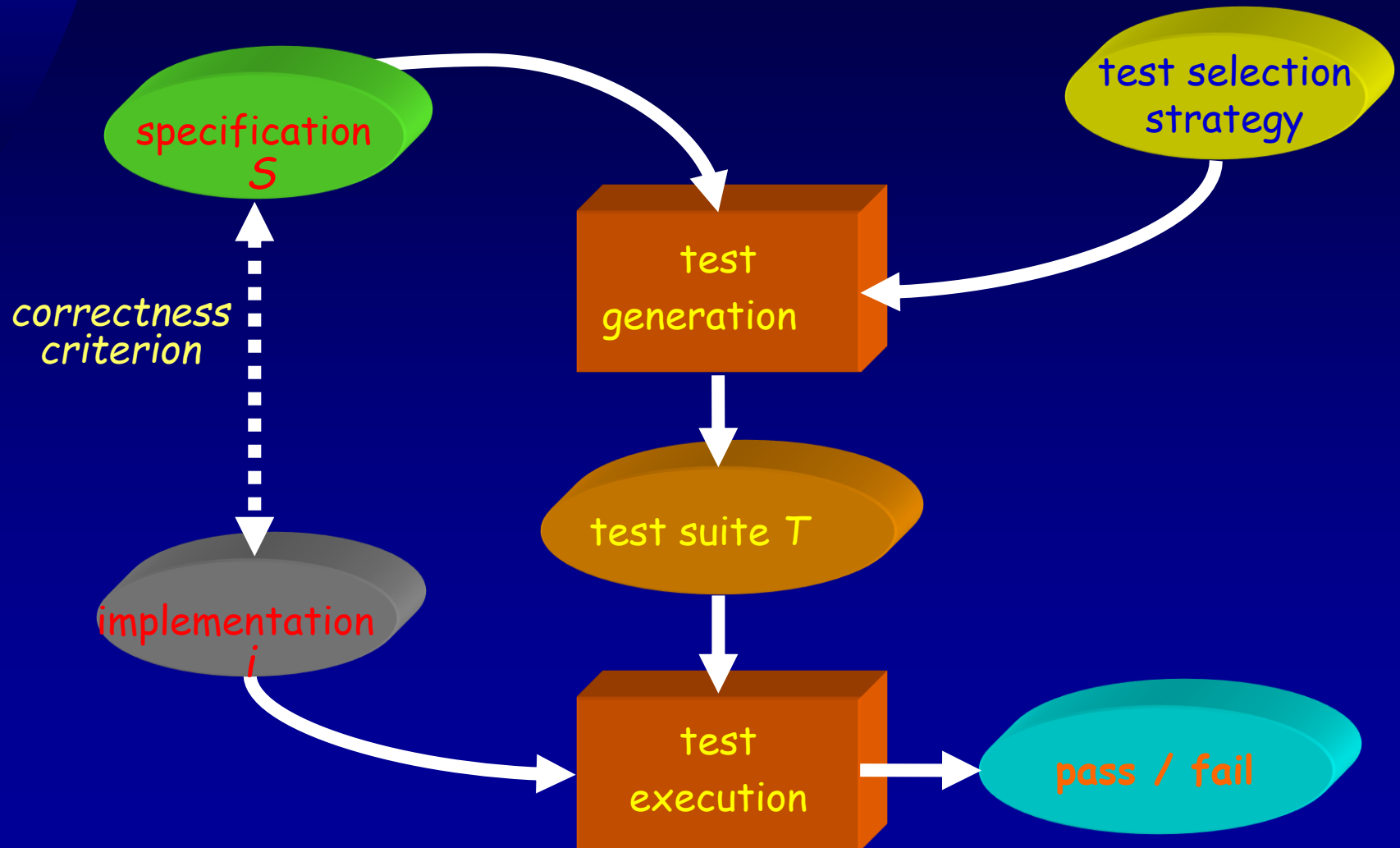




# Test Selection

# Test Selection



# Test Selection

(Infinitely) many sound test cases can be generated,  
but : no time and resources to execute them all

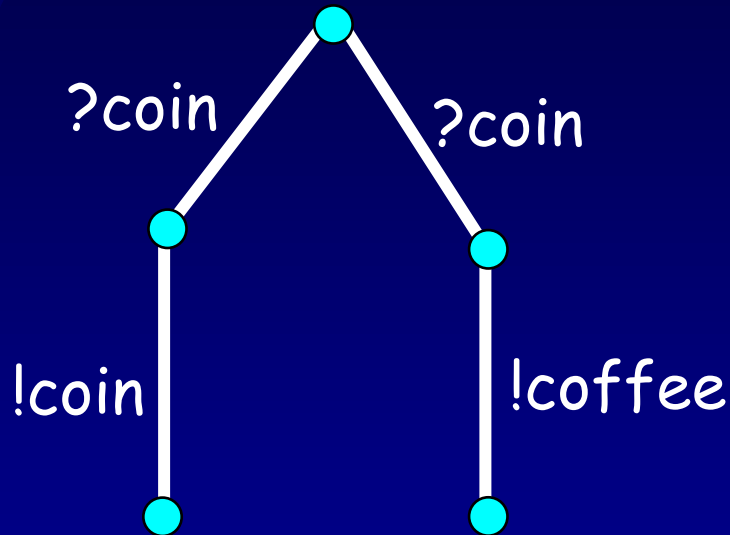
- ☞ Which are the best ones ?
- ☞ How many ?

## *Problem of Test Selection*

Test selection :

- ☞ guided by user : test purposes
- ☞ automatic by test tool : selection strategy
- ☞ bottom line : random

# Test Purpose : Example



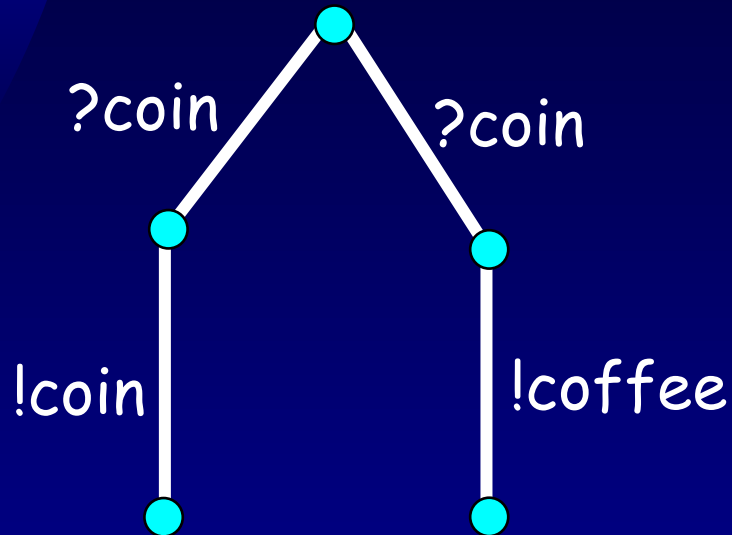
Test: can the machine  
deliver coffee?

Desired observation:  
coffee after coin

More confidence in correctness

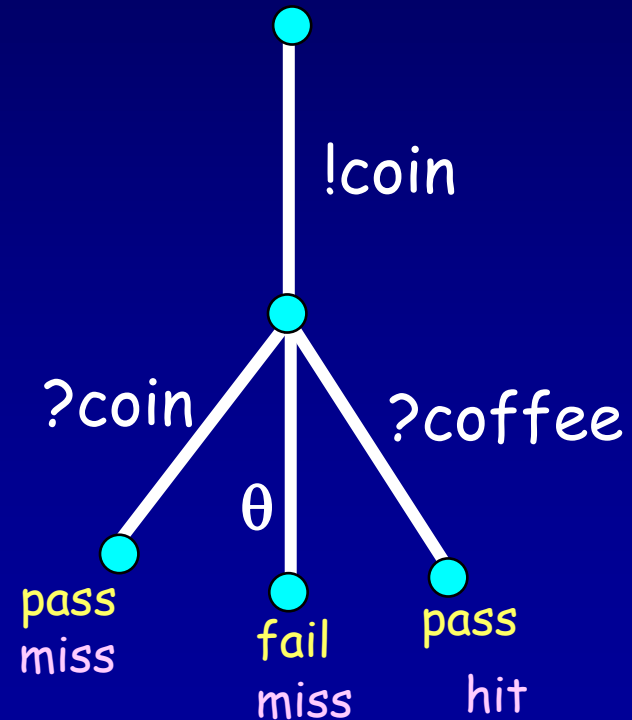
We can only draw conclusions based on observations

# Test Purpose : Example



Desired observation  
= observation objective :  
?coin . !coffee

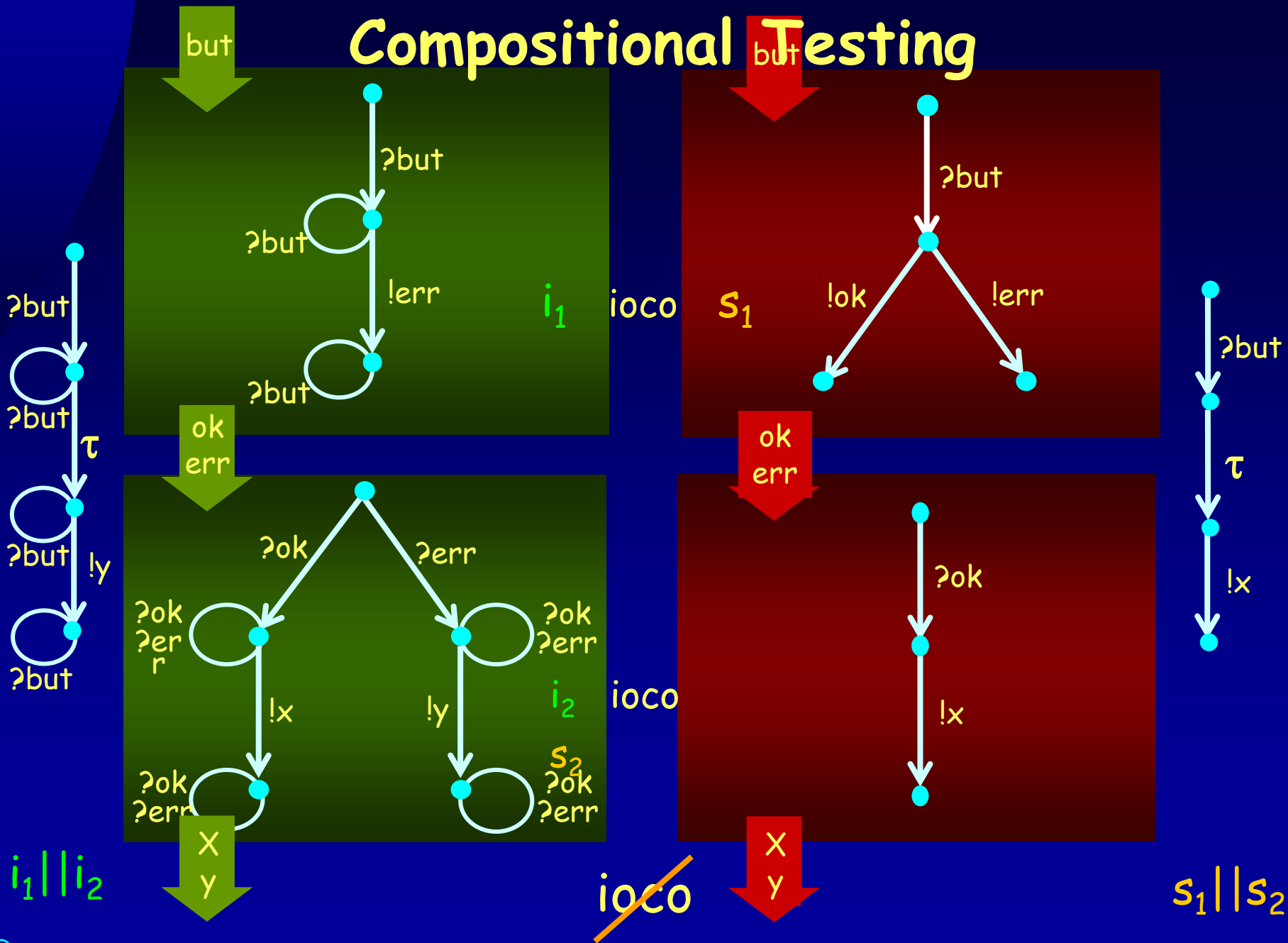
Test purpose:  
can the machine  
deliver coffee?



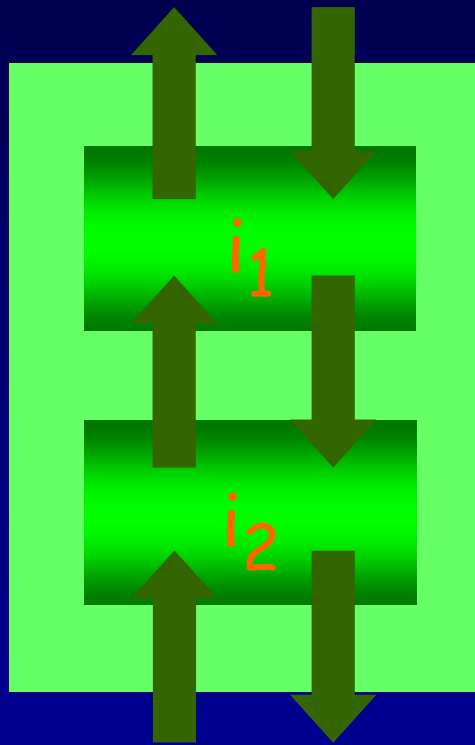


# Compositionality

# Compositional Testing



# Compositional Testing

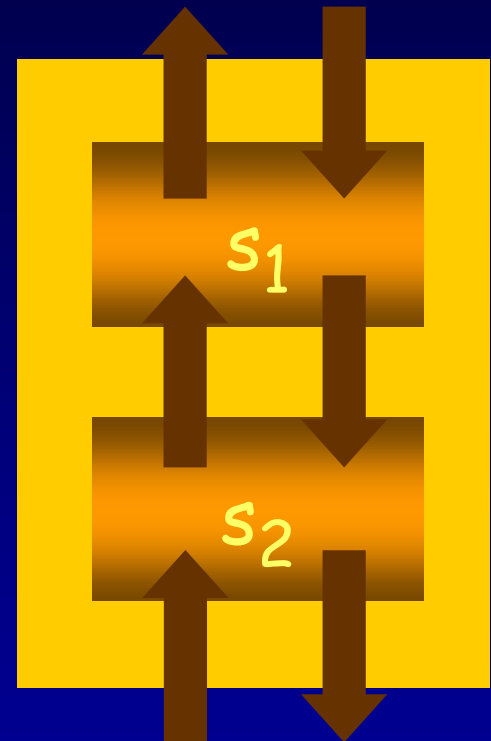


$i_1 \parallel i_2$

$i_1 \text{ ioco } s_1$

$i_2 \text{ ioco } s_2$

~~ioco~~



$s_1 \parallel s_2$

If  $s_1, s_2$  input enabled -  $s_1, s_2 \in \text{IOTS}$  - then  $\text{ioco}$  is preserved !





# Variations of (u)ioco

# Testing Transition Systems: Extensions

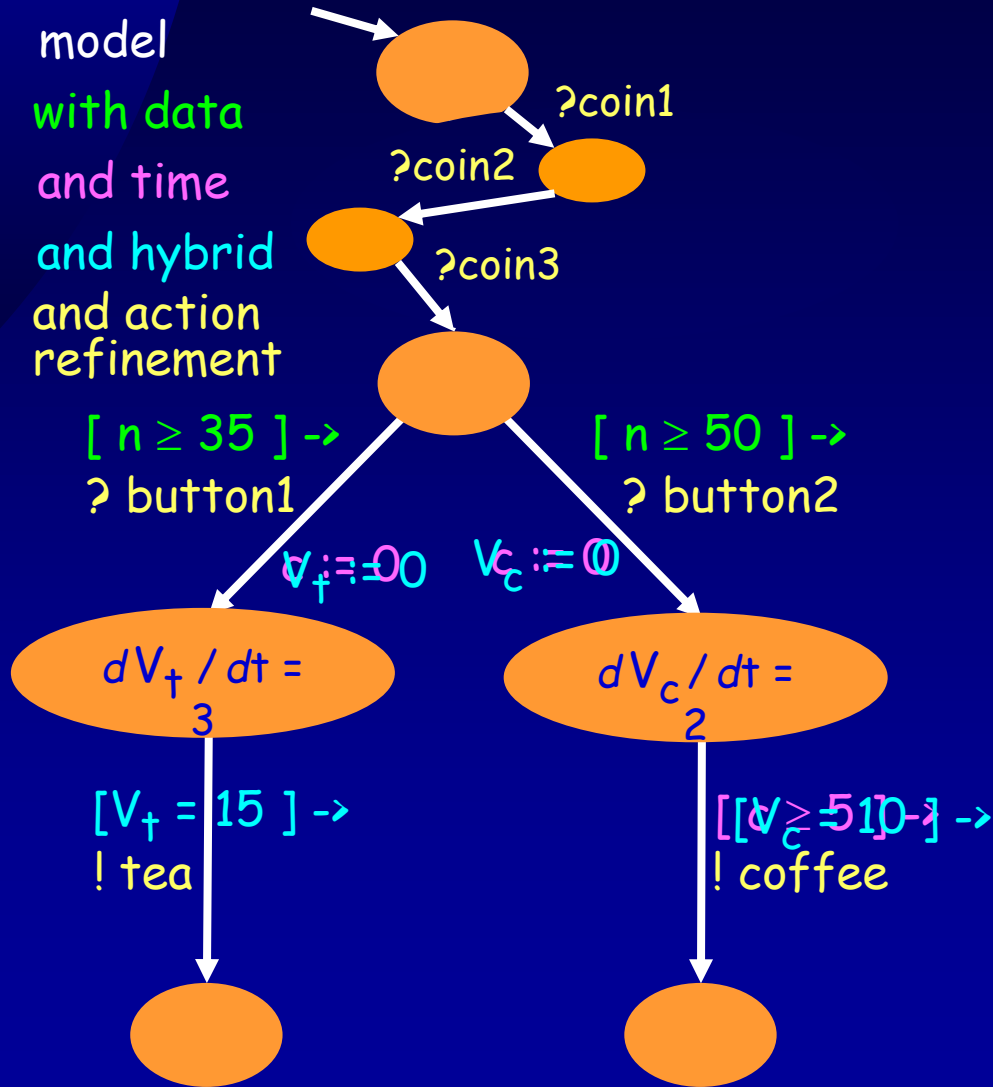
model

with data

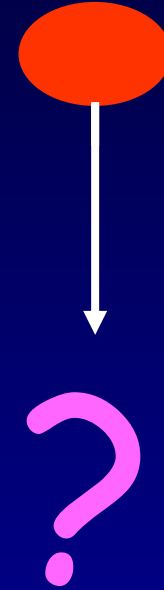
and time

and hybrid

and action  
refinement



test case



# Variations on a Theme

$i \text{ ioco } s \Leftrightarrow \forall \sigma \in \text{Straces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$

$i \leq_{ior} s \Leftrightarrow \forall \sigma \in (L \cup \{\delta\})^* : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$

$i \text{ ioconf } s \Leftrightarrow \forall \sigma \in \text{traces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$

$i \text{ ioco}_F s \Leftrightarrow \forall \sigma \in F : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$

$i \text{ uioco } s \Leftrightarrow \forall \sigma \in \text{Utraces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$

$i \text{ mioco } s$  multi-channel ioco

$i \text{ wioco } s$  non-input-enabled ioco

$i \text{ eco } e$  environmental conformance

$i \text{ sioco } s$  symbolic ioco

$i \text{ (r)tioco } s$  (real) timed tioco (Aalborg, Twente, Grenoble, Bordeaux, . . .)

$i \text{ ioco}_r s$  refinement ioco

$i \text{ hioco } s$  hybrid ioco

$i \text{ qioco } s$  quantified ioco



$ioco_F$  : Varying  $S$ -trace sets

# Variations on a Theme

$$i \text{ ioco}_F s \Leftrightarrow \forall \sigma \in F : \quad \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$$

$$i \text{ ioco } s \Leftrightarrow \forall \sigma \in \text{Straces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$$

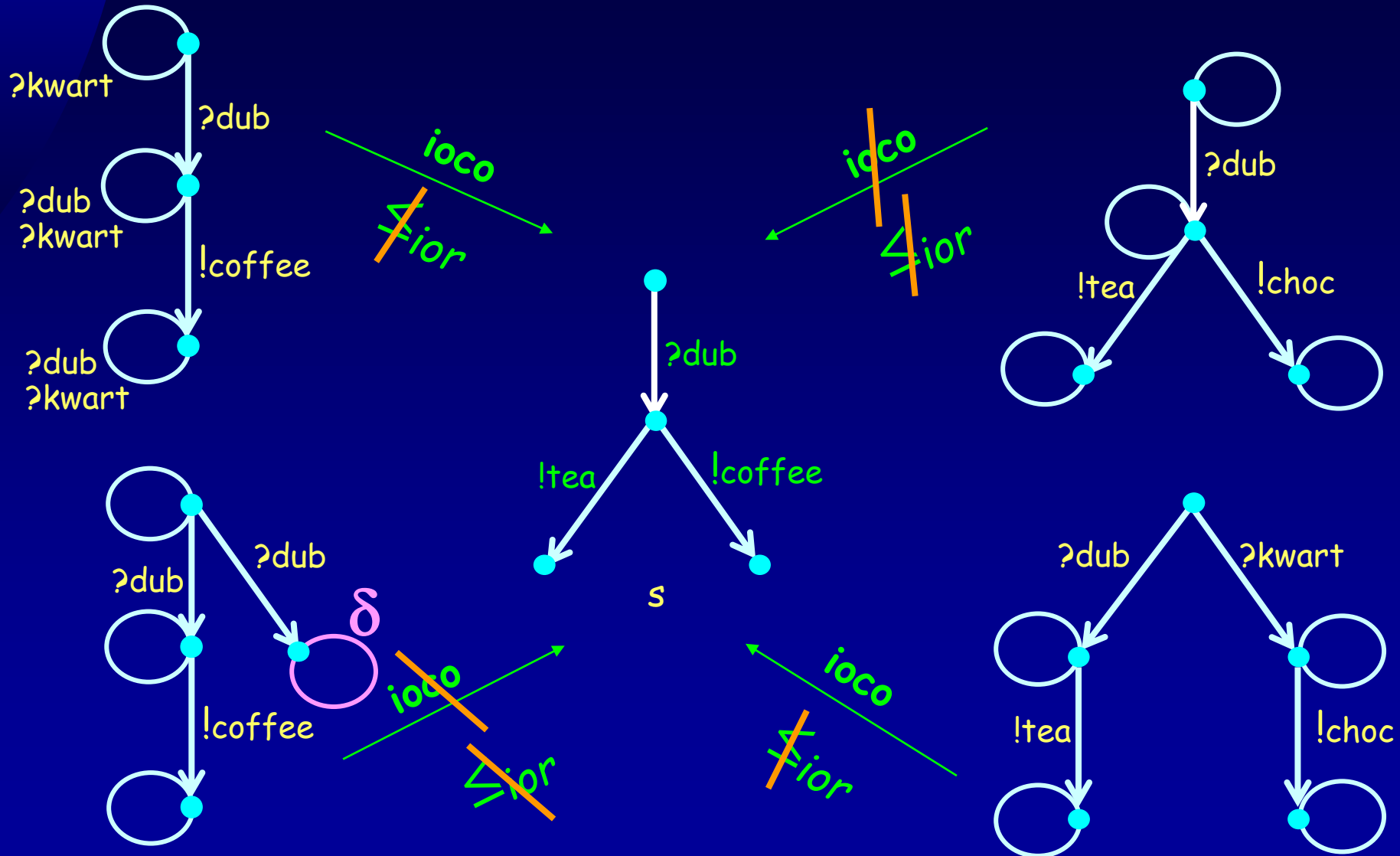
$$i \leq_{iot} s \Leftrightarrow \forall \sigma \in L^* : \quad \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$$

$$i \leq_{ior} s \Leftrightarrow \forall \sigma \in (L \cup \{\delta\})^* : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$$

$$i \text{ ioconf } s \Leftrightarrow \forall \sigma \in \text{traces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$$

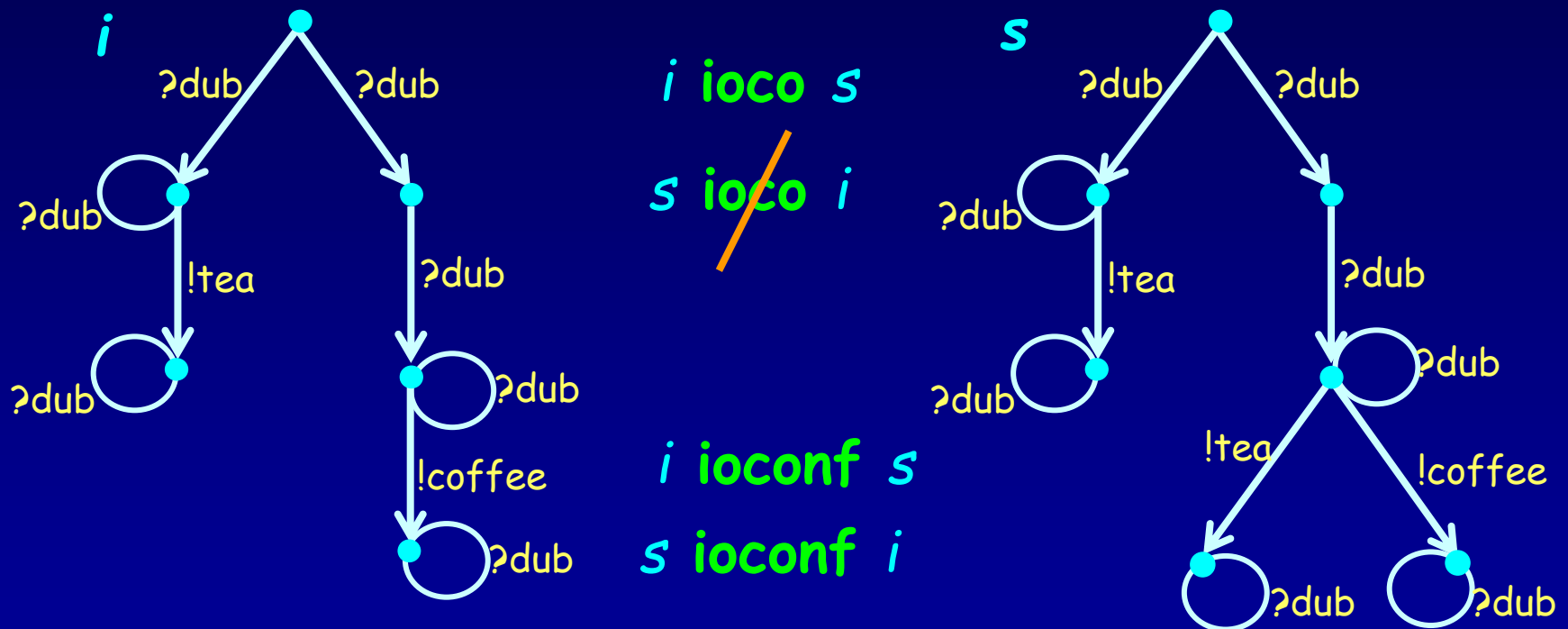
$$i \text{ uioco } s \Leftrightarrow \forall \sigma \in \text{Utraces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$$

# Implementation Relation $\leq_{ior}$



# Implementation Relation $\text{ioco}(\text{nf})$

$i \text{ ioco nf } s \stackrel{\text{def}}{=} \forall \sigma \in \text{traces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$



$\text{out}(i \text{ after } ?\text{dub}.?\text{dub}) = \text{out}(s \text{ after } ?\text{dub}.?\text{dub}) = \{ !\text{tea}, !\text{coffee} \}$

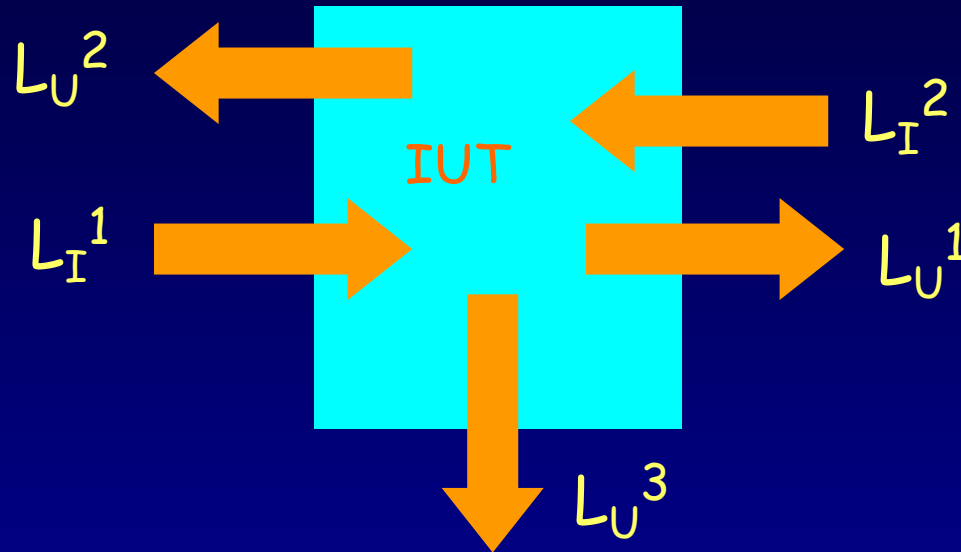
$\text{out}(i \text{ after } ?\text{dub}.\delta.?\text{dub}) = \{ !\text{coffee} \} \neq \text{out}(s \text{ after } ?\text{dub}.\delta.?\text{dub}) = \{ !\text{tea}, !\text{coffee} \}$



# mioco : Multiple Channels



# Variations on a Theme: mioco



$$i \text{ mioco } s \Leftrightarrow \forall \sigma \in \text{Straces}'(s) : \text{out}'(i \text{ after } \sigma) \subseteq \text{out}'(s \text{ after } \sigma)$$

$$p \xrightarrow{\delta_k} p = \forall !x \in L^k \cup \{\tau\} . p \not\xrightarrow{!x}$$

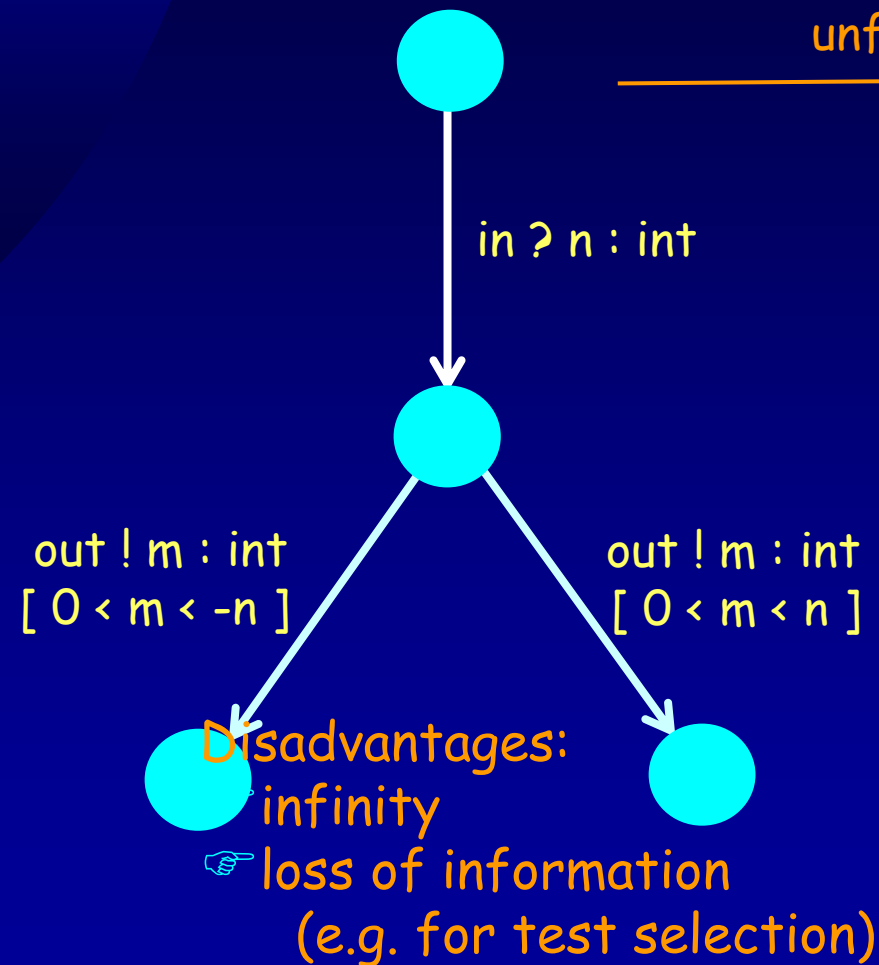
$$\text{Straces}'(s) = \{ \sigma \in (L \cup \{\delta_k\})^* \mid s \xRightarrow{\sigma} \}$$

$$\text{out}'(P) = \{ !x \in L_U \mid p \xrightarrow{!x}, p \in P \} \cup \{ \delta_k \mid p \xrightarrow{\delta_k} p, p \in P \}$$

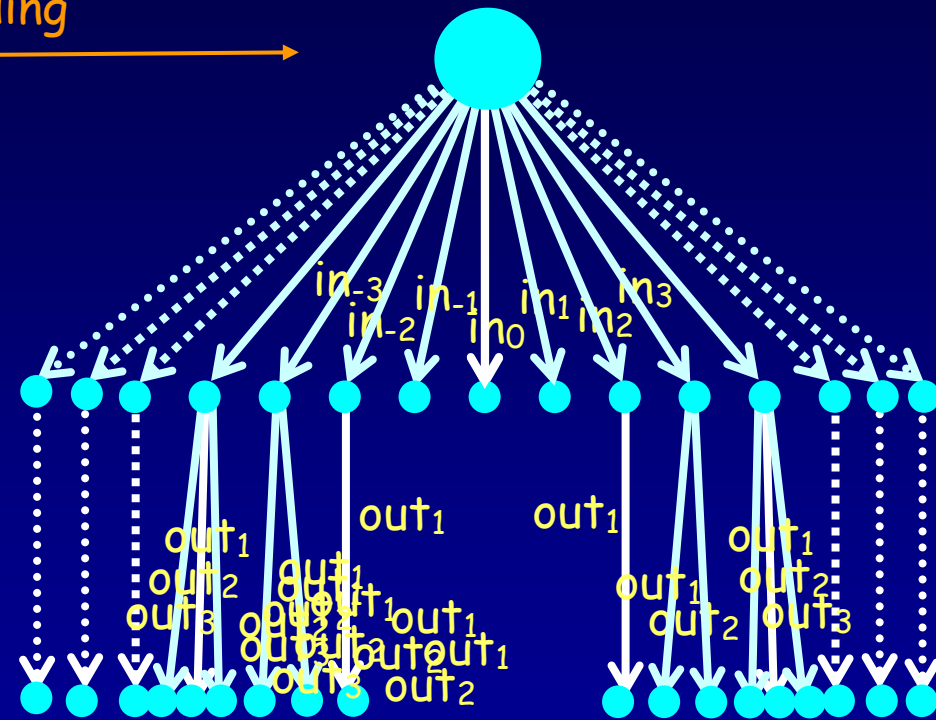


sioco : ioco with data

# Transition System with Data



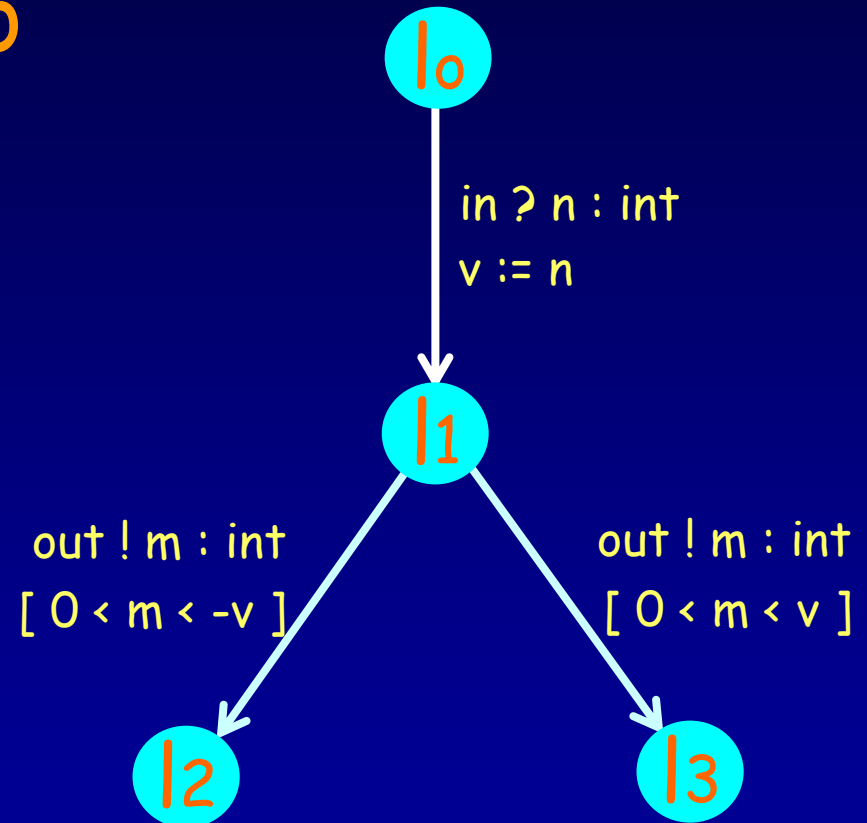
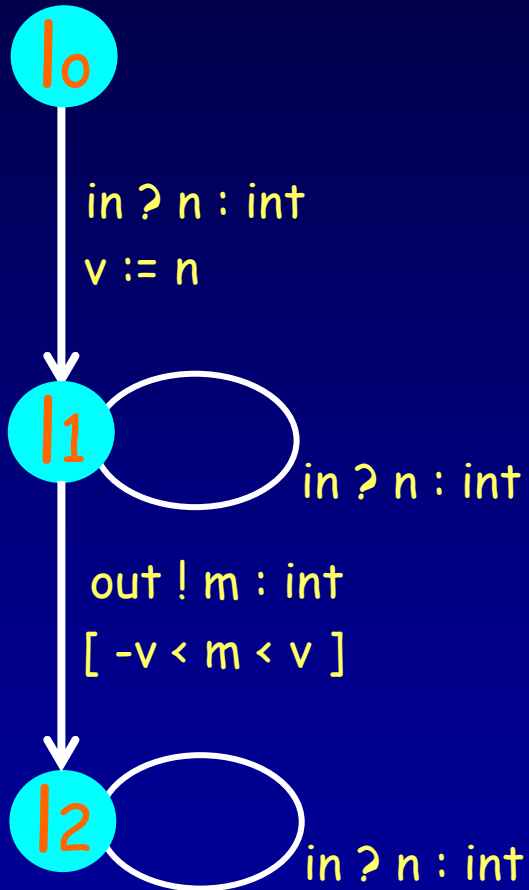
unfolding



Disadvantages:  
 infinity  
 loss of information  
 (e.g. for test selection)

# Symbolic ioco

sioco  
?



# Symbolic ioco

Specification: IOSTS  $\mathcal{S}(\iota_S) = \langle L_S, l_S, \mathcal{V}_S, \mathcal{I}, \Lambda, \rightarrow_S \rangle$

Implementation: IOSTS  $\mathcal{P}(\iota_P) = \langle L_P, l_P, \mathcal{V}_P, \mathcal{I}, \Lambda, \rightarrow_P \rangle$

both initialised, implementation input-enabled,  $\mathcal{V}_S \cap \mathcal{V}_P = \emptyset$

$\mathcal{F}_s$ : a set of symbolic extended traces satisfying  $\llbracket \mathcal{F}_s \rrbracket_{\iota_S} \subseteq \text{Straces}((l_0, \iota));$

$\mathcal{P}(\iota_P) \text{ sioco}_{\mathcal{F}_s} \mathcal{S}(\iota_S)$  iff

$$\forall (\sigma, \chi) \in \mathcal{F}_s \quad \forall \lambda_\delta \in \Lambda_U \cup \{\delta\} : \iota_P \cup \iota_S \models \bar{\nabla}_{\hat{\mathcal{I}} \cup \mathcal{I}} (\Phi(l_P, \lambda_\delta, \sigma) \wedge \chi \rightarrow \Phi(l_S, \lambda_\delta, \sigma))$$

$$\text{where } \Phi(\xi, \lambda_\delta, \sigma) = \bigvee \{ \varphi \wedge \psi \mid (\lambda_\delta, \varphi, \psi) \in \text{out}_s((\xi, \top, \text{id})_0 \text{after}_s(\sigma, \top)) \}$$

**Theorem 1.**

$$\mathcal{P}(\iota_P) \text{ sioco}_{\mathcal{F}_s} \mathcal{S}(\iota_S) \quad \text{iff} \quad \llbracket \mathcal{P} \rrbracket_{\iota_P} \text{ ioco}_{\llbracket \mathcal{F}_s \rrbracket_{\iota_S}} \llbracket \mathcal{S} \rrbracket_{\iota_S}$$



(r)tioco : ioco with time

# Timed Model-Based Testing

☞ In many systems real-time properties are crucial

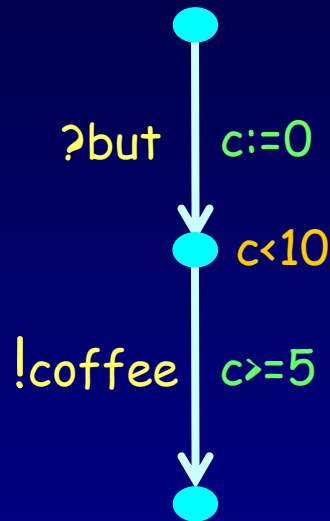
☞ Approach:

- ◆ Extension of IOTS/ioco theory
  - Timed Input Output Transition Systems (TIOTS)
  - Timed Implementation Relations: build on ioco

☞ Challenges:

- ◆ Is time input or output ?
- ◆ Quiescence: How long is there never eventually no output?

# Timed Input-Output Transition Systems



TIOTS :  $\langle Q, L_I, L_U, R_{\geq 0}, T, q_0 \rangle$

Observable actions:  $L_I, L_U$

delay  $d \in R_{\geq 0}$

Unobservable action:  $\tau$

Specifications are TIOTS

Implementations are assumed  
to behave as input-enabled TIOTS



# The Untimed Implementation Relation $ioco$

$$i \text{ ioco } s \quad =_{\text{def}} \quad \forall \sigma \in \text{Straces}(s) : \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma)$$

$\downarrow$   
*Straces*

$\downarrow$   
*after*

$\downarrow$   
*out*

$$\delta(p) = \forall !x \in L_U \cup \{\tau\}. \quad p \not\stackrel{!x}{\longrightarrow}$$

$$\text{Straces}(s) = \{ \sigma \in (L \cup \{\delta\})^* \mid s \stackrel{\sigma}{\Longrightarrow} \}$$

$$\text{out}(p) = \{ !x \in L_U \mid p \stackrel{!x}{\Longrightarrow} \} \cup \{ \delta \mid \delta(p) \}$$

$$\text{out}(P) = \bigcup \{ \text{out}(p) \mid p \in P \}$$

$$p \text{ after } \sigma = \{ p' \mid p \stackrel{\sigma}{\Longrightarrow} p' \}$$

# Some Timed Implementation Relations

$$\begin{array}{ccccccc} i \text{ ioco } s & =_{\text{def}} & \forall \sigma \in \text{Straces}(s) : & \text{out}(i \text{ after } \sigma) & \subseteq & \text{out}(s \text{ after } \sigma) \\ \downarrow & & \downarrow & \downarrow & & \downarrow & \\ \text{tioco}_X & & ? & ? & & ? & \end{array}$$

# A Timed Implementation Relation

$$\begin{array}{cccc}
 i \text{ tioco } s & =_{\text{def}} & \forall \sigma \in \text{Straces}(s) : & \text{out}(i \text{ after } \sigma) \subseteq \text{out}(s \text{ after } \sigma) \\
 \downarrow & & \downarrow & \downarrow \quad \downarrow \\
 \text{tioco} & & \text{ttraces} & \text{after}_t \quad \text{out}_{AG}
 \end{array}$$

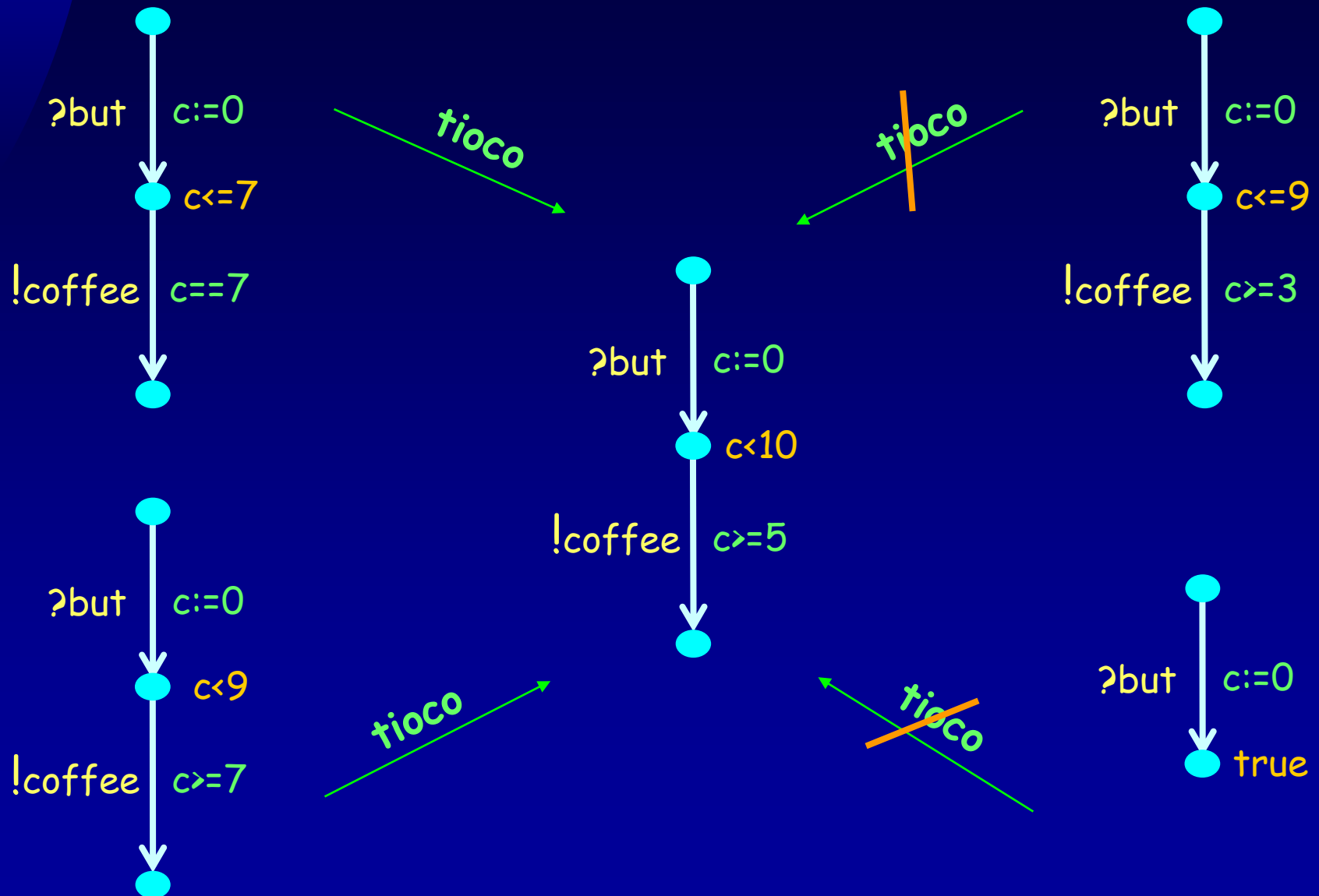
$$\delta(p) = \text{X}$$

$$\text{ttraces}(s) = \{ \sigma \in (L \cup \mathbf{R}_{\geq 0})^* \mid s \xRightarrow{\sigma} \}$$

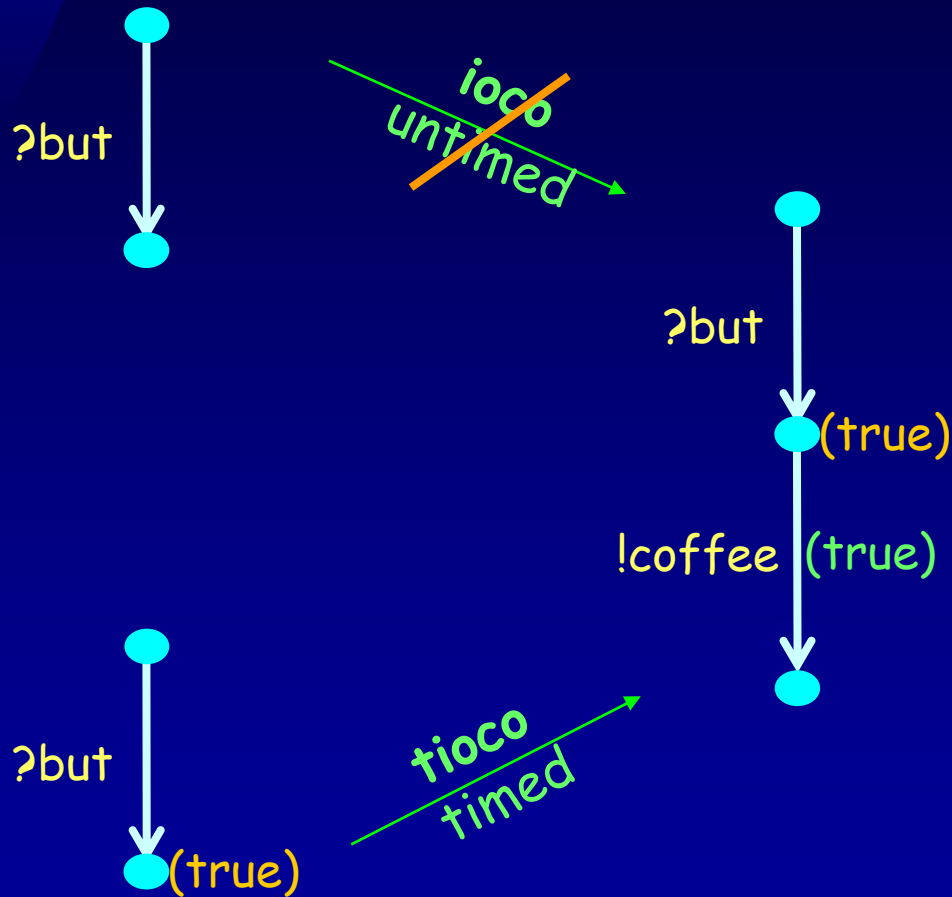
$$\text{out}_{AG}(p) = \{ x \in L \cup \mathbf{R}_{\geq 0} \mid p \xRightarrow{x} \}$$

$$p \text{ after}_t \sigma = \{ p' \mid p \xRightarrow{\sigma} p', \sigma \in (L \cup \mathbf{R}_{\geq 0})^* \}$$

# A Timed Implementation Relation tioco



# Unbounded Delay

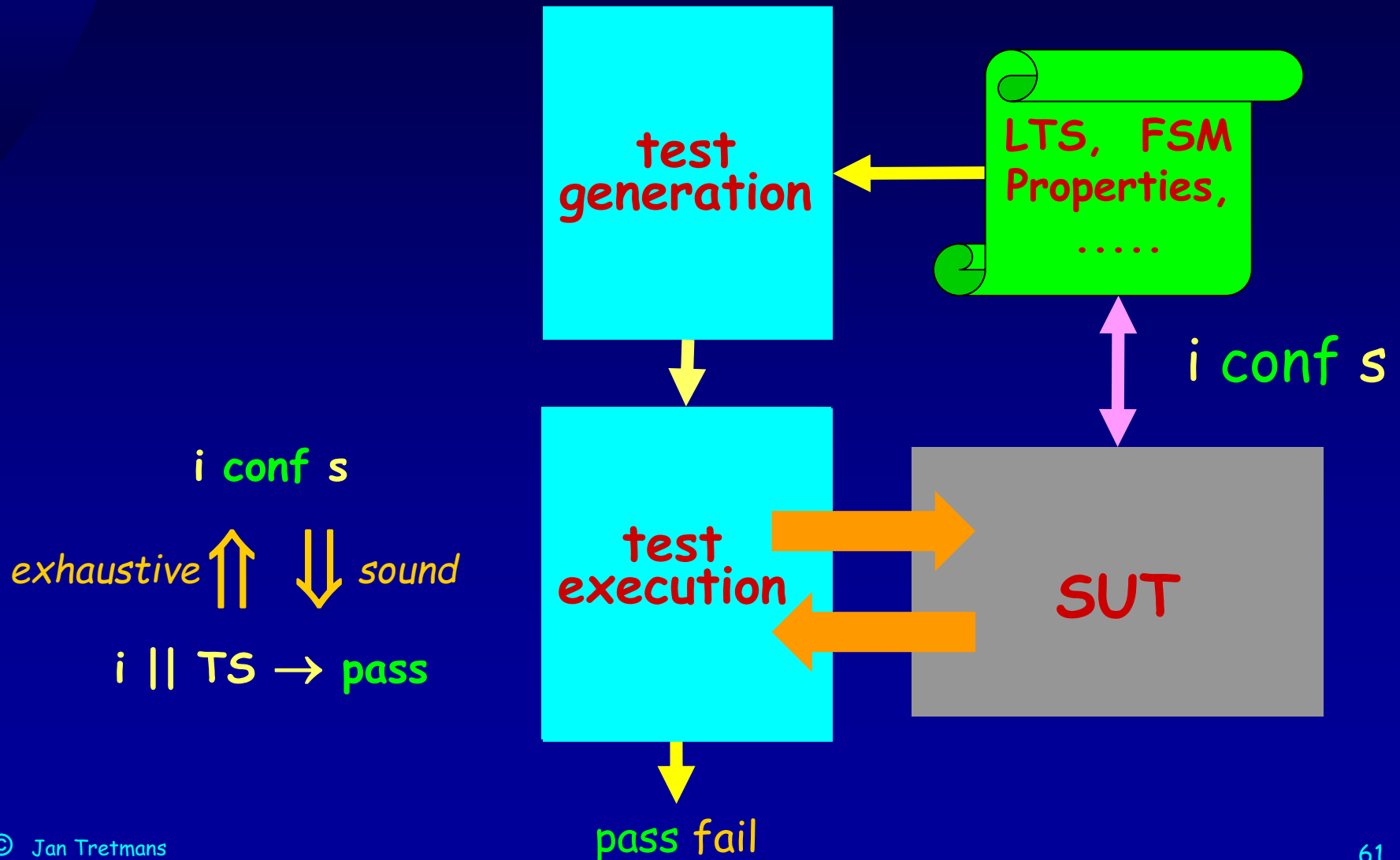


- And suppose you wish to reject this IUT: how long would you wait ?
- Untimed ioco: quiescence to express that there eventually is `!coffee`
- But when is eventually ?



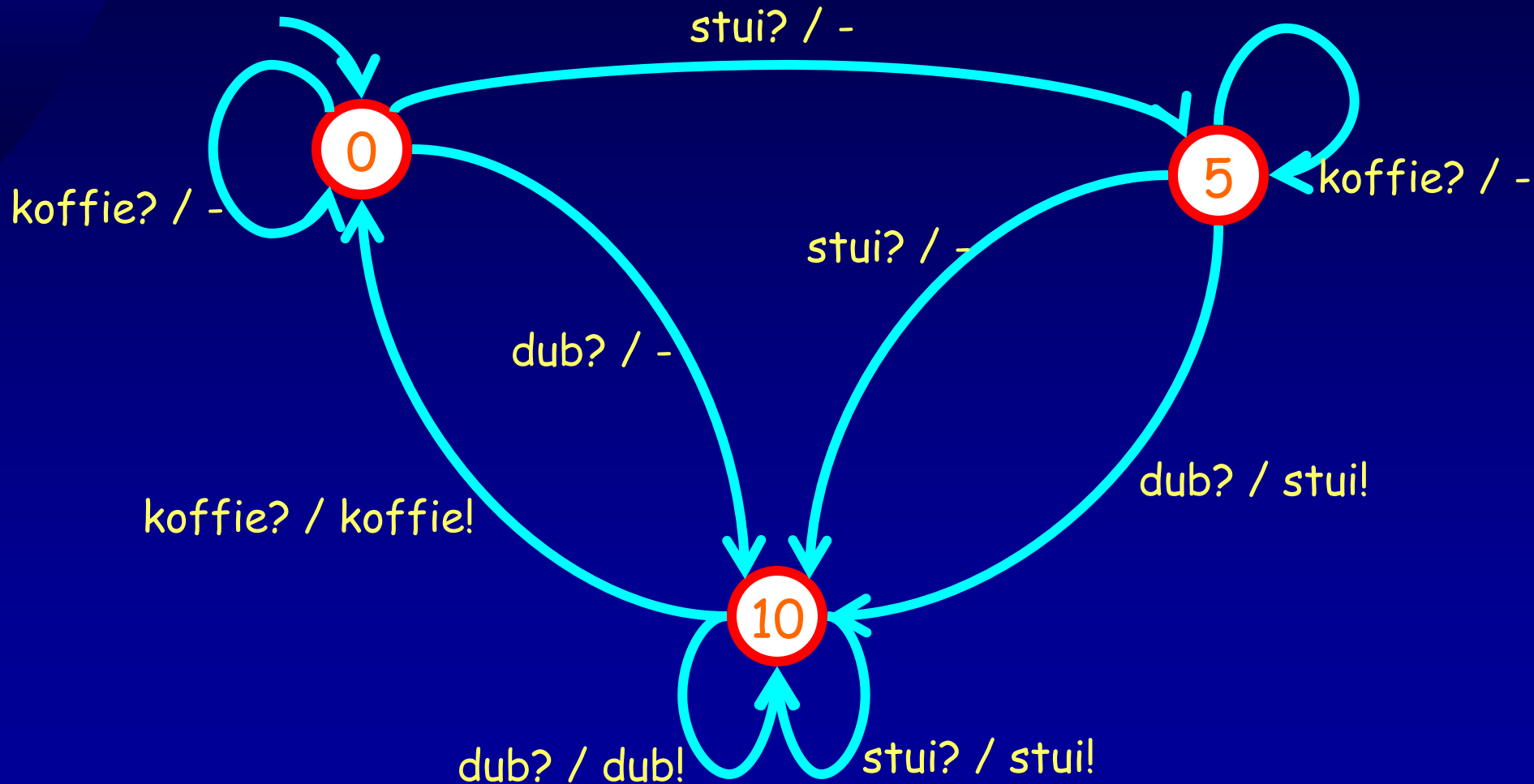
But There is More than LTS . . .

# Model Based Testing



# Finite State Machine (FSM)

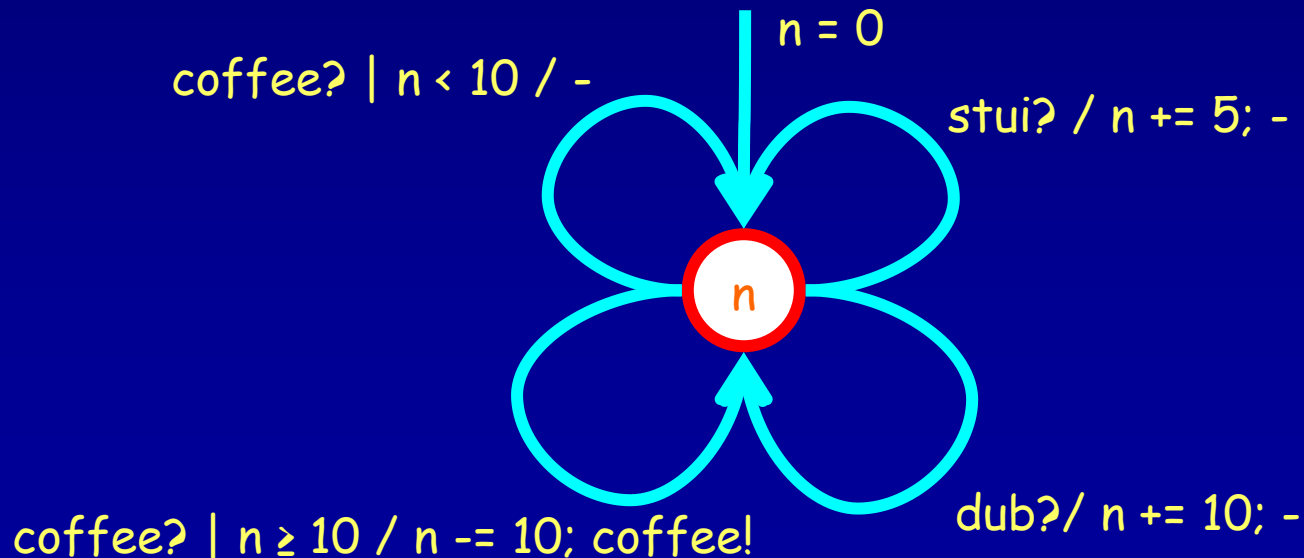
## Mealy Machine



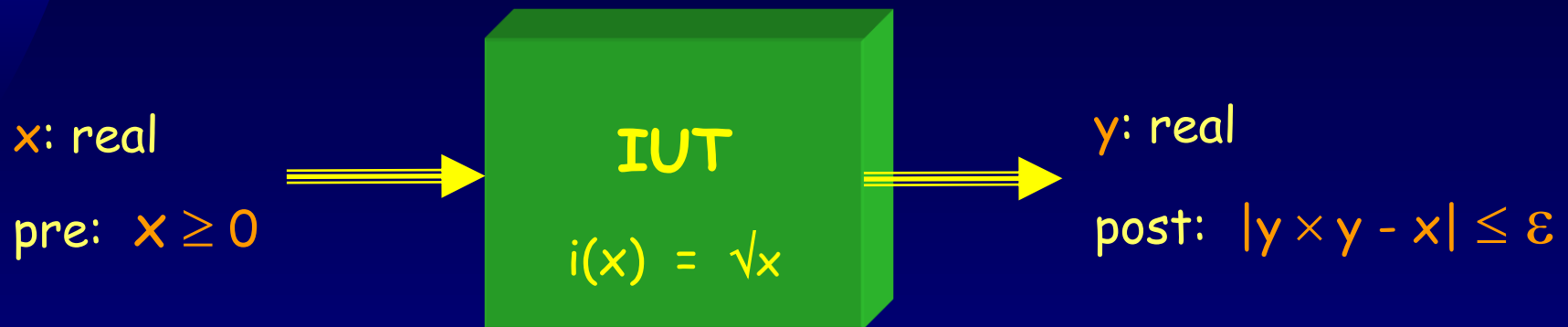


# Coffee Machine: FSM with Data

- ☞ use a variable to record the amount of money inserted
  - note this is a different machine; no money back
  - you can insert money for 3 coffee before retrieving coffee
    - dub? dub? dub? coffee? coffee? coffee?
  - infinitely many states; same inputs and outputs



# Testing Properties of Sequential Input/Output Programs



👉 Specification: property over  $x$  and  $y$

◆  $\text{property}(x, y) = x \geq 0 \Rightarrow |y \times y - x| \leq \varepsilon$

👉 Implementation is function  $i :: X \rightarrow Y$

👉 Test set  $T \subseteq X$

- ◆ Tools like **G $\forall$ ST** and **QuickCheck** generate thousands of tests by systematic traversal of all values of type  $X$
- ◆ But still: what is a "good" set ?