

---

```
procedure BINARY_EXP( $x, n$ )  
  if  $n = 0$  then return 1  
  if  $n = 1$  then return  $x$   
  
  if  $n \% 2 = 0$  then  
     $y \leftarrow \text{Binary\_Exp}(x, n/2)$   
    return  $y * y$   
  else  
     $y \leftarrow \text{Binary\_Exp}(x, \lfloor n/2 \rfloor)$   
    return  $x * y * y$ 
```

---

---

**Karatsuba's algorithm for integer multiplication**

**procedure** MULTIPLY( $num_1, num_2$ )  
  **if**  $num_1 < 10$  OR  $num_2 < 10$  **then**  
    **return**  $num_1 * num_2$

$m \leftarrow \min(\text{size\_base10}(num_1), \text{size\_base10}(num_2))$   
 $mid \leftarrow \lfloor m/2 \rfloor$

$x_1 \leftarrow mid$  higher order bits of  $num_1$   
 $x_0 \leftarrow$  remaining (after  $x_1$  split) lower order bits of  $num_1$   
 $y_1 \leftarrow mid$  higher order bits of  $num_2$   
 $y_0 \leftarrow$  remaining (after  $y_1$  split) lower order bits of  $num_2$

$xSum = x_0 + x_1$   
 $ySum = y_0 + y_1$

$x_0y_0 = \text{multiply}(x_0, y_0)$   
 $x_1y_1 = \text{multiply}(x_1, y_1)$   
 $xyProd = \text{multiply}(xSum, ySum)$   
 $xyTerm = xyProd - x_0y_0 - x_1y_1$

**return**  $x_1y_1 * 10^{2*mid} + xyTerm * 10^{mid} + x_0y_0$

---