

Non-Equivalence Relations on Labelled Transition Systems

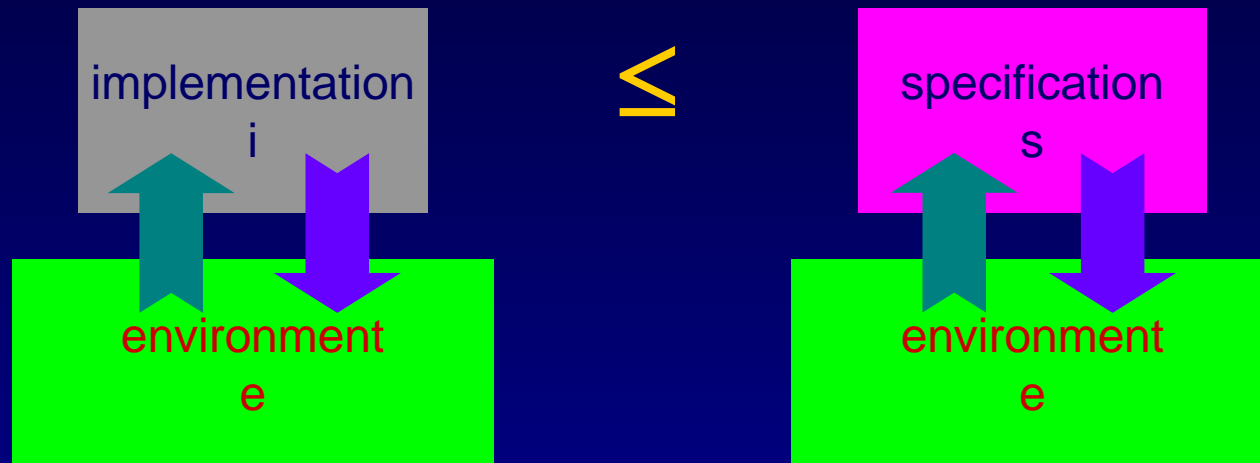
Implementation Relations

Conformance Relations

Refinement Relations

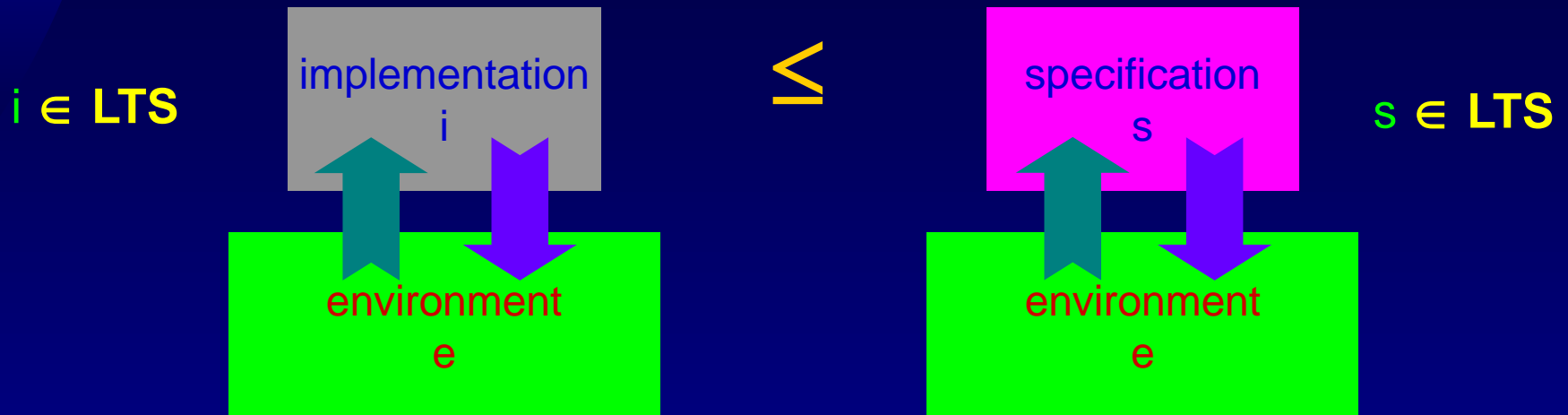
Pre-Orders

Preorders on Transition Systems

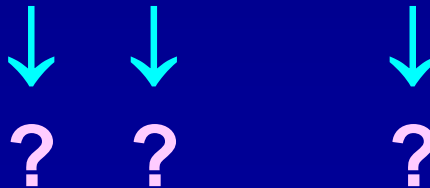


- 👉 Suppose environment e interacts with implementation i and with the specification s :
- ♦ i correctly implements s
iff all observations of i can be related to observations of s

Preorders on Transition Systems



$$i \leq s \iff \forall e \in E. \text{obs}(e, i) \subseteq \text{obs}(e, s)$$



Implementation Relation

$i \text{ imp } s$: implementation i implements specification s

☞ imp reflexive ?	$s \text{ imp } s$	Yes
☞ imp symmetric ?	$i \text{ imp } s \Rightarrow s \text{ imp } i$	No
☞ imp transitive ?	$i \text{ imp } s, s \text{ imp } t \Rightarrow i \text{ imp } t$	Prefer
☞ imp anti-symmetric ?	$i \text{ imp } s, s \text{ imp } i \Rightarrow i = s$	No
☞ imp linear ?	$i \text{ imp } s \text{ or } s \text{ imp } i$	No

equivalence :

reflexive, symmetric, transitive

preorder :

reflexive, transitive

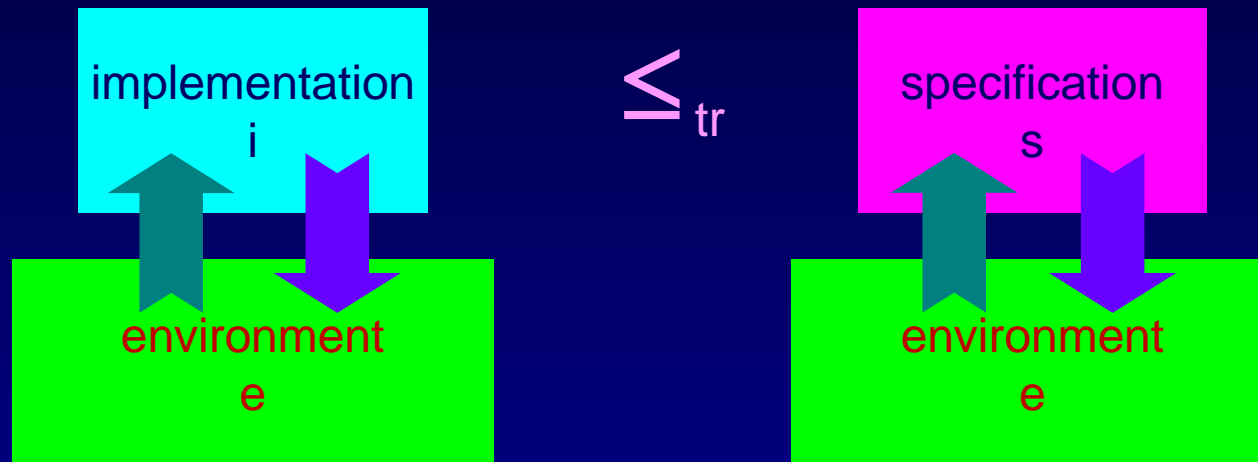
partial order :

anti-symmetric preorder

linear/total order :

linear partial order

Trace Preorder

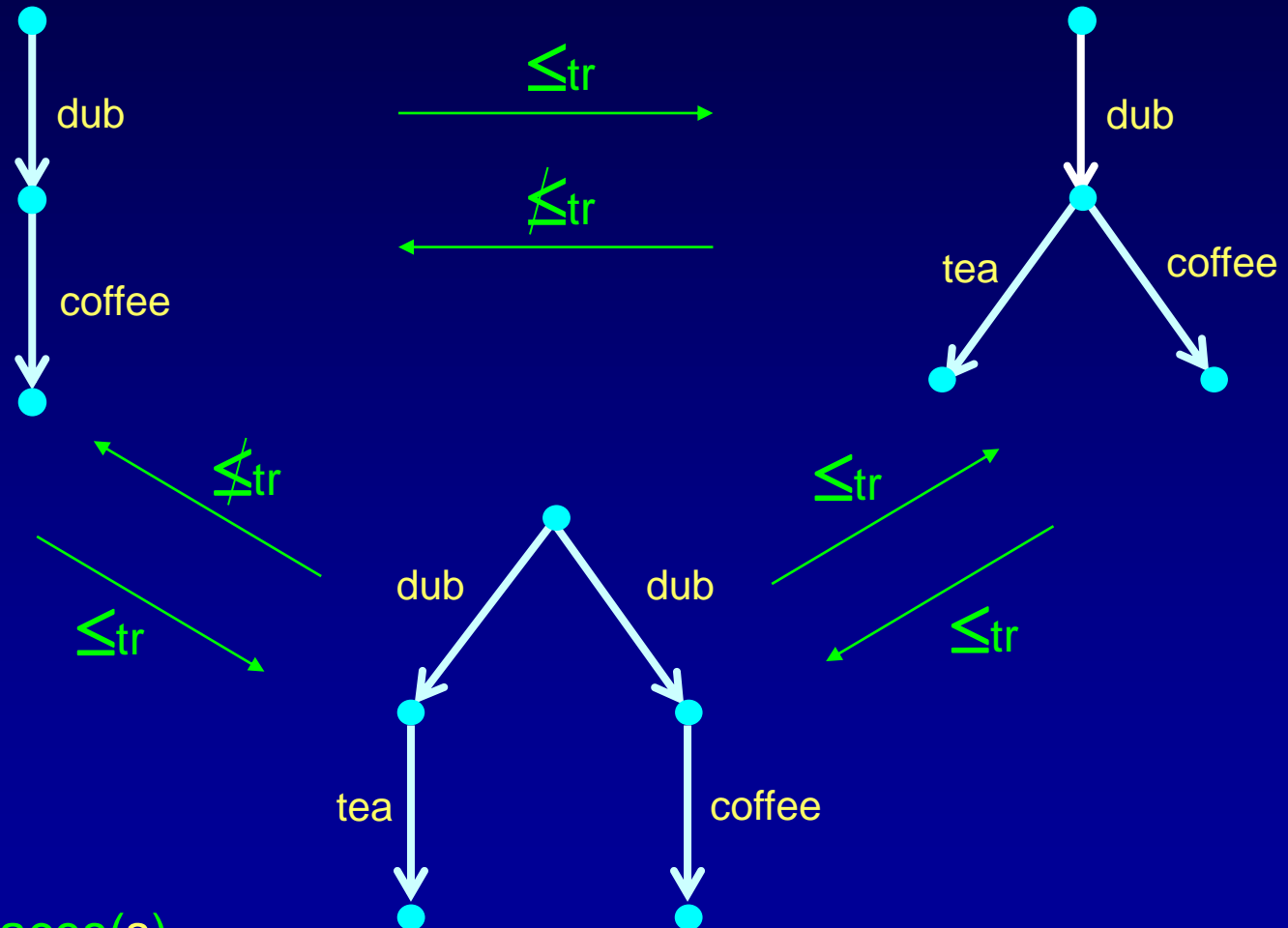


$$i \leq_{tr} s \iff \text{traces}(i) \subseteq \text{traces}(s)$$

$$i \approx_{tr} s \iff i \leq_{tr} s \wedge s \leq_{tr} i \qquad \approx = \leq_{tr} \cap \geq_{tr}$$

Traces: $\text{traces}(s) = \{ \sigma \in L^* \mid s \xRightarrow{\sigma} \}$

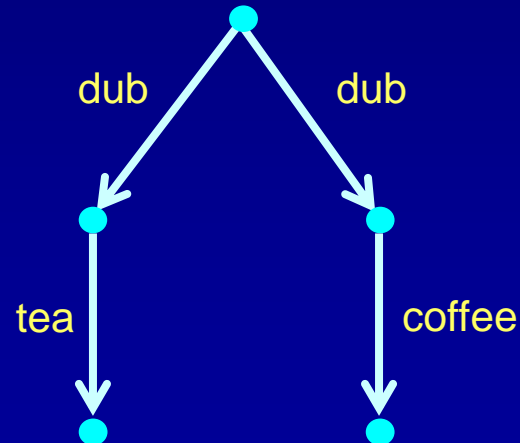
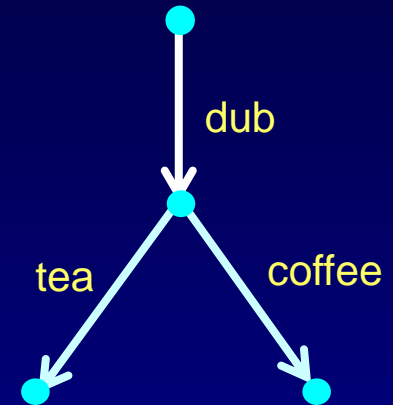
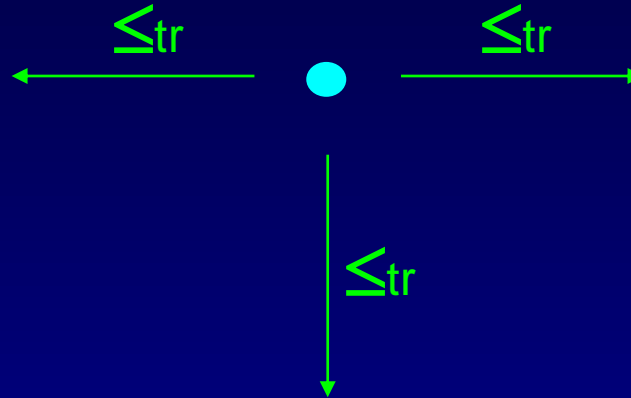
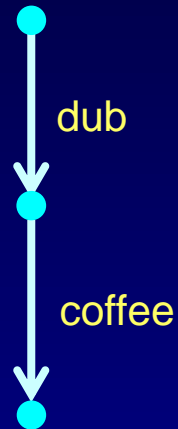
Trace Preorder



$i \leq_{tr} s =$

$\text{traces}(i) \subseteq \text{traces}(s)$

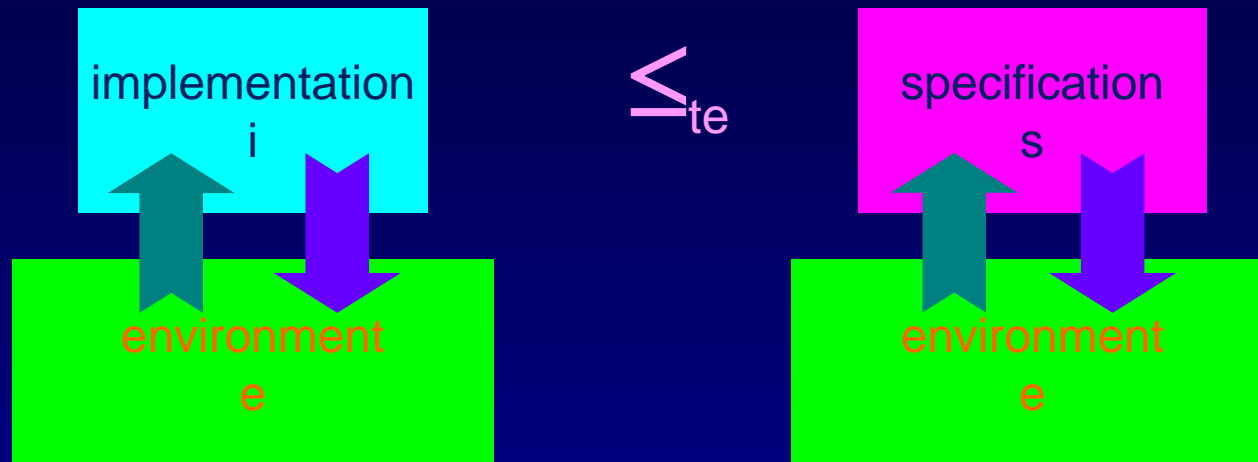
Trace Preorder



$i \leq_{tr} s =$

$\text{traces}(i) \subseteq \text{traces}(s)$

Testing Preorder



$$i \leq_{te} s \iff \forall e \in E. \text{obs}(e, i) \subseteq \text{obs}(e, s)$$

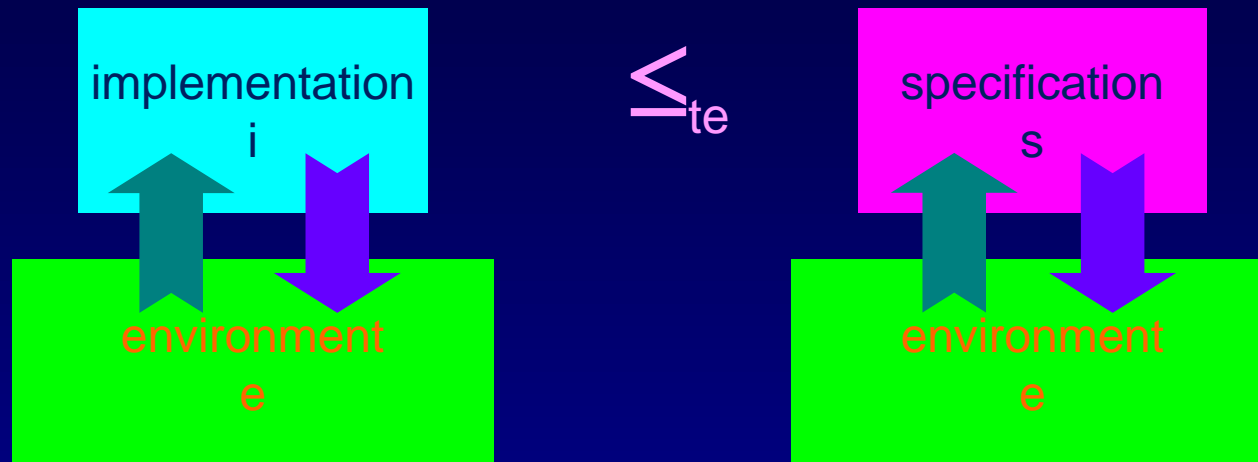


LTS(L)



Ctraces(e||s)

Testing Preorder

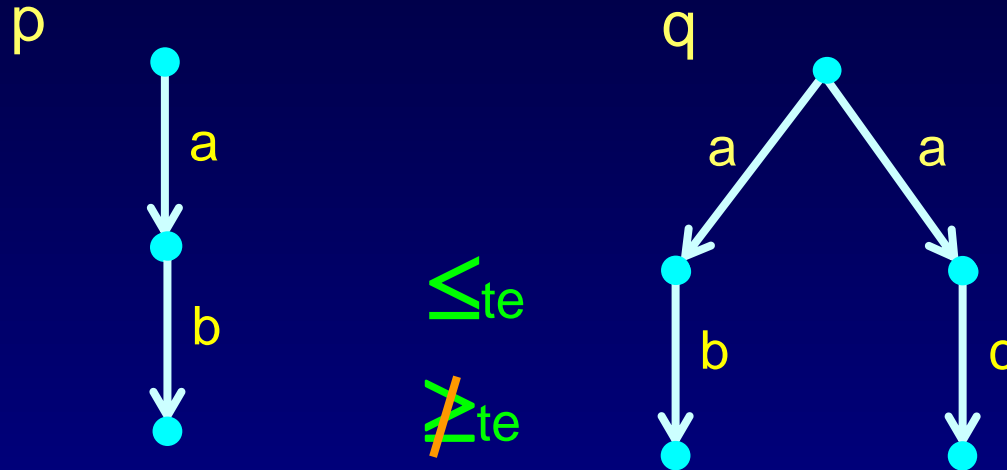


$$p \leq_{te} q \Leftrightarrow$$

$$\forall A \subseteq L, \forall \sigma \in L^* :$$

$$p \text{ after } \sigma \text{ refuses } A \Rightarrow q \text{ after } \sigma \text{ refuses } A$$

Testing Preorder



p after a refuses $\{a\}$

~~p after a refuses $\{b\}$~~

p after a refuses $\{c\}$

~~p after a refuses $\{b, c\}$~~

~~p after a refuses $\{a, b, c\}$~~

q after a refuses $\{a\}$

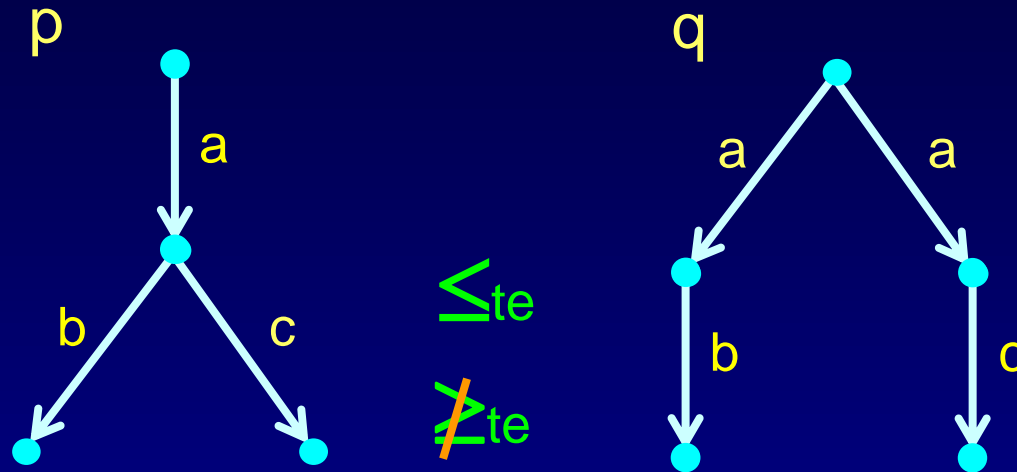
q after a refuses $\{b\}$

q after a refuses $\{c\}$

~~q after a refuses $\{b, c\}$~~

~~q after a refuses $\{a, b, c\}$~~

Testing Preorder



~~p after a refuses $\{b\}$~~

~~p after a refuses $\{c\}$~~

~~p after a refuses $\{b, c\}$~~

q after a b refuses L

q after a refuses $\{b\}$

q after a refuses $\{c\}$

~~q after a refuses $\{b, c\}$~~

Refusal Equivalence

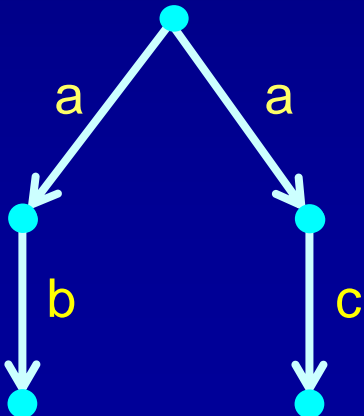
Failure A : $s \xrightarrow{A} s \iff \forall \mu \in A \cup \{\tau\} : s \not\xrightarrow{\mu}$

Failure trace σ : $\sigma \in (L \cup \emptyset(L))^* : s \xRightarrow{\sigma}$

Failure traces of p : $\text{Ftraces}(p) = \{ \sigma \in (L \cup \emptyset(L))^* \mid p \xRightarrow{\sigma} \}$

Failure trace equivalence
= refusal equivalence :

$$P \leq_{\text{rf}} q \iff \text{Ftraces}(p) \subseteq \text{Ftraces}(q)$$



Ftraces :

$\{b,c\} \ a \ \{a,c\} \ b \ L$

$a \ \{c\} \ b \ \{a\} \ \{b\} \ \{c\}$

$\emptyset \ a \ \{b\} \ \{b\} \ c$

Not Ftraces :

$\{a,b,c\} \ a \ \{a,c\} \ b \ L$

$a \ \{c\} \ c \ L$

$a \ a \ \emptyset$