LEARNING A COFFEE MACHINE MODEL WITH $L^{\#}$

We show how the $L^{\#}$ algorithm learns the following Mealy machine $\mathcal{M}$:

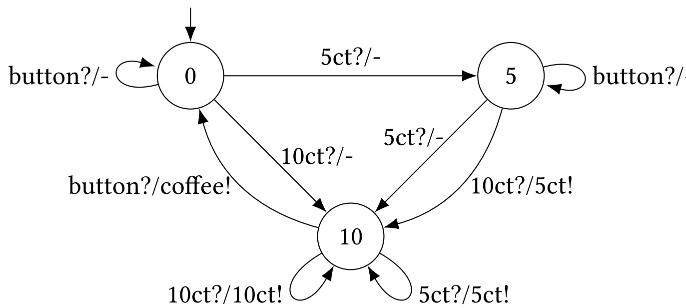

Fig. 1. Mealy machine $\mathcal{M}$ for coffee machine

The $L^{\#}$ algorithm constructs the observation tree $\mathcal{T}$ of Figure 4 by performing the steps below. Note that this is just one possible run of the algorithm as the rules may be applied in different orders and the teacher may provide different counterexamples. For convenience, we abbreviate the inputs to 5, 10 and b, and the outputs to $-$, 5, 10 and c.

(1) The initial observation tree has a single state 1, which constitutes the basis.
(2) Rule (R2) is applied three times to explore the outgoing transitions of state 1, for inputs 5, 10 and b, leading to new frontier states 2, 3 and 4, respectively.
(3) Since the frontier has no isolated states and the basis is complete, the learner applies rule (R4) and submits a first hypothesis $\mathcal{H}_1$ to the teacher:

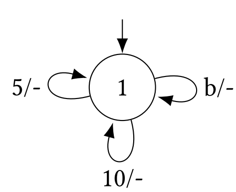

Fig. 2. Hypothesis $\mathcal{H}_1$

Suppose the teacher returns counterexample $\rho = 10\ b$. Then the observation tree is extended with an outgoing b-transition from state 3 to a new state 5. Since $b \vdash 1\#3$, the shortest prefix σ of ρ that leads to a conflict, i.e. $\delta^{\mathcal{H}}(1,\sigma)\ \#\ \delta^{\mathcal{T}}(1,\sigma)$, is the sequence 10. Since σ leads to a state on the frontier, counterexample processing finishes immediately.

(4) Rule (R1) is applied to add state 3 to the basis.
(5) Rule (R2) is applied to explore the outgoing transitions of state 3 for inputs 5 and 10, leading to new frontier states 6 and 7, respectively.
(6) Rule (R3) is applied with witness b to identify frontier states 2, 4, 5, 6 and 7, leading to new states 8, 9, 10, 11 and 12, respectively. (Note: If instead we would choose witness 10 then learning would be faster and the first hypothesis would already be correct. Learning is quite a nondeterministic process!)

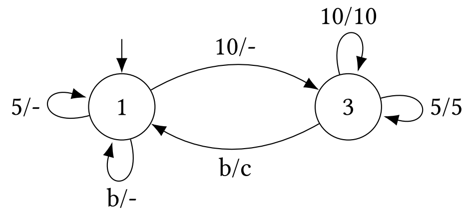

Fig. 3. Hypothesis $\mathcal{H}_2$

(7) Rule (R4) is applied with the hypothesis $\mathcal{H}_2$ from Figure 3.
Now suppose the teacher returns the (somewhat longer) counterexample $\rho = b\ b\ 5\ 5\ b$. The observation tree is extended with an outgoing path $5\ 5\ b$ from state 9 with new states 13, 14 and 15. Since $b \vdash 1\#15$, the shortest prefix of ρ that leads to a conflict is the sequence $\sigma = b\ b\ 5\ 5$. The learner now needs to figure out whether the problem occurs within the first part or the second part of σ. Using Algorithm 3, the learner decides to do an output query $5\ 5\ b$. A path $5\ b$ is added from state 2, with new states 16 and 17. Since $\neg(1\#5)$, the learner concludes that the problem occurs in the second part of the counterexample, and $5\ 5$ leads to a conflict. Indeed, we have $b \vdash 1\#16$. Thus the learner decides to process the counterexample $5\ 5$. There is no need to do an output query $5\ b$, since this information is already contained in the tree. Next, as $5\ b \vdash 1\#2$, decides to process counterexample 5. Since 5 leads to a state on the frontier, counterexample processing finishes.

(8) Rule (R1) is applied to add state 2 to the basis.

(9) Rule (R2) is applied for state 2 and input 10, leading to a new frontier state 18.

(10) Frontier states 6, 7 and 16 are already identified. In order to identify the remaining frontier states 4, 5, 8 and 18, the learner uses witness 10 (even though this sequence is not adaptive, the ADS version of $L^{\#}$ will find it, since it maximizes the number of expected apartness pairs). This leads to new states 19, 20, 21 and 22, respectively.

(11) The resulting hypothesis $\mathcal{H}_3$ is equivalent to $\mathcal{M}$.

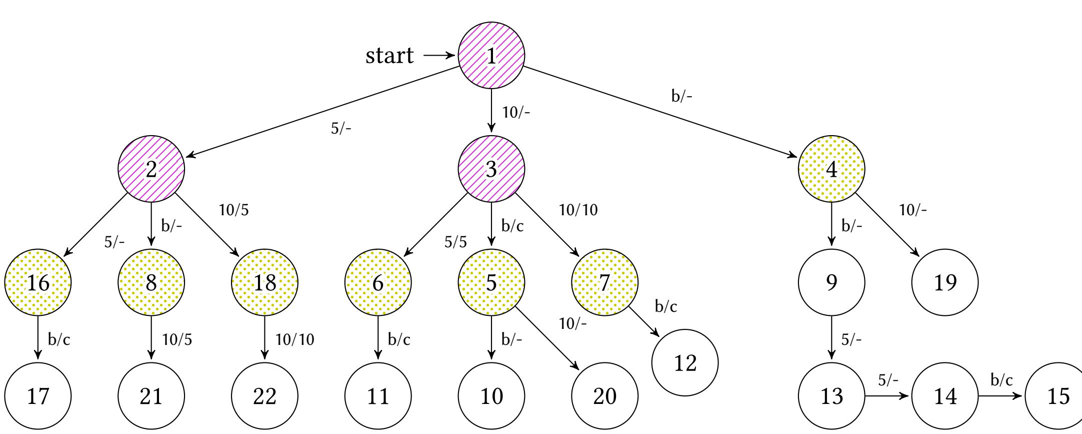

Fig. 4. Final observation tree