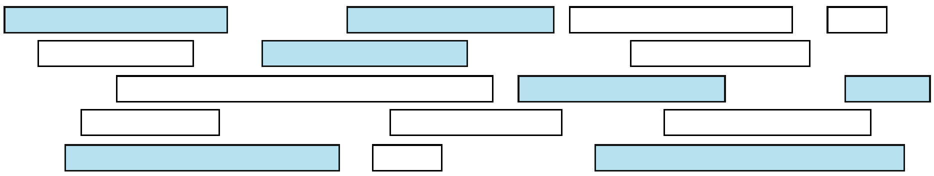
1. Describe an efficient algorithm that, given a set $\{ x_1, x_2, \ldots, x_n \}$ of points on the real line, determines the minimum number of unit-length closed intervals that contains all of the given points. Prove the correctness of your algorithm.

2. Consider the following scheduling problem: n jobs are given as input. Job j ($1 \leq j \leq n$) has a processing time p_j ($p_j > 0$) and a non-negative weight w_j ($w_j \geq 0$). We must construct a schedule for these jobs on a single machine such that at most one job is processed at each point in time, and each job must be processed non-preemptively; that is, once a job begins to be processed, it must be processed completely before any other job begins its processing. The objective is to find a schedule that minimizes the weighted sum of completion times: $\sum_{j=1}^{n} w_j C_j$. Suppose that jobs are indexed such that $\frac{w_1}{p_1} \geq \frac{w_2}{p_2} \geq \ldots \geq \frac{w_j}{p_j} \geq \ldots \geq \frac{w_n}{p_n}$. Then prove that it is optimal to schedule the jobs in the order $(1, 2, \ldots, j, \ldots, n)$ (job 1 first, job 2 second, and so on).

3. There exists an $O(n)$-time deterministic algorithm (M) for finding median of n given numbers. Using this algorithm as a subroutine, design an $O(n)$-time deterministic algorithm for solving the fractional knapsack problem (items are $(I_i)_{i=1}^{n}$, weight of items are $(w_i)_{i=1}^{n}$, profit of items are $(p_i)_{i=1}^{n}$, and knapsack capacity is W), and also prove the correctness and its time complexity.

4. Suppose we are given a set I of n intervals on the real line. We define a subset of intervals $J \subseteq I$ covers I if the union of all intervals in J is equal to the union of all intervals in I. The size of a cover is the number of intervals. Describe an efficient algorithm to compute the smallest cover of I. Use greedy algorithm, and prove the correctness of your algorithm.



A set of intervals, with a cover (shaded) of size 7.

5. **To reach n from 1:** Let learn a new process to reach from 1 to n. Initially start with 1, and in each step you can either increment by 1 or double the integer. Your goal is to reach n. For example to reach from 1 to 10,

$$1$$
$$1 + 1 = 2$$
$$2 * 2 = 4$$
$$4 + 1 = 5$$
$$5 * 2 = 10$$

In 4 steps, we can reach from 1 to 10. Describe and analyze an algorithm to compute the minimum number of steps required to produce any given integer n from 1 only using two strategies, i.e. incrementing by 1 or doubling.

6. Write the correctness of the algorithm discussed in class for Maximum independent set problem and minimum vertex cover on trees. Use exchange trick.

7. Let C be a unit radius circle. An arc of C is given by a pair $[\theta_1, \theta_2]$, where $\theta_1 < \theta_2$ are angles between 0 and 360 degrees. You are given a set of n arcs in the circle and would like to select a subset of arcs of maximum cardinality so that no two of them overlap. Give an efficient algorithm to find an optimal solution.

8. **Fill in the blanks** Suppose for the problem P, some greedy algorithm proposed and it returns a solution A. To prove the .... of the algorithm, we use .... arguments for the problem. We assumed a fact that there exists an optimal solution OPT such that............... elements of A. Now, we exchange (swap) some elements of OPT to get .... solution. After swapping in OPT, we might reach two different types of contradiction to the fact, and they are 1)................. or 2)..................

9. Solve the following instance of the *Fractional Knapsack Problem*, by applying the *Greedy Algorithm*:
   There are 8 items with profit of the items given by $(p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8) = (14, 9, 11, 8, 13, 12, 4, 10)$, weight of the items given by $(w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8) = (3, 2, 4, 8, 5, 9, 7, 6)$, and the knapsack capacity given by $W = 16$.