Idea behind the greedy choice of the knapsack

P8. Fractional Knapsack Problem

I/P: n items, each item i has profit p_i and size s_i

: a bag with capacity B

O/P: maximize the profit without violating the capacity constraint.

Need to fit items in bag, here we can use fraction of items…

Find x_1 for item i_i , x_2 for item i_2 , ….. , x_n for item i_n s.t.

$$x_1 s_1 + x_2 s_2 + \ldots + x_n s_n \leq B$$

$$\text{Each } 0 \leq x_i \leq 1$$

GOAL: Max $x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$

Lets find a better greedy choice….

Let suppose that bag is full.

Can we remove some fraction of an item i by adding some fraction of item j **(bag must be full after doing changes in fractions of two items I and j)**

Such that profit increases?

Lets find a better greedy choice….

Let suppose that bag is full.

Can we remove some fraction of an item k by adding some fraction of item ι

Such that profit increases?

let y_1 for item i_i , y_2 for item i_2 , ….. , y_n for item i_n

$$y_1 s_1 + y_2 s_2 + \ldots + y_n s_n = B \quad \text{(by assumption)}$$

y_1	y_2	y_k	y_ι	y_n
y_1	y_2	$(y_k - e')$	$(y_\iota + e)$	y_n

$$y_1 s_1 + y_2 s_2 + \ldots + (y_k - e')s_k + \ldots + (y_\iota + e)s_\iota + y_n s_n = B$$

Lets find a better greedy choice….

Let suppose that bag is full.

Can we remove some fraction of an item i by adding some fraction of item j

Such that profit increases?

y_1 for item i_i , y_2 for item i_2 , ….. , y_n for item i_n

$y_1 s_1 + y_2 s_2 + \dots + y_n s_n = B$ (by assumption)

y_1	y_2	y_k	y_l	y_n
y_1	y_2	$(y_k - e')$	$(y_l + e)$	y_n

$$y_1 s_1 + y_2 s_2 + \dots + (y_k - e')s_k + \dots + (y_l + e)s_l + y_n s_n = B$$

$$y_1 s_1 + y_2 s_2 + \dots + y_n s_n + (-e's_k + es_l) = B$$

$$B + (-e's_k + es_l) = B$$

$$es_l = e's_k \Rightarrow \frac{s_l}{s_k} = \frac{e'}{e}$$

Lets find a better greedy choice....

Let suppose that bag is full.

Can we remove some fraction of an item i by adding some fraction of item j

Such that profit increases?

y_1 for item i_i , y_2 for item i_2 , ….. , y_n for item i_n

$y_1 s_1 + y_2 s_2 + \ldots + y_n s_n = B$ (by assumption)

y_1	y_2	y_k	y_l	y_n
y_1	y_2	$(y_k - e')$	$(y_l + e)$	y_n

$$y_1 s_1 + y_2 s_2 + \ldots + (y_k - e')s_k + \ldots + (y_l + e)s_l + y_n s_n = B$$

$$y_1 s_1 + y_2 s_2 + \ldots + y_n s_n + (-e's_k + es_l) = B$$

$$B + (-e's_k + es_l) = B$$

$$es_l = e's_k \Rightarrow \frac{s_l}{s_k} = \frac{e'}{e}$$

- Profit

New profit

$$= y_1 p_1 + y_2 p_2 + \ldots + (y_k - e')p_k + \ldots + (y_l + e)p_l + y_n p_n$$

$= \text{old profit} - e'p_k + ep_l$

$= \text{old profit} + ep_l - e'p_k$

New profit > old profit ??

if $ep_l - e'p_k > 0$

$p_l / p_k > e'/e$

$p_l / p_k > e'/e = s_l / s_k$

$p_l / s_l > p_k / s_k$

Algorithm

For each item i, $score_i$ = $profit_i$ / $size_i$

Order items by decreasing scores.

Pick them in this order till bag is filled

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Fractions of items by our method : 1, 1 , 1, 1, 1, α , 0, 0, 0, 0, 0

Last item may not be picked full (0, 1)