
Input: List P of points (x, y)

Output: A pair of 2 points that have the smallest distance

Note: Assume we have some distance function $d(x, y)$ which returns the distance between points x and y . An example of such function would be the Euclidean distance.

procedure CLOSEST_PAIR(P)

$P_x \leftarrow$ list of points in P sorted by x coordinate

$P_y \leftarrow$ list of points in P sorted by y coordinate

return $Closest_Pair(P_x, P_y)$

procedure CLOSEST_PAIR(P_x, P_y)

if $|P_x| \leq 3$ **then** \triangleright For the base case, we will use brute force

\triangleright Yields constant time, since $|P_x|$ is very small

return $Closest_Pair_Brute_Force(P_x)$

$mid \leftarrow \lfloor |P|/2 \rfloor$

$L_x \leftarrow$ elements in index range $[1, mid]$ from P_x

$L_y \leftarrow$ elements from L_x sorted by y coordinate (construct from P_y)

$R_x \leftarrow$ elements in index range $[mid + 1, |P_x|]$ from P_x

$R_y \leftarrow$ elements from R_x sorted by y coordinate (construct from P_y)

$(l_1, l_2) \leftarrow Closest_Pair(L_x, L_y)$

$(r_1, r_2) \leftarrow Closest_Pair(R_x, R_y)$

$x_{mid} \leftarrow P[mid]$

$\delta \leftarrow \min(d(l_1, l_2), d(r_1, r_2))$

$C \leftarrow$ points from P with $|x_{mid} - x| < \delta$, sorted by y coordinate (construct from P_y)

$c \leftarrow (l_1, l_2)$

if $d(r_1, r_2) = \delta$ **then**

$c \leftarrow (r_1, r_2)$

for $i \leftarrow 0$ **to** $|C| - 1$ **do** \triangleright The inner loop will actually only check at most 6 points (proven, but not in this document), so we have constant time for the inner loop.

$j \leftarrow i + 1$

while $j < |C|$ AND $((C[j].y - C[i].y) < \delta)$ **do**

if $d(C[j], C[i]) < \delta$ **then**

$\delta \leftarrow d(C[j], C[i])$

$c \leftarrow (C[i], C[j])$ 1

return c

procedure CLOSEST_PAIR_BRUTE_FORCE(P)

$d \leftarrow \infty$

$c \leftarrow \text{UNDEFINED}$

for $p_1 \in P$ **do**

for $p_2 \in P \neq p_1$ **do**

$d_{12} \leftarrow d(p_1, p_2)$

if $d_{12} < d$ **then**

$d \leftarrow d_{12}$

$c \leftarrow (p_1, p_2)$

return c
