 = sum of squares of other two sides.

(*i*) Longest side is 6.5 cm.

Here, $(2.5)^2 + 6^2 = 6.25 + 36 = 42.25;$

$(6.5)^2 = 42.25$

$\therefore \quad (6.5)^2 = (2.5)^2 + 6^2.$

Hence, the given sides form a right angled triangle and the angle opposite to the side of length 6.5 cm is a right angle.

(*ii*) Longest side = 6 cm.

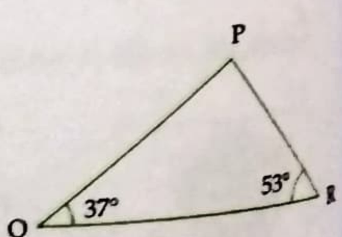
Here, $4^2 + 5^2 = 16 + 25 = 41 \neq 6^2.$

So, Pythagoras property is not satisfied. Hence, the given sides do not form a right angled triangle.

Example 3. *In ΔPQR, $\angle Q = 37°$ and $\angle R = 53°$. Which of the following is true?*

(*i*) $RP^2 + QR^2 = PQ^2$ (*ii*) $PQ^2 + PR^2 = QR^2$

(*iii*) $PQ^2 + QR^2 = PR^2$.



Solution. In ΔPQR, $\angle P + \angle Q + \angle R = 180°$

(sum of angles of a triangle = 180°)

$\Rightarrow \quad \angle P + 37° + 53° = 180°$

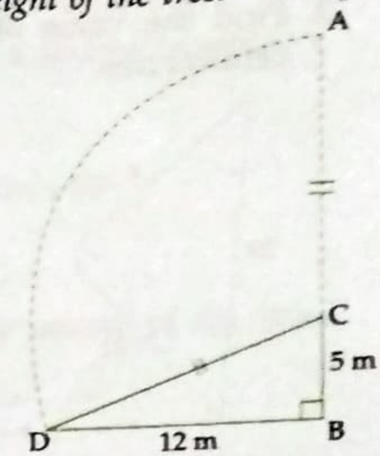
$\Rightarrow \quad \angle P = 180° - 37° - 53° = 90°.$

So, PQR is a right angled triangle at P.

$\therefore$ By Pythagoras property, $PQ^2 + PR^2 = QR^2$.

Hence, (*ii*) is true.

Example 4. *A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.*



Solution. Let AB be the tree. When broken at C, let its top touch the ground at D. Then BC = 5m and BD = 12m. Note that CD = AC. In ΔBCD, $\angle B = 90°$. By Pythagoras property,

$$CD^2 = BD^2 + BC^2$$

$\Rightarrow$ $CD^2 = 12^2 + 5^2$

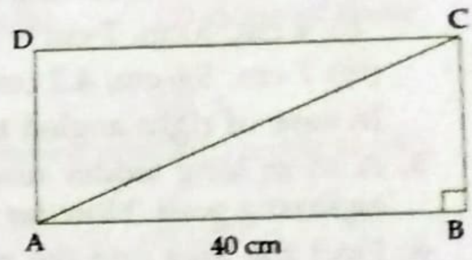
$\Rightarrow$ $CD^2 = 144 + 25 = 169$

$\Rightarrow$ $CD = 13$ m.

$\therefore$ Original height of tree = AB = BC + CA = BC + CD

= 5 m + 13 m = 18 m.

Example 5. *Find the perimeter of the rectangle whose length is 40 cm and length of one diagonal is 41 cm.*



Solution. Let ABCD be a rectangle with length AB = 40 cm and diagonal AC = 41 cm. In ΔABC, $\angle B = 90°$. By Pythagoras property,

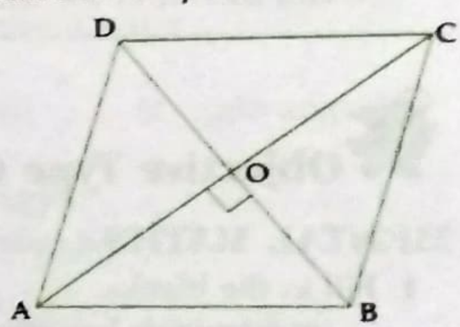
$$AC^2 = AB^2 + BC^2 \Rightarrow 41^2 = 40^2 + BC^2$$

$$BC^2 = 41^2 - 40^2 = 1681 - 1600 = 81$$

$\therefore$ BC = 9 cm

Perimeter of rectangle ABCD = 2(AB + BC) = 2(40 + 9) cm = (2 × 49) cm = 98 cm.

Example 6. *The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.*



Solution. Let ABCD be a rhombus with diagonals BD = 16 cm and AC = 30 cm. Let O be their point of intersection. We know that the diagonals of a rhombus bisect each other at right angles. Then

$$OA = \frac{1}{2}AC = \left(\frac{1}{2}\times 30\right) \text{ cm} = 15 \text{ cm and}$$

$$OB = \frac{1}{2}BD = \left(\frac{1}{2}\times 16\right) \text{ cm} = 8 \text{ cm}.$$

In ΔOAB, $\angle AOB = 90°$. By Pythagoras property,

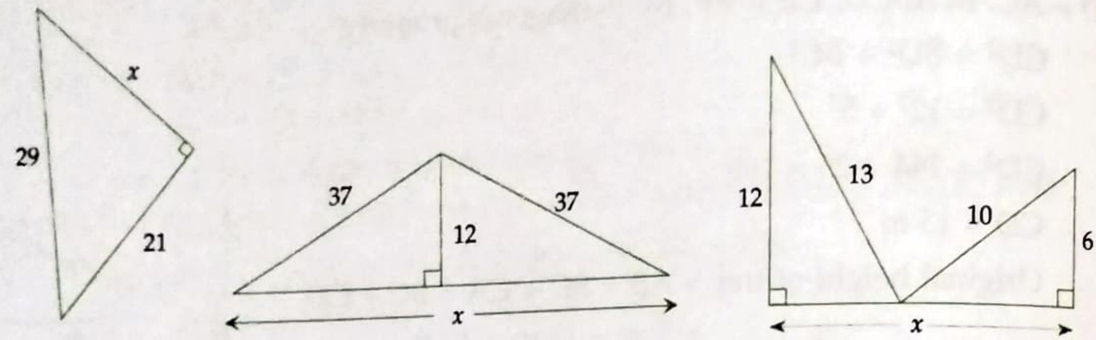
$AB^2 = OA^2 + OB^2 \Rightarrow AB^2 = 15^2 + 8^2 \Rightarrow AB^2 = 225 + 64 = 289$

$\Rightarrow$ AB = 17 cm.

$\therefore$ Perimeter of rhombus = 4 × AB = (4 × 17) cm = 68 cm.

Exercise 11.5

1. PQR is a triangle, right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.
2. ABC is a triangle, right angled at C. If AB = 25 cm and AC = 7 cm, find BC.
3. Find the value of x in each of the following figures. All measurements are in centimetres.



(i) (ii) (iii)

4. Which of the following can be the sides of a right angled triangle?
   (i) 4 cm, 5 cm, 7 cm
   (ii) 1.5 cm, 2 cm, 2.5 cm
   (iii) 7 cm, 5.6 cm, 4.2 cm
   In case of right angled triangles, identify the right angles.
5. A 15 m long ladder reaches a window 12 m high from the ground on placing it against a wall. How far is the foot of the ladder from the wall?
6. Find the area and the perimeter of the rectangle whose length is 15 cm and the length of one diagonal is 17 cm.
7. If the diagonals of a rhombus measure 10 cm and 24 cm, find its perimeter.
8. The side of a rhombus is 5 cm. If the length of one diagonal of the rhombus is 8 cm, then find the length of the other diagonal.

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
   (i) A triangle has atleast .... acute angles.
   (ii) A triangle cannot have more than ... right angle.
   (iii) A triangle cannot have more than ... obtuse angle.
   (iv) In every triangle, the sum of (interior) angles of a triangle = ... right angles.
   (v) In every triangle, an exterior angle + adjacent interior angle = ... degrees.
   (vi) In every triangle, an exterior angle = sum of the ... interior opposite angles.
   (vii) In a right angled triangle, if one of the acute angle measures 25° then the measure of the other acute angle is ......
   (viii) If one of the angles of a triangle is equal to the sum of the other two angles, then the measure of this angle is ... .

2. State whether the following statements are true (T) or false (F):
(i) A triangle can have all three angles with measure greater than 60°.
(ii) A triangle can have all three angles with measure less than 60°.
(iii) If an exterior angle of a triangle is a right angle, then each of its interior opposite angle is acute.
(iv) In a right angled triangle, the sum of two acute angles is 90°.
(v) If all the three sides of a triangle are equal, then it is called a scalene triangle.
(vi) Every equilateral triangle is an isosceles triangle.
(vii) Every isosceles triangle must be an equilateral triangle.
(viii) Each acute angle of an isosceles right angled triangle measures 60°.
(ix) A median of a triangle always lies inside the triangle.
(x) An altitude of a triangle always lies outside the triangle.
(xi) In a triangle, sum of squares of two sides is equal to the square of the third side.

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 17):

3. A triangle formed by the sides of lengths 4.5 cm, 6 cm and 4.5 cm is
(a) scalene (b) isosceles ✓ (c) equilateral (d) none of these
4. The number of medians in a triangle is
(a) 1 (b) 2 (c) 3 ✓ (d) 4
5. An exterior angle of a triangle is 125°. If one of the two interior opposite angle is 55°, then the other interior opposite angle is
(a) 70° ✓ (b) 55° (c) 60° (d) 80°
6. In a ΔABC, if $\angle A = 40°$ and $\angle B = 55°$ then $\angle C$ is
(a) 75° (b) 80° (c) 95° (d) 85° ✓
7. If the angles of a triangle are 35°, 35° and 110°, then it is
(a) an isosceles triangle ✓ (b) an equilateral triangle
(c) a scalene triangle (d) right angled triangle
8. A triangle whose two angles measure 30° and 120° is
(a) scalene (b) isosceles ✓ (c) equilateral (d) right angled
9. A triangle can have two
(a) right angles (b) obtuse angles
(c) acute angles ✓ (d) straight angles
10. A triangle whose angles measure 35°, 55° and 90° is
(a) acute angled (b) right angled (c) obtuse angled ✓ (d) isosceles
11. A triangle is not possible whose angles measure
(a) 40°, 65°, 75° (b) 50°, 56°, 74° (c) 72°, 63°, 45° (d) 67°, 42°, 81° ✓
12. If in an isosceles triangle, each of the base angle is 40°, then the triangle is
(a) right angled triangle (b) acute angled triangle
(c) obtuse angled triangle ✓ (d) isosceles right angled triangle
13. A triangle is not possible with sides of lengths (in cm)
(a) 6, 4, 10 ✓ (b) 5, 3, 7 (c) 7, 8, 9 (d) 3.6, 5.4, 8