

A cost function is needed for partial solutions to help in larger decisions. For instance, let's define $m[i][j]$ as the minimum cost of serving all the riders using exactly j stops, with the last stop being on floor i .

Can this function help us place the $(j+1)$ th stop based on smaller values? Yes, because the $(j+1)$ th stop must be higher than the previous j th stop on floor i . Additionally, this new stop will primarily benefit passengers aiming to travel beyond floor i .

We must carefully allocate passengers between the new stop and floor i according to their proximity to each. This approach leads to the following recurrence:

$$m_{i,j+1} = \min_{k=0}^i (m_{k,j} - (\text{cost of serving passengers above } k) + (\text{cost of introducing stop } i))$$

Note that: $\text{cost of introducing stop } i = \text{cost of } (k, i) + \text{cost of } (i, \text{infinity})$

the recurrence implies that if the last stop is at floor i , the previous stop should be at floor $k < i$. What is the cost for such solution? Subtract the cost of serving all passengers above k (cost of $(k, \text{infinity})$) from $m_{k,j}$. Then replace it with the potentially lower cost of adding a stop at floor i ($\text{cost of } (k, i)$ and $\text{cost of } (i, \text{infinity})$).