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Deber # 2 calculo vectorical

Calcular los valores máximos y mínimos locales, así como los puntos de silla de la función

1). $x^2 + y^2 + 4x - 6y = f(x,y)$

$fx(x,y)$

$fx(x,y) = 2x^2 + 4$

$fxx(x,y) = 2$

$fy(x,y)$

$fy(x,y) = 2y - 6$

$fyy(x,y) = 2$

$fxy(x,y) = 0$

$2x + 4 = 0$

$x = -2$

$fy(x,y) = 0$

$2y - 6 = 0$

$y = 3$

Punto critico $(-2, 3)$.

Calcular: $fxx(x,y) \cdot fyy(x,y) - (fxy(x,y))^2 = D$

$2 \cdot 2 - (0)^2 = 4 \quad D$

Si: $\begin{cases} D < 0 & \text{punto silla} \\ D > 0 \text{ y } fxx < 0 & \text{maximo local} \\ D > 0 \text{ y } fxx > 0 & \text{minimo local.} \end{cases}$

$(-2, 3)$ es un mínimo local

$f(x,y) = (-2)^3 + (3)^2 + (4)(2) - 6(-3) =$

$4 + 9 - 8 - 18 = -13$

$(-2, 3, -13)$ mínimo local.

2. $2x^2 + y^2 + 2xy + 24x + 2y = f(x,y)$

$f_x(x,y)$ $\quad$ $f_y(x,y)$

$4x + 2y + 24$ $\quad$ $2y + 2x + 2$

$f_x(x,y) = 4x + 2y + 24 = 0$ $\quad$ $2x + 2y + 2 = 0$

$$\begin{cases} 4x + 2y = 24 \\ 2x + 2y = -2 \quad (-1) \end{cases}$$

$$\begin{array}{rl} 4x + 2y &= -24 \\ -2x - 2y &= 2 \end{array}$$

$x = -11$

$2(x) + 2y = -2$

$2(-11) + 2y = -2$

$2y = -2 + 22$

$y = 10$

punto de corte $(-11, 10)$

$D = (f_{xx}(x,y))(f_{yy}(x,y)) - (f_{xy}(x,y)^2) \rightarrow$ si $\begin{cases} D < 0 \text{ punto silla} \\ D > 0,\ f_{xx} > 0 \text{ Mínimo} \\ D > 0,\ f_{xx} < 0 \text{ maximo} \end{cases}$

$D = (4)(2) - 2^2 = 8 - 4 = 4 = D$

$D > 0$

$(-11, 10)$ punto mínimo local

$f(x,y) = 2(-11)^2 + (10)^2 + 2(-11)(10) + 24(-11) + 2(10) = -122$

Punto mínimo local $(-11, 10, -122)$ //

3. $x^2 + y^2 + x^2y + 4$

$f_x(x,y) = 2x + 2xy$ $\quad$ $f_y(x,y) = 2y + x^2$

$f_{xx}(x,y) = 2 + 2y$ $\quad$ $f_{yy}(x,y) = 2$

$f_{xy}(x,y) = 2x$

$f_x(x,y) = 0$ $\quad$ $f_y(x,y) = 0$

$2x + 2xy = 0$ $\quad$ $2y + x^2 = 0$

$2x(1+y) = 0$ $\quad x = 0 \quad y = -1$ $\quad$ $y = \dfrac{-x^2}{2}$

$y: -\frac{x^2}{2} \Rightarrow y_1: \frac{0}{2} : y_1 = 0 \quad (0,0)$

$y: -\frac{x^2}{2} : -1 = -\frac{x^2}{2} \Rightarrow 2 = x^2$

$x_1 := \sqrt{2} \qquad (\sqrt{2}; -1)$

$x_2 : -\sqrt{2} \qquad (-\sqrt{2}; -1)$

Puntos de corte $(0,0)$ $(\sqrt{2}, -1)$; $(-\sqrt{2}; -1)$

$D: (fxx(x,y)\ fyy(x,y) - (fxy(x,y))^2 \quad (0,0)$

$D: (2+2(0)(2) - (2(0))^2 = 4$

$D = 4 \qquad D > 0$

$(0,0)$ es Minimo local

$D: fxx(x,y)\ fyy(x,y) - (fxy(x,y))^2 \mid (\sqrt{2}, -1)$

$D: 2+2(-1)\ 2 - ((2)(-\sqrt{2})^2 = -4(2) = -8$

$D: -8 < 0 \qquad fxx: 2+2(-1) = 0$

$fxx: 0$

Punto de silla

$D: fxx(x,y)\ fyy(x,y)) - fxy(x,y)^2$

$D: 2+2(-1)(2) - (2(-\sqrt{2})^2 \quad -4(2) \quad -8$

$-8 = D \qquad D < 0$

$fxx: 2+2(-1) = 0 \qquad$ Punto de Silla.

$P_1\ (0,0) : 0^2 + 0^2 + 0^2 0 + 4 = 4$

P_1 minimo local : $(0, 0, 4)$ //

$P_2\ (\sqrt{2}, -1) = (\sqrt{2})^2 + (-1)^2 + (\sqrt{2})^2(-1) + 4) = 0$

$\cancel{2} + 1 - \cancel{2} + 4 = 5$

P_2 Punto de silla $(\sqrt{2}, -1, 5)$ //

$P_3\ (-\sqrt{2}; -1) = (-\sqrt{2})^2 + (-1)^2 + (-\sqrt{2})^2 (-1) + 4$

$\cancel{2} + 1 - \cancel{2} + 4 = 5$

P_3 Punto de silla $(-\sqrt{2}; -1; 5)$ //

4) $x^3 - 3xy + y^3$

$f_x(x,y) = 3x^2 - 3y$ $\qquad$ $f_y(x,y) = 3y^2 - 3x$

$f_{xx} = 6x$ $\qquad$ $f_{yy}(x,y) = 6y$

$f_{xy} = -3$

$f_x(x,y) = 0$ $\qquad$ $f_y(x,y) = 0$

$3x^2 - 3y = 0$ $\qquad$ $3y^2 - 3x = 0$

$x^2 = y$ $\qquad$ $y^2 = x$

$x^4 = y^2$ $\qquad$ Para $x = 0$

$x^4 = x$ $\qquad$ $y^2 = 0$ $\qquad$ Para $x = 1$

$x^4 - x = 0$ $\qquad$ $y = 0$ $\qquad$ $y^2 = 1$

$x(x^3 - 1) = 0$ $\qquad$ $(0,0)$ $\qquad$ $y = \pm 1$

$x(x-1)(x^2 + x + 1) = 0$

$x_1 = 0$ $\quad$ $x_2 = 1$ $\quad$ $x_{3,4} \notin \mathbb{R}$ $\qquad$ $(1,1)$; $(1,-1)$

Puntos de corte $(0,0)$ $(1,1)$ $(1,-1)$

$D = f_{xx}(x,y)\, f_{yy}(x,y) - [f_{xy}(x,y)]^2 \rightarrow (0,0)$

$D = (6(0)) \cdot (6(0)) - (-3)^2 = -9$

$D = -9$ $\qquad$ $D < 0$

$f_{xx} = (6)(0) = 0$ $\qquad$ Punto Silla

$D = f_{xx}(x,y)\, f_{yy}(x,y) - [(f_{xy}(x,y))]^2 \rightarrow (1,1)$

$D = (6(1))\, 6(1) - (-3)^2 = 36 - 9 = 27$

$D > 0$;

$f_{xx} = 6(1) = 6$ $\qquad$ $6 > 0$ mínimo

$D = f_{xx}(x,y)\, f_{yy}(x,y) - [f_{xy}(x,y)]^2 \rightarrow (1,-1)$

$6(1)\, 6(-1) - (-3)^2 = 0 = -36 - 9 = -45$

$D < 0$ Punto silla

Entonces

Punto silla $(0,0)$

$f(x,y)\ x^3-3xy+y^3=$

$=0^3-3(0)(0)+(0)^3$

$=0$

Punto de silla $(0,0,0)$

Punto mínimo $(1,1)$

$f(x,y)\ x^3-3xy+y^3$

$=1-3(1)(1)+1$

$=-1$

$(1,1,-1)$ Punto mínimo.

5). $xy-2x-y$

$fx(x,y)=y-2$

$fxx(x,y)=-2$

$fx=0$

$y-2=0$

$y=2$

$fy(x,y)=x-1$

$fyy(x,y)=1$

$fy=0$

$x-1=0$

$x=1$

$fxy(x,y)=1$

Punto de corte $(1,2)$

$D=fxx(fyy)-[fxy]^2\rightarrow(1,2)$

$D=(0)(0)-1^2=-1\quad D<0$

Punto Silla

$f(x,y)=xy-2x-y$

$=1(2)-2(1)-2$

$=-2$

Punto Silla $(1,2,-2)$

6) $x^2y^2 - 8x + y \,/\, xy$

$f(x,y) = \frac{x^2y^2}{xy} - \frac{8x}{xy} + \frac{y}{xy} = xy - \frac{8}{y} + \frac{1}{x} = f(x,y)$

$f_x(x,y) = 0$

$y - \frac{1}{x^2} = 0$

$y^2 = \frac{1}{x^2}$

$x^4 = \frac{1}{y^2}$

$(-\frac{1}{2})^4 = \frac{1}{y^2}$

$\frac{1}{16} = \frac{1}{y^2}$

$16 = y^2$

$4 = y$

$f_y = 0$

$x + \frac{8}{y^2} = 0$

$x = -\frac{8}{y^2}$

$x = -8 \cdot \frac{1}{y^2}$

$x = -8x^4$

$8x^4 + x = 0$

$(2x+1)(2x)^2 - (2x(1) + 1) = 0$

$(2x+1)(4x^2 - 2x + 1) = 0$

$2x + 1 = 0 \quad 4x^2 - 2x + 1 \in \mathbb{R}$

$x = -\frac{1}{2}$

$P(-\frac{1}{2}, 4)$

$f_{xx}(x,y) = \frac{2}{x^3}$

$f_{yy}(x,y) = -\frac{16}{y^3}$ $\quad f_{xy}(x,y) = 1$

$D = f_{xx}(x,y)\, f_{yy}(x,y) - [f_{xy}(x,y)]^2 \rightarrow (-\frac{1}{2}, 4)$

$\frac{2}{(-1/2)^3}\left(-\frac{16}{4^3}\right) - (1)^2 = 3$

$D > 0$

$f_{xx} = \frac{2}{(-1/2)^3} = \frac{2}{-\frac{1}{8}} = -16 < 0$

$f_{xx} < 0$

Punto Máximo

$f(x,y) = xy - \frac{8}{y} + \frac{1}{x}$

$= (-\frac{1}{2})(4) - \frac{8}{4} + \frac{1}{-1/2}$

$= -2 - 2 - 2$

$= -6$

$(-1/2, 4, -6)$ Punto máximo //

1) $e^x \cos(y) = f(x,y)$

$D_x f(x,y) = 0$

$= e^x \cos(y)$

$f_{xx}(x,y) = e^x \cos(y)$

$e^x \cos(y) = 0$

$e^x \neq 0 \quad \cos(y) = 0$

$y = \frac{\pi}{2} + k\pi \quad k \in \mathbb{R}$

$D_y f(x,y) = 0$

$= -e^x \sin(y)$

$f_{yy} = -e^x \cos(y) \quad D_{xy} = -e^x \sin(y)$

$-e^x \sin(y) = 0$

$e^x \neq 0 \quad \sin(y) = 0$

$y = k\pi \quad k \in \mathbb{R}$

Puntos críticos

$(x, k\pi)$

evaluando puntos

$f_{xx} = e^x \cos(k\pi) \quad (x, k\pi)$

$= e^x (-1)^k$

$f_{yy} = -e^x \cos(k\pi)$

$D_{yy} f(x, k\pi) = -e^x (-1)^k$

$D_{xy}(x, k\pi) = -e^x \sin(k\pi)$

$D_{xy} f(x,y) = 0$

Hallar $D = f_{xx} f_{yy} f(x,y) - [f_{xy}(f(x,y))]^2$

$D = e^x (-1)^k (-e^x (-1)^k) - (0)^2$

$D = -e^{2x} \cancel{(-1)^{2k}}$

$D = -e^{2x}$

$D < 0$

Todos los puntos de $(x, k\pi)$ son puntos de silla

8) $x\sin(y) = f(x,y)$

$D_x f(x,y) = 0$

$\sin(y) = 0$

$y = k\pi, \ k \in \mathbb{Z}$

$D_y (x,y) = 0$

$x \cos(y) = 0$

$x = 0 \quad \cos(y) = 0$

$y = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$

Punto de corte

$(0, k\pi)$

$D_{xx}\, f(x,y) = \cos(y)$; $D_{yy}\, f(x,y) = -x\sin(y)$; $D_{xy}\, f(x,y) = \cos(y)$

$D_{xx}\, f(x,y) = \cos(K\pi)$; $D_{yy}\, f(x,y) = -(0)\sin(K\pi)$; $D_{xy} = \cos(K\pi)$

$D_{xx}\, f(x,y) = (-1)^K$; $D_{yy}\, f(x,y) = 0$; $D_{xy} = (-1)^K$

$D = D_{xx} f(x,y)\; D_{yy}(x,y) - [D_{xy} f(X,y)]^2$

$D = (-1)^K (0) \quad (-1)^{K \cdot 2}$

$D = -1$

Como $D<0$

Punto de silla $\rightarrow (0, K\pi)$

9) $\sqrt{x^2+y^2+1}$

$D_x\, f(x,y) = 0$; $D_y\, f(x,y) = 0$

$\dfrac{x}{\sqrt{x^2+y^2+1}} = 0$; $\dfrac{y}{\sqrt{x^2+y^2+1}} = 0$

$x = 0$; $y = 0$

Punto Crítico $(0,0)$

$D_{xx}\, f(x,y) = \dfrac{(x)'\sqrt{x^2+y^2+1} - (x)\sqrt{x^2+y^2+1}'}{(\sqrt{x^2+y^2+1})^2}$

$D_{xx}\, f(x,y) = \dfrac{\sqrt{x^2+y^2+1} - x\left(\dfrac{x}{\sqrt{x^2+y^2+1}}\right)}{x^2+y^2+1}$

$D_{xx}\, f(x,y) = \dfrac{y^2+1}{(x^2+y^2+1)^{3/2}}$

$D_{yy}\, f(x,y) = \dfrac{2(y)'\sqrt{x^2+y^2+1} - y\left(\dfrac{y}{\sqrt{x^2+y^2+1}}\right)}{(\sqrt{x^2+y^2+1})^2}$

$D_{yy}\, f(x,y) = \dfrac{x^2+1}{x^2+y^2+1}$

$D_{xy} = \dfrac{-xy}{(\sqrt{x^2+y^2+1})^3}$

$D_{xx}(0,0) = \dfrac{0^2+1}{(0^2+0^2+1)^{3/2}} = 1$; $D_{yy}(0,0) = \dfrac{0^2+1}{(0+0+1)^{3/2}} = 1$; $D_{xy}(0,0) = \dfrac{-0}{(0^2+0^2+1)^{3/2}} = 0$

$D = 1(1) - 0^2 = 1$

$1 > 0$
$f_{xx} > 0$

Punto Mínimo Local

$f(x,y) = \sqrt{x^2+y^2+1}$

$= \sqrt{0^2+0^2+1}$

$= 1$

Punto Mínimo Local $(0,0,1)$ //

10) $-x^2 - y^2 + 10x + 12y - 64$

$D_x f(x,y) = 0$ $D_y f(x,y) = 0$

$-2x + 10 = 0$ $-2y + 12 = 0$

$x = 5$ $y = 6$

$(5,6)$ Pto de Corte

$D_{xx} = -2$ $D_{yy} = -2$ $D_{xy} = 0$

$D = (-2)(-2) - 0^2 = 4$

$D > 0$
$D_{xx} < 0$

Máximo Local

$f(5,6) = -(5)^2 - (6)^2 + 10(5) + 12(6) - 64 = -3$

Punto máximo Local $= (5,6,-3)$ //

11) $-x^2 - 5y^2 + 10x - 10y - 28$

$D_x f(x,y) = 0$ $D_y = 0$

$-2x + 10 = 0$ $-10y - 10 = 0$

$x = 5$ $y = -1$

Punto de corte $(5,-1)$

$D_{xx} = -2$ $D_{yy} = -10$ $D_{xy} = 0$

$D = (-2)(-10) - (0)^2 = 20$

$D > 0$
$f_{xx} < 0$

Punto máximo local $(5, -1)$

$f(5,-1) = -(5)^2 - 5(-1)^2 + 10(5) - 10(-1) - 20 = 2$

Punto máximo Local $(5, -1, 2)$ //

12) $-5x^2 + 4xy - y^2 + 16x + 10$

$D_x f(x,y) = 0$ $\qquad$ $D_y = 0$

$-10x + 4y + 16 = 0$ $\qquad$ $4x - 2y = 0$

$5x - 2y - 8 = 0$ $\qquad$ $2x - y = 0$

$\begin{cases} 5x - \cancel{2y} - 8 = 0 \\ \cancel{2x} - y = 0 \end{cases}$

$5x - \cancel{2y} - 8 = 0$ $\qquad$ $2x - y = 0$
$x - 8 = 0$ $\qquad$ $2(8) - y = 0$
$x = 8$ $\qquad$ $16 = y$

$D_{xx} = -10$ $\qquad$ $D_{yy} = -2$ $\qquad$ $D_{xy} = 4$

$D = (-10)(-2) - (4)^2 = 20 - 16 = 4$

$D > 0$
$f_{xx} = -10 < 0$

Punto máximo Local $(8, 16)$

$f(8,16) = -5(8)^2 + 4(8)(16) - (16)^2 + 16(8) + 10 =$
$= 74$

máximo Local $(8, 16, 74)$ //

13) $(x^2 + y^2)^{1/3} + 2$

$D_x (x,y) = 0$ $\qquad$ $D_y f(x,y) = 0$

$\frac{2x}{3(x^2 + y^2)^{1/3}} = 0$ $\qquad$ $\frac{2y}{3(x^2 + y^2)^{2/3}} = 0$

$x = 0$ $\qquad$ $y = 0$

Punto crítico $(0,0)$

$$D_{xx}\, f(x,y) = -\frac{2(x^2-3y^2)}{9(x^2+y^2)^{5/3}} \qquad D_{yy} : \frac{2(3x^2-y^2)}{9(x^2+y^2)^{5/3}}$$

$$D_{xy} : \frac{-8xy}{9(x^2+y^2)^{5/3}}$$

$$D_{xx}(0,0) = -\frac{2(0^2-3(0)^2)}{9(0^2+0^2)^{5/3}} \; , \quad D_{yy}(0,0) = \frac{2(0^2-0^2)}{9(0^2+0^2)^{5/3}} \; ; \quad D_{xy}\, f(0,0) = \frac{8(0)}{9(0^2+0^2)^{5/3}}$$

$D_{xx}(0,0) = 0$ $\quad$ $D_{yy}\, f(0,0) = 0$ $\quad$ $D_{xy}(0,0) = 0$

$D = (0)(0) - (0)^2 = 0$

$D = 0$

Existe un mínimo global en el punto $(0,0)$

$f(x,y) = (0^2+0^2)^{1/3} + 2 = 2$

$(0,0,2)$ Punto mínimo global

14) $y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$

$D_x = 0$

$-3y(2x) - 3(2x) = 0$

$-6xy - 6x = 0$

$-6x(y-1) = 0$

$x = 0 \quad y = -1$

$D_y = 0$

$3y^2 - 3x^2 - 6y = 0$

$x^2 - y^2 + 2y = 0$

Si $x = 0$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0 \; , \; y = 2$

$(0,0) \quad (0,2)$

Si $y = -1$

$x^2 - (-1)^2 + 2(-1) = 0$

$x^2 = 3$

$x_1 = \sqrt{3}$

$x_2 = -\sqrt{3}$

$(\sqrt{3}\,;-1)\,;\;(-\sqrt{3}\,,-1)$

Puntos de corte $(0,0)\,;\,(0,2)\,;\,(\sqrt{3}\,,-1)\,;\,(-\sqrt{3}\,,-1)$

ESTILO

$D_{xx}: f(x,y) = -6y - 6$ $\quad D_{yy}: 6y - 6$ $\quad D_{xy}: -6x$

$D_{xx}(0,0) = -6$ $\quad D_{yy}(0,0) = -6$ $\quad D_{xy}(0,0) = 0$

$D = +(-6)(-6) - (0)^2 = 36$

$D > 0$
$f_{xx} < 0$ Punto Máximo local

$f(0,0) = -(0)^3 - 3(0)(0)^2 - 3(0)^2 - 3(0)^2 + 1 = 1$

Pto Máximo Local: $(0,0,1)$

$(0,2)$

$D_{xx} = -18$ $\quad D_{yy} = 6$ $\quad D_{xy} = 0$

$D = (-18)(6) - (0)^2 = -108$

$D < 0$

Punto de Silla

$f(0,2) = (2)^3 - 3(2)(0)^2 - 3(2)^2 - 3(0)^2 + 1 = -3$

Pto de Silla $\rightarrow (0, 2, -3)$

$(\sqrt{3}, -1)$

$D_{xx} = 0$ $\quad D_{yy} = -12$ $\quad D_{xy} = -6\sqrt{3}$

$D: (0)(-12) - (-6\sqrt{3})^2 = -108 < 0$

Punto de Silla

$f(\sqrt{3}; -1) = (-1)^3 - 3(-1)^2 - 3(\sqrt{3})^2 (-1) = -3$

Punto Silla $\rightarrow (\sqrt{3}, -1, -3)$ //

$(-\sqrt{3}; -1)$

$D_{xx} = 0$ $\quad D_{yy} = -12$ $\quad D_{xy} = 6\sqrt{3}$

$D: 0(-12) - (6\sqrt{3})^2 = -108 < 0 \rightarrow$ Punto Silla

$f(-\sqrt{3}; -1) = (-1)^3 - 3(-1)(\sqrt{3})^2 - 3(-1) - 3(-\sqrt{3})^2 + 1 = -1 + 9 - 3 - 9 + 1 = -3$

Punto Silla $(-\sqrt{3}; -1, -3)$ //

15) $x^2 - xy - y^2 - 3x - y$

$Dx = 0$ $\qquad\qquad$ $Dy = 0$

$2x - y - 3 = 0$ $\qquad\qquad$ $-x - 2y - 1 = 0$

$$\begin{cases} 2x - y - 3 = 0 \\ -x - 2y - 1 = 0 \end{cases}$$

$$\begin{array}{r} -4x + \cancel{2y} + 6 = 0 \\ -x - \cancel{2y} - 1 = 0 \\ \hline -5x + 5 = 0 \end{array}$$

$x = 1$

$2(1) - y - 3 = 0$

$y = -1$

Punto Critico $(1, -1)$

$Dxx = 2$ $\qquad$ $Dyy = -2$ $\qquad$ $Dxy = -1$

$D = 2(-2) - (-1)^2 = -5 < 0 \rightarrow$ Punto Silla

$f(1, -1) = (1)^2 - (1)(-1) - (1)^2 - 3(1) - (1) = -1$

Punto Silla $(1, -1, -1)$ //